Mass spectrum of 1^{--} heavy quarkonium

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We calculate the masses and leptonic decay widths of the bottomonium $b\bar{b}$ and charmonium $c\bar{c}$ states in a constituent quark model where the Cornell-like potential and spin-dependent interaction are employed, with all model parameters predetermined by studying ground and first radial excited states of S- and P-wave heavy quarkonium mesons. By comparing the theoretical predictions for $J^{PC} = 1^{--}$ quarkonium states with experimental data and considering possible mixtures of nS and $(n-1)D$ states, we provide tentative assignments for all observed $J^{PC} = 1^{--}$ heavy quarkonia. The work suggests that the $\Upsilon(10860)$ and $\Upsilon(11020)$ are $b\bar{b}$ $5S-4D$ mixture states, and the $\psi(4360)$ and $\psi(4415)$ are largely $4S$ and $3D$ $c\bar{c}$ states, respectively. The $\psi(4230)$ may not be accommodated with the conventional meson picture in the present work.

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I. INTRODUCTION

Over 20 charmoniumlike and bottomoniumlike XYZ states have been observed in the past two decades. The charged states (Z states), which might be good exotic state candidates in the tetraquark or molecule picture, have inspired extensive interests of theorists in revealing their underlying structures [1]. However, distinguishing the real exotic neutral X and Y states from conventional meson states is still a challenging work, and underlying structures of X and Y states are still wildly discussed and debated in the past decade [2,3].

The exotic states with $J^{PC} = 1^{--}$, also known as Y states, are named Υ in the bottomonium region, and ψ in the charmonium region according to the latest Particle Data Group (PDG) naming scheme [4]. It is significant to separate these exotic neutral states from the conventional meson picture before treating them in other exotic pictures.

In the bottomonium region, the PDG states $\Upsilon(9460)$, $\Upsilon(10023)$, $\Upsilon(10355)$, and $\Upsilon(10579)$ are assigned to be $\Upsilon(1S)$ to $\Upsilon(4S)$, respectively [4]. Meanwhile, in the charmonium region, the J/ψ, $\psi(3686)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ are assigned to be $\psi(1S)$, $\psi(2S)$ [4], $\psi(1D)$ [5–8], $\psi(3S)$ [5–8], and $\psi(2D)$ [5–8], respectively. Theoretical pictures of $c\bar{c}$ bound states including S-D mixings [6,9–11], hybrid charmonium $c\bar{c}g$ [12–14], compact tetraquark $(qc\bar{q}\bar{c})$ [15–17], and molecule $(q\bar{c})(\bar{q}c)$ [18–21] have been proposed for studying the higher excited states, $\psi(4230)$, $\psi(4360)$, $\psi(4660)$, and $Y(4500)$ observed by BESIII recently [22].

Meanwhile, experimental new values of mass and leptonic decay width have been reported for these $J^{PC} = 1^{--}$ states, and the understanding of these states has been also improved since many theoretical works have been done. However, theoretical predictions of leptonic widths for higher excited states are still not consistent with the latest experimental data [6–9,23–27]. All established heavy quarkonium states with $J^{PC} = 1^{--}$ are listed in Table I, with experimental data of mass and leptonic decay width from PDG [4], and also with assignments from cited theoretical works. We briefly review the model of those works here and discuss their results in Sec. III.

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TABLE I. Mass and leptonic decay width of bottomonium and charmonium 1^{--} states from PDG [4], and their assignments from cited sources.

State	$M^{\exp}$ (MeV)	$\Gamma^{\exp}$ (keV)	Assignment
$\Upsilon(1S)$	9460	1.340 ± 0.018	1S $b\bar{b}$ [4]
$\Upsilon(2S)$	10023	0.612 ± 0.011	2S $b\bar{b}$ [4]
$\Upsilon(3S)$	10355	0.443 ± 0.008	3S $b\bar{b}$ [4]
$\Upsilon(4S)$	10579	0.272 ± 0.029	4S $b\bar{b}$ [4]
$\Upsilon(10860)$	$10885.2^{+2.6}_{-1.6}$	0.31 ± 0.07	5S $b\bar{b}$ [23–25,28,29]
$\Upsilon(11020)$	11000 ± 4	0.13 ± 0.03	6S [23–25,28,29]
			7S $b\bar{b}$ [9]
$\psi(1S)$	3097	5.55 ± 0.14	1S $c\bar{c}$ [4]
$\psi(2S)$	3686	2.33 ± 0.04	2S $c\bar{c}$ [4]
$\psi(3770)$	3773	0.26 ± 0.02	1D $c\bar{c}$ [5–8,12]
$\psi(4040)$	4039 ± 1	0.86 ± 0.07	3S $c\bar{c}$ [5–8]
$\psi(4160)$	4191 ± 5	0.48 ± 0.22	2D $c\bar{c}$ [5–8]
$\psi(4230)$	4230 ± 8	$\cdots$	4S $c\bar{c}$ [6,9,30]
			3D $c\bar{c}$ [5,31]
			$c\bar{c}g$ [12–14]
			$(qc\bar{q}\bar{c})$ [15–17]
			$(q\bar{c})(\bar{q}c)$ [18–21]
$\psi(4360)$	4368 ± 13	$\cdots$	4S $c\bar{c}$ [7], 3D $c\bar{c}$ [6]
$\psi(4415)$	4421 ± 4	0.58 ± 0.07	4S $c\bar{c}$ [5], 3D $c\bar{c}$ [7]
			5S $c\bar{c}$ [6,9]
$\psi(4660)$	4643 ± 9	$\cdots$	5S $c\bar{c}$ [7,32]
			6S $c\bar{c}$ [6,9]

Masses and leptonic widths of heavy quarkonium are estimated in a Martin-like potential model where a non-Coulombic power law potential is employed [9]. Different parameters are applied for bottomonium and charmonium mesons.

In Refs. [23,24], masses and decay properties of excited bottomonium states are studied in a relativized quark model (Godfrey-Isgur model) developed from Refs. [33–37] where a Cornell-like potential is employed. Masses, radiative transitions, annihilation decays, hadronic transitions, and production cross sections of excited bottomonium states are evaluated.

