Cosmological collider signal from non-Bunch-Davies initial states

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We investigate the cosmological collider (CC) signal arising from the tree-level exchange of a scalar spectator particle with a non-Bunch-Davies (BD) initial state. We decompose the inflaton correlators into seed integrals, which we compute analytically by solving the bootstrap equations. We show that the non-BD initial state eliminates the Hubble-scale Boltzmann suppression $e^{-\pi m/H}$ that usually affects the CC signal. Consequently, in this scenario, the CC can probe an energy scale much higher than the inflationary Hubble scale H.

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I. INTRODUCTION

Inflation is a widely accepted paradigm of the early Universe that resolves various cosmological puzzles, such as the horizon problem [1,2]. Moreover, inflation predicts the cosmic non-Gaussianity (NG) [3] that offers a window into the deep UV physics that is inaccessible to any terrestrial particle collider. This paradigm, known as cosmological collider (CC) [4–51], has attracted broad attention in recent years. In CC, the inflaton is coupled to other spectator fields whose masses are above the inflationary Hubble scale. The interaction between the spectator fields and the inflaton leaves a distinctive imprint on the NG, which encodes the information of the mass and spin of the spectator fields. Despite its potential, CC is limited by the observational capability. The *Planck* satellite has constrained NG to a level of $f_{\rm NL} \lesssim \mathcal{O}(10)$ [52], and in the coming decades, SPHEREx can improve the sensitivity up to $f_{\rm NL} \sim \mathcal{O}(1)$ [53]. Furthermore, through the probe of the 21 cm line, we can eventually reach the resolution of $f_{\rm NL} \sim \mathcal{O}(0.01)$ that touches the gravitational floor [54]. Therefore, inflation models that predict large CC signals become more interesting experimentally, as they can be tested by CC.

The CC is subject to two constraints. In the standard inflation models, the initial state is assumed to be the Bunch-Davies (BD) vacuum, and the particles observed at late times are solely produced by the cosmic expansion with an energy scale set by the inflationary Hubble scale H. Hence, the magnitude of the CC signals, often denoted as $f_{\rm NL}$, is suppressed by the Hubble-scale Boltzmann factor $f_{\rm NL} \propto e^{-\pi m/H}$. Moreover, one-particle states (here we consider scalar particles with $s = 0$) in de Sitter (dS) are divided into two series: the principal series for particles with $m > 3H/2$ and the complementary series for particles with $0 < m < 3H/2$. Only the particles in the principal series generate oscillatory signals. Therefore, the CC is insensitive to light particles. These two factors limit the detection window of the CC to the energy scale around the Hubble scale H.

In this article, we explore a class of inflation models that predict large cosmological collider signals for heavy spectator fields with mass much larger than the Hubble scale H. Unlike previous models that rely on an effective chemical potential [55–58] to overcome the Hubble-scale Boltzmann factor $e^{-\pi m/H}$ that suppresses the CC signals, we consider the scenario where inflation begin with the non-BD state. The initial state we consider is the α vacuum [59–61], which is related to the BD state [62] by a Bogoliubov transformation with a parameter α. These initial states preserve the dS symmetry, and the case $\alpha = 0$ corresponds to the BD state. We show that, in such inflation models, the cosmological collider signals are free from the Hubble-scale Boltzmann suppression. This implies a higher energy scale for the CC in this scenario. Moreover, this inflation model has a theoretical advantage of evading the swampland conjecture [63], which imposes constraints on low-energy effective theories compatible with quantum gravity. However, for a general α vacuum, the deviation from the BD state implies that it is excited. For a genuine α vacuum, the condition that the extra energy from the excited states is smaller than the vacuum energy imposes a strict constraint on α. In this paper, we will adopt a flexible point of view as in [64], where α depends on the momentum of the mode considered. Physically, such an initial state can be realized in inflation models such as warm inflation [65], where a thermal bath is generated during inflation due to rapid interactions.

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In this paper, we adopt the Schwinger-Keldysh (SK) formalism [66] to compute the inflaton correlator in de Sitter space. We show that the choice of the non-BD initial state modifies the Schwinger-Keldysh propagators of the inflaton. With the modified SK propagators, we can, in principle, calculate the non-BD inflaton correlators that contain the CC signals.

Apart from the phenomenological implication, the analytical properties of the inflaton correlators have attracted considerable attention in recent years, along with the interest in exploring the quantum field theory in dS. However, the dS correlators are still poorly understood. The computation of the inflaton correlators analytically faces several challenges. First, the dS propagator of massive field typically involves special functions that are difficult to integrate analytically. Second, the SK formalism introduces complex time ordering into the calculation.

In recent years, some progress has been made in computing the inflaton correlators analytically. Two main methods are available: the Melin-Barnes (MB) formalism [67] and the bootstrap method [68–71]. The MB formalism transforms the conformal time integral into an integral over the MB variables, which can be solved using the residue theorem. However, this method usually yields an infinite series expansion as the final result. The bootstrap method, which we adopt in this article, converts the integration problem into a differential equation. The inflaton correlator can be obtained analytically in certain limits, and these results serve as the boundary condition for the bootstrap equation. By solving this equation, we can obtain the closed-form expression for the inflaton correlator. These two methods have been used to compute the inflaton correlators involving tree exchange of scalar particles and gauge bosons under the BD initial condition. In this article, we extend the result to the non-BD case.

The rest of this paper is structured as follows. In Sec. II, we provide a brief overview of the model and the initial state. We also derive the mode function of the real scalar spectator field in a non-BD scenario, which differs from the BD case by an additional negative frequency term. In Sec. III, we briefly review the SK formalism and compute the non-BD propagators of the spectator field. Next, we demonstrate that the seed integrals are the basic components of the tree-level inflaton correlators. By calculating the CC signal contributions from the seed integrals, we can extract the general features of the tree-level inflaton correlators that are insensitive to the specific interactions. We employ the bootstrap method to calculate the tree-level exchange of the scalar particle. Section IV includes our concluding remark. The technical details of the calculation are presented in the Appendix.

II. THE MODEL

In our model, the inflaton interacts with the scalar spectator field which has a non-BD initial state through the following action:

$$S = S_{\text{gravity}} - \int \mathrm{d}^4x \{\mathcal{L}_\phi + \mathcal{L}_\sigma + \mathcal{L}_{\text{int}}\}, \tag{1}$$

where $\mathcal{L}_\phi$, $\mathcal{L}_\sigma$, and $\mathcal{L}_{\text{int}}$ denote the Lagrangian for the inflaton, the spectator field, and the interaction, respectively. The Lagrangian of the inflaton takes the simplest form, which reads

$$\mathcal{L}_\phi = \sqrt{-g}\left[\frac{1}{2}(\partial_\mu\phi)^2 + V(\phi)\right], \tag{2}$$

and the S_{gravity} is the Hilbert-Einstein action of general relativity that reads

$$S_{\text{gravity}} = \frac{M_{\text{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R. \tag{3}$$

We assume the inflaton has the BD initial state; therefore, at leading order, our model give the conventional scale invariant power spectrum

$$P_\zeta(k) = \frac{H^4}{(2\pi)^2\dot{\phi}^2}, \tag{4}$$

whose value was measured through CMB by Planck [52] to be $P_\zeta \sim 2\times10^{-9}$.

Most of the previous studies on inflation that started from a non-BD initial state focused on the scenario where the inflaton field had a α-vacuum-type initial state [72–85]. Our scenario differs from theirs, as we consider the initial state of the inflaton to be BD while the spectator field has the non-BD initial state. This is based on the physical consideration that a non-BD initial condition for the inflaton would alter the leading-order expression for the power spectrum and entail additional constraints on the α parameter. To explore the scenario where the α parameter and, hence, the magnitude of the CC signal are least restricted, we adopt the scenario where the inflaton has BD initial condition and the spectator has non-BD condition. However, we anticipate that our results can also be extended to the scenario where both the inflaton and the spectator have non-BD initial states, as will be clear through our calculation.

A. Massive scalar in dS

We assume the simplest scenario in which the spectator field is a scalar with a nonzero mass term. Therefore, the action for the free spectator field has the following expression:

$$S_\sigma = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma + m^2\sigma^2\right] \tag{5}$$

$$= \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3\mathbf{x}\left[a^2\sigma'^2 - a^2(\partial_i\sigma)^2 - a^4m^2\sigma^2\right]. \tag{6}$$

The equation of the motion can be derived by taking the variation of the action (6). The result is

$$\sigma''(\tau,\mathbf{x}) - \frac{2}{\tau}\sigma'(\tau,\mathbf{x}) - \partial_i^2\sigma(\tau,\mathbf{x}) + \frac{m^2}{H^2\tau^2}\sigma(\tau,\mathbf{x}) = 0. \quad (7)$$

Subsequently, the equation of motion for each Fourier mode can be obtained by substituting $\sigma = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\sigma_\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{x}}$ into (7), which reads

$$\sigma''_\mathbf{k}(\tau) - \frac{2}{\tau}\sigma_\mathbf{k}(\tau) + \left(k^2 + \frac{m^2}{H^2\tau^2}\right)\sigma_\mathbf{k}(\tau) = 0. \quad (8)$$

In the case of BD vacuum, we keep only the solution with positive frequency as the mode function. Once the mode function is normalized by the Wronskian condition, it eventually takes the form of

$$u(k,\tau) = \frac{\pi}{2}e^{-\tilde{\nu}\pi/2}H(-\tau)^{3/2}\mathrm{H}^{(1)}_{i\tilde{\nu}}(-k\tau), \quad (9)$$

where $\tilde{\nu} \equiv \sqrt{m^2/H^2 - 9/4}$. Canonically, the Fourier mode of the scalar field then can be quantized as

$$\sigma_\mathbf{k}(\tau) = u(k,\tau)b_\mathbf{k} + u^*(k,\tau)b^\dagger_{-\mathbf{k}}, \quad (10)$$

where $b_\mathbf{k}$ is the annihilation operator that annihilates the BD vacuum, $b_\mathbf{k}|0\rangle_{\mathrm{BD}} = 0$. In this paper, we instead consider the non-BD initial state for the spectator field. Note that there are infinitely many states that satisfy the dS symmetry; therefore, they can also become the candidates of the vacuum state. These vacuum, often called α vacuum, are linked to the BD vacuum by a Bogoliubov transformation as follows:

$$v(k,\tau) = \cosh\alpha u(k,\tau) + e^{i\phi}\sinh\alpha u^*(k,\tau), \quad (11)$$

$$a_\mathbf{k} = \cosh\alpha b_\mathbf{k} - e^{-i\phi}\sinh\alpha b^\dagger_{-\mathbf{k}}, \quad (12)$$

where $a_\mathbf{k}$ is the annihilation operator that annihilates the α vacuum, $a_\mathbf{k}|0\rangle_\alpha = 0$.

