Holographic $T\bar{T}$ deformed entanglement entropy in dS_3/CFT_2

Deyou Chen,[1,2,*] Xin Jiang,[3,†] and Haitang Yang[3,‡]

[1]*School of Science, Xihua University, Chengdu 610039, China*
[2]*Key Laboratory of High Performance Scientific Computation, Xihua University, Chengdu 610039, China*
[3]*College of Physics, Sichuan University, Chengdu, 610065, China*

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In this paper, based on the $T\bar{T}$ deformed version of dS_3/CFT_2 correspondence, we calculate the pseudoentropy for an entangling surface consisting of two antipodal points on a sphere and find it is exactly dual to the complex geodesic in the bulk.

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I. INTRODUCTION

The study of quantum gravity in de Sitter space has generated much interest in recent years, particularly due to its potential relevance for inflationary cosmology and cosmic acceleration. One promising method to comprehend de Sitter (dS) space is through the dS/conformal field theory (CFT) correspondence [1]. It is a conjectured equivalence between a gravitational theory in de Sitter space and a conformal field theory residing on its boundary. The dS/CFT correspondence is a generalization of the well-known AdS/CFT correspondence [2,3], which has been extensively studied in string theory and provided numerous insights into extracting the nature of quantum gravity from its dual CFT. However, the dS/CFT correspondence is not as well understood as the AdS/CFT correspondence, as there are only limited explicit examples of CFTs that are dual to de Sitter spacetime. Recently, a remarkable and explicit example has been constructed for the dS_3/CFT_2 correspondence [4,5], where the dual CFT resides on the past/future boundary of de Sitter spacetime.

Starting with the three-dimensional de Sitter spacetime, we take a static compact slice Σ_t of constant time. Clearly, each Σ_t has a Riemannian metric γ and a second fundamental form K. In the canonical formalism for gravity, the quantum state residing on Σ_t can be described by the Hartle-Hawking wave function $\Psi_{dS}[\gamma]$. Following Refs. [4,5], and neglecting the contributions of bulk matter fields, we could obtain a calculable example of dS_3/CFT_2 correspondence described by

$$\Psi_{dS}[\gamma] = Z_{CFT}[\gamma], \qquad t \to \infty, \tag{1.1}$$

where Z_{CFT} is the partition function of the dual CFT_2 living on Σ_∞.

In this paper, we aim to further explore the scenario described above. Typically, it is not necessary to confine the slice Σ_t at the future infinity, which leads to a natural extension of the dS_3/CFT_2 correspondence,

$$\Psi_{dS}[\gamma] = Z_{QFT}[\gamma], \tag{1.2}$$

where Z_{QFT} is the partition function of the dual quantum field theory (QFT) living on a finite-volume slice Σ_t. The dual QFT could be defined as a two dimensional (2D) CFT deformed by the $T\bar{T}$ operator [6–8] that generates a trajectory in the space of field theory,

$$\frac{\partial}{\partial\lambda}\log Z(\lambda) = -2\pi\int_\Sigma d^2x\sqrt{\gamma}\langle T\bar{T}\rangle_\lambda. \tag{1.3}$$

At the first order of the deformation parameter λ, the deformed theory, perturbatively, could be written as

$$\log Z(\lambda) = \log Z(\lambda=0) - 2\pi\lambda\int_\Sigma d^2x\sqrt{\gamma}\langle T\bar{T}\rangle_{\lambda=0} + \mathcal{O}(\lambda^2), \tag{1.4}$$

where $\langle T\bar{T}\rangle_{\lambda=0}$ is defined by the stress tensor of the undeformed theory as $\langle T\bar{T}\rangle_{\lambda=0} \equiv \langle T\bar{T}\rangle = \frac{1}{8}[\langle T^{ab}\rangle\langle T_{ab}\rangle - \langle T^a_a\rangle^2]$. In recent years, the $T\bar{T}$ deformation has been widely studied [9–45], due to its integrability and its applications in holography. Our proposal is a natural extension of the cutoff-AdS/$T\bar{T}$-deformed-CFT correspondence [46] to de Sitter spacetime. Additionally, the $T\bar{T}$-deformed