Bottomonium mass spectrum, electromagnetic, strong and hadronic decays are also studied in a non-relativistic quark model [25] developed from [38] and their previous work [39].

Charmonium spectrum, and electromagnetic decays are estimated in a nonrelativistic model with a Coulomb potential plus a screened linear potential [6], and also are studied in a constituent quark model with a screened confinement potential [7]. In Ref. [8], higher charmonium mass spectra are calculated in a nonrelativistic model with a Cornell-like potential, and the corresponding leptonic widths are estimated in Ref. [26].

In this work, we apply a model developed from Refs. [40,41] to predict the masses and leptonic decay widths of higher excited 1^{--} bottomonium $b\bar{b}$ and charmonium $c\bar{c}$ states. By considering possible *S*-*D* mixtures, and comparing the theoretical results with experimental data, we present a possible conventional meson interpretation for the higher excited 1^{--} heavy quarkonium states. The states which cannot be accommodated in the present picture will be studied in our future work by applying exotic pictures.

The paper is organized as follows. In Sec. II, a constituent quark model [40–44] is developed to include a spin-dependent interaction [45] for studying higher orbital excited quarkonium states. In Sec. III, theoretical masses and leptonic decay widths of the 1^{--} heavy quarkonium states are calculated and compared with experimental data. Tentative assignments for higher excited heavy quarkonium states are suggested in *S*-*D* mixture picture. A summary is given in Sec. IV.

II. THEORETICAL MODEL

The nonrelativistic Hamiltonian for studying the meson system takes the form,

$$H = H_0 + H_{SD}, \tag{1}$$

with

$$H_0 = M_{\text{ave}} + \frac{p^2}{2m_r} + \left(Ar - \frac{B}{r}\right),$$
$$H_{SD} = C_{SS}(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + C_{LS}(r)\vec{L}\cdot\vec{S} + C_T(r)S_{12}, \tag{2}$$

where H_0 is taken from the previous work [40–42]. $\vec{r}$ is the relative coordinate between the two quarks, M_{ave} is the spin-averaged mass taken from experimental data [4], and m_r stands for the reduced quark mass taking the form $m_1m_2/(m_1+m_2)$. In the work, we employ $m_c = 1270$ MeV and $m_b = 4180$ MeV [4]. $\vec{L}$, $\vec{S}$, and $\vec{J}$ are the operators of orbital angular momentum, total spin, and total angular momentum, respectively. The tensor operator S_{12} is defined as $S_{12} = (3(\vec{\sigma}_1\cdot r)(\vec{\sigma}_2\cdot r) - \vec{\sigma}_1\cdot\vec{\sigma}_2)$.

$C_{SS}(r)$, $C_{LS}(r)$, and $C_T(r)$ in Eq. (2) are derived by following the Breit-Fermi interaction, that is,

$$C_{SS}(r) = \frac{1}{6m_i^2}\Delta V_V(r) = \frac{2B\sigma^3 e^{-\sigma^2 r^2}}{3\sqrt{\pi}m_i^2},$$
$$C_{LS}(r) = \frac{1}{2m_i^2}\frac{1}{r}\left[3\frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr}\right]$$
$$= -\frac{A}{2m_i^2}\frac{1}{r} - \frac{3B\sigma}{\sqrt{\pi}m_i^2}\frac{e^{-\sigma^2 r^2}}{r^2} + \frac{3B}{2m_i^2}\frac{Erf[\sigma r]}{r^3},$$
$$C_T(r) = \frac{1}{12m_i^2}\left[\frac{1}{r}\frac{dV_V(r)}{dr} - \frac{d^2V_V(r)}{dr^2}\right]$$
$$= -\frac{B\sigma e^{-\sigma^2 r^2}}{2\sqrt{\pi}m_i^2 r^2} - \frac{B\sigma^3 e^{-\sigma^2 r^2}}{3\sqrt{\pi}m_i^2} + \frac{BErf[\sigma r]}{4m_i^2 r^3}. \tag{3}$$

Note that we have employed $V_V(r) = -BErf[\sigma r]/r$ and $V_S(r) = Ar$, taken from Ref. [45].

$\vec{\sigma}_i$ in Eq. (2) are quark spin operators, and the contribution of $\vec{\sigma}_i \cdot \vec{\sigma}_j$ is -3 for $S = 0$ and $+1$ for $S = 1$ mesons. The matrix elements of $\vec{L} \cdot \vec{S}$ and S_{12} in the $|JMLS\rangle$ basis read

$$\langle \vec{L} \cdot \vec{S} \rangle = [J(J+1) - L(L+1) - S(S+1)]/2,$$
$$\langle S_{12} \rangle = \begin{cases} -\frac{2L}{2L+3} & J = L+1 \\ +2 & J = L \\ -\frac{2(L+1)}{2L-1} & J = L-1 \end{cases} \quad (4)$$

The tensor operator S_{12} has nonvanishing matrix elements between the two orbital parts of spin-triplet states.

The string tension coefficient A and Coulomb coefficient B in the Cornell potential $V(r) = Ar - B/r$ may take different values when A and B are fitted to charmonium and bottomonium experimental data. This indicates that A and B might be flavor dependent parameters. Inspired by lattice QCD studies [46,47], A and B are proposed to be mass dependent coupling parameters. For more detailed discussion, one may refer to Ref. [40]. The hyperfine coefficient σ is also proposed to be mass dependent [45].

In this work, parameters A, B, and σ are assumed to take the following mass dependent form:

$$A = a + bm_i, \qquad B = B_0\sqrt{\frac{1}{m_i}}, \qquad \sigma = \sigma_0 m_i, \quad (5)$$

with a, b, B_0, and σ_0 being constants. Four model coupling constants are determined by comparing the theoretical mass results with experimental data of conventional mesons,

$$a = 78650\ \text{MeV}^2, \qquad b = 28\ \text{MeV},$$
$$B_0 = 30.86\ \text{MeV}^{1/2}, \qquad \sigma_0 = 0.7. \quad (6)$$

The fitting results M^{cal} for the S- and P-wave ground and first radial excited bottomonium and charmonium meson states are listed in Table II, together with experimental data M^{exp} from PDG [4]. Some typical theoretical mass results from other works for bottomonium mesons [9,23,25] and charmonium mesons [6,8] are collected for comparison. Our results are fairly compatible with experimental data.

