Relativistic spin dynamics for vector mesons

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(Received 12 July 2023; accepted 2 January 2024; published 5 February 2024)

We propose a relativistic theory for spin density matrices of vector mesons based on Kadanoff-Baym equations in the closed-time-path formalism. The theory puts the calculation of spin observables such as the spin density matrix element ρ_{00} for vector mesons on a solid ground. Within the theory we formulate ρ_{00} for ϕ mesons into a factorization form in separation of momentum and spacetime variables. We argue that the main contribution to ρ_{00} at lower energies should be from the ϕ fields that can polarize the strange quark and antiquark in the same way as electromagnetic fields. The key observation is that there is correlation inside the ϕ meson wave function between the ϕ field that polarizes the strange quark and that polarizes the strange antiquark. This is reflected by the fact that the contributions to ρ_{00} are all in squares of fields that are nonvanishing even if the fields may strongly fluctuate in spacetime. The fluctuation of strong force fields can be extracted from ρ_{00} of unflavored vector mesons as links to fundamental properties of quantum chromodynamics.

DOI: 10.1103/PhysRevD.109.036004

I. INTRODUCTION

There is an intrinsic connection between rotation and spin polarization since they are related to the conservation of total angular momentum and can be converted from one to another, as demonstrated in the Barnett effect [1] and the Einstein–de Haas effect [2] in materials. A recent example is the observation of a spin current from the vortical motion in a liquid metal [3]. The same effects also exist in high energy heavy-ion collisions in which the huge orbital angular momentum along the direction normal to the reaction plane can be partially converted to the global spin polarization of hadrons [4–9] (see, e.g., [10–14] for recent reviews). The global spin polarization of Λ (including $\bar{\Lambda}$ hereafter) has been measured through their weak decays in Au + Au collisions at $\sqrt{s_{NN}} = 7.7$–200 GeV [15,16].

As spin-one particles, vector mesons can also be polarized in heavy-ion collisions in the same way as hyperons. Normally the spin states of vector mesons are described by the spin density matrix element $\rho_{\lambda_1\lambda_2}$ with $\lambda_1, \lambda_2 = 0, \pm 1$ labeling three spin states along the spin quantization direction. The vector mesons mainly decay through strong interaction that respects parity symmetry. So their spin polarization proportional to $\rho_{11} - \rho_{-1,-1}$ cannot be measured through their decays. Instead, ρ_{00} can be measured through the angular distribution of its decay daughters [5,17–20]. If $\rho_{00} = 1/3$, the spin states are equally populated in three spin states which may imply that the vector meson is not polarized. If $\rho_{00} \neq 1/3$, three spin states are not equally populated, so the polarization vector (not the spin) of the vector meson is aligned in either the spin quantization direction or the transverse direction perpendicular to it. In 2008, the STAR Collaboration measured ρ_{00} for the vector meson $\phi(1020)$ in Au + Au collisions at 200 GeV, but the result is consistent with $1/3$ within errors due to statistics [21]. Recently STAR has measured the ϕ meson's ρ_{00} at lower energies, which shows a significant deviation from $1/3$ [22]. It can hardly be explained by conventional mechanism [17,23–25].

In Ref. [26], some of us proposed that a large deviation of ρ_{00} from $1/3$ for the ϕ meson may possibly arise from the ϕ vector field, a strong force field in connection with the current of pseudo-Goldstone bosons [27] and vacuum properties of quantum chromodynamics [28–31]. Such a proposal is based on a nonrelativistic quark coalescence model for the spin density matrix of vector mesons [17,32].

In this paper we will present a relativistic theory for the spin density matrix of vector mesons from the Kadanoff-Baym (KB) equation [33] in the closed-time-path (CTP) formalism [34,35] (for reviews of the KB equation and the CTP formalism, we refer the readers to Refs. [36–39]). Then we can derive the spin Boltzmann equation for vector mesons with their spin density matrices being expressed in terms of the matrix valued spin dependent distributions (MVSD) of the quarks and antiquarks [40]. This puts the calculation of spin observables such as ρ_{00} for vector mesons on a solid ground [41].

The paper is organized as follows. In Sec. II we will give an introduction to Green's functions on the CTP for vector mesons that can be expressed in MVSD. In Sec. III the KB equations for vector mesons are derived. In Sec. IV the spin density matrices for vector mesons will be formulated from the spin Boltzmann equations. In Sec. V the spin density matrices for ϕ mesons will be evaluated. Discussions on the main results and conclusions are given in the final section, Sec. VI.

We adopt the sign convention for the metric tensor $g^{\mu\nu} = g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ where μ, $\nu = 0$, 1, 2, 3. The sign convention for the Levi-Civita symbol is $\epsilon^{0123} = -\epsilon_{0123} = 1$. We can write the spacetime coordinate as $x = x^\mu = (x^0, \mathbf{x}) = (t, \mathbf{x})$ and $x_\mu = (x_0, -\mathbf{x})$ with $x_0 = x^0 = t$. The four-momentum for a particle is denoted as $p = p^\mu = (p^0, \mathbf{p})$ or $p_\mu = (p_0, -\mathbf{p})$, and if it is on-shell we have $p_0 = p^0 = \sqrt{\mathbf{p}^2 + m^2} \equiv E_p = E_{\mathbf{p}}$. Normally we use Greek letters to denote four-dimensional indices of four-vectors and four-tensors and Latin letters to denote their spatial components.

II. GREEN'S FUNCTIONS ON CTP FOR VECTOR MESONS

The massive spin-one particle, such as the vector meson with the mass m_V, can be described by the vector field $A_V^\mu(x)$ with the classical Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^V F_V^{\mu\nu} + \frac{m_V^2}{2} A_\mu^V A_V^\mu - A_\mu^V j^\mu, \tag{1}$$

where j^μ is the source current, $F_V^{\mu\nu} = \partial^\mu A_V^\nu - \partial^\nu A_V^\mu$ is the field strength tensor, and A_V^μ is assumed to be the real classical field for the charge (including flavor) neutral particle. From $\mathcal{L}$ one can obtain the Proca equation [42,43]

$$L^{\mu\nu}(x) A_\nu^V(x) = j^\mu(x), \tag{2}$$

where the differential operator is defined as

$$L^{\mu\nu}(x) = (\partial_x^2 + m_V^2) g^{\mu\nu} - \partial_x^\mu \partial_x^\nu. \tag{3}$$

A constraint equation can be derived by contracting the above equation with ∂_μ as

$$\partial_\mu A_V^\mu(x) = \frac{1}{m_V^2} \partial_\mu j^\mu(x) = 0, \tag{4}$$

if the source current is conserved $\partial_\mu j^\mu = 0$. The above equation means that the longitudinal component of $A_V^\mu(x)$ is vanishing for the conserved current.

The free vector field can be quantized as

$$A_V^\mu(x) = \sum_{\lambda=0,\pm1} \int \frac{d^3k}{(2\pi\hbar)^3} \frac{1}{2E_k^V} \times \Big[\epsilon^\mu(\lambda, \mathbf{k}) a_V(\lambda, \mathbf{k}) e^{-ik\cdot x/\hbar} + \epsilon^{\mu*}(\lambda, \mathbf{k}) a_V^\dagger(\lambda, \mathbf{k}) e^{ik\cdot x/\hbar}\Big], \tag{5}$$

where $E_k^V = \sqrt{\mathbf{k}^2 + m_V^2}$ and λ denote the energy and the spin state in the spin quantization direction, respectively, the creation and annihilation operators $a_V(\lambda, \mathbf{k})$ and $a_V^\dagger(\lambda, \mathbf{k})$ satisfy the commutator

$$\Big[a_V(\lambda, \mathbf{k}), a_V^\dagger(\lambda', \mathbf{k}')\Big] = \delta_{\lambda\lambda'} 2E_{\mathbf{k}}^V (2\pi\hbar)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}'), \tag{6}$$

and the polarization vector $\epsilon^\mu(\lambda, \mathbf{k})$ obeys

$$k_\mu \epsilon^\mu(\lambda, \mathbf{k}) = 0,$$
$$\epsilon(\lambda, \mathbf{k}) \cdot \epsilon^*(\lambda', \mathbf{k}) = -\delta_{\lambda\lambda'},$$
$$\sum_\lambda \epsilon^\mu(\lambda, \mathbf{k}) \epsilon^{\nu*}(\lambda, \mathbf{k}) = -\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_V^2}\right). \tag{7}$$

In the above relations, the first one follows the constraint (4), and $k^\mu = (E_k^V, \mathbf{k})$ denotes the on-shell four-momentum for the vector meson. By the field quantization in (5), one can check that $A_V^\mu(x)$ is Hermitian, i.e., $A_V^{\mu\dagger}(x) = A_V^\mu(x)$.

One can define two-point Green's function for the vector meson on the CTP

$$G_{\mathrm{CTP}}^{\mu\nu}(x_1, x_2) \equiv \Big\langle T_C A_V^\mu(x_1) A_V^{\nu\dagger}(x_2) \Big\rangle, \tag{8}$$

where x_1 and x_2 are two spacetime points whose time components are defined on the CTP contour and T_C represents the time ordering on the CTP contour. We can write $G_{\mu\nu}^{\mathrm{CTP}}(x_1, x_2)$ in a matrix form:

$$G_{\mu\nu}^{\mathrm{CTP}}(x_1, x_2) = \begin{pmatrix} G_{\mu\nu}^{++}(x_1, x_2) & G_{\mu\nu}^{+-}(x_1, x_2) \\ G_{\mu\nu}^{-+}(x_1, x_2) & G_{\mu\nu}^{--}(x_1, x_2) \end{pmatrix} = \begin{pmatrix} G_{\mu\nu}^{F}(x_1, x_2) & G_{\mu\nu}^{<}(x_1, x_2) \\ G_{\mu\nu}^{>}(x_1, x_2) & G_{\mu\nu}^{\bar{F}}(x_1, x_2) \end{pmatrix}. \tag{9}$$

The "++" component of $G^{\rm CTP}_{\mu\nu}$ with both t_1 and t_2 (time components of x_1 and x_2) on the positive time branch is just the Feynman propagator $G^F_{\mu\nu}(x_1, x_2)$ as shown in Fig. 1(a). The "+−" component with t_1 on the positive time branch while t_2 is on the negative time branch is denoted as $G^<_{\mu\nu}(x_1, x_2)$ meaning that t_2 is always later than t_1 on the CTP contour as shown in Fig. 1(b). Analogously, $G^>_{\mu\nu}(x_1, x_2)$ denotes the "−+" component with t_1 on the negative time branch and t_2 on the positive time branch as shown in Fig. 1(c), while $G^{\bar{F}}_{\mu\nu}(x_1, x_2)$ denotes the "−−" component with both t_1 and t_2 on the negative time branch as shown in Fig. 1(d).

The Wigner function can be defined from $G^<_{\mu\nu}(x_1, x_2)$ by taking a Fourier transform with respect to the relative position $y = x_1 - x_2$ at the center point $x = (x_1 + x_2)/2$,

$$G^<_{\mu\nu}(x, p) \equiv \int d^4y e^{ip\cdot y/\hbar} G^<_{\mu\nu}(x_1, x_2) = \int d^4y e^{ip\cdot y/\hbar} \langle A^{V\dagger}_\nu(x_2) A^V_\mu(x_1)\rangle. \quad (10)$$

Inserting the quantized field (5) into the definition of the Wigner function (10), we obtain

$$G^<_{\mu\nu}(x, p) = 2\pi\hbar \sum_{\lambda_1,\lambda_2} \delta(p^2 - m_V^2) \times \{\theta(p^0)\epsilon_\mu(\lambda_1, \mathbf{p})\epsilon^*_\nu(\lambda_2, \mathbf{p}) f_{\lambda_1\lambda_2}(x, \mathbf{p}) + \theta(-p^0)\epsilon^*_\mu(\lambda_1, -\mathbf{p})\epsilon_\nu(\lambda_2, -\mathbf{p}) \times [\delta_{\lambda_2\lambda_1} + f_{\lambda_2\lambda_1}(x, -\mathbf{p})]\}, \quad (11)$$

where the gradient expansion has been taken with spatial gradients of the MVSD (defined below) being dropped, and the MVSD for the vector meson is defined as

$$f_{\lambda_1\lambda_2}(x, \mathbf{p}) \equiv \int \frac{d^4u}{2(2\pi\hbar)^3} \delta(p\cdot u) e^{-iu\cdot x/\hbar} \times \left\langle a^\dagger_V\left(\lambda_2, \mathbf{p} - \frac{\mathbf{u}}{2}\right) a_V\left(\lambda_1, \mathbf{p} + \frac{\mathbf{u}}{2}\right)\right\rangle. \quad (12)$$

The derivation of the two-point function in (11) for spin-one particles is similar to that for spin-1/2 particles which is given in Eqs. (115) and (116) of Ref. [40]. One can check that $f_{\lambda_1\lambda_2}(x, \mathbf{p})$ is a Hermitian matrix, i.e., $f^*_{\lambda_1\lambda_2}(x, \mathbf{p}) = f_{\lambda_2\lambda_1}(x, \mathbf{p})$. Similarly we can define another Wigner function from $G^>_{\mu\nu}(x_1, x_2)$,

$$G^>_{\mu\nu}(x, p) \equiv \int d^4y e^{ip\cdot y/\hbar} \langle A^V_\mu(x_1) A^{V\dagger}_\nu(x_2)\rangle. = 2\pi\hbar \sum_{\lambda_1,\lambda_2} \delta(p^2 - m_V^2) \times \{\theta(p^0)\epsilon_\mu(\lambda_1, \mathbf{p})\epsilon^*_\nu(\lambda_2, \mathbf{p})[\delta_{\lambda_1\lambda_2} + f_{\lambda_1\lambda_2}(x, \mathbf{p})] + \theta(-p^0)\epsilon^*_\mu(\lambda_1, -\mathbf{p})\epsilon_\nu(\lambda_2, -\mathbf{p}) f_{\lambda_2\lambda_1}(x, -\mathbf{p})\}. \quad (13)$$

Note that $G^>_{\mu\nu}(x, p)$ can be obtained by replacing $f_{\lambda_1\lambda_2} \to \delta_{\lambda_1\lambda_2} + f_{\lambda_1\lambda_2}$ and $\delta_{\lambda_2\lambda_1} + f_{\lambda_2\lambda_1} \to f_{\lambda_2\lambda_1}$ from $G^<_{\mu\nu}(x, p)$.

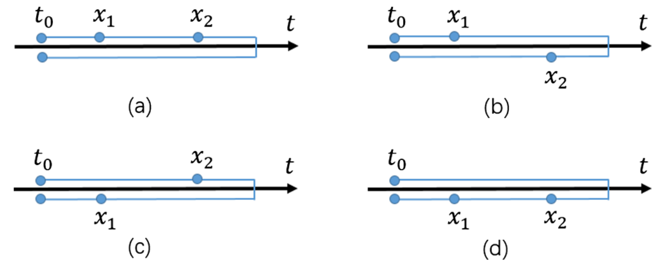

FIG. 1. The closed-time path and four components of two-point Green's function on CTP. The positive and negative time branches are denoted as C_+ and C_-, respectively. (a) $x^0_1 = t_1 \in C_+$, $x^0_2 = t_2 \in C_+$; (b) $x^0_1 = t_1 \in C_+$, $x^0_2 = t_2 \in C_-$; (c) $x^0_1 = t_1 \in C_-$, $x^0_2 = t_2 \in C_+$; and (d) $x^0_1 = t_1 \in C_-$, $x^0_2 = t_2 \in C_-$.

III. KADANOFF-BAYM EQUATIONS FOR VECTOR MESONS

The Wigner functions for massless vector particles such as gluons and photons [25,37,44–47] have been studied for many years, but to our knowledge there are few works about Wigner functions for massive vector mesons in the context of spin polarization (see Ref. [48] for a recent one). In this section, we will derive the Boltzmann equation for vector meson's Wigner functions from two-point Green's functions on the CTP. The starting point is the KB equations

$$L^\mu_\eta(x_1) G^{<,\eta\nu}(x_1, x_2) = -\frac{i\hbar}{2}\int d^4x' [g^{\mu\gamma}\Sigma^<_{\gamma\alpha}(x_1, x') G^{>,\alpha\nu}(x', x_2) - g^{\mu\gamma}\Sigma^>_{\gamma\alpha}(x_1, x') G^{<,\alpha\nu}(x', x_2)] \quad (14)$$

and

$$L^\nu_\eta(x_2) G^{<,\mu\eta}(x_1, x_2) = -\frac{i\hbar}{2}\int d^4x' [g^{\mu\gamma} G^<_{\gamma\alpha}(x_1, x')\Sigma^{>,\alpha\nu}(x', x_2) - g^{\mu\gamma} G^>_{\gamma\alpha}(x_1, x')\Sigma^{<,\alpha\nu}(x', x_2)]. \quad (15)$$

Equations (14) and (15) are the result of the quasiparticle approximation [40]. Note that the integrations over x in Eqs. (14) and (15) are ordinary ones.

