


2P-wave charmed baryons from QCD sum rulesHui-Min Yang^{1,*} and Hua-Xing Chen^{2,†}¹*School of Physics and Center of High Energy Physics, Peking University, Beijing 100871, China*²*School of Physics, Southeast University, Nanjing 210094, China* (Received 10 November 2023; accepted 22 January 2024; published 26 February 2024)

We conduct an investigation on the $1P$ - and $2P$ -wave charmed baryons using the methods of QCD sum rules and light-cone sum rules within the framework of heavy quark effective theory. Our results suggest that the $\Lambda_c(2910)^+$, $\Lambda_c(2940)^+$, and $\Xi_c(3123)^+$ can be well interpreted as the $2P$ -wave charmed baryons of $J^P = 1/2^-$ and $3/2^-$, belonging to the $SU(3)$ flavor $\mathbf{\bar{3}}_F$ representation. Moreover, the $\Xi_c(3123)^+$ possesses a partner state characterized by $J^P = 1/2^-$, denoted as $\Xi_c(1/2^-, 2P)$. Our analysis predicts its mass and width to be $m_{\Xi_c(1/2^-, 2P)} - m_{\Xi_c(3123)^+} = -18 \pm 7$ MeV and $\Gamma_{\Xi_c(2P, 1/2^-)} = 31_{-27}^{+170}$ MeV, with $m_{\Xi_c(3123)^+} = 3122.9 \pm 1.3$ MeV. We propose to search for it in the $\Xi_c(1/2^-, 2P) \rightarrow \Sigma_c K$ decay channel.

DOI: [10.1103/PhysRevD.109.036032](https://doi.org/10.1103/PhysRevD.109.036032)**I. INTRODUCTION**

The electromagnetic interaction within the deuterium atom contributes to its spectral fine structure, as supported by several studies [1–5]. Similarly, the strong interaction within the singly heavy baryon, constituted by a heavy quark and two light quarks orbiting around the relatively motionless heavy quark, results in the splitting of the hadron spectroscopy. Hence, this system serves as an excellent framework for investigating this phenomenon [6–12] and has attracted significant attentions from both experimental and theoretical communities.

In recent years, substantial strides have been made in the field of heavy baryons. Through both experimental inquiry and theoretical exploration, all ground-state charmed baryons have been accurately identified [13]. Furthermore, various research collaborations have determined the spin-parity quantum numbers of the lowest-lying orbitally excited charmed baryons, namely, $\Lambda_c(2595)$ [14], $\Lambda_c(2625)$ [15], $\Xi_c(2790)$ [16], and $\Xi_c(2815)$ [17]. Subsequently, several newly discovered excited charmed baryons, such as $\Lambda_c(2765)$ [18], $\Lambda_c(2860)$ [19], $\Lambda_c(2880)$ [18], $\Lambda_c(2910)$ [20], $\Lambda_c(2940)$ [19], $\Xi_c(2980)$ [21], $\Xi_c(3055)$ [22], $\Xi_c(3080)$ [23], and $\Xi_c(3123)$ [22], arising from both B -decay and $e^+e^- \rightarrow c\bar{c}$ scattering processes, have been observed by the *BABAR*, *Belle*, and *LHCb* Collaborations. We extract some experimental measurements as follows:

- (i) In 2006 the *BABAR* Collaboration observed two narrow charmed baryons, $\Lambda_c(2880)^+$ and $\Lambda_c(2940)^+$ [24], in the $D^0 p$ mass distribution. Particularly, the *LHCb* Collaboration subsequently confirmed the $\Lambda_c(2940)$ [19], and its decay to $\Sigma_c(2455)^{0,++}\pi^\pm$ was also observed by *Belle* [25]. Its mass and width were measured to be

$$\Lambda_c(2940)^+: M = 2939.8 \pm 5.6 \pm 3.8 \text{ MeV},$$

$$\Gamma = 17.5 \pm 5.2 \pm 5.9 \text{ MeV}; \quad (1)$$

- (ii) In 2007 the *BABAR* Collaboration reported two charm-strange baryons, $\Xi_c(3055)^+$ and $\Xi_c(3123)^+$ [22], in the $\Lambda_c^+ K^- \pi^+$ mass distribution, with the parameters of $\Xi_c(3123)^+$ measured as

$$\Xi_c(3123)^+: M = 3122.9 \pm 1.3 \pm 0.3 \text{ MeV},$$

$$\Gamma = 4.4 \pm 3.4 \pm 1.7 \text{ MeV}; \quad (2)$$

- (iii) In 2022 the *Belle* Collaboration discovered a new structure in the $\Sigma_c(2455)^{0,++}\pi^\pm$ spectrum with a significance of 4.2σ [20]. This state is tentatively named $\Lambda_c(2910)^+$. Its mass and width were measured to be

$$\Lambda_c(2910)^0: M = 2913.8 \pm 5.6 \pm 3.8 \text{ MeV},$$

$$\Gamma = 51.8 \pm 20.0 \pm 18.8 \text{ MeV}. \quad (3)$$

The Particle Data Group (PDG) have assigned accurate J^P values to the $\Lambda_c(2860)(3/2^+)$, $\Lambda_c(2880)(5/2^+)$, $\Lambda_c(2940)(3/2^-)$, and $\Xi_c(2980)(1/2^+)$, while the properties of $\Lambda_c(2765)$, $\Lambda_c(2910)$, $\Xi_c(3055)$, $\Xi_c(3080)$, and $\Xi_c(3123)$ remain uncertain. A rough estimation of their

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TABLE I. The experimental information of Λ_c and Ξ_c baryons. Their masses and decay widths (in units of MeV) are taken from PDG [13]. The mass differences between the Λ_c and Ξ_c baryons are listed in the last column.

Baryons	J^P	Mass	Width	Baryons	J^P	Mass	Width	ΔM
$\Lambda_c(2286)^+$	$1/2^+$	2286.46 ± 0.14	...	$\Xi_c(2468)^0$	$1/2^+$	$2470.88^{+0.34}_{-0.80}$...	$184.42^{+0.37}_{-0.81}$
$\Lambda_c(2595)^+$	$1/2^-$	2592.25 ± 0.28	2.6 ± 0.6	$\Xi_c(2790)^0$	$1/2^-$	2791.8 ± 3.3	10.0 ± 1.1	199.6 ± 3.3
$\Lambda_c(2625)^+$	$3/2^-$	2628.11 ± 0.19	<0.97	$\Xi_c(2815)^0$	$3/2^-$	2819.6 ± 1.2	2.5 ± 0.2	191.5 ± 1.2
$\Lambda_c(2765)^+$?	2766.6 ± 2.4	50	$\Xi_c(2980)^0$	$1/2^+$	2968.0 ± 2.6	20 ± 7	201.4 ± 3.5
$\Lambda_c(2860)^+$	$3/2^+$	$2856.1^{+2.3}_{-5.9}$	$67.6^{+11.8}_{-21.6}$	$\Xi_c(3055)^+$?	3054.2 ± 1.3	17 ± 13	$198.1^{+2.6}_{-6.0}$
$\Lambda_c(2880)^+$	$5/2^+$	2881.53 ± 0.35	5.8 ± 1.1	$\Xi_c(3080)^0$?	3079.9 ± 1.4	5.6 ± 2.2	198.4 ± 1.4
$\Lambda_c(2940)^+$	$3/2^-$	$2939.3^{+1.4}_{-1.5}$	20^{+6}_{-5}	$\Xi_c(3123)^+$?	3122.9 ± 1.3	4 ± 4	$183.6^{+1.9}_{-2.0}$

possible spin-parity values can be made based on the mass discrepancy between the Ξ_c baryons and their corresponding Λ_c baryons. Table I presents all the Λ_c and Ξ_c baryons recorded in PDG, with the final column indicating an approximate mass difference of 200 MeV between Ξ_c and Λ_c baryons. Hence, it is logical to assign $J^P = 1/2^+$, $3/2^+$, $5/2^+$, and $3/2^-$ to the $\Lambda_c(2765)$, $\Xi_c(3055)$, $\Xi_c(3080)$, and $\Xi_c(3123)$ baryons, respectively. Considering that the $\Lambda_c(2910)$ recently observed by Belle likely corresponds to a companion state of $\Lambda_c(2940)$, it follows that there may also exist a corresponding missing companion state of $\Xi_c(3123)$.

The singly charmed baryon system has attracted numerous experimentalists and theorists to study them. However, it remains a challenging issue to fully comprehend their internal structures. Various theoretical methods and models have been applied in this field, including various quark models [26–41], the chiral perturbation theory [42–47], the chiral effective field theory [48–50], various molecular interpretations [51–57], the Regge trajectory [35,58], the 3P_0 model [59–61], the relativistic flux tube model [62,63], QCD sum rules [64–73], and the lattice QCD [74–77]c. Their production and decay properties have been extensively studied in Refs. [59,60,78,79], and various reviews [80–84] provide insights into the recent progress.

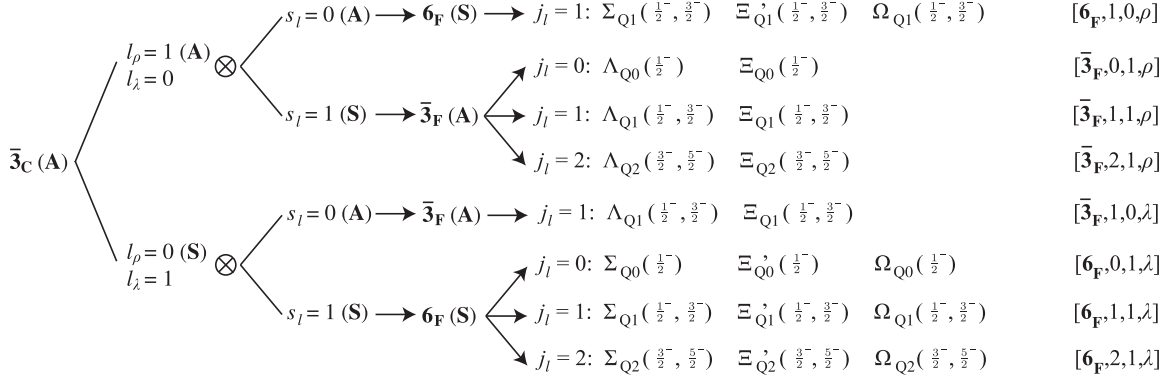
The higher states $\Lambda_c(2910)^+$ and $\Lambda_c(2940)^+$ among the Λ_c family have sparked considerable interest among theorists due to certain perplexing issues. The quantum number $J^P = 3/2^-$ is favored for $\Lambda_c(2940)^+$ according to the LHCb result, while its mass is roughly 60 MeV smaller than the expected $\Lambda_c(3/2^-, 2P)$ state in the quark models [35,36,63]. Furthermore, the mass of its partner state $\Lambda_c(1/2^-, 2P)$ is only slightly lighter than that of $\Lambda_c(3/2^-, 2P)$, suggesting that their masses are expected to exceed 3 GeV. Alternative interpretations of $\Lambda_c(2940)^+$ could involve the introduction of the D^*N channel contribution, as proposed in Refs. [41,57]. These studies propose that the $\Lambda_c(2940)^+$ is probably the isoscalar D^*N molecule encompassing both spin- $\frac{1}{2}$ and spin- $\frac{3}{2}$ structures. However, the higher mass of the $\Lambda_c(1/2^-)$ state leads to a mass inversion. Consequently, it is crucial to verify the quantum numbers of $\Lambda_c(2910)$ and $\Lambda_c(2940)$ through further experimental measurements and theoretical research.

In the present study we primarily focus on the analyses of the mass spectral and decay properties of the $\Lambda_c(2910)$, $\Lambda_c(2940)$, and $\Xi_c(3123)$ charmed baryons using the QCD sum rules and light-cone sum rules within the heavy quark effective theory (HQET) framework. We examine their S - and D -wave decays into the ground-state charmed baryons with light pseudoscalar or vector mesons, and calculate the relevant partial widths. The results obtained further support the identification of $\Lambda_c(2910)$, $\Lambda_c(2940)$, and $\Xi_c(3123)$ as $2P$ -wave charmed baryons of the $SU(3)$ flavor $\bar{\mathbf{3}}_F$. Furthermore, based on our previous work [85], we designated the $\Lambda_c(2595)$, $\Lambda_c(2625)$, $\Xi_c(2790)$, and $\Xi_c(2815)$ as the $1P$ -wave charmed baryons. Considering that the P -wave interpolating currents derived from Ref. [86] can couple into both $1P$ - and $2P$ -wave states, it is reasonable to regard the $\Lambda_c(2910)$, $\Lambda_c(2940)$, and $\Xi_c(3123)$ as the first radial excited states of the $\Lambda_c(2595)$, $\Lambda_c(2625)$, and $\Xi_c(2815)$. Additionally, it is noteworthy that a $2P$ -wave charmed baryon, the partner state of the $\Xi_c(3123)$, remains undiscovered. In this paper we shall calculate its mass and decay properties.

This paper is organized as follows. In Sec. II we briefly introduce our notations and apply the QCD sum rule method to calculate the masses of $\Lambda_c(2910)$, $\Lambda_c(2940)$, and $\Xi_c(3123)$ as $2P$ -wave charmed baryons of the $SU(3)$ flavor $\bar{\mathbf{3}}_F$. The obtained parameters are further used to study their decay properties through the light-cone sum rule method in Sec. III. In Sec. IV we discuss the results and conclude this paper.

II. MASS ANALYSES FROM QCD SUM RULES

A charmed baryon consists of a heavy charm quark and two light up/down/strange quarks. Its internal structure encompasses various properties, such as the color, flavor, spin, and orbital degrees of freedom. Notably, the orbital excitation of a P -wave charmed baryon can occur between the charmed quark and the light quarks, and it can also occur within the two light quarks. The former is known as the λ -mode excitation, characterized by $l_\lambda = 1$ and $l_\rho = 0$, while the latter is termed as the ρ -mode excitation with $l_\lambda = 0$ and $l_\rho = 1$, as depicted in Fig. 1. When investigating the singly charmed baryon, it is crucial to consider the properties of the two light up/down/strange quarks:

FIG. 1. P -wave heavy baryons belonging to the $SU(3)$ flavor $\bar{\mathbf{3}}_F$ representation.

- (i) Their color structure is antisymmetric ($\bar{\mathbf{3}}_C$).
- (ii) Their flavor structure is either symmetric ($\mathbf{6}_F$) or antisymmetric ($\bar{\mathbf{3}}_F$).
- (iii) Their spin structure is either symmetric ($s_l = 1$) or antisymmetric ($s_l = 0$).
- (iv) Their orbital structure is either symmetric (λ -mode) or antisymmetric (ρ -mode).

Applying the Pauli principle to the two light quarks, we can categorize the P -wave charmed baryons into eight multiplets, denoted as $[F(\text{flavor}), j_l, s_l, \rho/\lambda]$. In this notation $j_l = l_\lambda \otimes l_\rho \otimes s_l$ represents the total angular momentum of the light components. Each multiplet encompasses one or two baryons with the total angular momenta given by $j = j_l \otimes s_c = |j_l \pm 1/2|$. Within the framework of QCD sum rules, we can construct the corresponding interpolating currents $J_{j,P,F,j_l,s_l,\rho/\lambda}^{\alpha_1 \dots \alpha_{j-1/2}}$, which couple to the charmed baryons belonging to the $[F, j_l, s_l, \rho/\lambda]$ multiplet through

$$\langle 0 | J_{j,P,F,j_l,s_l,\rho/\lambda}^{\alpha_1 \dots \alpha_{j-1/2}} | j, P, F, j_l, s_l, \rho/\lambda \rangle = f_{F,j_l,s_l,\rho/\lambda} u^{\alpha_1 \dots \alpha_{j-1/2}},$$

with $f_{F,j_l,s_l,\rho/\lambda}$ the decay constant.

As shown in Fig. 1, there are four multiplets belonging to the $SU(3)$ flavor $\bar{\mathbf{3}}_F$ representation. In this work we assume that the $\Lambda_c(2910)$, $\Lambda_c(2940)$, and $\Xi_c(3123)$ are the $2P$ -wave charmed baryons of $J^P = 1/2^-$ and $J^P = 3/2^-$ belonging to the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublet. Accordingly, we use the following currents to study them [86]:

$$J_{1/2,-,\bar{\mathbf{3}}_F,1,1,\rho} = i\epsilon_{abc} ([D_t^\mu q^{aT}] C \gamma_t^\nu q^b - q^{aT} C \gamma_t^\nu [D_t^\mu q^b]) \sigma^{\mu\nu} h_v^c, \quad (4)$$

$$\begin{aligned} J_{3/2,-,\bar{\mathbf{3}}_F,1,1,\rho}^\alpha &= i\epsilon_{abc} ([D_t^\mu q^{aT}] C \gamma_t^\nu q^b - q^{aT} C \gamma_t^\nu [D_t^\mu q^b]) \\ &\times \left(g_t^{\alpha\mu} \gamma_t^\nu \gamma_5 - g_t^{\alpha\nu} \gamma_t^\mu \gamma_5 - \frac{1}{3} \gamma_t^\alpha \gamma_t^\mu \gamma_t^\nu \gamma_5 + \frac{1}{3} \gamma_t^\alpha \gamma_t^\nu \gamma_t^\mu \gamma_5 \right) h_v^c. \end{aligned} \quad (5)$$

Here, $a \dots c$ represent color indices, C denotes the charge-conjugation operator, $D_t^\mu = D^\mu - v \cdot D v^\mu$, $\gamma_t^\nu = \gamma^\nu - v \not{x}^\nu$,

and $g_t^{\rho\beta} = g^{\rho\beta} - v^\rho v^\beta$. The covariant derivative operator has been explicitly added to these currents.

