QED contributions to $\Xi_c^+ - \Xi_c'^+$ mixing

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We explore the quantum electrodynamic (QED) corrections to the $\Xi_c^+ - \Xi_c'^+$ mixing within the framework of light-front quark model (LFQM) in the three-quark picture. After explicitly investigating the relation between the $\Xi_c - \Xi_c'$ mixing and the flavor SU(3) and heavy quark symmetry breaking, we derive the QED contributions to the $\Xi_c^+ - \Xi_c'^+$ mixing angle. Numerical results indicate the QED contribution is smaller than the one from the mass difference between the strange and up/down quark provided by a recent lattice quantum chromodynamics analysis. Adding these contributions together we find that at this stage the $\Xi_c^+ - \Xi_c'^+$ mixing is small and still incapable to account for the large SU(3) symmetry breaking in the semileptonic Ξ_c decays.

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I. INTRODUCTION

Weak decays of heavy baryons provide an ideal platform for understanding the strong interaction and searching for new physics beyond the standard model. One of the widely used theoretical methods in this area is the flavor $SU(3)$ symmetry, where the u, d, s quark are treated as identical [1–10]. Using the flavor $SU(3)$ symmetry one can classify the singly heavy baryons into antitriplet and sextet, with the two light quarks inside the baryon forming a scalar and an axial-vector, respectively. However, since the u, d, s quarks have different masses and electric charges, the flavor $SU(3)$ symmetry is broken. Therefore, the antitriplet and sextet defined by $SU(3)$ symmetry are not the physical states. A physical singly heavy baryon state must be a mixture of the corresponding antitriplet and sextet states.

In the study of semi-leptonic charmed baryons decays, recent experimental measurements on the decay width by BESIII and Belle collaborations imply significant flavor $SU(3)$ symmetry breaking [11–14], while a recent lattice QCD calculation of transition form factors finds less severe symmetry breaking [15]. To understand this phenomenon, various possible mechanisms were explored in Ref. [16], with a very compelling contender being the incorporation of $\Xi_c - \Xi_c'$ mixing [17]. It was found that to realize the sizable flavor $SU(3)$ breaking in the semi-leptonic Ξ_c decays, a large $\Xi_c - \Xi_c'$ mixing angle is required: $\theta = 24.66° \pm 0.90°$ [17,18] and $\theta = 16.27° \pm 2.30°$ [19]. Based on these observations, a method to measure this mixing angle in a four-body Ξ_c decay has been proposed by Ref. [20]. On the other hand, direct calculation using QCD sum rules gives $\theta = 5.5° \pm 1.8°$ [21] and $\theta = 2.0° \pm 0.8°$ [22]; A recent lattice QCD calculation gives $\theta = 1.2° \pm 0.1°$ [23], confirmed by an improved determination [24]. Heavy quark effective theory gives $\theta = 8.12° \pm 0.80°$ [25].

In this work, we will concentrate on the QED corrections for the $\Xi_c^+ - \Xi_c'^+$ mixing which have not been taken into account in the previous investigations. It should be noticed that since QED does not break symmetry between the d and s quarks, i.e., U-spin, the $\Xi_c^0 - \Xi_c'^0$ mixing will not be affected by the QED contribution. Unlike QCD, the QED contributions cannot be directly calculated on the lattice. Therefore, we will employ the light-front quark model (LFQM) in the three-quark picture [26–29] to evaluate the corresponding matrix element at the leading order in $\alpha_{\rm EM}$. We will also explore the dependence of the mixing angle on the LFQM parameters to ensure the result is stable.

The rest of this paper is organized as follows. In Sec. II, we will introduce the $SU(3)$ breaking Lagrangian, and construct the necessary matrix elements of the $SU(3)$ breaking Hamiltonian for extracting the $\Xi_c - \Xi_c'$ mixing

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angle. An introduction to the three-quark light-front quark model and the calculation of QED contributions will be given in Secs. III and IV. Numerical results for the $\Xi_c^+ - \Xi_c'^+$ mixing angle and the dependence on the input parameters are given in Sec. V. The last paragraph contains a brief summary of this work.

II. THE $\Xi_c - \Xi_c'$ MIXING

The flavor SU(3) symmetry is very convenient to analyse the weak decay amplitudes and for some applications, one can see Refs. [1–3,30–34]. Despite of its success, the flavor $SU(3)$ symmetry is an approximate symmetry which are broken by the u, d, s quark mass differences and QED contributions. In the following we assume the same mass for up and down quarks, and accordingly the full QCD + QED Lagrangian contains both the terms conserving and breaking the flavor $SU(3)$ symmetry: $\mathcal{L}_{\rm QCD+QED} = \mathcal{L}_0 + \Delta\mathcal{L}$. The $SU(3)$ conserving term $\mathcal{L}_0$ reads as

$$\mathcal{L}_0=\sum_q\bar{\psi}_q(i\not{D}-m_u)\psi_q+e\sum_q e_s\bar{\psi}_q\not{A}\psi_q+ee_c\bar{\psi}_c\not{A}\psi_c, \quad (1)$$

where D is the covariant derivative of QCD. $q = u, d, s$, e_q is the electric charge of q and $m_u = m_d = m_s$ is assumed in the flavor $SU(3)$ symmetry. The $SU(3)$ symmetry breaking term $\Delta\mathcal{L}$ arises from the mass difference and charge difference the s quark and the d and/or u quarks. Explicitly, $\Delta\mathcal{L}$ can be divided into a mass term and a charge term:

$$\Delta\mathcal{L} = \bar{\psi}_s(m_u - m_s)\psi_s + e(e_u - e_s)\bar{\psi}_u\not{A}\psi_u. \quad (2)$$