[*]deyouchen@hotmail.com
[†]domoki@stu.scu.edu.cn
[‡]hyanga@scu.edu.cn

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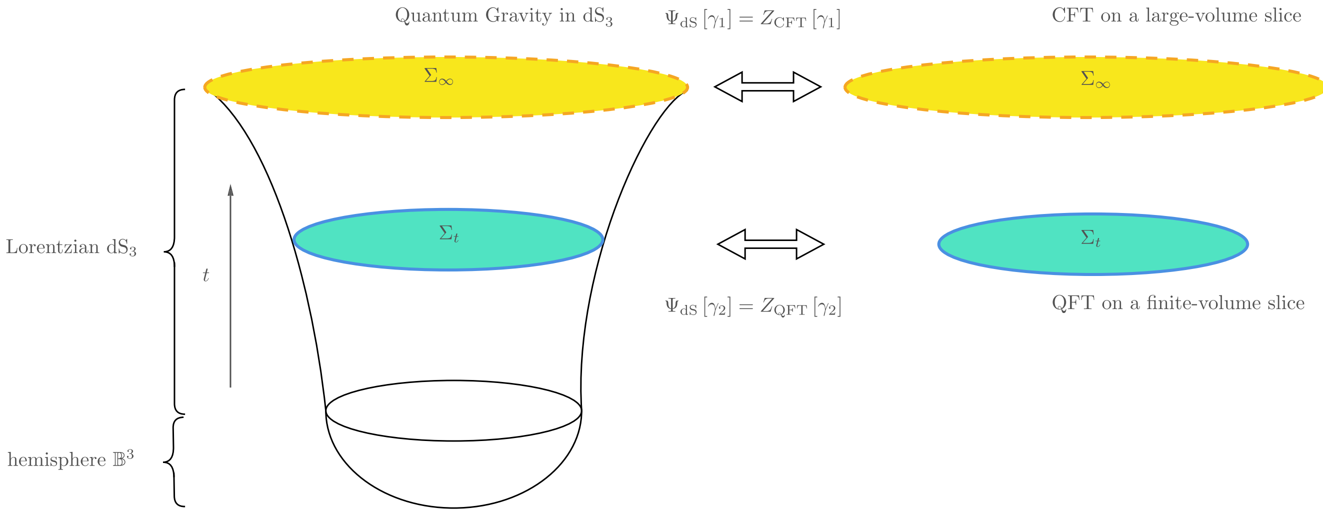

FIG. 1. The $T\bar{T}$-deformed version of the $\mathrm{dS}_3/\mathrm{CFT}_2$ correspondence, with the Lorentzian time t in the global coordinates of the dS_3 spacetime.

version (1.2) of $\mathrm{dS}_3/\mathrm{CFT}_2$ is remarkably coincident with the Cauchy slice holography [47,48], where time serves as the emergent direction. The $T\bar{T}$-deformed version (1.2) of $\mathrm{dS}_3/\mathrm{CFT}_2$ is illustrated in Fig. 1. Note that the deformation parameter λ is on the order of $\mathcal{O}(1/c)$, and in the limit of large c, the deformed theory is simply defined by Eq. (1.4). In this scenario, the $T\bar{T}$ flow has been demonstrated to nonperturbatively match with bulk computations of dS_3 with a finite temporal cutoff, analogous to the situation in AdS/CFT [46]. This justifies our choice of the time t on the hypersurface Σ_t to be finite. However, beyond the large c limit, one must define the $T\bar{T}$ deformed theory using Eq. (1.3), which generically cannot be solved completely.

It is clear that the $T\bar{T}$ deformation is irrelevant in the renormalization group sense. This implies that the $T\bar{T}$ deformation leads to no consequence in IR but does affect UV physics. Among the various deformable physical quantities in the UV region, a particularly important one is the entanglement entropy. In the dS/CFT correspondence, the dual CFTs turn to be nonunitary [1,4,5,49]. To characterize the degrees of freedom in a nonunitary CFT, complex-valued entanglement entropies, namely, pseudoentropy [41–43,50–59], are needed. In other words, the pseudoentropy can be viewed as a well-defined entanglement entropy in the $T\bar{T}$-deformed version of the $\mathrm{dS}_3/\mathrm{CFT}_2$ correspondence. It is then interesting to explore how the holographic entanglement entropy [60,61] behaves in the $T\bar{T}$-deformed version of the $\mathrm{dS}_3/\mathrm{CFT}_2$. In this paper, our main goal is to calculate the entanglement entropy in the $T\bar{T}$-deformed field theory and compare it with geodesics in the dS_3 bulk.

