Hidden supersymmetric dark sectors

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The attractive feature of supersymmetry is predictive power, due to the large number of calculable properties and to coupling nonrenormalization. This power can be fully expressed in hidden sectors where supersymmetry may be exact, as these sectors are secluded from the visible one where instead supersymmetry must be broken. This suggests a new paradigm for supersymmetric dark sectors, where supersymmetry is exact at the dark matter scale, implying that many properties of hidden supersymmetric dark sectors can be fully computed. As a proof of concept we discuss a concrete example based on $\mathcal{N} = 1$ super Yang-Mills.

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I. INTRODUCTION

Supersymmetry [1–3] is arguably the most natural extension of Minkowsky space-time symmetries, beyond the Poincaré algebra [4]. It links bosonic and fermionic particles, hence relating integer and semi-integer spins [5]. Furthermore, supersymmetric field theories enjoy special properties that make for an exciting theoretical playground. For instance, the nonrenormalization of a set of operators [3,6] renders supersymmetry a prime candidate to explain the quantum stability of scalar masses [7], in particular for the Higgs boson. As a second example, theories with extended supersymmetries can be solved exactly without the need for a perturbative expansion [8]. This idyllic theoretical situation contrasts with the physical real world, where the Standard Model of particle physics (SM) lacks any supersymmetric feature, *in primis* a mirror fermion-boson symmetric particle content. Henceforth, if supersymmetry is realized, it must be at energy scales well beyond the ones already probed at accelerators, like the Large Hadron Collider (LHC) at CERN, or in secluded sectors, at most feebly connected to the SM. However, there are phenomena that are not captured by the SM, specifically the presence of dark matter (DM), which constitutes 75% of the matter budget of the present-day Universe [9]. The only DM direct evidence involves its gravitational interactions, which have been observed at a variety of scales: from galaxies, via the rotational velocity of luminous stars and gas, to gravitational lensing of galaxy clusters, to ultimately the global properties of the observable Universe, revealed via the cosmic microwave background. Many models propose a particle candidate to describe DM, including particles predicted by (broken) supersymmetry [10,11] and hidden sectors [12]. See Refs. [13–15] for comprehensive reviews on DM models and properties. For a long time, the minimal supersymmetric SM (MSSM) has offered a standard DM candidate in the form of the lightest supersymmetric particle (LSP), which is usually a neutralino or gravitino. Its stability, however, is not guaranteed directly by supersymmetry as it requires an imposed matter symmetry (R–parity), which is also used to forbid proton decay [16]. As the DM candidate is the lightest partner of the SM states, its mass and nature in the LSP picture is strongly related to the breaking of supersymmetry. It should be remarked that a more minimal solution to the proton decay problem in supersymmetry is the imposition of baryon number

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conservation, hence without the emergence of a LSP DM candidate. Furthermore, the neutralino LSP, having electroweak interactions, leads to detectable signatures both at colliders and at direct detection experiments. The absence of a signal so far has imposed severe constraints on the MSSM parameter space.

In this article we propose a new paradigm of supersymmetric DM (sDM), alternative to the LSP scenario, as it is characterized by an almost exact supersymmetry at the scale of the DM mass. The realization of the sDM scenario requires the addition of a dark sector to the MSSM, while R-parity could be explicitly broken or, if preserved, the LSP may play the role of a subleading DM component with small thermal relic abundance. In order to preserve the supersymmetric properties of the sDM, the new supersymmetric states in the dark sector must either be heavier than the breaking scale of the MSSM, or be feebly coupled to the MSSM fields. The latter possibility features a hidden sector that has the advantage of benefiting from supersymmetric properties, primarily its calculability. This property is of particular interest if the new sector confines, as it is the case for the simplest models involving new gauge symmetries. Models of hidden supersymmetric sectors have been considered in the literature under various motivations [17–20]. Henceforth, the main model building ingredient is a hierarchy between the supersymmetry breaking scales in the visible sector, needed to be above a few TeV due to collider bounds, and in the Dark sector, where supersymmetry is assumed to be valid at the DM mass scale. Also, the properties of the hidden supersymmetric sector are disjointed from phenomenological requirements related to the SM physics, opening a new set of simple and attractive possibilities.