We consider the coupling between the vector meson and the quark-antiquark in the quark-meson model [27,49–53]. Then at lowest order in the coupling constant, the self-energies are from quark loops as shown in Fig. 2,

$$\Sigma^{<,\mu\nu}(x_1,x_2) = -\mathrm{Tr}\big[i\Gamma^\mu S^<(x_1,x_2) i\Gamma^\nu S^>(x_2,x_1)\big],$$
$$\Sigma^{>,\mu\nu}(x_1,x_2) = -\mathrm{Tr}\big[i\Gamma^\mu S^>(x_1,x_2) i\Gamma^\nu S^<(x_2,x_1)\big], \qquad (16)$$

where $S^>(x_1,x_2)$ and $S^<(x_1,x_2)$ are two-point Green's functions of quarks, $i\Gamma^\mu$ denotes the vertex of the vector meson and quark-antiquark, and the overall minus signs arise from quark loops. Inserting the self-energies (16) into

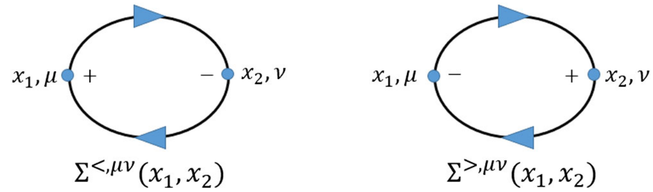

FIG. 2. The self-energies $\Sigma^{<,\mu\nu}$ and $\Sigma^{>,\mu\nu}$ of vector mesons from quark loops in the quark-meson model. Two quark propagators in the loop may have different flavors corresponding to the vector meson that is not flavor neutral.

Eq. (14) and taking a Fourier transform with respect to the difference $y = x_1 - x_2$, we obtain the KB equation for the Wigner function as

$$\Big\{g^\mu_\eta\Big[-\Big(p^2 - m_V^2 - \frac{\hbar^2}{4}\partial_x^2\Big) - i\hbar p\cdot\partial_x\Big] - \frac{\hbar^2}{4}\partial_x^\mu\partial^x_\eta + p^\mu p_\eta + \frac{1}{2}i\hbar(p_\eta\partial_x^\mu + p^\mu\partial^x_\eta)\Big\}G^{<,\eta\nu}(x,p)$$
$$= -\frac{i\hbar}{2}\int\frac{d^4p'}{(2\pi\hbar)^4}\big\{\mathrm{Tr}\big[\Gamma^\mu S^<(x,p+p')\Gamma_\alpha S^>(x,p')\big]G^{>,\alpha\nu}(x,p) - \mathrm{Tr}\big[\Gamma^\mu S^>(x,p+p')\Gamma_\alpha S^<(x,p')\big]G^{<,\alpha\nu}(x,p)\big\}$$
$$-\frac{\hbar^2}{4}\int\frac{d^4p'}{(2\pi\hbar)^4}\big[\big\{\mathrm{Tr}\big[\Gamma^\mu S^<(x,p+p')\Gamma_\alpha S^>(x,p')\big], G^{>,\alpha\nu}(x,p)\big\}_{\mathrm{P.B.}}$$
$$-\big\{\mathrm{Tr}\big[\Gamma^\mu S^>(x,p+p')\Gamma_\alpha S^<(x,p')\big], G^{<,\alpha\nu}(x,p)\big\}_{\mathrm{P.B.}}\big], \qquad (17)$$

where the Poisson bracket involves spacetime and momentum gradients and is defined as

$$\{A,B\}_{\mathrm{P.B.}} \equiv (\partial_x^\mu A)(\partial^p_\mu B) - (\partial_p^\mu A)(\partial^x_\mu B). \qquad (18)$$

In the same way, we obtain from Eq. (15) another KB equation for the Wigner function

$$\Big\{g^\nu_\eta\Big[-\Big(p^2 - m_V^2 - \frac{\hbar^2}{4}\partial_x^2\Big) + i\hbar p\cdot\partial_x\Big] - \frac{\hbar^2}{4}\partial_x^\nu\partial^x_\eta + p^\nu p_\eta - \frac{1}{2}i\hbar(p_\eta\partial_x^\nu + p^\nu\partial^x_\eta)\Big\}G^{<,\mu\eta}(x,p)$$
$$= -\frac{i\hbar}{2}\int\frac{d^4p'}{(2\pi\hbar)^4}\big\{g^{\mu\gamma}G^<_{\gamma\alpha}(x,p)\mathrm{Tr}\big[\Gamma^\alpha S^>(x,p+p')\Gamma^\nu S^<(x,p')\big] - g^{\mu\gamma}G^>_{\gamma\alpha}(x,p)\mathrm{Tr}\big[\Gamma^\alpha S^<(x,p+p')\Gamma^\nu S^>(x,p')\big]\big\}$$
$$-\frac{\hbar^2}{4}\int\frac{d^4p'}{(2\pi\hbar)^4}\big[\big\{g^{\mu\gamma}G^<_{\gamma\alpha}(x,p), \mathrm{Tr}\big[\Gamma^\alpha S^>(x,p+p')\Gamma^\nu S^<(x,p')\big]\big\}_{\mathrm{P.B.}}$$
$$-\big\{g^{\mu\gamma}G^>_{\gamma\alpha}(x,p), \mathrm{Tr}\big[\Gamma^\alpha S^<(x,p+p')\Gamma^\nu S^>(x,p')\big]\big\}_{\mathrm{P.B.}}\big]. \qquad (19)$$

Taking the difference between Eqs. (17) and (19), we are able to derive the Boltzmann equation for the Wigner function at the leading order in $\hbar$ (or in the spacetime gradient)

$$p\cdot\partial_x G^{<,\mu\nu}(x,p) - \frac{1}{4}\big[p^\mu\partial^x_\eta G^{<,\eta\nu}(x,p) + p^\nu\partial^x_\eta G^{<,\mu\eta}(x,p)\big] = \frac{1}{4}\int\frac{d^4p'}{(2\pi\hbar)^4}\big\{\mathrm{Tr}\big[\Gamma^\mu S^<(x,p+p')\Gamma_\alpha S^>(x,p')\big]G^{>,\alpha\nu}(x,p)$$
$$-\mathrm{Tr}\big[\Gamma^\mu S^>(x,p+p')\Gamma_\alpha S^<(x,p')\big]G^{<,\alpha\nu}(x,p)\big\}$$
$$+\frac{1}{4}\int\frac{d^4p'}{(2\pi\hbar)^4}\big\{g^{\mu\gamma}G^>_{\gamma\alpha}(x,p)\mathrm{Tr}\big[\Gamma^\alpha S^<(x,p+p')\Gamma^\nu S^>(x,p')\big]$$
$$-g^{\mu\gamma}G^<_{\gamma\alpha}(x,p)\mathrm{Tr}\big[\Gamma^\alpha S^>(x,p+p')\Gamma^\nu S^<(x,p')\big]\big\}, \qquad (20)$$

where we have neglected terms with Poisson brackets and those proportional to p_η in the left-hand side since their contraction with the leading-order $G^{<,\eta\nu}(x,p)$ and $G^{<,\mu\eta}(x,p)$ is vanishing. In the next section we will rewrite the above Boltzmann equation in terms of MVSDs for vector mesons, quarks, and antiquarks.

IV. SPIN DENSITY MATRIX FOR QUARK COALESCENCE AND DISSOCIATION

Two-point Green's functions $S^{>}(x,p)$ and $S^{<}(x,p)$ for quarks are given by [40]

$$\begin{aligned} S^{<}(x,p) &= -(2\pi\hbar)\theta(p_0)\delta(p^2-m_q^2)\sum_{r,s}u(r,\mathbf{p})\bar{u}(s,\mathbf{p})f_{rs}^{(+)}(x,\mathbf{p}) \\ &\quad -(2\pi\hbar)\theta(-p_0)\delta(p^2-m_q^2)\sum_{r,s}v(s,-\mathbf{p})\bar{v}(r,-\mathbf{p})\left[\delta_{rs}-f_{rs}^{(-)}(x,-\mathbf{p})\right], \\ S^{>}(x,p) &= (2\pi\hbar)\theta(p_0)\delta(p^2-m_q^2)\sum_{r,s}u(r,\mathbf{p})\bar{u}(s,\mathbf{p})\left[\delta_{rs}-f_{rs}^{(+)}(x,\mathbf{p})\right] \\ &\quad +(2\pi\hbar)\theta(-p_0)\delta(p^2-m_q^2)\sum_{r,s}v(s,-\mathbf{p})\bar{v}(r,-\mathbf{p})f_{rs}^{(-)}(x,-\mathbf{p}), \end{aligned} \tag{21}$$

where $f_{rs}^{(+)}$ and $f_{rs}^{(-)}$ are MVSD for quarks and antiquarks, respectively. Here u and $\bar{u}$ are Dirac spinors for quarks, and v and $\bar{v}$ are Dirac spinors for antiquarks. We can parametrize them as

$$\begin{aligned} f_{rs}^{(+)}(x,\mathbf{p}) &= \frac{1}{2}f_q(x,\mathbf{p})\left[\delta_{rs}-P_\mu^q(x,\mathbf{p})n_j^{(+)\mu}(\mathbf{p})\tau_{rs}^j\right], \\ f_{rs}^{(-)}(x,-\mathbf{p}) &= \frac{1}{2}f_{\bar{q}}(x,-\mathbf{p})\left[\delta_{rs}-P_\mu^{\bar{q}}(x,-\mathbf{p})n_j^{(-)\mu}(\mathbf{p})\tau_{rs}^j\right], \end{aligned} \tag{22}$$

where τ^j with $j=1,2,3$ are Pauli matrices, $f_q(x,\mathbf{p})$ and $f_{\bar{q}}(x,-\mathbf{p})$ are MVSDs for quarks and antiquarks. respectively, and $P_q^\mu(x,\mathbf{p})$ and $P_{\bar{q}}^\mu(x,-\mathbf{p})$ are polarization four-vectors for quarks and antiquarks. respectively. The spin direction four-vectors for quarks and antiquarks are given by

$$\begin{aligned} n_j^{(+)\mu}(\mathbf{p}) &\equiv n^\mu(\mathbf{n}_j,\mathbf{p},m_q) = \left(\frac{\mathbf{n}_j\cdot\mathbf{p}}{m_q},\mathbf{n}_j+\frac{(\mathbf{n}_j\cdot\mathbf{p})\mathbf{p}}{m_q(E_\mathbf{p}^q+m_q)}\right), \\ n_j^{(-)\mu}(\mathbf{p}) &\equiv n^\mu(\mathbf{n}_j,-\mathbf{p},m_{\bar{q}}) = \left(-\frac{\mathbf{n}_j\cdot\mathbf{p}}{m_{\bar{q}}},\mathbf{n}_j+\frac{(\mathbf{n}_j\cdot\mathbf{p})\mathbf{p}}{m_{\bar{q}}(E_\mathbf{p}^{\bar{q}}+m_{\bar{q}})}\right), \end{aligned} \tag{23}$$

where $\mathbf{n}_j$ for $j=1,2,3$ are three basis unit vectors that form a Cartesian coordinate system in the particle's rest frame with $\mathbf{n}_3$ being the spin quantization direction, and $n_j^{(+)\mu}$ and $n_j^{(-)\mu}$ are the Lorentz transformed four-vectors of $\mathbf{n}_j$ for quarks and antiquarks, respectively, which obey the sum rules

$$\begin{aligned} n_j^{(+)\mu}(\mathbf{p})n_j^{(+)\nu}(\mathbf{p}) &= -\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{m_q^2}\right), \\ n_j^{(-)\mu}(\mathbf{p})n_j^{(-)\nu}(\mathbf{p}) &= -\left(g^{\mu\nu}-\frac{\bar{p}^\mu \bar{p}^\nu}{m_{\bar{q}}^2}\right), \end{aligned} \tag{24}$$

where $p^\mu=(E_p^q,\mathbf{p})$ and $\bar{p}^\mu=(E_p^{\bar{q}},-\mathbf{p})$. We note that $f_{rs}^{(+)}(x,\mathbf{p})$ and $f_{rs}^{(-)}(x,-\mathbf{p})$ are actually the transpose of those MVSDs defined in Eqs. (117) and (118) of Ref. [40] in spin indices. We can flip the sign of the three-momentum, $\mathbf{p}\to-\mathbf{p}$, in $f_{rs}^{(-)}(x,-\mathbf{p})$ to obtain

$$f_{rs}^{(-)}(x,\mathbf{p}) = \frac{1}{2}f_{\bar{q}}(x,\mathbf{p})\left[\delta_{rs}-P_\mu^{\bar{q}}(x,\mathbf{p})n_j^{(-)\mu}(-\mathbf{p})\tau_{rs}^j\right], \tag{25}$$

where $n_j^{(-)\mu}(-\mathbf{p})$ has the same form as $n_j^{(+)\mu}(\mathbf{p})$ except the quark mass. Note that in the self-energy (16) of the vector meson that is not flavor neutral, $S^{<}(x,p)$ and $S^{>}(x,p)$ may involve different flavors of quarks and antiquarks.

Inserting $S^{<}(x,p)$, $S^{>}(x,p)$, $G^{<,\mu\nu}(x,p)$, and $G^{>,\mu\nu}(x,p)$ in Eqs. (11), (13), and (21) into Eq. (20), the Boltzmann equation can be put into the following form:

$$\begin{aligned} & p\cdot\partial_x G^{<,\mu\nu}(x,p)-\frac{1}{4}\left[p^\mu\partial_\eta^x G^{<,\eta\nu}(x,p)+p^\nu\partial_\eta^x G^{<,\mu\eta}(x,p)\right] \\ &= \frac{1}{4(2\pi\hbar)}\int d^4p'\delta(p'^2-m_{\bar{q}}^2)\delta\left[(p+p')^2-m_q^2\right]\delta(p^2-m_V^2) \\ &\quad\times\left\{\theta(p_0')\theta(p_0+p_0')\theta(p_0)I_{+++}+\theta(p_0')\theta(p_0+p_0')\theta(-p_0)I_{++-}\right. \\ &\quad+\theta(p_0')\theta(-p_0-p_0')\theta(-p_0)I_{+--}+\theta(-p_0')\theta(p_0+p_0')\theta(p_0)I_{-++} \\ &\quad\left.+\theta(-p_0')\theta(-p_0-p_0')\theta(p_0)I_{--+}+\theta(-p_0')\theta(-p_0-p_0')\theta(-p_0)I_{---}\right\}. \end{aligned} \tag{26}$$

TABLE I. Collision terms in the Boltzmann equation. All terms except I_{-++} and I_{+--} are vanishing for on-shell quarks, antiquarks, and mesons at the one-loop level of the self-energy.

I_{+++}	I_{++-}	I_{+--}	I_{-++}	I_{--+}	I_{---}
$q \to q + M$	$q + \bar{M} \to q$	$\bar{M} \to q + \bar{q}$	$q + \bar{q} \to M$	$\bar{q} \to \bar{q} + M$	$\bar{q} + \bar{M} \to \bar{q}$
$q + M \to q$	$q \to q + \bar{M}$	$q + \bar{q} \to \bar{M}$	$M \to q + \bar{q}$	$\bar{q} + M \to \bar{q}$	$\bar{q} \to \bar{q} + \bar{M}$

The terms I_{ijk}, with $i, j, k = \pm$ representing the positive/negative energy, correspond to all possible processes at lowest order in the coupling constant, as shown in Table I. In Eq. (26), I_{-+-} and I_{+-+} are absent due to incompatibility of theta functions, and I_{-++} and I_{+--} contain the coalescence of quark-antiquark to the vector meson and vice versa, but I_{-++} corresponds to the positive energy sector of the two-point function for the vector meson while I_{+--} corresponds to the negative energy sector. All terms except I_{-++} and I_{+--} are vanishing for on-shell quarks, antiquarks, and mesons at the one-loop level of the self-energy. We distinguish m_q from $m_{\bar{q}}$ in Eq. (26) since the quark and antiquark may have different flavors for the vector meson that is not flavor neutral so the meson and its antiparticle are not the same particle.