A. The sum rules at the leading order ($m_Q \rightarrow \infty$)

In this study we assume that the current $J_{1/2,-,\bar{\mathbf{3}}_F,1,1,\rho}$ can couple into both the $1P$ -wave charmed baryons $\Lambda_c(1/2^-, 1P)$ and $\Xi_c(1/2^-, 1P)$ as well as the $2P$ -wave excitations $\Lambda_c(1/2^-, 2P)$ and $\Xi_c(1/2^-, 2P)$. Similarly, the current $J_{3/2,-,\bar{\mathbf{3}}_F,1,1,\rho}$ can also couple into both $1P$ - and $2P$ -wave charmed baryons with $J^P = 3/2^-$. If the two light quarks inside a charmed baryon are the up and down quarks, the currents are associated with the Λ_c ; if one of the two light quarks is a strange quark, the currents correspond to the Ξ_c . Then, the relation between the state and the relevant interpolating field is

$$\begin{aligned} \langle 0 | J_{1/2,-,\bar{\mathbf{3}}_F,1,1,\rho}(x) | \Lambda_c(1/2^-, 1P/2P) \rangle \\ = f_{\Lambda_c(1/2^-, 1P/2P)} u(x), \end{aligned} \quad (6)$$

$$\begin{aligned} \langle 0 | J_{1/2,-,\bar{\mathbf{3}}_F,1,1,\rho}(x) | \Xi_c(1/2^-, 1P/2P) \rangle \\ = f_{\Xi_c(1/2^-, 1P/2P)} u(x), \end{aligned} \quad (7)$$

$$\begin{aligned} \langle 0 | J_{3/2,-,\bar{\mathbf{3}}_F,1,1,\rho}(x) | \Lambda_c(3/2^-, 1P/2P) \rangle \\ = f_{\Lambda_c(3/2^-, 1P/2P)} u^\alpha(x), \end{aligned} \quad (8)$$

$$\begin{aligned} \langle 0 | J_{3/2,-,\bar{\mathbf{3}}_F,1,1,\rho}(x) | \Xi_c(3/2^-, 1P/2P) \rangle \\ = f_{\Xi_c(3/2^-, 1P/2P)} u^\alpha(x), \end{aligned} \quad (9)$$

where $f_{\Lambda_c(\frac{1}{2}^-, 1P/2P)}$, $f_{\Xi_c(\frac{1}{2}^-, 1P/2P)}$, $f_{\Lambda_c(\frac{3}{2}^-, 1P/2P)}$, and $f_{\Xi_c(\frac{3}{2}^-, 1P/2P)}$ are the decay constants, and $u(x)$ and $u^\alpha(x)$ denote the Dirac and Rarita-Schwinger spinors. These currents can be used to construct the two-point correlation function:

$$\begin{aligned}
 & \Pi_{j,P,F,j_1,s_1,\rho/\lambda}^{\alpha_1 \cdots \alpha_{j-1/2}, \beta_1 \cdots \beta_{j-1/2}}(\omega) \\
 &= i \int d^4x e^{ikx} \langle 0 | T [J_{j,P,F,j_1,s_1,\rho/\lambda}^{\alpha_1 \cdots \alpha_{j-1/2}}(x) \bar{J}_{j,P,F,j_1,s_1,\rho/\lambda}^{\beta_1 \cdots \beta_{j-1/2}}(0)] | 0 \rangle \\
 &= \mathbb{S} [g_r^{\alpha_1 \beta_1} \cdots g_r^{\alpha_{j-1/2} \beta_{j-1/2}}] \frac{1 + \not{x}}{2} \Pi_{j,P,F,j_1,s_1,\rho/\lambda}(\omega), \quad (10)
 \end{aligned}$$

where $\omega = v \cdot k$ is external off-shell energy and \mathbb{S} denotes symmetrization and subtracting the trace terms in the sets $(\alpha_1 \cdots \alpha_{j-1/2})$ and $(\beta_1 \cdots \beta_{j-1/2})$.

We employ the four distinct currents for QCD sum rule analyses: $J_{1/2,-,\bar{3}_F,1,1,\rho}$, $J_{3/2,-,\bar{3}_F,1,1,\rho}$, $J_{1/2,-,\bar{3}_F,1,0,\lambda}$, and $J_{3/2,-,\bar{3}_F,1,0,\lambda}$. As an illustration of our methodology, we choose the current $J_{1/2,-,\bar{3}_F,1,1,\rho}$ to elucidate how to extract the masses of $\Lambda_c(\frac{1}{2}^-, 1P)$ and $\Lambda_c(\frac{1}{2}^-, 2P)$. We can write Eq. (10) at the hadron level as

$$\begin{aligned}
 & \Pi_{1/2,-,\bar{3}_F,1,1,\rho}(\omega, \omega') \\
 &= \frac{f_{\Lambda_c(\frac{1}{2}^-, 1P)}^2}{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega} + \frac{f_{\Lambda_c(\frac{1}{2}^-, 2P)}^2}{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega'} + \text{higher states.} \quad (11)
 \end{aligned}$$

In this equation $\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P/2P)}$ is defined to be $\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P/2P)} \equiv \lim_{m_c \rightarrow \infty} (m_{\Lambda_c(1/2^-, 1P/2P)} - m_c)$. To depress the influence of higher-order power, a Borel transformation is performed as

$$\begin{aligned}
 \Pi_{1/2,-,\bar{3}_F,1,1,\rho} &= f_{\Lambda_c(\frac{1}{2}^-, 1P)}^2 e^{-\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)}/T} \\
 &+ f_{\Lambda_c(\frac{1}{2}^-, 2P)}^2 e^{-\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)}/T}. \quad (12)
 \end{aligned}$$

At the quark-gluon level Eq. (10) can be calculated by the method of operator product expansion (OPE). We first insert Eq. (4) into Eq. (10), and then perform the Borel transformation:

$$\begin{aligned}
 & \Pi_{1/2,-,\bar{3}_F,1,1,\rho}(\omega'_c, T) \\
 &= \int_0^{\omega'_c} e^{-\omega/T} \left(\frac{3\omega^7}{35\pi^4} - \frac{\langle g_s^2 GG \rangle \omega^3}{48\pi^4} \right) d\omega \\
 &\quad - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle \bar{q} q \rangle}{4} - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{q} \sigma G q \rangle}{64T^2} \\
 &= \int_0^{\omega'_c} e^{-\omega/T} \left(\frac{3\omega^7}{35\pi^4} - \frac{\langle g_s^2 GG \rangle \omega^3}{48\pi^4} \right) d\omega \\
 &\quad - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle \bar{q} q \rangle}{4} - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{q} \sigma G q \rangle}{64T^2} \\
 &\quad + \int_{\omega_c}^{\omega'_c} e^{-\frac{\omega}{T}} \left(\frac{3\omega^7}{35\pi^4} - \frac{\langle g_s^2 GG \rangle \omega^3}{48\pi^4} \right) d\omega \\
 &= \Pi_{\Lambda_c(\frac{1}{2}^-, 1P)}(\omega_c, T) + \Pi_{\Lambda_c(\frac{1}{2}^-, 2P)}(\omega_c, \omega'_c, T). \quad (13)
 \end{aligned}$$

Finally, we differentiate Eqs. (11) and (13) with respect to $(-1/T)$ to obtain $\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P/2P)}$ and $f_{\Lambda_c(\frac{1}{2}^-, 1P/2P)}$:

$$\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} = \frac{\frac{\partial}{\partial(-1/T)} \Pi_{\Lambda_c(\frac{1}{2}^-, 1P)}}{\Pi_{\Lambda_c(\frac{1}{2}^-, 1P)}}, \quad (14)$$

$$f_{\Lambda_c(\frac{1}{2}^-, 1P)} = \sqrt{\Pi_{\Lambda_c(\frac{1}{2}^-, 1P)} e^{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)}/T}}, \quad (15)$$

$$\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} = \frac{\frac{\partial}{\partial(-1/T)} \Pi_{\Lambda_c(\frac{1}{2}^-, 2P)}}{\Pi_{\Lambda_c(\frac{1}{2}^-, 2P)}}, \quad (16)$$

$$f_{\Lambda_c(\frac{1}{2}^-, 2P)} = \sqrt{\Pi_{\Lambda_c(\frac{1}{2}^-, 2P)} e^{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)}/T}}. \quad (17)$$

Quark and gluon condensates, in fact, serve as tangible manifestations of the distinctive nature of QCD vacuum properties in contrast to the perturbative vacuum. The QCD physical vacuum is wholly shaped by nonperturbative interactions, consistently interacting with quarks and gluons, thus exerting influence on the propagation of these particles. This vacuum interaction with quarks or gluons entails zero momentum transfer and manifests large-distance behavior, defined by a scale denoted as Λ_{QCD} . The reliability of vacuum condensates, up to dimension five ($D = 5$), is firmly anchored in empirical observations. Regarding dimension six ($D = 6$) three-gluon condensate, a correlation can be established with dimension four ($D = 4$) two-gluon condensate within the diluted instanton gas model. Higher-dimensional condensates can be deduced by amalgamating fundamental dimension condensates, albeit with a subsequent escalation in numerical uncertainty. To ensure result reliability, the OPE must confirm the predominance of contributions up to $D = 6$, relegating those beyond to minor corrections. Undoubtedly, the discussion about the convergence of the OPE emphasizes its crucial significance. In the calculations we work at the renormalization scale of 1 GeV and use the condensates and other parameters with the following values [87–93]:

$$\begin{aligned}
 \langle \bar{q} q \rangle &= -(0.24 \pm 0.01 \text{ GeV})^3, \\
 \langle \bar{s} s \rangle &= (0.8 \pm 0.1) \times \langle \bar{q} q \rangle, \\
 \langle g_s \bar{q} \sigma G q \rangle &= M_0^2 \times \langle \bar{q} q \rangle, \\
 \langle g_s \bar{s} \sigma G s \rangle &= M_0^2 \times \langle \bar{s} s \rangle, \\
 M_0^2 &= 0.8 \text{ GeV}^2, \\
 \langle g_s^2 GG \rangle &= (0.48 \pm 0.14) \text{ GeV}^4, \\
 m_c &= 1.275 \pm 0.035 \text{ GeV}, \\
 m_s &= 128_{-4}^{+12} \text{ MeV}. \quad (18)
 \end{aligned}$$

Equations (14)–(17) imply that the mass and decay constant depend on three free parameters: the threshold values ω_c and ω'_c as well as the Borel mass T . There are three criteria to restrict these three parameters: (a) the convergence (CVG) of OPE requires that the high-order power corrections ($D = 8$ and $D = 10$) are less than 10%; (b) we require the pole contribution (PC) of OPE to be

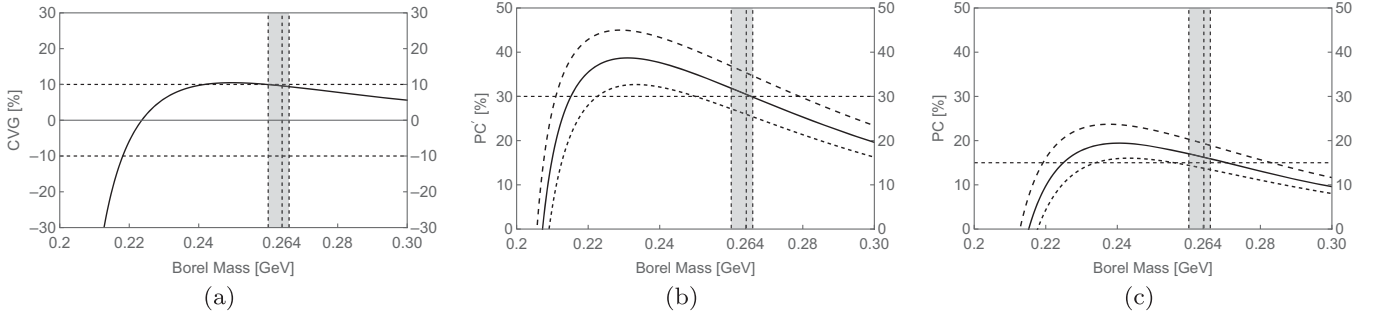


FIG. 2. Variations of CVG (left), PC' (middle), and PC (right), as defined in Eqs. (19)–(21), with respect to Borel mass T , when $J_{1/2,-\bar{3}_F,1,1,\rho}$ is adopted. In the middle panel the short-dashed, solid, and long-dashed curves are obtained by fixing $\omega'_c = 1.47$ GeV, 1.57 GeV, and 1.67 GeV, respectively. In the right panel the short-dashed, solid, and long-dashed curves are gained by setting $\omega_c = 1.10$ GeV, 1.20 GeV, and 1.30 GeV, respectively.

larger than 15% and 30% with respect to the thresholds ω_c and ω'_c , respectively; and (c) the mass dependence on these three parameters is sufficiently stable:

$$CVG \equiv \left| \frac{\Pi_{1/2,-\bar{3}_F,1,1,\rho}^{\text{high-order}}(T)}{\Pi_{1/2,-\bar{3}_F,1,1,\rho}(\infty, T)} \right| \leq 10\%, \quad (19)$$

$$PC \equiv \frac{\Pi_{1/2,-\bar{3}_F,1,1,\rho}(\omega'_c, T)}{\Pi_{1/2,-\bar{3}_F,1,1,\rho}(\infty, T)} \geq 30\%, \quad (20)$$

$$PC' \equiv \frac{\Pi_{1/2,-\bar{3}_F,1,1,\rho}(\omega_c, T)}{\Pi_{1/2,-\bar{3}_F,1,1,\rho}(\infty, T)} \geq 15\%, \quad (21)$$

where $\Pi_{1/2,-\bar{3}_F,1,1,\rho}^{\text{high-order}}(T)$ is used to denote the high-order power corrections

$$\Pi_{1/2,-\bar{3}_F,1,1,\rho}^{\text{high-order}}(T) = -\frac{\langle g_s \bar{q} \sigma G q \rangle \langle \bar{q} q \rangle}{4} - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle \bar{q} g_s \sigma G q \rangle}{64T^2}. \quad (22)$$

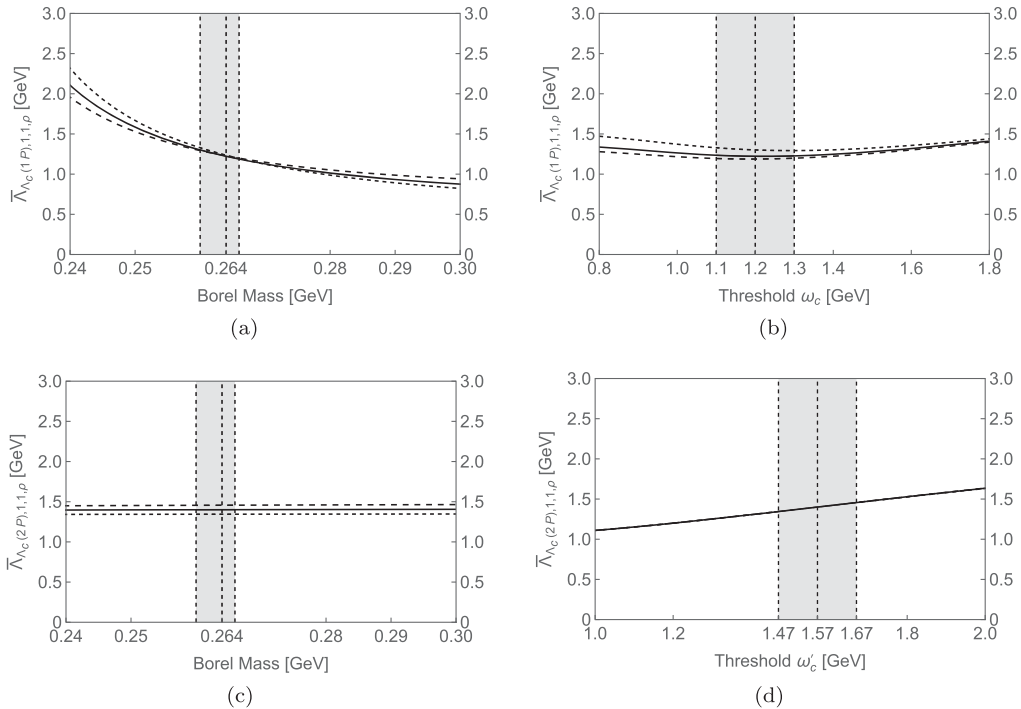


FIG. 3. Variations of $\bar{\Lambda}_{\Lambda_c(1P),1,1,\rho}$ with respect to (a) the Borel mass T and (b) the threshold value ω_c , as well as variations of $\bar{\Lambda}_{\Lambda_c(2P),1,1,\rho}$ with respect to (c) the Borel mass T and (d) the threshold value ω'_c , when $J_{1/2,-\bar{3}_F,1,1,\rho}$ is adopted. In (a) the short-dashed, solid, and long-dashed curves are obtained by fixing $\omega_c = 1.10$ GeV, 1.20 GeV, and 1.30 GeV, respectively. In (c) the short-dashed, solid, and long-dashed curves are obtained by fixing $\omega'_c = 1.47$ GeV, 1.57 GeV, and 1.67 GeV, respectively. In (b) and (d) the short-dashed, solid, and long-dashed curves are gained by setting $T = 0.260$ GeV, 0.264 GeV, and 0.266 GeV, respectively.

We determine a Borel window $T_{\min} \leq T \leq T_{\max}$ with fixed values of ω'_c and ω_c . These parameters, ω'_c and ω_c , are two independent variables setting at 1.57 GeV and 1.20 GeV, respectively, in order to accurately accommodate the masses of $\Lambda_c(1/2^-, 1P)$ and $\Lambda_c(1/2^-, 2P)$. In this study we derive an interval of $0.260 \text{ GeV} \leq T \leq 0.266 \text{ GeV}$ for $\omega'_c = 1.57 \text{ GeV}$ and $\omega_c = 1.20 \text{ GeV}$.