B. Backreaction constraints

The α-vacuum initial state poses a challenge for inflation models, as it deviates from the BD state and implies an excited state. To prevent the energy of the α vacuum from spoiling inflation, a stringent constraint on α is required. Following [64], we assume that the parameter α is accompanied with a cutoff; that is, the α vacuum applies only to modes with $-k\tau < z_\Lambda/H$. Here, we will comment on the role of the cutoff z_Λ that is usually considered in the context of the non-BD initial state [64,73]. Depending on the inflation model with non-BD initial condition, the cutoff z_Λ can have a clear physical meaning that is connected to the origin of the non-BD vacuum. For instance, in the case of warm inflation, where the initial state is thermal, the cutoff z_Λ is set by the physical scale of the thermal bath [65]. However, our aim is not to investigate the physical origin of the α vacuum but to explore the phenomenological consequences of the α vacuum. Thus, we will treat z_Λ as a free parameter of our model, and we will show in Fig. 4 that the magnitude of the CC signal is robust to the choice of the cutoff. Taking the cutoff into account, the energy density can be evaluated as

$$\mathcal{E} \sim \int_{|\mathbf{k}|<z_\Lambda} \mathrm{d}^3\mathbf{k}\sqrt{\mathbf{k}^2 + m^2}\sinh 2\alpha. \quad (13)$$

As argued, the inflation requires that the vacuum energy dominates. Therefore, we reach the first constraint:

$$\mathcal{E} \lesssim 3M_{\mathrm{Pl}}^2H^2. \quad (14)$$

Using (13), the constraint (14) can be rewritten as

$$e^{2\alpha} \lesssim \frac{H^2M_{\mathrm{Pl}}^2}{z_\Lambda^4}, \quad (15)$$

which is the constraint derived in [64]. The constraint on the parameter α depends on the Hubble scale. For example, we consider the scenario where $H/M_{\mathrm{Pl}} \sim 10^{-10}$, that is, $H \sim 10^8$ GeV. As in [64], we denote $e^R \equiv z_\Lambda/H$, which is required to be much greater than unity to make our calculation valid, and it is severed as the second constraint. Then (15) implies that

$$e^\alpha \ll 10^{10}e^{-2R}; \quad (16)$$

that is, $\alpha \ll 23 - 2R$. Therefore, the constraint of the small backreaction still leaves us a large parameter space for α given that $R \gg 1$. In the next section, we will show that the cosmological collider signals obtain significant enhancement even with $\alpha \sim 1$.

III. COSMOLOGICAL COLLIDER SIGNALS

A. Schwinger-Keldysh formalism

Conventionally, the cosmic correlator is calculated canonically by adopting the in-in formalism, the expectation value of an arbitrary operator Q can be computed through

$$\langle Q(\tau)\rangle = \langle\Omega|\left[\bar{T}\exp\left(i\int_{\tau_0}^{\tau}H_I(\tau)\mathrm{d}t\right)Q(\tau)\right.$$
$$\left.\times T\exp\left(i\int_{\tau_0}^{\tau}H_I(\tau)\mathrm{d}t\right)\right]|\Omega\rangle, \quad (17)$$

where H_I is the Hamiltonian in the interaction picture. The correlator can be also calculated in the Lagrangian formalism as follows:

$$\begin{aligned}&\langle\Omega|\varphi(\tau_f,\mathbf{x}_1)...\varphi(\tau_f,\mathbf{x}_n)|\Omega\rangle\\&=\int\mathcal{D}\varphi_+\mathcal{D}\varphi_-\varphi_+(\tau_f,\mathbf{x}_1)\varphi_+(\tau_f,\mathbf{x}_n)\\&\quad\times\exp\left\{i\int_{\tau_0}^{\tau_f}\mathrm{d}^4x(\mathcal{L}[\phi_+]-\mathcal{L}[\phi_-])\right\}\\&\quad\times\prod_{\mathbf{x}}\delta[\varphi_+(\tau_f,\mathbf{x})-\varphi_-(\tau_f,\mathbf{x})].\end{aligned}\tag{18}$$

The calculation admits a diagrammatic approach, which is analogous to the Feynman diagram. Similar to the flat spacetime, the basic ingredient of the calculation is the following SK propagators:

$$D_{++}(k;\tau_1,\tau_2)(2\pi)^3\delta^3(\mathbf{k}+\mathbf{q})=\langle0_\alpha|T\{\sigma_{\mathbf{k}}(\tau_1)\sigma_{\mathbf{q}}(\tau_2)\}|0_\alpha\rangle,\tag{19}$$

$$D_{--}(k;\tau_1,\tau_2)(2\pi)^3\delta^3(\mathbf{k}+\mathbf{q})=\langle0_\alpha|\bar{T}\{\sigma_{\mathbf{k}}(\tau_1)\sigma_{\mathbf{q}}(\tau_2)\}|0_\alpha\rangle,\tag{20}$$

$$D_{+-}(k;\tau_1,\tau_2)(2\pi)^3\delta^3(\mathbf{k}+\mathbf{q})=\langle0_\alpha|\sigma_{\mathbf{k}}(\tau_1)\sigma_{\mathbf{q}}(\tau_2)|0_\alpha\rangle,\tag{21}$$

$$D_{-+}(k;\tau_1,\tau_2)(2\pi)^3\delta^3(\mathbf{k}+\mathbf{q})=\langle0_\alpha|\sigma_{\mathbf{k}}(\tau_1)\sigma_{\mathbf{q}}(\tau_2)|0_\alpha\rangle,\tag{22}$$

where $T\{...\}$ and $\bar{T}\{...\}$ denote the time ordering and anti–time ordering. In contrast to the conventional BD initial state, the propagators here are obtained by taking the expectation value with respect to the α vacuum. Note that the four SK propagators are not independent but related through

$$\begin{aligned}D_{++}(k;\tau_1,\tau_2)&=D_>(k;\tau_1,\tau_2)\theta(\tau_1-\tau_2)\\&\quad+D_<(k;\tau_1,\tau_2)\theta(\tau_2-\tau_1),\end{aligned}\tag{23}$$

$$\begin{aligned}D_{--}(k;\tau_1,\tau_2)&=D_<(k;\tau_1,\tau_2)\theta(\tau_1-\tau_2)\\&\quad+D_>(k;\tau_1,\tau_2)\theta(\tau_2-\tau_1),\end{aligned}\tag{24}$$

$$D_{+-}(k;\tau_1,\tau_2)=D_<(k;\tau_1,\tau_2),\tag{25}$$

$$D_{-+}(k;\tau_1,\tau_2)=D_>(k;\tau_1,\tau_2),\tag{26}$$

where the Wightman functions $D_>$ and $D_<$ are defined as

$$D_>(k;\tau_1,\tau_2)=v(k,\tau_1)v^*(k,\tau_2),\tag{27}$$

$$D_<(k;\tau_1,\tau_2)=v^*(k,\tau_1)v(k,\tau_2),\tag{28}$$

respectively, where $v(k,\tau)$ is the mode function defined in (11). In terms of the BD mode function, the Wightman function can be rewritten as

$$\begin{aligned}D_>(k;\tau_1,\tau_2)&=\cosh^2\alpha u(k,\tau_1)u^*(k,\tau_2)\\&\quad+\sinh^2\alpha u^*(k,\tau_1)u(k,\tau_2)\\&\quad+e^{-i\theta}\cosh\alpha\sinh\alpha u(k,\tau_1)u(k,\tau_2)\\&\quad+e^{i\theta}\cosh\alpha\sinh\alpha u^*(k,\tau_1)u^*(k,\tau_2),\end{aligned}\tag{29}$$

$$\begin{aligned}D_<(k;\tau_1,\tau_2)&=\cosh^2\alpha u^*(k,\tau_1)u(k,\tau_2)\\&\quad+\sinh^2\alpha u(k,\tau_1)u^*(k,\tau_2)\\&\quad+e^{i\theta}\cosh\alpha\sinh\alpha u^*(k,\tau_1)u^*(k,\tau_2)\\&\quad+e^{-i\theta}\cosh\alpha\sinh\alpha u(k,\tau_1)u(k,\tau_2).\end{aligned}\tag{30}$$

For clarity, we will decompose the propagator into four parts and analyze them respectively. We define the following four Wightman functions:

$$D_>^{(1)}(k;\tau_1,\tau_2)\equiv u(k,\tau_1)u^*(k,\tau_2),\tag{31}$$

$$D_>^{(2)}(k;\tau_1,\tau_2)\equiv u^*(k,\tau_1)u(k,\tau_2),\tag{32}$$

$$D_>^{(3)}(k;\tau_1,\tau_2)\equiv e^{-i\theta}u(k,\tau_1)u(k,\tau_2),\tag{33}$$

$$D_>^{(4)}(k;\tau_1,\tau_2)\equiv e^{i\theta}u^*(k,\tau_1)u^*(k,\tau_2).\tag{34}$$

In the following, we will compute the contribution of the four parts of the propagator separately. For simplicity, we will take $\theta=0$ from now on.