The S-wave and D-wave 1^{--} quarkonium leptonic decay widths given by the Van Royen-Weisskopf formula [48], including radiative QCD corrections for the S wave [49], takes the same form with [25]

$$\Gamma(n^3S_1 \to e^+e^-) = \frac{4\alpha^2 e_q^2 |R_{nS}(0)|^2}{M_n^2}\left(1 - \frac{16\alpha_s}{3\pi}\right),$$
$$\Gamma(n^3D_1 \to e^+e^-) = \frac{25\alpha^2 e_q^2 |R''_{nD}(0)|^2}{2m_i^4 M_n^2}, \quad (7)$$

TABLE II. Masses of ground and first radial excited bottomonium and charmonium meson states, with the unit in MeV. M^{cal} are our fitting results. Experimental data M^{exp} are taken from PDG [4], and other theoretical results for comparison are from cited sources.

$b\bar{b}$	J^{PC}	nL	M^{exp}	M^{cal}	[25]	[23]	[9]
η_b	0^{-+}	1S	9399	9394	9455	9402	9392
		2S	9999	9989	9990	9976	9991
Υ	1^{--}	1S	9460	9461	9502	9465	9460
		2S	10023	10017	10015	10003	10024
h_b	1^{+-}	1P	9899	9894	9879	9882	9896
		2P	10260	10270	10240	10250	10260
χ_{b0}	0^{++}	1P	9859	9859	9855	9847	9862
		2P	10232	10244	10221	10226	10240
χ_{b1}	1^{++}	1P	9893	9888	9874	9876	9888
		2P	10255	10266	10236	10261	10256
χ_{b2}	2^{++}	1P	9912	9905	9886	9897	9908
		2P	10269	10280	10246	10261	10268

$c\bar{c}$	J^{PC}	nL	M^{exp}	M^{cal}	[8]NR	[8]GI	[6]
η_c	0^{-+}	1S	2984	2987	2982	2975	2979
		2S	3638	3633	3630	3623	3623
ψ	1^{--}	1S	3097	3110	3090	3098	3097
		2S	3686	3673	3672	3676	3673
h_c	1^{+-}	1P	3525	3533	3516	3517	3519
χ_{c0}	0^{++}	1P	3415	3460	3424	3445	3433
		2P	3860	3884	3852	3916	3842
χ_{c1}	1^{++}	1P	3510	3528	3505	3510	3510
χ_{c2}	2^{++}	1P	3556	3566	3556	3550	3554
		2P	3930	3949	3972	3979	3537

where the fine-structure constant $\alpha \simeq 1/137$. e_q is the charge of quarks, M_n is the mass of the decaying quarkonium states, $R_{nS}(0)$ and $R_{nD}(0)$ are the radial wave functions of the $^3S_1$ and $^3D_1$ states at the origin respectively. α_s is the running strong coupling constant, where $\alpha_s(b\bar{b}) = 0.118$ for bottomonium [9], and $\alpha_s(c\bar{c}) = 0.26$ for charmonium [6].

The difference between performing full integration for leptonic width and applying the lowest order approximation is about 50% for light mesons, but is about 10% for charmonium mesons and 4% for bottomonium mesons. Thus the Van's formula with the first order approximation is reliable to be employed for estimating heavy quarkonium leptonic widths.

III. RESULTS AND DISCUSSION

A. Masses and leptonic widths

We evaluate the masses and leptonic widths of the bottomonium and charmonium meson states using the Hamiltonian in Eq. (1) and the leptonic widths formula in Eq. (7). The theoretical results for the 1^{--} $1S$ to $5D$ states are listed in Table III, with M^{cal} for masses and Γ^{cal} for

TABLE III. Present predictions of bottomoium $b\bar{b}$ and charmonium $c\bar{c}$ 1^{--} state masses (MeV) and leptonic widths (keV) compared with experimental data from PDG [4] and others theoretical works from cited sources.

$nL(b\bar{b})$	$M^{\rm exp}$ (MeV)	$M^{\rm cal}$ (MeV)	[9]	[23]	[24]	[25]	$\Gamma^{\rm exp}$ (keV)	$\Gamma^{\rm cal}$ (keV)	[9]	[23]	[24]	[25]
1S	9460.30 ± 0.26	9461	9460	9465	9463	9502	1.340 ± 0.018	1.370	1.203	1.44	1.650	0.71
2S	10023.26 ± 0.31	10017	10024	10003	10017	10015	0.612 ± 0.011	0.626	0.519	0.73	0.821	0.37
1D	···	10143	10147	10138	10153	10117	···	0.002	···	0.001	0.002	0.001
3S	10355.2 ± 0.5	10379	10346	10354	10356	10349	0.443 ± 0.008	0.468	0.330	0.53	0.569	0.27
2D	···	10461	10427	10441	10442	10414	···	0.003	···	0.002	0.003	0.003
4S	10579.4 ± 1.2	10678	10576	10635	10612	10607	0.272 ± 0.029	0.393	0.242	0.39	0.431	0.21
3D	···	10739	10637	10698	10675	10653	···	0.005	···	0.002	0.003	···
5S	···	10942	10755	10878	10822	10818	···	0.346	0.191	0.33	0.348	0.18
4D	···	10991	10805	10928	10871	10853	···	0.006	···	0.002	0.003	···
6S	···	11184	10904	11102	11001	10995	···	0.313	0.158	0.27	0.286	0.15
5D	···	11224	10946	···	11041	11023	···	0.008	···	···	0.003	···

