

Observational constraints on interactions between dark energy and dark matter with momentum and energy transfers

Xiaolin Liu^{1,5,6}, Shinji Tsujikawa,² and Kiyotomo Ichiki^{3,4,5}

¹*Department of Astronomy, Beijing Normal University, Beijing 100875, China,*

²*Department of Physics, Waseda University, 3-4-1 Okubo, Shinjuku, Tokyo 169-8555, Japan,*

³*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe, Nagoya University, Furocho, Chikusa-ku, Nagoya, Aichi 464-8602, Japan,*

⁴*Institute for Advanced Research, Nagoya University, Furocho, Chikusa-ku, Nagoya, Aichi 464-8602, Japan,*

⁵*Graduate School of Science, Division of Particle and Astrophysical Science, Nagoya University, Furocho, Chikusa-ku, Nagoya, Aichi 464-8602, Japan,*

⁶*Instituto de Física Teórica UAM-CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain*



(Received 28 September 2023; accepted 29 January 2024; published 27 February 2024)

We place observational constraints on a dark energy (DE) model in which a quintessence scalar field ϕ is coupled to dark matter (DM) through momentum and energy exchanges. The momentum transfer is weighed by an interaction between the field derivative and DM four velocity with a coupling constant β , whereas the energy exchange is characterized by an exponential scalar-field coupling to the DM density with a coupling constant Q . A positive coupling β leads to the suppression for the growth of DM density perturbations at low redshifts, whose property offers a possibility for resolving the σ_8 tension problem. A negative coupling Q gives rise to a ϕ -matter-dominated epoch, whose presence can reduce the sound horizon around the cosmic microwave background (CMB) decoupling epoch. Using the data of the Planck 2018, 12th Sloan Digital Sky Survey, Pantheon supernovae samples, and 1-year dark energy survey, we find that the two couplings are constrained to be $\beta = 0.332^{+1.246}_{-0.237}$ and $Q = -0.0312^{+0.0312}_{-0.0085}$ at a 68% confidence level (CL). Thus, there is an interesting observational signature of the momentum exchange ($\beta \neq 0$) between DE and DM, with a peak of the probability distribution of the energy transfer coupling at $Q < 0$.

DOI: [10.1103/PhysRevD.109.043533](https://doi.org/10.1103/PhysRevD.109.043533)

I. INTRODUCTION

Revealing the origin of the dark sector in our Universe is an important challenge for the modern cosmology [1–7]. Dark energy (DE) accelerates the current Universe, while cold dark matter (CDM) is the main source for the formation of large-scale structures. The origin of DE can be a cosmological constant Λ [8–11], but it is theoretically challenging to naturally explain its small value from the vacuum energy arising from particle physics [12,13]. Instead, there have been many attempts for constructing DE models with dynamical propagating degrees of freedom such as scalar fields, vector fields, and massive gravitons (see Refs. [14–19] for reviews). Among them, the scalar-field DE, which is dubbed quintessence [20–27], is one of the simplest models which can be distinguished from the cosmological constant through its time-varying equation of state (EOS) w_{DE} .

From the observational side, we have not yet found compelling evidence that quintessence is favored over the cosmological constant. In particular, the joint analysis based on the data of supernovae Ia (SN Ia), baryon acoustic oscillations, and the cosmic microwave background (CMB)

showed that the quintessence EOS needs to be close to -1 at low redshifts [28–32]. Hence it is difficult to distinguish between quintessence and Λ from the information of w_{DE} alone. At the level of perturbations, the Λ CDM model has a so-called σ_8 tension for the amplitude of matter density contrast between the Planck CMB data [31] and low-redshift probes like shear-lensing [33–35] and redshift-space distortions [36,37]. For both Λ and quintessence, the effective gravitational coupling G_{eff} on scales relevant to the growth of large-scale structures is equivalent to the Newton constant G . Then, the problem of the σ_8 tension cannot be addressed by quintessence either. Moreover, for both Λ and quintessence, there is the tension of today's Hubble expansion rate H_0 between the CMB data and low-redshift measurements [38–45].

If we allow for a possibility of interactions between DE and dark matter (DM), the cosmic expansion and growth histories can be modified in comparison to the Λ CDM model. One example of such couplings corresponds to an energy exchange between DE and DM through an interacting Lagrangian $L_E = -(e^{Q\phi/M_{\text{Pl}}} - 1)\rho_c$ [46–49], where Q is a coupling constant, M_{Pl} is the reduced Planck mass, and ρ_c

is the CDM density. The similar type of couplings arises from Brans-Dicke theories [50] after transforming the Jordan-frame action to that in the Einstein frame [51–53]. In the presence of such an energy transfer, it is possible to realize a so-called ϕ -matter-dominated epoch (ϕ MDE) [47] in which the DE (scalar field) density parameter takes a nonvanishing constant value $\Omega_{\text{DE}} = 2Q^2/3$. The presence of the ϕ MDE can reduce the sound horizon at CMB decoupling [54–56], which may offer a possibility for alleviating the H_0 tension. On the other hand, the effective gravitational coupling of CDM is given by $G_{\text{eff}} = G(1 + 2Q^2)$ [57,58], which is larger than G . This property is not welcome for reducing the σ_8 tension, as we require that $G_{\text{eff}} < G$ to address this problem.

The scalar field can also mediate the momentum exchange with CDM through a scalar product $Z = u_c^\mu \nabla_\mu \phi$ [49,59–70], where u_c^μ is a CDM four velocity and $\nabla_\mu \phi$ is a covariant derivative of ϕ . If we consider an interacting Lagrangian of the form $L_M = \beta Z^2$, where β is a coupling constant, the modification to the background equations arises only through a change of the kinetic term $\dot{\phi}^2/2 \rightarrow (1 + 2\beta)\dot{\phi}^2/2$ in the density and pressure of ϕ [59,63]. At the level of perturbations, the Euler equation is modified by the momentum transfer, while the continuity equation is not affected. For $\beta > 0$, the conditions for the absence of ghosts and Laplacian instabilities of scalar and tensor perturbations are consistently satisfied [66]. In this case, the effective gravitational coupling of CDM is smaller than G at low redshifts [49,59,63,66]. Then, there is an intriguing possibility for reducing the σ_8 tension by the momentum transfer [63,65,67,70].

An interacting model of DE and DM with both momentum and energy transfers was proposed in Ref. [68] as a possible solution to the problems of σ_8 and H_0 tensions. This is described by the interacting Lagrangian $L_{\text{int}} = \beta Z^2 - (e^{Q\phi/M_{\text{pl}}} - 1)\rho_c$ with a canonical scalar field ϕ having a potential $V(\phi)$. Since the model has an explicit Lagrangian, the perturbation equations of motion are unambiguously fixed by varying the corresponding action with respect to the perturbed variables. We would like to stress that this is not the case for many interacting DE and DM models in which the background equations alone are modified by introducing phenomenological couplings [71–86]. We note however that there are some other models with concrete Lagrangians or energy-momentum tensors based on interacting fluids of DE and DM [87–90] or on vector-tensor theories [91].

In Ref. [68], it was anticipated that the momentum transfer associated with the coupling β may address the σ_8 tension due to the suppression of growth of matter perturbations and that the energy transfer characterized by the coupling Q may ease the H_0 tension by the presence of the ϕ MDE. While the gravitational attraction is enhanced by the energy transfer, the decrease of G_{eff} induced by the coupling β can overwhelm the increase

of G_{eff} induced by the coupling Q [68,69]. We also note that the coupling β does not remove the existence of the ϕ MDE at the background level. These facts already imply that nonvanishing values of couplings may be favored, but we require a statistical analysis with actual observational data to see the signatures of those couplings.

In this paper, we perform the Markov chain Monte Carlo (MCMC) analysis of the interacting model of DE and DM with momentum and energy transfers mentioned above. For this purpose, we exploit the recent data of the Planck CMB [92], 12th Sloan Digital Sky Survey (SDSS) [93], Pantheon supernovae samples [94], and 1-year dark energy survey (DES) [95]. We show that the nonvanishing value of β is statistically favored over the case $\beta = 0$, so there is an interesting signature of the momentum transfer between DE and DM. For the energy transfer, the probability distribution of the coupling has a peak at $Q < 0$. The $Q = 0$ case is also consistent with the data at 68% confidence level (CL), so the signature of energy transfer is not so significant compared to that of momentum transfer. Today's Hubble constant is constrained to be $H_0 = 68.20^{+0.54}_{-0.55}$ km/s/Mpc (68% CL), which is not much different from the bound derived for the Λ CDM model with the above datasets. Like most of the models proposed in the literature, our coupled DE-DM scenario does not completely resolve the Hubble tension problem present in the current observational data.

This paper is organized as follows. In Sec. II, we revisit the background dynamics in our interacting model of DE and DM. In Sec. III, we show the full linear perturbation equations of motion and discuss the stability and the effective gravitational couplings of nonrelativistic matter. In Sec. IV, we explain the methodology of how to implement the background and perturbation equations in the CAMB code. We also discuss the impact of our model on several observables. In Sec. V, we present our MCMC results and interpret constraints on the model parameters. Section VI is devoted to conclusions. Throughout the paper, we work in the natural unit system, i.e., $c = \hbar = k_B = 1$.

II. BACKGROUND EQUATIONS OF MOTION

We consider a DE scalar field ϕ interacting with CDM through energy and momentum transfers. We assume that ϕ is a canonical field with the kinetic term $X = -(1/2)\nabla^\mu \phi \nabla_\mu \phi$ and the exponential potential $V(\phi) = V_0 e^{-\lambda\phi/M_{\text{pl}}}$, where V_0 and λ are constants. The choice of the exponential potential is not essential for the purpose of probing the DE-DM couplings, but we can choose other quintessence potentials like the inverse power-law type $V(\phi) = V_0 \phi^{-p}$ [54–56]. The energy transfer is described by the interacting Lagrangian $L_E = -(e^{Q\phi/M_{\text{pl}}} - 1)\rho_c$, where Q is a coupling constant and ρ_c is the CDM density. In the limit that $Q \rightarrow 0$, we have $L_E \rightarrow 0$. The momentum transfer is weighed by the interacting Lagrangian $L_M = \beta Z^2$, where β is a coupling constant and Z is defined by

$$Z = u_c^\mu \nabla_\mu \phi, \quad (2.1)$$

where u_c^μ is the CDM four velocity. For the gravity sector, we consider Einstein gravity described by the Lagrangian of a Ricci scalar R . Then, the total action is given by [68]

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R + X - V_0 e^{-\lambda\phi/M_{\text{pl}}} - (e^{Q\phi/M_{\text{pl}}} - 1)\rho_c + \beta Z^2 \right] + \mathcal{S}_m, \quad (2.2)$$

where g is a determinant of the metric tensor $g_{\mu\nu}$, \mathcal{S}_m is the matter action containing the contributions of CDM, baryons, and radiation with the energy densities ρ_I , EOSs w_I , and squared sound speeds c_I , which are labeled by $I = c, b, r$ respectively. We assume that neither baryons nor radiation are coupled to the scalar field. The action \mathcal{S}_m of perfect fluids can be expressed as a form of the Schutz-Sorkin action [96–98],

$$\mathcal{S}_m = - \sum_{I=c,b,r} \int d^4x [\sqrt{-g}\rho_I(n_I) + J_I^\mu \partial_\mu \ell_I], \quad (2.3)$$

where ρ_I depends on the number density n_I of each fluid. The current vector field J_I^μ is related to n_I as $n_I = \sqrt{g_{\mu\nu} J_I^\mu J_I^\nu} / g$, with ℓ_I being the Lagrange multiplier. The fluid four velocity is given by

$$u_I^\mu = \frac{J_I^\mu}{n_I \sqrt{-g}}, \quad (2.4)$$

which satisfies the normalization $u_I^\mu u_{I\mu} = -1$. Varying the action (2.2) with respect to ℓ_I , it follows that $\partial_\mu J_I^\mu = 0$. In terms of the four velocity, this current conservation translates to

$$u_I^\mu \partial_\mu \rho_I + (\rho_I + P_I) \nabla_\mu u_I^\mu = 0, \quad (2.5)$$

where $P_I = n_I \rho_{I,n} - \rho_I$ is the pressure of each fluid.