To provide a clear visualization, we show the variations of CVG, PC', and PC with respect to the Borel mass T in Fig. 2. Additionally, the fluctuations of $\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)}$ and $\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)}$ are exhibited with respect to T , ω_c , and ω'_c in Fig. 3. We find that these curves remain sufficiently stable inside the regions $0.260 \text{ GeV} \leq T \leq 0.266 \text{ GeV}$, $1.10 \text{ GeV} \leq \omega_c \leq 1.30 \text{ GeV}$, and $1.47 \text{ GeV} \leq \omega'_c \leq 1.67 \text{ GeV}$, where the associated numerical results are obtained:

$$\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} = 1.22 \pm 0.06 \text{ GeV}, \quad (23)$$

$$f_{\Lambda_c(\frac{1}{2}^-, 1P)} = 0.043 \pm 0.008 \text{ GeV}^4, \quad (24)$$

$$\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} = 1.40 \pm 0.07 \text{ GeV}, \quad (25)$$

$$f_{\Lambda_c(\frac{1}{2}^-, 2P)} = 0.057 \pm 0.016 \text{ GeV}^4. \quad (26)$$

B. The sum rules at the $\mathcal{O}(1/m_Q)$ order

In this subsection we perform analysis at the $\mathcal{O}(1/m_Q)$ order. To do this we utilize the following Lagrangian of HQET:

$$\mathcal{L} = \bar{h}_v i v \cdot D h_v + \frac{1}{2m_Q} \mathcal{K} + \frac{1}{2m_Q} \mathcal{S}. \quad (27)$$

In this Lagrangian we designate \mathcal{K} to represent the operator of nonrelativistic kinetic energy,

$$\mathcal{K} = \bar{h}_v (iD_t)^2 h_v, \quad (28)$$

and use \mathcal{S} to denote the Pauli term describing the chromomagnetic interaction,

$$\mathcal{S} = \frac{g}{2} C_{\text{mag}}(m_Q/\mu) \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v, \quad (29)$$

where $C_{\text{mag}}(M_Q/\mu) = [\alpha_s(m_Q)/\alpha_s(\mu)]^{3/\beta_0}$ and $\beta_0 = 11 - 2n_f/3$.

The two pole terms arise at the $\mathcal{O}(1/m_Q)$ order when we perform the Taylor expansion for Eq. (11), with $\delta m_{\Lambda_c(\frac{1}{2}^-, 1P)}$, $\delta m_{\Lambda_c(\frac{1}{2}^-, 2P)}$, $\delta f_{\Lambda_c(\frac{1}{2}^-, 1P)}$, and $\delta f_{\Lambda_c(\frac{1}{2}^-, 2P)}$ representing the corrections to $m_{\Lambda_c(\frac{1}{2}^-, 1P)}$, $m_{\Lambda_c(\frac{1}{2}^-, 2P)}$, $f_{\Lambda_c(\frac{1}{2}^-, 1P)}$, and $f_{\Lambda_c(\frac{1}{2}^-, 2P)}$, respectively:

$$\begin{aligned} \Pi(\omega, \omega')_{\text{pole}} &= \frac{(f_{\Lambda_c(\frac{1}{2}^-, 1P)} + \delta f_{\Lambda_c(\frac{1}{2}^-, 1P)})^2}{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} + \delta m_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega} + \frac{(f_{\Lambda_c(\frac{1}{2}^-, 2P)} + \delta f_{\Lambda_c(\frac{1}{2}^-, 2P)})^2}{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} + \delta m_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega'} \\ &= \frac{f_{\Lambda_c(\frac{1}{2}^-, 1P)}^2}{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega} - \frac{\delta m_{\Lambda_c(\frac{1}{2}^-, 1P)} f_{\Lambda_c(\frac{1}{2}^-, 1P)}^2}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega)^2} + \frac{2f_{\Lambda_c(\frac{1}{2}^-, 1P)} \delta f_{\Lambda_c(\frac{1}{2}^-, 1P)}}{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega} \\ &\quad + \frac{f_{\Lambda_c(\frac{1}{2}^-, 2P)}^2}{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega'} - \frac{\delta m_{\Lambda_c(\frac{1}{2}^-, 2P)} f_{\Lambda_c(\frac{1}{2}^-, 2P)}^2}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega')^2} + \frac{2f_{\Lambda_c(\frac{1}{2}^-, 2P)} \delta f_{\Lambda_c(\frac{1}{2}^-, 2P)}}{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega'}, \end{aligned} \quad (30)$$

where $\delta m_{\Lambda_c(\frac{1}{2}^-, 1P)}$ and $\delta m_{\Lambda_c(\frac{1}{2}^-, 2P)}$ can be calculated through the following three-point correlation functions:

$$\begin{aligned} \delta_O \Pi_{j,P,F,j_1,s_1,\rho/\lambda}^{\alpha_1 \dots \alpha_{j-1/2} \beta_1 \dots \beta_{j-1/2}} &= i^2 \int d^4x d^4y e^{ik \cdot x - ik' \cdot y} \langle 0 | T [J_{i,P,F,j_1,s_1,\rho/\lambda}^{\alpha_1 \dots \alpha_{j-1/2}}(x) O(0) J_{j_1,s_1,\rho/\lambda}^{\beta_1 \dots \beta_{j-1/2}}(y)] | 0 \rangle \\ &= \mathbb{S} [g_t^{\alpha_1 \beta_1} \dots g_t^{\alpha_{j-1/2} \beta_{j-1/2}}] \frac{1 + \not{v}}{2} \delta_O \Pi_{j,P,F,j_1,s_1,\rho/\lambda}, \end{aligned} \quad (31)$$

with $O = \mathcal{K}$ or \mathcal{S} .

We can write Eq. (31) at the hadron level as

$$\begin{aligned} \delta_{\mathcal{K}} \Pi &= \frac{f_{\Lambda_c(\frac{1}{2}^-, 1P)}^2 K_{\Lambda_c(\frac{1}{2}^-, 1P)}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega_1)(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega'_1)} + \frac{f_{\Lambda_c(\frac{1}{2}^-, 2P)}^2 K_{\Lambda_c(\frac{1}{2}^-, 2P)}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega_2)(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega'_2)} \\ &\quad + \frac{f_{\Lambda_c(\frac{1}{2}^-, 1P)} f_{\Lambda_c(\frac{1}{2}^-, 2P)} K'_{\Lambda_c(\frac{1}{2}^-)}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega_1)(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega'_2)} + \frac{f_{\Lambda_c(\frac{1}{2}^-, 2P)} f_{\Lambda_c(\frac{1}{2}^-, 1P)} K'_{\Lambda_c(\frac{1}{2}^-)}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega_2)(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega'_1)} + \dots, \end{aligned} \quad (32)$$

$$\begin{aligned} \delta_S \Pi = & \frac{d_M f_{\Lambda_c(\frac{1}{2}^-, 1P)}^2 \Sigma_{\Lambda_c(\frac{1}{2}^-, 1P)}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega_1)(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega'_1)} + \frac{d_M f_{\Lambda_c(\frac{1}{2}^-, 2P)}^2 \Sigma_{\Lambda_c(\frac{1}{2}^-, 2P)}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega_2)(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega'_2)} \\ & + \frac{d_M f_{\Lambda_c(\frac{1}{2}^-, 1P)} f_{\Lambda_c(\frac{1}{2}^-, 2P)} \Sigma'_{\Lambda_c(\frac{1}{2}^-)}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega_1)(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega'_2)} + \frac{d_M f_{\Lambda_c(\frac{1}{2}^-, 2P)} f_{\Lambda_c(\frac{1}{2}^-, 1P)} \Sigma'_{\Lambda_c(\frac{1}{2}^-)}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega_2)(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega'_1)} + \dots, \end{aligned} \quad (33)$$

where $K_{\Lambda_c(\frac{1}{2}^-, 1P/2P)}$, $K'_{\Lambda_c(\frac{1}{2}^-)}$, $\Sigma_{\Lambda_c(\frac{1}{2}^-, 1P/2P)}$, and $\Sigma'_{\Lambda_c(\frac{1}{2}^-)}$ are the matrix elements formed from the operators \mathcal{K} and \mathcal{S} sandwiched between the $\Lambda_c(\frac{1}{2}^-, 1P/2P)$ states:

$$\begin{pmatrix} \langle \Lambda_c(\frac{1}{2}^-, 1P) | \mathcal{K} | \Lambda_c(\frac{1}{2}^-, 1P) \rangle & \langle \Lambda_c(\frac{1}{2}^-, 1P) | \mathcal{K} | \Lambda_c(\frac{1}{2}^-, 2P) \rangle \\ \langle \Lambda_c(\frac{1}{2}^-, 2P) | \mathcal{K} | \Lambda_c(\frac{1}{2}^-, 1P) \rangle & \langle \Lambda_c(\frac{1}{2}^-, 2P) | \mathcal{K} | \Lambda_c(\frac{1}{2}^-, 2P) \rangle \end{pmatrix} = \begin{pmatrix} K_{\Lambda_c(\frac{1}{2}^-, 1P)} & K'_{\Lambda_c(\frac{1}{2}^-)} \\ K'_{\Lambda_c(\frac{1}{2}^-)} & K_{\Lambda_c(\frac{1}{2}^-, 2P)} \end{pmatrix}, \quad (34)$$

$$\begin{pmatrix} \langle \Lambda_c(\frac{1}{2}^-, 1P) | \mathcal{S} | \Lambda_c(\frac{1}{2}^-, 1P) \rangle & \langle \Lambda_c(\frac{1}{2}^-, 1P) | \mathcal{S} | \Lambda_c(\frac{1}{2}^-, 2P) \rangle \\ \langle \Lambda_c(\frac{1}{2}^-, 2P) | \mathcal{S} | \Lambda_c(\frac{1}{2}^-, 1P) \rangle & \langle \Lambda_c(\frac{1}{2}^-, 2P) | \mathcal{S} | \Lambda_c(\frac{1}{2}^-, 2P) \rangle \end{pmatrix} = C_{\text{mag}} \begin{pmatrix} d_M \Sigma_{\Lambda_c(\frac{1}{2}^-, 1P)} & d_M \Sigma'_{\Lambda_c(\frac{1}{2}^-)} \\ d_M \Sigma'_{\Lambda_c(\frac{1}{2}^-)} & d_M \Sigma_{\Lambda_c(\frac{1}{2}^-, 2P)} \end{pmatrix}, \quad (35)$$

where $d_M = d_{j,j_l}$, $d_{j_l-1/2,j_l} = 2j_l + 2$, and $d_{j_l+1/2,j_l} = -2j_l$.

During the computations, it is observed that the nondiagonal elements are not equal to zero. The ratio of the nondiagonal elements to the diagonal elements is approximately 15% for $|K'_{\Lambda_c(\frac{1}{2}^-)}/K_{\Lambda_c(\frac{1}{2}^-, 2P)}|$ and about 20% for $|d_M \Sigma'_{\Lambda_c(\frac{1}{2}^-)}/d_M \Sigma_{\Lambda_c(\frac{1}{2}^-, 2P)}|$. This observation suggests that the nondiagonal elements exert a comparatively minor influence on the mass compared to the diagonal elements. Therefore, the contribution of nondiagonal elements can be considered negligible. We note that the term \mathcal{S} can induce a mass splitting within the same doublet, and thus, one can obtain the masses of $\Lambda_c(\frac{1}{2}^-, 1P/2P)$ through the following expressions:

$$\delta m = -\frac{1}{4m_c} (K + d_{j,j_l} C_{\text{mag}} \Sigma), \quad (36)$$

$$m = m_c + \bar{\Lambda} + \delta m, \quad (37)$$

where m_c is the charmed quark mass in the $\overline{\text{MS}}$ scheme, $\bar{\Lambda}$ is the sum rule result at the leading order, and δm is the sum rule result at the $\mathcal{O}(1/m_Q)$ order.

We can also calculate Eq. (31) at the quark-gluon level using the method of OPE. After inserting Eqs. (4), (28), and (29) into Eq. (31) and making a double Borel transformation for ω_1 , ω'_1 , ω_2 , and ω'_2 , we obtain the four Borel parameters T_1 , T_2 , T_3 , and T_4 . We choose $T_1 = T_2 = T_3 = T_4 = 2T$, and obtain the following sum rules for $K_{\Lambda_c(\frac{1}{2}^-, 1P/2P)}$ and $\Sigma_{\Lambda_c(\frac{1}{2}^-, 1P/2P)}$:

$$\begin{aligned} & f_{\Lambda_c(\frac{1}{2}^-, 1P)}^2 K_{\Lambda_c(\frac{1}{2}^-, 1P)} e^{-\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)}/T} + f_{\Lambda_c(\frac{1}{2}^-, 2P)}^2 K_{\Lambda_c(\frac{1}{2}^-, 2P)} e^{-\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)}/T} + 2f_{\Lambda_c(\frac{1}{2}^-, 1P)} f_{\Lambda_c(\frac{1}{2}^-, 2P)} K'_{\Lambda_c(\frac{1}{2}^-)} e^{-\frac{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} + \bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)}}{2T}} \\ & = \int_0^{\omega'_c} \left[-\frac{8\omega^9}{105\pi^4} + \frac{9\langle g_s^2 GG \rangle \omega^5}{240\pi^4} \right] e^{-\omega/T} d\omega - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{q} \sigma G q \rangle}{16} - \frac{\langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle \langle g_s^2 GG \rangle}{256T^2} - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{q} \sigma G q \rangle \langle g_s^2 GG \rangle}{4096T^4} \\ & = \int_0^{\omega_c} \left[-\frac{8\omega^9}{105\pi^4} + \frac{9\langle g_s^2 GG \rangle \omega^5}{240\pi^4} \right] e^{-\omega/T} d\omega - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{q} \sigma G q \rangle}{16} - \frac{\langle \bar{q} q \rangle \langle g_s \bar{q} \sigma G q \rangle \langle g_s^2 GG \rangle}{256T^2} \\ & \quad - \frac{\langle g_s \bar{q} \sigma G q \rangle \langle g_s \bar{q} \sigma G q \rangle \langle g_s^2 GG \rangle}{4096T^4} + \int_{\omega_c}^{\omega'_c} \left[-\frac{8\omega^9}{105\pi^4} + \frac{9\langle g_s^2 GG \rangle \omega^5}{240\pi^4} \right] e^{-\omega/T} d\omega \\ & = \delta_{\mathcal{K}} \Pi_{\Lambda_c(\frac{1}{2}^-, 1P)}(\omega_c, T) + \delta_{\mathcal{K}} \Pi_{\Lambda_c(\frac{1}{2}^-, 2P)}(\omega_c, \omega'_c, T), \end{aligned} \quad (38)$$

$$\begin{aligned} & f_{\Lambda_c(\frac{1}{2}^-, 1P)}^2 d_M \Sigma_{\Lambda_c(\frac{1}{2}^-, 1P)} e^{-\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)}/T} + f_{\Lambda_c(\frac{1}{2}^-, 2P)}^2 d_M \Sigma_{\Lambda_c(\frac{1}{2}^-, 2P)} e^{-\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)}/T} + 2f_{\Lambda_c(\frac{1}{2}^-, 1P)} f_{\Lambda_c(\frac{1}{2}^-, 2P)} d_M \Sigma'_{\Lambda_c(\frac{1}{2}^-)} e^{-\frac{\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} + \bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)}}{2T}} \\ & = \int_0^{\omega'_c} \left[\frac{3\langle g_s^2 GG \rangle \omega^5}{120\pi^4} \right] e^{-\omega/T} d\omega \\ & = \int_0^{\omega_c} \left[\frac{3\langle g_s^2 GG \rangle \omega^5}{120\pi^4} \right] e^{-\omega/T} d\omega + \int_{\omega_c}^{\omega'_c} \left[\frac{3\langle g_s^2 GG \rangle \omega^5}{120\pi^4} \right] e^{-\omega/T} d\omega \\ & = \delta_{\mathcal{S}} \Pi_{\Lambda_c(\frac{1}{2}^-, 1P)}(\omega_c, T) + \delta_{\mathcal{S}} \Pi_{\Lambda_c(\frac{1}{2}^-, 2P)}(\omega_c, \omega'_c, T), \end{aligned} \quad (39)$$

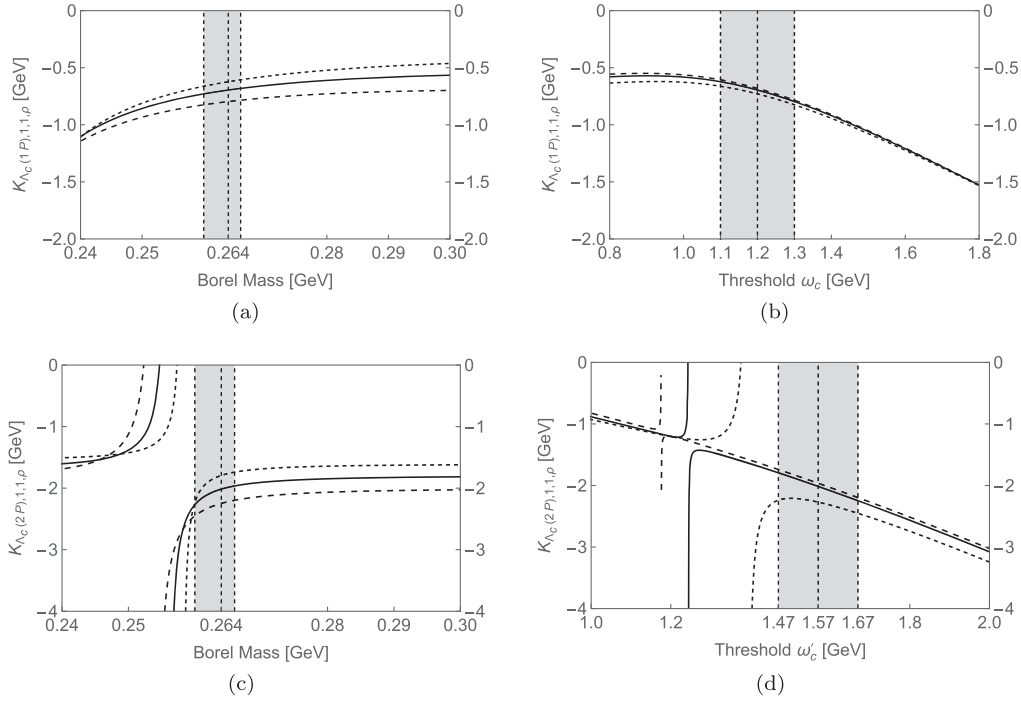


FIG. 4. Variations of $K_{\Lambda_c(1P),1,1,\rho}$ with respect to (a) the Borel mass T and (b) the threshold value ω_c as well as variations of $K_{\Lambda_c(2P),1,1,\rho}$ with respect to (c) the Borel mass T and (d) the threshold value ω'_c , when $J_{1/2,-\bar{3}_F,1,1,\rho}$ is adopted. In (a) the short-dashed, solid, and long-dashed curves are obtained by fixing $\omega_c = 1.10$ GeV, 1.20 GeV, and 1.30 GeV, respectively. In (c) the short-dashed, solid, and long-dashed curves are obtained by fixing $\omega'_c = 1.47$ GeV, 1.57 GeV, and 1.67 GeV, respectively. In (b) and (d) the short-dashed, solid, and long-dashed curves are gained by setting $T = 0.260$ GeV, 0.264 GeV, and 0.266 GeV, respectively.