In this paper, we focus on the coalescence and dissociation processes corresponding to I_{-++}. It is equivalent to consider I_{+--}. The coalescence is regarded as one of the main processes for particle production in heavy-ion collisions [54–59]. So the spin Boltzmann equation for the vector meson's MVSD reads

$$
\begin{aligned}
p\cdot\partial_x f_{\lambda_1\lambda_2}(x,\mathbf{p}) = {} & \frac{1}{16}\sum_{r_1,s_1,r_2,s_2,\lambda_1',\lambda_2'}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E^{\bar{q}}_{p'}E^{q}_{\mathbf{p}-\mathbf{p}'}}2\pi\hbar\delta(E^V_p - E^{\bar{q}}_{p'} - E^{q}_{\mathbf{p}-\mathbf{p}'}) \\
& \times\left\{\delta_{\lambda_2\lambda_2'}\epsilon^*_\mu(\lambda_1,\mathbf{p})\epsilon^\alpha(\lambda_1',\mathbf{p})\mathrm{Tr}\left[\Gamma_\alpha v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\mu u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\right]\right. \\
& \left.+\,\delta_{\lambda_1\lambda_1'}\epsilon_\nu(\lambda_2,\mathbf{p})\epsilon^*_\alpha(\lambda_2',\mathbf{p})\mathrm{Tr}\left[\Gamma^\nu v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\alpha u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\right]\right\} \\
& \times\left\{f^{(-)}_{r_1s_1}(x,\mathbf{p}')f^{(+)}_{r_2s_2}(x,\mathbf{p}-\mathbf{p}')\left[\delta_{\lambda_1'\lambda_2'} + f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right]\right. \\
& \left.-\left[\delta_{r_1s_1} - f^{(-)}_{r_1s_1}(x,\mathbf{p}')\right]\left[\delta_{r_2s_2} - f^{(+)}_{r_2s_2}(x,\mathbf{p}-\mathbf{p}')\right]f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right\},
\end{aligned}
\tag{27}
$$

where λ_1, λ_2, λ_1', and λ_2' denote the spin states along the spin quantization direction and Γ^α is the $q\bar{q}V$ vertex given by

$$\Gamma^\alpha \approx g_V B(\mathbf{p}-\mathbf{p}',\mathbf{p}')\gamma^\alpha, \tag{28}$$

where g_V is the coupling constant of the vector meson and quark-antiquark and $B(\mathbf{p}-\mathbf{p}',\mathbf{p}')$ denotes the Bethe-Salpeter wave function of the ϕ meson [60,61] in the following parametrization form:

$$B(\mathbf{p}-\mathbf{p}',\mathbf{p}') = \frac{1-\exp\left\{-\left[(E^s_{\mathbf{p}-\mathbf{p}'} - E^{\bar{s}}_{\mathbf{p}'})^2 - (\mathbf{p}-2\mathbf{p}')^2\right]/\sigma^2\right\}}{\left[(E^s_{\mathbf{p}-\mathbf{p}'} - E^{\bar{s}}_{\mathbf{p}'})^2 - (\mathbf{p}-2\mathbf{p}')^2\right]/\sigma^2}, \tag{29}$$

with $\sigma \approx 0.522$ GeV being the width parameter of the wave function. The derivation of Eq. (27) is given in Appendix A. We see that there is a gain term and a loss term in Eq. (27). In heavy-ion collisions, the distribution functions are normally much less than 1, $f_{\lambda_1\lambda_2}(x,\mathbf{p}) \sim f^{(+)}_{rs} \sim f^{(-)}_{rs} \ll 1$, so Eq. (27) can be approximated as

$$\frac{p}{E^V_p}\cdot\partial_x f_{\lambda_1\lambda_2}(x,\mathbf{p}) \approx R^{\mathrm{coal}}_{\lambda_1\lambda_2}(\mathbf{p}) - R^{\mathrm{diss}}(\mathbf{p})f_{\lambda_1\lambda_2}(x,\mathbf{p}), \tag{30}$$

where $R^{\mathrm{coal}}_{\lambda_1\lambda_2}$ and R^{diss} denote the coalescence and dissociation rates for the vector meson, i.e., the rates of $q+\bar{q}\to M$ and $M \to q + \bar{q}$, respectively, defined as

$$
\begin{aligned}
R^{\mathrm{coal}}_{\lambda_1\lambda_2}(\mathbf{p}) = {} & \frac{1}{8(2\pi\hbar)^2}\sum_{r_1,s_1,r_2,s_2}\int d^3\mathbf{p}'\frac{1}{E^{\bar{q}}_{p'}E^{q}_{\mathbf{p}-\mathbf{p}'}E^V_p}\delta(E^V_p - E^{\bar{q}}_{p'} - E^{q}_{\mathbf{p}-\mathbf{p}'})\epsilon^*_\alpha(\lambda_1,\mathbf{p})\epsilon_\beta(\lambda_2,\mathbf{p}) \\
& \times\mathrm{Tr}\left[\Gamma^\beta v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\alpha u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\right]f^{(-)}_{r_1s_1}(x,\mathbf{p}')f^{(+)}_{r_2s_2}(x,\mathbf{p}-\mathbf{p}'),
\end{aligned}
\tag{31}
$$

$$R^{\rm diss}(\mathbf{p}) = -\frac{1}{12(2\pi\hbar)^2}\sum_{r_1,r_2}\int d^3\mathbf{p}'\frac{1}{E_{p'}^{\bar{q}}E_{\mathbf{p}-\mathbf{p}'}^{q}E_p^V}\delta(E_p^V - E_{p'}^{\bar{q}} - E_{\mathbf{p}-\mathbf{p}'}^{q})\left(g_{\alpha\beta} - \frac{p_\alpha p_\beta}{m_V^2}\right) \times \mathrm{Tr}\left\{\Gamma^\beta(p'\cdot\gamma - m_{\bar{q}})\Gamma^\alpha\left[(p-p')\cdot\gamma + m_q\right]\right\}. \tag{32}$$

Note that $R^{\rm diss}(\mathbf{p})$ does not depend on the MVSDs of the quark, the antiquark, and the vector meson; therefore, it is independent of the quark polarization. Schematically the formal solution to Eq. (30) reads

$$f_{\lambda_1\lambda_2}(x,\mathbf{p}) \sim \frac{R^{\rm coal}_{\lambda_1\lambda_2}(\mathbf{p})}{R^{\rm diss}(\mathbf{p})}\left[1 - \exp\left(-R^{\rm diss}(\mathbf{p})\Delta t\right)\right] \sim \begin{cases} R^{\rm coal}_{\lambda_1\lambda_2}(\mathbf{p})\Delta t, & \text{for } \Delta t \ll 1/R^{\rm diss}(\mathbf{p}) \\ \frac{R^{\rm coal}_{\lambda_1\lambda_2}(\mathbf{p})}{R^{\rm diss}(\mathbf{p})}, & \text{for } \Delta t \gg 1/R^{\rm diss}(\mathbf{p}) \end{cases} \tag{33}$$

if $f_{\lambda_1\lambda_2}(x,\mathbf{p})$ for the vector meson at the initial time is assumed to be zero, where Δt is the formation time of the vector meson.

The spin density matrix element $\rho^V_{\lambda_1\lambda_2}$ is assumed to be proportional to $f_{\lambda_1\lambda_2}(x,\mathbf{p})$, which is $R^{\rm coal}_{\lambda_1\lambda_2}(\mathbf{p})\Delta t$ if $\Delta t \ll 1/R^{\rm diss}(\mathbf{p})$ or $R^{\rm coal}_{\lambda_1\lambda_2}(\mathbf{p})/R^{\rm diss}(\mathbf{p})$ if $\Delta t \gg 1/R^{\rm diss}(\mathbf{p})$. In both cases, $\rho^V_{\lambda_1\lambda_2}$ is proportional to $R^{\rm coal}_{\lambda_1\lambda_2}(\mathbf{p})$ times a constant independent of the spin states of the vector meson, which is a good feature for the spin density matrix. For a finite formation time between these two limits, the result of $\rho^V_{\lambda_1\lambda_2}$ can be regarded as averaged over the formation time. Here we assume that the coalescence of the vector meson takes place in a relatively short time, so we have

$$\rho^V_{\lambda_1\lambda_2}(x,\mathbf{p}) \approx \frac{\Delta t}{8}\sum_{r_1,s_1,r_2,s_2}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E_{p'}^{\bar{q}}E_{\mathbf{p}-\mathbf{p}'}^{q}E_p^V}2\pi\hbar\delta(E_p^V - E_{p'}^{\bar{q}} - E_{\mathbf{p}-\mathbf{p}'}^{q})\epsilon^*_\alpha(\lambda_1,\mathbf{p})\epsilon_\beta(\lambda_2,\mathbf{p}) \times \mathrm{Tr}\left[\Gamma^\beta v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\alpha u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\right]f^{(\bar{q})}_{r_1s_1}(x,\mathbf{p}')f^{(q)}_{r_2s_2}(x,\mathbf{p}-\mathbf{p}'), \tag{34}$$

where we have changed the notation to $f^{(q/\bar{q})}_{rs}$ from $f^{(\pm)}_{rs}$. The spin density matrix element (34) can be put into a compact form with an explicit dependence on the polarization vector of the quark and antiquark

$$\rho^V_{\lambda_1\lambda_2}(x,\mathbf{p}) = \frac{\Delta t}{32}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E_{p'}^{\bar{q}}E_{\mathbf{p}-\mathbf{p}'}^{q}E_p^V}f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{p}-\mathbf{p}')2\pi\hbar\delta(E_p^V - E_{p'}^{\bar{q}} - E_{\mathbf{p}-\mathbf{p}'}^{q})\epsilon^*_\alpha(\lambda_1,\mathbf{p})\epsilon_\beta(\lambda_2,\mathbf{p}) \times \mathrm{Tr}\left\{\Gamma^\beta(p'\cdot\gamma - m_{\bar{q}})\left[1+\gamma_5\gamma\cdot P^{\bar{q}}(x,\mathbf{p}')\right]\Gamma^\alpha\left[(p-p')\cdot\gamma + m_q\right]\left[1+\gamma_5\gamma\cdot P^q(x,\mathbf{p}-\mathbf{p}')\right]\right\}, \tag{35}$$

where $p^\mu = (E_p^q, \mathbf{p})$ and $p'^\mu = (E_{p'}^{\bar{q}}, \mathbf{p}')$. The derivation of the expression inside the trace is given in Appendix B. The contraction of $\epsilon^*_\alpha(\lambda_1,\mathbf{p})$ and $\epsilon_\beta(\lambda_2,\mathbf{p})$ with the trace can be worked out, and the result is given by Eq. (B9). The normalized $\rho^V_{\lambda_1\lambda_2}(x,\mathbf{p})$ is defined as

$$\bar{\rho}^V_{\lambda_1\lambda_2}(x,\mathbf{p}) = \frac{\rho^V_{\lambda_1\lambda_2}(x,\mathbf{p})}{\mathrm{Tr}(\rho_V)}, \tag{36}$$

where $\mathrm{Tr}(\rho_V)$ is the trace of the spin density matrix and is evaluated using Eq. (B10) and $\rho^V_{\lambda_1\lambda_2}(x,\mathbf{p})$ is evaluated using Eq. (B9).

For unflavored vector mesons such as ϕ mesons with $m_q = m_{\bar{q}}$, $\rho^V_{\lambda_1\lambda_2}(x,\mathbf{p})$, and $\mathrm{Tr}(\rho_V)$ can be simplified as

$$\rho^V_{\lambda_1\lambda_2}(x,\mathbf{p}) = -\frac{\Delta t}{8}g_V^2\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E_{p'}^{\bar{q}}E_{\mathbf{p}-\mathbf{p}'}^{q}E_p^V}f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{p}-\mathbf{p}')2\pi\hbar\delta(E_p^V - E_{p'}^{\bar{q}} - E_{\mathbf{p}-\mathbf{p}'}^{q})B^2(\mathbf{p}-\mathbf{p}',\mathbf{p}')\epsilon^*_\alpha(\lambda_1,\mathbf{p})\epsilon_\beta(\lambda_2,\mathbf{p}) \times \left\{(p'^\alpha P^\beta_{\bar{q}} + p'^\beta P^\alpha_{\bar{q}})(p'\cdot P_q) - (p'^\alpha P^\beta_q + p'^\beta P^\alpha_q)(p\cdot P_{\bar{q}}) + 2p'^\alpha p'^\beta(1 - P_{\bar{q}}\cdot P_q) + g^{\alpha\beta}\left[p'\cdot p + (p'\cdot P_q)(p\cdot P_{\bar{q}})\right] + (p\cdot p')(P^\alpha_{\bar{q}}P^\beta_q + P^\alpha_q P^\beta_{\bar{q}} - g^{\alpha\beta}P_{\bar{q}}\cdot P_q) - im_q\epsilon^{\alpha\beta\mu\nu}p_\mu(P^q_\nu + P^{\bar{q}}_\nu)\right\}, \tag{37}$$

$$\mathrm{Tr}(\rho_V) = \frac{\Delta t}{8}g_V^2\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E_{p'}^{\bar{q}}E_{\mathbf{p}-\mathbf{p}'}^{q}E_p^V}f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{p}-\mathbf{p}')2\pi\hbar\delta(E_p^V - E_{p'}^{\bar{q}} - E_{\mathbf{p}-\mathbf{p}'}^{q})B^2(\mathbf{p}-\mathbf{p}',\mathbf{p}') \times\left[-2m_q^2(P_{\bar{q}}\cdot P_q) + m_V^2 + 2m_q^2\right], \tag{38}$$

where we have used the shorthand notation $P_q \equiv P_q(x, \mathbf{p}-\mathbf{p}')$ and $P_{\bar{q}} \equiv P_{\bar{q}}(x, \mathbf{p}')$. Equations (37) and (38) will be used in the next section for evaluating spin density matrix elements for ϕ mesons.

V. SPIN DENSITY MATRIX ELEMENTS FOR ϕ MESONS

Now we consider the vector meson made of a quark and its antiquark, the so-called unflavored meson. For the unflavored vector meson such as the ϕ meson, the polarization distributions in phase space in Eq. (35) are given by [17,62–65]

$$P_s^\mu(x,\mathbf{p}) = \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} + \frac{g_\phi}{E_p^s T_{\mathrm{eff}}}F_{\rho\sigma}^\phi\right)p_\nu\left[1-f_s(x,\mathbf{p})\right],$$
$$P_{\bar{s}}^\mu(x,\mathbf{p}) = \frac{1}{4m_s}\epsilon^{\mu\nu\rho\sigma}\left(\omega_{\rho\sigma} - \frac{g_\phi}{E_p^{\bar{s}} T_{\mathrm{eff}}}F_{\rho\sigma}^\phi\right)p_\nu\left[1-f_{\bar{s}}(x,\mathbf{p})\right], \quad (39)$$

where $p^\mu = (E_p^s, \mathbf{p})$ and $p^\mu = (E_p^{\bar{s}}, \mathbf{p})$ denote the four-momenta of the strange quark s and antiquark $\bar{s}$, respectively, with $E_p^s = E_p^{\bar{s}} = \sqrt{|\mathbf{p}|^2 + m_s^2}$ and $m_{\bar{s}} = m_s$. We have assumed that s and $\bar{s}$ are polarized by the thermal vorticity (tensor) field $\omega_{\rho\sigma} = (1/2)\left[\partial_\rho(\beta u_\sigma) - \partial_\sigma(\beta u_\rho)\right]$ and ϕ field strength tensor $F_{\rho\sigma}^\phi = \partial_\rho A_\sigma^\phi - \partial_\sigma A_\rho^\phi$ [26], where u_σ is the fluid velocity, $\beta = 1/T_{\mathrm{eff}}$ is the inverse effective temperature, and A_σ^ϕ is the vector potential of the ϕ field. Note that in some literature the definition of $\omega_{\rho\sigma}$ may differ by a sign [14,62,63]. In Eq. (39) $f_s(x,\mathbf{p})$ and $f_{\bar{s}}(x,\mathbf{p})$ are unpolarized phase-space distributions of s and $\bar{s}$, respectively, and given by the Fermi-Dirac distribution

$$f_{s/\bar{s}}(x,\mathbf{p}) = \frac{1}{1+\exp(\beta E_p^{s/\bar{s}} \mp \beta\mu_s)}, \quad (40)$$

where μ_s is the chemical potential for s ($-\mu_s$ for $\bar{s}$). In most cases $f_{s/\bar{s}}$ are negligible relative to 1 in $P_{s/\bar{s}}^\mu$ in Eq. (39). The spin-field coupling in (39) can be derived from the Wigner functions for massive fermions [65] and has a clear physical meaning: one contribution is from the magnetic field through the magnetic moment and the other contribution from the electric field through the spin-orbit coupling; the former is always there while the latter is only present for moving fermions. The mean field effects of vector mesons have been studied in the context of spin polarization of Λ hyperons [66] and different elliptic flows between hadrons of some species and their antiparticles [67] in heavy-ion collisions.

The polarization four-vector for the ϕ meson is given by

$$\epsilon^\mu(\lambda,\mathbf{p}) = \left(\frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_\lambda}{m_\phi}, \boldsymbol{\epsilon}_\lambda + \frac{\mathbf{p}\cdot\boldsymbol{\epsilon}_\lambda}{m_\phi(E_p^\phi + m_\phi)}\mathbf{p}\right), \quad (41)$$

where $E_p^\phi \equiv \sqrt{m_\phi^2 + \mathbf{p}^2}$ is the energy of the ϕ meson, $\lambda = 0, \pm 1$ denotes the spin states, and $\boldsymbol{\epsilon}_\lambda$ denotes the polarization three-vector of the spin state in the ϕ meson's rest frame. To calculate the spin alignment along the direction of the global orbital angular momentum (the y direction) in heavy-ion collisions, we choose the y direction as the spin quantization direction. So the corresponding polarization three-vectors are

$$\boldsymbol{\epsilon}_0 = (0,1,0),$$
$$\boldsymbol{\epsilon}_{+1} = -\frac{1}{\sqrt{2}}(i,0,1),$$
$$\boldsymbol{\epsilon}_{-1} = \frac{1}{\sqrt{2}}(-i,0,1). \quad (42)$$

The 00-component of the spin density matrix is what can be measured in experiments that concerns the real vector $\boldsymbol{\epsilon}_0$ satisfying $\boldsymbol{\epsilon}_0 = \boldsymbol{\epsilon}_0^*$.