Similarly, the Hamiltonian is decomposed as:

$$H = H_0 + \Delta H, \quad (3)$$

with

$$\Delta H = \int d^3x\Delta\mathcal{H}(x) = -\int d^3x\Delta\mathcal{L}(x). \quad (4)$$

The Ξ_c^+ baryons are composed of c, u, s quarks. In the $SU(3)$ symmetry limit, they are classified into an antitriplet $\Xi_c^{\bar{3}}$ and a sextet Ξ_c^6, which are also eigenstates of the $SU(3)$ conserved Hamiltonian H_0. The corresponding eigenstates are defined as

$$H_0|\Xi_c^{\bar{3}}\rangle = m_{\Xi_c^{\bar{3}}}|\Xi_c^{\bar{3}}\rangle, \qquad H_0|\Xi_c^6\rangle = m_{\Xi_c^6}|\Xi_c^6\rangle. \quad (5)$$

On the other hand, the full Hamiltonian is diagonalized by the two physical mass eigenstates:

$$H|\Xi_c\rangle = m_{\Xi_c}|\Xi_c\rangle, \qquad H|\Xi_c'\rangle = m_{\Xi_c'}|\Xi_c'\rangle. \quad (6)$$

The mixing between the physical baryons $|P\rangle = (|\Xi_c\rangle, |\Xi_c'\rangle)^T$ and the $SU(3)$ states $|S\rangle = (|\Xi_c^{\bar{3}}\rangle, |\Xi_c^6\rangle)^T$ is expressed by a unitary transforming matrix U parameterized by a mixing angle θ:

$$|P\rangle = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}|S\rangle = U|S\rangle. \quad (7)$$

Now we consider the matrix element for the $SU(3)$ states $|S\rangle = (|\Xi_c^{\bar{3}}(P_{\bar{3}})\rangle, |\Xi_c^6(P_6)\rangle)^T$: $\langle S|H|S\rangle$, and set both the initial and final states to be static $\vec{P} = \vec{P}' = 0$ and on-shell $P_{\bar{3}}^0 = m_{\Xi_c^{\bar{3}}}, P_6^0 = m_{\Xi_c^6}$. Using the unitary transformation U defined in Eq. (7) as well as the physical masses defined in Eq. (6), we have

$$\begin{pmatrix} \langle\Xi_c^{\bar{3}}(S_z')|H|\Xi_c^{\bar{3}}(S_z)\rangle & \langle\Xi_c^6(S_z')|H|\Xi_c^{\bar{3}}(S_z)\rangle \\ \langle\Xi_c^{\bar{3}}(S_z')|H|\Xi_c^6(S_z)\rangle & \langle\Xi_c^6(S_z')|H|\Xi_c^6(S_z)\rangle \end{pmatrix}$$
$$= 2(2\pi)^3\delta^{(3)}(\vec{0})\delta_{S_zS_z'} \times \begin{pmatrix} m_{\Xi_c}^2\cos^2\theta + m_{\Xi_c'}^2\sin^2\theta & (m_{\Xi_c}^2 - m_{\Xi_c'}^2)\cos\theta\sin\theta \\ (m_{\Xi_c}^2 - m_{\Xi_c'}^2)\cos\theta\sin\theta & m_{\Xi_c}^2\sin^2\theta + m_{\Xi_c'}^2\cos^2\theta \end{pmatrix}, \quad (8)$$

where the momentum dependence of the $\Xi_c^{\bar{3},6}$ states are not shown.

Choosing the upper-right off-diagonal component in the Eq. (8), we have

$$\langle\Xi_c^6(S_z')|H|\Xi_c^{\bar{3}}(S_z)\rangle = (2\pi)^3\delta^{(3)}(\vec{0})\delta_{S_zS_z'}(m_{\Xi_c}^2 - m_{\Xi_c'}^2)\sin 2\theta. \quad (9)$$

To extract the mixing angle θ, one has to calculate the matrix element on the left-hand side above, which can be further expressed as

$$\langle\Xi_c^6(S_z')|H|\Xi_c^{\bar{3}}(S_z)\rangle = (2\pi)^3\delta^{(3)}(\vec{0})\langle\Xi_c^6(S_z')|\Delta\mathcal{H}(0)|\Xi_c^{\bar{3}}(S_z)\rangle. \quad (10)$$

Relating Eqs. (9) and (10) we have

$$\frac{\langle\Xi_c^6(S_z')|\Delta\mathcal{H}(0)|\Xi_c^{\bar{3}}(S_z)\rangle}{m_{\Xi_c}^2 - m_{\Xi_c'}^2} = \delta_{S_zS_z'}\sin 2\theta. \quad (11)$$

Since the mixing angle is independent of the baryon spin, we can average it and obtain

$$\sin 2\theta = \frac{1}{2}\sum_{S_z}\frac{\langle\Xi_c^6(S_z)|\Delta\mathcal{H}(0)|\Xi_c^{\bar{3}}(S_z)\rangle|}{m_{\Xi_c}^2 - m_{\Xi_c'}^2}, \quad (12)$$

where the 1/2 comes from the average of baryon spin.

III. LIGHT-FRONT QUARK MODEL FOR BARYON

The mixing angle of $\Xi_c^+ - \Xi_c'^+$ can be extracted from Eq. (12). Here we will use LFQM in the three quark picture [26,27] to calculate the matrix element on the right hand side of Eq. (12). We will first introduce the three quark picture and list the necessary formulas for the calculation in this work. In LFQM a $\Xi_c^{\bar{3}/6}$ baryon state can be expressed as

$$|\Xi_c^{\bar{3}/6}\rangle = \int (\Pi_{i=1}^3\{d^3\tilde{p}_i\})2(2\pi)^3\frac{\delta^{(3)}(\tilde{P}-\tilde{p}_1-\tilde{p}_2-\tilde{p}_3)}{\sqrt{P^+}} \times \sum_{\lambda_i}\Psi_{\bar{3}/6}^{S,S_z}\frac{\epsilon^{ijk}}{\sqrt{6}}|c^i(p_1,\lambda_1)s^j(p_2,\lambda_2)u^k(p_3,\lambda_3)\rangle, \tag{13}$$

where P denotes the baryon momentum, p_i and λ_i are the momentums and helicities of the constituent quarks. $\Psi_{\bar{3}/6}^{S,S_z}$ denotes the spin and momentum wave function.