where $K_{\Lambda_c(\frac{1}{2}^-,1P)}$, $K_{\Lambda_c(\frac{1}{2}^-,2P)}$, $S_{\Lambda_c(\frac{1}{2}^-,1P)}$, and $S_{\Lambda_c(\frac{1}{2}^-,2P)}$ can be extracted as

$$K_{\Lambda_c(\frac{1}{2}^-,1P)} = \frac{\delta_{\mathcal{K}}\Pi_{\Lambda_c(\frac{1}{2}^-,1P)}}{\Pi_{\Lambda_c(\frac{1}{2}^-,1P)}}, \quad d_M\Sigma_{\Lambda_c(\frac{1}{2}^-,1P)} = \frac{\delta_S\Pi_{\Lambda_c(\frac{1}{2}^-,1P)}}{\Pi_{\Lambda_c(\frac{1}{2}^-,1P)}}, \quad (40)$$

$$K_{\Lambda_c(\frac{1}{2}^-,2P)} = \frac{2T^2 \frac{\partial \delta_{\mathcal{K}}\Pi_{\Lambda_c(\frac{1}{2}^-,2P)}}{\partial T} - (\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-,1P)} + \bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-,2P)})\delta_{\mathcal{K}}\Pi_{\Lambda_c(\frac{1}{2}^-,2P)}}{\Pi_{\Lambda_c(\frac{1}{2}^-,2P)}(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-,2P)} - \bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-,1P)})}, \quad (41)$$

$$d_M\Sigma_{\Lambda_c(\frac{1}{2}^-,2P)} = \frac{2T^2 \frac{\partial \delta_S\Pi_{\Lambda_c(\frac{1}{2}^-,2P)}}{\partial T} - (\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-,1P)} + \bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-,2P)})\delta_S\Pi_{\Lambda_c(\frac{1}{2}^-,2P)}}{\Pi_{\Lambda_c(\frac{1}{2}^-,2P)}(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-,2P)} - \bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-,1P)}}). \quad (42)$$

Their variations are depicted in Figs. 4 and 5 with respect to the Borel mass T as well as the threshold values ω_c and ω'_c . We find that their dependence on T , ω_c , and ω'_c remains weak in the working regions $0.260 \text{ GeV} \leq T \leq 0.266 \text{ GeV}$, $1.10 \text{ GeV} \leq \omega_c \leq 1.30 \text{ GeV}$, and $1.47 \text{ GeV} \leq \omega'_c \leq 1.67 \text{ GeV}$, where the corresponding numerical results are obtained:

$$K_{\Lambda_c(\frac{1}{2}^-,1P)} = -0.69 \pm 0.10 \text{ GeV}, \quad (43)$$

$$d_M\Sigma_{\Lambda_c(\frac{1}{2}^-,1P)} = 0.08 \pm 0.03 \text{ GeV}, \quad (44)$$

$$K_{\Lambda_c(\frac{1}{2}^-,2P)} = -2.01 \pm 0.35 \text{ GeV}, \quad (45)$$

$$d_M\Sigma_{\Lambda_c(\frac{1}{2}^-,2P)} = 0.06 \pm 0.02 \text{ GeV}. \quad (46)$$

Finally, we can obtain the masses of $\Lambda_c(\frac{1}{2}^-/\frac{3}{2}^-, 1P/2P)$ belonging to the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublet:

$$m_{\Lambda_c(\frac{1}{2}^-,1P)} = 2.61 \pm 0.07 \text{ GeV}, \quad (47)$$

$$m_{\Lambda_c(\frac{3}{2}^-,1P)} = 2.63 \pm 0.07 \text{ GeV}, \quad (48)$$

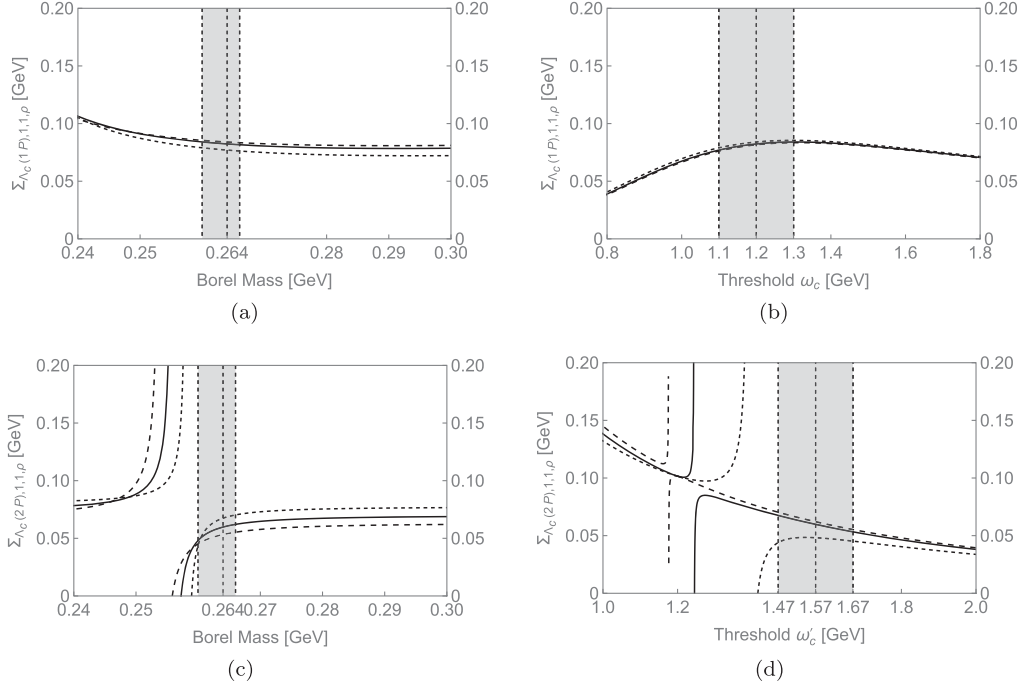


FIG. 5. Variations of $\Sigma_{\Lambda_c(1P),1,1,\rho}$ with respect to (a) the Borel mass T and (b) the threshold value ω_c as well as variations of $\Sigma_{\Lambda_c(2P),1,1,\rho}$ with respect to (c) the Borel mass T and (d) the threshold value ω'_c , when $J_{1/2,-\bar{3}_F,1,1,\rho}$ is adopted. In (a) the short-dashed, solid, and long-dashed curves are obtained by fixing $\omega_c = 1.10$ GeV, 1.20 GeV, and 1.30 GeV, respectively. In (c) the short-dashed, solid, and long-dashed curves are obtained by fixing $\omega'_c = 1.47$ GeV, 1.57 GeV, and 1.67 GeV, respectively. In (b) and (d) the short-dashed, solid, and long-dashed curves are gained by setting $T = 0.260$ GeV, 0.264 GeV, and 0.266 GeV, respectively.

$$m_{\Lambda_c(\frac{3}{2}^-,1P)} - m_{\Lambda_c(\frac{1}{2}^-,1P)} = 24 \pm 9 \text{ MeV}, \quad (49)$$

$$m_{\Lambda_c(\frac{1}{2}^-,2P)} = 3.05 \pm 0.12 \text{ GeV}, \quad (50)$$

$$m_{\Lambda_c(\frac{3}{2}^-,2P)} = 3.07 \pm 0.11 \text{ GeV}, \quad (51)$$

$$m_{\Lambda_c(\frac{3}{2}^-,2P)} - m_{\Lambda_c(\frac{1}{2}^-,2P)} = 18 \pm 7 \text{ MeV}. \quad (52)$$

In Fig. 6 we present the variations of $m_{\Lambda_c(1P),1,1,\rho}$ and $m_{\Lambda_c(2P),1,1,\rho}$ with respect to T , ω_c , and ω'_c . It is noteworthy that there exist significant theoretical uncertainties in our

results for the masses of the charmed baryons, but their differences within the same doublet are determined with less uncertainty since they do not depend much on the charm quark mass.

The results obtained are summarized in Table II. We utilize the charmed baryons belonging to the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublet to fit the $\Lambda_c(2595)(J^P = \frac{1}{2}^-, 1P)$, $\Lambda_c(2625)(J^P = \frac{3}{2}^-, 1P)$, $\Lambda_c(2910)(J^P = \frac{1}{2}^-, 2P)$, $\Lambda_c(2940) \times (J^P = \frac{3}{2}^-, 2P)$, $\Xi_c(2790)(J^P = \frac{1}{2}^-, 1P)$, $\Xi_c(2815) \times (J^P = \frac{3}{2}^-, 1P)$, and $\Xi_c(3123)(J^P = \frac{3}{2}^-, 2P)$. Within our

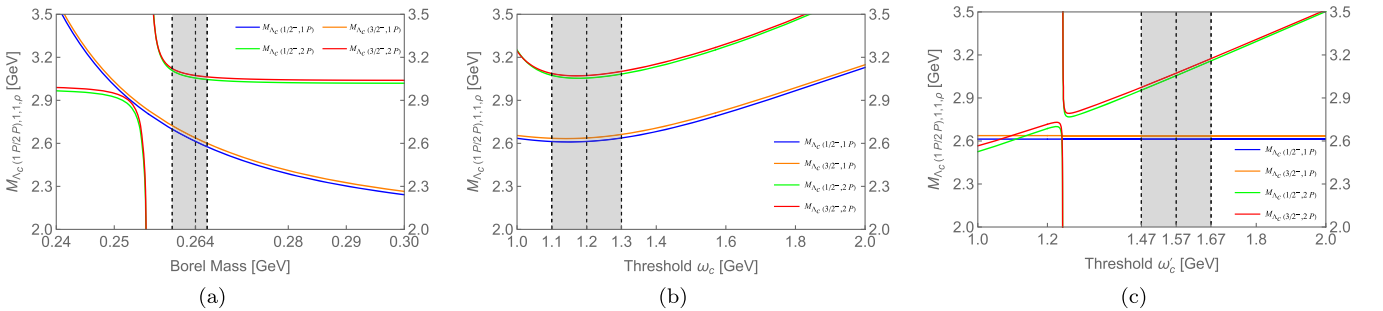


FIG. 6. Variations of $m_{\Lambda_c(1P),1,1,\rho}$ and $m_{\Lambda_c(2P),1,1,\rho}$ with respect to (a) the Borel mass T as well as the threshold values (b) ω_c and (c) ω'_c , calculated using the charmed baryon doublet $[\Lambda_c, \bar{\mathbf{3}}_F, 1, 1, \rho]$. The blue, orange, green, and red curves represent the mass functions of $\Lambda_c(\frac{1}{2}^-, 1P)$, $\Lambda_c(\frac{3}{2}^-, 1P)$, $\Lambda_c(\frac{1}{2}^-, 2P)$, and $\Lambda_c(\frac{3}{2}^-, 2P)$, respectively. The curves in the left panel are obtained by fixing $\omega_c = 1.20$ GeV and $\omega'_c = 1.57$ GeV, respectively. The curves in the middle panel are obtained by fixing $T = 0.264$ GeV and $\omega'_c = 1.47$ GeV, respectively. The curves in the right panel are gained by setting $T = 0.264$ GeV and $\omega_c = 1.20$ GeV, respectively.

TABLE II. Parameters of the $1P$ - and $2P$ -wave charmed baryons belonging to the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublets, calculated using the method of QCD sum rules within the framework of heavy quark effective theory. Decay constants in the last column satisfy $f_{\Xi_c^+} = f_{\Xi_c^0}$.

B	ω_c [GeV]	Working region [GeV]	$\bar{\Lambda}$ [GeV]	Baryon (j^P)	Mass [GeV]	Difference [MeV]	Decay constant [GeV ⁴]
$\Lambda_c(1P)$	1.20	$0.260 \leq T \leq 0.266$	1.22 ± 0.06	$\Lambda_c(1/2^-)$	2.61 ± 0.07	24 ± 9	$0.043 \pm 0.008(\Lambda_c^+(1/2^-))$
				$\Lambda_c(3/2^-)$	2.63 ± 0.07		$0.020 \pm 0.004(\Lambda_c^+(3/2^-))$
$\Xi_c(1P)$	1.45	$0.248 \leq T \leq 0.257$	1.43 ± 0.13	$\Xi_c(1/2^-)$	2.78 ± 0.14	29 ± 10	$0.069 \pm 0.023(\Xi_c^0(1/2^-))$
				$\Xi_c(3/2^-)$	2.81 ± 0.14		$0.033 \pm 0.011(\Xi_c^0(3/2^-))$
$\Lambda_c(2P)$	1.57	$0.260 \leq T \leq 0.266$	1.40 ± 0.07	$\Lambda_c(1/2^-)$	3.05 ± 0.12	18 ± 7	$0.057 \pm 0.016(\Lambda_c^+(1/2^-))$
				$\Lambda_c(3/2^-)$	3.07 ± 0.11		$0.027 \pm 0.007(\Lambda_c^+(3/2^-))$
$\Xi_c(2P)$	1.60	$0.248 \leq T \leq 0.257$	1.53 ± 0.07	$\Xi_c(1/2^-)$	3.16 ± 0.12	18 ± 7	$0.048 \pm 0.026(\Xi_c^+(1/2^-))$
				$\Xi_c(3/2^-)$	3.18 ± 0.12		$0.022 \pm 0.012(\Xi_c^+(3/2^-))$

QCD sum rule framework, the mass spectra of $1P$ -wave charmed baryons are consistent with the experimental measurements, while the mass spectra of $2P$ -wave charmed baryons are higher than experimental measurements, which align with the results in quark models. Apart from the mass spectra, we also need to analyze their decay properties. The decay constant f is an important input parameter, which will be used to study the decay widths of $\Lambda_c(\frac{1}{2}^-/\frac{3}{2}^-, 1P/2P)$ and $\Xi_c(\frac{1}{2}^-/\frac{3}{2}^-, 1P/2P)$ in the next section. For completeness, we have also investigated the $1P$ - and $2P$ -wave charmed baryons belonging to the $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ doublet using the following interpolating currents:

$$J_{1/2^-, -\bar{\mathbf{3}}_F, 1, 0, \lambda} = i\epsilon_{abc}([\mathcal{D}_i^\mu q^{aT}]\mathcal{C}\gamma_5 q^b + q^{aT}\mathcal{C}\gamma_5[\mathcal{D}_i^\mu q^b])\gamma_i^\mu \gamma_5 h_v^c, \quad (53)$$

$$J_{3/2^-, -\bar{\mathbf{3}}_F, 1, 0, \lambda}^\alpha = i\epsilon_{abc}([\mathcal{D}_i^\mu q^{aT}]\mathcal{C}\gamma_5 q^b + q^{aT}\mathcal{C}\gamma_5[\mathcal{D}_i^\mu q^b]) \times \left(g_i^{\alpha\mu} - \frac{1}{3}\gamma_i^\alpha \gamma_i^\mu\right) h_v^c. \quad (54)$$

The obtained results are summarized in Appendix. We find that the ρ -mode doublet $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ appears to be lower than the λ -mode doublet $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$. This behavior aligns with our previous QCD sum rule results for their corresponding doublets of the $SU(3)$ flavor $\mathbf{6}_F$ [94–96], but contradicts the expectation from the quark model [97,98]. However, this discrepancy could be attributed to the significant uncertainties associated with the mass difference between different multiplets within our QCD sum rule framework.