$nL(c\bar{c})$	$M^{\rm exp}$ (MeV)	$M^{\rm cal}$ (MeV)	[9]	[6]	[7]	[8]	$\Gamma^{\rm exp}$ (keV)	$\Gamma^{\rm cal}$ (keV)	[9]	[6]	[7]	[26]
1S	3096.90 ± 0.01	3110	3097	3097	3096	3090	5.55 ± 0.14	6.02	4.95	6.60	3.93	12.13
2S	3686.10 ± 0.03	3673	3690	3673	3703	3672	2.33 ± 0.04	2.33	1.69	2.40	1.78	5.03
1D	3773.13 ± 0.35	3782	3729	3787	3796	3785	0.26 ± 0.02	0.14	···	0.03	0.22	0.06
3S	4039 ± 1	4046	4030	4022	4097	4072	0.86 ± 0.07	1.55	0.96	1.42	1.11	3.48
2D	···	4114	4056	4089	4153	4142	···	0.22	···	0.04	0.30	0.10
4S	···	4355	4273	4273	4389	4406	···	1.19	0.65	0.97	0.78	2.63
3D	···	4404	4293	4317	4426	···	···	0.26	···	0.04	0.33	···
5S	···	4628	4464	4463	4614	···	···	0.97	0.49	0.70	0.57	···
4D	···	4667	4480	···	4641	···	···	0.20	···	···	0.31	···
6S	···	4879	4622	4608	4791	···	···	0.82	0.39	0.49	0.42	···
5D	···	4910	4634	···	4810	···	···	0.23	···	···	0.28	···

leptonic widths. The experimental data $M^{\rm exp}$ and $\Gamma^{\rm exp}$ of $\Upsilon(1S)$ to $\Upsilon(4S)$ and $\psi(1S)$, $\psi(2S)$, $\psi(4040)$, and $\psi(3770)$ are taken from PDG [4]. These states are widely believed to be conventional meson states.

For comparison, we also briefly discuss the results of several works reviewed in Sec. I, and show their predictions in Table III.

For bottomonium states, the fitting results of masses [9] can be matched very well with experimental data, but the leptonic widths are all smaller than experimental data especially for $\Upsilon(2S)$, $\Upsilon(3S)$, and $\psi(2S)$.

The theoretical mass results of 1^{--} bottomonium states from Refs. [23,24] are roughly compatible with experimental data, and the mass of 3S states has a very nice match with $\Upsilon(3S)$. However, both leptonic width results are significantly larger than experimental data from $\Upsilon(1S)$ to $\Upsilon(4S)$.

On the other hand, mass results in Ref. [25] are roughly compatible but leptonic width results are significantly smaller than the data.

For charmonium states, the collected theoretical results of Ref. [6] show that the 1S mass agrees well with the data of J/ψ, and the masses of 2S, 1D, and 3S are compatible with the data of $\psi(2S)$, $\psi(3770)$, and $\psi(4040)$. But theoretical leptonic width results are all larger than the corresponded data. The results of Ref. [7] show that the theoretical masses are roughly compatible with the data, but the leptonic widths of 1S and 2S states are much smaller than the data.

The theoretical mass results [8] are compatible with the data, but the leptonic width results [26] are too large due to only the leading order contribution in the leptonic width formula considered.

It can be seen from Table III that the predictions of the mass and leptonic width for higher excited 1^{--} states do not simultaneously match well with experimental data when one considers the meson states in either S-wave or D-wave states only.

B. Possible mixtures of nS and $(n-1)D$ states

As reviewed above, it is difficult to simultaneously reproduce masses and leptonic widths of experimental data for higher excited quarkonia under the assumption of pure S- and D-wave states.

Based on the results in Table III, it is natural to consider altering the theoretical masses and leptonic widths simultaneously by mixing the S and D waves. Dynamically, the coupling of S and D waves may stem from tensor forces. However, detailed calculations reveal that the tensor force in the Hamiltonian in Eqs. (1) and (3) is not strong enough to mix the S and D waves considerably. One may expect that coupled-channel effects, stemming from the mixing via decay channels, might be the source of the large S-D

mixing [50–54]. Or, the meson exchange or multigluon exchange may contribute stronger tensor force interactions [55,56].

The mixing probability is proportional to $1/|\delta E|^2$ in perturbation calculations, where δE is the energy difference between the two mixed states, and hence only the nearest states may mix up considerably. Based on the results in Table III, we estimate that the probability for the $(n-2)D$ and nS mixing as well as the nD and nS mixing is less than 10% of the probability for the nS and $(n-1)D$ mixing. It is a reasonable approximation to consider only the mixing between the nearest nS and $(n-1)D$ states. The mixed states may take the form,

$$|\psi_1\rangle = \cos\theta|nS\rangle + \sin\theta|(n-1)D\rangle,$$
$$|\psi_2\rangle = -\sin\theta|nS\rangle + \cos\theta|(n-1)D\rangle, \tag{8}$$

where θ is mixing angle.

The charmonium states, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4360)$, and $\psi(4415)$, and bottomonium states $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$, $\Upsilon(10860)$, and $\Upsilon(11020)$ are considered to be S-D mixture candidates. The masses, M_{ψ_1} and M_{ψ_2}, and the leptonic decay widths, Γ_{ψ_1} and Γ_{ψ_2}, of the states $|\psi_1\rangle$ and $|\psi_2\rangle$ are derived,

$$M_{\psi_1} = \frac{1}{2}\left(M_{nS} + M_{(n-1)D} + (M_{nS} - M_{(n-1)D})\frac{1}{\cos 2\theta}\right),$$
$$M_{\psi_2} = \frac{1}{2}\left(M_{nS} + M_{(n-1)D} + (M_{(n-1)D} - M_{nS})\frac{1}{\cos 2\theta}\right),$$
$$\Gamma_{\psi_1} = (\sqrt{F_c}F_{nS}R_{nS}(0)\cos\theta + F_{(n-1)D}R''_{(n-1)D}(0)\sin\theta)^2,$$
$$\Gamma_{\psi_2} = (-\sqrt{F_c}F_{nS}R_{nS}(0)\sin\theta + F_{(n-1)D}R''_{(n-1)D}(0)\cos\theta)^2 \tag{9}$$

with

$$F_{nS} = \frac{2\alpha e_q}{M_{nS}}, \quad F_{nD} = \frac{5\alpha e_q}{\sqrt{2}m_i^2 M_{nD}}, \quad F_c = \left(1 - \frac{16\alpha_s}{3\pi}\right). \tag{10}$$

Fitting the theoretical leptonic widths (Γ_{ψ_1} and Γ_{ψ_2}) of the S-D mixture states to experimental data leads to two mixing angles $\theta°$, as shown in the fourth column of Table IV. By applying the two angles to Eq. (9), we derive two masses for each mixing state shown in the fifth column. It is found that the masses derived with the first angle in column four are more consistent with experimental data.