In Sec. II, we give a brief review of the $T\bar{T}$ deformed version of dS/CFT. In Sec. III, we calculate the pseudoentropy for an entangling surface consisting of two antipodal points on a sphere $\mathbb{S}^2$, and we find that the entanglement entropy does perfectly match the length of the complex geodesic connecting these antipodal points in dS_3.

II. WHEELER-DEWITT EQUATION AND $T\bar{T}$ FLOW

We first briefly review the $T\bar{T}$ deformed version of dS/CFT in this section. In the canonical formalism of three-dimensional pure gravity, the Hartle-Hawking wave function $\tilde{\Psi}[\gamma]$ should obey the Wheeler-DeWitt equation

$$\mathcal{H}\tilde{\Psi}[\gamma] = \left\{\frac{16\pi G_N}{\sqrt{\gamma}}(\Pi^{ab}\Pi_{ab} - \Pi^a_a\Pi^b_b) - \frac{\sqrt{\gamma}}{16\pi G_N}(\mathcal{R}[\gamma] - 2\Lambda)\right\}\tilde{\Psi}[\gamma] = 0, \tag{2.1}$$

where $\mathcal{R}[\gamma]$ is the Ricci scalar, $\Lambda = \ell_{\mathrm{dS}}^{-2}$ is the cosmological constant of the de Sitter spacetime, and Π^{ab} is the momentum conjugate to the metric γ_{ab}:

$$\Pi^{ab} = -\mathrm{i}\frac{\delta}{\delta\gamma_{ab}} = \frac{\sqrt{\gamma}}{16\pi G_N}(K^{ab} - K^c_c\gamma^{ab}). \tag{2.2}$$

The standard quasilocal stress tensor can be defined as

$$T^{ab} = -\frac{2}{\sqrt{\gamma}}\frac{\delta}{\delta\gamma_{ab}} = -\frac{2\mathrm{i}}{\sqrt{\gamma}}\Pi^{ab}, \tag{2.3}$$

which coincides with the field-theoretic definition. To require the finiteness of the quasilocal stress tensor at the future infinity Σ_∞, one needs to perform a canonical transformation [48,62],

$$\tilde{\Psi} = \exp\left(-\frac{\mathrm{i}}{8\pi G_N \ell_{\mathrm{dS}}}\int_\Sigma d^2x\sqrt{\gamma}\right)\Psi, \tag{2.4}$$

which leads a shift on the momentum

$$\Pi^{ab} \to \Pi^{ab} + \frac{\sqrt{\gamma}}{16\pi G_N \ell_{\mathrm{dS}}}\gamma^{ab}. \tag{2.5}$$

Therefore, the Wheeler-DeWitt equation could be rewritten as

$$\left\{-\frac{2}{\sqrt{\gamma}}\Pi^a_a + \frac{16\pi G_N \ell_{\mathrm{dS}}}{\det\gamma}(\Pi^{ab}\Pi_{ab} - \Pi^a_a\Pi^b_b) - \frac{\ell_{\mathrm{dS}}}{16\pi G_N}\mathcal{R}[\gamma]\right\}\Psi[\gamma] = 0. \tag{2.6}$$

By using the quasilocal stress tensor, the equation is simply

$$T^a_a = \frac{\mathrm{i}\ell_{\mathrm{dS}}}{16\pi G_N}\mathcal{R}[\gamma] + \mathrm{i}4\pi G_N \ell_{\mathrm{dS}}(T^{ab}T_{ab} - T^a_a T^b_b). \tag{2.7}$$

On the other hand, in the $T\bar{T}$ deformed field theory, when the deformation parameter λ is small,[1] one can rewrite Eq. (1.3) as

$$\log Z_{\mathrm{QFT}} = \log Z_{\mathrm{CFT}} - 2\pi\lambda\int_\Sigma d^2x\sqrt{\gamma}\langle T\bar{T}\rangle. \tag{2.8}$$