III. DECAY ANALYSES FROM LIGHT-CONE SUM RULES

In this section we investigate the decay properties of $\Lambda_c(1P/2P)$ and $\Xi_c(1P/2P)$ belonging to the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublet. To achieve this we apply the light-cone sum rule method to study their S - and D -wave decays into the ground-state charmed baryons and light pseudoscalar/vector mesons. The relevant decay channels include the following:

$$(a1) \Gamma[\Lambda_c[1/2^-, 1P/2P] \rightarrow \Lambda_c + \pi] = \Gamma[\Lambda_c^+[1/2^-, 1P/2P] \rightarrow \Lambda_c^+ + \pi^0], \quad (55)$$

$$(a2) \Gamma[\Lambda_c[1/2^-, 1P] \rightarrow \Sigma_c + \pi] = \Gamma[\Lambda_c^+[1/2^-, 1P] \rightarrow \Sigma_c^+ + \pi^0] + 2 \times \Gamma[\Lambda_c^+[1/2^-, 1P] \rightarrow \Sigma_c^{++} + \pi^- \rightarrow \Lambda_c^+ + \pi^+ + \pi^-], \quad (56)$$

$$(a3) \Gamma[\Lambda_c[1/2^-, 2P] \rightarrow \Sigma_c + \pi] = 3 \times \Gamma[\Lambda_c[1/2^-, 2P] \rightarrow \Sigma_c^{++} + \pi^-], \quad (57)$$

$$(a4) \Gamma[\Lambda_c[1/2^-, 1P] \rightarrow \Sigma_c^* + \pi \rightarrow \Lambda_c + \pi + \pi] = 3 \times \Gamma[\Lambda_c^+[1/2^-, 1P] \rightarrow \Sigma_c^{*++} + \pi^- \rightarrow \Lambda_c^+ + \pi^+ + \pi^-], \quad (58)$$

$$(a5) \Gamma[\Lambda_c[1/2^-, 2P] \rightarrow \Sigma_c^* + \pi] = 3 \times \Gamma[\Lambda_c^+[1/2^-, 2P] \rightarrow \Sigma_c^{*++} + \pi^-], \quad (59)$$

$$(a6) \Gamma[\Lambda_c[1/2^-, 1P/2P] \rightarrow \Lambda_c + \rho \rightarrow \Lambda_c + \pi + \pi] = \Gamma[\Lambda_c^+[1/2^-, 1P/2P] \rightarrow \Lambda_c^+ + \pi^+ + \pi^-], \quad (60)$$

$$(a7) \Gamma[\Lambda_c[1/2^-, 1P/2P] \rightarrow \Sigma_c + \rho \rightarrow \Sigma_c + \pi + \pi] = 3 \times \Gamma[\Lambda_c^+[1/2^-, 1P/2P] \rightarrow \Sigma_c^+ + \pi^+ + \pi^-], \quad (61)$$

$$(a8) \Gamma[\Lambda_c[1/2^-, 1P/2P] \rightarrow \Sigma_c^* + \rho \rightarrow \Sigma_c^* + \pi + \pi] = 3 \times \Gamma[\Lambda_c^+[1/2^-, 1P/2P] \rightarrow \Sigma_c^{*+} + \pi^+ + \pi^-], \quad (62)$$

- (b1) $\Gamma[\Lambda_c[3/2^-, 1P/2P] \rightarrow \Lambda_c + \pi] = \Gamma[\Lambda_c^+[3/2^-] \rightarrow \Lambda_c^+ + \pi^0]$, (63)
- (b2) $\Gamma[\Lambda_c[3/2^-, 1P] \rightarrow \Sigma_c + \pi \rightarrow \Lambda_c + \pi + \pi] = 3 \times \Gamma[\Lambda_c^+[3/2^-, 1P] \rightarrow \Sigma_c^{*++} + \pi^- \rightarrow \Lambda_c^+ + \pi^+ + \pi^-]$, (64)
- (b3) $\Gamma[\Lambda_c[3/2^-, 2P] \rightarrow \Sigma_c + \pi] = 3 \times \Gamma[\Lambda_c^+[3/2^-, 2P] \rightarrow \Sigma_c^{*++} + \pi^-]$, (65)
- (b4) $\Gamma[\Lambda_c[3/2^-, 1P] \rightarrow \Sigma_c^* + \pi \rightarrow \Lambda_c + \pi + \pi] = 3 \times \Gamma[\Lambda_c^+[3/2^-, 1P] \rightarrow \Sigma_c^{*++} + \pi^- \rightarrow \Lambda_c^+ + \pi^+ \pi^-]$, (66)
- (b5) $\Gamma[\Lambda_c[3/2^-, 2P] \rightarrow \Sigma_c^* + \pi] = 3 \times \Gamma[\Lambda_c^+[3/2^-, 2P] \rightarrow \Sigma_c^{*++} + \pi^-]$, (67)
- (b6) $\Gamma[\Lambda_c[3/2^-, 1P/2P] \rightarrow \Lambda_c + \rho \rightarrow \Lambda_c + \pi + \pi] = \Gamma[\Lambda_c^+[3/2^-, 1P/2P] \rightarrow \Lambda_c^+ + \pi^+ + \pi^-]$, (68)
- (b7) $\Gamma[\Lambda_c[3/2^-, 1P/2P] \rightarrow \Sigma_c + \rho \rightarrow \Sigma_c + \pi + \pi] = 3 \times \Gamma[\Lambda_c^+[3/2^-, 1P/2P] \rightarrow \Sigma_c^+ + \pi^+ + \pi^-]$, (69)
- (b8) $\Gamma[\Lambda_c[3/2^-, 1P/2P] \rightarrow \Sigma_c^* + \rho \rightarrow \Sigma_c^* + \pi + \pi] = 3 \times \Gamma[\Lambda_c^0[3/2^-, 1P/2P] \rightarrow \Sigma_c^{*+} + \pi^+ + \pi^-]$, (70)
- (c1) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Lambda_c + \bar{K}] = \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Lambda_c^+ + K^-]$, (71)
- (c2) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Xi_c + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Xi_c^+ + \pi^-]$, (72)
- (c3) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Sigma_c + \bar{K}] = 3 \times \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Sigma_c^+ + K^-]$, (73)
- (c4) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Xi_c' + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Xi_c'^+ + \pi^-]$, (74)
- (c5) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Sigma_c^* + K] = 3 \times \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Sigma_c^{*+} + K^-]$, (75)
- (c6) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Xi_c^* + \pi \rightarrow \Xi_c + \pi + \pi] = \frac{9}{2} \times \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Xi_c^{*+} + \pi^- \rightarrow \Xi_c^+ + \pi^0 + \pi^-]$, (76)
- (c7) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Lambda_c + \bar{K}^* \rightarrow \Lambda_c + \bar{K} + \pi] = 3 \times \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Lambda_c^+ + K^- + \pi^0]$, (77)
- (c8) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Xi_c + \rho \rightarrow \Xi_c + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Xi_c^+ + \pi^0 + \pi^-]$, (78)
- (c9) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Sigma_c^* + \bar{K}^* \rightarrow \Sigma_c^* + \bar{K} + \pi] = 9 \times \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Sigma_c^{*+} + K^- + \pi^0]$, (79)
- (c10) $\Gamma[\Xi_c[1/2^-, 1P] \rightarrow \Xi_c^* + \rho \rightarrow \Xi_c^* + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^0[1/2^-, 1P] \rightarrow \Xi_c^{*+} + \pi^0 + \pi^-]$, (80)
- (c11) $\Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Lambda_c + \bar{K}] = \Gamma[\Xi_c^+[1/2^-, 2P] \rightarrow \Lambda_c^+ + \bar{K}^0]$, (81)
- (c12) $\Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Xi_c + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^+[1/2^-, 2P] \rightarrow \Xi_c^0 + \pi^+]$, (82)
- (c13) $\Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Sigma_c + \bar{K}] = \frac{3}{2} \times \Gamma[\Xi_c^+[1/2^-, 2P] \rightarrow \Sigma_c^{*+} + K^-]$, (83)
- (c14) $\Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Xi_c' + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^+[1/2^-, 2P] \rightarrow \Xi_c'^0 + \pi^+]$, (84)
- (c15) $\Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Sigma_c^* + K] = \frac{3}{2} \times \Gamma[\Xi_c^+[1/2^-, 2P] \rightarrow \Sigma_c^{*++} + K^-]$, (85)
- (c16) $\Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Xi_c^* + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^+[1/2^-, 2P] \rightarrow \Xi_c^{*0} + \pi^+]$, (86)

$$(c17) \Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Lambda_c + \bar{K}^* \rightarrow \Lambda_c + \bar{K} + \pi] = 3 \times \Gamma[\Xi_c^+[1/2^-, 2P] \rightarrow \Lambda_c^+ + \bar{K}^0 + \pi^0], \quad (87)$$

$$(c18) \Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Xi_c + \rho \rightarrow \Xi_c + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^+[1/2^-, 2P] \rightarrow \Xi_c^0 + \pi^+ + \pi^0], \quad (88)$$

$$(c19) \Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Sigma_c^* + \bar{K}^* \rightarrow \Sigma_c^* + \bar{K} + \pi] = \frac{9}{2} \times \Gamma[\Xi_c^+[1/2^-, 2P] \rightarrow \Sigma_c^{*++} + K^- + \pi^0], \quad (89)$$

$$(c20) \Gamma[\Xi_c[1/2^-, 2P] \rightarrow \Xi_c^* + \rho \rightarrow \Xi_c^* + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^+[1/2^-, 1P] \rightarrow \Xi_c^{*0} + \pi^0 + \pi^+], \quad (90)$$

$$(d1) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Lambda_b + \bar{K}] = \Gamma[\Xi_c^0[3/2^-, 1P] \rightarrow \Lambda_c^+ + K^-], \quad (91)$$

$$(d2) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Xi_c + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^0[3/2^-, 1P] \rightarrow \Xi_c^+ + \pi^-], \quad (92)$$

$$(d3) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Sigma_c + \bar{K}] = 3 \times \Gamma[\Xi_c^0[3/2^-, 1P] \rightarrow \Sigma_c^+ + K^-], \quad (93)$$

$$(d4) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Xi_c' + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^0[3/2^-, 1P] \rightarrow \Xi_c'^+ + \pi^-], \quad (94)$$

$$(d5) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Sigma_c^* + \bar{K}] = 3 \times \Gamma[\Xi_c^0[3/2^-, 1P] \rightarrow \Sigma_c^{*+} + K^-], \quad (95)$$

$$(d6) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Xi_c^* + \pi \rightarrow \Xi_c + \pi + \pi] = \frac{9}{2} \times \Gamma[\Xi_c^{*0}[3/2^-, 1P] \rightarrow \Xi_c^{*+} + \pi^- \rightarrow \Xi_c^+ + \pi^0 + \pi^-], \quad (96)$$

$$(d7) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Lambda_c + \bar{K}^* \rightarrow \Lambda_b + \bar{K} + \pi] = 3 \times \Gamma[\Xi_c^0[3/2^-, 1P] \rightarrow \Lambda_c^+ + K^- + \pi^0], \quad (97)$$

$$(d8) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Xi_c + \rho \rightarrow \Xi_b + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^0[3/2^-, 1P] \rightarrow \Xi_c^+ + \pi^0 + \pi^-], \quad (98)$$

$$(d9) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Sigma_c^* + \bar{K}^* \rightarrow \Sigma_c^{*0} + \bar{K} + \pi] = 9 \times \Gamma[\Xi_c^0[3/2^-, 1P] \rightarrow \Sigma_c^{*+} + K^- + \pi^0], \quad (99)$$

$$(d10) \Gamma[\Xi_c[3/2^-, 1P] \rightarrow \Xi_c^* + \rho \rightarrow \Xi_c^* + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^0[3/2^-, 1P] \rightarrow \Xi_c^{*+} + \pi^0 + \pi^-], \quad (100)$$

$$(d11) \Gamma[\Xi_c[3/2^-, 2P] \rightarrow \Lambda_c + \bar{K}] = \Gamma[\Xi_c^+[3/2^-, 2P] \rightarrow \Lambda_c^+ + \bar{K}^0], \quad (101)$$

$$(d12) \Gamma[\Xi_c[3/2^-, 2P] \rightarrow \Xi_c + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^+[3/2^-, 2P] \rightarrow \Xi_c^0 + \pi^+], \quad (102)$$

$$(d13) \Gamma[\Xi_c[3/2^-, 2P] \rightarrow \Sigma_c + \bar{K}] = \frac{3}{2} \times \Gamma[\Xi_c^+[3/2^-, 2P] \rightarrow \Sigma_c^{++} + K^-], \quad (103)$$

$$(d14) \Gamma[\Xi_c[3/2^-, 2P] \rightarrow \Xi_c' + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^+[3/2^-, 2P] \rightarrow \Xi_c'^0 + \pi^+], \quad (104)$$

$$(d15) \Gamma[\Xi_c[3/2^-, 2P] \rightarrow \Sigma_c^* + \bar{K}] = \frac{3}{2} \times \Gamma[\Xi_c^+[3/2^-, 2P] \rightarrow \Sigma_c^{*++} + K^-], \quad (105)$$

$$(d16) \Gamma[\Xi_b[3/2^-, 2P] \rightarrow \Xi_c^* + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^{*+}[3/2^-, 2P] \rightarrow \Xi_c^{*0} + \pi^+], \quad (106)$$

$$(d17) \Gamma[\Xi_c[3/2^-, 2P] \rightarrow \Lambda_c + \bar{K}^* \rightarrow \Lambda_b + \bar{K} + \pi] = 3 \times \Gamma[\Xi_c^+[3/2^-, 2P] \rightarrow \Lambda_c^+ + \bar{K}^0 + \pi^0], \quad (107)$$

$$(d18) \Gamma[\Xi_c[3/2^-, 2P] \rightarrow \Xi_c + \rho \rightarrow \Xi_b + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^+[3/2^-, 2P] \rightarrow \Xi_c^0 + \pi^0 + \pi^+], \quad (108)$$

$$(d19) \Gamma[\Xi_c[3/2^-, 2P] \rightarrow \Sigma_c^* + \bar{K}^* \rightarrow \Sigma_c^* + \bar{K} + \pi] = \frac{9}{2} \times \Gamma[\Xi_c^+[3/2^-, 1P] \rightarrow \Sigma_c^{*++} + K^- + \pi^0], \quad (109)$$

$$(d20) \Gamma[\Xi_c[3/2^-, 2P] \rightarrow \Xi_c^* + \rho \rightarrow \Xi_c^* + \pi + \pi] = \frac{3}{2} \times \Gamma[\Xi_c^+[3/2^-, 1P] \rightarrow \Xi_c^{*0} + \pi^0 + \pi^+]. \quad (110)$$

We shall calculate their partial decay widths through the following Lagrangians:

$$\mathcal{L}_{X_c(1/2^-) \rightarrow Y_c(1/2^+) \mathcal{P}}^S = g \bar{X}_c Y_c \mathcal{P}, \quad (111)$$

$$\mathcal{L}_{X_c(3/2^-) \rightarrow Y_c(3/2^+) \mathcal{P}}^S = g \bar{X}_{c\mu} Y_c^\mu \mathcal{P}, \quad (112)$$

$$\mathcal{L}_{X_c(1/2^-) \rightarrow Y_c(1/2^+) V}^S = g \bar{X}_c \gamma_\mu \gamma_5 Y_c V^\mu, \quad (113)$$

$$\mathcal{L}_{X_c(1/2^-) \rightarrow Y_c(3/2^+) V}^S = g \bar{X}_c Y_c^\mu V_\mu, \quad (114)$$

$$\mathcal{L}_{X_c(3/2^-) \rightarrow Y_c(1/2^+) V}^S = g \bar{X}_c^\mu Y_c V_\mu, \quad (115)$$

$$\mathcal{L}_{X_c(3/2^-) \rightarrow Y_c(3/2^+) V}^S = g \bar{X}_c^\nu \gamma_\mu \gamma_5 Y_{c\nu} V^\mu. \quad (116)$$

$$\mathcal{L}_{X_c(1/2^-) \rightarrow Y_c(3/2^+) \mathcal{P}}^D = g \bar{X}_c \gamma_\mu \gamma_5 Y_{c\nu} \partial^\mu \partial^\nu \mathcal{P}, \quad (117)$$

$$\mathcal{L}_{X_c(3/2^-) \rightarrow Y_c(1/2^+) \mathcal{P}}^D = g \bar{X}_{c\mu} \gamma_\nu \gamma_5 Y_c \partial^\mu \partial^\nu \mathcal{P}, \quad (118)$$

$$\mathcal{L}_{X_c(3/2^-) \rightarrow Y_c(3/2^+) \mathcal{P}}^D = g \bar{X}_{c\mu} Y_{c\nu} \partial^\mu \partial^\nu \mathcal{P}. \quad (119)$$

Here the superscripts S and D indicate the S - and D -wave decays, respectively; the fields $X_c^{(\mu)}$ and $Y_c^{(\mu)}$ represent the P -wave charmed baryons and ground-state charmed baryons, respectively; the fields \mathcal{P} and V^μ denote the light pseudoscalar mesons and light vector mesons, respectively. To facilitate a comprehensive analysis of the aforementioned decay channels, we employ the pertinent parameters of ground-states, excited states, pseudoscalar, and vector mesons as follows:

(i) Relevant parameters of ground-state charmed baryons are the following:

$$\begin{aligned} \Lambda_c(1/2^+): M &= 2286.46 \text{ MeV}, \\ \Xi_c(1/2^+): M &= 2469.34 \text{ MeV}, \\ \Sigma_c(1/2^+): M &= 2453.54 \text{ MeV}, \\ \Sigma_c^*(3/2^+): M &= 2518.1 \text{ MeV}, \\ \Xi_c'(1/2^+): M &= 2576.8 \text{ MeV}, \\ \Xi_c^*(3/2^+): M &= 2645.9 \text{ MeV}, \\ \Omega_c(1/2^+): M &= 2695.2 \text{ MeV}, \\ \Omega_c^*(1/2^+): M &= 2765.9 \text{ MeV}. \end{aligned}$$

(ii) Relevant parameters of excited state charmed baryons are the following:

$$\begin{aligned} \Lambda_c(1/2^-, 1P): m &= 2592.25 \text{ MeV}, \\ \Lambda_c(3/2^-, 1P): m &= 2628.11 \text{ MeV}, \\ \Xi_c(1/2^-, 1P): m &= 2790.45 \text{ MeV}, \\ \Xi_c(3/2^-, 1P): m &= 2818.1 \text{ MeV}, \\ \Lambda_c(1/2^-, 2P): m &= 2913.8 \text{ MeV}, \\ \Lambda_c(3/2^-, 2P): m &= 2939.6 \text{ MeV}, \\ \Xi_c(3/2^-, 2P): m &= 3122.9 \text{ MeV}, \\ \Xi_c(1/2^-, 2P): m &= m[\Xi_c(3/2^-, 2P)] - \Delta m \\ &= (3122.9 - 18) \text{ MeV}. \end{aligned}$$

