

Electromagnetic waves generated by null cosmic strings passing pulsars

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Null cosmic strings disturb electromagnetic (EM) fields of charged sources and sources with magnetic moments. As has been recently shown by the authors, these perturbations result in a self-force acting on the sources and create EM waves outgoing from the sources. We develop an analytic approximation for the asymptotic of the EM waves at the future null infinity and calculate radiation fluxes for sources of both types. For magnetic-dipole-like sources the radiation flux depends on orientation of the magnetic moment with respect to the string. Estimates show that the peak power of the radiation can be quite large for null strings moving near pulsars and considerably larger in the case of magnetars. The string generated variations of the luminosities of the stars can be used as a potential experimental signature of null cosmic strings.

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I. INTRODUCTION

As has been recently shown [1], null cosmic strings disturb electric fields of charged sources and produce electromagnetic (EM) pulses outgoing from the sources. The aim of the present work, based on the approach developed in [1], is twofold. First, it is to describe in more detail properties of EM waves at future null infinity generated by straight null cosmic strings in a locally Minkowski space-time. Second, it is to study analogous effects for sources with magnetic moments, such as pulsars and magnetars. As opposed to electrically charged objects, the case of pulsars and magnetars is more interesting from an astrophysical point of view. We show that variations of luminosities of such objects induced by the null strings can be large enough to be experimentally observable.

The null strings are one-dimensional objects whose points move along trajectories of light rays, orthogonally to strings [2]. As a result, the null strings exhibit optical properties: they behave as one-dimensional null geodesic congruences governed by an analogue of the Sachs' optical equation [3]. The world sheets of null strings may develop caustics [4].

The null cosmic strings, like tensile cosmic strings [5,6], are hypothetical astrophysical objects, which might have been produced in the very early Universe, at the Planckian epoch [7–10]. Possible astrophysical and cosmological effects of null cosmic strings such as deviations of light rays and trajectories of particles in the gravitational field of strings [11–12], as well as scattering of strings by massive sources [4], are similar to those of the tensile strings.

Gravitational fields of null cosmic strings can be described in terms of holonomies around the string world

sheet. The holonomies are null rotations of the Lorentz group [11,13] with the group parameter determined by the string energy per unit length. The null rotations have fixed points on the string world sheet $\mathcal{S}$. The world sheet belongs to a null hypersurface $\mathcal{H}$, which is the string event horizon.

To find the trajectories of particles and light rays near null cosmic strings, one should set on the string horizon $\mathcal{H}$ the “initial” data that ensure the required holonomy transformations [11]. Such an approach has been generalized in [1] to describe the evolution of classical fields under the gravity of the null strings. Finding solutions to wave equations in a free field theory on a space-time of a null string, in the domain above $\mathcal{H}$, is equivalent to solving a characteristic problem with initial data on $\mathcal{H}$ determined by incoming data. The initial data are null rotated (or, better to say, transformed by the Carroll group of $\mathcal{H}$) so to ensure the required holonomy. It is this approach we use in the present work to describe perturbations caused by null cosmic strings on EM fields of pointlike sources.

In [1] we found a useful integral representation for perturbations of EM fields of point electric charges and straight strings. We generalize this result to point magnetic-dipole-like sources (MD sources) and develop an approximation for the perturbations at future null infinity. The asymptotic for angular components of the vector potential at large distances r from the source look as follows:

$$A_B(r, U, \Omega) \simeq a_B(U, \Omega) + b_B(\Omega) \ln r / \varrho + O(r^{-1} \ln r), \quad (1.1)$$

where U is a retarded time, vector fields a_B , b_B are in the tangent space of S^2 , and ϱ is a dimensional parameter related to the approximation. The asymptotic holds if r is

large enough with respect to U and with respect to the impact parameter between the string and the source. Equation (1.1) yields a finite energy flux at large r ,

$$\lim_{r \rightarrow \infty} \partial_t E(r, t) = \int d\Omega \dot{a}_A \dot{a}^A, \quad (1.2)$$

where $\dot{a}_A \equiv \partial_U a_A$, and index A is risen with the help of the metric on S^2 . In the found approximation, we analytically calculate $\dot{a}_A \dot{a}^A$, the density of the flux, for different sources and study its properties. The logarithmic term in (1.1) appears since the Sommerfeld radiation condition is violated in the presence of null strings. This term, however, does not affect the flux (1.2).

The paper is organized as follows. In Sec. II we introduce necessary notions and describe the method. In Sec. II A we pay a special attention to the isometry group near the string world sheet $\mathcal{S}$. On the string horizon, the null rotations induce the Carroll transformations [14,15,16,17], which we use to set initial data on $\mathcal{H}$. The approach [11] and the characteristic problem for EM fields [1] are formulated in Sec. II B. The problem is characteristic since $\mathcal{H}$ is a null hypersurface. Pointlike sources we consider are defined in Sec. III A. Fields of MD sources may be viewed as a far-distance approximation of fields of finite size bodies with magnetic moments. Solutions for characteristic problems of vector fields can be formulated in terms of solutions for the auxiliary scalar problem defined in Sec. III B. Asymptotic properties of EM waves and energy fluxes generated from electric and MD sources are considered in detail in Secs. III C and III D, respectively. Here we provide explicit expressions for quantities in (1.1) and (1.2). Possibilities to detect EM pulses generated by null cosmic strings near pulsars and magnetars are discussed in Sec. IV. A summary and discussion of our results can be found in Sec. V. Approximations for integral representations of perturbations caused by null strings are developed in Appendix A. Some technicalities related to perturbations for MD sources are given in Appendix B.

II. DESCRIPTION OF THE METHOD

A. Null rotations and Carroll transformations

We consider classical electrodynamics,

$$\partial_\mu F^{\mu\nu} = j^\nu, \quad (2.1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The current j^μ , $\partial j = 0$, will correspond to a pointlike source, either of electric or MD type. We study solutions of (2.1) near a straight cosmic string, which is stretched along the z axis and moves along the x axis. The space-time is locally Minkowski $R^{1,3}$, and we use the light cone coordinates $v = t + x$, $u = t - x$, where the metric is

$$ds^2 = -dvdu + dy^2 + dz^2. \quad (2.2)$$

The string world sheet $\mathcal{S}$ can be defined by equations $u = y = 0$.

A number of definitions are needed for further purposes. We use the null rotations $x^\mu = M^\mu_\nu(\lambda)\bar{x}^\nu$, which leave invariant (2.2),

$$u = \bar{u}, \quad v = \bar{v} + 2\lambda\bar{y} + \lambda^2\bar{u}, \quad y = \bar{y} + \lambda\bar{u}, \quad z = \bar{z}, \quad (2.3)$$

where λ is some real parameter. Transformations of quantities with lower indices are

$$V_u = \bar{V}_u - \lambda\bar{V}_y + \lambda^2\bar{V}_v, \quad V_v = \bar{V}_v, \quad V_y = \bar{V}_y - 2\lambda\bar{V}_v, \quad V_z = \bar{V}_z, \quad (2.4)$$

or $V_\mu = M_\mu^\nu(\lambda)\bar{V}_\nu$, where $M_\mu^\nu = \eta_{\mu\mu'}\eta^{\nu\nu'}M^{\mu'}_{\nu'}$. The null rotations make a parabolic subgroup of the Lorentz group.

For a null string with the world sheet $u = y = 0$, a parallel transport of a vector V along a closed contour around the string results in a null rotation, $V' = M(\omega)V$ with ω defined as, see [13],

$$\omega \equiv 8\pi GE. \quad (2.5)$$

The world sheet is a fixed point set of (2.3).

The null hypersurface $u = 0$ is the event horizon of the string. We denote it by $\mathcal{H}$. The properties of null hypersurfaces are reviewed, for example, in [18]. We enumerate coordinates v, y, z on $\mathcal{H}$ by indices $a, b, \dots$. Coordinate transformations (2.3) at $u = 0$ (with $\lambda = \omega$) induce the change of coordinates on $\mathcal{H}$:

$$x^a = C^a_b \bar{x}^b \quad \text{or} \quad \mathbf{x} = \bar{\mathbf{x}} + 2\omega y \mathbf{q}, \quad q^a = \delta^a_v, \quad (2.6)$$

where $\mathbf{x} \equiv \{v, y, z\}$. Matrices C^a_b can be defined as

$$C^a_b = M^a_b(\omega). \quad (2.7)$$

Transformations (2.6) and (2.7) make the Carroll group of symmetries of the string horizon. An introduction to the Carroll transformations can be found in [14,15]. The components of one-forms $\theta = \theta_a dx^a$ and vector fields $V = V^a \partial_a$ on $\mathcal{H}$, in a coordinate basis, change as

$$\theta_a(\mathbf{x}) = C^b_a \bar{\theta}_b(\bar{\mathbf{x}}), \quad V^a(\mathbf{x}) = C^a_b \bar{V}^b(\bar{\mathbf{x}}), \quad (2.8)$$

where $C^b_a = M^b_a$. Since $M^b_a = 0$ the matrix C^b_a is inverse of C^a_b . Forms like $\theta(V) = \theta_a V^a$ are invariant with respect to (2.8). It should be noted, however, that indices a, b cannot be risen or lowered since the metric of $\mathcal{H}$ is degenerate.

By following the method suggested in [11] one can construct a string space-time, which is locally flat but has the required holonomy on $\mathcal{S}$. One starts with $R^{1,3}$, which is decomposed into two parts: $u < 0$ and $u > 0$. Trajectories of particles and light rays at $u < 0$ and $u > 0$ can be called

ingoing and outgoing trajectories, respectively. To describe outgoing trajectories, one introduces two types of coordinate charts: R and L charts, with cuts on the horizon either on the left ($u = 0, y < 0$) or on the right ($u = 0, y > 0$). The initial data on the string horizon are related to the ingoing data via null rotations (2.3) taken at $u = 0$. For brevity the right ($u = 0, y > 0$) and the left ($u = 0, y < 0$) parts of $\mathcal{H}$ will be denoted as $\mathcal{H}_+$ and $\mathcal{H}_-$.