B. Trispectrum

We consider the simplest possibility of the interaction between the inflaton and the spectator and the inflaton which reserves the shift symmetry of the inflaton, which is given by

$$\mathcal{L}_{\mathrm{int}}\supset\frac{1}{2}\lambda a^2(\varphi')^2\sigma+\kappa a^3\varphi'\sigma,\tag{35}$$

where φ denotes the inflaton fluctuation. Note that the dimension-5 operator $\frac{1}{2}\lambda_5(\partial_\mu\phi)^2\sigma$ gives rise to both of the

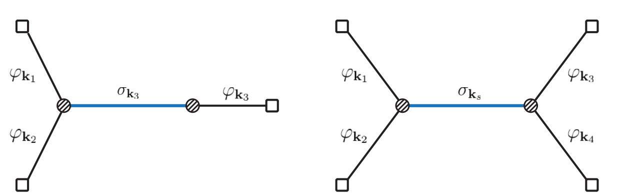

FIG. 1. The Feynman diagrams of the three-point correlator (77) and the four-point correlator (36) following the Feynman rules listed in [66].

couplings we considered in (35). Specifically, the κ term emerges from the evaluation of the λ_5 coupling in the presence of the inflaton background. However, the coupling $\frac{1}{2}\lambda_5(\partial_\mu\phi)^2\sigma$ is not complete, as it also generates a tadpole term $\frac{1}{2}\lambda_5\dot{\phi}^2\sigma$, which destabilizes the potential of the σ field. Nevertheless, we can regard it as an effective description of a more comprehensive model. For instance, a model that incorporates a quartic term $\frac{1}{4}\lambda_4\sigma^4$, where the σ field is fixed at some background value σ_0, and we are simply perturbing the full action around σ_0 [39]. Such interaction generates the tree-level four-point correlator (see Fig. 1) that can be computed using the SK formalism and takes the following form:

$$\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle'_\sigma = -\lambda^2\sum_{\mathsf{a,b}=\pm}\mathsf{ab}\int_{-k_s/(Hz_\Lambda)}^0\frac{d\tau_1}{(-H\tau_1)^2}\frac{d\tau_2}{(-H\tau_2)^2}\times G'_{\mathsf{a}}(k_1,\tau_1)G'_{\mathsf{a}}(k_2,\tau_1)G'_{\mathsf{b}}(k_3,\tau_2)\times G'_{\mathsf{b}}(k_4,\tau_2)D_{\mathsf{ab}}(k_s;\tau_1,\tau_2), \tag{36}$$

where $G_{\mathsf{a}} = \frac{H^2}{2k^3}(1-i\mathsf{a}k\tau)e^{i\mathsf{a}k\tau}$ is the inflaton propagator. As we previously argued, we have to impose a cutoff on the integration (36), implying the k dependence of α. To avoid the unphysical and cumbersome hard cutoff on the α-vacuum contribution of the correlator, we adopt a soft cutoff instead. We define the seed integral with a soft cutoff as follows:

$$\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle'_\sigma = -\frac{H^4\lambda^2}{16k_1k_2k_3k_4}\sum_{\mathsf{a,b}=\pm}\mathsf{ab}\times\int_{-\infty}^0 d\tau_1 d\tau_2 e^{i\mathsf{a}k_{12\mathsf{a}}\tau_1+i\mathsf{b}k_{34\mathsf{b}}\tau_2}\times D_{\mathsf{ab}}(k_s;\tau_1,\tau_2), \tag{37}$$

where we have denoted $k_{12\mathsf{a}}\equiv k_{12}-i\mathsf{a}Hk_s/z_\Lambda$ and $k_{34\mathsf{b}}\equiv k_{34}-i\mathsf{b}Hk_s/z_\Lambda$ with $k_{12}\equiv|\mathbf{k}_1|+|\mathbf{k}_2|$ and $k_{34}\equiv|\mathbf{k}_3|+|\mathbf{k}_4|$. Note that the imaginary part of the new external momentum suppresses the contribution from the region $-k_s\tau_{1,2}\gg z_\Lambda/H$. The integration appears in (37) is generally encountered in the tree-level calculation of the inflaton four-point correlator and is not restricted to the interaction (35). Therefore, it motivate us to analyze the following seed integrals instead:

$$\mathcal{I}^{(n),p_1p_2}_{\mathsf{ab}} \equiv -\mathsf{ab}k_s^{5+p_1+p_2}\int_{-\infty}^0 d\tau_1 d\tau_2(-\tau_1)^{p_1}(-\tau_2)^{p_2}\times e^{i\mathsf{a}k_{12\mathsf{a}}\tau_1+i\mathsf{b}k_{34\mathsf{b}}\tau_2}D^{(n)}_{\mathsf{ab}}(k_s;\tau_1,\tau_2). \tag{38}$$

The seed integrals determine the tree-level inflaton correlator of any interaction, and we can infer its properties from them. The seed $\mathcal{I}^{(1),p_1p_2}_{\mathsf{ab}}$ is the integral that will be encountered in BD calculation, while others are non-BD contribution. For instance, in term of the seed integrals, the four-point correlator (37) can be written as

$$\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle'_\sigma = \frac{H^4\lambda^2}{16k_1k_2k_3k_4k_s^5}\sum_{\mathsf{a,b}=\pm}\Big\{\cosh^2\alpha\mathcal{I}^{(1),00}_{\mathsf{ab}} + \sinh^2\alpha\mathcal{I}^{(2),00}_{\mathsf{ab}} + \cosh\alpha\sinh\alpha\mathcal{I}^{(3),00}_{\mathsf{ab}} + \cosh\alpha\sinh\alpha\mathcal{I}^{(4),00}_{\mathsf{ab}}\Big\}. \tag{39}$$

We obtain these seed integrals by solving the bootstrap equation, which we derive in detail in the Appendix. The general solution of the bootstrap equation has the form

$$\mathcal{I}^{(n),p_1p_2}_{\mathsf{ab}}(u_{1\mathsf{a}},u_{2\mathsf{b}}) = \mathcal{V}^{(n),p_1p_2}_{\mathsf{ab}}(u_{1\mathsf{a}},u_{2\mathsf{b}}) + \sum_{\mathsf{c,d}=\pm}\alpha^{(n),p_1p_2}_{\mathsf{ab|cd}}\mathcal{Y}^{p_1}_{\mathsf{a}}(u_{1\mathsf{a}})\mathcal{Y}^{p_2}_{\mathsf{b}}(u_{2\mathsf{b}})\times(n=1,\ldots,4), \tag{40}$$

where $u_{1\mathsf{a}}\equiv u_1(k_{12}\to k_{12\mathsf{a}})$, $u_{2\mathsf{b}}\equiv u_2(k_{34}\to k_{34\mathsf{b}})$ [we also define $u_{1,2}\equiv 2r_{1,2}/(r_{1,2}+1)$ and $r_{1,2}\equiv k_s/k_{12,34}$], and $\mathcal{Y}^{p_1}_\pm(u)$ are the homogeneous solution of the bootstrapping equations that are defined as follows:

$$\mathcal{Y}^p_\pm(u) = 2^{\mp i\tilde{\nu}}\left(\frac{u}{2}\right)^{5/2+p\pm i\tilde{\nu}}\Gamma(5/2+p\pm i\tilde{\nu})\Gamma(\mp i\tilde{\nu})\times {}_2\mathrm{F}_1\left[\begin{matrix}5/2+p\pm i\tilde{\nu},1/2\pm i\tilde{\nu}\\1\pm 2i\tilde{\nu}\end{matrix}\middle|u\right], \tag{41}$$

where

$${}_2\mathrm{F}_1\left[\begin{matrix}5/2+p_1\pm i\tilde{\nu},1/2\pm i\tilde{\nu}\\1\pm 2i\tilde{\nu}\end{matrix}\middle|u\right]$$

denotes the hypergeometric function. $\mathcal{V}^{(n),p_1p_2}_{\mathsf{ab}}$ denotes the particular solution, and the α coefficients are the integration constants of the bootstrap equation. Following the Appendix, we can further decompose (40) into

$$\mathcal{I}^{(n),p_1p_2}_{\mathsf{ab|bg}}(u_{1\mathsf{a}},u_{2\mathsf{b}}) \equiv \mathcal{V}^{(n),p_1p_2}_{\mathsf{ab}}(u_{1\mathsf{a}},u_{2\mathsf{b}}), \tag{42}$$

that exclusively contribute to the background, and

$$\mathcal{I}^{(n),p_1p_2}_{\mathsf{ab|signal}}(u_{1\mathsf{a}},u_{2\mathsf{b}}) \equiv \sum_{\mathsf{c,d}=\pm}\alpha^{(n),p_1p_2}_{\mathsf{ab|cd}}\mathcal{Y}^{p_1}_{\mathsf{a}}(u_{1\mathsf{a}})\mathcal{Y}^{p_2}_{\mathsf{b}}(u_{2\mathsf{b}}), \tag{43}$$

which contain all the contributions to the oscillatory signal. We can uniquely determine the particular solutions and the integration constants by imposing the collapsed limit result as the boundary condition (see the Appendix); results are as follows.