In the light-cone coordinates, the momentum and the corresponding integration measure are written as

$$p=(p^+,p^-,p_\perp),\quad \tilde{p}=(p^+,p_\perp),\quad p^\pm=p^0\pm p^3,$$
$$\{d^3\tilde{p}\}\equiv\frac{dp^+d^2p_\perp}{2(2\pi)^3\sqrt{p^+}},\quad \frac{d^4p}{(2\pi)^3}\equiv\frac{dp^-dp^+d^2p_\perp}{2(2\pi)^3}. \tag{14}$$

The three-momentum in the light-cone frame is defined as $\tilde{p}=(p^+,\vec{p}_\perp)$. We introduce two intrinsic variables for the constituent quarks, the light-cone momentum fraction x_i and the transversal momentum $k_{i\perp}$:

$$p_i^+=x_iP^+,\quad \vec{p}_{i\perp}=x_i\vec{P}_\perp+\vec{k}_{i\perp},\quad \sum_{i=1}^3\vec{k}_{i\perp}=0, \tag{15}$$

where $0<x_i<1$, the constraints: $\sum_{i=1}^3 x_i=1$ and $\sum_{i=1}^3\vec{k}_{i\perp}=0$ are imposed due to the momentum conservation. The total momentum of the constituent quarks is denoted as $\bar{P}=\sum_{i=1}^3 p_i$. The invariant mass M_0 is defined as $M_0^2\equiv\bar{P}^2$. It should be noted that $\bar{P}$ is not equal to the baryon momentum P since the momentum of baryon and its constituent quarks cannot be on-shell simultaneously. Choosing a frame where $P_\perp=0$, we can express the invariant mass M_0 as

$$M_0^2=\sum_{i=1}^3\frac{\vec{k}_{i\perp}^2+m_i^2}{x_i}. \tag{16}$$

The internal momentum of the constituent quarks is defined as

$$k_i=(k_i^+,k_i^-,k_{i\perp})=\left(x_iM_0,\frac{\vec{k}_{i\perp}^2+m_i^2}{x_iM_0},k_{i\perp}\right). \tag{17}$$

Then in the Cartesian coordinate the components of $k_i=(e_i,\vec{k}_{i\perp},k_{iz})$ can be written as

$$e_i=\frac{k_i^++k_i^-}{2}=\frac{x_iM_0}{2}+\frac{\vec{k}_{i\perp}^2+m_i^2}{2x_iM_0},$$
$$k_{iz}=\frac{k_i^+-k_i^-}{2}=\frac{x_iM_0}{2}-\frac{\vec{k}_{i\perp}^2+m_i^2}{2x_iM_0}. \tag{18}$$

The wave function Ψ^{S,S_z} for the antitriplet state $\Xi_c^{\bar{3}}$ with momentum P is

$$\Psi_{\bar{3}}=A\bar{u}_{\lambda_3}(p_3)(\bar{\not{P}}+M_0)(-\gamma_5)C\bar{u}_{\lambda_2}^T(p_2)\bar{u}_{\lambda_1}(p_1)u(\bar{P}) \times\Phi(x_i,k_{i\perp}), \tag{19}$$

where the u, s quark form a 0^+ diquark. The wave function for the antitriplet state Ξ_c^6 with momentum P' is

$$\Psi_6=A'\bar{u}_{\lambda_3}(p_3)(\bar{\not{P}}'+M_0')(\gamma^\mu-v^\mu)C\bar{u}_{\lambda_2}^T(p_2)\bar{u}_{\lambda_1}(p_1) \times\left(\frac{1}{\sqrt{3}}\gamma_\mu\gamma_5\right)u(\bar{P}')\Phi(x_i,k'_{i\perp}), \tag{20}$$

where the u, s quarks form a 1^+ diquark, and $v^\mu=\bar{P}'^\mu/M_0'$. The three-particle momentum wave function Φ describes the relative motion between two of the three constituent quarks as well as the relative motion between the third one and the center of other two quarks. Their explicit expressions are

$$\Phi(x_i,k_{i\perp})=\sqrt{\frac{e_1e_2e_3}{x_1x_2x_3M_0}}\phi(\vec{k}_1,\beta_1)\phi\left(\frac{\vec{k}_2-\vec{k}_3}{2},\beta_{23}\right),$$
$$\phi(\vec{k},\beta)=4\left(\frac{\pi}{\beta^2}\right)^{\frac{3}{4}}e^{\frac{-k_\perp^2-k_z^2}{2\beta^2}}, \tag{21}$$

where β_1 and β_{23} are the shape parameters. The normalization factor A is given as

$$A=\frac{1}{4\sqrt{M_0^3(e_1+m_1)(e_2+m_2)(e_3+m_3)}}, \tag{22}$$

where m_1, m_2 and m_3 represent the masses of the charm quark, the strange quark, and the up quark in Eq. (13), respectively.