Substituting Eq. (39) into Eq. (35), we obtain

$$\rho_{\lambda_1\lambda_2}^\phi = \rho_{\lambda_1\lambda_2}^\phi(0) + \rho_{\lambda_1\lambda_2}^\phi(\omega^1) + \rho_{\lambda_1\lambda_2}^\phi(F_\phi^1) + \rho_{\lambda_1\lambda_2}^\phi(\omega^2) + \rho_{\lambda_1\lambda_2}^\phi(F_\phi^2), \quad (43)$$

where ω^i and F_ϕ^i with $i = 0$, 1, 2 denote the zeroth, first, and second order terms in the vorticity and ϕ field, respectively. The zeroth order term $\rho_{\lambda_1\lambda_2}^\phi(0)$ represents the unpolarized contribution. In (43), we neglected mixing terms of $\omega_{\mu\nu}$ and $F_{\mu\nu}^\phi$ since we assume that there is no correlation between them in spacetime so these terms are vanishing after taking a spacetime average of $\rho_{\lambda_1\lambda_2}^\phi$. For $\lambda_1 = \lambda_2 = 0$, $\epsilon_\alpha^*(0,\mathbf{p})\epsilon_\beta(0,\mathbf{p}) = \epsilon_\alpha(0,\mathbf{p})\epsilon_\beta(0,\mathbf{p})$ is symmetric in α and β, and then one can verify that the first order terms $\rho_{00}^\phi(\omega^1)$ and $\rho_{00}^\phi(F_\phi^1)$ are vanishing. The zeroth order term $\rho_{00}^\phi(0)$ is given by

$$\rho_{00}^\phi(0) = \frac{\Delta t}{8}g_\phi^2\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E_{p'}^{\bar{s}}E_{\mathbf{p}-\mathbf{p}'}^s E_p^\phi}f_{\bar{s}}(\mathbf{p}')f_s(\mathbf{p}-\mathbf{p}')B^2(\mathbf{p}-\mathbf{p}',\mathbf{p}')$$
$$\times 2\pi\hbar\delta(E_p^\phi - E_{p'}^{\bar{s}} - E_{\mathbf{p}-\mathbf{p}'}^s)\left\{(p'\cdot p) - 2\left[p'\cdot\epsilon(0,\mathbf{p})\right]^2\right\}, \quad (44)$$

where we have used the second relation of Eq. (7). The second order terms $\rho^{\phi}_{\lambda_1\lambda_2}(\omega^2)$ and $\rho^{\phi}_{\lambda_1\lambda_2}(F^2_{\phi})$ read

$$\rho^{\phi}_{\lambda_1\lambda_2}(\omega^2) \approx -\frac{\Delta t}{32}\frac{1}{4m_s^2}g_{\phi}^2\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E^{\bar{s}}_{p'}E^{s}_{\mathbf{p}-\mathbf{p}'}E^{\phi}_{p}}B^2(\mathbf{p}-\mathbf{p}',\mathbf{p}')f_{\bar{s}}(\mathbf{p}')f_s(\mathbf{p}-\mathbf{p}')2\pi\hbar\delta(E^{\phi}_p-E^{\bar{s}}_{p'}-E^{s}_{\mathbf{p}-\mathbf{p}'})$$
$$\times\,\epsilon^*_{\alpha}(\lambda_1,\mathbf{p})\epsilon_{\beta}(\lambda_2,\mathbf{p})\tilde{\omega}_{\rho\xi}(x)\tilde{\omega}_{\sigma\gamma}(x)p'^{\xi}(p-p')^{\gamma}\mathrm{Tr}\{\gamma^{\beta}(p'\cdot\gamma+m_s)\gamma^{\rho}\gamma^{\alpha}[(p-p')\cdot\gamma+m_s]\gamma^{\sigma}\} \tag{45}$$

and

$$\rho^{\phi}_{\lambda_1\lambda_2}(F^2_{\phi}) \approx \frac{\Delta t}{32}\frac{1}{4m_s^2T^2_{\mathrm{eff}}}g_{\phi}^4\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{(E^{\bar{s}}_{p'})^2(E^{s}_{\mathbf{p}-\mathbf{p}'})^2E^{\phi}_{p}}B^2(\mathbf{p}-\mathbf{p}',\mathbf{p}')f_{\bar{s}}(\mathbf{p}')f_s(\mathbf{p}-\mathbf{p}')2\pi\hbar\delta(E^{\phi}_p-E^{\bar{s}}_{p'}-E^{s}_{\mathbf{p}-\mathbf{p}'})$$
$$\times\,\epsilon^*_{\alpha}(\lambda_1,\mathbf{p})\epsilon_{\beta}(\lambda_2,\mathbf{p})\tilde{F}^{\phi}_{\rho\xi}(x)\tilde{F}^{\phi}_{\sigma\gamma}(x)p'^{\xi}(p-p')^{\gamma}\mathrm{Tr}\{\gamma^{\beta}(p'\cdot\gamma+m_s)\gamma^{\rho}\gamma^{\alpha}[(p-p')\cdot\gamma+m_s]\gamma^{\sigma}\}. \tag{46}$$

In Eqs. (45) and (46) we have used $\tilde{\omega}_{\rho\xi}=(1/2)\epsilon_{\rho\xi\alpha\beta}\omega^{\alpha\beta}$, $\tilde{F}^{\phi}_{\rho\xi}=(1/2)\epsilon_{\rho\xi\alpha\beta}F^{\alpha\beta}_{\phi}$, and neglected $f_{s/\bar{s}}$ relative to 1 in $P^{\mu}_{s/\bar{s}}$. The tensor part of $\rho^{\phi}_{\lambda_1\lambda_2}(\omega^2)$ and $\rho^{\phi}_{\lambda_1\lambda_2}(F^2_{\phi})$ that is contracted with $\epsilon^*_{\alpha}\epsilon_{\beta}\tilde{\omega}_{\rho\xi}\tilde{\omega}_{\sigma\gamma}$ and $\epsilon^*_{\alpha}\epsilon_{\beta}\tilde{F}^{\phi}_{\rho\xi}\tilde{F}^{\phi}_{\sigma\gamma}$, respectively, can be evaluated as

$$\begin{aligned}I^{\alpha\beta;\rho\xi;\sigma\gamma} &= p'^{\xi}(p-p')^{\gamma}\mathrm{Tr}\{\gamma^{\beta}(p'\cdot\gamma+m_s)\gamma^{\rho}\gamma^{\alpha}[(p-p')\cdot\gamma+m_s]\gamma^{\sigma}\}\\
&= 2p'^{\xi}p^{\gamma}[m_{\phi}^2(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})+2p^{\rho}(g^{\alpha\beta}p'^{\sigma}-g^{\alpha\sigma}p'^{\beta}-g^{\beta\sigma}p'^{\alpha})\\
&\quad+2(g^{\beta\rho}p'^{\alpha}p'^{\sigma}+g^{\alpha\rho}p'^{\beta}p'^{\sigma}-2g^{\rho\sigma}p'^{\alpha}p'^{\beta})]-2p'^{\xi}p'^{\gamma}[m_{\phi}^2(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})\\
&\quad-2p^{\rho}(g^{\alpha\sigma}p'^{\beta}+g^{\beta\sigma}p'^{\alpha})-4g^{\rho\sigma}p'^{\alpha}p'^{\beta}].\end{aligned} \tag{47}$$

With the above tensor and the quantity inside the curly brackets in (44), $\rho^{\phi}_{00}(0)$, $\rho^{\phi}_{\lambda_1\lambda_2}(\omega^2)$, and $\rho^{\phi}_{\lambda_1\lambda_2}(F^2)$ involve the following moments of momenta:

$$\{I_0,I_0^{\mu},I_0^{\mu\nu},I_0^{\mu\nu\rho},I_0^{\mu\nu\rho\sigma}\}=\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E^{\bar{s}}_{p'}E^{s}_{\mathbf{p}-\mathbf{p}'}}B^2(\mathbf{p}-\mathbf{p}',\mathbf{p}')f_{\bar{s}}(\mathbf{p}')f_s(\mathbf{p}-\mathbf{p}')2\pi\hbar\delta(E^{\phi}_p-E^{\bar{s}}_{p'}-E^{s}_{\mathbf{p}-\mathbf{p}'})$$
$$\times\,\{1,p'^{\mu},p'^{\mu}p'^{\nu},p'^{\mu}p'^{\nu}p'^{\rho},p'^{\mu}p'^{\nu}p'^{\rho}p'^{\sigma}\}, \tag{48}$$

$$\{I_F^{\mu},I_F^{\mu\nu},I_F^{\mu\nu\rho},I_F^{\mu\nu\rho\sigma}\}=\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{(E^{\bar{s}}_{p'})^2(E^{s}_{\mathbf{p}-\mathbf{p}'})^2}B^2(\mathbf{p}-\mathbf{p}',\mathbf{p}')f_{\bar{s}}(\mathbf{p}')f_s(\mathbf{p}-\mathbf{p}')2\pi\hbar\delta(E^{\phi}_p-E^{\bar{s}}_{p'}-E^{s}_{\mathbf{p}-\mathbf{p}'})$$
$$\times\,\{p'^{\mu},p'^{\mu}p'^{\nu},p'^{\mu}p'^{\nu}p'^{\rho},p'^{\mu}p'^{\nu}p'^{\rho}p'^{\sigma}\}. \tag{49}$$

The tensors in (48) with the subscript "0" are those in $\rho^{\phi}_{00}(0)$ and $\rho^{\phi}_{\lambda_1\lambda_2}(\omega^2)$, and the tensors in (49) with the subscript "F" are those in $\rho^{\phi}_{\lambda_1\lambda_2}(F^2_{\phi})$. The difference between Eqs. (48) and (49) is in the powers of $E^{\bar{s}}_{p'}$ and $E^{s}_{\mathbf{p}-\mathbf{p}'}$ in the denominators. Note that all of the above tensors with two or more indices are symmetric with respect to the interchange of any two indices.

Using Eqs. (47)–(49), the zeroth and second order terms of the spin density matrix in (44)–(46) can be expressed in terms of moments of momenta

$$\rho^{\phi}_{00}(0)=\frac{\Delta t}{16}g_{\phi}^2m_{\phi}^2\frac{1}{E^{\phi}_p}I_0\left[1-4\epsilon_{\alpha}(0,\mathbf{p})\epsilon_{\beta}(0,\mathbf{p})\frac{I_0^{\alpha\beta}}{m_{\phi}^2I_0}\right], \tag{50}$$

$$\begin{aligned}\rho^{\phi}_{00}(\omega^2) = &-\frac{\Delta t}{64}\frac{g_{\phi}^2}{m_s^2}\frac{1}{E^{\phi}_p}\epsilon_{\alpha}(0,\mathbf{p})\epsilon_{\beta}(0,\mathbf{p})\tilde{\omega}_{\rho\xi}(x)\tilde{\omega}_{\sigma\gamma}(x)\Big[p^{\gamma}m_{\phi}^2(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})I_0^{\xi}\\
&+2p^{\gamma}p^{\rho}(g^{\alpha\beta}I_0^{\xi\sigma}-g^{\alpha\sigma}I_0^{\xi\beta}-g^{\beta\sigma}I_0^{\xi\alpha})+2p^{\gamma}(g^{\beta\rho}I_0^{\xi\sigma\alpha}+I_0^{\xi\sigma\beta}g^{\alpha\rho}-2g^{\rho\sigma}I_0^{\xi\alpha\beta})\\
&-m_{\phi}^2(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})I_0^{\xi\gamma}+2p^{\rho}(g^{\alpha\sigma}I_0^{\xi\beta\gamma}+g^{\beta\sigma}I_0^{\xi\alpha\gamma})+4g^{\rho\sigma}I_0^{\xi\alpha\beta\gamma}\Big],\end{aligned} \tag{51}$$

$$\rho_{00}^{\phi}(F_{\phi}^{2}) = \frac{\Delta t}{64}\frac{g_{\phi}^{4}}{m_s^2 T_{\rm eff}^2}\frac{1}{E_p^{\phi}}\epsilon_{\alpha}(0,\mathbf{p})\epsilon_{\beta}(0,\mathbf{p})\tilde{F}^{\phi}_{\rho\xi}(x)\tilde{F}^{\phi}_{\sigma\gamma}(x)\Big[p^{\gamma}m_{\phi}^{2}(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})I_F^{\xi} + 2p^{\gamma}p^{\rho}(g^{\alpha\beta}I_F^{\xi\sigma}-g^{\alpha\sigma}I_F^{\xi\beta}-g^{\beta\sigma}I_F^{\xi\alpha}) + 2p^{\gamma}(g^{\beta\rho}I_F^{\xi\sigma\alpha}+I_F^{\xi\sigma\beta}g^{\alpha\rho}-2g^{\rho\sigma}I_F^{\xi\alpha\beta}) - m_{\phi}^{2}(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})I_F^{\xi\gamma}+2p^{\rho}(g^{\alpha\sigma}I_F^{\xi\beta\gamma}+g^{\beta\sigma}I_F^{\xi\alpha\gamma})+4g^{\rho\sigma}I_F^{\xi\alpha\beta\gamma}\Big]. \tag{52}$$

From Eq. (38), the trace of the spin density matrix for the ϕ meson reads

$$\mathrm{Tr}(\rho_{\phi}) = \frac{\Delta t}{8}g_{\phi}^{2}(m_{\phi}^{2}+2m_s^2)\frac{1}{E_p^{\phi}}I_0\Big[1-\frac{1}{2(m_{\phi}^2+2m_s^2)}\tilde{\omega}_{\rho\xi}(x)\tilde{\omega}_{\sigma\gamma}(x)g^{\rho\sigma}\frac{1}{I_0}(p^{\gamma}I_0^{\xi}-I_0^{\xi\gamma}) + \frac{g_{\phi}^{2}}{2(m_{\phi}^2+2m_s^2)T_{\rm eff}^2}\tilde{F}^{\phi}_{\rho\xi}(x)\tilde{F}^{\phi}_{\sigma\gamma}(x)g^{\rho\sigma}\frac{1}{I_0}(p^{\gamma}I_F^{\xi}-I_F^{\xi\gamma})\Big]. \tag{53}$$

Here we have neglected mixing terms of $\omega_{\mu\nu}$ and $F^{\phi}_{\mu\nu}$ since we assume that there is no correlation in spacetime between them.

From Eqs. (50)–(53) we obtain the 00-component of the normalized spin density matrix for the ϕ meson defined in (36)

$$\bar{\rho}_{00}^{\phi}(x,\mathbf{p}) = c_0(\mathbf{p}) + c_{\omega}(x,\mathbf{p}) + c_F(x,\mathbf{p}), \tag{54}$$

where c_0, c_{ω}, and c_F are given by

$$c_0(\mathbf{p}) = \frac{m_{\phi}^2}{2(m_{\phi}^2+2m_s^2)}\Big[1-4\epsilon_{\alpha}(0,\mathbf{p})\epsilon_{\beta}(0,\mathbf{p})\frac{I_0^{\alpha\beta}}{m_{\phi}^2 I_0}\Big], \tag{55}$$

$$c_{\omega}(x,\mathbf{p}) = -\frac{1}{8m_s^2(m_{\phi}^2+2m_s^2)}\epsilon_{\alpha}(0,\mathbf{p})\epsilon_{\beta}(0,\mathbf{p})\tilde{\omega}_{\rho\xi}(x)\tilde{\omega}_{\sigma\gamma}(x)\frac{1}{I_0}\Big[p^{\gamma}m_{\phi}^{2}(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})I_0^{\xi} + 2p^{\gamma}p^{\rho}(g^{\alpha\beta}I_0^{\xi\sigma}-g^{\alpha\sigma}I_0^{\xi\beta}-g^{\beta\sigma}I_0^{\xi\alpha}) + 2p^{\gamma}(g^{\beta\rho}I_0^{\xi\sigma\alpha}+I_0^{\xi\sigma\beta}g^{\alpha\rho}-2g^{\rho\sigma}I_0^{\xi\alpha\beta}) - m_{\phi}^{2}(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})I_0^{\xi\gamma}+2p^{\rho}(g^{\alpha\sigma}I_0^{\xi\beta\gamma}+g^{\beta\sigma}I_0^{\xi\alpha\gamma})+4g^{\rho\sigma}I_0^{\xi\alpha\beta\gamma}\Big] + \frac{c_0(\mathbf{p})}{2(m_{\phi}^2+2m_s^2)}\tilde{\omega}_{\rho\xi}(x)\tilde{\omega}_{\sigma\gamma}(x)g^{\rho\sigma}\frac{1}{I_0}(p^{\gamma}I_0^{\xi}-I_0^{\xi\gamma}), \tag{56}$$

and

$$c_F(x,\mathbf{p}) = \frac{1}{8m_s^2(m_{\phi}^2+2m_s^2)}\frac{g_{\phi}^2}{T_{\rm eff}^2}\epsilon_{\alpha}(0,\mathbf{p})\epsilon_{\beta}(0,\mathbf{p})\tilde{F}^{\phi}_{\rho\xi}(x)\tilde{F}^{\phi}_{\sigma\gamma}(x)\frac{1}{I_0}\Big[p^{\gamma}m_{\phi}^{2}(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})I_F^{\xi} + 2p^{\gamma}p^{\rho}(g^{\alpha\beta}I_F^{\xi\sigma}-g^{\alpha\sigma}I_F^{\xi\beta}-g^{\beta\sigma}I_F^{\xi\alpha}) + 2p^{\gamma}(g^{\beta\rho}I_F^{\xi\sigma\alpha}+I_F^{\xi\sigma\beta}g^{\alpha\rho}-2g^{\rho\sigma}I_F^{\xi\alpha\beta}) - m_{\phi}^{2}(g^{\beta\rho}g^{\alpha\sigma}-g^{\alpha\beta}g^{\rho\sigma}+g^{\beta\sigma}g^{\alpha\rho})I_F^{\xi\gamma}+2p^{\rho}(g^{\alpha\sigma}I_F^{\xi\beta\gamma}+g^{\beta\sigma}I_F^{\xi\alpha\gamma})+4g^{\rho\sigma}I_F^{\xi\alpha\beta\gamma}\Big] - \frac{g_{\phi}^2 c_0(\mathbf{p})}{2(m_{\phi}^2+2m_s^2)T_{\rm eff}^2}\tilde{F}^{\phi}_{\rho\xi}(x)\tilde{F}^{\phi}_{\sigma\gamma}(x)g^{\rho\sigma}\frac{1}{I_0}(p^{\gamma}I_F^{\xi}-I_F^{\xi\gamma}). \tag{57}$$

We see in Eqs. (55)–(57) that the momentum moments always come with the factor $1/I_0$, so they can be understood as normalized moments by I_0, a kind of momentum averages.