For the R charts the cut is along $\mathcal{H}_-$. If x^μ and $\bar{x}^\mu$ are, respectively, the coordinates above and below the horizon, then the transition conditions on $\mathcal{H}$ in the R charts look as

$$x^\mu = \bar{x}^\mu|_{\mathcal{H}_+} \quad \text{or} \quad x^a = \bar{x}^a, \quad (2.9)$$

$$x^\mu = M^\mu{}_\nu(\omega)\bar{x}^\nu|_{\mathcal{H}_-} \quad \text{or} \quad x^a = C^a{}_b\bar{x}^b. \quad (2.10)$$

Analogously, three components of 4-velocities of particles and light rays (with the lower indices) change on $\mathcal{H}_-$ as $u_a = C_a{}^b\bar{u}_b$. Change of the u_u component can be found by requiring the invariance of $u^\mu u_\mu$, which implies that u^μ experiences the four-dimensional null rotation on $\mathcal{H}_-$.

Coordinate transformations (2.10) are reduced to a linear supertranslation

$$v = \bar{v} + 2\omega y, \quad y < 0. \quad (2.11)$$

Hence the gravitational field of a null string is a particular example of a gravitational shockwave background. The method we follow is close to a more general approach suggested by Penrose [19].

As a result of the Lorenz invariance of the theory the descriptions based on R or L charts are equivalent. Without loss of the generality from now on we work with the R charts. In this case all field variables experience Carroll transformations on $\mathcal{H}_-$, and one needs to solve field equations in the domain $u > 0$ with the initial data changed on $\mathcal{H}_-$.

B. Characteristic problem for EM fields

Consider the Maxwell theory on the string space-time. Let $\bar{A}_\mu$ be a solution to the problem in the region $u < 0$,

$$\partial_\mu \bar{F}^{\mu\nu} = \bar{j}^\nu. \quad (2.12)$$

The currents at $u > 0$ and $u < 0$ are related on $\mathcal{H}$ as

$$j^\mu(\mathbf{x})|_{\mathcal{H}_+} = \bar{j}^\mu(\mathbf{x}), \quad j^\mu(\mathbf{x})|_{\mathcal{H}_-} = M^\mu{}_\nu(\omega)\bar{j}^\nu(\bar{\mathbf{x}}). \quad (2.13)$$

We need to solve (2.1) at $u > 0$ by setting the following initial conditions at $u = 0$:

$$A_b(x)|_{\mathcal{H}} = a_b(\mathbf{x}), \quad \bar{A}_b(x)|_{\mathcal{H}} = \bar{a}_b(\mathbf{x}) \quad (2.14)$$

$$a_b(\mathbf{x}) = \bar{a}_b(\mathbf{x})|_{\mathcal{H}_+}, \quad a_b(\mathbf{x}) = C_b{}^c(\omega)\bar{a}_c(\bar{\mathbf{x}})|_{\mathcal{H}_-}. \quad (2.15)$$

In the theory of hyperbolic second-order partial differential equations a problem is called the characteristic initial

value problem [20] if the initial data are set on a null hypersurface. In this case the number of initial data are twice less, and setting just a_b is enough to determine the solution (see discussion in [1]).

The variation of the action on the string space-time, if we take into account (2.1) and (2.12), has the form

$$\begin{aligned} \delta_A \left(-\frac{1}{4} \int \sqrt{g} d^4x F^{\mu\nu} F_{\mu\nu} + I[A] \right) \\ = \frac{1}{2} \int_{\mathcal{H}} d^3\mathbf{x} (\pi^b(\mathbf{x}) \delta a_b(\mathbf{x}) - \bar{\pi}^b(\mathbf{x}) \delta \bar{a}_b(\mathbf{x})), \end{aligned} \quad (2.16)$$

$$\pi^b = F^{ub}, \quad \bar{\pi}^b = \bar{F}^{ub}, \quad b = v, y, z. \quad (2.17)$$

Here $I[A]$ is the action of a charged matter, and $\pi^b, \bar{\pi}^b$ can be interpreted as canonical momenta conjugated with $a_b, \bar{a}_b$.

This variation vanishes given conditions (2.15) since the momenta transform under the Carroll group as components of a three-vector,

$$\pi^b(\mathbf{x}) = \bar{\pi}^b(\mathbf{x})|_{\mathcal{H}_+}, \quad \pi^b(\mathbf{x}) = C^b{}_c \bar{\pi}^c(\bar{\mathbf{x}})|_{\mathcal{H}_-}. \quad (2.18)$$

This transformation law follows, for example, from the constraints

$$\partial_b \pi^b = -j^u|_{\mathcal{H}}, \quad \partial_b \bar{\pi}^b = -\bar{j}^u|_{\mathcal{H}}, \quad (2.19)$$

and the fact that $j^u = \bar{j}^u$. The given arguments, of course, mean that the Maxwell strength is continuous across $\mathcal{H}$ on the string space-time.

In what follows we suppose that charged particles do not cross the left part of the horizon, that is $j^\mu = \bar{j}^\mu$ on $\mathcal{H}$. One can write a solution to (2.1), (2.14), and (2.15) in the form

$$A_\mu(x) = \bar{A}_\mu(x) + A_{S,\mu}(x). \quad (2.20)$$

Here $\bar{A}_\mu$ is an extension of the solution (2.12) to the region $u > 0$, and $A_{S,\mu}$ is a solution to a homogeneous characteristic initial value problem

$$\partial_\mu F_S^{\mu\nu} = 0, \quad A_{S,b}(x)|_{\mathcal{H}} = a_{S,b}(\mathbf{x}), \quad (2.21)$$

$$a_{S,b}(\mathbf{x})|_{\mathcal{H}_+} = 0, \quad a_{S,b}(\mathbf{x})|_{\mathcal{H}_-} = C_b{}^c(\omega)\bar{a}_c(\bar{\mathbf{x}}) - \bar{a}_b(\mathbf{x}), \quad (2.22)$$

where $F_{S,\mu\nu} = \partial_\mu A_{S,\nu} - \partial_\nu A_{S,\mu}$.

One can interpret $\bar{A}$ as a solution in the absence of the string, while A_S can be considered as a perturbation caused by the string. This perturbation:

- (i) vanishes in the limit $\omega \rightarrow 0$, where ω is the energy of the string,
- (ii) depends on the choice of the source j_μ through initial data (2.22), and

(iii) can be written as

$$A_{S,\mu}(x) = \mathcal{A}_\mu^\omega(x) - \mathcal{A}_\mu(x), \quad (2.23)$$

$$\mathcal{A}_\mu^\omega(x) = M_\mu^\nu(\omega) \mathcal{A}_\nu(\bar{x}), \quad x^\mu = M_\nu^\mu(\omega) \bar{x}^\nu, \quad (2.24)$$

where $\mathcal{A}_\mu(x)$ is a solution to homogeneous problem $\partial_\mu \mathcal{F}^{\mu\nu} = 0$ with the following initial data on $\mathcal{H}$:

$$\mathcal{A}_b(\mathbf{x})|_{\mathcal{H}_+} = 0, \quad \mathcal{A}_b(\mathbf{x})|_{\mathcal{H}_-} = \bar{a}_b(\mathbf{x}). \quad (2.25)$$

A geometrical interpretation of the perturbation is that at small ω it is the Lie derivative,

$$A_S = -\omega \mathcal{L}_\xi A, \quad (2.26)$$

generated by vector $\zeta^\mu = 2y\delta_v^\mu + u\delta_y^\mu$ associated to null rotations (2.3).

For the subsequent analysis, it is convenient to use the Lorentz gauge condition $\partial A = 0$ since it is invariant under null rotations and can be imposed globally on the cosmic string space-time. Thus, equations for the perturbations caused by the string are reduced to the problem

$$\square A_{S,a} = 0, \quad A_{S,a}(x)|_{\mathcal{H}} = a_{S,a}(\mathbf{x}), \quad (2.27)$$

with initial data (2.22). Component A_u is determined by condition $\partial A_S = 0$. The rest of the paper is devoted to studying solutions of (2.27) for different sources.

III. ASYMPTOTICS AND RADIATION ENERGY FLUX

A. Definitions

Perturbations $A_{S,\mu}$ will be considered for pointlike sources of electric and magnetic types. Without loss of the generality we suppose that the source is at rest at a point with coordinates $x_o = z_o = 0, y_o = a > 0$. Since the string trajectory is $x = t, y = 0$, we interpret a as an impact parameter between the string and the source. As we agreed, the source crosses $\mathcal{H}_+$, see Fig. 1.