For $\mathcal{I}^{(1)}_{\mathsf{ab}}$,

$$\alpha^{(1),p_1p_2}_{+-|\mathrm{cd}} = \alpha^{(1),p_1p_2*}_{-+|\mathrm{cd}} = \frac{H^2 e^{-i\pi(p_1-p_2)/2}}{4\pi}, \tag{44}$$

$$\alpha^{(1),p_1p_2}_{++|+\pm} = \alpha^{(1),p_1p_2*}_{--|-\pm} = \frac{iH^2 e^{-i\pi(p_1+p_2)/2} e^{\pi\tilde{\nu}}}{4\pi}, \tag{45}$$

$$\alpha^{(1),p_1p_2}_{++|-\pm} = \alpha^{(1),p_1p_2*}_{--|+\pm} = \frac{iH^2 e^{-i\pi(p_1+p_2)/2} e^{-\pi\tilde{\nu}}}{4\pi}. \tag{46}$$

For $\mathcal{I}^{(2)}_{\mathsf{ab}}$,

$$\alpha^{(2),p_1p_2}_{+-|\pm\pm} = \alpha^{(2),p_1p_2*}_{-+|\pm\pm} = \frac{H^2 e^{-i\pi(p_1-p_2)/2}}{4\pi}, \tag{47}$$

$$\alpha^{(2),p_1p_2}_{+-|\pm\mp} = \alpha^{(2),p_1p_2*}_{-+|\mp\pm} = \frac{H^2 e^{-i\pi(p_1-p_2)/2} e^{\pm 2\pi\tilde{\nu}}}{4\pi}, \tag{48}$$

$$\alpha^{(2),p_1p_2}_{++|\pm+} = \alpha^{(2),p_1p_2*}_{--|\pm-} = \frac{iH^2 e^{-i\pi(p_1+p_2)/2} e^{\pi\tilde{\nu}}}{4\pi}, \tag{49}$$

$$\alpha^{(2),p_1p_2}_{++|\pm-} = \alpha^{(2),p_1p_2*}_{--|\pm+} = \frac{iH^2 e^{-i\pi(p_1+p_2)/2} e^{-\pi\tilde{\nu}}}{4\pi}. \tag{50}$$

For $\mathcal{I}^{(3)}_{\mathsf{ab}}$,

$$\alpha^{(3),p_1p_2}_{+-|\pm\pm} = \alpha^{(3),p_1p_2*}_{-+|\mp\mp} = -\frac{H^2 e^{i\pi(p_1-p_2)/2} e^{\mp\pi\tilde{\nu}}}{4\pi}, \tag{51}$$

$$\alpha^{(3),p_1p_2}_{+-|\pm\mp} = \alpha^{(3),p_1p_2*}_{-+|\mp\pm} = -\frac{H^2 e^{-i\pi(p_1-p_2)/2} e^{\pm\pi\tilde{\nu}}}{4\pi}, \tag{52}$$

$$\alpha^{(3),p_1p_2}_{++|\pm\pm} = \alpha^{(3),p_1p_2*}_{--|\mp\mp} = -\frac{iH^2 e^{-i\pi(p_1+p_2)/2} e^{\pm 2\pi\tilde{\nu}}}{4\pi}, \tag{53}$$

$$\alpha^{(3),p_1p_2}_{++|\pm\mp} = \alpha^{(3),p_1p_2*}_{--|\pm\mp} = -\frac{iH^2 e^{-i\pi(p_1+p_2)/2}}{4\pi}. \tag{54}$$

For $\mathcal{I}^{(4)}_{\mathsf{ab}}$,

$$\alpha^{(4),p_1p_2}_{+-|\pm\pm} = \alpha^{(4),p_1p_2*}_{-+|\pm\pm} = -\frac{H^2 e^{-i\pi(p_1-p_2)/2} e^{\pm\pi\tilde{\nu}}}{4\pi}, \tag{55}$$

$$\alpha^{(4),p_1p_2}_{+-|\pm\mp} = \alpha^{(4),p_1p_2*}_{-+|\pm\mp} = -\frac{H^2 e^{-i\pi(p_1-p_2)/2} e^{\pm\pi\tilde{\nu}}}{4\pi}, \tag{56}$$

$$\alpha^{(4),p_1p_2}_{++|\pm\pm} = \alpha^{(4),p_1p_2*}_{--|\pm\pm} = -\frac{iH^2 e^{-i\pi(p_1+p_2)/2}}{4\pi}, \tag{57}$$

$$\alpha^{(4),p_1p_2}_{++|\pm\mp} = \alpha^{(4),p_1p_2*}_{--|\pm\mp} = -\frac{iH^2 e^{-i\pi(p_1+p_2)/2}}{4\pi}. \tag{58}$$

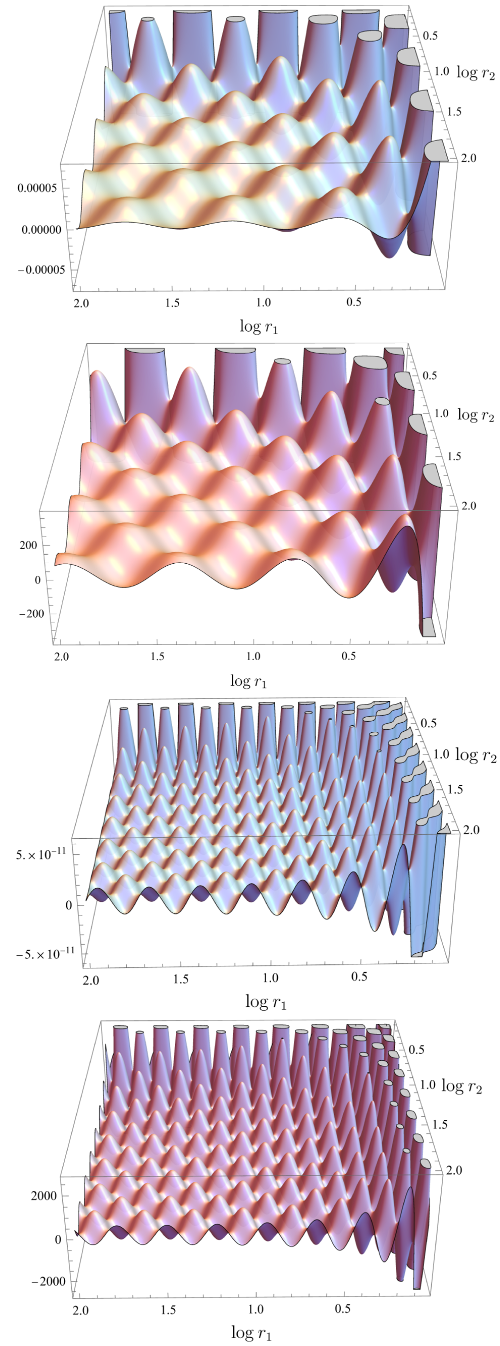

FIG. 2. The effect of different values of α and m on the rescaled seed integral $(\frac{2r_1}{r_1+1})^{-5/2}(\frac{2r_2}{r_2+1})^{-5/2}\mathcal{I}^{00}_{\text{signal}}$. The blue and magenta curves correspond to $\alpha = 0$ and $\alpha = 1$ with $z_\Lambda/H = 20$, respectively. The first and second plots show the case of $m/H = 5$, while the third and fourth plots show the case of $m/H = 10$. The seed integral is a measure of the cosmological collider signal from the non-BD initial state. We have taken $H = 1$ in these plots.

We plot the signal contribution (43) from the seed integral in Fig. 2, where we define $\mathcal{I}^{00}_{\text{signal}} \equiv \sum_{n,\mathsf{a},\mathsf{b}} \mathcal{I}^{(n),00}_{\mathsf{ab}|\text{signal}}$. We find that the seed integral is significantly enhanced by the non-BD contribution.

1. Signal size estimation

We will estimate the size of the CC signal from the bispectrum in the following. Conventionally, the trispectrum is captured through a dimenionless shape function defined as follows [86]:

$$\mathcal{T}(k_1,k_2,k_3,k_4) = \frac{1}{(2\pi)^6 P_\zeta^3}\frac{H^4}{\dot{\phi}^4}\frac{(k_1k_2k_3k_4)^3}{K^3} \times \langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle', \tag{59}$$

where P_ζ is the power spectrum. The trispectrum is quantified by $t_{\rm NL} \equiv |\mathcal{T}|$, which depends on the shape of the quadrangle formed by $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4$. Therefore, different values of $t_{\rm NL}$ should be used for different quadrangle configurations. We focus on the case where one of the wave vectors is much smaller than the others, i.e., $k_s \ll k_{12}, k_{34}$, i.e., the collapsed limit; see Fig. 3.

This limit exhibits nonanalytic oscillations in the momentum ratios, which can be used to probe the presence of beyond standard model particles from the CMB/LSS observations. We call this the CC signal and denote its amplitude by $t_{\rm NL}^{\rm (osc)}$. From these definitions, it follows that $t_{\rm NL}^{\rm (osc)} \sim \mathcal{T}(k_s \ll k_{12}, k_{34})$ and can be estimated as [56]

$$t_{\rm NL}^{\rm (osc)} \sim \frac{1}{(2\pi)^2 P_\zeta}\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\varphi_{\mathbf{k}_4}\rangle'. \tag{60}$$

At the tree level, the four-point correlators can be generally decomposed into seed integrals, which have signal and background parts as shown in (42) and (43). Therefore, the magnitude of $t_{\rm NL}^{\rm (osc)}$ depends exclusively on the signal parts of the seed integrals (43). We will focus on analyzing these seed integrals as proxies for the oscillatory signals. We will use the following asymptotic behavior of the gamma function:

$$\Gamma(a \pm ib) \simeq \sqrt{2\pi} b^{a-1/2} e^{-\pi b/2} e^{\pm i(b\log b + \pi a/2)}, \quad a, b \in \mathbb{R}, \qquad b \gg 1. \tag{61}$$

Note that, even for moderately large values of $|b|$ greater than 1, this asymptotic behavior remains valid. For BD initial state, $t_{\rm NL}^{\rm (osc)}$ is suppressed by the factor $e^{-\pi m/H}$ at large mass which set the energy scale of the cosmological collider to H. This suppression can be found in the BD seed $\mathcal{I}_{\mathsf{ab}}^{(1),p_1p_2}$. In the collapsed limit $k_s \ll k_{12}, k_{34}$, we can perform the following large mass expansion:

$$\mathcal{Y}_\pm^{p_1}(u) \sim \Gamma\left(\frac{5}{2} + p_1 \pm i\tilde{\nu}\right)\Gamma(\mp i\tilde{\nu}) \propto e^{-\pi m/H} \times (m \gg H), \tag{62}$$

where in the second step of (62) we have applied (61). Among the coefficients in the BD seeds, the leading terms in the asymptotic expansion for large mass are given by

$$\alpha_{++|+\pm}^{(1),p_1p_2} \sim \alpha_{--|-\pm}^{(1),p_1p_2} \sim e^{\pi m/H}, \tag{63}$$

while the rest of the coefficients are further suppressed by the Boltzmann factor. Note that any physical calculation will require the summation over the Schwinger-Keldysh indices a and b. Hence, from the combination of equations (62) and (63), it is evident that the signal contribution from the BD seed is always governed by the factor $e^{-\pi m/H}$, which is consistent with our expectation.