(iii) Relevant parameters of light pseudoscalar and vector mesons are the following:

$$\begin{aligned} \pi(0^-): m &= 138.04 \text{ MeV}, \\ K(0^-): m &= 495.65 \text{ MeV}, \\ \rho(1^-): m &= 775.21 \text{ MeV}, \\ \Gamma &= 148.2 \text{ MeV}, \\ g_{\rho\pi\pi} &= 5.94 \text{ GeV}^{-2}, \\ K^*(1^-): m &= 893.57 \text{ MeV}, \\ \Gamma &= 49.1 \text{ MeV}, \\ g_{K^*k\pi} &= 3.20 \text{ GeV}^{-2}. \end{aligned}$$

We use the $\Lambda_c(1P, 1/2^-)$ and $\Lambda_c(2P, 1/2^-)$ from the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublet as an example, and study their S -wave decays into the ground-state charmed baryon Σ_c^{++} and the light pseudoscalar meson π^- . To do this we investigate the two-point correlation function:

$$\begin{aligned} \Pi(\omega_1, \omega_2, \omega') &= \int d^4x e^{-ik \cdot x} \langle 0 | J_{1/2^-, \Lambda_c^-}^\alpha(0) \bar{J}_{\Sigma_c^{++}}(x) | \pi^-(q) \rangle \\ &= \frac{1+x}{2} G_{\Lambda_c^+(\frac{1}{2}^-) \rightarrow \Sigma_c^{++} \pi^-}^\alpha(\omega_1, \omega_2, \omega'). \quad (120) \end{aligned}$$

At the hadron level we write $G_{\Lambda_c^+(\frac{1}{2}^-) \rightarrow \Sigma_c^{++} \pi^-}$ as

$$\begin{aligned} &G_{\Lambda_c^+(\frac{1}{2}^-) \rightarrow \Sigma_c^{++} \pi^-}(\omega_1, \omega_2, \omega') \\ &= g_{\Lambda_c^+(\frac{1}{2}^-, 1P) \rightarrow \Sigma_c^{++} \pi^-} \frac{f_{\Lambda_c(\frac{1}{2}^-, 1P)} f_{\Sigma_c^{++}}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 1P)} - \omega_1)(\bar{\Lambda}_{\Sigma_c^{++}} - \omega')} \\ &\quad + g_{\Lambda_c^+(\frac{1}{2}^-, 2P) \rightarrow \Sigma_c^{++} \pi^-} \frac{f_{\Lambda_c(\frac{1}{2}^-, 2P)} f_{\Sigma_c^{++}}}{(\bar{\Lambda}_{\Lambda_c(\frac{1}{2}^-, 2P)} - \omega_2)(\bar{\Lambda}_{\Sigma_c^{++}} - \omega')} \dots, \quad (121) \end{aligned}$$

where \dots contains other possible amplitudes.

At the quark-gluon level we calculate $G_{\Lambda_c^+(1/2^-) \rightarrow \Sigma_c^{++}\pi^-}$ using the method of OPE. Then we perform the Borel transformation to both the hadron level and the quark-gluon level:

$$\begin{aligned}
& g_{\Lambda_c^+(1/2^-, 1P) \rightarrow \Sigma_c^{++}\pi^-}^S f_{\Lambda_c^+(1/2^-, 1P)} f_{\Sigma_c^{++}} e^{-\frac{\bar{\Lambda}_{\Lambda_c^+(1/2^-, 1P)}}{T_1}} e^{-\frac{\bar{\Lambda}_{\Sigma_c^{++}}}{T'}} + g_{\Lambda_c^+(1/2^-, 2P) \rightarrow \Sigma_c^{++}\pi^-}^S f_{\Lambda_c^+(1/2^-, 2P)} f_{\Sigma_c^{++}} e^{-\frac{\bar{\Lambda}_{\Lambda_c^+(1/2^-, 2P)}}{T_2}} e^{-\frac{\bar{\Lambda}_{\Sigma_c^{++}}}{T'}} \\
& = 4\sqrt{2} \times \left(- \int_0^{\frac{1}{2}} du_1 \frac{3f_\pi}{2\pi^2} T^4 f_3 \left(\frac{\omega'_c}{T} \right) \psi_{4;\pi}(u_1) - \frac{3f_\pi}{2\pi^2} T^6 f_5 \left(\frac{\omega'_c}{T} \right) \frac{\partial}{\partial u_0} \phi_{2;\pi}(u_0) \Big|_{u_0=\frac{1}{2}} + \frac{f_\pi}{2\pi^2} T^6 f_5 \left(\frac{\omega'_c}{T} \right) \frac{\partial^2}{\partial u_0^2} u_0 \phi_{2;\pi}(u_0) \Big|_{u_0=\frac{1}{2}} \right. \\
& + \frac{3f_\pi}{32\pi^2} T^4 f_3 \left(\frac{\omega'_c}{T} \right) \frac{\partial}{\partial u_0} \phi_{4;\pi}(u_0) \Big|_{u_0=\frac{1}{2}} + \frac{f_\pi m_\pi^2 m_s}{8(m_u + m_d)\pi^2} T^4 f_3 \left(\frac{\omega'_c}{T} \right) \frac{\partial}{\partial u_0} \phi_{3;\pi}^\sigma(u_0) \Big|_{u_0=\frac{1}{2}} \\
& + \frac{f_\pi}{32\pi^2} T^4 f_3 \left(\frac{\omega'_c}{T} \right) \frac{\partial^2}{\partial u_0^2} u_0 \phi_{4;\pi}(u_0) \Big|_{u_0=\frac{1}{2}} - \frac{f_\pi m_\pi^2 m_s}{24(m_u + m_d)\pi^2} T^4 f_3 \left(\frac{\omega'_c}{T} \right) \frac{\partial^2}{\partial u_0^2} u_0 \phi_{3;\pi}^\sigma(u_0) \Big|_{u_0=\frac{1}{2}} \\
& - \frac{f_\pi m_s}{16} \langle \bar{q}q \rangle T^2 f_1 \left(\frac{\omega'_c}{T} \right) \frac{\partial}{\partial u_0} \phi_{2;\pi}(u_0) \Big|_{u_0=\frac{1}{2}} - \frac{f_\pi m_\pi^2}{24(m_u + m_d)} \langle \bar{q}q \rangle T^2 f_1 \left(\frac{\omega'_c}{T} \right) \frac{\partial}{\partial u_0} \phi_{3;\pi}^\sigma(u_0) \Big|_{u_0=\frac{1}{2}} \\
& - \int_0^{\frac{1}{2}} du_1 \frac{f_\pi m_s}{16} \langle \bar{q}q \rangle \psi_{4;\pi}(u_1) + \frac{f_\pi m_\pi^2}{72(m_u + m_d)} \langle \bar{q}q \rangle T^2 f_1 \left(\frac{\omega'_c}{T} \right) \frac{\partial^2}{\partial u_0^2} u_0 \phi_{3;\pi}^\sigma(u_0) \Big|_{u_0=\frac{1}{2}} \\
& - \frac{f_\pi m_s}{48} \langle \bar{q}q \rangle T^2 f_1 \left(\frac{\omega'_c}{T} \right) \frac{\partial^2}{\partial u_0^2} u_0 \phi_{2;\pi}(u_0) \Big|_{u_0=\frac{1}{2}} + \frac{f_\pi m_\pi^2}{384(m_u + m_d)} \langle g_s \bar{q} \sigma G q \rangle \frac{\partial}{\partial u_0} \phi_{3;\pi}^\sigma(u_0) \Big|_{u_0=\frac{1}{2}} \\
& + \frac{f_\pi m_s}{256} \langle \bar{q}q \rangle \frac{\partial}{\partial u_0} \phi_{4;\pi}(u_0) \Big|_{u_0=\frac{1}{2}} - \frac{f_\pi m_\pi^2}{1152(m_u + m_d)} \langle g_s \bar{q} \sigma G q \rangle \frac{\partial^2}{\partial u_0^2} u_0 \phi_{3;\pi}^\sigma(u_0) \Big|_{u_0=\frac{1}{2}} + \frac{f_\pi m_s}{768} \langle \bar{q}q \rangle \frac{\partial^2}{\partial u_0^2} u_0 \phi_{4;\pi}(u_0) \Big|_{u_0=\frac{1}{2}} \Big) \\
& - \frac{\sqrt{2}}{2} \times \left(- \frac{f_\pi}{4\pi^2 u_0^2} T^4 f_3 \left(\frac{\omega'_c}{T} \right) \int_0^{\frac{1}{2}} d\alpha_2 \int_{\frac{1}{2}-\alpha_2}^{1-\alpha_2} \left(- \frac{u_0^2}{\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \alpha_3 \Phi_{4;\pi}(\alpha) - \frac{u_0 \alpha_2}{\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \Phi_{4;\pi}(\alpha) - \frac{u_0}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \alpha_3 \Phi_{4;\pi}(\alpha) \right. \right. \\
& - \frac{u_0}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \alpha_3 \tilde{\Phi}_{4;\pi}(\alpha) + \frac{u_0}{\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \Phi_{4;\pi}(\alpha) - \frac{\alpha_2}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \Phi_{4;\pi}(\alpha) - \frac{\alpha_2}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \tilde{\Phi}_{4;\pi}(\alpha) + \frac{1}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \Phi_{4;\pi}(\alpha) \\
& + \left. \frac{1}{2\alpha_3} \frac{\partial^2}{\partial \alpha_3^2} \tilde{\Phi}_{4;\pi}(\alpha) \right) - \frac{f_\pi}{2\pi^2 u_0} T^4 f_3 \left(\frac{\omega'_c}{T} \right) \int_0^{\frac{1}{2}} d\alpha_2 \int_{\frac{1}{2}-\alpha_2}^{1-\alpha_2} d\alpha_3 \left(\frac{u_0}{2\alpha_3} \frac{\partial}{\partial \alpha_3} \Phi_{4;\pi}(\alpha) + \frac{u_0}{\alpha_3} \frac{\partial}{\partial \alpha_3} \Psi_{4;\pi}(\alpha) \right. \\
& + \left. \frac{3}{4\alpha_3} \frac{\partial}{\partial \alpha_3} \Phi_{4;\pi}(\alpha) + \frac{1}{4\alpha_3} \frac{\partial}{\partial \alpha_3} \tilde{\Phi}_{4;\pi}(\alpha) - \frac{1}{2\alpha_3} \frac{\partial}{\partial \alpha_3} \Psi_{4;\pi}(\alpha) + \frac{1}{2\alpha_3} \frac{\partial}{\partial \alpha_3} \tilde{\Psi}_{4;\pi}(\alpha) \right) \Big).
\end{aligned}$$

In the above expressions, $f_n(x) \equiv 1 - e^{-x} \sum_{k=0}^n \frac{x^k}{k!}$; the parameters ω_1 , ω_2 , and ω' are transformed to be T_1 , T_2 , and T' , respectively; we choose $T_1 = T_2 = T' = 2T$ so that $u_0 = \frac{T_1}{T_1 + T'} = \frac{1}{2}$; we choose $\omega_c = 1.20$ GeV to be the average threshold value of the $\Lambda_c^+(1/2^-, 1P)$ and $\Sigma_c^{++}(1/2^+)$ mass sum rules; we choose $\omega_c = 1.47$ GeV

to be the average threshold value of the $\Lambda_c^+(1/2^-, 2P)$ and $\Sigma_c^{++}(1/2^+)$ mass sum rules; we choose 0.260 GeV $< T < 0.266$ GeV to be the Borel window of the $\Lambda_c^+(1/2^-, 1P/2P)$ mass sum rule. The light-cone distribution amplitudes contained in the above sum rule expressions can be found in Refs. [99–106].

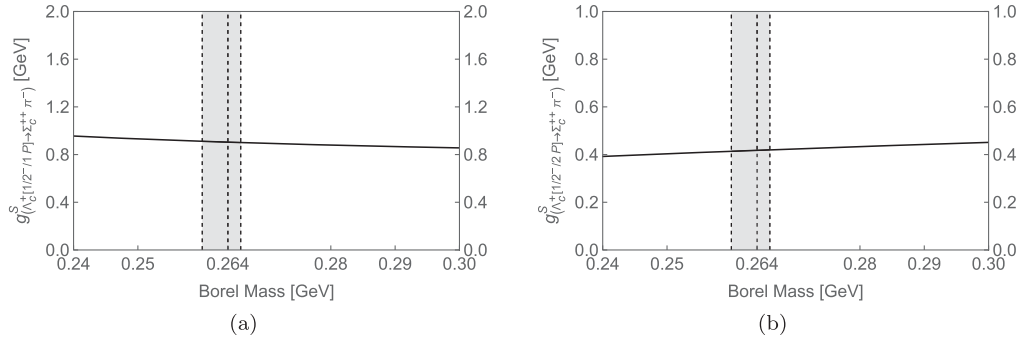


FIG. 7. Coupling constants as functions of the Borel mass T : (a) $g_{\Lambda_c^+(1/2^-, 1P) \rightarrow \Sigma_c^{++}\pi^-}^S$, and (b) $g_{\Lambda_c^+(1/2^-, 2P) \rightarrow \Sigma_c^{++}\pi^-}^S$.

We extract the coupling constant from Eq. (122) to be

$$g_{\Lambda_c^+[\frac{1}{2}^-, 1P] \rightarrow \Sigma_c^{++} \pi^-}^S = 0.90_{-0.90}^{+2.39},$$

$$g_{\Lambda_c^+[\frac{1}{2}^-, 2P] \rightarrow \Sigma_c^{++} \pi^-}^S = 0.42_{-0.42}^{+0.62}, \quad (122)$$

where the uncertainties are due to T , ω_c , ω'_c , parameters of $\Lambda_c(1/2^+, 1P)$, parameters of $\Sigma_c^{++}(1/2^+)$, and various QCD parameters given in Eq. (18). We depict $g_{\Lambda_c^+[\frac{1}{2}^-, 1P/2P] \rightarrow \Sigma_c^{++} \pi^0}^S$ in Fig. 7 as a function of the Borel mass T . Its dependence on T is weak inside the Borel window $0.260 \text{ GeV} < T < 0.266 \text{ GeV}$.

The S -wave decay widths of P -wave charmed baryons can be calculated through the Lagrangians expressed in Eq. (119):

- (i) Since the $\Lambda_c(1/2^-, 1P)$ is below the $\Sigma_c \pi$ threshold but above the $\Lambda_c \pi \pi$ threshold, we use the following formula for the three-body S -wave decay process $\Lambda_c(1/2^-, 1P) \rightarrow \Sigma_c + \pi \rightarrow \Lambda_c + \pi + \pi$:

$$\begin{aligned} & \Gamma(\Lambda_c(1/2^-, 1P) \rightarrow \Sigma_c + \pi) \\ &= \Gamma(0 \rightarrow 2 + 1) + 2 \times \Gamma(0 \rightarrow 6 + 3 \rightarrow 5 + 4 + 3) \\ &\equiv \Gamma(\Lambda_c^+(1/2^-, 1P) \rightarrow \Sigma_c^+ + \pi^0) + 2 \\ &\quad \times \Gamma(\Lambda_c^+(1/2^-, 1P) \rightarrow \Sigma_c^{++} + \pi^-) \\ &\rightarrow \Lambda_c^+ + \pi^+ + \pi^- \\ &= \frac{|\vec{p}_1|}{8\pi m_0^2} \times g_{0 \rightarrow 2+1}^2 \times \frac{1}{2} \text{Tr}[(\not{p}_0 + m_0)(\not{p}_2 + m_2)] \\ &\quad + 2 \times \frac{1}{(2\pi)^3} \times \frac{1}{32m_0^3} \times g_{0 \rightarrow 6+3}^2 \times g_{6 \rightarrow 5+4}^2 \\ &\quad \times \int dm_{34} dm_{45} \times \frac{1}{2} \text{Tr}[(\not{p}_5 + m_5) \gamma'_\nu \gamma_5 (\not{p}_6 + m_6) \\ &\quad \times (\not{p}_0 + m_0) (\not{p}_6 + m_6) \gamma_\mu \gamma_5] \\ &\quad \times \frac{1}{|p_6^2 - m_6^2 + im_6 \Gamma_6|^2} \times P_{4,\mu} P_{4,\mu'}. \end{aligned} \quad (123)$$

TABLE III. Decay properties of the $1P$ - and $2P$ -wave charmed baryons belonging to the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublets, calculated using the method of light-cone sum rules within the framework of heavy quark effective theory. Experimental candidates are given in the last column.