We see in Eqs. (55)–(57) that c_0, c_{ω}, and c_F are all Lorentz scalars, so it is convenient to evaluate them in the rest frame of the vector meson. All nonvanishing moments of momenta in c_{ω} and c_F in Eqs. (56) and (57) that are evaluated in the rest frame of the vector meson are listed in Table II. We can also evaluate c_0 in the rest frame of the vector meson using Eq. (41) and $I_0^{00} = (m_{\phi}^2/4)I_0$ and $I_0^{ij} = (1/3)\delta_{ij}(m_{\phi}^2/4 - m_s^2)I_0$, which gives $c_0 = 1/3$. Finally, the result for $\bar{\rho}_{00}^{\phi}$ is

TABLE II. All nonvanishing moments of momenta normalized by I_0 in $\bar{\rho}_{00}^{\phi}$ from contributions of the vorticity and the ϕ field, which are evaluated in the rest frame of the vector meson. Note that I represents either I_0 or I_F. The definition for some quantities are $I^{aa} \equiv I^{11} + I^{22} + I^{33}$, $I^{0aa} \equiv I^{011} + I^{022} + I^{033}$, $I^{00aa} \equiv I^{0011} + I^{0022} + I^{0033}$, $I^{aabb} \equiv I^{1122} + I^{2233} + I^{3311}$, and $I^{aaaa} \equiv I^{1111} + I^{2222} + I^{3333}$. The constant d_0 is defined as $d_0 \equiv 1\text{–}4m_s^2/m_\phi^2$. The tensors in Eqs. (56) and (57) are linear combinations of the quantities listed in this table, for example, $I_F^{ij} = (1/3)\delta_{ij}I_F^{aa}$, $I_F^{0ij} = (1/3)\delta_{ij}I_F^{0aa}$, etc.

	I^μ	I^{00}	I^{aa}	I^{000}	I^{0aa}	I^{0000}	I^{00aa}	I^{aabb}	I^{aaaa}
ω	$(m_\phi/2)g^{\mu 0}$	$m_\phi^2/4$	$d_0 m_\phi^2/4$	$m_\phi^3/8$	$d_0 m_\phi^3/8$	$m_\phi^4/16$	$d_0 m_\phi^4/16$	$d_0^2 m_\phi^4/80$	$3d_0^2 m_\phi^4/80$
F_ϕ	$(2/m_\phi)g^{\mu 0}$	1	d_0	$m_\phi/2$	$d_0 m_\phi/2$	$m_\phi^2/4$	$d_0 m_\phi^2/4$	$d_0^2 m_\phi^2/20$	$3d_0^2 m_\phi^2/20$

$$\bar{\rho}_{00}^{\phi}(x,\mathbf{p}) \approx \frac{1}{3} + C_1\left[\frac{1}{3}\left(\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi\right) - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\omega}')^2 + \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}(\boldsymbol{\epsilon}_0\cdot\mathbf{B}'_\phi)^2\right] + C_2\left[\frac{1}{3}\left(\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi\right) - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\varepsilon}')^2 + \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}(\boldsymbol{\epsilon}_0\cdot\mathbf{E}'_\phi)^2\right], \tag{58}$$

where the fields with primes are in the rest frame of the vector meson, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\omega}$ denote the electric and magnetic parts of the vorticity tensor $\omega^{\mu\nu}$, respectively, $\mathbf{E}_\phi$ and $\mathbf{B}_\phi$ denote the electric and magnetic parts of the ϕ field tensor $F_\phi^{\mu\nu}$, respectively, and C_1 and C_2 are two coefficients depending on masses of the quark and vector mesons defined as

$$C_1 = \frac{8m_s^4 + 16m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)},$$
$$C_2 = \frac{8m_s^4 - 14m_s^2 m_\phi^2 + 3m_\phi^4}{120m_s^2(m_\phi^2 + 2m_s^2)}. \tag{59}$$

The result for $\bar{\rho}_{00}^{\phi}(x,\mathbf{p})$ in Eq. (58) is rigorous and remarkable since all contributions are in squares of the fields. This is a clear piece of evidence that there exists in the ϕ meson an exact correlation between the strong force field coupled to the s quark and that coupled to the $\bar{s}$ quark. This feature makes ρ_{00} for unflavored vector mesons very different from that for other vector mesons carrying net charges or flavors.

In evaluating the integrals in momentum moments in the rest frame of the vector meson, we assume a simple form for the fluid four-velocity $u'^\mu = (1,\mathbf{0})$ so that the quark and antiquark distributions depend only on energies. Then we obtain the simple form of $\bar{\rho}_{00}^{\phi}(x,\mathbf{p})$ in Eq. (58) with C_1 and C_2 depending only on masses as shown in (59). In general, the fluid four-velocity has also a spatial component or three-velocity, and in this case $\bar{\rho}_{00}^{\phi}(x,\mathbf{p})$ should have a much more complicated form than Eq. (58) where the coefficients also depend on the three-velocity of the fluid in a more sophisticated way.

One can approximate $\bar{\rho}_{00}^{\phi}$ by expanding C_1 and C_2 in terms of the average quark momentum inside the vector meson as

$$C_1 \approx \frac{1}{6} + \frac{1}{9}d_0 + O(d_0^2),$$
$$C_2 \approx \frac{1}{18}d_0 + O(d_0^2), \tag{60}$$

with $d_0 \equiv 1\text{–}4m_s^2/m_\phi^2$, and the result is

$$\bar{\rho}_{00}^{\phi}(x,\mathbf{p}) \approx \frac{1}{3} + \left(\frac{1}{6} + \frac{1}{9}d_0\right)\left\{\frac{1}{3}\left(\boldsymbol{\omega}'\cdot\boldsymbol{\omega}' - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}\mathbf{B}'_\phi\cdot\mathbf{B}'_\phi\right) - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\omega}')^2 + \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}(\boldsymbol{\epsilon}_0\cdot\mathbf{B}'_\phi)^2\right\} + \frac{1}{18}d_0\left\{\frac{1}{3}\left(\boldsymbol{\varepsilon}'\cdot\boldsymbol{\varepsilon}' - \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}\mathbf{E}'_\phi\cdot\mathbf{E}'_\phi\right) - (\boldsymbol{\epsilon}_0\cdot\boldsymbol{\varepsilon}')^2 + \frac{4g_\phi^2}{m_\phi^2 T_{\text{eff}}^2}(\boldsymbol{\epsilon}_0\cdot\mathbf{E}'_\phi)^2\right\} + O(d_0^2). \tag{61}$$

The above result can be compared with that in the nonrelativistic limit (see Appendix C). To recover the momentum dependence, one can express $\bar{\rho}_{00}^{\phi}$ in terms of lab-frame fields. The transformation of the fields between the lab and rest frame reads

$$\mathbf{B}'_\phi = \gamma\mathbf{B}_\phi - \gamma\mathbf{v}\times\mathbf{E}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{B}_\phi}{v^2}\mathbf{v},$$
$$\mathbf{E}'_\phi = \gamma\mathbf{E}_\phi + \gamma\mathbf{v}\times\mathbf{B}_\phi + (1-\gamma)\frac{\mathbf{v}\cdot\mathbf{E}_\phi}{v^2}\mathbf{v},$$
$$\boldsymbol{\omega}' = \gamma\boldsymbol{\omega} - \gamma\mathbf{v}\times\boldsymbol{\varepsilon} + (1-\gamma)\frac{\mathbf{v}\cdot\boldsymbol{\omega}}{v^2}\mathbf{v},$$
$$\boldsymbol{\varepsilon}' = \gamma\boldsymbol{\varepsilon} + \gamma\mathbf{v}\times\boldsymbol{\omega} + (1-\gamma)\frac{\mathbf{v}\cdot\boldsymbol{\varepsilon}}{v^2}\mathbf{v}, \tag{62}$$

where $\gamma = E_{\mathbf{p}}^{\phi}/m_{\phi}$ is the Lorentz factor and $\mathbf{v} = \mathbf{p}/E_{\mathbf{p}}^{\phi}$ is the velocity of the ϕ meson. Taking the y direction as the spin quantization direction, $\boldsymbol{\epsilon}_0 = (0,1,0)$, we obtain $\bar{\rho}_{00}^{\phi}$ in terms of the fields in the lab frame

$$\bar{\rho}_{00}^{\phi}(x,\mathbf{p}) \approx \frac{1}{3} + \frac{1}{3}\sum_{i=1,2,3} I_{B,i}(\mathbf{p})\frac{1}{m_{\phi}^2}\left[\boldsymbol{\omega}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\mathrm{eff}}^2}(\mathbf{B}_i^{\phi})^2\right] + \frac{1}{3}\sum_{i=1,2,3} I_{E,i}(\mathbf{p})\frac{1}{m_{\phi}^2}\left[\boldsymbol{\varepsilon}_i^2 - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\mathrm{eff}}^2}(\mathbf{E}_i^{\phi})^2\right], \tag{63}$$

where the coefficients are given by

$$\begin{aligned}
I_{B,x}(\mathbf{p}) &= C_1\left[(E_{\mathbf{p}}^{\phi})^2 - \left(1 + \frac{3p_y^2}{(m_{\phi}+E_{\mathbf{p}}^{\phi})^2}\right)p_x^2\right] + C_2(p_y^2 - 2p_z^2),\\
I_{E,x}(\mathbf{p}) &= C_1(p_y^2 - 2p_z^2) + C_2\left[(E_{\mathbf{p}}^{\phi})^2 - \left(1 + \frac{3p_y^2}{(m_{\phi}+E_{\mathbf{p}}^{\phi})^2}\right)p_x^2\right],\\
I_{B,y}(\mathbf{p}) &= C_1\left[6\frac{E_{\mathbf{p}}^{\phi}}{m_{\phi}+E_{\mathbf{p}}^{\phi}}p_y^2 - 2(E_{\mathbf{p}}^{\phi})^2 - p_y^2 - \frac{3p_y^4}{(m_{\phi}+E_{\mathbf{p}}^{\phi})^2}\right] + C_2(p_x^2 + p_z^2),\\
I_{E,y}(\mathbf{p}) &= C_1(p_x^2 + p_z^2) + C_2\left[6\frac{E_{\mathbf{p}}^{\phi}}{m_{\phi}+E_{\mathbf{p}}^{\phi}}p_y^2 - 2(E_{\mathbf{p}}^{\phi})^2 - p_y^2 - \frac{3p_y^4}{(m_{\phi}+E_{\mathbf{p}}^{\phi})^2}\right],\\
I_{B,z}(\mathbf{p}) &= C_1\left[(E_{\mathbf{p}}^{\phi})^2 - \left(1 + \frac{3p_y^2}{(m_{\phi}+E_{\mathbf{p}}^{\phi})^2}\right)p_z^2\right] + C_2(p_y^2 - 2p_x^2),\\
I_{E,z}(\mathbf{p}) &= C_1(p_y^2 - 2p_x^2) + C_2\left[(E_{\mathbf{p}}^{\phi})^2 - \left(1 + \frac{3p_y^2}{(m_{\phi}+E_{\mathbf{p}}^{\phi})^2}\right)p_z^2\right].
\end{aligned} \tag{64}$$

The result in Eq. (63) is remarkable in its factorization form: the momentum functions are separated from spacetime functions. This has an advantage that the momentum functions can be determined by experimental data on momentum spectra while unknown spacetime functions can be extracted from data on $\bar{\rho}_{00}^{\phi}$.

One can take an average of $\bar{\rho}_{00}^{\phi}(x,\mathbf{p})$ over the local spacetime volume in which the vector meson is formed as

$$\langle\bar{\rho}_{00}^{\phi}(x,\mathbf{p})\rangle_x \approx \frac{1}{3} + \frac{1}{3}\sum_{i=1,2,3} I_{B,i}(\mathbf{p})\frac{1}{m_{\phi}^2}\left[\langle\boldsymbol{\omega}_i^2\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\mathrm{eff}}^2}\left\langle(\mathbf{B}_i^{\phi})^2\right\rangle\right] + \frac{1}{3}\sum_{i=1,2,3} I_{E,i}(\mathbf{p})\frac{1}{m_{\phi}^2}\left[\langle\boldsymbol{\varepsilon}_i^2\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\mathrm{eff}}^2}\left\langle(\mathbf{E}_i^{\phi})^2\right\rangle\right]. \tag{65}$$

These averaged field squares can play as parameters and be determined by comparing $\langle\bar{\rho}_{00}^{\phi}(x,\mathbf{p})\rangle_x$ with the data of ρ_{00}^{ϕ} as functions of transverse momenta. One can further take a momentum average of $\langle\bar{\rho}_{00}^{\phi}(x,\mathbf{p})\rangle_{\mathbf{p}}$ and compare with the data as functions of collision energies,

$$\langle\bar{\rho}_{00}^{\phi}(x,\mathbf{p})\rangle_{x,\mathbf{p}} \approx \frac{1}{3} + \frac{1}{3}\sum_{i=1,2,3}\langle I_{B,i}(\mathbf{p})\rangle\frac{1}{m_{\phi}^2}\left[\langle\boldsymbol{\omega}_i^2\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\mathrm{eff}}^2}\left\langle(\mathbf{B}_i^{\phi})^2\right\rangle\right] + \frac{1}{3}\sum_{i=1,2,3}\langle I_{E,i}(\mathbf{p})\rangle\frac{1}{m_{\phi}^2}\left[\langle\boldsymbol{\varepsilon}_i^2\rangle - \frac{4g_{\phi}^2}{m_{\phi}^2 T_{\mathrm{eff}}^2}\left\langle(\mathbf{E}_i^{\phi})^2\right\rangle\right], \tag{66}$$

where the momentum average can be defined as

$$\langle O(\mathbf{p})\rangle = \frac{\int \frac{d^3\mathbf{p}}{E_p^{\phi}} O(\mathbf{p}) f_{\phi}(\mathbf{p})}{\int \frac{d^3\mathbf{p}}{E_p^{\phi}} f_{\phi}(\mathbf{p})}, \tag{67}$$

if we want to obtain momentum-integrated data for $\bar{\rho}_{00}^{\phi}$. Here E_p is the ϕ meson's energy, and $f_{\phi}(\mathbf{p})$ is its momentum distribution which may contain information about collective flows such as v_1 and v_2, etc. If we want to obtain the transverse momentum spectra of $\bar{\rho}_{00}^{\phi}$, we have to integrate over the azimuthal angle and rapidity in the average $\bar{\rho}_{00}^{\phi}$ and keep p_T, i.e., to replace $d^3\mathbf{p}/E_p^{\phi}$ in (67) by $dyd\varphi$. The theoretical results for $\langle\bar{\rho}_{00}^{\phi}\rangle$ as functions of transverse momenta, collision energies and centralities are presented in Ref. [41], which are in a good agreement with recent STAR data [22].

VI. DISCUSSIONS

In this section we will discuss the main results as well as approximations or assumptions that have been made in this paper.

The Lagrangian (1) is for real vector fields since we are concerned about the charge or flavor neutral particles such as quarkonia made of a quark and its antiquark. To describe those particles that carry net charge or flavor, we have to consider complex vector fields. The generalization of the formalism to complex vector fields is straightforward.