We discuss the cosmological dynamics on the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) background given by the line element

$$ds^2 = -dt^2 + a^2(t) \delta_{ij} dx^i dx^j, \quad (2.6)$$

where $a(t)$ is the time-dependent scale factor. On this background we have $u_I^\mu = (1, 0, 0, 0)$ and $\nabla_\mu u_I^\mu = 3H$, where $H = \dot{a}/a$ is the expansion rate of the Universe and a dot denotes the derivative with respect to the cosmic time t . From Eq. (2.5), we have

$$\dot{\rho}_I + 3H(\rho_I + P_I) = 0, \quad (2.7)$$

which holds for each $I = c, b, r$. We consider the cosmological dynamics after the CDM and baryons started

to behave as nonrelativistic particles. At this epoch, we have $w_c = 0$, $w_b = 0$, $c_c^2 = 0$, and $c_b^2 = 0$. The radiation has a usual relativistic EOS $w_r = 1/3$ with $c_r^2 = 1/3$. The gravitational field equations of motion are given by

$$3M_{\text{pl}}^2 H^2 = \rho_\phi + e^{Q\phi/M_{\text{pl}}} \rho_c + \rho_b + \rho_r, \quad (2.8)$$

$$M_{\text{pl}}^2 (2\dot{H} + 3H^2) = -P_\phi - \frac{1}{3} \rho_r, \quad (2.9)$$

where ρ_ϕ and P_ϕ are the scalar-field density and pressure defined, respectively, by

$$\rho_\phi = \frac{1}{2} q_s \dot{\phi}^2 + V_0 e^{-\lambda\phi/M_{\text{pl}}}, \quad P_\phi = \frac{1}{2} q_s \dot{\phi}^2 - V_0 e^{-\lambda\phi/M_{\text{pl}}}, \quad (2.10)$$

with

$$q_s \equiv 1 + 2\beta. \quad (2.11)$$

We require that $q_s > 0$ to have a positive kinetic term in ρ_ϕ .

The scalar-field equation can be expressed in the form

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = -\frac{Q\dot{\phi}}{M_{\text{pl}}} \hat{\rho}_c, \quad (2.12)$$

where

$$\hat{\rho}_c \equiv e^{Q\phi/M_{\text{pl}}} \rho_c. \quad (2.13)$$

Note that $\hat{\rho}_c$ is the CDM density containing the effect of an energy transfer, and the energy flows from CDM to ϕ if $\dot{\phi} > 0$ with $Q < 0$. From Eq. (2.7), CDM obeys the continuity equation $\dot{\rho}_c + 3H(\rho_c + P_c) = 0$. In terms of $\hat{\rho}_c$, this equation can be expressed as

$$\dot{\hat{\rho}}_c + 3H\hat{\rho}_c = +\frac{Q\dot{\phi}}{M_{\text{pl}}} \hat{\rho}_c. \quad (2.14)$$

From Eqs. (2.12) and (2.14), it is clear that there is the energy transfer between the scalar field and CDM, but the momentum exchange between DE and DM does not occur at the background level. The effect of the coupling β appears only as the modification to the coefficient of $\dot{\phi}^2$.

To study the background cosmological dynamics, it is convenient to introduce the following dimensionless variables:

$$x_1 = \frac{\dot{\phi}}{\sqrt{6} M_{\text{pl}} H}, \quad x_2 = \sqrt{\frac{V_0}{3}} \frac{e^{-\lambda\phi/(2M_{\text{pl}})}}{M_{\text{pl}} H}, \quad (2.15)$$

and

$$\begin{aligned}\Omega_\phi &= q_s x_1^2 + x_2^2, & \Omega_c &= \frac{e^{Q\phi/M_{\text{pl}}}\rho_c}{3M_{\text{pl}}^2 H^2}, \\ \Omega_b &= \frac{\rho_b}{3M_{\text{pl}}^2 H^2}, & \Omega_r &= \frac{\rho_r}{3M_{\text{pl}}^2 H^2}.\end{aligned}\quad (2.16)$$

From Eq. (2.8), the density parameters are subject to the constraint

$$\Omega_c = 1 - \Omega_\phi - \Omega_b - \Omega_r. \quad (2.17)$$

The variables x_1 , x_2 , Ω_b , and Ω_r obey the differential equations

$$\begin{aligned}\frac{dx_1}{dN} &= \frac{1}{2}x_1(6q_s x_1^2 - 6 + 3\Omega_c + 3\Omega_b + 4\Omega_r) \\ &+ \frac{\sqrt{6}}{2q_s}(\lambda x_2^2 - Q\Omega_c),\end{aligned}\quad (2.18)$$

$$\frac{dx_2}{dN} = \frac{1}{2}x_2(6q_s x_1^2 - \sqrt{6}\lambda x_1 + 3\Omega_c + 3\Omega_b + 4\Omega_r), \quad (2.19)$$

$$\frac{d\Omega_b}{dN} = \Omega_b(6q_s x_1^2 - 3 + 3\Omega_c + 3\Omega_b + 4\Omega_r), \quad (2.20)$$

$$\frac{d\Omega_r}{dN} = \Omega_r(6q_s x_1^2 - 4 + 3\Omega_c + 3\Omega_b + 4\Omega_r), \quad (2.21)$$

where $N = \ln a$. The scalar-field EOS $w_\phi = P_\phi/\rho_\phi$ and effective EOS $w_{\text{eff}} = -1 - 2\dot{H}/(3H^2)$ are

$$w_\phi = \frac{q_s x_1^2 - x_2^2}{q_s x_1^2 + x_2^2}, \quad w_{\text{eff}} = -1 + 2q_s x_1^2 + \Omega_c + \Omega_b + \frac{4}{3}\Omega_r. \quad (2.22)$$

The fixed points with constant values of x_1 , x_2 , Ω_b , and Ω_r relevant to the radiation, matter, and dark-energy dominated epochs are given, respectively, by

(1) Radiation point (A)

$$x_1 = 0, \quad x_2 = 0, \quad \Omega_b = 0, \quad \Omega_r = 1, \quad \Omega_\phi = 0, \quad w_{\text{eff}} = \frac{1}{3}. \quad (2.23)$$

(2) ϕ MDE point (B)

$$x_1 = -\frac{\sqrt{6}Q}{3q_s}, \quad x_2 = 0, \quad \Omega_b = 0, \quad \Omega_r = 0, \quad \Omega_\phi = w_{\text{eff}} = \frac{2Q^2}{3q_s}, \quad w_\phi = 1. \quad (2.24)$$

(3) Accelerated point (C)

$$x_1 = \frac{\lambda}{\sqrt{6}q_s}, \quad x_2 = \sqrt{1 - \frac{\lambda^2}{6q_s}}, \quad \Omega_b = 0, \quad \Omega_r = 0, \quad \Omega_\phi = 1, \quad w_\phi = w_{\text{eff}} = -1 + \frac{\lambda^2}{3q_s}. \quad (2.25)$$

The coupling Q modifies the standard matter era through the nonvanishing values of Ω_ϕ and w_{eff} . To avoid the dominance of the scalar-field density over the CDM and baryon densities during the ϕ MDE, we require that $\Omega_\phi \ll 1$, i.e.,

$$Q^2 \ll \frac{3}{2}(1 + 2\beta). \quad (2.26)$$

To have the epoch of late-time cosmic acceleration driven by point (C), we need the condition $w_{\text{eff}} < -1/3$, i.e.,

$$\lambda^2 < 2(1 + 2\beta). \quad (2.27)$$

Under this condition, we can show that point (C) is stable against the homogeneous perturbation if [68]

$$\lambda(\lambda + Q) < 3(1 + 2\beta). \quad (2.28)$$

Provided that the conditions (2.26)–(2.28) hold, the cosmological sequence of fixed points (A) \rightarrow (B) \rightarrow (C) can be realized. We refer the reader to Ref. [68] for the numerically integrated background solution. Taking the limits $Q \rightarrow 0$, $\beta \rightarrow 0$, and $\lambda \rightarrow 0$, we recover the background evolution in the Λ CDM model.

III. PERTURBATION EQUATIONS OF MOTION

In Ref. [68], the scalar perturbation equations of motion were derived without fixing particular gauges. The perturbed line element containing four scalar perturbations α , χ , ζ , and E on the spatially flat FLRW background is given by

$$\begin{aligned}ds^2 &= -(1 + 2\alpha)dt^2 + 2\partial_i\chi dt dx^i \\ &+ a^2(t)[(1 + 2\zeta)\delta_{ij} + 2\partial_i\partial_j E]dx^i dx^j.\end{aligned}\quad (3.1)$$

Tensor perturbations propagate in the same manner as in the Λ CDM model, so we do not consider them in the following. The scalar field ϕ is decomposed into the background part $\bar{\phi}(t)$ and the perturbed part $\delta\phi$, as

$$\phi = \bar{\phi}(t) + \delta\phi(t, x^i), \quad (3.2)$$

where we omit the bar from background quantities in the following.

The spatial components of four velocities $u_{Ii} = J_{Ii}/(n_I\sqrt{-g})$ in perfect fluids are related to the scalar velocity potentials v_I , as

$$u_{Ii} = -\partial_i v_I. \quad (3.3)$$

The fluid density is given by $\rho_I = \rho_I(t) + \delta\rho_I(t, x^i)$, where the perturbed part is [49,66,69]

$$\delta\rho_I = \frac{\rho_{I,n_I}}{a^3} [\delta J_I - \mathcal{N}_I(3\zeta + \partial^2 E)], \quad (3.4)$$

where $\rho_{I,n_I} = \partial\rho_I/\partial n_I$, and $\mathcal{N}_I = n_I a^3$ is the background particle number of each fluid (which is conserved).