By the definition of the trace of the stress tensor

$$T^a_a = -2\frac{\gamma_{ab}}{\sqrt{\gamma}}\frac{\delta}{\delta\gamma_{ab}}\log Z, \tag{2.9}$$

and the famous Weyl anomaly

$$(T^a_a)_{\mathrm{CFT}} = -\frac{c}{24\pi}\mathcal{R}[\gamma], \tag{2.10}$$

the trace flow equation for deformed theory is

$$\langle T^a_a\rangle = -\frac{c}{24\pi}\mathcal{R}[\gamma] - \frac{\pi\lambda}{2}(\langle T^{ab}\rangle\langle T_{ab}\rangle - \langle T^a_a\rangle\langle T^b_b\rangle). \tag{2.11}$$

Here, all stress tensors emanate from the deformed theory residing on a 2-sphere. This alignment is in accordance with the capability of $T\bar{T}$ deformation to be defined on compact backgrounds [7,10,37]. Relating to Eq. (2.7), we immediately find the identifications between field-theoretic quantities and gravitational quantities

$$c = -\mathrm{i}\frac{3\ell_{\mathrm{dS}}}{2G_N} = -\mathrm{i}c_{\mathrm{dS}}, \qquad \lambda = -\mathrm{i}8G_N\ell_{\mathrm{dS}} = -\mathrm{i}\lambda_{\mathrm{dS}}, \tag{2.12}$$

where the Brown-Henneaux central charge [63] turns to be imaginary valued in the de Sitter context [49] and the deformation parameter is also imaginary valued. The deformation parameter λ remains unrelated to the time variable t due to our selection of the seed CFT residing on the hypersurface Σ_t, as opposed to Σ_∞. It is noteworthy that these distinct choices of the seed CFT are connected through a straightforward Weyl rescaling on the hypersurface. Furthermore, the momentum constraint for the Hartle-Hawking wave function,

$$\mathcal{D}^a\Psi[\gamma] = -2(\nabla_b\Pi^{ab})\Psi[\gamma] = 0, \tag{2.13}$$

can be easily interpreted as the conversation law of the stress tensor in field theory,

$$\nabla_b\langle T^{ab}\rangle = 0. \tag{2.14}$$

The wave function $\Psi[\gamma]$ should be invariant under diffeomorphisms of Σ_t, given that $\mathcal{D}^a$ serves as the generator of diffeomorphisms. In simpler terms, $\Psi[\gamma]$ is a function on the space of metrics modulo diffeomorphisms. Even though the dual field theory is nonunitary, the dynamical inner product $\langle\Psi|\Psi\rangle$ is Hermitian and positive semidefinite, which indicates that we still have the bulk unitarity [47,48].

III. HOLOGRAPHIC ENTANGLEMENT ENTROPY

First, we briefly introduce the pseudoentropy. Dividing the total system into two subsystems A and B, the pseudoentropy is defined by the von Neumann entropy,

$$S_A = -\mathrm{Tr}[\tau_A\log\tau_A], \tag{3.1}$$

of the reduced transition matrix

$$\tau_A = \mathrm{Tr}_B\left[\frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}\right], \tag{3.2}$$

where $|\psi\rangle$ and $|\varphi\rangle$ are two different quantum states in the total Hilbert space that is factorized as $\mathcal{H} = \mathcal{H}_A\otimes\mathcal{H}_B$. For a generic QFT living on a curved surface Σ, the pseudoentropy could be captured by the replica method [64,65] in path integral formalism. Denoting the manifold

[1]Our approach is limited to perturbation theory because a nonperturbative completion of the $T\bar{T}$ deformation (1.3) is unknown. In our analysis, we operate under the assumption that the Zamolodchikov's factorization formula remains valid in the context of dS/CFT, particularly when considering the large c limit.

corresponding to $\langle\varphi|\psi\rangle$ as Σ and the manifold corresponding to $\mathrm{Tr}_A(\tau_A)^n$ as Σ_n, the pseudoentropy for the subsystem A reads

$$S_A = \lim_{n\to 1} \frac{1}{1-n} \log\left[\frac{Z_{\Sigma_n}}{(Z_\Sigma)^n}\right], \tag{3.3}$$

where Z_Σ is the path integral over the manifold Σ and S_A can be regarded as a well-defined entanglement entropy in the dS_3 context. For an example, one can compute the pseudoentropy for subsystem A, which corresponds to an interval, in a nonunitary QFT residing on a sphere $\mathbb{S}^2$, as depicted in Fig. 2.