II. GENERAL FRAMEWORK

A schematic realization of the sDM scenario is illustrated in Fig. 1: both the visible and hidden sectors consist of supersymmetric theories, connected to each other via feeble interactions (a similar setup was considered in [19]). Those can consist of a small supersymmetric coupling [20] or interactions due to a heavy mediator. As such, the dark sector can be populated via the freeze-in mechanism [21] to generate the correct DM relic density. Both sectors are connected to the same supersymmetry breaking source (~~SUSY~~), however the breaking is mediated by different mechanisms. To generate a clear hierarchy between the ~~SUSY~~ scales in the two sectors, we choose gravity mediation [22–24] for the hidden sector and gauge mediation [25–27] for the visible sector. Hence, the model setup proposed in Fig. 1 draws a connection between the Planck scale $M_P = 10^{19}$ GeV and the supersymmetry breaking scales in the visible sector, $m_{\rm VS}$, and in the hidden sector, $m_{\rm HS}$. We recall that the latter must be smaller than the DM mass, $m_{\rm HS} \ll m_{\rm DM}$. Assuming, as an illustration, that the

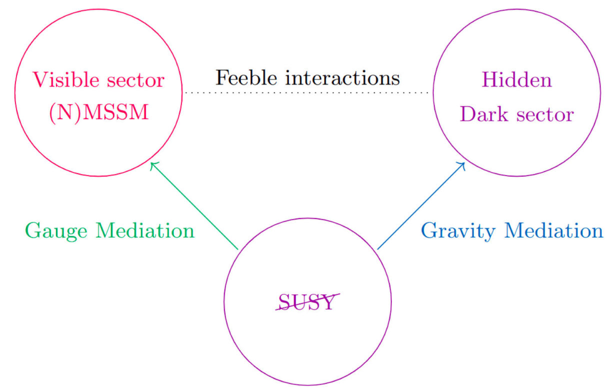

FIG. 1. Illustration of the different sectors of the model with a hidden sDM. Purple circles are hidden sectors while the red circle is the visible sector of the theory. The source of supersymmetry breaking (~~SUSY~~) is transmitted to the visible and the hidden sectors through gauge and gravity mediation, respectively. There are also feeble interactions between the visible and the hidden sectors.

~~SUSY~~ scale is generated via F-term breaking, the scales are related as follows:

$$m_{\rm HS} \sim \frac{\langle F_X \rangle}{M_P}, \qquad m_{\rm VS} \sim \frac{g_G^2}{16\pi^2} \frac{\langle F_X \rangle}{M_G}, \tag{1}$$

where g_G is the gauge coupling of the heavy gauge mediators of mass M_G, which needs to sit well below the Planck scale for consistency. Structure formation typically requires the DM mass to be above the 100 keV scale [28,29]. Hence, imposing $m_{\rm HS} < 100$ keV and as a sample scale $m_{\rm VS} \sim 1$ TeV yields the following estimates:

$$\langle F_X \rangle \lesssim 10^{15}\ \mathrm{GeV}^2, \qquad M_G \lesssim 10^{10}\ \mathrm{GeV}, \tag{2}$$

for $g_G \sim 1$. We shall retain them as typical orders of magnitude for the model building. Note that $m_{\rm VS}$ also contributes to $m_{\rm HS}$ via the interactions between these two sectors, thus requiring such interactions to be feeble. This scenario potentially suffers from the cosmological gravitino problem [30]. Gravitinos are the spin-$3/2$ super-partners of gravitons, and they are produced in the early Universe via gravitational interactions. Their presence at late times can efficiently suppress structure formation. One possible solution is to lower their mass at the eV scale [31], with most recent bounds reading $m_{3/2} \lesssim 4.7$ eV [32]. As $m_{3/2} \sim m_{\rm HS}$, this mass limit fits well within our scenario if

$$\langle F_X \rangle \lesssim 10^{10}\ \mathrm{GeV}^2. \tag{3}$$

The sDM candidate, which is the lightest state in the hidden supersymmetric sector, could decay into gravitinos, hence repopulating them in the late universe. There are two kinds of decays: channels involving the visible sector, doubly suppressed by the Planck mass and the feeble interactions;

and decays among components of the same multiplet in the hidden sector, suppressed by the small mass splitting and on-threshold masses (as $\Delta m \sim m_{3/2} \sim m_{\rm HS}$). This deems our sDM scenario generally free from the cosmological gravitino problem.

III. HIDDEN SYM: AN EXAMPLE WITH ULTRAVIOLET FREEZE-IN

As a concrete, simple and calculable example of the hidden dark sector, we consider a $\mathcal{N}=1$ supersymmetric Yang-Mills (SYM) theory [3], based on the gauge symmetry $\mathrm{SU}(N_c)$. This class of theories possesses classical scale-invariance, which is however broken at quantum level (except for $\mathcal{N}=4$ SYM, which is fully integrable). Hence, like quantum chromodynamics, the theory is expected to confine at low energies and generate a dynamical mass scale Λ. This scale, which is fully supersymmetric, controls the mass of the lightest states in the low energy theory. As no SM states are charged under the hidden $\mathrm{SU}(N_c)$ gauge symmetry, heavy mediators need to be introduced to couple the hidden sector to the visible one. As freeze-in is due to a higher dimensional operator, the relic density is ultraviolet sensitive [33] and will be determined by the reheating temperature at the end of inflation, $T_{\rm rh}$. This observation confirms the necessity of a very light gravitino mass, as discussed in the previous section, else a strong upper bound must be imposed on the reheating temperature to suppress gravitino production.