The decay widths of E1 radiative transitions are calculated for the S-D mixture states, since the radiative transitions are sensitive to the internal structure of states. The decay width for E1 transitions between an initial state $n^{2S+1}L_J$ and final state $n'^{2S'+1}L'_{J'}$ can be written as [25]

$$\Gamma(n^{2S+1}L_J \to n'^{2S'+1}L'_{J'} + \gamma) = \frac{4\alpha^2 e_q^2 k^3}{3}(2J'+1)S^E_{fi}\delta_{SS'}|\epsilon_{fi}|^2\frac{E_f}{M_i}, \tag{11}$$

$$S^E_{fi} = \max(L, L')\begin{Bmatrix} J & 1 & J' \\ L' & S & L \end{Bmatrix}^2, \tag{12}$$

$$\epsilon_{fi} = \frac{3}{k}\int_0^\infty R_i(r)\left[\frac{kr}{2}j_0\left(\frac{kr}{2}\right) - j_1\left(\frac{kr}{2}\right)\right]R_f(r)r^2 dr \tag{13}$$

where k is the emitted photon momentum. M_i is the mass of the initial state and E_f is the energy of the final state, which are taken from established experimental data. S^E_{fi} is a the statistical factor. $j_i(x)$ is the spherical Bessel functions of the first kind. $R_i(r)$ and $R_f(r)$ are the radial wave functions of initial and final states, respectively.

TABLE IV. The mixtures of nS and $(n-1)D$ 1^{--} charmonium and bottomonium states. $M^{\rm exp}$ and $\Gamma^{\rm exp}$ are from PDG [4].

Quark content	Mixture states	$M^{\rm cal}$ (MeV)	$\theta°$	M_{ψ_1} M_{ψ_2}	Assignment	$M^{\rm exp}$ (MeV)	Γ_{ψ_1} Γ_{ψ_2}	$\Gamma^{\rm exp}$ (keV)
$b\bar{b}$	2S	10017	−9.0°, 15.1°	10014, 10007	$\Upsilon(2S)$	10023.26 ± 0.31	0.601	0.612 ± 0.011
	1D	10143		10146, 10153	⋯	⋯	0.027	⋯
$b\bar{b}$	3S	10379	−12.5°, 22.2°	10375, 10363	$\Upsilon(3S)$	10355.2 ± 0.5	0.430	0.443 ± 0.008
	2D	10461		10465, 10477	⋯	⋯	0.042	⋯
$b\bar{b}$	4S	10678	38.0°, −25.3°	10583, 10661	$\Upsilon(4S)$	10579.4 ± 1.2	0.288	0.272 ± 0.029
	3D	10739		10834, 10756	$\Upsilon(10753)$?	10753 ± 6	0.109	⋯
$b\bar{b}$	5S	10942	34.9°, −19.6°	10897, 10935	$\Upsilon(10860)$	$10885.2^{+2.6}_{-1.6}$	0.278	0.31 ± 0.07
	4D	10991		11036, 10998	$\Upsilon(11020)$	11000 ± 4	0.074	0.13 ± 0.03
$c\bar{c}$	2S	3673	−2.5°, 30.6°	3673, 3615	$\psi(2S)$	3686.10 ± 0.03	2.27	2.33 ± 0.04
	1D	3782		3782, 3840	$\psi(3770)$	3773.13 ± 0.35	0.20	0.26 ± 0.02
$c\bar{c}$	3S	4046	−21.2°, 62.6°	4034, 4139	$\psi(4040)$	4039 ± 1	0.98	0.86 ± 0.07
	2D	4114		4125, 4021	$\psi(4160)$	4191 ± 5	0.79	0.48 ± 0.22
$c\bar{c}$	4S	4355	−18.1°, 68.3°	4349, 4413	$\psi(4360)$	4368 ± 13	0.77	⋯
	3D	4404		4410, 4346	$\psi(4415)$	4421 ± 4	0.68	0.58 ± 0.07

Since the E1 transition branching fractions in PDG of $\Upsilon(2S)$, $\Upsilon(3S)$, $\psi(2S)$, and $\psi(3770)$ are clear, E1 transition decay processes, $\Upsilon(2S-1D) \to \gamma\chi_{b_J}(1P)$, $\Upsilon(3S-2D) \to \gamma\chi_{b_J}(2P)$ and $\psi(2S-1D) \to \gamma\chi_{b_J}(1P)$, are studied. The E1 decay widths are calculated by applying the *S-D* mixed radial wave function Eq. (8) for initial states to Eq. (13). M_i in Eq. (11) for Υ *D*-wave mixture states are taken from the mass spectrum in Table IV due to no available data. The theoretical results, compared with experimental data, are listed in Table V. In this case, experimental data of E1 decay widths are derived from the experimental data of total decay widths and E1 branching fractions of PDG 2022 [4].

TABLE V. Theoretical results and experimental data of E1 radiative transition decay widths of Υ and ψ mixture states.