We can construct the following gauge-invariant combinations:

$$\begin{aligned} \delta\phi_N &= \delta\phi + \dot{\phi}(\chi - a^2\dot{E}), & \delta\rho_{IN} &= \delta\rho_I + \dot{\rho}_I(\chi - a^2\dot{E}), & v_{IN} &= v_I + \chi - a^2\dot{E}, \\ \Psi &= \alpha + \frac{d}{dt}(\chi - a^2\dot{E}), & \Phi &= \zeta + H(\chi - a^2\dot{E}). \end{aligned} \quad (3.5)$$

We also introduce the dimensionless variables

$$\delta_{IN} = \frac{\delta\rho_{IN}}{\rho_I}, \quad \delta\varphi_N = \frac{H}{\dot{\phi}} \delta\phi_N, \quad V_{IN} = H v_{IN}, \quad \mathcal{K} = \frac{k}{aH}, \quad (3.6)$$

where k is a comoving wave number. In Fourier space, the linear perturbation equations of motion are given by [68]

$$\begin{aligned} 6q_s x_1^2 \frac{d\delta\varphi_N}{dN} - 6 \frac{d\Phi}{dN} + 6(1 - q_s x_1^2)(\xi\delta\varphi_N + \Psi) - 2\mathcal{K}^2\Phi + 3(3\Omega_c + 3\Omega_b + 4\Omega_r)\delta\varphi_N \\ + 3(\Omega_c\delta_{cN} + \Omega_b\delta_{bN} + \Omega_r\delta_{rN}) = 0, \end{aligned} \quad (3.7)$$

$$\frac{d\Phi}{dN} - \Psi - \xi\delta\varphi_N + \frac{3}{2}(\Omega_c + 4\beta x_1^2)(V_{cN} - \delta\varphi_N) + \frac{3}{2}\Omega_b(V_{bN} - \delta\varphi_N) + 2\Omega_r(V_{rN} - \delta\varphi_N) = 0, \quad (3.8)$$

$$\frac{d\delta_{IN}}{dN} + 3(c_I^2 - w_I)\delta_{IN} + (1 + w_I) \left(\mathcal{K}^2 V_{IN} + 3 \frac{d\Phi}{dN} \right) = 0, \quad (\text{for } I = c, b, r), \quad (3.9)$$

$$\begin{aligned} (\Omega_c + 4\beta x_1^2) \frac{dV_{cN}}{dN} - [\xi(\Omega_c + 4\beta x_1^2) - 4\beta x_1^2(3 + 2\epsilon_\phi) - \sqrt{6}Qx_1\Omega_c]V_{cN} - \Omega_c\Psi \\ - 4\beta x_1^2 \frac{d\delta\varphi_N}{dN} + [4\beta x_1(\xi - 3 - 2\epsilon_\phi) - \sqrt{6}Q\Omega_c]x_1\delta\varphi_N = 0, \end{aligned} \quad (3.10)$$

$$\frac{dV_{IN}}{dN} - (\xi + 3c_I^2)V_{IN} - \Psi - \frac{c_I^2}{1 + w_I}\delta_{IN} = 0, \quad (\text{for } I = b, r), \quad (3.11)$$

$$\begin{aligned} \frac{d^2\delta\varphi_N}{dN^2} + (3 - \xi + 2\epsilon_\phi) \frac{d\delta\varphi_N}{dN} + \left[\hat{c}_s^2 \mathcal{K}^2 - \frac{d\xi}{dN} - 3\xi + \frac{d\epsilon_\phi}{dN} + \epsilon_\phi^2 + (3 - \xi)\epsilon_\phi + \frac{3}{q_s}(\lambda^2 x_2^2 + Q^2\Omega_c) \right] \delta\varphi_N \\ + 3\hat{c}_s^2 \frac{d\Phi}{dN} - \frac{d\Psi}{dN} - 2(3 + \epsilon_\phi)\Psi - \frac{2\beta}{q_s} \frac{d\delta_{cN}}{dN} + \frac{\sqrt{6}Q\Omega_c}{2q_s x_1} \delta_{cN} = 0, \end{aligned} \quad (3.12)$$

$$\Psi = -\Phi, \quad (3.13)$$

where

$$\begin{aligned}\xi &= -3q_s x_1^2 - \frac{3}{2}\Omega_c - \frac{3}{2}\Omega_b - 2\Omega_r, \\ \epsilon_\phi &= -3 + \frac{\sqrt{6}}{2q_s x_1}(\lambda x_2^2 - Q\Omega_c), \quad \hat{c}_s^2 = \frac{1}{q_s}.\end{aligned}\quad (3.14)$$

We can choose any convenient gauges at hand in the perturbation of Eqs. (3.7)–(3.13). For example, the Newtonian gauge corresponds to $\chi = 0 = E$, in which case Eqs. (3.7)–(3.13) can be directly solved for the gravitational potentials Ψ , Φ and the scalar-field perturbation $\delta\varphi_N$. For the unitary gauge $\delta\phi = 0 = E$, we can introduce the curvature perturbation $\mathcal{R} = \Phi - \delta\varphi_N$ and the CDM density perturbation $\delta\rho_{cu} = \delta\rho_{cN} - \dot{\rho}_c \delta\phi_N / \dot{\phi}$ as two propagating degrees of freedom. These dynamical perturbations have neither ghost nor Laplacian instabilities under the following conditions [49,66,69]:

$$q_s \equiv 1 + 2\beta > 0, \quad (3.15)$$

$$q_c \equiv 1 + \frac{4\beta x_1^2}{\Omega_c} > 0, \quad (3.16)$$

$$c_s^2 \equiv \hat{c}_s^2 + \frac{8\beta^2 x_1^2}{q_s(4\beta x_1^2 + \Omega_c)} > 0. \quad (3.17)$$

Since the CDM effective sound speed vanishes for $c_s^2 \rightarrow +0$, it does not provide an additional Laplacian stability condition. The conditions (3.15)–(3.17) are independent of the gauge choices.

The evolution of perturbations after the onset of the ϕ MDE can be analytically estimated for the modes deep inside the sound horizon. Under the quasistatic approximation, the dominant terms in Eqs. (3.7)–(3.13) are those containing \mathcal{K}^2 , δ_{cN} , $d\delta_{cN}/dN$, and δ_{bN} . From Eqs. (3.7), (3.12), and (3.13), it follows that

$$\begin{aligned}\Psi &= -\Phi \simeq -\frac{3}{2\mathcal{K}^2}(\Omega_c \delta_{cN} + \Omega_b \delta_{bN}), \\ \delta\varphi_N &\simeq \frac{1}{q_s \hat{c}_s^2 \mathcal{K}^2} \left(2\beta \frac{d\delta_{cN}}{dN} - \frac{\sqrt{6}Q\Omega_c}{2x_1} \delta_{cN} \right).\end{aligned}\quad (3.18)$$

We differentiate Eq. (3.9) with respect to N and then use Eqs. (3.10) and (3.11) for CDM and baryons, respectively. On using Eq. (3.18) together with the quasistatic approximation, we obtain the second-order differential equations of CDM and baryons, as [68]

$$\frac{d^2 \delta_{cN}}{dN^2} + \nu \frac{d\delta_{cN}}{dN} - \frac{3}{2G} (G_{cc} \Omega_c \delta_{cN} + G_{cb} \Omega_b \delta_{bN}) \simeq 0, \quad (3.19)$$

$$\frac{d^2 \delta_{bN}}{dN^2} + (2 + \xi) \frac{d\delta_{bN}}{dN} - \frac{3}{2G} (G_{bc} \Omega_c \delta_{cN} + G_{bb} \Omega_b \delta_{bN}) \simeq 0, \quad (3.20)$$

where

$$G_{cc} = \frac{1+r_1}{1+r_2} G, \quad G_{cb} = \frac{1}{1+r_2} G, \quad G_{bc} = G_{bb} = G, \quad (3.21)$$

with

$$\begin{aligned}r_1 &= \frac{2Q[3Q\Omega_c + 2\sqrt{6}\beta x_1(2 + \epsilon_\phi + \sqrt{6}Qx_1)]}{3\Omega_c}, \\ r_2 &= \frac{4\beta(1+2\beta)x_1^2}{\Omega_c},\end{aligned}\quad (3.22)$$

and

$$\nu = \frac{4\beta(1+2\beta)(5 + \xi + 2\epsilon_\phi)x_1^2 + (2 + \xi + \sqrt{6}Qx_1)\Omega_c}{4\beta(1+2\beta)x_1^2 + \Omega_c}. \quad (3.23)$$

Since G_{bc} and G_{bb} are equivalent to G , the baryon perturbation is not affected by the DE-DM couplings. On the other hand, G_{cc} and G_{cb} are different from G for nonvanishing values of Q and β .

During the ϕ MDE, we obtain

$$\begin{aligned}G_{cc} &= \left(1 + \frac{2Q^2}{1+2\beta} \right) G, \\ G_{cb} &= \left[1 - \frac{8\beta Q^2}{3 - 2Q^2 + 2(3+4Q^2)\beta} \right] G.\end{aligned}\quad (3.24)$$

Under the no-ghost condition (3.15), we have $G_{cc} > G$. So long as the coupling Q is in the range $Q^2 \ll 1$, G_{cb} is smaller than G .

After the end of the ϕ MDE, we do not have a simple formula for G_{cc} . However, assuming that $|\beta| \ll 1$ and $|Q| \ll 1$, we find

$$G_{cc} \simeq \left(1 + 2Q^2 - \frac{4\beta x_1^2}{\Omega_c} \right) G. \quad (3.25)$$

Since Ω_c decreases and x_1^2 increases at low redshifts, the third term in the parenthesis of Eq. (3.25) dominates over $2Q^2$ to realize the value of G_{cc} smaller than G . Indeed, the numerical simulation in Ref. [68] shows that the growth rate of δ_{cN} can be less than the value for $\beta = 0$ even in the presence of the coupling Q . This suppressed growth of δ_{cN} at low redshifts should allow the possibility of reducing the σ_8 tension.

IV. METHODOLOGY

We implement our model into the public code CAMB [99] and simulate the evolution of density perturbations with the

background equations to compute the CMB and matter power spectra. In this section, we rewrite the background and perturbation equations of motion in the language of the CAMB code. For this purpose, we use the conformal time defined by $\tau = \int a^{-1} dt$. The background Eqs. (2.7)–(2.9) and (2.12) can be expressed as

$$\rho'_I + 3\mathcal{H}(\rho_I + P_I) = 0, \quad (\text{for } I = c, b, r), \quad (4.1)$$

$$3M_{\text{Pl}}^2 \mathcal{H}^2 = \frac{1}{2} q_s \phi'^2 + a^2 (V_0 e^{-\lambda\phi/M_{\text{Pl}}} + e^{Q\phi/M_{\text{Pl}}} \rho_c + \rho_b + \rho_r), \quad (4.2)$$

$$2M_{\text{Pl}}^2 (\mathcal{H}' - \mathcal{H}^2) = -q_s \phi'^2 - a^2 \left(e^{Q\phi/M_{\text{Pl}}} \rho_c + \rho_b + \frac{4}{3} \rho_r \right), \quad (4.3)$$

$$q_s (\phi'' + 2\mathcal{H}\phi') + \frac{a^2}{M_{\text{Pl}}} (Q\rho_c e^{Q\phi/M_{\text{Pl}}} - \lambda V_0 e^{-\lambda\phi/M_{\text{Pl}}}) = 0, \quad (4.4)$$

where a prime represents the derivative with respect to τ , and we have introduced the conformal Hubble parameter \mathcal{H} as

$$\mathcal{H} \equiv aH = \dot{a} = \frac{a'}{a}. \quad (4.5)$$

For perturbations, we adopt the synchronous gauge conditions

$$\alpha = 0, \quad \chi = 0. \quad (4.6)$$

Following Ma and Bertschinger [100], we use the notations

$$\zeta = -\eta, \quad E = -\frac{h + 6\eta}{2k^2}, \quad \theta_I = \frac{k^2}{a} v_I. \quad (4.7)$$