Baryon (j^P)	Mass [GeV]	Difference [MeV]	Decay channels	S -wave width [MeV]	D -wave width [MeV]	Total width [MeV]	Candidate	
$\Lambda_c(1P, \frac{1}{2}^-)$	2.61 ± 0.07	24 ± 9	$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c \pi \rightarrow \Lambda_c \pi \pi$	$4.9_{-4.9}^{+60.0}$...	5_{-5}^{+60}	$\Lambda_c(2595)$	
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c^* \pi \rightarrow \Lambda_c \pi \pi$...	4×10^{-8}			
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$		1×10^{-3}			
$\Lambda_c(1P, \frac{3}{2}^-)$	2.64 ± 0.07		$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c \pi$...	4×10^{-3}	$0.2_{-0.2}^{+2.4}$	$\Lambda_c(2625)$	
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c^* \pi \rightarrow \Lambda_c \pi \pi$	$0.20_{-0.20}^{+2.44}$	6×10^{-7}			
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$		0.02			
$\Lambda_c(2P, \frac{1}{2}^-)$	3.05 ± 0.12	18 ± 7	$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c \pi$	29_{-29}^{+148}	...	35_{-29}^{+148}	$\Lambda_c(2910)$	
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c^* \pi$...	$0.11_{-0.10}^{+0.26}$			
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$		$0.47_{-0.47}^{+1.51}$			
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c \rho \rightarrow \Sigma_c \pi \pi$		$5.0_{-4.1}^{+9.6}$			
$\Lambda_c(2P, \frac{3}{2}^-)$	3.07 ± 0.11		$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c \pi$...	$0.19_{-0.17}^{+0.45}$	29_{-26}^{+140}	$\Lambda_c(2940)$	
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c^* \pi$	26_{-26}^{+140}	0.02			
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$		$0.53_{-0.53}^{+1.59}$			
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c \rho \rightarrow \Sigma_c \pi \pi$		$1.4_{-1.1}^{+2.6}$			
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c^* \rho \rightarrow \Sigma_c^* \pi \pi$		$0.76_{-0.63}^{+1.47}$			
$\Xi_c(1P, \frac{1}{2}^-)$	2.78 ± 0.14	29 ± 10	$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi'_c \pi$	$3.1_{-3.1}^{+36.5}$...	3_{-3}^{+37}	$\Xi_c(2790)$	
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c^* \pi \rightarrow \Xi_c \pi \pi$...	2×10^{-5}			
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$		3×10^{-3}			
$\Xi_c(1P, \frac{3}{2}^-)$	2.81 ± 0.14		$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c^* \pi \rightarrow \Xi_c \pi \pi$	$0.54_{-0.54}^{+18.85}$	1×10^{-4}	$0.6_{-0.6}^{+18.9}$	$\Xi_c(2815)$	
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi'_c \pi$...	0.05			
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$		0.02			

(Table continued)

TABLE III. (Continued)

Baryon (J^P)	Mass [GeV]	Difference [MeV]	Decay channels	S-wave width [MeV]	D-wave width [MeV]	Total width [MeV]	Candidate
$\Xi_c(2P, \frac{1}{2}^-)$	3.16 ± 0.12	18 ± 7	$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi'_c \pi$	$2.7^{+22.5}_{-2.7}$...	31^{+170}_{-27}	...
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Sigma_c K$	27^{+168}_{-27}	...		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c^* \pi$...	0.02		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Sigma_c^* K$...	3×10^{-3}		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$...	$0.11^{+0.71}_{-0.11}$		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Lambda_c K^* \rightarrow \Lambda_c K \pi$...	0.02		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi'_c \rho \rightarrow \Xi'_c \pi \pi$...	$1.2^{+6.9}_{-1.2}$		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Sigma_c K^* \rightarrow \Sigma_c K \pi$...	1×10^{-3}		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c^* \rho \rightarrow \Xi_c^* \pi \pi$...	0.09		
			$\Xi_c(2P, \frac{3}{2}^-)$	3.18 ± 0.12	18 ± 7		
$\Xi_c(\frac{3}{2}^-) \rightarrow \Sigma_c^* K$	20^{+129}_{-20}	2×10^{-3}					
$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi'_c \pi$...	0.03					
$\Xi_c(\frac{3}{2}^-) \rightarrow \Sigma_c K$...	0.02					
$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$...	$0.10^{+0.64}_{-0.10}$					
$\Xi_c(\frac{3}{2}^-) \rightarrow \Lambda_c K^* \rightarrow \Lambda_c K \pi$...	0.03					
$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi'_c \rho \rightarrow \Xi'_c \pi \pi$...	$0.27^{+1.52}_{-0.27}$					
$\Xi_c(\frac{3}{2}^-) \rightarrow \Sigma_c K^* \rightarrow \Sigma_c K \pi$...	2×10^{-3}					
$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c^* \rho \rightarrow \Xi_c^* \pi \pi$...	$0.18^{+1.01}_{-0.18}$					

- (ii) Since the $\Lambda_c(1/2^-, 2P)$ is above the $\Sigma_c \pi$ threshold, we use the following formula for the two-body S-wave decay process $\Lambda_c(1/2^-, 2P) \rightarrow \Sigma_c + \pi$:

$$\begin{aligned}
& \Gamma(\Lambda_c(1/2^-, 2P) \rightarrow \Sigma_c + \pi) \\
&= 3 \times \Gamma(0 \rightarrow 6 + 3) \\
&\equiv 3 \times \Gamma(\Lambda_c^+(1/2^-, 2P) \rightarrow \Sigma_c^{++} + \pi^-) \\
&= \frac{|\vec{P}_3|}{8\pi m_0^2} \times g_{0 \rightarrow 6+3}^2 \times \frac{1}{2} \text{Tr}[(p_\emptyset + m_0)(p_\emptyset + m_6)],
\end{aligned} \tag{124}$$

where 0 denotes the initial $\Lambda_c(1/2^-, 1P/2P)$; 1 and 2 denote the final states π^0 and $\Sigma_c^+(1/2^+)$, respectively; 3, 4, and 5 denote the final states π^-, π^+ , and $\Lambda_c(1/2^+)$, respectively; and 6 denotes the middle (final) state $\Sigma_c^{++}(1/2^+)$.

We calculate their partial decay widths to be

$$\begin{aligned}
\Gamma_{\Lambda_c^+[\frac{1}{2}^-, 1P] \rightarrow \Sigma_c \pi}^S &= 4.9^{+60.0}_{-4.9} \text{ MeV}, \\
\Gamma_{\Lambda_c^+[\frac{1}{2}^-, 2P] \rightarrow \Sigma_c \pi}^S &= 29^{+148}_{-29} \text{ MeV}.
\end{aligned} \tag{125}$$

Similarly, we study the other decay channels for the charmed baryons belonging to the $[\bar{3}_F, 1, 1, \rho]$ and $[\bar{3}_F, 1, 0, \lambda]$ doublets. Their partial decay widths are evaluated and summarized in Tables III and V.

IV. DISCUSSIONS AND SUMMARY

In this paper we investigate the $\Lambda_c(2910)^+$, $\Lambda_c(2940)^+$, and $\Xi_c(3123)^+$ as potential 2P-wave charmed baryons with

$J^P = 1/2^-$ and $3/2^-$, all of which belong to the $[\bar{3}_F, 1, 1, \rho]$ doublet. This doublet contains four P-wave charmed baryons, namely, $\Lambda_c(1/2^-)$, $\Lambda_c(3/2^-)$, $\Xi_c(1/2^-)$, and $\Xi_c(3/2^-)$. We make the assumption that the current $J_{\bar{3}_F, 1, 1, \rho}$ can simultaneously couple to 1P- and 2P-wave charmed baryons, consequently leading to the possible production of 1P- and 2P-wave excitations such as $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Xi_c(2790)^+$, $\Xi_c(2815)^+$, $\Lambda_c(2910)^+$, $\Lambda_c(2940)^+$, and $\Xi_c(3123)^+$. The masses of these baryons are computed utilizing the QCD sum rule method within the framework of heavy quark effective theory. Additionally, we analyze their decays into ground-state charmed baryons as well as the light pseudoscalar mesons π/K and vector mesons ρ/K^* using the light-cone sum rule method for S- and D-wave decays. Based on these results, we derive the masses and total widths of the P-wave charmed baryons, facilitating a comprehensive understanding of their properties as a whole:

$$\begin{aligned}
M_{\Lambda_c(\frac{1}{2}^-, 1P)} &= 2.61 \pm 0.07 \text{ GeV}, \\
\Gamma_{\Lambda_c(\frac{1}{2}^-, 1P)} &= 5^{+60}_{-5} \text{ MeV}, \\
M_{\Lambda_c(\frac{3}{2}^-, 1P)} &= 2.63 \pm 0.07 \text{ GeV}, \\
\Gamma_{\Lambda_c(\frac{3}{2}^-, 1P)} &= 0.2^{+2.4}_{-0.2} \text{ MeV}, \\
M_{\Lambda_c(\frac{3}{2}^-, 1P)} - M_{\Lambda_c(\frac{1}{2}^-, 1P)} &= 24 \pm 9 \text{ MeV},
\end{aligned} \tag{126}$$

$$\begin{aligned}
M_{\Lambda_c(\frac{1}{2}^-, 2P)} &= 3.05 \pm 0.12 \text{ GeV}, \\
\Gamma_{\Lambda_c(\frac{1}{2}^-, 2P)} &= 35_{-29}^{+148} \text{ MeV}, \\
M_{\Lambda_c(\frac{3}{2}^-, 2P)} &= 3.07 \pm 0.11 \text{ GeV}, \\
\Gamma_{\Lambda_c(\frac{3}{2}^-, 2P)} &= 29_{-26}^{+140} \text{ MeV}, \\
M_{\Lambda_c(\frac{3}{2}^-, 2P)} - M_{\Lambda_c(\frac{1}{2}^-, 2P)} &= 18 \pm 7 \text{ MeV}, \\
M_{\Xi_c(\frac{1}{2}^-, 1P)} &= 2.78 \pm 0.14 \text{ GeV}, \\
\Gamma_{\Xi_c(\frac{1}{2}^-, 1P)} &= 3_{-3}^{+37} \text{ MeV}, \\
M_{\Xi_c(\frac{3}{2}^-, 1P)} &= 2.81 \pm 0.14 \text{ GeV}, \\
\Gamma_{\Xi_c(\frac{3}{2}^-, 1P)} &= 0.6_{-0.6}^{+18.9} \text{ MeV}, \\
M_{\Xi_c(\frac{3}{2}^-, 1P)} - M_{\Xi_c(1P/\frac{1}{2}^-)} &= 29 \pm 10 \text{ MeV}, \\
M_{\Xi_c(\frac{1}{2}^-, 2P)} &= 3.16 \pm 0.12 \text{ GeV}, \\
\Gamma_{\Xi_c(\frac{1}{2}^-, 2P)} &= 31_{-27}^{+170} \text{ MeV}, \\
M_{\Xi_c(\frac{3}{2}^-, 2P)} &= 3.18 \pm 0.12 \text{ GeV}, \\
\Gamma_{\Xi_c(\frac{3}{2}^-, 2P)} &= 24_{-20}^{+131} \text{ MeV}, \\
M_{\Xi_c(\frac{3}{2}^-, 2P)} - M_{\Xi_c(\frac{1}{2}^-, 2P)} &= 18 \pm 7 \text{ MeV}. \tag{127}
\end{aligned}$$

Our results suggest that the $\Lambda_c(2910)^+$, $\Lambda_c(2940)^+$, and $\Xi_c(3123)^+$ can be suitably explained as the $2P$ -wave charmed baryon of $J^P = 1/2^-$ and $3/2^-$, belonging to the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublet. However, the expected masses of $\Lambda_c(2910)$ and $\Lambda_c(2940)$ surpass experimental measurements by approximately 130 MeV, although consistent with the results obtained in the quark models, which suggests that their masses are expected to exceed 3 GeV [35,36,63]. Additionally, our analysis of their decay properties is detailed in Table III. For the $\Lambda_c(2910)$ and $\Lambda_c(2940)$, in fact, they not only can decay into charmed baryons and light mesons but also can decay into charmed mesons and light baryons. In this work, we only investigate the former so the total width of $\Lambda_c(2P, \frac{1}{2}^-)$ is about 20 MeV smaller than the measurement of $\Lambda_c(2910)^+$. Accounting for its S -wave decay into the $D^0 p$ final state would result in a greater width. Similarly, the $\Lambda_c(\frac{3}{2}^-, 2P)$ can also decay into $D^0 p$ through a D -wave decay process, resulting in a slight increase in its total width, which is consistent with the experimental result of $\Lambda_c(2940)$ within the margin of error. The $\Xi_c(3123)^+$ could be interpreted as the $\Lambda_c(2940)^+$'s charmed-strange partner state with $J^P = 3/2^-$. Furthermore, a $2P$ -wave charmed-strange baryon, the $\Xi_b(\frac{1}{2}^-, 2P)$, with a mass $\Delta M = 18 \pm 7$ MeV smaller than the $\Xi_c(3123)^+$, remains undiscovered. We propose to explore its presence in the $\Xi_c(\frac{1}{2}^-, 2P) \rightarrow \Sigma_c K$ decay channel.

Apart from the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublet, we also investigate $1P$ - and $2P$ -wave charmed baryons belonging to the $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ doublet. The results, summarized in Appendix, indicate that the masses of $\Lambda_c(2910)^+$, $\Lambda_c(2940)^+$,

$\Xi_c(2790)$, $\Xi_c(2815)$, and $\Xi_c(3123)^+$ alone cannot distinguish whether they belong to the $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublet or the $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ doublet. However, considering their decay properties, the $1P$ - and $2P$ -wave charmed baryons belonging to the $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ doublet exhibit wide widths, rendering them unobservable experimentally due to their extremely large coupling constants at the vertexes of their S -wave decay processes. Furthermore, our QCD sum rule results for the $\Lambda_c(2595)$, $\Lambda_c(2625)$, $\Lambda_c(2910)$, $\Lambda_c(2940)$, $\Xi_c(2790)$, $\Xi_c(2815)$, and $\Xi_c(3123)$ appear inconsistent with the quark model's expectations, where the ρ -mode multiplet $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ is anticipated to be higher in mass than the λ -mode multiplet $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$. This discrepancy could be attributed to considerable uncertainties in the mass differences between different multiplets within our framework. Furthermore, our previous study [85] suggests that the $\Lambda_c(2595)$, $\Lambda_c(2635)$, $\Xi_c(2790)$, and $\Xi_c(2815)$ could potentially be explained as singly heavy baryons belonging to $[\bar{\mathbf{3}}_F, 1, 1, \rho]$ doublet. Thus, it is crucial to verify the existence of ρ -mode heavy baryons, and we recommend to conduct an investigation into the existence of the ρ -mode. For instance, these exist four excited Ω_c baryons that are more likely to belong to the λ -mode multiplets in five Ω_c baryons observed by LHCb [107], while the possible assignment for the rest particle is either the radial $2S$ -wave excitation or the orbital $1P$ -wave excitation of the ρ -mode [97,98].

Summarizing the above results, we study the $1P$ and $2P$ -wave charmed baryons using the methods of QCD sum rules and light-cone sum rules within the framework of heavy quark effective theory. Our results suggest that the $\Lambda_c(2910)^+$, $\Lambda_c(2940)^+$, and $\Xi_c(3123)^+$ can be well interpreted as the $2P$ -wave charmed baryon of $J^P = 1/2^-$ and $3/2^-$, belonging to the $SU(3)$ flavor $\bar{\mathbf{3}}_F$ representation. The $\Xi_c(3123)^+$ has a partner state of $J^P = 1/2^-$, labeled as $\Xi_c(1/2^-, 2P)$, whose mass and width are calculated to be $m_{\Xi_c(1/2^-, 2P)} - m_{\Xi_c(3123)^+} = -18 \pm 7$ MeV and $\Gamma_{\Xi_c(2P, 1/2^-)} = 31_{-27}^{+170}$ MeV, with $m_{\Xi_c(3123)^+} = 3122.9 \pm 1.3$ MeV. We suggest to search for this state in the $\Xi_c(2P, 1/2^-) \rightarrow \Sigma_c K$ decay channel.

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APPENDIX: 1P AND 2P-WAVE CHARMED BARYONS FROM DOUBLET $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$

In this appendix we study the $1P$ - and $2P$ -wave charmed baryons belonging to the $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ doublets. We apply the

TABLE IV. Parameters of the $1P$ - and $2P$ -wave charmed baryons belonging to the $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ doublets, calculated using the method of QCD sum rules within the framework of heavy quark effective theory. Decay constants in the last column satisfy $f_{\Xi_c^+} = f_{\Xi_c^0}$.

B	ω_c [GeV]	Working region [GeV]	$\bar{\Lambda}$ [GeV]	Baryon (j^P)	Mass [GeV]	Difference [MeV]	Decay constant [GeV ⁴]
$\Lambda_c(1P)$	1.00	$T = 0.279$	0.99 ± 0.01	$\Lambda_c(1/2^-)$	2.76 ± 0.03	7 ± 3	$0.016 \pm 0.002(\Lambda_c^+(1/2^-))$
				$\Lambda_c(3/2^-)$	2.77 ± 0.03		$0.007 \pm 0.001(\Lambda_c^+(3/2^-))$
$\Xi_c(1P)$	1.28	$T = 0.271$	1.07 ± 0.03	$\Xi_c(1/2^-)$	2.81 ± 0.04	9 ± 3	$0.022 \pm 0.003(\Xi_c^0(1/2^-))$
				$\Xi_c(3/2^-)$	2.82 ± 0.04		$0.010 \pm 0.001(\Xi_c^0(3/2^-))$
$\Lambda_c(2P)$	1.53	$T = 0.279$	1.31 ± 0.07	$\Lambda_c(1/2^-)$	3.08 ± 0.12	13 ± 4	$0.026 \pm 0.007(\Lambda_c^+(1/2^-))$
				$\Lambda_c(3/2^-)$	3.09 ± 0.12		$0.012 \pm 0.003(\Lambda_c^+(3/2^-))$
$\Xi_c(2P)$	1.46	$T = 0.271$	1.37 ± 0.07	$\Xi_c(1/2^-)$	3.12 ± 0.11	14 ± 4	$0.019 \pm 0.009(\Xi_c^+(1/2^-))$
				$\Xi_c(3/2^-)$	3.13 ± 0.11		$0.009 \pm 0.004(\Xi_c^+(3/2^-))$

QCD sum rule method to study their mass spectrum, and the obtained results are summarized in Table IV. We apply the light-cone sum rule method to study their decay properties, and the obtained results are summarized in Table V. These results suggest that the charmed

baryons $\Lambda_c(\frac{1}{2}^-, 1P/2P)$, $\Lambda_c(\frac{3}{2}^-, 1P/2P)$, $\Xi_c(\frac{1}{2}^-, 1P)$ and $\Xi_c(\frac{3}{2}^-, 1P/2P)$ belonging to the $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ doublet cannot be easily used to explain the $\Lambda_c(2595)$, $\Lambda_c(2625)$, $\Lambda_c(2910)$, $\Lambda_c(2940)$, $\Xi_c(2790)$, $\Xi_c(2815)$, and $\Xi_c(3123)$.