About the kinetic parts of the Boltzmann equations (20) or (27) one may ask if there are additional vorticity terms arising from noninertia force such as Coriolis and centrifugal force as in Ref. [68] for massless fermions. The answer is no. The effect of local rotation of the fluid in terms of vorticity does not depend on whether one sees the system in noninertia frame or inertia frame. What is essential is that the fluid is really rotating relative to an inertia frame but not just caused by the rotation of the observer. This means one can describe the same system either in a rotating frame or in an inertia frame. In the inertia frame, such a term does not appear in the kinetic term of the Boltzmann equation for a simple reason: there is no noninertia force. This has been shown in several independent investigations, for example, in Eq. (19) of Ref. [69] for massless fermions. One may wonder where the vorticity effect (such as the chiral vortical effect) goes. It actually comes from an equilibrium distribution function with spin-vorticity coupling in Eqs. (32) and (33) that finally enters the current in Eq. (34). Both approaches in Refs. [68,69] are equivalent in describing the vorticity effect for massless fermions.

For massive fermions, the covariant Wigner functions (in an inertia frame) are powerful tools to describe phase-space evolution of spin-$1/2$ fermions. Several groups have developed this approach and made significant progress. For example, the collisionless kinetic equations are given in Eq. (60) of Ref. [65]. We can see that once the external electromagnetic field is turned off, the kinetic field is just in the form of $p\cdot\partial_x V(x,p)=0$ (V is proportional to the vector component of the Wigner function) and $p\cdot\partial_x \Sigma_{\mu\nu}(x,p)=0$ ($\Sigma_{\mu\nu}$ is the tensor component of the Wigner function), and both V and $\Sigma_{\mu\nu}$ depend on distribution functions of fermions. The vorticity effect is actually encoded in the vector and axial vector components of Wigner functions, as shown in Eqs. (101) and (111). Note that the axial vector component of the Wigner function gives the spin (pseudo)vector. The absence of such a vorticity force term in the collisionless part of the kinetic equation can also be confirmed in Eqs. (94)–(97) of Ref. [70].

The vector fields that polarize s and $\bar{s}$ are assumed to be the ϕ field according to the chiral quark model [27]. It is the effective (color singlet) vector field induced by the current of pseudo-Goldstone bosons. The local averaged field squares $\langle(\mathbf{B}_i^{\phi})^2\rangle$ and $\langle(\mathbf{E}_i^{\phi})^2\rangle$ are also related to gluon fluctuation of instantons [30,31] according to the quark model based on instanton vacuum [71]. If quarks and antiquarks are polarized by gluon fields, the local averaged field squares are related to the gluon condensate that contributes to the trace anomaly of the energy momentum tensor. Therefore, the local averaged field squares are in connection with fundamental properties of the QCD vacuum which play an important role in hadron structures [28,29].

As an input to the general formula (35) we assume that P_q^{μ} and $P_{\bar{q}}^{\mu}$ have the linear form in Eq. (39) in the vorticity and ϕ fields. The coupling between the spin and fluid velocity field is assumed to be through the vorticity. One can also introduce other coupling forms such as spin-shear couplings [72–75]. The spin polarization by the ϕ field for q and $\bar{q}$ is assumed to have a covariant and linear form $\sim\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}^{\phi}p_{\nu}$. There may be contributions to P_q^{μ} and $P_{\bar{q}}^{\mu}$ from the ϕ field in quadratic or higher orders in $F_{\alpha\beta}^{\phi}$, but we neglected these contributions in this paper for simplicity because the purpose of this paper is to illustrate the effects of field fluctuations on the spin alignment. This is our main assumption. Furthermore, one can use other forms of spin-field couplings or add more terms to Eq. (39). An alternative choice is to use the coupling of the spin and gluon field as in the nonrelativistic quantum chromodynamics (NRQCD) [76,77]. But the Hamiltonian of NRQCD is not covariant at all and may be different from the covariant from $\sim\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}^{c}p_{\nu}$ where $F_{\alpha\beta}^{c}$ is the gluon field with adjoint color c. In this case the final result may be different from the result in this paper.

We can generalize the current relativistic coalescence model to the spin alignment of heavy quarkonia such as J/ψ. Then the ϕ vector field should be replaced by the gluon field. The generalization is straightforward.

VII. SUMMARY AND CONCLUSION

In summary, a relativistic transport theory with spin degrees of freedom for vector mesons is constructed based on KB equations from which the spin Boltzmann equations are derived. With the spin Boltzmann equations we formulate the spin density matrix element ρ_{00} for the ϕ meson. The dominant contributions to ρ_{00}^{ϕ} at lower energies are assumed to come from the ϕ field, a kind of the vector force field in strong interaction that can polarize the strange and antistrange quarks in the same way as the electromagnetic field polarizes charged particles. The key observation is that there is correlation inside the ϕ meson's wave function between the ϕ field that polarizes the strange antistrange quarks. This is reflected by the fact that the contributions to ρ_{00}^{ϕ} are all in squares of the fields which are nonvanishing even if the fields may strongly fluctuate. There are six parameters for fluctuations of strong force fields, $\langle(\mathbf{B}_i^{\phi})^2\rangle$ and $\langle(\mathbf{E}_i^{\phi})^2\rangle$ with $i=x,\ y,\ z$, that can be extracted from experimental data. Constrained by the number and precision of data and considering the geometry of heavy-ion collisions, one can approximately reduce the number of parameters to two: longtitudinal and transverse field squares as was done in Ref. [41]. These parameters of strong force field fluctuations in ρ_{00} for unflavored vector mesons reflect fundamental properties of QCD.

ACKNOWLEDGMENTS

The authors thank C. D. Roberts for providing us with the Bethe-Salpeter wave function of the ϕ meson. The authors thank X. G. Huang, J. F. Liao, S. Pu, A. H. Tang, D. L. Yang, and Y. Yin for helpful discussion. This work is supported in part by the National Natural Science Foundation of China (NSFC) under Grants No. 12135011, No. 11890713 (a subgrant of No. 11890710), the Strategic Priority Research Program of the Chinese Academy of Sciences (CAS) under Grant No. XDB34030102, and by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231. L. O. acknowledges the Next Generation action of the European Commission and the MUR funding (PNRR Missione 4) under the Heavy flavour dynamics in the early stage of ultrarelativistic collisions (HEFESTUS) project.

APPENDIX A: COLLISION TERMS FOR COALESCENCE AND DISSOCIATION

In this appendix, we will derive the collision term for the coalescence process of the quark and antiquark into the vector meson corresponding to I_{-++} in Eq. (26).

The explicit form of I_{-++} is

$$\begin{aligned}
I_{-++} = &\sum_{r_1,s_1,r_2,s_2,\lambda_1',\lambda_2'} \left\{\epsilon^{\alpha}(\lambda_1',\mathbf{p})\epsilon^{\nu*}(\lambda_2',\mathbf{p})\mathrm{Tr}\left[\Gamma_{\alpha}v(s_1,-\mathbf{p}')\bar{v}(r_1,-\mathbf{p}')\Gamma^{\mu}u(r_2,\mathbf{p}+\mathbf{p}')\bar{u}(s_2,\mathbf{p}+\mathbf{p}')\right] + \epsilon^{\mu}(\lambda_1',\mathbf{p})\epsilon^{*}_{\alpha}(\lambda_2',\mathbf{p})\right.\\
&\times \mathrm{Tr}\left[\Gamma^{\nu}v(s_1,-\mathbf{p}')\bar{v}(r_1,-\mathbf{p}')\Gamma^{\alpha}u(r_2,\mathbf{p}+\mathbf{p}')\bar{u}(s_2,\mathbf{p}+\mathbf{p}')\right]\Big\}\\
&\times\Big\{\left[\delta_{r_1s_1}-f^{(-)}_{r_1s_1}(x,-\mathbf{p}')\right]\left[\delta_{r_2s_2}-f^{(+)}_{r_2s_2}(x,\mathbf{p}+\mathbf{p}')\right]f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\\
&-f^{(-)}_{r_1s_1}(x,-\mathbf{p}')f^{(+)}_{r_2s_2}(x,\mathbf{p}+\mathbf{p}')\left[\delta_{\lambda_1'\lambda_2'}+f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right]\Big\}. \qquad \text{(A1)}
\end{aligned}$$

The corresponding collision term in the left-hand side of Eq. (26) reads

$$\begin{aligned}
I^{\mu\nu}_{\mathrm{l.h.s}} = &\frac{1}{4(2\pi\hbar)}\sum_{r_1,s_1,r_2,s_2,\lambda_1',\lambda_2'}\int d^3\mathbf{p}'\frac{1}{8E^{\bar{q}}_{p'}E^{q}_{\mathbf{p}-\mathbf{p}'}E^{V}_{p}}\delta(E^V_p-E^{\bar{q}}_{p'}-E^q_{\mathbf{p}-\mathbf{p}'})\delta(p^0-E^V_p)\\
&\times\left\{\epsilon^{\alpha}(\lambda_1',\mathbf{p})\epsilon^{\nu*}(\lambda_2',\mathbf{p})\mathrm{Tr}\left[\Gamma_{\alpha}v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^{\mu}u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\right]\right.\\
&+\epsilon^{\mu}(\lambda_1',\mathbf{p})\epsilon^{*}_{\alpha}(\lambda_2',\mathbf{p})\mathrm{Tr}\left[\Gamma^{\nu}v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^{\alpha}u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\right]\Big\}\\
&\times\Big\{\left[\delta_{r_1s_1}-f^{(-)}_{r_1s_1}(x,\mathbf{p}')\right]\left[\delta_{r_2s_2}-f^{(+)}_{r_2s_2}(x,\mathbf{p}-\mathbf{p}')\right]f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\\
&-f^{(-)}_{r_1s_1}(x,\mathbf{p}')f^{(+)}_{r_2s_2}(x,\mathbf{p}-\mathbf{p}')\left[\delta_{\lambda_1'\lambda_2'}+f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\right]\Big\}, \qquad \text{(A2)}
\end{aligned}$$

where we have changed the sign of the antiquark's three-momentum as $\mathbf{p}'\to-\mathbf{p}'$ in the integral, used Eq. (7), and the relation

$$\begin{aligned}
&\delta(p'^2-m_q^2)\delta\left[(p+p')^2-m_q^2\right]\delta(p^2-m_V^2)\theta(-p_0')\theta(p_0+p_0')\theta(p_0)\\
&=\frac{1}{8E^{\bar{q}}_{p'}E^{q}_{\mathbf{p}+\mathbf{p}'}E^V_p}\delta(p_0'+E^{\bar{q}}_{p'})\delta(p_0+p_0'-E^q_{\mathbf{p}+\mathbf{p}'})\delta(p_0-E^V_p)\\
&=\frac{1}{8E^{\bar{q}}_{p'}E^{q}_{\mathbf{p}+\mathbf{p}'}E^V_p}\delta(p_0'+E^{\bar{q}}_{p'})\delta(E^V_p-E^{\bar{q}}_{p'}-E^q_{\mathbf{p}+\mathbf{p}'})\delta(p_0-E^V_p). \qquad \text{(A3)}
\end{aligned}$$

From (11), the particle sector of $p\cdot\partial_x G^{<,\mu\nu}(x,p)$ in the left-hand side of Eq. (26) becomes

$$p\cdot\partial_x G^{<,\mu\nu}(x,p) = 2\pi\hbar\frac{1}{2E^V_p}\delta(p_0-E^V_p)\sum_{\lambda_1',\lambda_2'}\epsilon^{\mu}(\lambda_1',\mathbf{p})\epsilon^{\nu*}(\lambda_2',\mathbf{p})p\cdot\partial_x f_{\lambda_1'\lambda_2'}(x,\mathbf{p}). \qquad \text{(A4)}$$

Using Eqs. (A2) and (A4) into Eq. (26), taking a contraction of the resulting equation with $\epsilon^*_\mu(\lambda_1,\mathbf{p})$ and $\epsilon_\nu(\lambda_2,\mathbf{p})$, and using the first identity in (7), we obtain

$$\begin{aligned}
p\cdot\partial_x f_{\lambda_1\lambda_2}(x,\mathbf{p}) = {}& \frac{1}{16}\sum_{r_1,s_1,r_2,s_2,\lambda_1',\lambda_2'}\int\frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}\frac{1}{E^{\bar{q}}_{p'}E^q_{\mathbf{p}-\mathbf{p}'}}2\pi\hbar\delta(E^V_p - E^{\bar{q}}_{p'} - E^q_{\mathbf{p}-\mathbf{p}'}) \\
&\times\big\{\delta_{\lambda_2\lambda_2'}\epsilon^*_\mu(\lambda_1,\mathbf{p})\epsilon^\alpha(\lambda_1',\mathbf{p})\mathrm{Tr}\big[\Gamma_\alpha v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\mu u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\big] \\
&+\delta_{\lambda_1\lambda_1'}\epsilon_\nu(\lambda_2,\mathbf{p})\epsilon^*_\alpha(\lambda_2',\mathbf{p})\mathrm{Tr}\big[\Gamma^\nu v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\alpha u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\big]\big\} \\
&\times\big\{f^{(-)}_{r_1s_1}(x,\mathbf{p}')f^{(+)}_{r_2s_2}(x,\mathbf{p}-\mathbf{p}')\big[\delta_{\lambda_1'\lambda_2'} + f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\big] \\
&-\big[\delta_{r_1s_1} - f^{(-)}_{r_1s_1}(x,\mathbf{p}')\big]\big[\delta_{r_2s_2} - f^{(+)}_{r_2s_2}(x,\mathbf{p}-\mathbf{p}')\big]f_{\lambda_1'\lambda_2'}(x,\mathbf{p})\big\},
\end{aligned} \tag{A5}$$

which reproduces Eq. (27). Note that the terms proportional to p^μ and p^ν in the left-hand side of Eq. (26) do not contribute since their contraction with $\epsilon^*_\mu(\lambda_1,\mathbf{p})$ and $\epsilon_\nu(\lambda_2,\mathbf{p})$ is vanishing.

We consider the coalescence process in heavy-ion collisions in which the MVSDs of quarks, antiquarks, and vector mesons are assumed to be much smaller than unity. So the term with $\delta_{\lambda_1\lambda_2}$ dominates the gain term that can be simplified as

$$\begin{aligned}
\mathrm{gain} \approx {}& \frac{1}{8(2\pi\hbar)^2}\sum_{r_1,s_1,r_2,s_2}\int d^3\mathbf{p}'\frac{1}{E^{\bar{q}}_{p'}E^q_{\mathbf{p}-\mathbf{p}'}}\delta(E^V_p - E^{\bar{q}}_{p'} - E^q_{\mathbf{p}-\mathbf{p}'})\epsilon^*_\mu(\lambda_1,\mathbf{p})\epsilon_\alpha(\lambda_2,\mathbf{p}) \\
&\times\mathrm{Tr}\big[\Gamma^\alpha v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\mu u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\big]f^{(-)}_{r_1s_1}(x,\mathbf{p}')f^{(+)}_{r_2s_2}(x,\mathbf{p}-\mathbf{p}'),
\end{aligned} \tag{A6}$$

which gives Eq. (31). The loss term can be simplified as

$$\begin{aligned}
\mathrm{loss} \approx {}& -\frac{1}{16(2\pi\hbar)^2}\sum_{r_1,s_1,r_2,s_2}\int d^3\mathbf{p}'\frac{1}{E^{\bar{q}}_{p'}E^q_{\mathbf{p}-\mathbf{p}'}}\delta(E^V_p - E^{\bar{q}}_{p'} - E^q_{\mathbf{p}-\mathbf{p}'}) \\
&\times\big\{\delta_{\lambda_2\lambda_2'}\epsilon^*_\mu(\lambda_1,\mathbf{p})\epsilon^\alpha(\lambda_1',\mathbf{p})\mathrm{Tr}\big[\Gamma_\alpha v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\mu u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\big] \\
&+\delta_{\lambda_1\lambda_1'}\epsilon_\nu(\lambda_2,\mathbf{p})\epsilon^*_\alpha(\lambda_2',\mathbf{p})\mathrm{Tr}\big[\Gamma^\nu v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\alpha u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\big]\big\}\delta_{r_1s_1}\delta_{r_2s_2}f_{\lambda_1'\lambda_2'}(x,\mathbf{p}) \\
= {}& -\frac{1}{16(2\pi\hbar)^2}\int d^3\mathbf{p}'\frac{1}{E^{\bar{q}}_{p'}E^q_{\mathbf{p}-\mathbf{p}'}}\delta(E^V_p - E^{\bar{q}}_{p'} - E^q_{\mathbf{p}-\mathbf{p}'}) \\
&\times\left\{\sum_{\lambda_1'}f_{\lambda_1'\lambda_2}(x,\mathbf{p})\epsilon^*_\mu(\lambda_1,\mathbf{p})\epsilon^\alpha(\lambda_1',\mathbf{p})\mathrm{Tr}\big[\Gamma_\alpha(p'\cdot\gamma - m_{\bar{q}})\Gamma^\mu((p-p')\cdot\gamma + m_q)\big]\right. \\
&\left.+\sum_{\lambda_2'}f_{\lambda_1\lambda_2'}(x,\mathbf{p})\epsilon_\nu(\lambda_2,\mathbf{p})\epsilon^*_\alpha(\lambda_2',\mathbf{p})\mathrm{Tr}\big[\Gamma^\nu(p'\cdot\gamma - m_{\bar{q}})\Gamma^\alpha((p-p')\cdot\gamma + m_q)\big]\right\} \\
= {}& -\frac{1}{16(2\pi\hbar)^2}\int d^3\mathbf{p}'\frac{1}{E^{\bar{q}}_{p'}E^q_{\mathbf{p}-\mathbf{p}'}}\delta(E^V_p - E^{\bar{q}}_{p'} - E^q_{\mathbf{p}-\mathbf{p}'}) \\
&\times\left[\sum_{\lambda_1'}f_{\lambda_1'\lambda_2}(x,\mathbf{p})\epsilon^*_\mu(\lambda_1,\mathbf{p})\epsilon_\alpha(\lambda_1',\mathbf{p}) + \sum_{\lambda_2'}f_{\lambda_1\lambda_2'}(x,\mathbf{p})\epsilon^*_\mu(\lambda_2',\mathbf{p})\epsilon_\alpha(\lambda_2,\mathbf{p})\right] \\
&\times\mathrm{Tr}\big\{\Gamma^\alpha(p'\cdot\gamma - m_{\bar{q}})\Gamma^\mu\big[(p-p')\cdot\gamma + m_q\big]\big\},
\end{aligned} \tag{A7}$$