Then, some of the gauge-invariant variables defined in Eqs. (3.5) and (3.6) reduce to

$$\begin{aligned} \Psi &= \frac{1}{2k^2} (h'' + \mathcal{H}h' + 6\eta'' + 6\mathcal{H}\eta'), & \Phi &= -\eta + \frac{\mathcal{H}}{2k^2} (h' + 6\eta'), \\ \delta_{\text{IN}} &= \delta_I - \frac{3\mathcal{H}}{2k^2} (1 + w_I) (h' + 6\eta'), & \delta\varphi_{\text{IN}} &= \mathcal{H} \left(\frac{\delta\phi}{\phi'} + \frac{h' + 6\eta'}{2k^2} \right), & V_{\text{IN}} &= \frac{\mathcal{H}}{k^2} \left(\theta_I + \frac{1}{2} h' + 3\eta' \right), \end{aligned} \quad (4.8)$$

where $\delta_I \equiv \delta\rho_I/\rho_I$ and $w_I \equiv P_I/\rho_I$. In the presence of perfect fluids of CDM ($w_c = 0 = c_c^2$), baryons ($w_b = 0 = c_b^2$), and radiation ($w_r = 1/3 = c_r^2$), we can express the perturbation Eqs. (3.7)–(3.13) in the forms

$$k^2 \eta - \frac{\mathcal{H}}{2} h' + \frac{a^2}{2M_{\text{Pl}}^2} \left[\frac{q_s}{a^2} \phi' \delta\phi' + \frac{1}{M_{\text{Pl}}} (Q\rho_c e^{Q\phi/M_{\text{Pl}}} - \lambda V_0 e^{-\lambda\phi/M_{\text{Pl}}}) \delta\phi + e^{Q\phi/M_{\text{Pl}}} \rho_c \delta_c + \rho_b \delta_b + \rho_r \delta_r \right] = 0, \quad (4.9)$$

$$k^2 \eta' - \frac{a^2}{2M_{\text{Pl}}^2} \left[\frac{k^2}{a^2} \phi' \delta\phi + \left(\rho_c e^{Q\phi/M_{\text{Pl}}} + \frac{2\beta\phi'^2}{a^2} \right) \theta_c + \rho_b \theta_b + \frac{4}{3} \rho_r \theta_r \right] = 0, \quad (4.10)$$

$$\delta'_c + \theta_c + \frac{1}{2} h' = 0, \quad (4.11)$$

$$\delta'_b + \theta_b + \frac{1}{2} h' = 0, \quad (4.12)$$

$$\delta'_r + \frac{4}{3} \theta_r + \frac{2}{3} h' = 0, \quad (4.13)$$

$$\begin{aligned} \theta'_c + \mathcal{H}\theta_c - \frac{1}{q_s q_c \phi'^2 M_{\text{Pl}}^2} [q_s (q_c - 1) \phi' M_{\text{Pl}} k^2 \delta\phi' + \{Q\phi'^2 + a^2 (q_c - 1) \lambda V_0 e^{-\lambda\phi/M_{\text{Pl}}}\} k^2 \delta\phi \\ + \{Q(q_s - 2)\phi'^3 + 3q_s (q_c - 1) \mathcal{H}\phi'^2 M_{\text{Pl}} - 2a^2 (q_c - 1) \phi' \lambda V_0 e^{-\lambda\phi/M_{\text{Pl}}}\} \theta_c] = 0, \end{aligned} \quad (4.14)$$

$$\theta'_b + \mathcal{H}\theta_b = 0, \quad (4.15)$$

$$\theta'_r - \frac{k^2}{4}\delta_r = 0, \quad (4.16)$$

$$\delta\phi'' + 2\mathcal{H}\delta\phi' + \frac{k^2 M_{\text{Pl}}^2 + a^2(\lambda^2 V_0 e^{-\lambda\phi/M_{\text{Pl}}} + Q^2 \rho_c e^{Q\phi/M_{\text{Pl}}})}{q_s M_{\text{Pl}}^2} \times \delta\phi + \frac{\phi'}{2}h' + \frac{2\beta}{q_s}\phi'\theta_c + \frac{a^2 Q \rho_c e^{Q\phi/M_{\text{Pl}}}}{q_s M_{\text{Pl}}} \delta_c = 0, \quad (4.17)$$

$$h'' + 6\eta'' + 2\mathcal{H}(h' + 6\eta') - 2\eta k^2 = 0, \quad (4.18)$$

where q_s and q_c are defined by Eqs. (3.15) and (3.16), respectively. The perturbation equations of motion for baryons and radiation are the same as those in the Λ CDM model. Thus we modify the equations for CDM and gravitational field equations in the CAMB code. We also take into account the background and perturbation equations of motion for the scalar field, i.e., Eqs. (4.4) and (4.17). Note that the CDM velocity is usually set to zero all the time as a result of the gauge fixing condition in CAMB based on the synchronous gauge. In the models considered here, CDM has nonzero velocity due to the coupling to ϕ in the late Universe. However, we will set $\theta_c = 0$ as the initial condition to eliminate the gauge degree of freedom, assuming that CDM streams freely in the early Universe (i.e., we neglect the interaction between DE and CDM) as in the standard scenario.

In the background Eqs. (4.2)–(4.4), the coupling β appears through the positive no-ghost parameter $q_s = 1 + 2\beta$. In the limit $q_s \rightarrow \infty$, Eq. (4.4) shows that ϕ approaches a constant after the onset of the ϕ MDE. This limit corresponds to the Λ CDM model with a constant potential energy. Since the parameter space for large values of q_s spreads widely, the MCMC chains tend to wander in such regions. This actually leads to the loss of information about the evolution of the scalar field itself. To avoid this, we introduce a set of new variables, $p_s, \hat{\lambda}, \hat{Q}$, defined by

$$p_s \equiv q_s^{-1/2} = \frac{1}{\sqrt{1+2\beta}}, \quad \hat{\lambda} \equiv p_s \lambda, \quad \hat{Q} \equiv p_s Q. \quad (4.19)$$

As we discussed in Sec. III, the growth of matter perturbations is suppressed for positive values of β . In the MCMC analysis, we will set the prior

$$\beta \geq 0. \quad (4.20)$$

In this case, the stability conditions (3.15)–(3.17) are automatically satisfied. Then, the parameter p_s is in the range $0 < p_s \leq 1$. We will choose the uniform prior for p_s . In this case, unlike the flat prior for β , the MCMC chain does not wander in the region of unphysically large values of β .

For the parameter λ , we choose the value

$$\lambda > 0, \quad (4.21)$$

without loss of generality. In Eq. (4.4), we observe that, for $Q > 0$, the background scalar field can approach the instantaneous minima characterized by the condition $Q\rho_c e^{Q\phi/M_{\text{Pl}}} = \lambda V_0 e^{-\lambda\phi/M_{\text{Pl}}}$ even during the matter era. Since we would like to study the case in which the ϕ MDE is present, we will focus on the coupling range

$$Q \leq 0. \quad (4.22)$$

The same prior was chosen in the MCMC analysis of Refs. [54–56]¹ for the coupled DE-DM model with $Q \neq 0$ and $\beta = 0$.

To implement our model in the CAMB code, we use the unit $M_{\text{Pl}} = 1$ and replace ϕ and $\delta\phi$ with the following new variables:

$$\phi \equiv p_s \hat{\phi}, \quad \delta\phi \equiv p_s \delta\hat{\phi}. \quad (4.23)$$

Then, the background scalar-field equation can be expressed as

$$\hat{\phi}'' + 2\mathcal{H}\hat{\phi}' + a^2(\hat{\rho}_{c,\hat{\phi}} + V_{,\hat{\phi}}) = 0, \quad (4.24)$$

where $\hat{\rho}_c = \rho_c e^{\hat{Q}\hat{\phi}}$ and $V_{,\hat{\phi}} = dV/d\hat{\phi}$. The energy density and pressure of $\hat{\phi}$ read as $\rho_{\hat{\phi}} = \hat{\phi}'^2/(2a^2) + V_0 e^{-\hat{\lambda}\hat{\phi}}$ and $P_{\hat{\phi}} = \hat{\phi}'^2/(2a^2) - V_0 e^{-\hat{\lambda}\hat{\phi}}$, respectively. This means that, at the background level, the effect of the momentum transfer can be absorbed into the redefined canonical scalar field $\hat{\phi}$. We note that $\hat{\phi}$ mediates the energy with CDM through the term $a^2 \hat{\rho}_{c,\hat{\phi}}$ in Eq. (4.24). Using the variables and parameters defined above, the perturbation equations of motion for θ_c and $\delta\phi$ are now expressed as

$$\theta'_c + \mathcal{H}\theta_c - \frac{1-p_s^2}{a^2 \hat{\rho}_{c,\hat{\phi}} q_c} [k^2 \hat{\phi}' \delta\hat{\phi}' - a^2 k^2 \delta\hat{\phi} V_{,\hat{\phi}} + (3\mathcal{H}\hat{\phi}' + 2a^2 V_{,\hat{\phi}}) \hat{\phi}' \theta_c] - \frac{\hat{Q}}{q_c} [k^2 p_s^2 \delta\hat{\phi} + (1-2p_s^2) \hat{\phi}' \theta_c] = 0, \quad (4.25)$$

$$\delta\hat{\phi}'' + 2\mathcal{H}\delta\hat{\phi}' + [p_s^2 k^2 + a^2(V_{,\hat{\phi}\hat{\phi}} + \hat{\rho}_{c,\hat{\phi}\hat{\phi}})] \delta\hat{\phi} + [k\mathcal{Z} + (1-p_s^2)\theta_c] \hat{\phi}' + a^2 \hat{\rho}_{c,\hat{\phi}} \delta_c = 0, \quad (4.26)$$

¹In these papers, the sign convention of Q is opposite to ours.

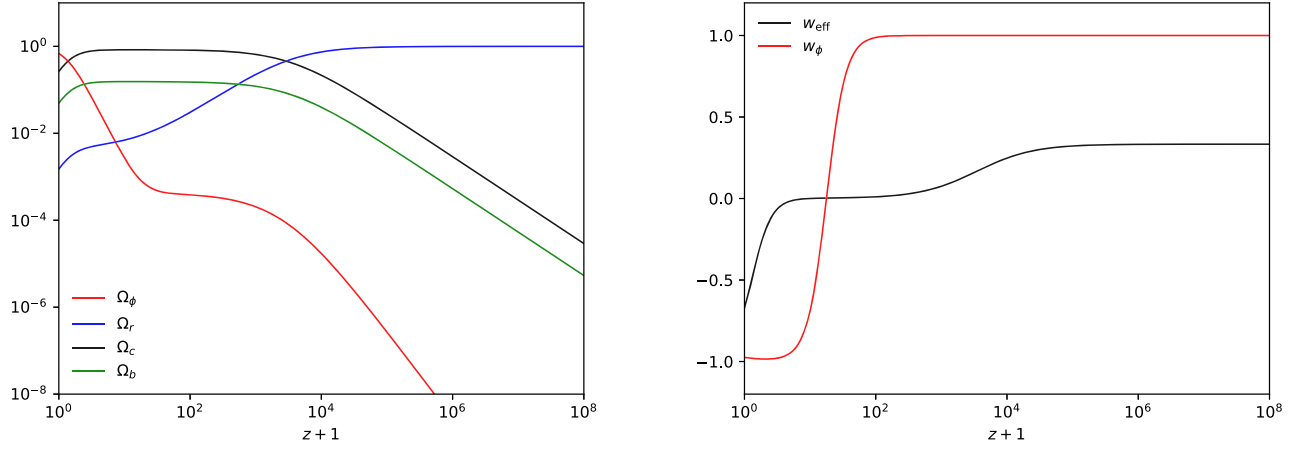


FIG. 1. Left: evolution of Ω_ϕ , Ω_r , Ω_c , Ω_b versus $z + 1$ (z is the redshift) for $Q = -0.04$, $\lambda = 0.5$, and $\beta = 0.4$ with today's density parameters $\Omega_{c0} = 0.25$, $\Omega_{b0} = 0.05$, and $\Omega_{r0} = 1.0 \times 10^{-4}$. Right: evolution of the effective equation of state w_{eff} and the scalar-field equation of state w_ϕ for the same model parameters and initial conditions as those used on the left.

where $\mathcal{Z} \equiv h'/(2k)$. We will also express the other perturbation equations of motion in terms of the new variables introduced above and numerically solve them with the background equations.