In contrast to the BD seeds, the leading coefficients from the non-BD seed are given by

$$\alpha_{+-|+-}^{(2),p_1p_2} = \alpha_{-+|-+}^{(2),p_1p_2} \sim e^{2\pi m/H}, \tag{64}$$

$$\alpha_{++|++}^{(3),p_1p_2} = \alpha_{--|--}^{(3),p_1p_2} \sim e^{2\pi m/H}. \tag{65}$$

These coefficients in the non-BD seed cancel out the suppression factors in (62). Hence, the non-BD seed $\mathcal{I}_{\mathsf{ab}}^{(2)}$, $\mathcal{I}_{\mathsf{ab}}^{(3)}$ does not suffer from the Hubble-scale Boltzmann suppression $e^{-\pi m/H}$. Here, we will comment on the role of the θ parameter that we have chosen to be 0. Observe that the parameter θ indeed influences the magnitude of the signal. As an example, consider the trispectrum in (39), where there are two terms that are enhanced, which correspond to the contributions from the seed integrals $\mathcal{I}_{\pm\mp}^{(2),p_1p_2}$ and $\mathcal{I}_{\pm\pm}^{(3),p_1p_2}$, respectively. Only the latter term oscillates with θ. However, by applying the late-time expansion for (41) and plugging this expansion into the seed integral (43), we can see that the enhancements of the CC signal from $\mathcal{I}_{\pm\mp}^{(2),p_1p_2}$ and $\mathcal{I}_{\pm\pm}^{(3),p_1p_2}$ belong to two different types. The former seed corresponds to the local type, where the signal oscillates like $(\frac{k_{12}}{k_{34}})^{\pm i\tilde{\nu}}$, and the latter seed corresponds to the nonlocal type, where the signal oscillates like $(\frac{k_s^2}{k_{12}k_{34}})^{\pm i\tilde{\nu}}$. Thus, it is clear that the choice of the θ parameter will not result in the cancellation of the CC signal in trispectrum. On the other hand, for the case of the bispectrum, the nonlocal and local signals combine together. However, only for the fine-tuned θ such that the coefficients $\sinh^2\alpha$ and $f(\theta)\cosh\alpha\sinh\alpha$ match can the

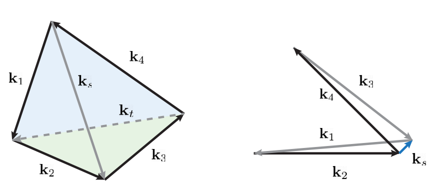

FIG. 3. Left panel: the momentum configuration for the four-point correlator. Right panel: the collapsed limit configuration.

potential cancellation of the CC signal occur. Therefore, without loss of generality, we will maintain $\theta = 0$ as the typical value of our analysis.

To elucidate this phenomenon, we investigate the asymptotic behavior of the scalar mode function in the late-time regime and insert it into the four-point seed integrals. *The BD mode function* has the following form in the late-time expansion:

$$\lim_{k\to 0} u(k,\tau) = -i\sqrt{\frac{2}{\pi k^3}}H\left[e^{\tilde{\nu}\pi/2}\Gamma(i\tilde{\nu})\left(-\frac{k\tau}{2}\right)^{3/2-i\tilde{\nu}} + e^{-\tilde{\nu}\pi/2}\Gamma(-i\tilde{\nu})\left(-\frac{k\tau}{2}\right)^{3/2+i\tilde{\nu}}\right]. \quad (66)$$

On the other hand, as in Sec. III A, the non-time-ordered BD propagators are defined as

$$D^{(1)}_{-+}(k;\tau_1,\tau_2) = u(k,\tau_1)u^*(k,\tau_2), \quad (67)$$

$$D^{(1)}_{+-}(k;\tau_1,\tau_2) = u^*(k,\tau_1)u(k,\tau_2). \quad (68)$$

Using (66), we observe that both (67) and (68) contain a term that is free from the exponential suppression in the late-time limit, which originates from the product of the first term in the bracket of (66) and its complex conjugate, that is,

$$D^{(1)}_{\mathsf{ab}}(k;\tau_1,\tau_2) \supset \mathcal{O}(1)(\tau_1)^{3/2-i\mathsf{a}\tilde{\nu}}(\tau_2)^{3/2-i\mathsf{b}\tilde{\nu}}, \quad (69)$$

where a and b take opposite sign. We take seed integral $\mathcal{I}^{(1)}_{\mathsf{ab}}$ as an example, which is defined in (38) as

$$\mathcal{I}^{(1),p_1p_2}_{\mathsf{ab}} \equiv -\mathsf{ab}k_s^{5+p_1+p_2}\int_{-\infty}^0 \mathrm{d}\tau_1\mathrm{d}\tau_2 e^{i\mathsf{a}k_{12\mathsf{a}}\tau_1+i\mathsf{b}k_{34\mathsf{b}}\tau_1} \times(-\tau_1)^{p_1}(-\tau_2)^{p_2}D^{(1)}_{\mathsf{ab}}(k_s,\tau_1,\tau_2). \quad (70)$$

Substituting (69) into (70), we obtain a term in the seed integral that originates from (69) and is given by

$$\mathcal{I}^{(1),p_1p_2}_{\mathsf{ab}} \supset \mathcal{O}(1)\Gamma(1+i\tilde{\nu})\Gamma(1-i\tilde{\nu}). \quad (71)$$

From (61), this indicates that the seed integral is exponentially suppressed by the mass much larger than H, even though the propagator is not. However, for non-BD initial condition, we will encounter the non-BD seed (here we consider $\mathcal{I}^{(2)}_{\mathsf{ab}}$ as an example; the same holds for $\mathcal{I}^{(3)}_{\mathsf{ab}}$) that has the following form:

$$\mathcal{I}^{(2),p_1p_2}_{\mathsf{ab}} \equiv -\mathsf{ab}k_s^{5+p_1+p_2}\int_{-\infty}^0 \mathrm{d}\tau_1\mathrm{d}\tau_2 e^{i\mathsf{a}k_{12\mathsf{a}}\tau_1+i\mathsf{b}k_{34\mathsf{b}}\tau_2} \times(-\tau_1)^{p_1}(-\tau_2)^{p_2}D^{(2)}_{\mathsf{ab}}(k_s,\tau_1,\tau_2). \quad (72)$$

In contrast to (69), in the non-BD propagator, we have

$$D^{(2)}_{\mathsf{ab}}(k;\tau_1,\tau_2) = D^{(1)}_{-\mathsf{a}-\mathsf{b}}(k;\tau_1,\tau_2) \supset \mathcal{O}(1)(\tau_1)^{3/2+i\mathsf{a}\tilde{\nu}}(\tau_2)^{3/2+i\mathsf{b}\tilde{\nu}}. \quad (73)$$

It follows that (73) gives rise to the term

$$\mathcal{I}^{(2),p_1p_2}_{\mathsf{ab}} \supset \mathcal{O}(1)e^{\pi\tilde{\nu}}\Gamma(1+i\tilde{\nu})\Gamma(1-i\tilde{\nu}), \quad (74)$$

which is free from the Hubble-scale Boltzmann suppression. In conclusion, the absence of the suppression in the non-BD seed is a result of the Schwinger-Keldysh index mismatch; namely, the BD propagator that carries inverse SK indices $D^{(1)}_{-\mathsf{a}-\mathsf{b}}$ is matched with $e^{i\mathsf{a}k_{12}\tau_1+i\mathsf{b}k_{34}\tau_1}$ in the non-BD seed (72). We remark that the late-time truncation of the propagator (66) is not applicable for computing the three-point seed integrals, but this analysis still yields the correct exponential factor.

2. *Physical interpretation*

In the following, we will elucidate the physical mechanism behind the amplification of the signal that was obtained from the preceding computation. Prior to investigating the case of non-BD initial condition, we will revisit the physical origin of the CC signal in the BD initial state. It is evident that the inflaton and the spectator fields have different dispersion relations at late time, which are time dependent and time independent, respectively, namely,

(i) the inflaton ($m \ll H$): $\omega = k/a$ and
(ii) the spectator ($m \gtrsim H$): $\omega = \sqrt{(\frac{k}{a})^2 + m^2} \sim m$.

The resonance between the inflaton and the spectator arises from the difference in their dispersion relations, which is reflected in the seed integral $\mathcal{I}^{(1),p_1p_2}_{\mathsf{ab}}$ that we will examine below. We can assume without loss of generality that we are dealing with the non-time-ordered seed, where the SK indices a and b have opposite signs. Applying the late-time expansion (66) of the spectator mode, we obtain the dominant term in the non-time-ordered seed $\mathcal{I}^{(1),p_1p_2}_{\mathsf{ab}}$ as the one that includes the saddle point and is proportional to the product of

$$\int \mathrm{d}\tau f_{\pm}(\tau)e^{\pm ik_{12,34}\tau}e^{\mp imt}, \quad (75)$$

where $f_\pm(\tau)$ represents some real power of τ. This term corresponds to the physical phenomenon of the resonance of the inflaton and the spectator at both of the vertices of the right panel in Fig. 1; that is, it includes the saddle point at $k_{12}/a(\tau_1) \sim m$ and $k_{34}/a(\tau_2) \sim m$. However, from (66) and the definition of the propagator (32), we can observe that only the positive frequency term of (66) participates in the resonance process, and, hence, such a process is suppressed by the coefficient

$$e^{-\tilde{\nu}\pi}|\Gamma(-i\tilde{\nu})|^2 \sim e^{-2\pi m/H}, \tag{76}$$

which indicates that the particle number of the spectator field generated by gravitational effects has a distribution similar to a thermal distribution for a massive state with mass $m \gg H$ and a temperature $T = m/(2\pi)$ in BD initial state.