To capture the pseudoentropy, we first need to calculate the partition function for a field theory. Specifically, one saddle solution for the Hartle-Hawking wave function $\Psi_{\mathrm{dS}}[\gamma]$ is the Euclidean sphere $\mathbb{S}^2$, where the corresponding metric of de Sitter spacetime is given by

$$ds^2 = \ell_{\mathrm{dS}}^2(-dt^2 + \cosh^2 t d\Omega_2^2), \tag{3.4}$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric of a 2D unit sphere and the spacelike boundary at time t is a Euclidean sphere $\mathbb{S}^2$ with the radius $r = \sqrt{\frac{\lambda_{\mathrm{dS}} c_{\mathrm{dS}}}{12}} \cosh t$. In this section, we will calculate the pseudoentropy of the $T\bar{T}$ deformed field theory living on a sphere $\mathbb{S}^2$ with a radius r. To be precise, we focus on the case in which an entangling surface consists of two antipodal points on this sphere, as shown in the right panel in Fig. 3. Following Ref. [10], for a field theory living on a sphere with the metric

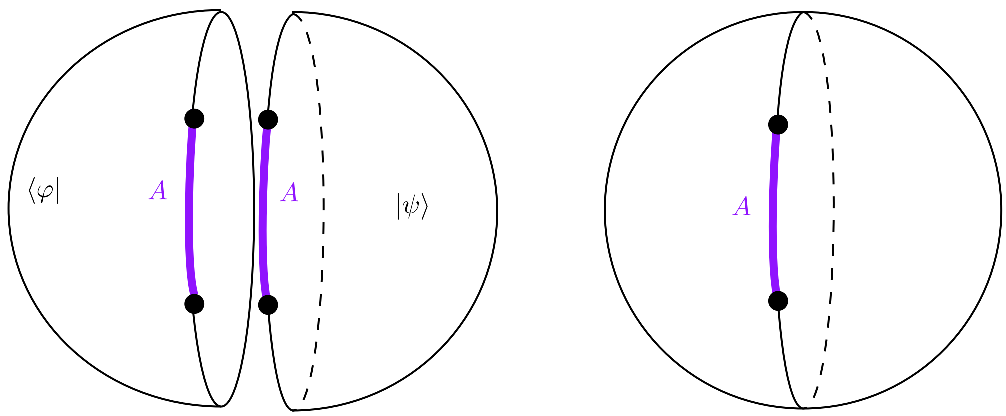

FIG. 2. The subsystem A within a nonunitary QFT residing on a 2-sphere, with black points indicating the codimension-2 entangling surface.