Initial state	Final state	$\mathcal{B}_{E1}^{\rm exp}$ [4] ($\Gamma_{E1}/\Gamma_{\rm tot}$)	$\Gamma_{E1}^{\rm exp}$ (keV)	$\Gamma_{E1}^{\rm the}$ (keV)
$\Upsilon(2S)$	$\gamma\chi_{b0}(1P)$	$(3.8 \pm 0.4)\%$	1.2 ± 0.2	1.0
$\Upsilon(1D)$		⋯	⋯	8.2
$\Upsilon(2S)$	$\gamma\chi_{b1}(1P)$	$(6.9 \pm 0.4)\%$	2.2 ± 0.3	1.8
$\Upsilon(1D)$		⋯	⋯	4.8
$\Upsilon(2S)$	$\gamma\chi_{b2}(1P)$	$(7.15 \pm 0.35)\%$	2.3 ± 0.3	2.1
$\Upsilon(1D)$		⋯	⋯	0.3
$\Upsilon(3S)$	$\gamma\chi_{b0}(2P)$	$(5.9 \pm 0.6)\%$	1.2 ± 0.2	1.1
$\Upsilon(2D)$		⋯	⋯	5.7
$\Upsilon(3S)$	$\gamma\chi_{b1}(2P)$	$(12.6 \pm 1.2)\%$	2.6 ± 0.5	2.1
$\Upsilon(2D)$		⋯	⋯	3.8
$\Upsilon(3S)$	$\gamma\chi_{b2}(2P)$	$(13.1 \pm 1.6)\%$	2.7 ± 0.6	2.4
$\Upsilon(2D)$		⋯	⋯	0.2
$\psi(2S)$	$\gamma\chi_{c0}(1P)$	$(9.79 \pm 0.20)\%$	28.8 ± 1.4	24.6
$\psi(1D)$		$(0.69 \pm 0.06)\%$	187.7 ± 23.8	138.9
$\psi(2S)$	$\gamma\chi_{c1}(1P)$	$(9.75 \pm 0.24)\%$	28.7 ± 1.5	35.7
$\psi(1D)$		$(0.249 \pm 0.023)\%$	67.7 ± 9.0	65.3
$\psi(2S)$	$\gamma\chi_{c2}(1P)$	$(9.52 \pm 0.20)\%$	28.0 ± 1.4	32.7
$\psi(1D)$		$<6.4 \times 10^{-4}$	<17.4	3.1

C. Assignments and discussion

The theoretical mass and leptonic width results of 1^{--} heavy quarkonium states are summarized in Table VI, where some possible *S-D* mixing states are listed in brackets, and the tentative assignments for the observed states are provided.

For excited bottomonium states 2S–1D, 3S–2D, 4S–3D, and 5S–4D mixtures are considered. The $\Upsilon(10023)$ and $\Upsilon(10355)$ are assigned to be largely 2S and 3S states, respectively, containing some *D*-wave component. The $\Upsilon(10579)$ is assigned a 4S–3D mixture state due to the large mixing angle.

The leptonic width data of $\Upsilon(11020)$, 0.13 ± 0.03 keV [4], are averaged from $0.095 \pm 0.03 \pm 0.035$ keV [57] and 0.156 ± 0.040 keV [58], which is too small to be a 5S state where the 5S leptonic width is predicted to be around 0.3 keV in Table III. Thus, the $\Upsilon(10860)$ and $\Upsilon(11020)$ are assigned to be 5S–4D mixed states due to a congruent matching for both masses and leptonic widths.

TABLE VI. Present predictions of bottomonium and charmonium 1^{--} state masses (MeV) and leptonic widths (keV) after possible *S-D* mixture compared with experimental data. The experimental data is taken from PDG [4].

nL	$M_{S-D}^{\rm cal}$ (MeV)	Assignment	$M^{\rm exp}$ (MeV)	$\Gamma_{S-D}^{\rm cal}$ (keV)	$\Gamma^{\rm exp}$ (keV)	Other assignments
1S	9461	$\Upsilon(1S)$	9460.30 ± 0.26	1.370	1.340 ± 0.018	1S $b\bar{b}$ [4]
2S	10014	$\Upsilon(2S)$	10023.26 ± 0.31	0.601	0.612 ± 0.011	2S $b\bar{b}$ [4]
1D	10146	⋯	⋯	0.027	⋯	⋯
3S	10375	$\Upsilon(3S)$	10355.2 ± 0.5	0.430	0.443 ± 0.008	3S $b\bar{b}$ [4]
2D	10465	⋯	⋯	0.042	⋯	⋯
4S	10583	$\Upsilon(4S)$	10579.4 ± 1.2	0.288	0.272 ± 0.029	4S $b\bar{b}$ [4]
3D	10834	$\Upsilon(10753)$?	10753 ± 6	0.109	⋯	⋯
5S	10897	$\Upsilon(10860)$	$10885.2^{+2.6}_{-1.6}$	0.278	0.31 ± 0.07	5S $b\bar{b}$ [23–25,28,29]
4D	11036	$\Upsilon(11020)$	11000 ± 4	0.074	0.13 ± 0.03	6S $b\bar{b}$ [23–25,28,29], 7S $b\bar{b}$ [9]
1S	3110	J/ψ	3096.90 ± 0.01	6.02	5.55 ± 0.14	1S $c\bar{c}$ [4]
2S	3673	$\psi(2S)$	3686.10 ± 0.03	2.27	2.33 ± 0.04	2S $c\bar{c}$ [4]
1D	3782	$\psi(3770)$	3773.13 ± 0.35	0.20	0.26 ± 0.02	1D $c\bar{c}$ [5–8,12]
3S	4034	$\psi(4040)$	4039 ± 1	0.98	0.86 ± 0.07	3S $c\bar{c}$ [5–8]
2D	4125	$\psi(4160)$	4191 ± 5	0.79	0.48 ± 0.22	2D $c\bar{c}$ [5–8]
⋯	⋯	$\psi(4230)$	4230 ± 8	⋯	⋯	4S $c\bar{c}$ [6,9,30], 3D $c\bar{c}$ [5,31], $c\bar{c}g$ [12–14], $(qc\bar{q}\bar{c})$ [15–17], $(q\bar{c})(\bar{q}c)$ [18–21]
4S	4349	$\psi(4360)$	4368 ± 13	0.77	⋯	4S $c\bar{c}$ [7], 3D $c\bar{c}$ [6]
3D	4410	$\psi(4415)$	4421 ± 4	0.68	0.58 ± 0.07	4S $c\bar{c}$ [5], 3D $c\bar{c}$ [7], 5S $c\bar{c}$ [6,9]
5S	4628	$\psi(4660)$	4643 ± 9	0.97	⋯	5S $c\bar{c}$ [7,32], 6S $c\bar{c}$ [6,9]
4D	4667	⋯	⋯	0.20	⋯	

The newly reported state $\Upsilon(10753)$ observed by the Belle [59] and Belle-II Collaborations [60] is tentatively assigned to be largely a 3D state. For a tetraquark mixture interpretation, one may refer to Ref. [61]. More experimental data and theoretical works are essential for making an unambiguous assignment for the $\Upsilon(10753)$.