In Fig. 1, we plot the density parameters Ω_ϕ , Ω_r , Ω_c , Ω_b (left panel) and w_{eff} , w_ϕ (right panel) for the model parameters $Q = -0.04$, $\lambda = 0.5$, and $\beta = 0.4$. We observe that the solution temporally approaches the ϕ MDE characterized by $\Omega_\phi = w_{\text{eff}} = 2Q^2/[3(1 + 2\beta)]$, which is a distinguished feature compared to the Λ CDM model. The ϕ MDE is followed by the epoch of cosmic acceleration ($w_{\text{eff}} < -1/3$) driven by the fixed point (C).

In left panel of Fig. 2, we show the CMB angular power spectra of temperature anisotropies for several different values of Q and β , with $\lambda = 0.3$. Compared to the uncoupled quintessence, there are two main effects on

CMB induced mostly by the coupling Q . The first is the shift of acoustic peaks toward larger multipoles ℓ . The multiple ℓ_A corresponding to the sound horizon r_{s*} at decoupling (redshift z_*) is given by

$$\ell_A = \pi \frac{D_A(z_*)}{r_{s*}}, \quad (4.27)$$

where

$$D_A(z_*) = \int_0^{z_*} \frac{1}{H(z)} dz \quad (4.28)$$

is the comoving angular diameter distance, and

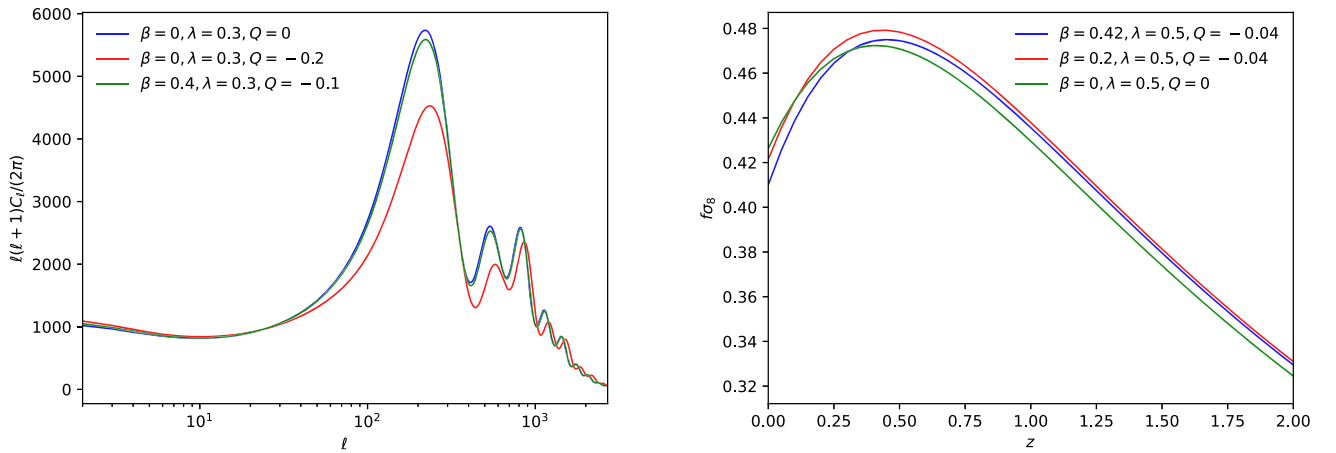


FIG. 2. Left: theoretical CMB temperature anisotropies versus the multipole ℓ for three different model parameter sets: (i) $\beta = 0$, $\lambda = 0.3$, $Q = 0$, (ii) $\beta = 0$, $\lambda = 0.3$, $Q = -0.2$, and (iii) $\beta = 0.4$, $\lambda = 0.3$, $Q = -0.1$. Right: evolution of $f\sigma_8$ versus the redshift z for three different cases: (i) $\beta = 0.42$, $\lambda = 0.5$, $Q = -0.04$, (ii) $\beta = 0.2$, $\lambda = 0.5$, $Q = -0.04$, and (iii) $\beta = 0$, $\lambda = 0.5$, $Q = 0$.

$$r_{s*} = \frac{1}{\sqrt{3}} \int_0^{a_*} \frac{da}{\sqrt{1 + R_s(a)a^2 H(a)}}, \quad (4.29)$$

with $R_s(a) = (3\Omega_{b0}/4\Omega_{\gamma0})a$ and $a_* = (1 + z_*)^{-1}$ [101,102]. Here, Ω_{b0} and $\Omega_{\gamma0}$ are today's density parameters of baryons and photons, respectively. In our model, there is the ϕ MDE in which the CDM density grows faster toward the higher redshift ($\rho_c \propto (1+z)^{3+2Q^2/(1+2\beta)}$) in comparison to the uncoupled case ($Q = 0$). Moreover, the scalar-field density ρ_ϕ scales in the same manner as ρ_c during the ϕ MDE. These properties lead to the larger Hubble expansion rate before the decoupling epoch, so that the sound horizon (4.29) gets smaller in comparison to the uncoupled case.

The coupling Q can increase the value of $H(z)$ from the end of the ϕ MDE toward the decoupling epoch $z = z_*$, which results in the decrease of $D_A(z_*)$. However, for fixed H_0 , the increase of $1/r_{s*}$ induced by the same coupling typically overwhelms the reduction of $D_A(z_*)$ in the estimation of ℓ_A in Eq. (4.27). For the model parameters $Q = 0$ with $\beta = 0$ and $\lambda = 0.5$, we obtain the numerical values $D_A(z_*) = 13.84$ Gpc and $r_{s*} = 144.40$ Mpc. If we change the coupling Q to -0.2 , the two distances change to $D_A(z_*) = 12.95$ Gpc and $r_{s*} = 127.20$ Mpc, respectively. Clearly, the reduction of r_{s*} induced by the coupling Q is stronger than the decrease of $D_A(z_*)$, which leads to the increase of ℓ_A from 301.17 (for $Q = 0$) to 319.85 (for $Q = -0.2$). Hence the larger coupling $|Q|$ leads to the shift of CMB acoustic peaks toward smaller scales. This effect tends to be significant especially for $|Q| \gtrsim 0.1$. We note that the positive coupling β works to suppress the factor $2Q^2/(1+2\beta)$ in the $(1+z)$ -dependent power of ρ_c during the ϕ MDE. In comparison to the case $\beta = 0$, we need to choose larger values of $|Q|$ to have the shift of acoustic peaks toward smaller scales.

The second effect of the coupling Q on the CMB temperature spectrum is the suppressed amplitude of acoustic peaks. The existence of the ϕ MDE gives rise to the larger CDM density ρ_c at decoupling, while the baryon density ρ_b is hardly affected. Then, the coupling Q gives rise to a smaller ratio ρ_b/ρ_c around $z = z_*$. For $Q = 0$ with $\beta = 0$ and $\lambda = 0.5$, we obtain the numerical value $\rho_b/\rho_c = 0.186$, while, for $Q = -0.2$ with the same values of β and λ , this ratio decreases to $\rho_b/\rho_c = 0.116$. This is the main reason for the reduction of the height of CMB acoustic peaks seen in Fig. 2. We note that, in the MCMC analysis performed in Sec. V, the best-fit value of today's density parameter Ω_{c0} is slightly smaller than the one in the Λ CDM model. However, for $Q \neq 0$, the increase of ρ_c toward the past during the ϕ MDE results in the larger CDM density at decoupling in comparison to the uncoupled case, suppressing the early integrated Sachs-Wolfe contribution around the first acoustic peak.

From the above discussion, the coupling $|Q|$ of order 0.1 can reduce the sound horizon at decoupling, whose

property is required to ease the H_0 tension. However, the same order of Q strongly suppresses the amplitude of CMB acoustic peaks. To avoid the latter suppression, we require that $|Q|$ does not exceed the order of 0.1. In Sec. V, we will show that the best-fit value of Q constrained from the observational data is around -0.03 . In this case, the sound horizon does not decrease significantly in comparison to the case $Q = 0$, and hence the complete resolution to the H_0 tension problem is limited.

In the right panel of Fig. 2, we show the evolution of $f\sigma_8$ for several different model parameters, where $f = \dot{\delta}_m/(H\delta_m)$ is the growth rate of matter density contrast (incorporating both CDM and baryons) and σ_8 is the amplitude of matter overdensity at the comoving $8 h^{-1}$ Mpc scale (h is the normalized Hubble constant $H_0 = 100 h$ km/s/Mpc). We find that the large coupling β induces the suppression for the growth rate of matter perturbations at low redshifts. This is also the case even in the presence of the coupling Q of order -0.01 . This result is consistent with the analytic estimation for the growth of perturbations discussed in Sec. III.

V. RESULTS AND DISCUSSION

We are now going to place observational constraints on our model by using the MCMC likelihood CosmoMC [103]. In our analysis, we will exploit the following datasets.

(i) The CMB data containing TT, TE, EE + lowE from Planck 2018 [92], and the full-shape large-scale structure data from the 12th data release of SDSS [93]. We also run the MCMC code by using the 16th SDSS data, but we did not find any obvious difference from the constraints derived below.

(ii) The Pantheon supernovae samples containing 1048 type Ia supernovae magnitudes with redshift in the range of $0.01 < z < 2.3$ [94], which are commonly used to constrain the property of late-time cosmic acceleration.

(iii) The 1st year DES results [95], which are the combined analyses of galaxy clustering and weak gravitational lensing.

We stop the calculations when the Gelman-Rubin statistic $R - 1 \sim 0.01$ is reached. In Fig. 3 and Table I, we present the results of observational constraints on our model parameters.