IV. THE QED CONTRIBUTIONS TO THE $\Xi_c - \Xi_c'$ MIXING

In this section, we calculate the QED effects on the $\Xi_c^+ - \Xi_c'^+$ mixing angle. According to Eq. (12), we need to consider the matrix element

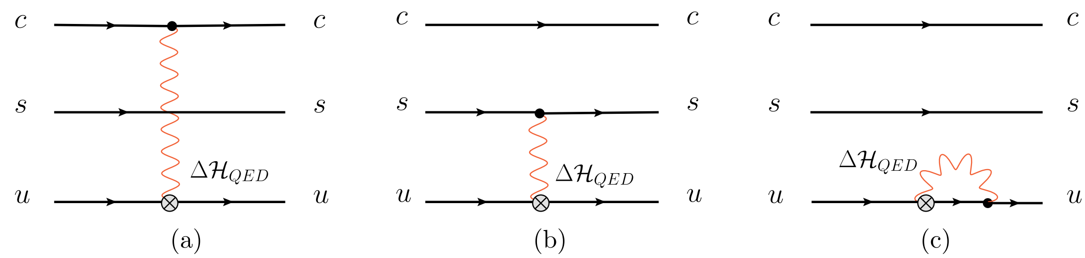

FIG. 1. Three leading-order diagrams for the matrix element in Eq. (23). Only the diagram (a) breaks the heavy quark spin symmetry. Both diagram (b) and diagram (c) hold the heavy quark spin symmetry.

$$\frac{1}{2}\sum_{S_z}\langle\Xi_c^6(P,S_z)|\Delta\mathcal{H}_{\rm QED}(0)|\Xi_c^{\bar{3}}(P,S_z)\rangle \tag{23}$$

with $\Delta\mathcal{H}_{\rm QED}=-e(e_u-e_s)\bar{\psi}_u A\!\!\!/\psi_u$. The calculation is performed by using the light front baryon state given in Eq. (13), as well as the explicit expressions of the wave functions given in Eqs. (19) and (20). The corresponding Feynman diagrams are shown in Fig. 1, where the photon emitted from the $\Delta\mathcal{H}_{\rm QED}$ vertex attaches on the constituent quark lines.

It should be noticed that the Fig. 1(b) and Fig. 1(c) where no photon attaches on the charm quark are zero. This can be seen from the amplitude contributed from the charm quark line:

$$\begin{aligned}&\sum_{S_z}\sum_{\lambda_1}\bar{u}_{\lambda_1}(p_1)u_{S_z}(\bar{P})\bar{u}_{S_z'}(\bar{P})\left(\frac{\gamma_\mu\gamma_5}{\sqrt{3}}\right)u_{\lambda_1}(p_1')\\&\quad={\rm Tr}\left[(p\!\!\!/_1+m_1)(\bar{P}\!\!\!\!/+M_0)\left(\frac{1}{\sqrt{3}}\gamma_\mu\gamma_5\right)\right]=0,\end{aligned} \tag{24}$$

which vanishes when the initial and final baryon spin are averaged. Note that the photon emission from the charm quark as shown in diagram (a) can change the charm quark spin, which breaks the heavy quark spin symmetry. In heavy quark effective theory, at the leading power the heavy quark spin is conserved, and the breaking effect occurs at the $\mathcal{O}(1/m_Q)$. Therefore, the QED induced $\Xi_c^+-\Xi_c^{\prime +}$ mixing should be at the order of $1/m_c$. This also applies to the contributions from the scalar operators $\bar{s}s$.

Now the only contributing diagram is shown in Fig. 2, with each quark momentum is explicitly denoted.

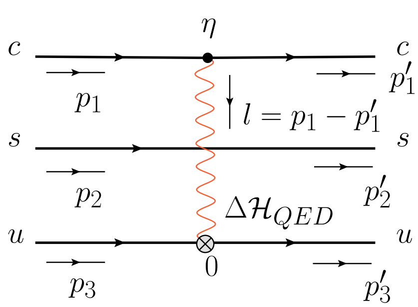

FIG. 2. The leading-order diagram that breaks the heavy quark spin symmetry for the matrix element in Eq. (23).

A detailed calculation on this diagram by LFQM with three-quark picture can be found in the Appendix. The obtained amplitude can be expressed as

$$\begin{aligned}&-e\sum_{S_z}\langle\Xi_c^6(P,S_z)|\bar{\psi}_u(0)A\!\!\!/(0)\psi_u(0)|\Xi_c^{\bar{3}}(P,S_z)\rangle\\&\quad=-Q_ce^2\int\frac{dx_1d^2k_{1\perp}}{2(2\pi)^3\sqrt{x_1}}dx_2d^2k_{2\perp}\frac{1}{\sqrt{1-x_1-x_2}}\\&\qquad\times\int\frac{dx_1'd^2k_{1\perp}'}{2(2\pi)^3\sqrt{x_1'}}\frac{A'A}{2(2\pi)^3\sqrt{(1-x_2-x_1')}}\\&\qquad\times\frac{\Phi(x_i,k_{i\perp})\Phi(x_i',k_{i\perp}')}{(p_1-p_1')^2+i\epsilon}{\rm Tr}_A{\rm Tr}_B,\end{aligned} \tag{25}$$

where we have used $e_u-e_s=1$, and $Q_c=+\frac{2}{3}$ denotes the electric charge of the charm quark, and the trace terms ${\rm Tr}_A$ and ${\rm Tr}_B$ are

$$\begin{aligned}{\rm Tr}_A&={\rm Tr}[(p\!\!\!/_3+m_3)(\bar{P}\!\!\!\!/+M_0)(-\gamma_5)(p\!\!\!/_2-m_2)\\&\quad\times(\gamma^\mu-v^\mu)(\bar{P}\!\!\!\!/+M_0')(p\!\!\!/_3'+m_3)\gamma^\nu],\\{\rm Tr}_B&={\rm Tr}\left[(\bar{P}\!\!\!\!/+M_0)\frac{1}{\sqrt{3}}\gamma_\mu\gamma_5(p\!\!\!/_1'+m_1)\gamma_\nu(p\!\!\!/_1+m_1)\right].\end{aligned} \tag{26}$$