The sources of the electric type are just electric charges. The corresponding current is

$$j_0 = e\delta^{(3)}(\vec{x} - \vec{x}_o), \quad j_i(x) = 0, \quad i = 1, 2, 3. \quad (3.1)$$

The field in the absence of the string is

$$\bar{A}_0(x) = e\phi(x), \quad \bar{A}_i(x) = 0, \quad (3.2)$$

$$\phi(x, y, z) = -\frac{1}{4\pi} \frac{1}{\sqrt{x^2 + (y-a)^2 + z^2}}. \quad (3.3)$$

The sources of the MD type are described by the current

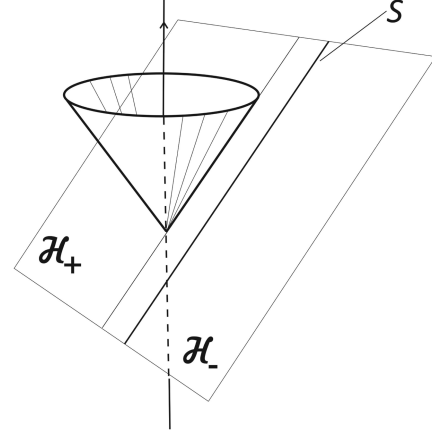


FIG. 1. Shows the world sheet of the string $\mathcal{S}$, the string horizon $\mathcal{H} = \mathcal{H}_+ \cup \mathcal{H}_-$, and a trajectory of the pointlike source (the dashed line) that crosses $\mathcal{H}_+$. At late times perturbation of the EM field of the source caused by the string propagates along a null cone, which is tangent to $\mathcal{H}$ and has the apex approximately at a point where the trajectory crosses the horizon.

$$j_0 = 0, \quad j_i(x) = \varepsilon_{ijk} M_j \partial_k \delta^{(3)}(\vec{x} - \vec{x}_o), \quad (3.4)$$

where M_i is a magnetic moment. The corresponding field in the absence of the string is

$$\bar{A}_0(x) = 0, \quad \bar{A}_i(x) = \varepsilon_{ijk} M_j \partial_k \phi, \quad (3.5)$$

which is the field of a magnetic dipole [21]. We consider (3.5) as an approximation for an EM field of a body with a magnetic moment. Under certain assumptions (3.5) can be used to describe the EM field of such objects as pulsars. Note that M_i in (3.5) has the dimension of length.

It is convenient to go from Minkowski coordinates (2.2) to retarded time coordinates

$$ds^2 = -dU^2 - 2dUdr + r^2 d\Omega^2, \quad (3.6)$$

where $U = t - r$ and $r = \sqrt{x^2 + y^2 + z^2}$. We denote coordinates θ, φ on the unit sphere by x^A , $d\Omega^2 = \gamma_{AB} dx^A dx^B = \sin^2 \theta d\varphi^2 + d\theta^2$. Our results will be expressed in terms of a unit vector $\vec{n}$ with components $n_x = x/r, n_y = y/r, n_z = z/r$. If $\vec{l}$ is a unit vector along the velocity of the string and $\vec{p}$ is another unit vector along the string axis ($l^i = \delta_x^i, p^i = \delta_z^i$), then $n_x = (\vec{n} \cdot \vec{l})$, $n_z = (\vec{n} \cdot \vec{p})$. We use coordinates (3.6) above the string horizon, $u > 0$. In this region $U > -r(1 - n_x)$. The region includes the domain $U > 0$.

As has been shown in [1] a null string moving near a pointlike charge disturbs its field and creates an outgoing EM-pulse traveling to the future null infinity $\mathcal{I}^+$, $r \rightarrow \infty$. The trajectory of the pulse is the light cone shown on Fig. 1.

The energy of the EM field inside the sphere of the radius R with the center at the point $x^i = 0$ is

$$E(R, t) = \int_{r < R} d^3x T_{00}, \quad (3.7)$$

where T_ν^μ is the stress-energy tensor of the EM field,

$$T_\nu^\mu = -F^{\mu\alpha} F_{\nu\alpha} + \delta_\nu^\mu \frac{1}{4} F^{\alpha\beta} F_{\alpha\beta}. \quad (3.8)$$

The energy density T_{00} is measured in the frame of reference where the charge is at rest. The conservation law implies that

$$\partial_t E(R, t) = R^2 \int d\Omega T_U^r. \quad (3.9)$$

We consider the energy flow at $r \gg a$, $r \gg U$. As is shown in the next section, in the given regime one has the following asymptotics, which hold for all angles Ω if $U > 0$:

$$A_{S,\mu}(r, U, \Omega) \simeq \frac{1}{r} (a_\mu(U, \Omega) + b_\mu(\Omega) \ln r/\varrho) + O(r^{-2} \ln r), \quad \mu = U, r, \quad (3.10)$$

$$A_{S,B}(r, U, \Omega) \simeq a_B(U, \Omega) + b_B(\Omega) \ln r/\varrho + O(r^{-1} \ln r), \quad (3.11)$$

where $A_{S,B}$ are components in the tangent space to S^2 , ϱ is a dimensional parameter related to the approximation. This yields

$$T_U^r = F_{U\mu} F^{r\mu} \simeq \frac{1}{r^2} \gamma^{AB} \dot{a}_A \dot{a}_B, \quad (3.12)$$

where $\dot{a}_A \equiv \partial_U a_A$, and guarantees a finite energy flux (3.9) at large r , for $U > 0$,

$$\lim_{R \rightarrow \infty} \partial_t E(R, t) = \int d\Omega \gamma^{AB} \dot{a}_A \dot{a}_B = \int d\Omega (\dot{\vec{a}}^2 - (\dot{\vec{a}} \cdot \vec{n})^2). \quad (3.13)$$

The rhs of (3.13) is given for components in Minkowski coordinates, $n^i = x^i/r$.

The calculation of (3.13) is our main goal. In next sections we derive the leading terms in (3.10) and (3.11) in an analytic form for sources of electric and magnetic types.

B. Master problem

As we will see, solutions to vector problem (2.27) can be generated by a solution of the following scalar problem:

$$\square \Phi_\omega(u, \mathbf{x}) = 0, \quad \Phi_\omega(0, \mathbf{x}) = \theta(-y) f_\omega(\mathbf{x}), \quad (3.14)$$

$$f_\omega(\mathbf{x}) \equiv f(\bar{\mathbf{x}}), \quad f(\mathbf{x}) \equiv \phi(v/2, y, z). \quad (3.15)$$

Here $\bar{\mathbf{x}}$ is defined in (2.6) and $\phi(x, y, z)$ in (3.3). Before we proceed it is helpful to discuss properties of $\Phi_\omega(x)$ in some detail. The key fact is the integral representation found in [1]:

$$\Phi_\omega(x) = -C \int_{S^2} d\Omega' \Re \left(\frac{\tilde{\Phi}_\omega(\Omega')}{x^\mu m_\mu(\Omega') + i a \varepsilon(\Omega')} \right), \quad \tilde{\Phi}_\omega(\Omega') \equiv \frac{\cos \varphi'}{g(\Omega', \omega)}, \quad (3.16)$$

where $C = 1/(8\pi^3)$. The integration goes over a unit sphere S^2 , with coordinates $\Omega' = (\theta', \varphi')$, $d\Omega = \sin \theta' d\theta' d\varphi'$. Other notations are

$$m_u = 1 - \sin^2 \theta' \cos^2 \varphi', \quad m_v = \sin^2 \theta' \cos^2 \varphi', \\ m_y = \sin 2\theta' \cos \varphi', \quad m_z = \sin^2 \theta' \sin 2\varphi', \quad (3.17)$$

$$g(\Omega', \omega) = e^{i\theta'} + \omega \sin \theta' \cos \varphi', \quad \varepsilon(\Omega') = 2 \sin^2 \theta' \cos \varphi'. \quad (3.18)$$

One can check that vector field m_μ is null, $m^2 = 0$, which guarantees that $\square \Phi_\omega = 0$.

Let $\Phi(x)$ be a solution to (3.14) and (3.15) for $\omega = 0$. The initial data for Φ_ω are obtained by the Carroll transformation of initial data for Φ . Let us show that this transformation generates the null rotation of the solution, that is

$$\Phi_\omega(x) = \Phi(\bar{x}), \quad x = M(\omega) \bar{x}. \quad (3.19)$$

To this aim we introduce a unit vector on S^2

$$l_x = \sin \theta' \cos \varphi', \quad l_y = \cos \theta', \quad l_z = \sin \theta' \sin \varphi', \quad (3.20)$$

and express quantities under the integral in (3.16) as

$$m_0 = 1, \quad m_x = 2l_x^2 - 1, \quad m_y = 2l_x l_y, \\ m_z = 2l_x l_z, \quad \varepsilon = 2l_x \sqrt{1 - l_y^2}, \quad (3.21)$$

where $m_0 = m_v + m_u$, $m_x = m_v - m_u$. According to (3.16)

$$\Phi(\bar{x}) = \Phi_{\omega=0}(\bar{x}) = -C \int_{S^2} d\Omega' \Re \left(\frac{\tilde{\Phi}(\Omega')}{x^\mu m'_\mu + i a \varepsilon} \right), \quad (3.22)$$

where $m' = M(\omega)m$ and

$$\tilde{\Phi} = \tilde{\Phi}_{\omega=0} = \frac{l_x}{\sqrt{1 - l_y^2} (l_y + i \sqrt{1 - l_y^2})}. \quad (3.23)$$

Vector m' can be further transformed to $\tilde{m} = m'/S$,

$$\tilde{m}_0 = 1, \quad \tilde{m}_x = 2\tilde{l}_x^2 - 1, \quad \tilde{m}_y = 2\tilde{l}_x\tilde{l}_y, \quad \tilde{m}_z = 2\tilde{l}_x\tilde{l}_z, \quad (3.24)$$

$$\begin{aligned} \tilde{l}_x &= S^{-1/2}l_x, & \tilde{l}_y &= S^{-1/2}(l_y - \omega l_x), \\ \tilde{l}_z &= S^{-1/2}l_z, & S &= 1 - 2\omega l_x l_y + \omega^2 l_x^2. \end{aligned} \quad (3.25)$$

This yields

$$\frac{\tilde{\Phi}}{(x')^\mu m'_\mu + ia\varepsilon} = S^{-3/2} \frac{\tilde{\Phi}_\omega}{(x')^\mu \tilde{m}_\mu + ia\tilde{\varepsilon}}, \quad (3.26)$$

where $\tilde{\Phi}_\omega, \tilde{\varepsilon}$ are determined in terms of $\tilde{l}_i$. Then (3.19) easily follows from (3.22) and (3.23) if one changes the integration measure.