where we have neglected $f^{(-)}_{r_1s_1}$ and $f^{(+)}_{r_2s_2}$ relative to $\delta_{r_1s_1}$ and $\delta_{r_2s_2}$, respectively. After completing the integral over $\mathbf{p}'$ in the vector meson's rest frame, one can prove

$$\epsilon^*_\mu(\lambda_1,\mathbf{p})\epsilon_\alpha(\lambda_1',\mathbf{p})\int d^3\mathbf{p}'\frac{1}{4E^{\bar{q}}_{p'}E^q_{\mathbf{p}-\mathbf{p}'}}\delta(E^V_p - E^{\bar{q}}_{p'} - E^q_{\mathbf{p}-\mathbf{p}'})\mathrm{Tr}\big\{\Gamma^\alpha(p'\cdot\gamma - m_{\bar{q}})\Gamma^\mu\big[(p-p')\cdot\gamma + m_q\big]\big\} \propto \delta_{\lambda_1\lambda_1'}, \tag{A8}$$

so we can replace

$$\sum_{\lambda_1'} f_{\lambda_1'\lambda_2}(x,\mathbf{p})\epsilon_\mu^*(\lambda_1,\mathbf{p})\epsilon_\alpha(\lambda_1',\mathbf{p}) + \sum_{\lambda_2'} f_{\lambda_1\lambda_2'}(x,\mathbf{p})\epsilon_\mu^*(\lambda_2,\mathbf{p})\epsilon_\alpha(\lambda_2',\mathbf{p}) \to -\frac{2}{3} f_{\lambda_1\lambda_2}(x,\mathbf{p})\left(g_{\mu\alpha} - \frac{p_\mu p_\alpha}{m_V^2}\right), \tag{A9}$$

and then the loss term becomes

$$\mathrm{loss} \approx \frac{1}{12(2\pi\hbar)^2} f_{\lambda_1\lambda_2}(x,\mathbf{p})\left(g_{\mu\alpha} - \frac{p_\mu p_\alpha}{m_V^2}\right)\int d^3\mathbf{p}' \frac{1}{E_{p'}^{\bar{q}} E_{\mathbf{p}-\mathbf{p}'}^q}\delta(E_p^V - E_{p'}^{\bar{q}} - E_{\mathbf{p}-\mathbf{p}'}^q)\mathrm{Tr}\left\{\Gamma^\alpha(p'\cdot\gamma - m_{\bar{q}})\Gamma^\mu\left[(p-p')\cdot\gamma + m_q\right]\right\}. \tag{A10}$$

This gives Eq. (32).

APPENDIX B: COLLISION KERNEL

The spin density matrix element for vector mesons is given by Eq. (35). In this appendix we evaluate the collision kernel in Eq. (35):

$$I_{\lambda_1\lambda_2}(\mathbf{p},\mathbf{p}') = I^{\alpha\beta}(\mathbf{p},\mathbf{p}')\epsilon_\alpha^*(\lambda_1,\mathbf{p})\epsilon_\beta(\lambda_2,\mathbf{p}), \tag{B1}$$

where $I^{\alpha\beta}(\mathbf{p},\mathbf{p}')$ is defined as

$$I^{\alpha\beta}(\mathbf{p},\mathbf{p}') \equiv \mathrm{Tr}\left[\Gamma^\beta v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\alpha u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\right] f_{r_1s_1}^{(\bar{q})}(x,\mathbf{p}') f_{r_2s_2}^{(q)}(x,\mathbf{p}-\mathbf{p}'). \tag{B2}$$

Now we use the following formula to simplify $I^{\alpha\beta}$. For quark spinors of particles and antiparticles we have

$$\begin{aligned}
u(r,\mathbf{p})\bar{u}(s,\mathbf{p}) &= \frac{1}{2}(m_q + \gamma^\mu p_\mu)\delta_{rs} + \frac{1}{2}m_q\gamma^5\gamma^\mu n_\mu(\mathbf{n}_{sr},\mathbf{p},m_q) - \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}\sigma^{\mu\nu}p^\alpha n^\beta(\mathbf{n}_{sr},\mathbf{p},m_q),\\
\sum_{r,s} u(r,\mathbf{p})\bar{u}(s,\mathbf{p})(\tau_j)_{rs} &= m_q\gamma^5\gamma^\mu n_\mu(\mathbf{n}_j,\mathbf{p},m_q) - \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\mu\nu}p^\alpha n^\beta(\mathbf{n}_j,\mathbf{p},m_q),\\
v(r,\mathbf{p})\bar{v}(s,\mathbf{p}) &= \frac{1}{2}(-m_{\bar{q}} + p^\mu\gamma_\mu)\delta_{rs} - \frac{1}{2}m_{\bar{q}}\gamma_5\gamma_\mu n^\mu(\mathbf{n}_{sr}^*,\mathbf{p},m_{\bar{q}}) - \frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\sigma_{\mu\nu}p_\alpha n_\beta(\mathbf{n}_{sr}^*,\mathbf{p},m_{\bar{q}}),\\
\sum_{r,s} v(r,\mathbf{p})\bar{v}(s,\mathbf{p})(\tau_j)_{sr} &= -m_{\bar{q}}\gamma_5\gamma_\mu n^\mu(\mathbf{n}_j,\mathbf{p},m_{\bar{q}}) - \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\mu\nu}p_\alpha n_\beta(\mathbf{n}_j,\mathbf{p},m_{\bar{q}}),
\end{aligned} \tag{B3}$$

where we have used $\mathbf{n}_{sr}^* = \mathbf{n}_{rs} = \mathbf{n}_i(\tau_i)_{rs}$ with $\mathbf{n}_i$ ($i = 1, 2, 3$) being three basis directions in Eq. (23). Inserting Eq. (22) into Eq. (B2) gives

$$\begin{aligned}
I^{\alpha\beta}(\mathbf{p},\mathbf{p}') &= \mathrm{Tr}\left[\Gamma^\beta v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\Gamma^\alpha u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}')\right]\frac{1}{2}f_{\bar{q}}(x,\mathbf{p}')\left[\delta_{r_1s_1} - P_\mu^{\bar{q}}(x,\mathbf{p}')n_j^{(-)\mu}(-\mathbf{p}')\tau_{r_1s_1}^j\right]\\
&\quad\times\frac{1}{2}f_q(x,\mathbf{p}-\mathbf{p}')\left[\delta_{r_2s_2} - P_\mu^q(x,\mathbf{p}-\mathbf{p}')n_j^{(+)\mu}(\mathbf{p}-\mathbf{p}')\tau_{r_2s_2}^j\right]\\
&= \frac{1}{4}f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{p}-\mathbf{p}')\mathrm{Tr}\left\{\Gamma^\beta(p'\cdot\gamma - m_{\bar{q}})\left[1+\gamma_5\gamma\cdot P^{\bar{q}}(x,\mathbf{p}')\right]\Gamma^\alpha\right.\\
&\quad\times\left.\left[(p-p')\cdot\gamma + m_q\right]\left[1+\gamma_5\gamma\cdot P^q(x,\mathbf{p}-\mathbf{p}')\right]\right\},
\end{aligned} \tag{B4}$$

where we have used

$$v(s_1,\mathbf{p}')\bar{v}(r_1,\mathbf{p}')\left[\delta_{r_1s_1} - P_\mu^{\bar{q}}(x,\mathbf{p}')n_j^{(-)\mu}(-\mathbf{p}')\tau_{r_1s_1}^j\right] = (p'\cdot\gamma - m_{\bar{q}})\left[1+\gamma_5\gamma\cdot P^{\bar{q}}(x,\mathbf{p}')\right] \tag{B5}$$

and

$$u(r_2,\mathbf{p}-\mathbf{p}')\bar{u}(s_2,\mathbf{p}-\mathbf{p}') \times\left[\delta_{r_2s_2}-P^q_\mu(x,\mathbf{p}-\mathbf{p}')n_j^{(+)\mu}(\mathbf{p}-\mathbf{p}')\tau^j_{r_2s_2}\right] =\left[(p-p')\cdot\gamma+m_q\right]\left[1+\gamma^5\gamma\cdot P^q(x,\mathbf{p}-\mathbf{p}')\right]. \quad (B6)$$

In deriving (B5) and (B6) we have used

$$\frac{1}{2}\epsilon^{\alpha\beta\mu\nu}\sigma_{\mu\nu}=\gamma_5\gamma^\alpha\gamma^\beta-g^{\alpha\beta}\gamma_5,$$
$$n_j^{(-)\mu}(-\mathbf{p}')=n^\mu(\mathbf{n}_j,\mathbf{p}',m_{\bar{q}}),$$
$$p'^\mu P^{\bar{q}}_\mu(x,\mathbf{p}')=(p-p')^\mu P^q_\mu(x,\mathbf{p}-\mathbf{p}')=0. \quad (B7)$$

Inserting (B4) into (B1) we obtain

$$I_{\lambda_1\lambda_2}=f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{p}-\mathbf{p}')\epsilon^*_\alpha(\lambda_1,\mathbf{p})\epsilon_\beta(\lambda_2,\mathbf{p}) \times\mathrm{Tr}\left\{\Gamma^\beta(p'\cdot\gamma-m_{\bar{q}})\left[1+\gamma_5\gamma\cdot P_{\bar{q}}(x,\mathbf{p}')\right]\Gamma^\alpha \times\left[(p-p')\cdot\gamma+m_q\right]\left[1+\gamma_5\gamma\cdot P_q(x,\mathbf{p}-\mathbf{p}')\right]\right\}. \quad (B8)$$

From Eq. (B8) one arrives at Eq. (35). Using (28), the trace in (B8) can be worked out and the result of $I_{\lambda_1\lambda_2}$ is

$$I_{\lambda_1\lambda_2}=-4g_V^2B^2(\mathbf{p}-\mathbf{p}',\mathbf{p}')f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{p}-\mathbf{p}')\epsilon^*_\alpha(\lambda_1)\epsilon_\beta(\lambda_2)\left\{(p'^\alpha P^\beta_{\bar{q}}+p'^\beta P^\alpha_{\bar{q}})(p'\cdot P_q)-(p'^\alpha P^\beta_q+p'^\beta P^\alpha_q)(p\cdot P_{\bar{q}}) +2p'^\alpha p'^\beta(1-P_{\bar{q}}\cdot P_q)+g^{\alpha\beta}\left[p'\cdot p+(p'\cdot P_q)(p\cdot P_{\bar{q}})\right]+\left[(m_q-m_{\bar{q}})m_{\bar{q}}+p\cdot p'\right](P^\alpha_{\bar{q}}P^\beta_q+P^\alpha_qP^\beta_{\bar{q}}-g^{\alpha\beta}P_{\bar{q}}\cdot P_q) -(m_{\bar{q}}-m_q)m_{\bar{q}}g^{\alpha\beta}-i(m_q-m_{\bar{q}})\epsilon^{\alpha\beta\mu\nu}p'_\mu(P^q_\nu+P^{\bar{q}}_\nu)-im_{\bar{q}}\epsilon^{\alpha\beta\mu\nu}p_\mu(P^q_\nu+P^{\bar{q}}_\nu)\right\}, \quad (B9)$$

where we have used shorthand notations $\epsilon(\lambda)\equiv\epsilon(\lambda,\mathbf{p})$, $P_q\equiv P_q(x,\mathbf{p}-\mathbf{p}')$, and $P_{\bar{q}}\equiv P_{\bar{q}}(x,\mathbf{p}')$. We can take the sum of $I_{\lambda\lambda}$ over λ as

$$\sum_\lambda I_{\lambda\lambda}=2g_V^2B^2(\mathbf{p}-\mathbf{p}',\mathbf{p}')f_{\bar{q}}(x,\mathbf{p}')f_q(x,\mathbf{p}-\mathbf{p}')\left\{\left[-(m_q+m_{\bar{q}})^2+\frac{1}{m_V^2}(m_{\bar{q}}^2-m_q^2)^2\right](P_{\bar{q}}\cdot P_q) +\frac{2}{m_V^2}(m_q-m_{\bar{q}})^2(p\cdot P_q)(p\cdot P_{\bar{q}})+2m_V^2+6m_qm_{\bar{q}}-m_{\bar{q}}^2-m_q^2-\frac{1}{m_V^2}(m_{\bar{q}}^2-m_q^2)^2\right\}. \quad (B10)$$

From Eq. (B10) one can obtain $\mathrm{Tr}(\rho_V)$ in Eq. (36).

APPENDIX C: SPIN DENSITY MATRIX IN NONRELATIVISTIC LIMIT

We consider $m_q=m_{\bar{q}}$ and assume $m_V\approx 2m_q$ in the nonrelativistic limit, which is a good approximation for heavy quarkonia and the constituent quark coalescence model. In this case we can approximate

$$p^\mu\approx(m_V,\mathbf{0}),$$
$$p'^\mu\approx(m_{\bar{q}},\mathbf{0})\approx(m_q,\mathbf{0}),$$
$$p^\mu-p'^\mu\approx(m_V-m_{\bar{q}},0)\approx(m_q,0),$$
$$P^\mu_{\bar{q}}(x,\mathbf{p}')\approx(0,\mathbf{P}_{\bar{q}}(x,\mathbf{p}')),$$
$$P^\mu_q(x,\mathbf{p}-\mathbf{p}')\approx(0,\mathbf{P}_q(x,\mathbf{p}-\mathbf{p}')),$$
$$\epsilon^\mu(\lambda_1)\approx(0,\boldsymbol{\epsilon}(\lambda_1)),$$
$$\epsilon^\mu(\lambda_2)\approx(0,\boldsymbol{\epsilon}(\lambda_2)), \quad (C1)$$

which leads to

$$p'\cdot\epsilon^*(\lambda_1)\approx 0,$$
$$p'\cdot\epsilon(\lambda_2)\approx 0,$$
$$p'\cdot p\approx m_Vm_q,$$
$$\epsilon^*(\lambda_1)\cdot\epsilon(\lambda_2)\approx-\boldsymbol{\epsilon}^*(\lambda_1)\cdot\boldsymbol{\epsilon}(\lambda_2). \quad (C2)$$

In Eqs. (C1) and (C2) we have used the shorthand notation $\epsilon(\lambda)\equiv\epsilon(\lambda,\mathbf{p})$. Using (C1) and (C2), $I_{\lambda_1\lambda_2}$ in Eq. (B9) has a simple form

$$I_{\lambda_1\lambda_2}=4g_V^2m_Vm_q\left\{\boldsymbol{\epsilon}^*(\lambda_1)\cdot\boldsymbol{\epsilon}(\lambda_2)(1+\mathbf{P}_{\bar{q}}\cdot\mathbf{P}_q) -\left[\mathbf{P}_{\bar{q}}\cdot\boldsymbol{\epsilon}^*(\lambda_1)\right]\left[\mathbf{P}_q\cdot\boldsymbol{\epsilon}(\lambda_2)\right]-\left[\mathbf{P}_q\cdot\boldsymbol{\epsilon}^*(\lambda_1)\right]\left[\mathbf{P}_{\bar{q}}\cdot\boldsymbol{\epsilon}(\lambda_2)\right] -i\left[\boldsymbol{\epsilon}^*(\lambda_1)\times\boldsymbol{\epsilon}(\lambda_2)\right]\cdot(\mathbf{P}_q+\mathbf{P}_{\bar{q}})\right\}. \quad (C3)$$

One can verify $I_{\lambda_2\lambda_1}=I^*_{\lambda_1\lambda_2}$.