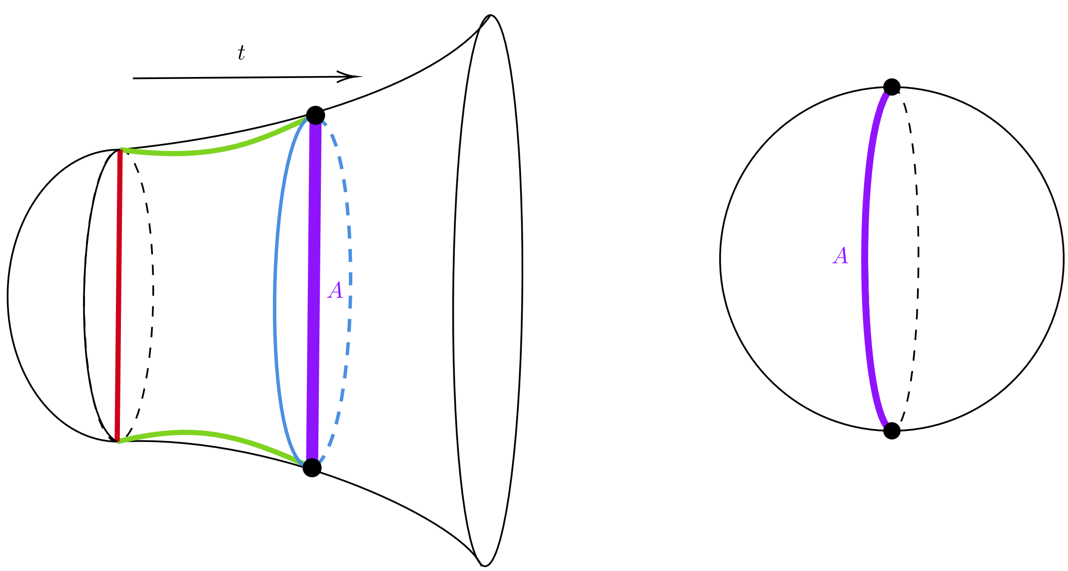

FIG. 3. Left panel: geodesics connecting to the entangling surface in the dS_3. The red line denotes one spacelike geodesic, and two green lines denote two timelike geodesics. The purple line denotes a spacelike interval A on the 2-sphere. Right panel: the entangling surface consists of two antipodal points on the 2-sphere.

$$ds^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{3.5}$$

the stress tensor takes the form[2]

$$T_{ab} = \alpha\gamma_{ab}, \tag{3.6}$$

where α could be determined by substituting Eq. (3.6) into the trace flow equation (2.11),

$$\begin{aligned}\alpha &= \frac{1}{\pi\lambda}\left(1 - \sqrt{1 + \frac{\lambda c}{12r^2}}\right)\\ &= \frac{\mathrm{i}}{\pi\lambda_{\mathrm{dS}}}\left(1 - \sqrt{1 - \frac{\lambda_{\mathrm{dS}}c_{\mathrm{dS}}}{12r^2}}\right).\end{aligned} \tag{3.7}$$

Noticing that $r\partial_r\gamma_{ab} = 2\gamma_{ab}$, one obtains the equation for the partition function

$$\begin{aligned}\frac{d\log Z_{\mathrm{QFT}}}{dr} &= -\frac{1}{r}\int_\Sigma d^2x\sqrt{\gamma}T^a_a\\ &= \mathrm{i}\frac{8}{\lambda_{\mathrm{dS}}}\left(\sqrt{r^2 - \frac{\lambda_{\mathrm{dS}}c_{\mathrm{dS}}}{12}} - r\right),\end{aligned} \tag{3.8}$$

and the partition function thus reads

$$\log Z_{\mathrm{QFT}} = \mathrm{i}\frac{4}{\lambda_{\mathrm{dS}}}\left[r\left(\sqrt{r^2 - \frac{\lambda_{\mathrm{dS}}c_{\mathrm{dS}}}{12}} - r\right) - \frac{\lambda_{\mathrm{dS}}c_{\mathrm{dS}}}{12}\tanh^{-1}\left(\frac{r}{\sqrt{r^2 - \lambda_{\mathrm{dS}}c_{\mathrm{dS}}/12}}\right)\right]. \tag{3.9}$$

It is worthy to note that, since $r/\sqrt{r^2 - \lambda_{\mathrm{dS}}c_{\mathrm{dS}}/12} > 1$, the function $\tanh^{-1}(r/\sqrt{r^2 - \lambda_{\mathrm{dS}}c_{\mathrm{dS}}/12})$ is indeed complex valued. Focusing on the principal branch of the inverse hyperbolic function, one then obtains

$$\log Z_{\mathrm{QFT}} = \mathrm{i}\frac{4}{\lambda_{\mathrm{dS}}}\left[r\left(\sqrt{r^2 - \frac{\lambda_{\mathrm{dS}}c_{\mathrm{dS}}}{12}} - r\right) - \frac{\lambda_{\mathrm{dS}}c_{\mathrm{dS}}}{12}\coth^{-1}\left(\frac{r}{\sqrt{r^2 - \lambda_{\mathrm{dS}}c_{\mathrm{dS}}/12}}\right)\right] + \frac{\pi c_{\mathrm{dS}}}{6}. \tag{3.10}$$

The real part $\frac{\pi c_{\mathrm{dS}}}{6}$ is consistent with the result in Ref. [5],

$$|Z_{\mathrm{CFT}}|^2 = \exp\left(\frac{\pi c_{\mathrm{dS}}}{3}\right), \tag{3.11}$$

since the deformation parameter is imaginary valued and the $T\bar{T}$ deformation only affects the imaginary part of $\log Z$.