There are eight independent integration variables in Eq. (25), which contains three transverse momentum module: $k_1=|\vec{k}_{1\perp}|,k_2=|\vec{k}_{2\perp}|,k_1'=|\vec{k}_{1\perp}'|$; three plus component momentum fractions: $x_1=k_1^+/P^+,x_2=k_2^+/P^+,x_1'=k_1'^+/P^+$; The angle between $\vec{k}_{1\perp}$ and $\vec{k}_{2\perp}$ is α; The angle between the momentum $\vec{k}_{1\perp}$ and $\vec{k}_{1\perp}'$ is β. Using those eight variables we can write the amplitude as

$$\begin{aligned}&-e\sum_{S_z}\langle\Xi_c^6(P,S_z)|\bar{\psi}_u(0)A\!\!\!/(0)\psi_u(0)|\Xi_c^{\bar{3}}(P,S_z)\rangle\\&\quad=-\frac{2e^2}{3}\int_0^1dx_1\int_0^{1-x_1}dx_2\int_0^{1-x_2}dx_1'\int_0^\infty\frac{dk_1dk_2dk_1'}{[2(2\pi)^3]^3}\\&\qquad\times2\pi\int_0^{2\pi}d\alpha d\beta\frac{k_1k_2k_1'A^2}{\sqrt{x_1x_1'(1-x_1-x_2)(1-x_1'-x_2)}}\\&\qquad\times\Phi(x_i,k_{i\perp})\Phi(x_i',k_{i\perp}')\frac{{\rm Tr}_A{\rm Tr}_B}{l^2+i\epsilon},\end{aligned} \tag{27}$$

where l is the photon momentum and $l^2=2m_1^2-2p_1\cdot p_1'$.

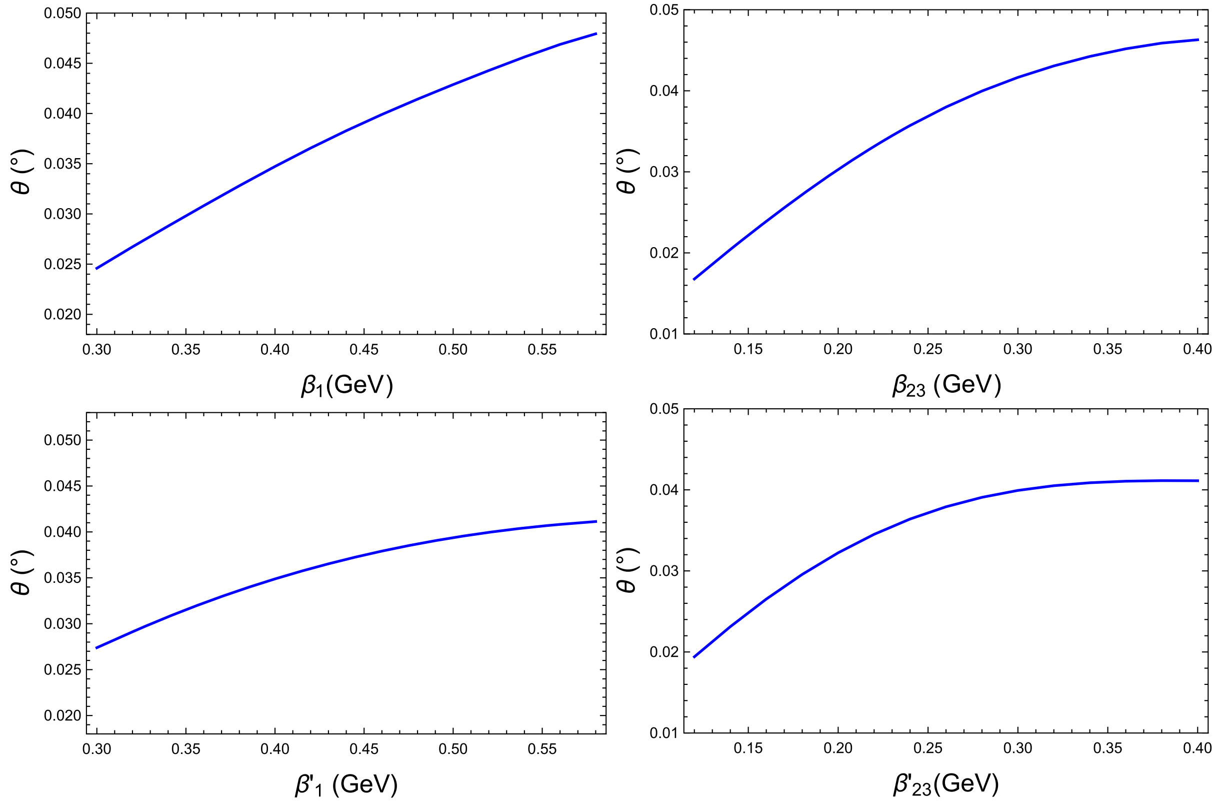

FIG. 3. The dependence of the mixing angle on the shape parameters.

It should be mentioned that the above integration is infrared (IR) and ultraviolet (UV) finite. In the LFQM calculation all the internal quark lines as shown in Fig. 2 are on shell. Therefore the photon emitted from an on shell quark cannot be on shell $l^2 = 0$ unless its momentum vanishes $l = 0$. Interestingly it can be found that the trace term in the nominator: $\mathrm{Tr}_A \mathrm{Tr}_B$ is proportional to quadratic forms of l such as l^2 or $l^\mu l^\nu$, which cancels the singularity from the denominator so that prevents the IR divergence in the integration. Since the exponent of the wave function Eq. (21) depresses the contribution which from large momentum, there is also no UV divergence.