TABLE V. Decay properties of the $1P$ - and $2P$ -wave charmed baryons belonging to the $[\bar{\mathbf{3}}_F, 1, 0, \lambda]$ doublets, calculated using the method of light-cone sum rules within the framework of heavy quark effective theory.

Baryon (j^P)	Mass [GeV]	Difference [MeV]	Decay channels	S-wave width [MeV]	D-wave width [MeV]	Total width [MeV]
$\Lambda_c(1P, \frac{1}{2}^-)$	2.76 ± 0.03	7 ± 3	$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c \pi \rightarrow \Lambda_c \pi \pi$	>500	\dots	>500
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c^* \pi \rightarrow \Lambda_c \pi \pi$	\dots	2×10^{-6}	
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$	1×10^{-3}		
$\Lambda_c(1P, \frac{3}{2}^-)$	2.77 ± 0.03		$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c \pi$	\dots	0.06	29_{-21}^{+41}
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c^* \pi \rightarrow \Lambda_c \pi \pi$	29_{-21}^{+41}	3×10^{-5}	
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$	0.01		
$\Lambda_c(2P, \frac{1}{2}^-)$	3.08 ± 0.12	13 ± 4	$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c \pi$	>2000	\dots	>2000
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c^* \pi$	\dots	$0.92_{-0.74}^{+1.77}$	
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$	$1.8_{-1.3}^{+2.7}$		
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c \rho \rightarrow \Sigma_c \pi \pi$	3×10^{-3}		
			$\Lambda_c(\frac{1}{2}^-) \rightarrow \Sigma_c^* \rho \rightarrow \Sigma_c^* \pi \pi$	1×10^{-4}		
$\Lambda_c(2P, \frac{3}{2}^-)$	3.09 ± 0.12		$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c \pi$	\dots	$1.5_{-1.2}^{+2.8}$	>500
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c^* \pi$	>500	$0.19_{-0.15}^{+0.36}$	
			$\Lambda_c(\frac{3}{2}^-) \rightarrow \Lambda_c \rho \rightarrow \Lambda_c \pi \pi$	$1.7_{-1.2}^{+2.6}$		
$\Lambda_c(\frac{3}{2}^-) \rightarrow \Sigma_c^* \rho \rightarrow \Sigma_c^* \pi \pi$	8×10^{-5}					
$\Xi_c(1P, \frac{1}{2}^-)$	2.81 ± 0.04	9 ± 3	$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c' \pi$	>1800	\dots	>1800
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c^* \pi \rightarrow \Xi_c \pi \pi$	\dots	2×10^{-4}	
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$	6×10^{-3}		
$\Xi_c(1P, \frac{3}{2}^-)$	2.82 ± 0.04		$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c^* \pi \rightarrow \Xi_c \pi \pi$	>300	1×10^{-3}	>300
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c' \pi$	\dots	$0.23_{-0.17}^{+0.35}$	
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$	0.02		

(Table continued)

TABLE V. (Continued)

Baryon (j^P)	Mass [GeV]	Difference [MeV]	Decay channels	S-wave width [MeV]	D-wave width [MeV]	Total width [MeV]
$\Xi_c(2P, \frac{1}{2}^-)$	3.12 ± 0.11	14 ± 4	$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi'_c \pi$	900^{+3700}_{-900}	...	2200^{+7100}_{-1600}
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Sigma_c K$	1300^{+6100}_{-1300}	...	
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi'_c \pi$...	$0.12^{+0.51}_{-0.12}$	
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Sigma'_c K$...	0.03	
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$	$4.7^{+14.0}_{-3.8}$		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Lambda_c K^* \rightarrow \Lambda_c K \pi$	$0.19^{+0.77}_{-0.19}$		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Xi'_c \rho \rightarrow \Xi'_c \pi \pi$	6×10^{-4}		
			$\Xi_c(\frac{1}{2}^-) \rightarrow \Sigma_c K^* \rightarrow \Sigma_c K \pi$	6×10^{-6}		
$\Xi_c(2P, \frac{3}{2}^-)$	3.13 ± 0.11		$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi'_c \pi$	200^{+700}_{-200}	0.02	500^{+1200}_{-400}
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Sigma'_c K$	300^{+1000}_{-300}	0.01	
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi'_c \pi$...	$0.15^{+0.62}_{-0.15}$	
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Sigma_c K$...	$0.17^{+0.72}_{-0.17}$	
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi_c \rho \rightarrow \Xi_c \pi \pi$	$0.42^{+1.64}_{-0.42}$		
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Lambda_c K^* \rightarrow \Lambda_c K \pi$	$0.20^{+0.78}_{-0.19}$		
			$\Xi_c(\frac{3}{2}^-) \rightarrow \Xi'_c \rho \rightarrow \Xi'_c \pi \pi$	1×10^{-5}		

- [1] S. Triebwasser, E. S. Dayhoff, and W. E. Lamb, Jr., *Phys. Rev.* **89**, 98 (1953).
- [2] E. S. Dayhoff, S. Triebwasser, and W. E. Lamb, *Phys. Rev.* **89**, 106 (1953).
- [3] H. C. Schroder, A. Badertscher, P. F. A. Goudsmit, M. Janousch, H. J. Leisi, E. Matsinos, D. Sigg, Z. G. Zhao, D. Chatellard, J. P. Egger *et al.*, *Eur. Phys. J. C* **21**, 473 (2001).
- [4] A. A. Krutov and A. P. Martynenko, *Phys. Rev. A* **84**, 052514 (2011).
- [5] U. G. Meißner, B. C. Metsch, and H. Meyer, *Eur. Phys. J. A* **59**, 223 (2023).
- [6] N. Isgur and G. Karl, *Phys. Rev. D* **19**, 2653 (1979); **23**, 817(E) (1981).
- [7] L. A. Copley, N. Isgur, and G. Karl, *Phys. Rev. D* **20**, 768 (1979); **23**, 817(E) (1981).
- [8] J. G. Korner, M. Kramer, and D. Pirjol, *Prog. Part. Nucl. Phys.* **33**, 787 (1994).
- [9] A. V. Manohar and M. B. Wise, *Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol.* **10**, 1 (2000).
- [10] S. Bianco, F. L. Fabbri, D. Benson, and I. Bigi, *Riv. Nuovo Cimento* **26**, 1 (2003).
- [11] M. Karliner, B. Keren-Zur, H. J. Lipkin, and J. L. Rosner, *Ann. Phys. (Amsterdam)* **324**, 2 (2009).
- [12] E. Klempt and J. M. Richard, *Rev. Mod. Phys.* **82**, 1095 (2010).
- [13] P. A. Zyla *et al.* (Particle Data Group), *Prog. Theor. Exp. Phys.* **2020**, 083C01 (2020).
- [14] K. W. Edwards *et al.* (CLEO Collaboration), *Phys. Rev. Lett.* **74**, 3331 (1995).
- [15] H. Albrecht *et al.* (ARGUS Collaboration), *Phys. Lett. B* **317**, 227 (1993).
- [16] S. E. Csorna *et al.* (CLEO Collaboration), *Phys. Rev. Lett.* **86**, 4243 (2001).
- [17] J. P. Alexander *et al.* (CLEO Collaboration), *Phys. Rev. Lett.* **83**, 3390 (1999).
- [18] M. Artuso *et al.* (CLEO Collaboration), *Phys. Rev. Lett.* **86**, 4479 (2001).
- [19] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **05** (2017) 030.
- [20] Y. B. Li *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **130**, 031901 (2023).
- [21] R. Chistov *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **97**, 162001 (2006).
- [22] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. D* **77**, 012002 (2008).
- [23] Y. Kato *et al.* (Belle Collaboration), *Phys. Rev. D* **94**, 032002 (2016).
- [24] B. Aubert *et al.* (BABAR Collaboration), *Phys. Rev. Lett.* **98**, 012001 (2007).
- [25] K. Abe *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **98**, 262001 (2007).
- [26] N. Isgur and G. Karl, *Phys. Rev. D* **18**, 4187 (1978).
- [27] D. Ebert, R. N. Faustov, and V. O. Galkin, *Phys. Rev. D* **72**, 034026 (2005).

- [28] W. Roberts and M. Pervin, *Int. J. Mod. Phys. A* **23**, 2817 (2008).
- [29] X. H. Zhong and Q. Zhao, *Phys. Rev. D* **77**, 074008 (2008).
- [30] R. Bijker, H. García-Tecocoatzí, A. Giachino, E. Ortiz-Pacheco, and E. Santopinto, *Phys. Rev. D* **105**, 074029 (2022).
- [31] B. Chen and X. Liu, *Phys. Rev. D* **98**, 074032 (2018).
- [32] B. Chen, K. W. Wei, X. Liu, and A. Zhang, *Phys. Rev. D* **98**, 031502 (2019).
- [33] K. L. Wang, Q. F. Lü, and X. H. Zhong, *Phys. Rev. D* **99**, 014011 (2019).
- [34] L. Y. Xiao and X. H. Zhong, *Phys. Rev. D* **102**, 014009 (2020).
- [35] D. Ebert, R. N. Faustov, and V. O. Galkin, *Phys. Rev. D* **84**, 014025 (2011).
- [36] B. Chen, K. W. Wei, X. Liu, and T. Matsuki, *Eur. Phys. J. C* **77**, 154 (2017).
- [37] P. Yang, J. J. Guo, and A. Zhang, *Phys. Rev. D* **99**, 034018 (2019).
- [38] Q. F. Lü, *Eur. Phys. J. C* **80**, 921 (2020).
- [39] K. L. Wang, L. Y. Xiao, and X. H. Zhong, *Phys. Rev. D* **102**, 034029 (2020).
- [40] K. L. Wang, Q. F. Lü, and X. H. Zhong, *Phys. Rev. D* **100**, 114035 (2019).
- [41] S. Q. Luo, B. Chen, Z. W. Liu, and X. Liu, *Eur. Phys. J. C* **80**, 301 (2020).
- [42] M. B. Wise, *Phys. Rev. D* **45**, R2188 (1992).
- [43] S. Scherer, *Adv. Nucl. Phys.* **27**, 277 (2003).
- [44] H. Y. Cheng and C. K. Chua, *Phys. Rev. D* **75**, 014006 (2007).
- [45] J. X. Lu, Y. Zhou, H. X. Chen, J. J. Xie, and L. S. Geng, *Phys. Rev. D* **92**, 014036 (2015).
- [46] H. Y. Cheng and C. K. Chua, *Phys. Rev. D* **92**, 074014 (2015).
- [47] N. Jiang, X. L. Chen, and S. L. Zhu, *Phys. Rev. D* **92**, 054017 (2015).
- [48] L. Meng, B. Wang, and S. L. Zhu, *Phys. Rev. C* **101**, 064002 (2020).
- [49] B. Wang, L. Meng, and S. L. Zhu, *Phys. Rev. D* **101**, 094035 (2020).
- [50] R. Machleidt and D. R. Entem, *Phys. Rep.* **503**, 1 (2011).
- [51] Y. Huang, C. j. Xiao, L. S. Geng, and J. He, *Phys. Rev. D* **99**, 014008 (2019).
- [52] W. H. Liang, J. M. Dias, V. R. Debastiani, and E. Oset, *Nucl. Phys.* **B930**, 524 (2018).
- [53] R. Chen, A. Hosaka, and X. Liu, *Phys. Rev. D* **97**, 036016 (2018).
- [54] W. H. Liang, C. W. Xiao, and E. Oset, *Phys. Rev. D* **89**, 054023 (2014).
- [55] C. S. An and H. Chen, *Phys. Rev. D* **96**, 034012 (2017).
- [56] V. R. Debastiani, J. M. Dias, W. H. Liang, and E. Oset, *Phys. Rev. D* **97**, 094035 (2018).
- [57] Z. L. Zhang, Z. W. Liu, S. Q. Luo, F. L. Wang, B. Wang, and H. Xu, *Phys. Rev. D* **107**, 034036 (2023).
- [58] X. H. Guo, K. W. Wei, and X. H. Wu, *Phys. Rev. D* **78**, 056005 (2008).
- [59] C. Chen, X. L. Chen, X. Liu, W. Z. Deng, and S. L. Zhu, *Phys. Rev. D* **75**, 094017 (2007).
- [60] D. D. Ye, Z. Zhao, and A. Zhang, *Phys. Rev. D* **96**, 114009 (2017).
- [61] Q. F. Lü and X. H. Zhong, *Phys. Rev. D* **101**, 014017 (2020).
- [62] C. Olson, M. G. Olsson, and K. Williams, *Phys. Rev. D* **45**, 4307 (1992).
- [63] B. Chen, K. W. Wei, and A. Zhang, *Eur. Phys. J. A* **51**, 82 (2015).
- [64] T. M. Aliev, K. Azizi, Y. Sarac, and H. Sundu, *Phys. Rev. D* **99**, 094003 (2019).
- [65] Z. G. Wang, *Int. J. Mod. Phys. A* **35**, 2050043 (2020).
- [66] S. S. Agaev, K. Azizi, and H. Sundu, *Eur. Phys. J. A* **57**, 201 (2021).
- [67] K. Azizi, Y. Sarac, and H. Sundu, *Phys. Rev. D* **101**, 074026 (2020).
- [68] G. L. Yu, Z. G. Wang, and X. W. Wang, *Chin. Phys. C* **46**, 093102 (2022).
- [69] K. Azizi, Y. Sarac, and H. Sundu, *Eur. Phys. J. C* **82**, 920 (2022).
- [70] X. Liu, H. X. Chen, Y. R. Liu, A. Hosaka, and S. L. Zhu, *Phys. Rev. D* **77**, 014031 (2008).
- [71] Q. Mao, H. X. Chen, W. Chen, A. Hosaka, X. Liu, and S. L. Zhu, *Phys. Rev. D* **92**, 114007 (2015).
- [72] H. X. Chen, Q. Mao, W. Chen, A. Hosaka, X. Liu, and S. L. Zhu, *Phys. Rev. D* **95**, 094008 (2017).
- [73] U. Ozdem, *Eur. Phys. J. C* **83**, 1077 (2023).
- [74] M. Padmanath, R. G. Edwards, N. Mathur, and M. Peardon, *arXiv:1311.4806*.
- [75] T. Burch, *arXiv:1502.00675*.
- [76] P. Pérez-Rubio, S. Collins, and G. S. Bali, *Phys. Rev. D* **92**, 034504 (2015).
- [77] M. Padmanath and N. Mathur, *Phys. Rev. Lett.* **119**, 042001 (2017).
- [78] Y. X. Yao, K. L. Wang, and X. H. Zhong, *Phys. Rev. D* **98**, 076015 (2018).
- [79] J. J. Guo, P. Yang, and A. Zhang, *Phys. Rev. D* **100**, 014001 (2019).
- [80] V. Crede and W. Roberts, *Rep. Prog. Phys.* **76**, 076301 (2013).
- [81] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, *Rep. Prog. Phys.* **80**, 076201 (2017).
- [82] Y. R. Liu, H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, *Prog. Part. Nucl. Phys.* **107**, 237 (2019).
- [83] H. Y. Cheng, *Chin. J. Phys.* **78**, 324 (2022).
- [84] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, *Rep. Prog. Phys.* **86**, 026201 (2023).
- [85] H. M. Yang, H. X. Chen, E. L. Cui, and Q. Mao, *Phys. Rev. D* **106**, 036018 (2022).
- [86] H. X. Chen, W. Chen, Q. Mao, A. Hosaka, X. Liu, and S. L. Zhu, *Phys. Rev. D* **91**, 054034 (2015).
- [87] K. C. Yang, W. Y. P. Hwang, E. M. Henley, and L. S. Kisslinger, *Phys. Rev. D* **47**, 3001 (1993).
- [88] W. Y. P. Hwang and K. C. Yang, *Phys. Rev. D* **49**, 460 (1994).
- [89] A. A. Ovchinnikov and A. A. Pivovarov, *Sov. J. Nucl. Phys.* **48**, 721 (1988).
- [90] M. Jamin, *Phys. Lett. B* **538**, 71 (2002).
- [91] B. L. Ioffe and K. N. Zyblyuk, *Eur. Phys. J. C* **27**, 229 (2003).

- [92] V. Gimenez, V. Lubicz, F. Mescia, V. Porretti, and J. Reyes, *Eur. Phys. J. C* **41**, 535 (2005).
- [93] P. Colangelo, F. De Fazio, and N. Paver, *Phys. Rev. D* **58**, 116005 (1998).
- [94] H. M. Yang and H. X. Chen, *Phys. Rev. D* **104**, 034037 (2021).
- [95] H. M. Yang and H. X. Chen, *Phys. Rev. D* **101**, 114013 (2020); **102**, 079901(E) (2020).
- [96] H. M. Yang, H. X. Chen, and Q. Mao, *Phys. Rev. D* **102**, 114009 (2020).
- [97] T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, and K. Sadato, *Phys. Rev. D* **92**, 114029 (2015).
- [98] H. Nagahiro, S. Yasui, A. Hosaka, M. Oka, and H. Noumi, *Phys. Rev. D* **95**, 014023 (2017).
- [99] P. Ball, *J. High Energy Phys.* 01 (1999) 010.
- [100] P. Ball, V. M. Braun, and A. Lenz, *J. High Energy Phys.* 05 (2006) 004.
- [101] P. Ball and R. Zwicky, *Phys. Rev. D* **71**, 014029 (2005).
- [102] P. Ball and V. M. Braun, *Phys. Rev. D* **58**, 094016 (1998).
- [103] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, *Nucl. Phys.* **B529**, 323 (1998).
- [104] P. Ball and V. M. Braun, *Nucl. Phys.* **B543**, 201 (1999).
- [105] P. Ball and G. W. Jones, *J. High Energy Phys.* 03 (2007) 069.
- [106] P. Ball, V. M. Braun, and A. Lenz, *J. High Energy Phys.* 08 (2007) 090.
- [107] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **118**, 182001 (2017).