On the other hand, from the Bogoliubov transformation (12), it follows that the spectator field in the α vacuum has an initial particle number $n_\sigma = \sinh^2\alpha$, which escapes the Boltzmann suppression (76). Therefore, for the α-vacuum case, the resonant term in the CC signal is not suppressed for masses much larger than the Hubble scale; rather, it is controlled by the parameter α.

C. Bispectrum

The Feynman diagram in the left panel in Fig. 1 leads to the CC signal in bispectrum and can be calculated using SK formalism. The expression for the three-point correlator reads

$$\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle'_\sigma = -\frac{H^4\lambda\kappa}{8k_1k_2k_3^4}\sum_{\mathsf{a,b}=\pm}\mathsf{ab}\int_{-\infty}^0 d\tau_1 d\tau_2(-\tau_2)^{-2} \times e^{i\mathsf{a}k_{12\mathsf{a}}\tau_1+i\mathsf{b}k_{34\mathsf{b}}\tau_2}D_{\mathsf{ab}}(k_s;\tau_1,\tau_2). \tag{77}$$

Analogous to the four-point case, the tree-level three-point correlator can be generally constructed from the three-point seed integral that takes the following form:

$$\mathcal{I}_{\mathsf{ab}}^{(n),p_1p_2} \equiv -\mathsf{ab}k_3^{5+p_1+p_2}\int_{-\infty}^0 d\tau_1 d\tau_2(-\tau_1)^{p_1}(-\tau_2)^{p_2} \times e^{i\mathsf{a}k_{12\mathsf{a}}\tau_1+i\mathsf{b}k_{3\mathsf{b}}\tau_2}D_{\mathsf{ab}}^{(n)}(k_3;\tau_1,\tau_2). \tag{78}$$

The three-point seed can be obtained from the four-point seed by applying the folded limit, $k_4 \to 0$. As in the trispectrum case, our result for the bispectrum is not restricted to the specific interaction (35), but rather it is applicable to any tree-level exchange process of the scalar particle, as it relies on seed integrals that can be combined in various ways.

As an example, we consider the seed integral that appeared in (77), where $p_1 = 0$ and $p_2 = -2$. We define a quantity that capture this seed integral's contribution to the CC signal as

$$\mathcal{I}_{\text{signal}}^{0,-2}(r) = \sum_{\mathsf{ab}}\mathcal{I}_{\mathsf{ab}|\text{signal}}^{0,-2}(u_{\mathsf{a}}(r), 1_{\mathsf{b}}), \tag{79}$$

where $u_{\mathsf{a}}(r) \equiv u_{1\mathsf{a}}(k_s \to k_3)$, $1_{\mathsf{b}} \equiv u_{2\mathsf{b}}(k_4 \to 0)$, and $r \equiv k_3/k_{12}$. We plot $\mathcal{I}_{\text{signal}}^{0,-2}$ in Fig. 4. We see that the magnitude of the CC signal is substantially enhanced by the non-BD contribution.

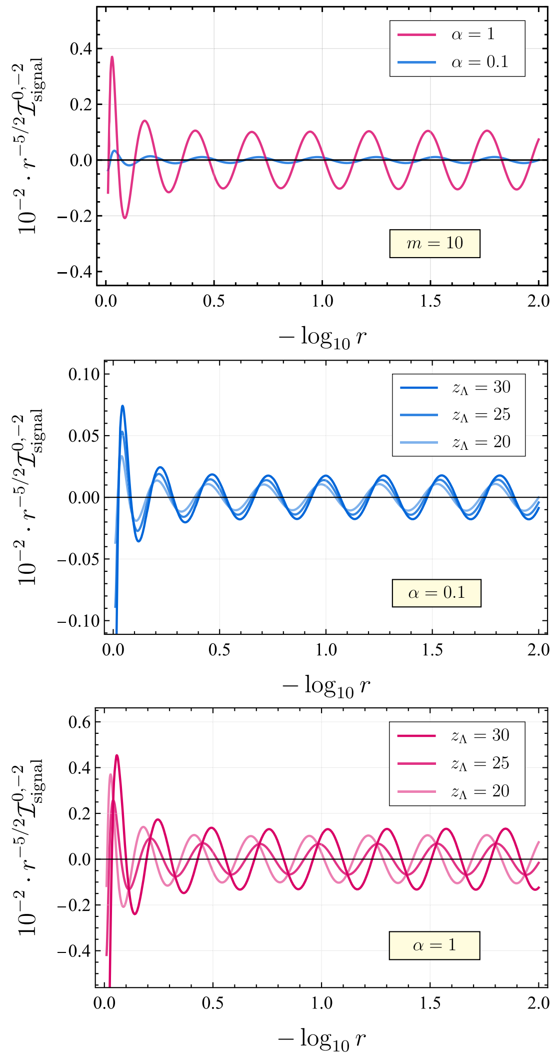

FIG. 4. Upper panel: the seed integral as a function of the momentum configuration with $m/H = 10$ and $z_\Lambda/H = 20$. Middle panel and lower panel: the seed integral with different cutoff scales z_Λ. We have taken $H = 1$ in these plots.

1. Signal size estimation

To quantify the oscillatory signal in the bispectrum, we can employ the same method that we utilized for the trispectrum. Analogous to to the trispectrum case, we can introduce a dimensionless shape function for the bispectrum as follows [56]:

$$\mathcal{S}(k_1,k_2,k_3) \equiv -\frac{(k_1k_2k_3)^2}{(2\pi)^4P_\zeta^2}\left(\frac{H}{\dot{\phi}}\right)^3\langle\varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle'. \tag{80}$$

We can also introduce the analog of t_{NL} for the bispectrum as $f_{\text{NL}} \equiv |\mathcal{S}|$. For the bispectrum, the oscillatory signal appears in the squeezed limit, namely, $k_3 \ll k_{12}$, which is illustrated in the right panel in Fig. 5. Hence, we can estimate the magnitude of this signal by using a

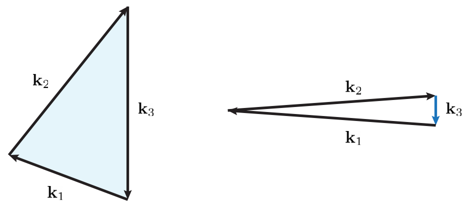

FIG. 5. Left panel: the momentum configuration for the three-point correlator. Right panel: the squeezed limit configuration.

parameter $f_{\rm NL}^{\rm osc}$ that satisfies $f_{\rm NL}^{\rm osc} \sim |\mathcal{S}(k_3 \ll k_{12})|$ and can be estimated as

$$f_{\rm NL}^{(\rm osc)} \sim \frac{1}{2\pi P_\zeta^{1/2}} \langle \varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2} \varphi_{\mathbf{k}_3} \rangle'. \tag{81}$$

As argued, the tree-level three-point correlator can be decomposed into seed integrals. Note that the three-point seed integral can be obtained from the four-point seed integral by applying the single folded limit, $k_4 \to 0$. Therefore, the non-BD contribution to the three-point seed inherits the leading coefficients from the four-point seed, given by (64) and (65). These coefficients cancel out the exponential suppression factor $e^{-\pi m/H}$ that appears in the mode function $\mathcal{Y}_\pm^p$.

IV. CONCLUSIONS AND DISCUSSIONS

Each collider has a characteristic energy scale, which also applies to the cosmological collider. Usually, the energy scale of CC is limited by the fact that $f_{\rm NL}^{(\rm osc)} \propto e^{-\pi m/H}$, which suppresses the signal from particles with mass much larger than the Hubble scale H.

The energy scale of CC can be enhanced by considering models where the spectator field has an effective chemical potential [55–57]. These models still have a Hubble-scale suppression, but it is offset by another exponential factor. We find that this suppression is absent if the spectator's initial state is α vacuum, a class of vacua that preserve dS symmetry and are characterized by a parameter α. We show that the magnitude of the oscillatory signal depends on α in this scenario. We argue that CC can probe a wider range of particle masses in this case.

The inflaton correlator captures the cosmic non-Gaussianity, which can be computed systematically by using the SK formalism. We demonstrate that the SK formalism can accommodate the non-BD initial state by replacing the SK propagators.

We have shown that the inflaton correlators can be expressed as a combination of seed integrals at tree level. By analyzing the seed integrals, we can reveal the general features of the CC signal that are independent of the interaction details. We obtained the analytical expression for the full momentum configuration by solving the bootstrap equation with the boundary condition given by the collapsed limit. Our analysis reveals that the absence of the Hubble-scale Boltzmann suppression originates from the non-BD part of the propagator. Since our result relies on the seed integrals, it is applicable to general tree-level process and not restricted to a specific model. We emphasize that the α vacuum of the spectator field favors low-scale inflation. However, our result indicates that the cosmological collider can probe a much broader range of particle masses if the spectator has an α-vacuum initial state.

Note that our calculation focus solely on tree-level processes. In contrast, in some prominent inflation models, such as axion inflation [57], features a spin-1 spectator field, and the dominant contribution comes from a bosonic loop. The generalization to loop calculation is intricate. Nevertheless, it has been shown that the loop process can also be tackled by the bootstrap method [87]; we expect that our results can be extended to such a process, which we will explore in future studies.

ACKNOWLEDGMENTS

We thank Qi Chen for useful discussions.