First, let us discuss constraints on the parameter β . In Table I, the bounds on β (68% CL) constrained by different datasets are presented in terms of the log prior. From the joint analysis based on the datasets (i) + (ii) + (iii), this bound translates to

$$\beta = 0.332_{-0.237}^{+1.246} \quad (68\% \text{ CL}), \quad (5.1)$$

where 0.332 is the mean value. Since β is constrained to be larger than 0.095 at 1σ , there is an interesting observational signature of the momentum exchange between DE and DM. Even with the analysis of the dataset (i) or with the

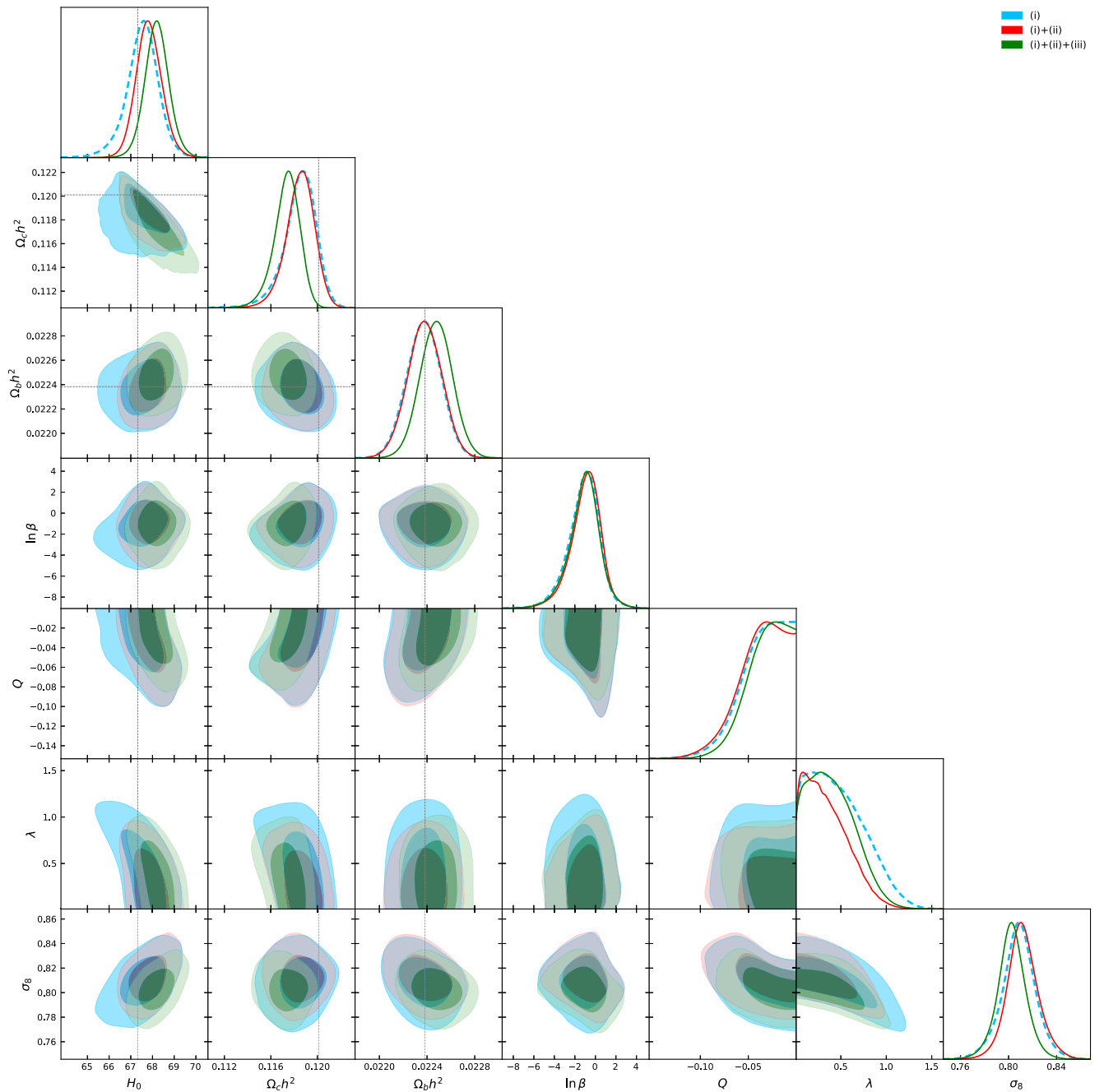


FIG. 3. Triangle plot for the one-dimensional marginalized distributions on individual parameters and the 1σ and 2σ two-dimensional contours. The blue dashed lines represent constraints by the Planck 2018 [104] and 12th SDSS datasets, which we call (i). The red and green solid lines correspond to constraints when the datasets (ii) and (ii) + (iii) are combined with (i), respectively.

datasets (i) + (ii), the 1σ lower limits on β are close to the value 0.1. In other words, adding the DES data to the datasets (i) + (ii) hardly modifies the constraint on β . Hence the Planck CMB data combined with the SDSS data already show the signature of the momentum transfer. We note that this result is consistent with the likelihood analysis of Refs. [63,65,67,70] performed for the model $Q = 0$, where the joint analysis based on the CMB and galaxy clustering data favor nonvanishing values of β .

With the datasets (i) + (ii) + (iii), we also obtain the following 2σ bound:

$$0.011 < \beta < 7.526 \quad (95\% \text{ CL}). \quad (5.2)$$

Since the lower limit of β is as small as 0.01, this value is not significantly distinguished from $\beta = 0$. This means that the evidence for the momentum transfer can be confirmed at 68% CL, but not firmly at 95% CL, with the current

TABLE I. Priors, mean values, best-fit values and 1σ errors of the model parameters $\ln\beta$, λ , Q and cosmological parameters H_0 , $\Omega_{c0}h^2$, $\Omega_{b0}h^2$, σ_8 , where Ω_{c0} and Ω_{b0} are today's density parameters of CDM and baryons respectively. The third, fourth, and fifth columns correspond to the constraints derived by the datasets (i), (i) + (ii), and (i) + (ii) + (iii), respectively.

Parameters	Priors	Mean (best fit) (i)	Mean (best fit) (i) + (ii)	Mean (best fit) (i) + (ii) + (iii)
H_0 [km/s/Mpc]	[20, 100]	67.52(67.17) $^{+0.76}_{-0.64}$	67.84(67.87) $^{+0.56}_{-0.61}$	68.20(67.84) $^{+0.54}_{-0.55}$
$\Omega_{c0}h^2$	[0.001, 0.99]	0.11848(0.11861) $^{+0.00155}_{-0.00105}$	0.11850(0.11904) $^{+0.00132}_{-0.00106}$	0.11735(0.11882) $^{+0.00122}_{-0.00092}$
$\Omega_{b0}h^2$	[0.005, 0.1]	0.02237(0.02240) $^{+0.00015}_{-0.00014}$	0.02238(0.02235) $^{+0.00015}_{-0.00014}$	0.02248(0.02241) $^{+0.00014}_{-0.00014}$
$\ln\beta$	*	-1.1546(-2.7365) $^{+1.6471}_{-1.2185}$	-0.9538(1.0231) $^{+1.5897}_{-1.1714}$	-1.1012(0.4111) $^{+1.5576}_{-1.2456}$
λ	[0.1, ∞]	0.4650(0.5043) $^{+0.1421}_{-0.4459}$	0.3517(0.2875) $^{+0.0987}_{-0.3391}$	0.4042(0.3376) $^{+0.1370}_{-0.3673}$
Q	$[-\infty, 0]$	-0.0347(-0.0209) $^{+0.0347}_{-0.0093}$	-0.0366(-0.0716) $^{+0.0366}_{-0.0096}$	-0.0312(-0.0036) $^{+0.0312}_{-0.0085}$
σ_8	*	0.8084(0.8065) $^{+0.0140}_{-0.0130}$	0.8118(0.8101) $^{+0.0118}_{-0.0132}$	0.8026(0.8002) $^{+0.0114}_{-0.0112}$

observational data. We note that the mean value of σ_8 constrained by the datasets (i) + (ii) + (iii) is 0.8026, which is smaller than the Planck 2018 bound $\sigma_8 = 0.8111 \pm 0.0060$ [31] derived for the Λ CDM model. Thus, in our model, the σ_8 tension between the CMB and other measurements is alleviated by the momentum transfer. This property is mostly attributed to the fact that the growth rate of δ_c at low redshifts is suppressed by the positive coupling β .

The other coupling constant Q , which mediates the energy transfer between DE and DM, is constrained to be

$$Q = -0.0312^{+0.0312}_{-0.0085} \quad (68\% \text{ CL}), \quad (5.3)$$

where -0.0312 is the mean value. With the datasets (i) alone, the one-dimensional probability distribution of Q does not have a particular peak at $Q < 0$. As we see in Fig. 3, the MCMC analysis based on the full datasets gives rise to a peak in the one-dimensional distribution of Q around -0.03 . Since the vanishing coupling ($Q = 0$) is within the 1σ contour, we do not have strong observational evidence that the nonvanishing value of Q is favored over the $Q = 0$ case. However, it is interesting to note that the current data give rise to the probability distribution of Q with a peak at $Q < 0$.

In Refs. [54–56], the couplings $|Q|$ slightly smaller than the mean value of (5.3) were obtained by the MCMC analysis with several datasets for the coupled dark energy model with $\beta = 0$. In our model, we have $\Omega_\phi = w_{\text{eff}} = 2Q^2/[3(1+2\beta)]$ during the ϕ MDE, so both Ω_ϕ and w_{eff} are suppressed by the positive coupling β . This allows the larger values of $|Q|$ in comparison to the case $\beta = 0$. Still, the coupling $|Q|$ exceeding the order 0.1 is forbidden from the data because of the significant changes of heights and positions in CMB acoustic peaks (see Fig. 2).

The parameter λ is related to the slope of the scalar-field potential. To realize the DE equation of state closer to -1 at late times, we require that λ cannot be significantly away from 0. From the MCMC analysis with the datasets (i) + (ii) + (iii), we obtain the upper limit

$$\lambda < 0.5412 \quad (68\% \text{ CL}). \quad (5.4)$$

The upper limit on λ is mostly determined by the Planck and SN Ia datasets. We also remark that, for larger λ , the distance to the CMB last scattering surface is reduced. To compensate this property, we require smaller values of H_0 . This explains the tendency for blue contours seen in the $\lambda - H_0$ plane. Thus, the smaller values of λ are favored from the viewpoint of increasing H_0 .

In Fig. 3, we find that today's CDM density parameter Ω_{c0} is constrained to be smaller than the Planck 2018 bound $\Omega_{c0}h^2 = 0.120 \pm 0.001$ derived for the Λ CDM model [31]. In spite of this decrease of Ω_{c0} , the CDM density evolves as $\rho_c \propto (1+z)^{3+2Q^2/(1+2\beta)}$ during the ϕ MDE and hence Ω_c at decoupling can be increased by the nonvanishing coupling Q . We note that today's baryon density parameter Ω_{b0} is only slightly larger than the Planck 2018 bound $\Omega_{b0} = 0.0224 \pm 0.0001$ (see Fig. 3). Then, the nonvanishing coupling Q hardly modifies the value of Ω_b at $z = z_*$ in comparison to the case $Q = 0$. Since the ratio Ω_b/Ω_c at decoupling is decreased by the coupling $|Q|$ larger than the order 0.01, this suppresses the height of CMB acoustic peaks. The MCMC analysis with the CMB data alone already places the bound $|Q| < 0.1$ at 95% CL.

As we discussed in Sec. IV, the nonvanishing coupling Q reduces the sound horizon r_{s*} at $z = z_*$. This leads to the shift of CMB acoustic peaks toward smaller scales. To keep the position of the multipole ℓ_A corresponding to the sound horizon, we require that the comoving angular diameter distance $D_A(z_*)$ appearing in the numerator of Eq. (4.27) should be also reduced. We can express Eq. (4.28) as $D_A(z_*) = H_0^{-1} \int_0^{z_*} E^{-1}(z) dz$, where $E(z) = H(z)/H_0$. In the Λ CDM model we have $E(z) = [\Omega_{m0}(1+z)^3 + \Omega_\Lambda + \Omega_{r0}(1+z)^4]^{1/2}$, where $\Omega_{m0} = \Omega_{c0} + \Omega_{b0}$. In our model, the CDM density parameter during the ϕ MDE has the dependence $\Omega_{c0}(1+z)^{3+2Q^2/(1+2\beta)}$ instead of $\Omega_{c0}(1+z)^3$, together with the scaling behavior of ρ_ϕ with ρ_c . Then, the coupling Q leads to the increase of $E(z)$ from the end of ϕ MDE to the

decoupling epoch, so that the integral $\int_0^{z_*} E^{-1}(z) dz$ is decreased. This property is different from the early DE scenario of Ref. [105], where the energy density of early DE quickly decays after the recombination epoch.