C. Radiation from an electric charge

For an electric source perturbation $A_{S,b}$ is a solution to (2.27) with

$$\bar{a}_b(\mathbf{x}) = \frac{1}{2} f(\mathbf{x}) \delta_b^v, \quad (3.27)$$

where $f(\mathbf{x})$ is defined in (3.15). Equation (3.27) is the consequence of (3.2) and (3.3). As is easy to see, $A_{S,b}$ is generated by the solution to scalar problem (3.14) in the following way:

$$\begin{aligned} A_{S,v}(x) &= \frac{e}{2} (\Phi_\omega(x) - \Phi(x)), & A_{S,y}(x) &= -e\omega\Phi_\omega(x), \\ A_{S,z}(x) &= 0. \end{aligned} \quad (3.28)$$

The rest component, $A_{S,u}$, is determined by the gauge condition $\partial A_S = 0$. The solution at $u > 0$ can be represented as [1]

$$A_{S,\mu}(x) = -eC \int_{S^2} d\Omega' \Re \left[\frac{\beta_\mu(\Omega')}{x^\nu m_\nu(\Omega') + ia\varepsilon(\Omega')} \right]. \quad (3.29)$$

The new notations used in (3.29) are

$$\begin{aligned} \beta_v &= -\frac{1}{2} \cos \varphi' (g^{-1}(\Omega', \omega) - g^{-1}(\Omega', 0)), \\ \beta_y &= \cos \varphi' \omega g^{-1}(\Omega', \omega), & \beta_z &= 0, \end{aligned} \quad (3.30)$$

$$\beta_u = \frac{m_y \beta_y - 2m_u \beta_v}{2m_v}, \quad (3.31)$$

with $g(\Omega', \omega), \varepsilon(\Omega')$ defined in (3.18). The gauge condition, which results in (3.31), allows residual gauge transformations. So solution to β_u is not unique. This arbitrariness, however, does not affect physical observables.

In coordinates U, r, x^A , see (3.6), the denominator in the integral in (3.29) can be written as

$$x^\nu m_\nu + ia\varepsilon = U + r((\vec{m} \cdot \vec{n}) + 1) + ia\varepsilon.$$

Since we are interested in a large r asymptotic, the integration in (3.29) can be decomposed into two parts: the integration over a domain, where the factor $(\vec{m} \cdot \vec{n}) + 1$ is small, that is, $\vec{m}$ is almost $-\vec{n}$, and the integration over the rest part of S^2 . If we introduce a dimensionless parameter Λ such that

$$\frac{\sqrt{U^2 + a^2}}{r} \ll \Lambda^2 \ll 1, \quad (3.32)$$

then the first region can be defined as $(\vec{m} \cdot \vec{n}) + 1 \leq \Lambda^2$ and the second as $(\vec{m} \cdot \vec{n}) + 1 > \Lambda^2$. Contributions from these regions to A_S will be denoted as A_1 and A_2 , respectively,

$$A_S = A_1 + A_2. \quad (3.33)$$

After some algebra one gets the following estimates at large r [for components in coordinates (2.2)]:

$$A_{1,\mu}(r, U, \Omega) \simeq \frac{eN(\Omega)}{r} \Re \left[\bar{\beta}_\mu \ln \left(\frac{U + ia\tilde{\varepsilon} + r\Lambda^2}{U + ia\tilde{\varepsilon}} \right) \right], \quad (3.34)$$

$$A_{2,\mu}(r, U, \Omega) \simeq \frac{b_{2,\mu}(\Omega, \Lambda)}{r}, \quad (3.35)$$

$$\bar{\beta}_\mu = \beta_\mu|_{\vec{m}=-\vec{n}}, \quad \tilde{\varepsilon} = \varepsilon|_{\vec{m}=-\vec{n}}, \quad (3.36)$$

$$N(\Omega) = \frac{1}{4\pi^2} \sqrt{\frac{2}{1-n_x}}, \quad (3.37)$$

$$b_{2,\mu}(\Omega, \Lambda) = -eC \int_{S_\Lambda^2} d\Omega' \cos \varphi \Re \left[\frac{\beta_\mu(\Omega')}{(\vec{m}(\Omega') \cdot \vec{n}) + 1} \right], \quad (3.38)$$

where S_Λ^2 is a part of S^2 with the restriction $(\vec{m} \cdot \vec{n}) + 1 > \Lambda^2$. Arguments which lead to (3.34) can be found in Appendix A. Calculation of (3.35) is trivial.

As a result of (3.34) and (3.35) the solution at large r is a sum of a static U independent part and a dynamical part. The dynamical part presents only in A_1 , and it is the only part which contributes to the flux (3.13). One can show that formula (3.34) is in a good agreement with numerical simulations for $n_x \neq 1$, so it provides a remarkable analytic tool to compute different characteristics of the EM field at future null infinity. We come to the following expression which leads to Eq. (1.1) announced in Sec. I:

$$A_{S,\mu}(r, U, \Omega) \simeq \frac{a_\mu(U, \Omega)}{r} + \frac{b_\mu(\Omega)}{r} \ln \varrho/r, \quad (3.39)$$

$$a_\mu(U, \Omega) = -eN(\Omega) \Re \left[\bar{\beta}_\mu \ln \left(\frac{U + ia\tilde{\varepsilon}}{a} \right) \right] + b_{2,\mu}(\Omega, \Lambda), \quad (3.40)$$

$$b_\mu(\Omega, \Lambda) = eN(\Omega) \Re \bar{\beta}_\mu(\Omega), \quad (3.41)$$

where $\varrho = a/\Lambda^2$.

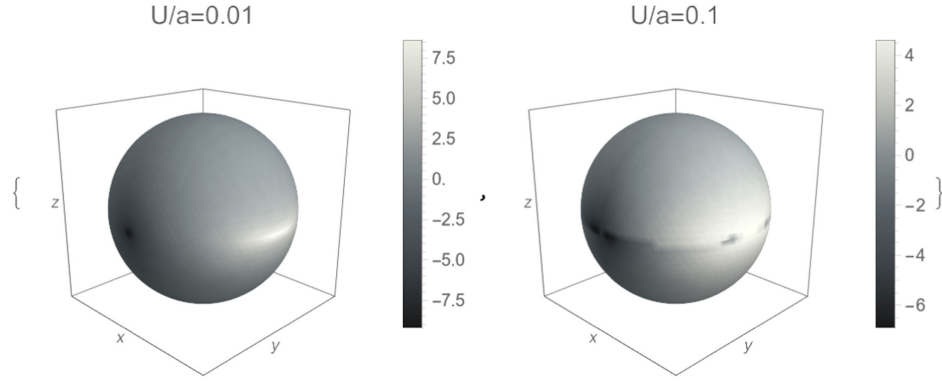


FIG. 2. The flux density [divided by $e^2\omega^2/((2\pi)^4 a^2)$] from an electric source. The flux is given in the logarithmic scale in 4π geometry for $U = 0.01, 0.1$ and impact parameter $a = 1$. The radiation has a form of EM burst which is directed mostly toward the velocity of the string and rapidly decays with increasing of U .

The power of the radiation is given by (3.13). The dynamical part is determined by $\tilde{\beta}_\mu$, with the condition $\tilde{\beta}^\mu \tilde{m}_\mu = 0$. It implies that $\dot{a}_0 = -(\dot{\vec{a}} \cdot \vec{n})$ and

$$\dot{E} = \int d\Omega \gamma^{AB} \dot{a}_A \dot{a}_B = \int d\Omega (\dot{a}_\mu \eta^{\mu\nu} \dot{a}_\nu) \equiv \int d\Omega f_E(U, \Omega), \quad (3.42)$$

where $\eta_{\mu\nu}$ is the flat metric. If we express $\dot{a}_v$ through the rest components by using the gauge condition $\dot{a}_\mu m^\mu = 0$, where $m^\mu = -(1 - n_x)$, $m^v = -(1 + n_x)$, $m^y = -n_y$, then the flux density (intensity of the radiation) takes the form

$$f_E(U, \Omega) = \dot{a}_\mu \eta^{\mu\nu} \dot{a}_\nu = \frac{4n_y \dot{a}_y \dot{a}_v + 4(1 + n_x) \dot{a}_v^2 + (1 - n_x) \dot{a}_y^2}{1 - n_x}. \quad (3.43)$$

Unit vector $\vec{n}$ is introduced after Eq. (3.6). The functions $\dot{a}$ are expressed in terms of real and imaginary parts of $\tilde{\beta}_\mu$,

$$\dot{a}_\mu = -\frac{eN(\Omega)}{U^2 + a^2 \bar{\epsilon}^2} [U \Re \tilde{\beta}_\mu + a \bar{\epsilon} \Im \tilde{\beta}_\mu], \quad (3.44)$$

see (3.40). We are interested in the limit when the string energy is small, $\omega \ll 1$. With the help of (A5)–(A7) one gets in the linear approximation in ω ,

$$\begin{aligned} \Re \tilde{\beta}_y &\simeq -\frac{\omega n_y \sqrt{1 - n_x}}{\sqrt{2} \bar{\epsilon}}, & \Re \tilde{\beta}_v &\simeq \frac{\omega}{2\sqrt{2}} \frac{\sqrt{1 - n_x}}{\bar{\epsilon}} (n_y^2 - (1 - n_x)), \\ \Im \tilde{\beta}_y &\simeq -\frac{\omega}{\sqrt{2}} \sqrt{1 - n_x}, & \Im \tilde{\beta}_v &\simeq \frac{\omega n_y}{2\sqrt{2}} \sqrt{1 - n_x}. \end{aligned} \quad (3.45)$$

Substitution of (3.44) and (3.45) into (3.43) yields

$$\begin{aligned} f_E(U, \Omega) &\simeq \frac{e^2 \omega^2}{(2\pi)^4 a^2} \frac{1}{[U^2/a^2 + \bar{\epsilon}^2]^2} \\ &\times \left\{ \frac{1 - n_x^2 - n_y^2 (1 - \bar{\epsilon}^2)}{\bar{\epsilon}^2} \frac{U^2}{a^2} \right. \\ &\left. + 2n_y (1 - n_x - n_y^2) \frac{U}{a} + (1 - n_y^2) \bar{\epsilon}^2 \right\}, \end{aligned} \quad (3.46)$$

where $\bar{\epsilon}^2 = 2(1 - n_x) - n_y^2$, $n_x = \cos \theta$, $n_y = \sin \theta \sin \varphi$. We consider (3.46) at $U > 0$.