APPENDIX: SCALAR SEED INTEGRALS

In this appendix, we present a detailed derivation of the second seed integral $\mathcal{I}_{\sf ab}^{(2)}$ that appeared in (39), following the method of [69,70]. The first seed integral $\mathcal{I}_{\sf ab}^{(1)}$ is derived in [70], and the rest of the seed integrals $\mathcal{I}_{\sf ab}^{(3)}$ and $\mathcal{I}_{\sf ab}^{(4)}$ can be derived in a similar way. The seed integral $\mathcal{I}_{\sf ab}^{(2)}$ is defined as

$$\begin{aligned}\mathcal{I}_{\sf ab}^{(2),p_1p_2}(u_1,u_2;z_\Lambda) \equiv -{\sf ab}k_s^{5+p_1+p_2}\int_{-\infty}^0 \mathrm{d}\tau_1\mathrm{d}\tau_2(-\tau_1)^{p_1} \\ \times(-\tau_2)^{p_2}e^{i{\sf a}k_{12{\sf a}}\tau_1+i{\sf b}k_{34{\sf b}}\tau_2} \\ \times D_{\sf ab}^{(2)}(k_s,\tau_1,\tau_2).\end{aligned} \tag{A1}$$

The propagator $D_{\sf ab}^{(2)}(k_s,\tau_1,\tau_2)$ satisfies the following equation of motion:

$$(\tau_1^2\partial_{\tau_1}^2 - 2\tau_1\partial_{\tau_1} + k_s^2\tau_1^2 + m^2)D_{\pm\mp}^{(2)}(k_s;\tau_1,\tau_2) = 0, \tag{A2}$$

$$\begin{aligned}(\tau_1^2\partial_{\tau_1}^2 - 2\tau_1\partial_{\tau_1} + k_s^2\tau_1^2 + m^2)D_{\pm\pm}^{(2)}(k_s;\tau_1,\tau_2) \\ = \pm i\tau_1^2\tau_2^2\delta(\tau_1-\tau_2).\end{aligned} \tag{A3}$$

To proceed, we define two new dimensionless variables as $z_{1{\sf a}} \equiv -k_{12{\sf a}}\tau_1$ and $z_{2{\sf b}} \equiv -k_{34{\sf b}}\tau_2$. In terms of those variables, the equations of motion become

$$\begin{aligned}(r_{1\pm}^2\partial_{r_{1\pm}}^2 - 2r_{1\pm}\partial_{r_{1\pm}} + r_{1\pm}^2z_{1\pm}^2 + m^2) \\ \times\mathcal{D}_{\pm\mp}^{(2)}(r_{1\pm}z_{1\pm}, r_{2\mp}z_{2\mp}) = 0,\end{aligned} \tag{A4}$$

$$(r_{1\pm}^2\partial_{r_{1\pm}}^2 - 2r_{1\pm}\partial_{r_{1\pm}} + r_{1\pm}^2 z_{1\pm}^2 + m^2)\mathcal{D}_{\pm\pm}^{(2)}(r_{1\pm}z_{1\pm}, r_{2\pm}z_{2\pm})$$
$$= \pm i r_{1\pm}^2 r_{2\pm}^2 z_{1\pm}^2 z_{2\pm}^2 \delta(r_{1\pm}z_{1\pm} - r_{2\pm}z_{2\pm}), \quad (A5)$$

where $\mathcal{D}_{\mathsf{ab}}^{(2)}(r_{1\mathsf{a}}z_{1\mathsf{a}}, r_{2\mathsf{b}}z_{2\mathsf{b}}) \equiv k_s^3 D^{(2)}(k_s;\tau_1,\tau_2)$ and $r_{1\mathsf{a}} \equiv k_s/k_{12\mathsf{a}}$, $r_{2\mathsf{b}} \equiv k_s/k_{34\mathsf{b}}$. To derive the differential equation satisfied by the seed integrals, we apply the operator $r_{1\mathsf{a}}^2\partial_{r_{1\mathsf{a}}}^2 - 2r_{1\mathsf{a}}\partial_{r_{1\mathsf{a}}} + r_{1\mathsf{a}}^2 z_{1\mathsf{a}}^2 + m^2$ to the propagator inside the seed integrals (A1) and extract it from the integral. The main challenge arises from the term $r_{1\mathsf{a}}^2 z_{1\mathsf{a}}^2$, which requires repeated integration by parts to be pulled out of the integral. The first step of this process is given by

$$\int_0^{k_{12\mathsf{a}}\cdot\infty} \mathrm{d}z_{1\mathsf{a}} z_{1\mathsf{a}}^p e^{-i\mathsf{a}z_{1\mathsf{a}}} z_{1\mathsf{a}} \mathcal{D}_{\mathsf{ab}}^{(2)}(r_{1\mathsf{a}}z_{1\mathsf{a}}, r_{2\mathsf{b}}z_{2\mathsf{b}})$$
$$= -i\mathsf{a}(p+1+r_{1\mathsf{a}}\partial_{r_{1\mathsf{a}}}) \int_0^{k_{12\mathsf{a}}\cdot\infty} \mathrm{d}z_{1\mathsf{a}} z_{1\mathsf{a}}^p e^{-i\mathsf{a}z_{1\mathsf{a}}} \times \mathcal{D}_{\mathsf{ab}}^{(2)}(r_{1\mathsf{a}}z_{1\mathsf{a}}, r_{2\mathsf{b}}z_{2\mathsf{b}}). \quad (A6)$$

Applying (A6) twice yields the following result:

$$\int_0^{k_{12\mathsf{a}}\cdot\infty} \mathrm{d}z_{1\mathsf{a}} z_{1\mathsf{a}}^p e^{-i\mathsf{a}z_{1\mathsf{a}}} z_{1\mathsf{a}}^2 \mathcal{D}_{\mathsf{ab}}^{(2)}(r_{1\mathsf{a}}z_{1\mathsf{a}}, r_{2\mathsf{b}}z_{2\mathsf{b}})$$
$$= -(p+2+r_{1\mathsf{a}}\partial_{r_{1\mathsf{a}}})(p+1+r_{1\mathsf{a}}\partial_{r_{1\mathsf{a}}}) \times \int_0^{k_{12\mathsf{a}}\cdot\infty} \mathrm{d}z z_{1\mathsf{a}}^p e^{-i\mathsf{a}z_{1\mathsf{a}}} \mathcal{D}_{\mathsf{ab}}^{(2)}(r_{1\mathsf{a}}z_{1\mathsf{a}}, r_{2\mathsf{b}}z_{2\mathsf{b}}). \quad (A7)$$

Therefore, by applying (A7), we can extract the term proportional to $r_{1\mathsf{a}}^2 z_{1\mathsf{a}}^2$ from the integral and obtain the following differential equation for the seed integral:

$$\big[(r_{1\pm}^2 - r_{1\pm}^4)\partial_{r_{1\pm}}^2 - (2r_{1\pm} + (4+2p_1)r_{1\pm}^3)\partial_{r_{1\pm}} + ((\tilde\nu^2 + 9/4) - (p_1+1)(p_1+2)r_{1\pm}^2)\big] \times \big[r_{1\pm}^{-1-p_1} r_{2\mp}^{-1-p_2} \mathcal{I}_{\pm\mp}^{(2),p_1p_2}(r_{1\pm}, r_{2\mp})\big] = 0, \quad (A8)$$

$$\big[(r_{1\pm}^2 - r_{1\pm}^4)\partial_{r_{1\pm}}^2 - (2r_{1\pm} + (4+2p_1)r_{1\pm}^3)\partial_{r_{1\pm}} + ((\tilde\nu^2 + 9/4) - (p_1+1)(p_1+2)r_{1\pm}^2)\big] \times \big[r_{1\pm}^{-1-p_1} r_{2\pm}^{-1-p_2} \mathcal{I}_{\pm\pm}^{(2),p_1p_2}(r_{1\pm}, r_{2\pm})\big]$$
$$= e^{\pm i p_{12}\pi/2} \frac{r_{1\pm}^{4+p_2} r_{2\pm}^{4+p_1}}{(r_{1\pm} + r_{2\pm})^{5+p_{12}}} \Gamma(5+p_{12}), \quad (A9)$$

where $p_{12} \equiv p_1 + p_2$. In terms of $u_{1\mathsf{a}} = 2r_{1\mathsf{a}}/(r_{1\mathsf{a}}+1)$, (A8) and (A9) can be rewritten as

$$\big\{(u_{1\pm}^2 - u_{1\pm}^3)\partial_{u_{1\pm}}^2 - \big[(4+2p_1) - (1+p_1)u_{1\pm}^2\big]\partial_{u_{1\pm}} + [\tilde\nu^2 + (p_1+5/2)^2]\big\}\mathcal{I}_{\pm\mp}^{(2),p_1p_2}(u_{1\pm}, u_{2\mp})$$
$$= 0, \quad (A10)$$

$$\big\{(u_{1\pm}^2 - u_{1\pm}^3)\partial_{u_{1\pm}}^2 - \big[(4+2p_1) - (1+p_1)u_{1\pm}^2\big]\partial_{u_{1\pm}} + [\tilde\nu^2 + (p_1+5/2)^2]\big\}\mathcal{I}_{\pm\pm}^{(2),p_1p_2}(u_{1\pm}, u_{2\pm})$$
$$= e^{\pm i p_{12}\pi/2}\Gamma(5+p_{12})\left(\frac{u_{1\pm}u_{2\pm}}{2(u_{1\pm} + u_{2\pm} - u_{1\pm}u_{2\pm})}\right)^{5+p_{12}}. \quad (A11)$$

To cover the whole parameter space, we have to consider another ratio $r_{2\mathsf{b}} \equiv k_s/k_{34\mathsf{b}}$. This leads to two more differential equations for the propagator, which have the same form as (A10) and (A11). We omit them in the following discussion for brevity.