V. NUMERICAL RESULTS

In this work we will adopt the following constituent quark mass parameters:

$$m_u = 0.25\ \mathrm{GeV}, \quad m_s = 0.37\ \mathrm{GeV}, \quad m_c = 1.4\ \mathrm{GeV},$$

which can be found from [35,36]. The shape parameters are extracted from [26]:

$$\beta_1^{\bar{3}} = 0.45 \pm 0.05\ \mathrm{GeV}, \quad \beta_{23}^{\bar{3}} = 0.27 \pm 0.03\ \mathrm{GeV},$$
$$\beta_1^{6} = 0.49 \pm 0.04\ \mathrm{GeV}, \quad \beta_{23}^{6} = 0.28 \pm 0.03\ \mathrm{GeV}. \tag{28}$$

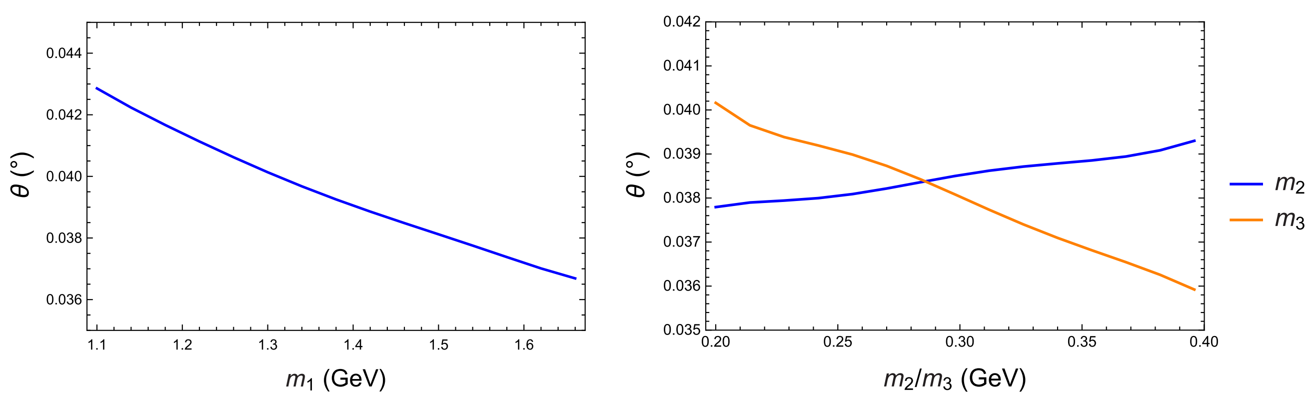

FIG. 4. The dependence of the mixing angle on quark mass.

The baryon masses are taken as [17,37]

$$m_{\Xi_c} = 2.47\ \text{GeV}, \qquad m_{\Xi'_c} = 2.58\ \text{GeV}. \tag{29}$$

According to Eqs. (12) and (27), the mixing angle from QED correction is obtained as

$$\theta_{\text{QED}} \approx 0.04°. \tag{30}$$

We have also investigated the dependence of θ on the input parameters: the shape parameters and the constituent heavy quark mass. The mixing angle as a functions of shape parameters are shown in Fig. 3, while the mixing angle as a function of heavy quark mass are shown in Fig. 4.

It should be noted that our result is smaller than the contributions induced by the mass difference from the most recent lattice QCD calculation [24]. The sum of these two kinds of contributions is still smaller than the experimental measurement and cannot explain the large $SU(3)$ symmetry breaking in experiment.

VI. CONCLUSION

In this work we have calculated the QED contribution to the $\Xi_c^+ - \Xi_c'^+$ mixing which has not been taken into account in the previous analyses. In the calculation, we have employed the light-front quark model. We have explicitly demonstrated that the mixing breaks the heavy quark symmetry.

Numerically, the QED contribution to the mixing angle is found about 0.04°, and the result is less sensitive to the quark masses and shape parameters. Our result is smaller than the contributions induced by the mass difference from the most recent lattice QCD. The sum of these two kinds of contributions is much smaller than the experimental measurement and cannot explain the large $SU(3)$ symmetry breaking in experiment.

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APPENDIX: DETAILED DERIVATION OF THE TRANSITION AMPLITUDES

In this appendix we present a detailed calculation of the diagram in Fig. 2. Using the light front baryon state given in Eq. (13), and inserting the first order QED interaction Hamiltonian, we can express the amplitude as

$$\begin{aligned}
&-e\sum_{S_z}\langle\Xi_c^6(P,S_z)|\bar\psi_u(0)A\!\!\!/(0)\psi_u(0)|\Xi_c^{\bar 3}(P,S_z)\rangle \\
&= -e\frac{1}{\sqrt{P'^+P^+}}\sum_{S_z}\int\{d^3\tilde p_1\}\{d^3\tilde p_2\}\{d^3\tilde p_3\}[2(2\pi)^3]^2\delta^3(\tilde P-\tilde p_1-\tilde p_2-\tilde p_3)\int\{d^3\tilde p_1'\}\{d^3\tilde p_2'\}\{d^3\tilde p_3'\} \\
&\quad\times\delta^3(\tilde P-\tilde p_1'-\tilde p_2'-\tilde p_3')\left[\sum_{\lambda_1',\lambda_2',\lambda_3'}\Psi_1^{1/2,S_z}(\tilde p_1',\tilde p_2',\tilde p_3',\lambda_1',\lambda_2',\lambda_3')\right]^\dagger\sum_{\lambda_1,\lambda_2,\lambda_3}\Psi_0^{1/2,S_z}(\tilde p_1,\tilde p_2,\tilde p_3,\lambda_1,\lambda_2,\lambda_3)\frac{\epsilon^{ijk}\epsilon^{i'j'k'}}{6} \\
&\quad\times\langle u^{k'}(p_3',\lambda_3')s^{j'}(p_2',\lambda_2')c^{i'}(p_1',\lambda_1')|\bar\psi_u(0)A\!\!\!/(0)\psi_u(0)iQ_ce\int d^4y\bar\psi_c(y)A\!\!\!/(y)\psi_c(y)|c^i(p_1,\lambda_1)s^j(p_2,\lambda_2)u^k(p_3,\lambda_3)\rangle.
\end{aligned}\tag{A1}$$