For $U/a \gg 1$ the flux density vanishes as $(U/a)^{-2}$. It follows from (3.40) and numerical simulations in [1] that the peak power is near $U = 0$. Such behavior is demonstrated in Fig. 2 which shows an angular distribution of the flux (intensity of the flux) at different moments of U . For small U the energy flux is focused in the direction of string motion. For $n_x \rightarrow \pm 1$ and finite U ,

$$\begin{aligned} f_E(U, \Omega)_{n_x \rightarrow 1} &= \frac{e^2 \omega^2}{(2\pi)^4 U^2}, \\ f_E(U, \Omega)_{n_x \rightarrow -1} &= \frac{e^2 \omega^2}{16\pi^4 (4 + U^2)^2}. \end{aligned} \quad (3.47)$$

At $U = 0$, expression (3.46) takes the simple form

$$f_E(0, \Omega) = \frac{e^2 \omega^2}{(2\pi)^4 a^2} \frac{(1 - n_y^2)}{2(1 - n_x) - n_y^2}. \quad (3.48)$$

It has a polelike singularity at $n_x = 1$, where our approximation is not applicable. Note that at $U = 0$ this point lies exactly on the string world sheet $\mathcal{S}$.

To better illustrate the intensity of the flux, we present in Fig. 3 polar radiation plots, which are common in antenna physics. They refer to the directional (angular) dependence of the flux density of the EM-waves coming from an electric source. The flux density (3.46) depends on two angles and retarded Bondi coordinate U . We fix the

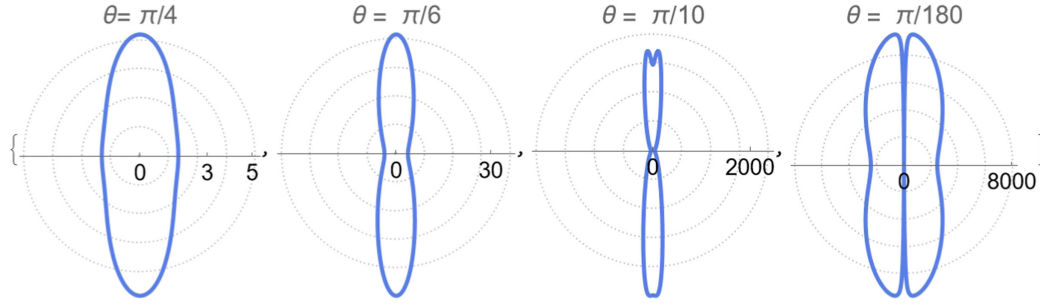


FIG. 3. Polar radiation plots (far-field patterns) in the planes $\theta = \pi/4, \pi/6, \pi/10, \pi/180$. The flux density is depicted for $U/a = 0.01$ when $e^2\omega^2/((2\pi)^4 a^2) = 1$. Angles θ and φ are defined by (A1). Angle φ changes clockwise. The intensity of the flux grows in the direction of the string velocity as $\theta \rightarrow 0$. The flux is orthogonal to the axis of the string, which corresponds to $\varphi = 0, \pi$.

distance from the source, $U = 0.01$, and one of the angles, θ , and draw the flux density in polar coordinates. Then the polar radiation graphs in Fig. 3 show the dependence of the flux density on the angle φ in different planes $\theta = \text{const}$ and $e^2\omega^2/((2\pi)^4 a^2) = 1$. We return to an estimate of the flux from a null cosmic string and a gas of charged particles in Sec. IV.

D. Radiation from a magnetic-dipole source

The case of pointlike magnetic-dipole sources has not been studied in [1]. Therefore, we consider this case in more detail by taking into account results of [1]. It follows from (3.5) that incoming data for problem (2.27) for a magnetic-dipole source are

$$\bar{a}_b(\mathbf{x}) = \mathcal{D}_b f(\mathbf{x}), \quad b = v, y, z, \quad (3.49)$$

where $f(\mathbf{x})$ is defined in (3.15), and

$$\begin{aligned} 2\mathcal{D}_v &= (M_y \partial_z - M_z \partial_y), \quad \mathcal{D}_y = -M_x \partial_z + 2M_z \partial_v, \\ \mathcal{D}_z &= M_x \partial_y - 2M_y \partial_v. \end{aligned} \quad (3.50)$$

If we introduce the operators,

$$\mathcal{D}_b^\omega = \mathcal{D}_b + \omega(-M_z \delta_b^v + 2M_x \delta_b^z) \partial_v, \quad (3.51)$$

then, according to (2.22), the initial data for perturbations induced by the string can be written as

$$a_b(\mathbf{x}) = \theta(-y)(\mathcal{D}_b^\omega f_\omega(\mathbf{x}) - 2\omega \delta_b^v \mathcal{D}_v^\omega f_\omega(\mathbf{x}) - \mathcal{D}_b f(\mathbf{x})). \quad (3.52)$$

Here $f_\omega(\mathbf{x})$ is introduced in (3.15) and operators $\mathcal{D}_b^\omega$ are obtained from $\mathcal{D}_b$ by replacing ∂_y to $\partial_y + 2\omega \partial_v$ since $(\partial_y + 2\omega \partial_v)f_\omega(\mathbf{x}) = \partial_y f(\mathbf{x})_{\mathbf{x}=\bar{\mathbf{x}}}$.

Let $\Psi_b^\omega(u, \mathbf{x})$ be a solution at $u > 0$ of the following problem:

$$\square \Psi_b^\omega(u, \mathbf{x}) = 0, \quad \Psi_b^\omega(0, \mathbf{x}) = \theta(-y) \mathcal{D}_b^\omega f_\omega(\mathbf{x}), \quad b = v, y, z. \quad (3.53)$$

Then the solution to (2.27) for the magnetic-dipole source has the components

$$A_{S,b}(x) = \Psi_b^\omega(x) - \Psi_b(x) - 2\omega \delta_b^v \Psi_v^\omega(x), \quad (3.54)$$

where $\Psi_b(x) = \Psi_b^{\omega=0}(x)$. To proceed we consider another auxiliary problem,

$$\square \chi(u, \mathbf{x}) = 0, \quad \chi(0, \mathbf{x}) = \delta(y) f(\mathbf{x}). \quad (3.55)$$

Given solutions to (3.14) and (3.55) one can easily construct the solution,

$$\tilde{\Psi}_b^\omega(x) = \partial_b \Phi_\omega + \delta_b^v \chi(x), \quad (3.56)$$

to the problem,

$$\square \tilde{\Psi}_b^\omega(u, \mathbf{x}) = 0, \quad \tilde{\Psi}_b^\omega(0, \mathbf{x}) = \theta(-y) \partial_b f_\omega(\mathbf{x}), \quad (3.57)$$

and represent Ψ_b^ω as

$$\Psi_b^\omega(x) = \mathcal{D}_b^\omega \Phi_\omega(x) + (M_x \delta_b^z - 2M_z \delta_b^v) \chi(x). \quad (3.58)$$

Now the final result for components can be written with the help of (3.54) and (3.58) in the form

$$\begin{aligned} A_{S,a}(x) &= \mathcal{D}_a^\omega \Phi_\omega(x) - \mathcal{D}_a \Phi(x) - 2\omega (\mathcal{D}_v^\omega \Phi_\omega(x) \\ &\quad - 2M_z \chi(x)) \delta_y^a, \end{aligned} \quad (3.59)$$

where $\Phi(x) = \Phi_{\omega=0}(x)$. Equation (3.59) is our starting point for computations of the flux.

By taking into account (3.59) and the gauge conditions perturbation of the vector potential of the magnetic source caused by the null string can be written as

$$A_{S,\nu}(x) = A_{1,\nu}(x) + A_{2,\nu}(x). \quad (3.60)$$

By its structure $A_{1,\nu}(x)$ is analogous to potential (3.29) for an electric source,

$$A_{1,\nu}(x) = C \int_{S^2} d\Omega' \Re \left\{ \frac{\alpha_\nu(\Omega')}{(x^\mu m_\mu + ia\epsilon)^2} \right\}, \quad (3.61)$$

$$\alpha_a(\Omega) = m_\mu ((\mathcal{D}_a^\omega x^\mu) - 2\omega \delta_a^y (\mathcal{D}_v^\omega x^\mu)) \tilde{\Phi}_\omega - m_\mu (\mathcal{D}_a x^\mu) \tilde{\Phi}, \quad a = v, y, z \quad (3.62)$$

$$\alpha_u(\Omega) = \frac{m_y \alpha_y + m_z \alpha_z - 2m_u \alpha_v}{2m_v}, \quad (3.63)$$

where $\tilde{\Phi}_\omega(\Omega) = \cos \varphi g^{-1}(\Omega, \omega)$, see (3.16). The rest part of the solution presents only if $M_z \neq 0$ and can be written as

$$A_{2,\nu}(x) = 2\omega M_z (2\delta_v^y \partial_v + \delta_v^u \partial_y) \tilde{\chi}. \quad (3.64)$$

Here $\partial_v \tilde{\chi} = \chi$. Properties of function χ are studied in detail in Appendix B. The u component of A_μ in (3.63) and (3.64) is determined by the gauge condition $\partial A = 0$. As has been pointed out the gauge condition fixes the solution up to residual gauge transformations, which do not change physical observables, like the energy flux.