1. Boundary conditions

To find the general solutions of (A10) and (A11) in the form of (40), we need to specify their boundary conditions. These can be obtained from the late-time expansion (66), which applies when k_s is much smaller than k_{12} and k_{34} (or, equivalently, when $u_{1\mathsf{a}}$ and $u_{2\mathsf{b}}$ approach zero). Using this expansion, we can obtain the asymptotic expression for the BD scalar Wightman function:

$$\lim_{k_s\to 0} D_>^{(2)}(k_s;\tau_1,\tau_2) = \frac{H^2}{4\tilde\nu}(\tau_1\tau_2)^{3/2}(\coth(\pi\tilde\nu)+1)\left(\frac{\tau_1}{\tau_2}\right)^{i\tilde\nu} + \frac{H^2}{4\tilde\nu}(\tau_1\tau_2)^{3/2}\Gamma^2(-i\tilde\nu)\left(\frac{k_s^2\tau_1\tau_2}{4}\right)^{i\tilde\nu} + (\tilde\nu \to -\tilde\nu). \quad (A12)$$

By substituting the late-time expansion into the seed integral, we are able to reach an analytical expression at the squeezed limit. The results are

$$\lim_{u_{1+},u_{2-}\to 0} \mathcal{I}_{+-}^{(2),p_1p_2} = \frac{H^2 e^{-i\bar p_{12}\pi/2} e^{3\pi\tilde\nu}}{4\tilde\nu} \cdot \mathcal{G}_{+-}^{p_1,p_2}(\tilde\nu)\left(\frac{k_{34-}}{k_{12+}}\right)^{i\tilde\nu}\left(\frac{k_s^2}{k_{12+}k_{34-}}\right)^{5/2+p_{12}} + \frac{H^2 e^{-i\bar p_{12}\pi/2} e^{-\pi\tilde\nu}}{4\tilde\nu} \cdot \mathcal{G}_{-+}^{p_1,p_2}(\tilde\nu)\left(\frac{k_{34-}}{k_{12+}}\right)^{-i\tilde\nu}$$
$$\times \left(\frac{k_s^2}{k_{12+}k_{34-}}\right)^{5/2+p_{12}} + \frac{H^2 4^{-i\tilde\nu} e^{-i\bar p_{12}\pi/2} e^{\pi\tilde\nu}}{4\tilde\nu}\frac{\Gamma(-i\tilde\nu)}{\Gamma(i\tilde\nu)} \cdot \mathcal{G}_{++}^{p_1,p_2}(\tilde\nu)\left(\frac{k_s^2}{k_{12+}k_{34-}}\right)^{i\tilde\nu+p_{12}+\frac{5}{2}}$$
$$+ \frac{H^2 4^{i\tilde\nu} e^{-i\bar p_{12}\pi/2} e^{\pi\tilde\nu}}{4\tilde\nu} \cdot \frac{\Gamma(i\tilde\nu)}{\Gamma(-i\tilde\nu)} \cdot \mathcal{G}_{--}^{p_1,p_2}(\tilde\nu)\left(\frac{k_s^2}{k_{12+}k_{34-}}\right)^{-i\tilde\nu+p_{12}+\frac{5}{2}}, \quad (A13)$$

where $\mathcal{G}_{\mathsf{ab}}^{p_1,p_2}(\tilde{\nu}) \equiv (\coth(\pi\tilde{\nu})-1)\Gamma(p_1+i\mathsf{a}\tilde{\nu}+5/2)\Gamma(p_2+i\mathsf{b}\tilde{\nu}+5/2)$, and for the time-ordered seed integral, we have

$$\lim_{u_{1+}\gg u_{2-}\to 0}\mathcal{I}_{++}^{(2),p_1p_2} = \frac{iH^2e^{-ip_{12}\pi/2}}{4\tilde{\nu}}\cdot\mathcal{A}_-(\tilde{\nu})\mathcal{B}_{+-}^{p_1,p_2}(\tilde{\nu})\left(\frac{k_{34-}}{k_{12+}}\right)^{i\tilde{\nu}}\left(\frac{k_s^2}{k_{12+}k_{34-}}\right)^{5/2+p_{12}} + \frac{iH^2e^{-ip_{12}\pi/2}}{4\tilde{\nu}}\cdot\mathcal{A}_+(\tilde{\nu})\mathcal{B}_{-+}^{p_1,p_2}(\tilde{\nu}) \times\left(\frac{k_{34-}}{k_{12+}}\right)^{-i\tilde{\nu}}\left(\frac{k_s^2}{k_{12+}k_{34-}}\right)^{5/2+p_{12}} + \frac{iH^24^{-i\tilde{\nu}}e^{-ip_{12}\pi/2}}{4\tilde{\nu}}\frac{\Gamma(-i\tilde{\nu})}{\Gamma(i\tilde{\nu})}\cdot\mathcal{A}_+(\tilde{\nu})\mathcal{B}_{++}^{p_1,p_2}(\tilde{\nu})\left(\frac{k_s^2}{k_{12+}k_{34-}}\right)^{i\tilde{\nu}+p_{12}+\frac{5}{2}} + \frac{iH^24^{i\tilde{\nu}}e^{-ip_{12}\pi/2}e^{-2\pi\tilde{\nu}}}{4\tilde{\nu}}\cdot\frac{\Gamma(i\tilde{\nu})}{\Gamma(-i\tilde{\nu})}\cdot\mathcal{A}_+(\tilde{\nu})\mathcal{B}_{--}^{p_1,p_2}(\tilde{\nu})\left(\frac{k_s^2}{k_{12+}k_{34-}}\right)^{-i\tilde{\nu}+p_{12}+\frac{5}{2}}, \tag{A14}$$

where $\mathcal{A}_\pm(\tilde{\nu}) \equiv \pm 1+\coth(\pi\tilde{\nu})$, $\mathcal{B}_{\mathsf{ab}}^{p_1,p_2}(\tilde{\nu}) \equiv \Gamma(p_1+i\mathsf{a}\tilde{\nu}+5/2)\Gamma(p_2+i\mathsf{b}\tilde{\nu}+5/2)$. The other seed integrals are obtained through

$$\mathcal{I}_{-+}^{(2),p_1p_2} = \mathcal{I}_{+-}^{(2),p_1p_2*}, \tag{A15}$$

$$\mathcal{I}_{--}^{(2),p_1p_2} = \mathcal{I}_{++}^{(2),p_1p_2*}. \tag{A16}$$

We note that the non-BD seed has a term that escapes the Hubble-scale Boltzmann suppression, as shown by the Gamma function asymptotics in (61). Equations (A13)–(A16) give the boundary conditions for the bootstrap equations and fix the integration constants in (40).

2. The particular solution

The last piece of the solution is the particular solution. This can be find by power expanding both sides and matching the coefficients. The Taylor expansion of the inhomogeneous term in the bootstrap equation (A11) takes the form of

$$e^{\pm ip_{12}\pi/2}\Gamma(5+p_{12})\left(\frac{u_{1\pm}u_{2\pm}}{2(u_{1\pm}+u_{2\pm}-u_{1\pm}u_{2\pm})}\right)^{5+p_{12}} = \sum_{n=0}^{\infty}\mathcal{C}_\pm\binom{-5-p_{12}}{n}u_{1\pm}^{5+p_{12}+n}\left(\frac{1}{u_{2\pm}}-1\right)^n, \tag{A17}$$

where

$$\binom{-5-p_{12}}{n}$$

denotes the combinatorial number. The constant $\mathcal{C}_\pm$ is defined as

$$\mathcal{C}_\pm \equiv \frac{e^{\pm ip_{12}\pi/2}}{2^{5+p_{12}}}\Gamma(5+p_{12}). \tag{A18}$$

Equation (A17) indicates us to take the following ansatz:

$$\mathcal{V}_\pm^{p_1,p_2}(u_{1\pm},u_{2\pm}) = \sum_{m,n=0}^{\infty}\mathcal{V}_{m,n|\pm}^{p_1,p_2}u_{1\pm}^{5+p_{12}+m+n}\left(\frac{1}{u_{2\pm}}-1\right)^n. \tag{A19}$$

Plugging the ansatz into (A11) and matching the coefficients with (A17) at each order, we reach the following recursion equations:

$$\left[(n+p_{12}+5)(n+p_{12}-2p_1)+\tilde{\nu}^2+\left(p_1+\frac{5}{2}\right)^2\right] \times\mathcal{V}_{0,n|\pm}^{p_1,p_2} = \mathcal{C}_\pm\binom{-5-p_{12}}{n} \tag{A20}$$

and

$$\mathcal{V}_{m+1,n|\pm}^{p_1,p_2} = \mathcal{R}_{m,n}^{p_1,p_2}\mathcal{V}_{m,n|\pm}^{p_1,p_2}, \tag{A21}$$

where

$$\mathcal{R}_{m,n}^{p_1,p_2} \equiv \frac{\mathcal{M}_{m,n}^{p_1p_2}}{\mathcal{N}_{m,n}^{p_1p_2}}, \tag{A22}$$

with $\mathcal{M}_{m,n}^{p_1p_2} = (m+n+p_{12}+5)(m+n+p_2+3)$ and $\mathcal{N}_{m,n}^{p_1p_2} = (m+n+p_{12}+6)(m+n+p_{12}-2p_1+1)+\tilde{\nu}^2+(p_1+\frac{5}{2})^2$. The solution of the recursion equations reads

$$\mathcal{V}_{m,n|\pm}^{p_1,p_2} = \frac{(n+p_2+3)(n+p_{12}+5)}{\tilde{\nu}^2+(n+p_{12}+6)(n-2p_1+p_{12}+1)+(p_1+\frac{5}{2})^2} \times\mathcal{C}_\pm\cdot\mathcal{F}_{m,n}^{p_1,p_2}\cdot\mathcal{V}_{0,n|\pm}^{p_1,p_2}, \tag{A23}$$

where

$$\mathcal{F}_{m,n}^{p_1,p_2} \equiv \frac{(n+p_2+4)_{m-1}(n+p_{12}+6)_{m-1}}{(n-i\tilde{\nu}+p_2+\frac{9}{2})_{m-1}(n+i\tilde{\nu}+p_2+\frac{9}{2})_{m-1}} \times\binom{-5-p_{12}}{n} \tag{A24}$$

and $(\ldots)_m$ denotes the Pochhammer function with $\mathcal{V}_{0,n|\pm}^{p_1,p_2}$ given in (A20). This completes the calculation for the particular solution. Note that there is no $(\ldots)^{i\tilde{\nu}}$ like non-analytical oscillation in the particular solution. Therefore, our separation of signal and background in (42) and (43) is justified.

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