From (35), the spin density matrix for the vector meson in the nonrelativistic limit is given by

$$\rho^V_{\lambda_1\lambda_2}(x,\mathbf{p}) = \frac{\Delta t}{8} g_V^2 m_V m_q \int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^3} \frac{1}{E^{\bar{q}}_{p'} E^q_{\mathbf{p}-\mathbf{p}'} E^V_p} f_{\bar{q}}(x,\mathbf{p}') f_q(x,\mathbf{p}-\mathbf{p}') 2\pi\hbar\delta(E^V_p - E^{\bar{q}}_{p'} - E^q_{\mathbf{p}-\mathbf{p}'})$$
$$\times \{\boldsymbol{\epsilon}^*(\lambda_1)\cdot\boldsymbol{\epsilon}(\lambda_2)[1+\mathbf{P}_q(x,\mathbf{p}-\mathbf{p}')\cdot\mathbf{P}_{\bar{q}}(x,\mathbf{p}')] - [\mathbf{P}_q(x,\mathbf{p}-\mathbf{p}')\cdot\boldsymbol{\epsilon}(\lambda_2)][\mathbf{P}_{\bar{q}}(x,\mathbf{p}')\cdot\boldsymbol{\epsilon}^*(\lambda_1)]$$
$$- [\mathbf{P}_q(x,\mathbf{p}-\mathbf{p}')\cdot\boldsymbol{\epsilon}^*(\lambda_1)][\mathbf{P}_{\bar{q}}(x,\mathbf{p}')\cdot\boldsymbol{\epsilon}(\lambda_2)] - i[\boldsymbol{\epsilon}^*(\lambda_1)\times\boldsymbol{\epsilon}(\lambda_2)]\cdot[\mathbf{P}_q(x,\mathbf{p}-\mathbf{p}')+\mathbf{P}_{\bar{q}}(x,\mathbf{p}')]\}. \quad \text{(C4)}$$

We can simplify the above formula by using the shorthand notation

$$Dp' \equiv \frac{\Delta t}{8} g_V^2 m_V m_q \int \frac{d^3\mathbf{p}'}{(2\pi\hbar)^3} \frac{1}{E^{\bar{q}}_{p'} E^q_{\mathbf{p}-\mathbf{p}'} E^V_p} \times f_{\bar{q}}(x,\mathbf{p}') f_q(x,\mathbf{p}-\mathbf{p}') 2\pi\hbar\delta(E^V_p - E^{\bar{q}}_{p'} - E^q_{\mathbf{p}-\mathbf{p}'}). \quad \text{(C5)}$$

We can put $\rho^V_{\lambda_1\lambda_2}$ into a matrix form

$$\rho^V = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1,-1} \\ \rho^*_{1,0} & \rho_{00} & \rho_{0,-1} \\ \rho^*_{1,-1} & \rho^*_{0,-1} & \rho_{-1,-1} \end{pmatrix}. \quad \text{(C6)}$$

Note that ρ^V is a Hermitian matrix and we have suppressed the index "V" in all elements.

For a given spin quantization direction $\mathbf{n}_3$, we can construct $\boldsymbol{\epsilon}(\lambda)$ as follows:

$$\boldsymbol{\epsilon}(0) = \mathbf{n}_3,$$
$$\boldsymbol{\epsilon}(1) = -\frac{1}{\sqrt{2}}(\mathbf{n}_1 + i\mathbf{n}_2),$$
$$\boldsymbol{\epsilon}(-1) = \frac{1}{\sqrt{2}}(\mathbf{n}_1 - i\mathbf{n}_2), \quad \text{(C7)}$$

where $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ form orthogonal basis vectors in the rest frame of the vector meson. From (C4) we obtain

$$\rho_{11} = \int Dp'(1+\mathbf{n}_3\cdot\mathbf{P}_q)(1+\mathbf{n}_3\cdot\mathbf{P}_{\bar{q}}),$$
$$\rho_{10} = \frac{1}{\sqrt{2}}\int Dp'\{[(\mathbf{n}_1 - i\mathbf{n}_2)\cdot\mathbf{P}_q](1+\mathbf{n}_3\cdot\mathbf{P}_{\bar{q}}) + [(\mathbf{n}_1 - i\mathbf{n}_2)\cdot\mathbf{P}_{\bar{q}}](1+\mathbf{n}_3\cdot\mathbf{P}_q)\},$$
$$\rho_{1,-1} = \int Dp'[(\mathbf{n}_1 - i\mathbf{n}_2)\cdot\mathbf{P}_q][(\mathbf{n}_1 - i\mathbf{n}_2)\cdot\mathbf{P}_{\bar{q}}],$$
$$\rho_{00} = \int Dp'\{1+\mathbf{P}_q\cdot\mathbf{P}_{\bar{q}} - 2(\mathbf{n}_3\cdot\mathbf{P}_q)(\mathbf{n}_3\cdot\mathbf{P}_{\bar{q}})\},$$
$$\rho_{-1,0} = -\frac{1}{\sqrt{2}}\int Dp'\{[(\mathbf{n}_1 + i\mathbf{n}_2)\cdot\mathbf{P}_q](1-\mathbf{n}_3\cdot\mathbf{P}_{\bar{q}}) + [(\mathbf{n}_1 + i\mathbf{n}_2)\cdot\mathbf{P}_{\bar{q}}](1-\mathbf{n}_3\cdot\mathbf{P}_q)\},$$
$$\rho_{-1,-1} = \int Dp'(1-\mathbf{n}_3\cdot\mathbf{P}_q)(1-\mathbf{n}_3\cdot\mathbf{P}_{\bar{q}}), \quad \text{(C8)}$$

where we have used shorthand notations $\mathbf{P}_q \equiv \mathbf{P}_q(x,\mathbf{p}-\mathbf{p}')$ and $\mathbf{P}_{\bar{q}} \equiv \mathbf{P}_{\bar{q}}(x,\mathbf{p}')$. The 00-element of the normalized density matrix is given by

$$\bar{\rho}_{00} = \frac{\rho_{00}}{\rho_{11}+\rho_{00}+\rho_{-1,-1}} = \frac{\int Dp'[1+\mathbf{P}_q\cdot\mathbf{P}_{\bar{q}} - 2(\mathbf{n}_3\cdot\mathbf{P}_q)(\mathbf{n}_3\cdot\mathbf{P}_{\bar{q}})]}{\int Dp'(3+\mathbf{P}_q\cdot\mathbf{P}_{\bar{q}})}, \quad \text{(C9)}$$

where $N \equiv \int Dp'$ is the normalization constant. If the magnitude of the polarization is much smaller than 1, we can make a Taylor expansion in it and obtain

$$\bar{\rho}_{00}(x,\mathbf{p}) \simeq \frac{1}{3} + \frac{2}{9N}\int Dp'[(\mathbf{n}_1\cdot\mathbf{P}_q)(\mathbf{n}_1\cdot\mathbf{P}_{\bar{q}}) + (\mathbf{n}_2\cdot\mathbf{P}_q)(\mathbf{n}_2\cdot\mathbf{P}_{\bar{q}}) - 2(\mathbf{n}_3\cdot\mathbf{P}_q)(\mathbf{n}_3\cdot\mathbf{P}_{\bar{q}})]. \quad \text{(C10)}$$

If we assume $\mathbf{P}_q$ and $\mathbf{P}_{\bar{q}}$ are only in the direction of $\mathbf{n}_3$, i.e.,

$$\mathbf{n}_x\cdot\mathbf{P}_q = \mathbf{n}_y\cdot\mathbf{P}_q = \mathbf{n}_x\cdot\mathbf{P}_{\bar{q}} = \mathbf{n}_y\cdot\mathbf{P}_{\bar{q}} = 0, \quad \text{(C11)}$$

we obtain

$$\bar{\rho}_{00}(x,\mathbf{p}) \approx \frac{1}{3} - \frac{4}{9N}\int Dp'[\mathbf{n}_3\cdot\mathbf{P}_q(x,\mathbf{p}-\mathbf{p}')] \times [\mathbf{n}_3\cdot\mathbf{P}_{\bar{q}}(x,\mathbf{p}')], \quad \text{(C12)}$$

which recovers a form similar to the previous result but expressed in terms of the weighted integral Dp' in (C5).

[1] S. Barnett, Rev. Mod. Phys. **7**, 129 (1935).
[2] A. Einstein and W. de Haas, Dtsch. Phys. Ges. Verh. **17**, 152 (1915).
[3] R. Takahashi, M. Matsuo, M. Ono, K. Harii, H. Chudo, S. Okayasu, J. Ieda, S. Takahashi, S. Maekawa, and E. Saitoh, Nat. Phys. **12**, 52 (2016).
[4] Z.-T. Liang and X.-N. Wang, Phys. Rev. Lett. **94**, 102301 (2005); **96**, 039901(E) (2006).
[5] Z.-T. Liang and X.-N. Wang, Phys. Lett. B **629**, 20 (2005).
[6] S. A. Voloshin, arXiv:nucl-th/0410089.
[7] B. Betz, M. Gyulassy, and G. Torrieri, Phys. Rev. C **76**, 044901 (2007).
[8] F. Becattini, F. Piccinini, and J. Rizzo, Phys. Rev. C **77**, 024906 (2008).
[9] J.-H. Gao, S.-W. Chen, W.-t. Deng, Z.-T. Liang, Q. Wang, and X.-N. Wang, Phys. Rev. C **77**, 044902 (2008).
[10] Q. Wang, Nucl. Phys. **A967**, 225 (2017).
[11] W. Florkowski, A. Kumar, and R. Ryblewski, Prog. Part. Nucl. Phys. **108**, 103709 (2019).
[12] F. Becattini and M. A. Lisa, Annu. Rev. Nucl. Part. Sci. **70**, 395 (2020).
[13] J.-H. Gao, Z.-T. Liang, Q. Wang, and X.-N. Wang, Lect. Notes Phys. **987**, 195 (2021).
[14] X.-G. Huang, J. Liao, Q. Wang, and X.-L. Xia, Lect. Notes Phys. **987**, 281 (2021).
[15] STAR Collaboration, Nature (London) **548**, 62 (2017).
[16] J. Adam *et al.* (STAR Collaboration), Phys. Rev. C **98**, 014910 (2018).
[17] Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys. Rev. C **97**, 034917 (2018).
[18] A. H. Tang, B. Tu, and C. S. Zhou, Phys. Rev. C **98**, 044907 (2018).
[19] K. J. Gonçalves and G. Torrieri, Phys. Rev. C **105**, 034913 (2022).
[20] P. H. De Moura, K. J. Goncalves, and G. Torrieri, Phys. Rev. D **108**, 034032 (2023).
[21] B. I. Abelev *et al.* (STAR Collaboration), Phys. Rev. C **77**, 061902 (2008).
[22] M. Abdallah *et al.* (STAR Collaboration), Nature (London) **614**, 244 (2023).
[23] X.-L. Xia, H. Li, X.-G. Huang, and H. Zhong Huang, Phys. Lett. B **817**, 136325 (2021).
[24] J.-H. Gao, Phys. Rev. D **104**, 076016 (2021).
[25] B. Müller and D.-L. Yang, Phys. Rev. D **105**, L011901 (2022).
[26] X.-L. Sheng, L. Oliva, and Q. Wang, Phys. Rev. D **101**, 096005 (2020).
[27] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[28] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
[29] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 448 (1979).
[30] E. V. Shuryak, Nucl. Phys. **B203**, 93 (1982).
[31] E. V. Shuryak, Nucl. Phys. **B203**, 116 (1982).
[32] X.-L. Sheng, Q. Wang, and X.-N. Wang, Phys. Rev. D **102**, 056013 (2020).
[33] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).
[34] P. C. Martin and J. S. Schwinger, Phys. Rev. **115**, 1342 (1959).
[35] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].
[36] K.-c. Chou, Z.-b. Su, B.-l. Hao, and L. Yu, Phys. Rep. **118**, 1 (1985).
[37] J.-P. Blaizot and E. Iancu, Phys. Rep. **359**, 355 (2002).
[38] J. Berges, AIP Conf. Proc. **739**, 3 (2004).
[39] W. Cassing, Eur. Phys. J. Spec. Top. **168**, 3 (2009).
[40] X.-L. Sheng, N. Weickgenannt, E. Speranza, D. H. Rischke, and Q. Wang, Phys. Rev. D **104**, 016029 (2021).
[41] X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, and X.-N. Wang, Phys. Rev. Lett. **131**, 042304 (2023).
[42] A. Proca, J. Phys. Radium **7**, 347 (1936).
[43] C. Itzykson and J. B. Zuber, *Quantum Field Theory*, International Series in Pure and Applied Physics (McGraw-Hill, New York, 1980), ISBN 978-0-486-44568-7.
[44] H. T. Elze, M. Gyulassy, and D. Vasak, Phys. Lett. B **177**, 402 (1986).
[45] Q. Wang, K. Redlich, H. Stoecker, and W. Greiner, Phys. Rev. Lett. **88**, 132303 (2002).
[46] X.-G. Huang, P. Mitkin, A. V. Sadofyev, and E. Speranza, J. High Energy Phys. 10 (2020) 117.
[47] K. Hattori, Y. Hidaka, N. Yamamoto, and D.-L. Yang, J. High Energy Phys. 02 (2021) 001.
[48] N. Weickgenannt, D. Wagner, and E. Speranza, Phys. Rev. D **105**, 116026 (2022).
[49] F. Fernandez, A. Valcarce, U. Straub, and A. Faessler, J. Phys. G **19**, 2013 (1993).
[50] Z.-p. Li, H.-x. Ye, and M.-h. Lu, Phys. Rev. C **56**, 1099 (1997).
[51] Q. Zhao, Z.-p. Li, and C. Bennhold, Phys. Rev. C **58**, 2393 (1998).
[52] A. Zacchi, R. Stiele, and J. Schaffner-Bielich, Phys. Rev. D **92**, 045022 (2015).
[53] A. Zacchi, L. Tolos, and J. Schaffner-Bielich, Phys. Rev. D **95**, 103008 (2017).
[54] V. Greco, C. M. Ko, and P. Levai, Phys. Rev. Lett. **90**, 202302 (2003).
[55] R. J. Fries, B. Muller, C. Nonaka, and S. A. Bass, Phys. Rev. Lett. **90**, 202303 (2003).
[56] V. Greco, C. M. Ko, and P. Levai, Phys. Rev. C **68**, 034904 (2003).
[57] R. J. Fries, B. Muller, C. Nonaka, and S. A. Bass, Phys. Rev. C **68**, 044902 (2003).
[58] V. Greco, C. M. Ko, and R. Rapp, Phys. Lett. B **595**, 202 (2004).
[59] W. Zhao, C. M. Ko, Y.-X. Liu, G.-Y. Qin, and H. Song, Phys. Rev. Lett. **125**, 072301 (2020).
[60] Y.-Z. Xu, D. Binosi, Z.-F. Cui, B.-L. Li, C. D. Roberts, S.-S. Xu, and H. S. Zong, Phys. Rev. D **100**, 114038 (2019).
[61] Y.-Z. Xu, S. Chen, Z.-Q. Yao, D. Binosi, Z.-F. Cui, and C. D. Roberts, Eur. Phys. J. C **81**, 895 (2021).
[62] F. Becattini, V. Chandra, L. Del Zanna, and E. Grossi, Ann. Phys. (Amsterdam) **338**, 32 (2013).
[63] F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin, Phys. Rev. C **95**, 054902 (2017).
[64] R.-h. Fang, L.-g. Pang, Q. Wang, and X.-n. Wang, Phys. Rev. C **94**, 024904 (2016).
[65] N. Weickgenannt, X.-L. Sheng, E. Speranza, Q. Wang, and D. H. Rischke, Phys. Rev. D **100**, 056018 (2019).

[66] L. P. Csernai, J. I. Kapusta, and T. Welle, Phys. Rev. C **99**, 021901 (2019).
[67] J. Xu, L.-W. Chen, C. M. Ko, and Z.-W. Lin, Phys. Rev. C **85**, 041901 (2012).
[68] Y.-C. Liu, L.-L. Gao, K. Mameda, and X.-G. Huang, Phys. Rev. D **99**, 085014 (2019).
[69] J.-Y. Chen, D. T. Son, and M. A. Stephanov, Phys. Rev. Lett. **115**, 021601 (2015).
[70] W. Florkowski, A. Kumar, and R. Ryblewski, Phys. Rev. C **98**, 044906 (2018).
[71] D. Diakonov and V. Y. Petrov, Nucl. Phys. **B272**, 457 (1986).
[72] S. Y. F. Liu and Y. Yin, J. High Energy Phys. 07 (2021) 188.
[73] F. Becattini, M. Buzzegoli, and A. Palermo, Phys. Lett. B **820**, 136519 (2021).
[74] B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, Phys. Rev. Lett. **127**, 142301 (2021).
[75] F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Phys. Rev. Lett. **127**, 272302 (2021).
[76] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997).
[77] E. Braaten, in *3rd International Workshop on Particle Physics Phenomenology* (1996), arXiv:hep-ph/9702225.