With the help of (3.16) one finds the asymptotic form of (3.61) for $n_x \neq 1$,

$$A_{1,\mu}(x) \simeq \frac{N(\Omega)}{r} \Re \left(\frac{\bar{\alpha}_\mu}{U + ia\bar{\epsilon}} \right), \quad (3.65)$$

$$\bar{\alpha}_\mu = \alpha_\mu|_{\vec{m}=-\vec{n}}. \quad (3.66)$$

Function N is defined in (3.37) and, as earlier, $\bar{\epsilon}^2 = 2(1 - n_x) - n_y^2$. Components of $\bar{\alpha}_\mu$ follow from (3.62)–(3.63) where one has to replace $\tilde{\Phi}_\omega$ to

$$\bar{\Phi}_\omega = \frac{\sqrt{2(1 - n_x)^3}}{\bar{\epsilon}(-n_y + \omega(1 - n_x) + i\bar{\epsilon})}. \quad (3.67)$$

Derivation of (3.65) can be performed along the lines of the electric case elaborated on in Appendix A. Asymptotic of χ at large r , which is derived in Appendix B, is

$$\bar{\chi}(x) = \frac{1}{2\pi^2 r} \left[\frac{1}{4\bar{\epsilon}} \ln \left(\frac{\sqrt{U^2 + a^2 \bar{\epsilon}^2}}{2Lr} \right) - X(\Omega) \right]. \quad (3.68)$$

Like in (3.42), we calculate the density of the energy flux, or the intensity, $f_M(U, \Omega)$,

$$\dot{E} \equiv \int d\Omega f_M(U, \Omega), \quad (3.69)$$

$$f_M(U, \Omega)$$

$$= \frac{4(n_y \dot{a}_y \dot{a}_v + n_z \dot{a}_z \dot{a}_v + (1 + n_x) \dot{a}_v^2) + (1 - n_x)(\dot{a}_y^2 + \dot{a}_z^2)}{(1 - n_x)}, \quad (3.70)$$

where $\dot{a}_\mu$ is defined by the asymptotic $\dot{A}_\mu(x) \simeq \dot{a}_\mu/r$ at large r . The difference of this expression, with respect to the case of the electric source, is in nonvanishing component $\dot{a}_z$. After some algebra one arrives at the following result valid at small ω :

$$\dot{a}_b \simeq -\frac{\omega}{8\pi^2 a^2 \bar{\epsilon}} \frac{\sigma_i \zeta_b^i}{((U/a)^2 + \bar{\epsilon}^2)^2}, \quad i = 1, 2, 3, \quad (3.71)$$

$$\sigma_1 = (n_y^2 - \bar{\epsilon}^2)((U/a)^2 - \bar{\epsilon}^2) + 4\bar{\epsilon}^2 n_y (U/a) \quad (3.72)$$

$$\sigma_2 = 2\bar{\epsilon}(n_y((U/a)^2 - \bar{\epsilon}^2) - (n_y^2 - \bar{\epsilon}^2)(U/a)) \quad (3.73)$$

$$\sigma_3 = (U/a)^2 - \bar{\epsilon}^2 \quad (3.74)$$

$$\zeta_v^1 = \frac{1}{2} M_y n_z, \quad \zeta_z^1 = M_y (1 - n_x),$$

$$\zeta_y^1 = -(M_x n_z + M_z (1 - n_x)) - \frac{n_y}{1 - n_x} (M_y n_z - M_z n_y),$$

$$\zeta_v^2 = \frac{\bar{\epsilon}}{2} M_z, \quad \zeta_z^2 = -\bar{\epsilon} M_x, \quad \zeta_y^2 = -\frac{\bar{\epsilon}}{1 - n_x} (M_y n_z - M_z n_y);$$

$$\zeta_b^3 = 2M_z (1 - n_x) \delta_b^y.$$

Here the presence of ζ_b^3 in (3.75) is related to the part of the solution defined in (3.64). The intensity of the radiation $f_M(U, \Omega)$ can be obtained with the help of (3.70) and (3.71).

Similarly to the case of the electric source $f_M(U, \Omega)$ is distributed predominantly in the direction of the string motion, however, now it depends on the direction of the magnetic moment $\vec{M}$ with respect to the string. Figure 4 shows the angular distribution of the flux density over a unit sphere for different directions of $\vec{M}$ in logarithmic scale. The highest flux density is achieved when the magnetic moment is aligned with the sting, $M = M_z$; the weakest effect is observed when the magnetic moment is orthogonal both to the string and to its velocity, $M = M_y$.

For illustrative purposes we give concrete expressions of the flux for some special cases at small ω . At finite $U > 0$ one can conclude that in the given approximation the flux in the direction $n_x = 1$ tends to zero. This direction corresponds to an observer with coordinates $x_o = r, y_o = z_o = 0$, that is the observer between the string and the source on trajectory of the string. It should be noted that at $n_x = 1$ our approximation (B20) is violated.

Another observer on the same trajectory positioned such that the source is between the observer and the string, with

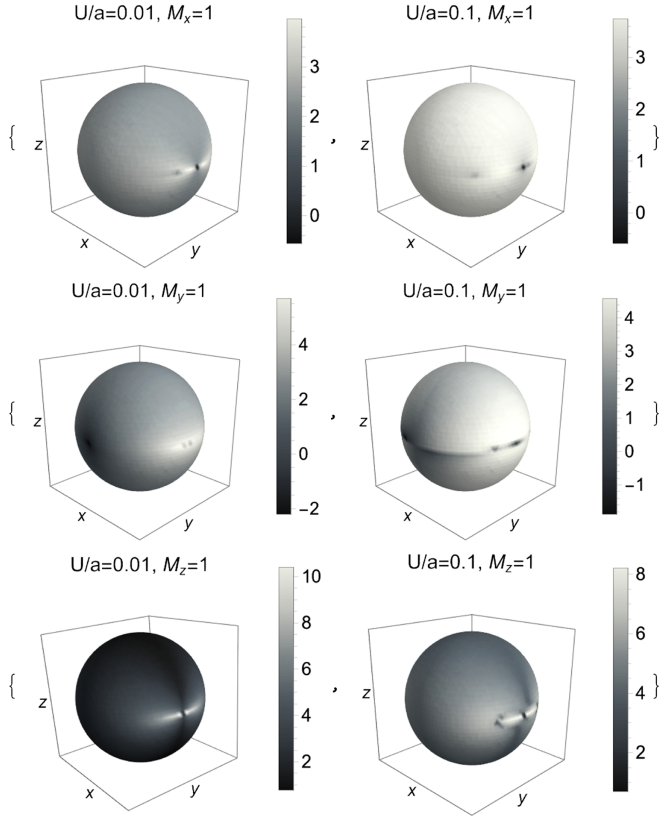


FIG. 4. Shows the flux density [divided by $\omega^2/(64\pi^4 a^4)$] from a magnetic-dipole source for $U/a = 0.1$ and $U/a = 0.01$, $|M| = 1$ in logarithmic scale. Three different directions of the magnetic moment are chosen: toward the string velocity, $M_x = 1$, orthogonally to the string and its velocity, $M_y = 1$, and along the string, $M_z = 1$.

coordinates $x_o = -r, y_o = z_o = 0$, sees the flux which depends on the direction of $\vec{M}$:

$$\vec{M} = (0, 0, M) \quad f_M(U) = \frac{9 \omega^2 M^2 (U^2/a^2 - 4)^2}{16 \pi^4 a^4 (U^2/a^2 + 4)^4}, \quad (3.75)$$

$$\vec{M} = (0, M, 0) \quad f_M(U) = \frac{1 \omega^2 M^2 (U^2/a^2 - 4)^2}{4 \pi^4 a^4 (U^2/a^2 + 4)^4}, \quad (3.76)$$

$$\vec{M} = (M, 0, 0) \quad f_M(U) = \frac{4 \omega^2 M^2}{\pi^4 a^4} \frac{U^2/a^2}{(U^2/a^2 + 4)^4}. \quad (3.77)$$

If $U = 0$, and $n_x \neq 1$, then one finds for $\vec{M} = (0, 0, M)$,

$$f_M(0, \Omega) = \frac{\omega^2 M^2}{16 \pi^4 a^4 \epsilon^6} \{ (2 - n_x)^2 (1 - n_x)^2 + 2(1 - n_x)(6 + 2n_x - n_x^2)n_y^2 + (n_x^2 + 2n_x - 8)n_y^4 + n_y^6 \}; \quad (3.78)$$

for $\vec{M} = (0, M, 0)$,

$$f_M(0, \Omega) = \frac{\omega^2 M^2}{16 \pi^4 a^4 \epsilon^2} (1 - n_y^2); \quad (3.79)$$

and for $\vec{M} = (M, 0, 0)$,

$$f_M(0, \Omega) = \frac{\omega^2 M^2}{16 \pi^4 a^4 \epsilon^6} (1 - n_x)^2 (1 - n_x^2 + (1 - 2n_x)n_y^2 - n_y^4). \quad (3.80)$$

At $n_x \rightarrow 1$ the flux density diverges since the observer intersects trajectory of the string.

For the case of maximal intensity, when the magnetic moment is parallel to the string axis, $M = M_z = 1$, the polar radiation plot of the flux density divided by $\omega^2/(64\pi^4 a^4)$ is shown in Fig. 5 for $U/a = 0.1$. The angle ϕ changes from 0 to 2π clockwise. The direction along the string velocity (along the x axes) is $\phi = \pi/2$. The plots are given for different angular positions of the observer: $\theta = \pi/3, \pi/6, \pi/8, \pi/9$. As θ decreases the plot is stretched along the x axis. Thus, the radiation is directed toward the motion of the string. Note that at $U/a = 0.01$ (the value at which similar graphs are plotted in the case of an electric source), the graphs stretch in the direction of the string velocity by several orders of magnitude.

IV. TOWARD EXPERIMENTAL OBSERVATIONS OF EM RADIATION GENERATED BY NULL STRINGS

To understand at which energies of null strings the pulses can be potentially observable we consider the power of the generated radiation. This power can be considered as the luminosity of the system which consists of the charge and the null string. We take $0 < U/a \ll 1$ for the peak power. As the preceding analysis shows, the flux densities are maximal at these values.