Inserting the explicit expressions of the wave functions given in Eqs. (19) and (20), one obtains the amplitude as

$$\begin{aligned}
&-e\sum_{S_z}\langle\Xi_c^6(P,S_z)|\bar\psi_u(0)A\!\!\!/(0)\psi_u(0)|\Xi_c^{\bar 3}(P,S_z)\rangle \\
&= -\frac{Q_ce^2}{\sqrt{P^+P'^+}}\sum_{S_z}\int\{d^3\tilde p_1\}\{d^3\tilde p_2\}\{d^3\tilde p_3\}[2(2\pi)^3]^2\delta^{(3)}(\tilde P-\tilde p_1-\tilde p_2-\tilde p_3)\int\{d^3\tilde p_1'\}\{d^3\tilde p_2'\}\{d^3\tilde p_3'\}AA' \\
&\quad\times\delta^{(3)}(\tilde P-\tilde p_1'-\tilde p_2'-\tilde p_3')2p_2^+(2\pi)^3\delta^{(3)}(\tilde p_2'-\tilde p_2)\frac{1}{(p_1-p_1')^2+i\epsilon}\Phi(x_i,k_{i\perp})\Phi(x_i',k_{i\perp}') \\
&\quad\times\text{Tr}[(p\!\!\!/_3+m_3)\mathcal{A}(p\!\!\!/_2+m_2)^T\mathcal{D}(p\!\!\!/_3'+m_3')\gamma^\nu]\bar u_{S_z}(\bar P)\mathcal{B}(p\!\!\!/_1'+m_1')\gamma_\nu(p\!\!\!/_1+m_1)u_{S_z}(\bar P),
\end{aligned}\tag{A2}$$

where $\mathcal{A}=(\bar P\!\!\!/+M_0)(-\gamma_5)C$, $\mathcal{B}=\frac{1}{\sqrt3}(-\gamma_5)\gamma_\mu$, $\mathcal{D}=C(\gamma^\mu-v^\mu)(\bar P\!\!\!/+M_0')$. Summing up the spin index S_z, and explicitly writing the integration measures by the plus and transverse components, we can express the amplitude as

$$
\begin{aligned}
&-e\sum_{S_z}\langle\Xi_c^6(P,S_z)|\bar\psi_u(0)A\!\!\!/(0)\psi_u(0)|\Xi_c^{\bar 3}(P,S_z)\rangle\\
&\quad=-\frac{Q_c e^2}{\sqrt{P^+P'^+}}\int\frac{dp_1^+d^2p_{1\perp}}{2(2\pi)^3\sqrt{p_1^+}}\frac{dp_2^+d^2p_{2\perp}}{2(2\pi)^3\sqrt{p_2^+}}\frac{dp_3^+d^2p_{3\perp}}{2(2\pi)^3\sqrt{p_3^+}}[2(2\pi)^3]^2\delta(\bar P^+-p_1^+-p_2^+-p_3^+)\delta^{(2)}(\bar P_\perp-p_{1\perp}-p_{2\perp}-p_{3\perp})\\
&\qquad\times\int\frac{dp_1'^+d^2p_{1\perp}'}{2(2\pi)^3\sqrt{p_1'^+}}\frac{dp_2'^+d^2p_{2\perp}'}{2(2\pi)^3\sqrt{p_2'^+}}\frac{dp_3'^+d^2p_{3\perp}'}{2(2\pi)^3\sqrt{p_3'^+}}\delta(\bar P^+-p_1'^+-p_2'^+-p_3'^+)\delta^{(2)}(\bar P_\perp-p_{1\perp}'-p_{2\perp}'-p_{3\perp}')AA'\\
&\qquad\times\Phi(x_i,k_{i\perp})\Phi(x_i',k_{i\perp}')2p_2^+(2\pi)^3\delta(p_2'^+-p_2^+)\delta^{(2)}(p_{2\perp}-p_{2\perp}')\frac{1}{(p_1-p_1')^2+i\epsilon}\mathrm{Tr}_A\mathrm{Tr}_B,
\end{aligned}\tag{A3}
$$

where the expressions of the trace terms Tr_A, Tr_B are

$$
\begin{aligned}
\mathrm{Tr}_A&=\mathrm{Tr}[(p\!\!\!/_3+m_3)(\bar P\!\!\!/+M_0)(-\gamma_5)(p\!\!\!/_2-m_2)(\gamma^\mu-v^\mu)\\
&\quad\times(\bar P\!\!\!/+M_0')(p\!\!\!/_3'+m_3)\gamma^\nu],\\
\mathrm{Tr}_B&=\mathrm{Tr}[(\bar P\!\!\!/+M_0)\frac{1}{\sqrt3}\gamma_\mu\gamma_5(p\!\!\!/_1'+m_1)\gamma_\nu(p\!\!\!/_1+m_1)].
\end{aligned}\tag{A4}
$$

Using the light front components of the momentums

$$
\begin{aligned}
p_i&=\left(x_iP^+,\frac{m_i^2+k_{i\perp}^2}{x_iP^+},k_{i\perp}\right),\\
p_j'&=\left(x_j'P^+,\frac{m_j^2+k_{j\perp}'^2}{x_j'P^+},k_{j\perp}'\right),
\end{aligned}\tag{A5}
$$

and integrating our p_2', p_3', and p_3, we arrive at

$$
\begin{aligned}
&-e\sum_{S_z}\langle\Xi_c^6(P,S_z)|\bar\psi_u(0)A\!\!\!/(0)\psi_u(0)|\Xi_c^{\bar 3}(P,S_z)\rangle\\
&\quad=-Q_ce^2\int\frac{dx_1d^2k_{1\perp}}{2(2\pi)^3\sqrt{x_1}}dx_2d^2k_{2\perp}\frac{1}{\sqrt{1-x_1-x_2}}\\
&\qquad\times\int\frac{dx_1'd^2k_{1\perp}'}{2(2\pi)^3\sqrt{x_1'}}\frac{1}{2(2\pi)^3\sqrt{(1-x_2-x_1')}}\\
&\qquad\times A'A\frac{\Phi(x_i,k_{i\perp})\Phi(x_i',k_{i\perp}')}{(p_1-p_1')^2+i\epsilon}\mathrm{Tr}_A\mathrm{Tr}_B.
\end{aligned}\tag{A6}
$$