For the higher excited charmonium states, 2S–1D, 3S–2D, and 4S–3D mixtures are considered. It is found that the $\psi(2\mathrm{S})$ possesses a small D-wave component, and $\psi(3770)$ possesses a small S-wave component, which is consistent with our previous work [62,63] and other theoretical work [6].

Since the theoretical results of the $\psi(4040)$ leptonic width (from 0.96–3.48 keV) in Table III are all larger than experimental data [4] (with $\Gamma_{ee} = 0.86 \pm 0.07$ keV) significantly, and the leptonic width of the widely believed 2D state $\psi(4160)$ [5–8] is measured to be 0.48 ± 0.22 keV [64], one may naturally consider the $\psi(4040)$ and $\psi(4160)$ to be S-D mixture states. The PDG mass, 4191 ± 5 MeV, of the $\psi(4160)$ [4] is collected from the BES Collaboration [64]. However, data analyses in Ref. [65] result in the mass and leptonic width, 4151 ± 4 MeV and 0.83 ± 0.08 keV from the Crystal Ball measurement [66], and 4155 ± 5 MeV and 0.84 ± 0.13 keV from the BES measurement [67]. Our theoretical results are compatible with the results in Ref. [65], and we suggest that the $\psi(4040)$ and $\psi(4160)$ are 3S and 2D mixed states.

In other conventional meson assignments, the $\psi(4360)$ is assigned to be 4S $c\bar{c}$ [7] and 3D $c\bar{c}$ [6] while the $\psi(4415)$ is assigned to be 4S $c\bar{c}$ [5], 3D $c\bar{c}$ [7], and 5S $c\bar{c}$ [6,9]. Considering the congruent matching for both masses and leptonic widths in the work, we assign the $\psi(4360)$ and $\psi(4415)$ to be 4S and 3D mixture states, where the $\psi(4360)$ and $\psi(4415)$ are largely 4S and 3D states, respectively.

$\psi(4660)$ is tentatively assigned to be a 5S state according to the good mass matching, which is consistent with Refs. [7,32]. The $\psi(4230)$ cannot be accommodated as a $c\bar{c}$ state in the present work. For other interpretations, one may refer to Refs. [12–14] for the charmonium hybrid, Refs. [15–17] for the tetraquark, and Refs. [18–21] for the molecule picture.

IV. SUMMARY

The masses and leptonic decay widths of S-wave and D-wave heavy quarkonium meson states with quantum number $J^{PC} = 1^{--}$ until 6S and 5D have been evaluated, with all model parameters predetermined by studying all ground and first radial excited S- and P-wave heavy quarkonium mesons. The theoretical results have been matched with experimental data by considering possible S-D mixtures, and the tentative assignments for higher excited states are provided. Based on the assignment, E1 radiative transition decay widths are calculated.

For the 1^{--} bottomonium states, this work suggests that the $\Upsilon(2S)$ and $\Upsilon(3S)$ may possesses some D-wave component, and $\Upsilon(4S)$ may be a 4S–3D mixture state. The $\Upsilon(10860)$ and $\Upsilon(11020)$ are assigned to be 5S–4D mixture states. The $\Upsilon(10753)$ is tentatively assigned to be 4S–3D mixture state, and more experimental data are required to make unambiguous assignment for this newly reported state.

For the 1^{--} charmonium states, the work suggests that the $\psi(2S)$ and $\psi(3770)$ may possesses some small D-wave and S-wave component, respectively, and the $\psi(4040)$ and $\psi(4160)$ are mainly 3S and 2D states, respectively. The $\psi(4360)$ and $\psi(4415)$ are largely 4S and 3D states, respectively. The $\psi(4660)$ is assigned to be a 5S state. The $\psi(4230)$ may not be accommodated with the conventional meson picture in the present work.

The work shows that a large S-D mixing is essential to understand the experimental data of higher excited quarkonia, but the tensor force in the widely applied Hamiltonian is not strong enough to mix the S and D waves considerably. It is expected that the coupled-channel effects, resulting from couplings to common decay channels, might be an important source of the large S-D mixing. Heavy quarkonia will be studied by considering the coupled channel induced S-D mixing in our future work.

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[1] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Phys. Rep. **639**, 1 (2016).
[2] E. Eichten, S. Godfrey, H. Mahlke, and J. L. Rosner, Rev. Mod. Phys. **80**, 1161 (2008).
[3] Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Prog. Part. Nucl. Phys. **107**, 237 (2019).
[4] R. L. Workman *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[5] A. Zhang, Phys. Lett. B **647**, 140 (2007).
[6] B.-Q. Li and K.-T. Chao, Phys. Rev. D **79**, 094004 (2009).
[7] J. Segovia, A. M. Yasser, D. R. Entem, and F. Fernandez, Phys. Rev. D **78**, 114033 (2008).
[8] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[9] M. Shah, A. Parmar, and P. C. Vinodkumar, Phys. Rev. D **86**, 034015 (2012).