Let us start with the radiation from electrically charged sources. Formulae (3.42) and (3.46) yield the crude estimate for the luminosity:

$$\dot{E} \sim \frac{e^2}{(2\pi)^4} \frac{\omega^2}{a^2}. \quad (4.1)$$

There is an upper limit on the tension of tensile strings $G\mu \leq 10^{-7}$, which follows from cosmic microwave background (CMB) [22–24]. Since the effects of null strings on the CMB spectrum are similar to those of the tensile strings [11] we use the energy parameter $\omega_0 = 1 \times 10^{-7}$ as some reference value to proceed with computations. With the help of this parameter the peak luminosity of the radiation emitted by N charged particles can be written as

$$\dot{E} \sim 0.6 \left(\frac{\omega}{\omega_0} \right)^2 \left(\frac{1 \text{ cm}}{a} \right)^2 \frac{N}{N_A} \frac{\text{erg}}{\text{s}}. \quad (4.2)$$

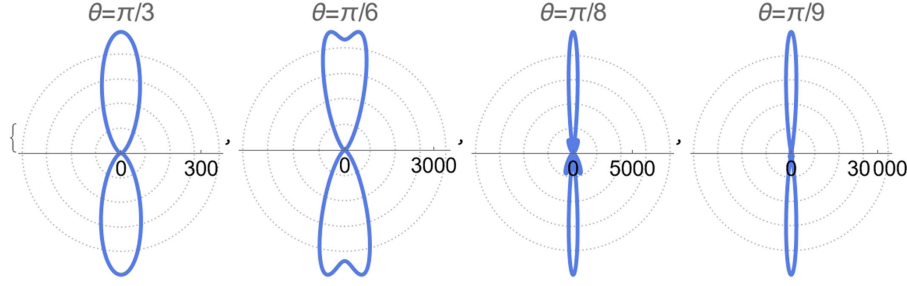


FIG. 5. Shows polar radiation plots (far-field patterns) in the planes $\theta = \pi/3, \pi/6, \pi/8, \pi/9$ for a MD source, for $U/a = 0.1$, when $e^2\omega^2/((2\pi)^4 a^2) = 1$. The magnetic moment is directed along the string. Angles θ and φ are defined as for Fig. 3, φ changes clockwise. Like in the case of the electric source the intensity of the flux grows in the direction of the string velocity $\theta = 0$. The flux is orthogonal to the axis of the string.

Here N_A is the Avogadro number, and we assumed that $e^2/(4\pi) \simeq 1/137$.

Suppose a null string with energy $\omega = \omega_0$ moves through 1 mole of ionized hydrogen atoms being under normal conditions. For the mole volume 14 cm^3 one can assume $a \simeq 1 \text{ cm}$ and get from (4.2) the estimate

$$\dot{E} \sim 1 \frac{\text{erg}}{\text{s}} = 1 \times 10^{-7} \text{ W}. \quad (4.3)$$

Corrections to (4.2) related to motion of particles with velocity v is negligible since under normal conditions $v/c \sim 10^{-6}$. Power (4.3) is extremely small. It is just 2 orders of magnitude larger than the power of radiation of a star which goes through 1 square meter of the Earth's surface. Thus, radiation from charged sources generated by null cosmic strings may have rather theoretical interests.

Consider now the radiation from magnetic-dipole sources. Equations (3.70) and (3.75) yield the following estimate for the luminosity at small U :

$$\dot{E} \sim \frac{1}{(2\pi)^4} \frac{\omega^2 M^2}{a^4}. \quad (4.4)$$

To proceed we need a physical source with a large magnetic moment M whose field can be approximated by magnetic-dipole potential (3.5). As such sources we consider pulsars, whose electromagnetic fields in a near zone [25] have form (3.5). The near zone of a pulsar is defined by distances from the source $r < c/\Omega$, where Ω is the pulsar rotation frequency. Say, if we take $\Omega \sim 200 \text{ s}^{-1}$, the near zone is $r < r_0 = 3.29 \times 10^2 \text{ km}$.

A typical value of the surface magnetic field of a pulsar is $B_0 \sim 10^{12} \text{ G}$, which corresponds to a magnetic moment $M_0 = 4.17 \times 10^{30} \text{ erg/G} = 3.7 \times 10^{52} \text{ GeV}^{-1}$, see [26]. One can write (4.4) roughly as

$$\dot{E} \sim 3.4 \times 10^{21} \left(\frac{\omega}{\omega_0} \right)^2 \left(\frac{r_0}{a} \right)^4 \left(\frac{B}{B_0} \right)^2 \text{ W}, \quad (4.5)$$

where B is the surface magnetic field of the pulsar, and B is assumed to be proportional to M . According to (4.5) a null

string passing a pulsar in the near zone at a distance $a = 10^2 \text{ km}$, generates an EM pulse with the peak power

$$\dot{E} \sim 10^{21} \left(\frac{\omega}{\omega_0} \right)^2 \text{ W}. \quad (4.6)$$

For $\omega = \omega_0$ (4.6) is still extremely small, if we compare it with the radiation power of the Sun, $\dot{E}_\odot \sim 10^{26} \text{ W}$, or with the rate of rotation energy loss of the pulsar, 10^{30} W . For a typical pulsar a null string may cause considerable variations of its luminosity if $\omega \simeq 10^2 \omega_0$.

Luminosity (4.5) can be increased if the string moves closer to the star or if the star has a larger magnetic moment. For instance, for a magnetar with the surface magnetic field $B \sim 10^{15} \text{ G}$ the estimate is

$$\dot{E} \sim 10^{27} \left(\frac{\omega}{\omega_0} \right)^2 \text{ W}, \quad (4.7)$$

which is comparable to the luminosity of a star even for $\omega \simeq \omega_0$.

Let us emphasize that the above estimates are quite rough. As has been shown in the previous sections the intensity of the radiation is not homogeneous and is maximal in the direction of the string velocity. In addition to the intensity and luminosity of the radiation the EM pulses generated by strings are specified by other physical parameters such as spectrum, duration of the pulse, etc. The systems we consider, the string and the point sources, are characterized by an effective size of the interaction, which is the impact parameter a . This parameter determines the duration of the generated EM pulse and its typical frequency. For instance, in the case of pulsars, the duration of the pulse is $a/c \simeq 3 \times 10^{-4} \text{ s}$.

It should be noted that for tensile cosmic strings there is a stronger limit on the energy, $G\mu \leq 10^{-12}$, which is imposed by studying stochastic gravitational-wave background [27]. If a similar restriction is applicable to null strings, then the EM pulses generated by these strings are hardly observable.

V. DISCUSSION

The aim of this work was to find an analytical description of EM perturbations at future null infinity caused by null cosmic strings which move near charged particles or point to magnetic-dipole-like sources. By using the explicit asymptotic form of the perturbations we proved that there is a nonvanishing flux of the radiation whose intensity is maximal in the direction of the velocity of the string. For strings with energies $\omega \sim 10^{-5}$ moving in the near zones of pulsars the generated perturbations of EM fields of the pulsars can cause considerable variations of the luminosities. For magnetars these variations can be caused by strings with even lower energies, $\omega < 10^{-7}$.

We studied a simplified model of a straight null string with a constant energy per unit length. The string, however, changes its form in the gravitational field of the source, and its energy density is not homogeneously distributed over the length [4]. This effect should be taken into account in the subsequent calculations.

Although our computations have been done for point sources they can be easily extended to sources of a finite size. Then numerical methods can be used, say, to describe the interaction of strings with more realistic fields of pulsars.

Another interesting development of our approach to asymptotic form of perturbations is application to perturbations of gravitational fields of massive sources generated by null cosmic strings. By analogy with EM perturbations one may expect that null strings create fluxes of gravitational waves. This work is in progress.

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APPENDIX A: ASYMPTOTIC FOR FIELDS OF ELECTRIC TYPE SOURCES

Here we present details of estimate (3.34) for A_1 in Sec. III C. Part A_1 of the vector potential, see (3.33), is determined by (3.29) when integration goes over a domain where $(\vec{m} \cdot \vec{n}) + 1$ is small. To proceed with computations it is convenient to set coordinates $x^A = \theta, \varphi$,

$$n_x = \cos \theta, \quad n_y = \sin \theta \sin \varphi, \quad n_z = \sin \theta \cos \varphi, \quad (\text{A1})$$

where $n_i = x^i/r$, and use parametrization (3.21) on the sphere S^2 in (3.29) with the help of the unit three-vector l^i . It is convenient to change parametrization (3.20) as

$$l_x = \cos \theta'', \quad l_y = \sin \theta'' \sin \varphi'', \quad l_z = \sin \theta'' \cos \varphi'' \quad (\text{A2})$$

to correspond to (A1). In terms of l the condition

$$(\vec{m} \cdot \vec{n}) = -1, \quad (\text{A3})$$

which determines A_1 , yields two solutions,

$$l_x^\pm = \pm \sqrt{\frac{1-n_x}{2}}, \quad l_y^\pm = \mp \frac{n_y}{\sqrt{2(1-n_x)}}, \quad l_z^\pm = \mp \frac{n_z}{\sqrt{2(1-n_x)}}, \quad (\text{A4})$$

or $\theta''_\pm = (\pi \mp \theta)/2$, $\varphi''_+ = \varphi + \pi$, $\varphi''_- = \varphi$. By using (A2) and (A4) one gets the corresponding quantities for these solutions:

$$g(\Omega_+, \omega) = \frac{1}{\sqrt{2(1-n_x)}} (-n_y + \omega(1-n_x) + i\varepsilon^+),$$

$$g(\Omega_-, \omega) = -(g(\Omega_+, \omega))^*, \quad (\text{A5})$$

$$\varepsilon^\pm(\Omega_\pm) = \varepsilon_\pm = \pm \sqrt{2(1-n_x) - n_y^2}. \quad (\text{A6})$$