In our model, increasing the value of H_0 also reduces $D_A(z_*)$, so it can compensate for the reduction of r_{s*} . However, the integral $\int_0^{z_*} E^{-1}(z) dz$ is already decreased at some extent by the existence of the ϕ MDE. In this sense, there is the limitation for realizing H_0 significantly larger than the value obtained for $Q = 0$. The observational constraint on H_0 derived by the dataset (i) for the model with $Q = 0$ is consistent with the Planck 2018 bound $H_0 = 67.27 \pm 0.60$ km/s/Mpc. In the presence of the negative coupling Q , the likelihood region in the $Q - H_0$ plane shown in Fig. 3 shifts toward larger values of H_0 . This is particularly the case for adding the datasets (ii) and (iii) to the Planck data. With the full datasets (i) + (ii) + (iii), the Hubble constant is constrained to be

$$H_0 = 68.20_{-0.55}^{+0.54} \text{ km/s/Mpc} \quad (68\% \text{ CL}), \quad (5.5)$$

whose mean value is larger than the one derived for the Λ CDM model with the Planck 2018 data alone. However, it is not possible to reach the region $H_0 > 70$ km/s/Mpc due to the limitation of reducing $D_A(z_*)$ by increasing the value of H_0 . We also carried out the MCMC analysis for the Λ CDM model and obtained the bound $H_0 = 68.16_{-0.38}^{+0.37}$ km/s/Mpc. The 1σ upper limit of the constraint (5.5) is only slightly larger than that of the Λ CDM bound. In Fig. 4, we plot the evolution of the Hubble expansion rate versus $z + 1$ for the best-fit model parameters constrained from the Planck 2018 data alone and from the SN Ia data alone. Clearly, there are discrepancies for the

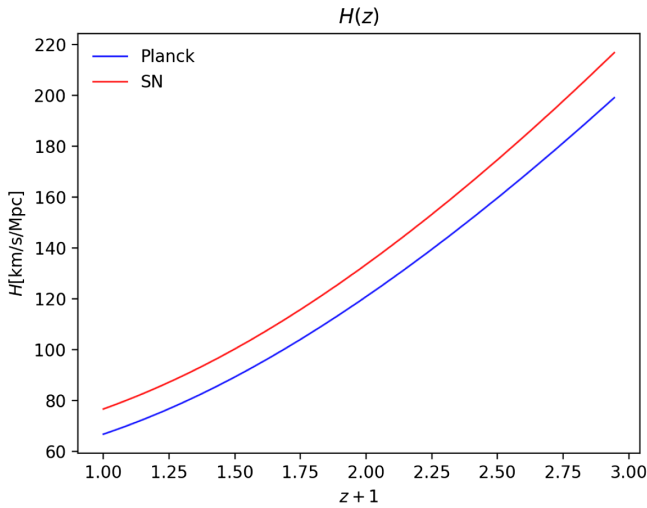


FIG. 4. Evolution of the Hubble expansion rate $H(z)$ at low redshifts for the best-fit model parameters constrained from the Planck 2018 data alone (blue) and from the SN Ia data alone (red).

low-redshift values of $H(z)$ obtained from the CMB and SN Ia data. This is related to the fact that the coupling $|Q|$ of order 0.01 is not sufficient to reduce the sound horizon at decoupling significantly for the purpose of obtaining large values of H_0 . Hence the Hubble tension problem between the Planck 2018 data and those constrained by the direct measurements of H_0 still persists in our coupled DE scenario.

Despite the difficulty of resolving the Hubble tension problem, the fact that the probability distribution of Q has a peak around -0.03 is an interesting property of our model. Moreover, there are observational signatures of the momentum transfer with $\beta > 0$ between DE and DM at 68% CL. The coupling β can alleviate the σ_8 tension without spoiling the existence of the ϕ MDE.

VI. CONCLUSIONS

In this paper, we put observational constraints on an interacting model of DE and DM given by the action (2.2). Since our model has a concrete Lagrangian, the background and perturbation equations of motion are unambiguously fixed by the variational principle. This is not the case for many coupled DE-DM models studied in the literature, in which the interacting terms are added to the background equations by hands. In our model, the DE scalar field ϕ and the CDM fluid mediate both energy and momentum transfers, whose coupling strengths are characterized by the constants Q and β , respectively. We considered an exponential potential $V(\phi) = V_0 e^{-\lambda\phi/M_{\text{pl}}}$ of the scalar field to derive late-time cosmic acceleration, but the different choice of quintessence potentials should not affect the observational constraints on Q and β significantly.

The coupling Q can give rise to the ϕ MDE during which the scalar-field density parameter Ω_ϕ and the effective equation of state w_{eff} are nonvanishing constants, such that $\Omega_\phi = w_{\text{eff}} = 2Q^2/[3(1+2\beta)]$. In this epoch, the CDM density grows as $\rho_c \propto (1+z)^{3+2Q^2/(1+2\beta)}$ toward the past, and hence the value of ρ_c at CMB decoupling can be increased by the coupling Q . Since this enhances the Hubble expansion rate in the past, the sound horizon r_{s*} at decoupling (redshift z_*) gets smaller. Moreover, the ratio between the baryon and CDM densities, ρ_b/ρ_c , is suppressed at $z = z_*$ due to the increase of ρ_c induced by the presence of the ϕ MDE. These modifications shift the positions and heights of acoustic peaks of CMB temperature anisotropies, so that the coupling Q can be tightly constrained from the CMB data.

The effect of momentum transfers on the dynamics of perturbations mostly manifests itself for the evolution of CDM density contrast δ_c at low redshifts. For $\beta > 0$, the growth of δ_c is suppressed due to the decrease of an effective gravitational coupling G_{eff} on scales relevant to the galaxy clustering. The coupling Q enhances the value of G_{eff} through the energy transfer between DE and DM. However, the reduction of G_{eff} induced by positive β

typically overwhelms the increase of G_{eff} for the redshift $z \lesssim 1$. Hence the growth rate of CDM perturbations is suppressed in comparison to the Λ CDM model.

We carried out the MCMC analysis for our model by using the observational data of Planck 2018 [92], 12th SDSS, Phantheon supernovae samples, and 1-year DES. The coupling β is constrained to be in the range $\beta = 0.332_{-0.237}^{+1.246}$ (68% CL) by using all the datasets. Since the $\beta = 0$ case is outside the 1σ observational contour, there is an interesting observational signature of the momentum transfer between DE and DM. This is an outcome of the suppressed growth of δ_c at low redshifts, thereby easing the σ_8 tension. Indeed, we found that the mean value of σ_8 constrained by the full data is 0.8026, which is smaller than the best-fit value 0.8111 derived for the Λ CDM model with the Planck data alone.

For the coupling characterizing the energy transfer, we obtained the bound $Q = -0.0312_{-0.0085}^{+0.0312}$ (68% CL) by the analysis with full datasets. While the $Q = 0$ case is within the 1σ observational contour, there is a peak for the probability distribution of the coupling at a negative value of Q . This result is consistent with the likelihood analysis performed for the model with $\beta = 0$ [54–56], but now the constrained values of $|Q|$ get larger. This increase of $|Q|$ is mostly attributed to the fact that the effective equation of state during the ϕ MDE is modified to $w_{\text{eff}} = 2Q^2/[3(1 + 2\beta)]$ through the coupling β . In comparison to the momentum transfer, we have not yet detected significant observational signatures of the energy transfer, but the future high-precision data will clarify this issue.

The presence of the coupling Q reduces the sound horizon r_{s*} at decoupling, thereby increasing the multipole ℓ_A defined in Eq. (4.27). To keep the position of CMB acoustic peaks, we require that the comoving angular

diameter distance $D_A(z_*)$ from $z = 0$ to $z = z_*$ decreases. During the ϕ MDE, the Hubble expansion rate increases due to the enhancement of ρ_c induced by the energy transfer. Since this leads to the decrease of $D_A(z_*)$, the further reduction of $D_A(z_*)$ by the choice of larger values of H_0 is quite limited in our model. From the MCMC analysis of full datasets we obtained the bound $H_0 = 68.20_{-0.55}^{+0.54}$ km/s/Mpc, whose mean value is larger than the one derived for the Λ CDM model with the Planck 2018 data alone. However, the Hubble constant H_0 does not exceed the value 70 km/s/Mpc, so the Hubble tension problem is not completely resolved in our scenario.

It is still encouraging that the current data support signatures of the interaction between DE and DM. We expect that upcoming observational data like those from the Euclid satellite will place further tight constraints on the couplings β and Q . Along with the H_0 tension problem, we hope that we will be able to approach the origins of DE and DM and their possible interactions in the foreseeable future.

ACKNOWLEDGMENTS

X.L. is supported by the National Natural Science Foundation of China under Grants No. 11920101003, No. 12021003, and No. 11633001, and the Strategic Priority Research Program of the Chinese Academy of Sciences, Grant No. XDB23000000. S.T. is supported by the Grant-in-Aid for Scientific Research Fund of the JSPS No. 22K03642 and Waseda University Special Research Project No. 2023C-473. K.I. is supported by the JSPS Grant No. 21H04467, JST FOREST Program No. JPMJFR20352935, and by JSPS Core-to-Core Program (Grants No. JPJSCCA20200002 and No. JPJSCCA20200003).

-
- [1] G. Jungman, M. Kamionkowski, and K. Griest, *Phys. Rep.* **267**, 195 (1996).
 - [2] G. Bertone, D. Hooper, and J. Silk, *Phys. Rep.* **405**, 279 (2005).
 - [3] P. J. E. Peebles and B. Ratra, *Rev. Mod. Phys.* **75**, 559 (2003).
 - [4] E. J. Copeland, M. Sami, and S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006).
 - [5] A. Silvestri and M. Trodden, *Rep. Prog. Phys.* **72**, 096901 (2009).
 - [6] R. Kase and S. Tsujikawa, *Int. J. Mod. Phys. D* **28**, 1942005 (2019).
 - [7] J. M. Ezquiaga and M. Zumalacárregui, *Front. Astron. Space Sci.* **5**, 44 (2018).
 - [8] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
 - [9] V. Sahni and A. A. Starobinsky, *Int. J. Mod. Phys. D* **09**, 373 (2000).
 - [10] T. Padmanabhan, *Phys. Rep.* **380**, 235 (2003).
 - [11] S. M. Carroll, *Living Rev. Relativity* **4**, 1 (2001).
 - [12] J. Martin, *C. R. Phys.* **13**, 566 (2012).
 - [13] A. Padilla, *arXiv:1502.05296*.
 - [14] A. De Felice and S. Tsujikawa, *Living Rev. Relativity* **13**, 3 (2010).
 - [15] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, *Phys. Rep.* **513**, 1 (2012).
 - [16] A. Joyce, B. Jain, J. Khoury, and M. Trodden, *Phys. Rep.* **558**, 1 (2015).
 - [17] K. Koyama, *Rep. Prog. Phys.* **79**, 046902 (2016).
 - [18] M. Ishak, *Living Rev. Relativity* **22**, 1 (2019).
 - [19] L. Heisenberg, *Phys. Rep.* **796**, 1 (2019).
 - [20] Y. Fujii, *Phys. Rev. D* **26**, 2580 (1982).
 - [21] L. H. Ford, *Phys. Rev. D* **35**, 2339 (1987).
 - [22] B. Ratra and P. J. E. Peebles, *Phys. Rev. D* **37**, 3406 (1988).