[10] J.-Z. Wang, D.-Y. Chen, X. Liu, and T. Matsuki, Phys. Rev. D **99**, 114003 (2019).
[11] J.-Z. Wang and X. Liu, Phys. Rev. D **107**, 054016 (2023).
[12] L. Liu, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, P. Vilaseca, J. J. Dudek, R. G. Edwards, B. Joo, and D. G. Richards (Hadron Spectrum Collaboration), J. High Energy Phys. 07 (2012) 126.
[13] X.-Q. Luo and Y. Liu, Phys. Rev. D **74**, 034502 (2006).
[14] S.-L. Zhu, Phys. Lett. B **625**, 212 (2005).
[15] L. Maiani, V. Riquer, F. Piccinini, and A. D. Polosa, Phys. Rev. D **72**, 031502(R) (2005).
[16] N. V. Drenska, R. Faccini, and A. D. Polosa, Phys. Rev. D **79**, 077502 (2009).
[17] D. Ebert, R. N. Faustov, and V. O. Galkin, Eur. Phys. J. C **58**, 399 (2008).
[18] T.-W. Chiu and T.-H. Hsieh (TWQCD Collaboration), Phys. Rev. D **73**, 094510 (2006).
[19] G.-J. Ding, Phys. Rev. D **79**, 014001 (2009).
[20] F. Close and C. Downum, Phys. Rev. Lett. **102**, 242003 (2009).
[21] F. Close, C. Downum, and C. E. Thomas, Phys. Rev. D **81**, 074033 (2010).
[22] M. Ablikim *et al.* (BESIII Collaboration), Chin. Phys. C **46**, 111002 (2022).
[23] S. Godfrey and K. Moats, Phys. Rev. D **92**, 054034 (2015).
[24] J.-Z. Wang, Z.-F. Sun, X. Liu, and T. Matsuki, Eur. Phys. J. C **78**, 915 (2018).
[25] J. Segovia, P. G. Ortega, D. R. Entem, and F. Fernández, Phys. Rev. D **93**, 074027 (2016).
[26] T. Barnes, in *Proceedings of the CLEO-C and BES-III Joint Workshop on Charm, QCD and Tau Physics* (2004), arXiv: hep-ph/0406327.
[27] H. Mutuk, Adv. High Energy Phys. **2018**, 5961031 (2018).
[28] W.-J. Deng, H. Liu, L.-C. Gui, and X.-H. Zhong, Phys. Rev. D **95**, 074002 (2017).
[29] X.-K. Dong, X.-H. Mo, P. Wang, and C.-Z. Yuan, Chin. Phys. C **44**, 083001 (2020).
[30] F. J. Llanes-Estrada, Phys. Rev. D **72**, 031503(R) (2005).
[31] L. Y. Dai, M. Shi, G.-Y. Tang, and H. Q. Zheng, Phys. Rev. D **92**, 014020 (2015).
[32] G.-J. Ding, J.-J. Zhu, and M.-L. Yan, Phys. Rev. D **77**, 014033 (2008).
[33] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[34] S. Godfrey, Phys. Rev. D **31**, 2375 (1985).
[35] S. Godfrey and N. Isgur, Phys. Rev. D **34**, 899 (1986).
[36] S. Godfrey, Phys. Rev. D **70**, 054017 (2004).
[37] S. Godfrey, Phys. Rev. D **72**, 054029 (2005).
[38] J. Vijande, F. Fernandez, and A. Valcarce, J. Phys. G **31**, 481 (2005).
[39] J. Segovia, D. R. Entem, and F. Fernandez, Phys. Lett. B **662**, 33 (2008).
[40] Z. Zhao, K. Xu, A. Kaewsnod, X. Liu, A. Limphirat, and Y. Yan, Phys. Rev. D **103**, 116027 (2021).
[41] Z. Zhao, K. Xu, A. Kaewsnod, X. Liu, A. Limphirat, and Y. Yan, Phys. Rev. D **105**, 036001 (2022).
[42] Z. Zhao, K. Xu, A. Kaewsnod, X. Liu, A. Limphirat, and Y. Yan, Few Body Syst. **62**, 80 (2021).
[43] K. Xu, A. Kaewsnod, X. Y. Liu, S. Srisuphaphon, A. Limphirat, and Y. Yan, Phys. Rev. C **100**, 065207 (2019).
[44] K. Xu, A. Kaewsnod, Z. Zhao, X. Y. Liu, S. Srisuphaphon, A. Limphirat, and Y. Yan, Phys. Rev. D **101**, 076025 (2020).
[45] F. Schoberl, Nuovo Cimento Soc. Ital. Fis. **94A**, 79 (1986).
[46] T. Kawanai and S. Sasaki, Phys. Rev. Lett. **107**, 091601 (2011).
[47] Y. Ikeda and H. Iida, Prog. Theor. Phys. **128**, 941 (2012).
[48] R. Van Royen and V. F. Weisskopf, Nuovo Cimento A **50**, 617 (1967); **51**, 583(E) (1967).
[49] R. Barbieri, E. d'Emilio, G. Curci, and E. Remiddi, Nucl. Phys. **B154**, 535 (1979).
[50] K. Heikkila, N. A. Tornqvist, and S. Ono, Phys. Rev. D **29**, 110 (1984); **29**, 2136(E) (1984).
[51] S. Ono, A. I. Sanda, and N. A. Tornqvist, Phys. Rev. D **34**, 186 (1986).
[52] S. Ono, N. A. Tornqvist, J. Lee-Franzini, and A. I. Sanda, Phys. Rev. Lett. **55**, 2938 (1985).
[53] Y. Lu, M. N. Anwar, and B.-S. Zou, Phys. Rev. D **94**, 034021 (2016).
[54] H.-F. Fu and L. Jiang, Eur. Phys. J. C **79**, 460 (2019).
[55] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).
[56] L. Y. Glozman and D. O. Riska, Phys. Rep. **268**, 263 (1996).
[57] D. Besson *et al.* (CLEO Collaboration), Phys. Rev. Lett. **54**, 381 (1985).
[58] D. M. J. Lovelock *et al.*, Phys. Rev. Lett. **54**, 377 (1985).
[59] R. Mizuk *et al.* (Belle Collaboration), J. High Energy Phys. 10 (2019) 220.
[60] I. Adachi *et al.* (Belle-II Collaboration), Phys. Rev. Lett. **130**, 091902 (2023).
[61] A. Ali, L. Maiani, A. Y. Parkhomenko, and W. Wang, Phys. Lett. B **802**, 135217 (2020).
[62] W. Sreethawong, K. Xu, and Y. Yan, J. Phys. G **42**, 025001 (2015).
[63] A. Limphirat, W. Sreethawong, K. Khosonthongkee, and Y. Yan, Phys. Rev. D **89**, 054030 (2014).
[64] M. Ablikim *et al.* (BES Collaboration), eConf **C070805**, 02 (2007).
[65] K. K. Seth, Phys. Rev. D **72**, 017501 (2005).
[66] A. Osterheld *et al.* (Crystal Ball Collaboration), SLAC Report No. SLAC-PUB-4160, 1986.
[67] J. Z. Bai *et al.* (BES Collaboration), Phys. Rev. Lett. **88**, 101802 (2002).