It then follows that vector $\beta_\mu(\Omega)$ defined by (3.30) and (3.31) has the property

$$\beta^\mu(l^-) = (\beta^\mu(l^+))^*, \quad (\text{A7})$$

where $\beta^\mu(l^\pm) \equiv \beta^\mu(\Omega_\pm)$. Since we are estimating integral (3.29) near $\theta''_\pm, \varphi''_\pm$, it is convenient to introduce, near each point, new coordinates $\zeta_1 = \theta'' - \theta''_\pm$, $\zeta_2 = \varphi'' - \varphi''_\pm$. At small ζ_A ,

$$(\vec{m} \cdot \vec{n}) + 1 \simeq 2\zeta_1^2 + \frac{1}{2} \sin^2 \theta_\pm \zeta_2^2. \quad (\text{A8})$$

The integration region is determined by the scale Λ , see (3.32). If $|\zeta_A| < \Lambda$, then

$$A_{1,\mu}(x) \equiv -C \int_{|\zeta_A| < \Lambda} d\Omega'' \Re \left[\frac{\beta_\mu(\Omega'')}{x^\nu m_\nu(\Omega'') + i a \varepsilon(\Omega'')} \right] \simeq -\sum_{\pm} C \int_{|\zeta_A| < \Lambda} \sin \theta''_\pm d\zeta_1 d\zeta_2 \Re \left[\frac{\beta_\mu(l^\pm)}{U + r(2\zeta_1^2 + \frac{1}{2} \sin^2 \theta_\pm \zeta_2^2) + i a \varepsilon^\pm} \right]$$

$$= -\frac{\pi C}{r} \sum_{\pm} \frac{\sin \theta''_\pm}{\sin \theta} \Re \left[\beta_\mu(l^\pm) \ln \left(\frac{U + i a \varepsilon^\pm + \Lambda^2 r}{U + i a \varepsilon^\pm} \right) \right]. \quad (\text{A9})$$

By taking into account (A7) and relation

$$\frac{\sin \theta''_{\pm}}{\sin \theta} = \frac{1}{\sin(\theta/2)} = \sqrt{\frac{2}{1-n_x}},$$

one gets

$$A_{1,\mu}(x) \simeq \frac{N(\Omega)}{r} \Re \left[\bar{\beta}_{\mu} \ln \left(\frac{U + ia\bar{\epsilon} + \Lambda^2 r}{U + ia\bar{\epsilon}} \right) \right], \quad (\text{A10})$$

$$\bar{\beta}_{\mu} = \beta_{\mu}(l^+), \quad \bar{\epsilon} = \epsilon^+, \quad (\text{A11})$$

$$N(\Omega) = -2\pi C \sqrt{\frac{2}{1-n_x}}. \quad (\text{A12})$$

Equation (A10) reproduces (3.34). The approximation is not applicable at $n_x = 1$.

APPENDIX B: ASYMPTOTIC FOR FIELDS OF MAGNETIC TYPE SOURCES

Here we give some details of our computations in Sec. III D. Consider the solution to the following problem:

$$\square \chi(u, \mathbf{x}) = 0, \quad \chi(0, \mathbf{x}) = \delta(y) f(\mathbf{x}), \quad (\text{B1})$$

which appears in the y component in Eq. (3.59). The solution can be written as

$$\chi(u, v, y, z) = \int dy' dz' dv' D(u, v - v', y - y', z - z') \times \delta(y') f(v', y', z'), \quad (\text{B2})$$

where the D function,

$$D(x) = \frac{1}{\pi} \frac{\partial}{\partial v} \delta(x^2), \quad x^2 = -uv + y^2 + z^2, \quad (\text{B3})$$

is the solution to the problem:

$$\square D(x) = 0, \quad D(u = 0, \mathbf{x}) = \delta^{(3)}(\mathbf{x}), \quad (\text{B4})$$

see details in [1]. This yields $\chi = \partial_v \bar{\chi}$, where

$$\bar{\chi}(u, v, y, z) = C \int_{-\infty}^{\infty} dz' [(uv - y^2 - (z - z')^2)^2 + 4u^2((z')^2 + a^2)]^{-1/2}, \quad (\text{B5})$$

and $C = -1/(2\pi^2)$. In coordinates (3.6)

$$\bar{\chi}(x) = \frac{C}{r} X(U, r, \Omega), \quad X(U, r, \Omega) = \int_{-\infty}^{\infty} \frac{dx}{f^{1/2}(x)} \quad (\text{B6})$$

$$f(x) = (-x^2 + 2xn_z + c_1(2 + c_1))^2 + 4(x^2 + c_2^2)(c_1 + (1 - n_x))^2, \quad (\text{B7})$$

where $x = z'/r$, $c_1 = U/r$, $c_2 = a/r$. To estimate X at large r we decompose $X = X_1 + X_2$,

$$X_1(U, r, \Omega) = \int_{|x| < L} \frac{dx}{f^{1/2}(x)}, \quad X_2(U, r, \Omega) = \int_{|x| \geq L} \frac{dx}{f^{1/2}(x)}. \quad (\text{B8})$$

L is some parameter which is assumed to be $L \gg c_i$. In the leading order X_2 is a quasistatic function,

$$X_2(U, r, \Omega) \simeq X_2(\Omega) = \int_{|x| \geq L} \frac{dx}{x((x - 2n_z)^2 + 4(1 - n_x)^2)^{1/2}}. \quad (\text{B9})$$

As for X_1 , the main contribution to the integral comes from x near the point $x_{\star}$, where $f(x)$ has a minimum. At large r

$$f'(x_{\star}) = 0, \quad x_{\star} \simeq -\frac{n_z c_1}{\bar{\epsilon}^2}, \quad (\text{B10})$$

$$f(x_{\star}) \simeq 4(1 - n_x)^2 \frac{c_1 + \bar{\epsilon}^2 c_2}{\bar{\epsilon}^2}, \quad f''(x_{\star}) \simeq 8\bar{\epsilon}^2. \quad (\text{B11})$$

To guarantee that $f''(x_{\star}) > 0$ we assume that $n_x \neq 1$. Then one has

$$X_1(U, r, \Omega) \simeq \int_{|x| < L} \frac{dx}{\sqrt{f(x_{\star}) + f''(x_{\star})(x - x_{\star})^2/2}} = \frac{1}{\sqrt{2f''(x_{\star})}} \ln \left(\frac{\sqrt{\bar{L}^2 + 1} + \bar{L}}{\sqrt{\bar{L}^2 + 1} - \bar{L}} \right), \quad (\text{B12})$$

$$\bar{L} = L \sqrt{\frac{f''(x_{\star})}{2|f(x_{\star})|}}.$$

This yields

$$X_1(U, r, \Omega) \simeq \frac{1}{4\bar{\epsilon}} \ln \left(\frac{2Lr}{(1 - n_x)\sqrt{U^2 + a^2\bar{\epsilon}^2}} \right), \quad (\text{B13})$$

$$\bar{\chi}(x) = \frac{1}{2\pi^2 r} \left[\frac{1}{4\bar{\epsilon}} \ln \left(\frac{\sqrt{U^2 + a^2\bar{\epsilon}^2}}{2Lr} \right) - X(\Omega) \right], \quad (\text{B14})$$

where $X(\Omega)$ is some static part.

To find asymptotic (3.64) of $A_2(x)$ we compute the derivatives of $\bar{\chi}$

$$\partial_y \bar{\chi} = n_y (\partial_r \bar{\chi} - \partial_U \bar{\chi}) + \frac{1}{r} \frac{n_z}{1 - n_x^2} \partial_{\phi} \bar{\chi} + \frac{1}{r} \frac{n_x n_y}{\sqrt{1 - n_x^2}} \partial_{\theta} \bar{\chi}, \quad (\text{B15})$$

$$\partial_v \bar{\chi} = \frac{n_x}{2} \partial_r \bar{\chi} + \frac{(1-n_x)}{2} \partial_U \bar{\chi} - \frac{\sqrt{1-n_x^2}}{2r} \partial_\theta \bar{\chi}, \quad (\text{B16})$$

where

$$\partial_r \bar{\chi}(x) = \frac{1}{2\pi^2 r^2} \left[X(\Omega) - \frac{1}{4\bar{\epsilon}} \left(\ln \left(\frac{\sqrt{U^2 + a^2 \bar{\epsilon}^2}}{2Lr} \right) + 1 \right) \right], \quad (\text{B17})$$

$$\partial_b \bar{\chi}(x) = \frac{1}{2\pi^2 r} \left[-\frac{1}{4\bar{\epsilon}^2} \ln \left(\frac{\sqrt{U^2 + a^2 \bar{\epsilon}^2}}{2Lr} \right) \frac{\partial \bar{\epsilon}}{\partial b} - \frac{a^2}{U^2 + a^2 \bar{\epsilon}^2} \frac{\partial \bar{\epsilon}}{\partial b} - \frac{\partial X}{\partial b} \right], \quad (\text{B18})$$

$$\partial_U \bar{\chi}(x) = \frac{1}{8\pi^2 r \bar{\epsilon}} \frac{U}{U^2 + a^2 \bar{\epsilon}^2}, \quad (\text{B19})$$

with $b = \phi, \theta$. With the help of (B15)–(B19) we finally obtain

$$A_{2,\nu}(x) = 2\omega M_z (2\delta_\nu^y \partial_\nu + \delta_\nu^u \partial_y) \bar{\chi} \approx \frac{\omega M_z (\delta_\nu^y (1-n_x) - \delta_\nu^u n_y)}{\pi^2 r} \frac{U}{4\bar{\epsilon} (U^2 + a^2 \bar{\epsilon}^2)}. \quad (\text{B20})$$

The terms in the right hand side, which decay as $r^{-2} \ln r$ or faster, are omitted.

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