Utilizing the replica method introduced in Ref. [10], in the case where the entangling surface consists of two antipodal points on the 2-sphere, the n-sheeted cover is simply

$$ds^2 = g_{ab}dx^a dx^b = r^2(d\theta^2 + n^2\sin^2\theta d\phi^2), \tag{3.12}$$

and the pseudoentropy reads

$$\begin{aligned}S_A &= \lim_{n\to1}\frac{1}{1-n}\log\left[\frac{Z_n}{(Z_1)^n}\right]\\ &= \left(1 - n\frac{\partial}{\partial n}\right)\log Z_n\Big|_{n=1}.\end{aligned} \tag{3.13}$$

Noting that $n\partial_n g_{\phi\phi} = 2g_{\phi\phi}$, the variation of $\log Z_n$ with respect to n can be expressed as

$$n\frac{\partial}{\partial n}\log Z_n\Big|_{n=1} = -\frac{1}{2}\int_\Sigma d^2x\sqrt{\gamma}T^a_a = \frac{r}{2}\frac{d}{dr}\log Z_{\mathrm{QFT}}. \tag{3.14}$$

We could thus obtain the pseudoentropy S_A for an entangling surface of two antipodal points on a sphere $\mathbb{S}^2$:

$$\begin{aligned}S_A &= \left(1 - \frac{r}{2}\frac{d}{dr}\right)\log Z_{\mathrm{QFT}}\\ &= -\mathrm{i}\frac{c_{\mathrm{dS}}}{3}\coth^{-1}\left(\frac{r}{\sqrt{r^2 - \lambda_{\mathrm{dS}}c_{\mathrm{dS}}/12}}\right) + \frac{\pi c_{\mathrm{dS}}}{6}.\end{aligned} \tag{3.15}$$

Substituting $r = \sqrt{\frac{\lambda_{\mathrm{dS}}c_{\mathrm{dS}}}{12}}\cosh t$, the entanglement entropy of two antipodal points on a $\mathbb{S}^2$ is thus given by

$$S_A = \frac{c_{\mathrm{dS}}}{6}\pi - \mathrm{i}\frac{c_{\mathrm{dS}}}{3}t. \tag{3.16}$$

On the other hand, in the dS_3 bulk, the geodesic distance between two points at the same time $(t, \theta_i = 0, \phi = 0)$ and $(t, \theta_f = \pi, \phi = 0)$ is given by

$$\begin{aligned}D(\theta_i, \theta_f) &= \ell_{\mathrm{dS}}\mathrm{arcos}\left[1 - 2\cosh^2 t\sin^2\left(\frac{\theta_f - \theta_i}{2}\right)\right]\\ &= \ell_{\mathrm{dS}}\pi - 2\mathrm{i}\ell_{\mathrm{dS}}t.\end{aligned} \tag{3.17}$$

According to the Ryu-Takayanagi formula [60], Refs. [5,55] have determined that the complex-valued extremal surface comprises one spacelike geodesic and two timelike geodesics, as illustrated in Fig. 3. The two timelike geodesics connect the entangling surface and the de Sitter horizon, respectively, while the spacelike geodesic

[2] Generally, for vacuum states in QFTs living in a d-dimensional maximally symmetric space, the stress tensor satisfies $\langle T_{\mu\nu}(x)\rangle = \frac{1}{d}\langle\Theta\rangle g_{\mu\nu}(x)$, where $\langle\Theta\rangle$ is an x-independent constant.

links the end points of the two timelike geodesics on the de Sitter horizon. Furthermore, the length of the spacelike geodesic is proportional to the real part of the pseudoentropy, whereas the total length of the two timelike geodesics is proportional to the imaginary part of the pseudoentropy. Related works on the complex-valued extremal surface have also been proposed in Refs. [66–68]. Using the identifications (2.12), the Ryu-Takayanagi formula gives

$$S_A = \frac{D(\theta_i, \theta_f)}{4G_N} = \frac{c_{\mathrm{dS}}}{6}\pi - \mathrm{i}\frac{c_{\mathrm{dS}}}{3}t, \tag{3.18}$$

which exactly is equal to the entanglement entropy equation (3.16).

Therefore, as promised, we verified that, for a static finite volume slice $\mathbb{S}^2$ in dS_3, the pseudoentropy for an entangling surface consisting of two antipodal points is precisely equal to the complex geodesic in the dS_3 bulk.

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