- [23] C. Wetterich, *Nucl. Phys.* **B302**, 668 (1988).
- [24] T. Chiba, N. Sugiyama, and T. Nakamura, *Mon. Not. R. Astron. Soc.* **289**, L5 (1997).
- [25] P. G. Ferreira and M. Joyce, *Phys. Rev. Lett.* **79**, 4740 (1997).
- [26] R. R. Caldwell, R. Dave, and P. J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998).
- [27] I. Zlatev, L.-M. Wang, and P. J. Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1999).
- [28] T. Chiba, A. De Felice, and S. Tsujikawa, *Phys. Rev. D* **87**, 083505 (2013).
- [29] S. Tsujikawa, *Classical Quantum Gravity* **30**, 214003 (2013).
- [30] J.-B. Durrive, J. Ooba, K. Ichiki, and N. Sugiyama, *Phys. Rev. D* **97**, 043503 (2018).
- [31] N. Aghanim *et al.* (Planck Collaboration), *Astron. Astrophys.* **641**, A6 (2020); **652**, C4(E) (2021).
- [32] Y. Akrami, R. Kallosh, A. Linde, and V. Vardanyan, *Fortschr. Phys.* **67**, 1800075 (2019).
- [33] C. Heymans *et al.*, *Mon. Not. R. Astron. Soc.* **427**, 146 (2012).
- [34] H. Hildebrandt *et al.*, *Mon. Not. R. Astron. Soc.* **465**, 1454 (2017).
- [35] T. Abbott *et al.* (DES Collaboration), *Phys. Rev. D* **98**, 043526 (2018).
- [36] L. Samushia *et al.*, *Mon. Not. R. Astron. Soc.* **439**, 3504 (2014).
- [37] E. Macaulay, I. K. Wehus, and H. K. Eriksen, *Phys. Rev. Lett.* **111**, 161301 (2013).
- [38] N. Aghanim *et al.* (Planck Collaboration), *Astron. Astrophys.* **641**, A6 (2020).
- [39] A. G. Riess *et al.*, *Astrophys. J.* **855**, 136 (2018).
- [40] K. C. Wong *et al.*, *Mon. Not. R. Astron. Soc.* **498**, 1420 (2020).
- [41] A. G. Riess, S. Casertano, W. Yuan, J. B. Bowers, L. Macri, J. C. Zinn, and D. Scolnic, *Astrophys. J. Lett.* **908**, L6 (2021).
- [42] L. Verde, T. Treu, and A. Riess, *Nat. Astron.* **3**, 891 (2019).
- [43] E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri, D. F. Mota, A. G. Riess, and J. Silk, *Classical Quantum Gravity* **38**, 153001 (2021).
- [44] L. Perivolaropoulos and F. Skara, *New Astron. Rev.* **95**, 101659 (2022).
- [45] W. L. Freedman, *Astrophys. J.* **919**, 16 (2021).
- [46] C. Wetterich, *Astron. Astrophys.* **301**, 321 (1995).
- [47] L. Amendola, *Phys. Rev. D* **62**, 043511 (2000).
- [48] N. Frusciante, R. Kase, K. Koyama, S. Tsujikawa, and D. Vernieri, *Phys. Lett. B* **790**, 167 (2019).
- [49] R. Kase and S. Tsujikawa, *Phys. Rev. D* **101**, 063511 (2020).
- [50] C. Brans and R. H. Dicke, *Phys. Rev.* **124**, 925 (1961).
- [51] L. Amendola, *Phys. Rev. D* **60**, 043501 (1999).
- [52] J. Khoury and A. Weltman, *Phys. Rev. D* **69**, 044026 (2004).
- [53] S. Tsujikawa, K. Uddin, S. Mizuno, R. Tavakol, and J. Yokoyama, *Phys. Rev. D* **77**, 103009 (2008).
- [54] V. Pettorino, *Phys. Rev. D* **88**, 063519 (2013).
- [55] P. A. R. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **594**, A14 (2016).
- [56] A. Gómez-Valent, V. Pettorino, and L. Amendola, *Phys. Rev. D* **101**, 123513 (2020).
- [57] L. Amendola, *Phys. Rev. D* **69**, 103524 (2004).
- [58] S. Tsujikawa, *Phys. Rev. D* **76**, 023514 (2007).
- [59] A. Poursidou, C. Skordis, and E. Copeland, *Phys. Rev. D* **88**, 083505 (2013).
- [60] C. G. Boehmer, N. Tamanini, and M. Wright, *Phys. Rev. D* **91**, 123003 (2015).
- [61] C. Skordis, A. Poursidou, and E. Copeland, *Phys. Rev. D* **91**, 083537 (2015).
- [62] T. S. Koivisto, E. N. Saridakis, and N. Tamanini, *J. Cosmol. Astropart. Phys.* **09** (2015) 047.
- [63] A. Poursidou and T. Tram, *Phys. Rev. D* **94**, 043518 (2016).
- [64] J. Dutta, W. Khylllep, and N. Tamanini, *Phys. Rev. D* **95**, 023515 (2017).
- [65] M. S. Linton, A. Poursidou, R. Crittenden, and R. Maartens, *J. Cosmol. Astropart. Phys.* **04** (2018) 043.
- [66] R. Kase and S. Tsujikawa, *Phys. Lett. B* **804**, 135400 (2020).
- [67] F. N. Chamings, A. Avgoustidis, E. J. Copeland, A. M. Green, and A. Poursidou, *Phys. Rev. D* **101**, 043531 (2020).
- [68] L. Amendola and S. Tsujikawa, *J. Cosmol. Astropart. Phys.* **06** (2020) 020.
- [69] R. Kase and S. Tsujikawa, *J. Cosmol. Astropart. Phys.* **11** (2020) 032.
- [70] M. S. Linton, R. Crittenden, and A. Poursidou, *J. Cosmol. Astropart. Phys.* **08** (2022) 075.
- [71] N. Dalal, K. Abazajian, E. E. Jenkins, and A. V. Manohar, *Phys. Rev. Lett.* **87**, 141302 (2001).
- [72] W. Zimdahl and D. Pavon, *Phys. Lett. B* **521**, 133 (2001).
- [73] L. P. Chimento, A. S. Jakubi, D. Pavon, and W. Zimdahl, *Phys. Rev. D* **67**, 083513 (2003).
- [74] B. Wang, Y.-G. Gong, and E. Abdalla, *Phys. Lett. B* **624**, 141 (2005).
- [75] L. Amendola, G. Camargo Campos, and R. Rosenfeld, *Phys. Rev. D* **75**, 083506 (2007).
- [76] Z.-K. Guo, N. Ohta, and S. Tsujikawa, *Phys. Rev. D* **76**, 023508 (2007).
- [77] J. Valiviita, E. Majerotto, and R. Maartens, *J. Cosmol. Astropart. Phys.* **07** (2008) 020.
- [78] V. Salvatelli, N. Said, M. Bruni, A. Melchiorri, and D. Wands, *Phys. Rev. Lett.* **113**, 181301 (2014).
- [79] S. Kumar and R. C. Nunes, *Phys. Rev. D* **94**, 123511 (2016).
- [80] E. Di Valentino, A. Melchiorri, and O. Mena, *Phys. Rev. D* **96**, 043503 (2017).
- [81] W. Yang, S. Pan, E. Di Valentino, R. C. Nunes, S. Vagnozzi, and D. F. Mota, *J. Cosmol. Astropart. Phys.* **09** (2018) 019.
- [82] S. Pan, W. Yang, E. Di Valentino, E. N. Saridakis, and S. Chakraborty, *Phys. Rev. D* **100**, 103520 (2019).
- [83] E. Di Valentino, A. Melchiorri, O. Mena, and S. Vagnozzi, *Phys. Dark Universe* **30**, 100666 (2020).
- [84] E. Di Valentino, A. Melchiorri, O. Mena, and S. Vagnozzi, *Phys. Rev. D* **101**, 063502 (2020).
- [85] V. Salzano *et al.*, *J. Cosmol. Astropart. Phys.* **09** (2021) 033.
- [86] V. Poulin, J. L. Bernal, E. D. Kovetz, and M. Kamionkowski, *Phys. Rev. D* **107**, 123538 (2023).
- [87] M. Asghari, J. Beltrán Jiménez, S. Khosravi, and D. F. Mota, *J. Cosmol. Astropart. Phys.* **04** (2019) 042.

- [88] J. Beltrán Jiménez, D. Bettoni, D. Figueruelo, and F. A. Teppa Pannia, *J. Cosmol. Astropart. Phys.* **08** (2020) 020.
- [89] J. Beltrán Jiménez, D. Bettoni, D. Figueruelo, F. A. Teppa Pannia, and S. Tsujikawa, *J. Cosmol. Astropart. Phys.* **03** (2021) 085.
- [90] J. Beltrán Jiménez, D. Bettoni, D. Figueruelo, F. A. Teppa Pannia, and S. Tsujikawa, *Phys. Rev. D* **104**, 103503 (2021).
- [91] A. De Felice, S. Nakamura, and S. Tsujikawa, *Phys. Rev. D* **102**, 063531 (2020).
- [92] N. Aghanim *et al.* (Planck Collaboration), *Astron. Astrophys.* **641**, A5 (2020).
- [93] S. Alam *et al.* (SDSS-III Collaboration), *Astrophys. J. Suppl. Ser.* **219**, 12 (2015).
- [94] D. M. Scolnic *et al.* (Pan-STARRS1 Collaboration), *Astrophys. J.* **859**, 101 (2018).
- [95] T. M. C. Abbott *et al.* (DES Collaboration), *Phys. Rev. D* **98**, 043526 (2018).
- [96] B. F. Schutz and R. Sorkin, *Ann. Phys. (N.Y.)* **107**, 1 (1977).
- [97] J. Brown, *Classical Quantum Gravity* **10**, 1579 (1993).
- [98] A. De Felice, J.-M. Gerard, and T. Suyama, *Phys. Rev. D* **81**, 063527 (2010).
- [99] A. Lewis, A. Challinor, and A. Lasenby, *Astrophys. J.* **538**, 473 (2000).
- [100] C.-P. Ma and E. Bertschinger, *Astrophys. J.* **455**, 7 (1995).
- [101] W. Hu and N. Sugiyama, *Astrophys. J.* **444**, 489 (1995).
- [102] L. Amendola and S. Tsujikawa, *Dark Energy: Theory and Observations* (Cambridge University Press, Cambridge, England, 2015).
- [103] A. Lewis and S. Bridle, *Phys. Rev. D* **66**, 103511 (2002).
- [104] Planck Collaboration, *Astron. Astrophys.* **641**, A6 (2020).
- [105] V. Poulin, T. L. Smith, T. Karwal, and M. Kamionkowski, *Phys. Rev. Lett.* **122**, 221301 (2019).