Now the expressions of $\mathrm{Tr}_A\mathrm{Tr}_B$ are written as:

$$
\begin{aligned}
p_1\cdot p_2&=\frac12\left[\frac{x_1}{x_2}(m_2^2+k_{2\perp}^2)+\frac{x_2}{x_1}(m_1^2+k_{1\perp}^2)\right]-k_{1\perp}k_{2\perp}\cos\alpha\\
p_1\cdot p_3&=\frac12\left[\frac{x_1}{x_3}(m_3^2+k_{1\perp}^2+k_{2\perp}^2+2k_{1\perp}k_{2\perp}\cos\alpha)+\frac{x_3}{x_1}(m_1^2+k_{1\perp}^2)\right]+k_{1\perp}^2+k_{1\perp}k_{2\perp}\cos\alpha\\
p_2\cdot p_3&=\frac12\left[\frac{x_2}{x_3}(m_3^2+k_{1\perp}^2+k_{2\perp}^2+2k_{1\perp}k_{2\perp}\cos\alpha)+\frac{x_3}{x_2}(m_2^2+k_{2\perp}^2)\right]+k_{2\perp}^2+k_{1\perp}k_{2\perp}\cos\alpha\\
p_1\cdot p_1'&=\frac12\left[\frac{x_1}{x_1'}(m_1^2+k_{1\perp}'^2)+\frac{x_1'}{x_1}(m_1^2+k_{1\perp}^2)\right]-k_{1\perp}k_{1\perp}'\cos\beta\\
p_2\cdot p_1'&=\frac12\left[\frac{x_2}{x_1'}(m_1^2+k_{1\perp}'^2)+\frac{x_1'}{x_2}(m_2^2+k_{2\perp}^2)\right]-k_{2\perp}k_{1\perp}'\cos(\alpha-\beta)\\
p_3\cdot p_1'&=\frac12\left[\frac{x_3}{x_1'}(m_1^2+k_{1\perp}'^2)+\frac{x_1'}{x_3}(m_3^2+k_{1\perp}^2+k_{2\perp}^2+2k_{1\perp}k_{2\perp}\cos\alpha)\right]+k_{2\perp}k_{1\perp}'\cos(\alpha-\beta)+k_{1\perp}k_{1\perp}'\cos\beta\\
p_1\cdot p_3'&=\frac12\left[\frac{x_3'}{x_1}(m_1^2+k_{1\perp}^2)+\frac{x_1}{x_3'}(m_3^2+k_{1\perp}'^2+k_{2\perp}'^2+2k_{1\perp}'k_{2\perp}'\cos(\alpha-\beta))\right]+k_{1\perp}k_{1\perp}'\cos\beta+k_{1\perp}k_{2\perp}'\cos\alpha\\
p_2\cdot p_3'&=\frac12\left[\frac{x_3'}{x_2}(m_2^2+k_{2\perp}^2)+\frac{x_2}{x_3'}(m_3^2+k_{1\perp}'^2+k_{2\perp}'^2+2k_{1\perp}'k_{2\perp}'\cos(\alpha-\beta))\right]+k_{2\perp}k_{1\perp}'\cos(\alpha-\beta)+k_{2\perp}k_{2\perp}'\\
p_3\cdot p_3'&=\frac12\left[\frac{x_3'}{x_3}(m_3^2+k_{1\perp}^2+k_{2\perp}^2+2k_{1\perp}k_{2\perp}\cos\alpha)+\frac{x_3}{x_3'}(m_3^2+k_{1\perp}'^2+k_{2\perp}'^2+2k_{1\perp}'k_{2\perp}'\cos(\alpha-\beta))\right]\\
&\quad-k_{1\perp}k_{1\perp}'\cos\beta-k_{1\perp}k_{2\perp}'\cos\alpha-k_{2\perp}k_{1\perp}'\cos(\alpha-\beta)-k_{2\perp}k_{2\perp}'\\
p_1'\cdot p_3'&=\frac12\left[\frac{x_3'}{x_1'}(m_1^2+k_{1\perp}'^2)+\frac{x_1'}{x_3'}(m_3^2+k_{1\perp}'^2+k_{2\perp}'^2+2k_{1\perp}'k_{2\perp}'\cos(\alpha-\beta))\right]+k_{1\perp}'^2+k_{1\perp}'k_{2\perp}'\cos(\alpha-\beta),
\end{aligned}\tag{A7}
$$

where $p_i^2 = p_i'^2 = m_i^2$. The angle between $\vec{k}_{1\perp}$ and $\vec{k}_{2\perp}$ is α, the angle between the momentum $\vec{k}_{1\perp}$ and $\vec{k}'_{1\perp}$ is β.

In our calculation, only the leading order contribution is taken into account, but we believe this is a common strategy in many analyses in the literature. If QCD corrections are considered, we need to insert at least two additional QCD Lagrangian. In this way, the QCD correction will introduce an additional suppression at the magnitude of α_s, and thus the leading order QCD correction contribution to the matrix element in Eq. (A1) is at the order of $e\alpha_s$. Only the leading order contribution is considered in our analysis while higher order QCD corrections that might be sizable are left for future analysis.

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