As the hidden sector consists of pure gauge fields, the simplest mediator fields consist in a set of matter superfields [34] $\mathcal{F}_i$ and $\tilde{\mathcal{F}}_i$ in conjugate representations of $\mathrm{SU}(N_c)$. To couple the mediators to the MSSM, we can include an $\mathrm{SU}(2)_w$ doublet $i=Q$ and an up-type singlet $i=U$ with superpotential couplings

$$\int d^2\theta[M_{\mathcal{F}}(\mathcal{F}_Q\tilde{\mathcal{F}}_Q+\mathcal{F}_U\tilde{\mathcal{F}}_U)+\lambda_M H_u\mathcal{F}_Q\tilde{\mathcal{F}}_U]+\mathrm{H.c.} \quad (4)$$

where, for simplicity, we assume the same mass for both multiplets. A similar scenario can be obtained with a down-type singlet, $i=D$, coupling to H_d. Integrating out the massive multiplets, we obtain an effective coupling between the SYM gauge superfields W^α and the singlet superfield in the form of a dimension-6 operator at leading order:

$$\mathcal{L}_{\text{dim-6}} \supset \frac{1}{32\pi}\int d^2\theta\left[\mathrm{Im}(\tau\mathrm{Tr}W^\alpha W_\alpha)\times\left(1+\frac{1}{\Lambda_M^2}H_uH_u^\dagger+\cdots\right)\right]. \quad (5)$$

A dimension-5 coupling can be obtained if the visible sector consists of the NMSSM, i.e., the MSSM with an additional gauge singlet $\hat{N}$ [35]. In this case, only one mediator is needed, singlet under the SM gauge interactions, with superpotential couplings

$$\int d^2\theta[M_{\mathcal{F}}\mathcal{F}\tilde{\mathcal{F}}+\lambda_M\hat{N}\mathcal{F}\tilde{\mathcal{F}}]+\mathrm{H.c.} \quad (6)$$

Integrating out the massive multiplet, we obtain a leading dimension-5 operator

$$\mathcal{L}_{\text{dim-5}} \supset \frac{1}{32\pi}\int d^4\theta\left[\mathrm{Im}(\tau\mathrm{Tr}\ W^\alpha W_\alpha)\times\left(1+\frac{1}{4\pi\Lambda_M}(\hat{N}+\hat{N}^\dagger)+\frac{1}{\Lambda_M^2}\hat{N}\hat{N}^\dagger+\cdots\right)\right]. \quad (7)$$

In both Eqs. (5) and (7), the dots represent higher dimensional operators, while the scale Λ_M is defined as

$$\frac{1}{\Lambda_M}\sim\frac{\lambda_M}{4\pi M_{\mathcal{F}}}. \quad (8)$$

In both cases, the effective scale Λ_M determines the production rate of the dark sector states from the thermal bath of the visible sector. As the freeze-in production is dominated by the reheating temperature, which we assume to be higher than the condensation scale of the SYM sector in order to be able to produce the DM states, to compute the relic density we can use directly the Lagrangians in Eqs. (5) or (7). The standard Boltzmann equation applies to the evolution of the number density of states in the hidden sector:

$$\frac{dn_{\rm HS}}{dt}+3Hn_{\rm HS}\simeq\frac{T}{512\pi^5}\int_0^\infty ds|\mathcal{M}|^2\sqrt{s}K_1(\sqrt{s}/T), \quad (9)$$

where T is the temperature in the visible sector, H the Hubble parameter and, in our model, the amplitude reads

$$|\mathcal{M}|^2=N_c\begin{cases}\frac{s}{4\Lambda_M^2} & \text{for dim-5,}\\ \frac{33s^2}{256\Lambda_M^4} & \text{for dim-6.}\end{cases} \quad (10)$$

An approximate solution of the Boltzmann equation (9) yields the following comoving number density

$$Y_{\rm HS}=\frac{n_{\rm HS}}{s_e}\simeq\begin{cases}\frac{45M_PT_{\rm rh}N_c}{128\pi^7 1.66g_*^s\sqrt{g_*^\rho}\Lambda_M^2} & \text{(dim-5),}\\ \frac{1485M_PT_{\rm rh}^3N_c}{1024\pi^7 1.66g_*^s\sqrt{g_*^\rho}\Lambda_M^4} & \text{(dim-6).}\end{cases} \quad (11)$$

This quantity depends dominantly on the physics at high scales, hence it can be computed directly in the SYM theory, without knowledge of the dynamics at low energies. Assuming that this number density is directly converted into a number density of DM candidates, we estimate

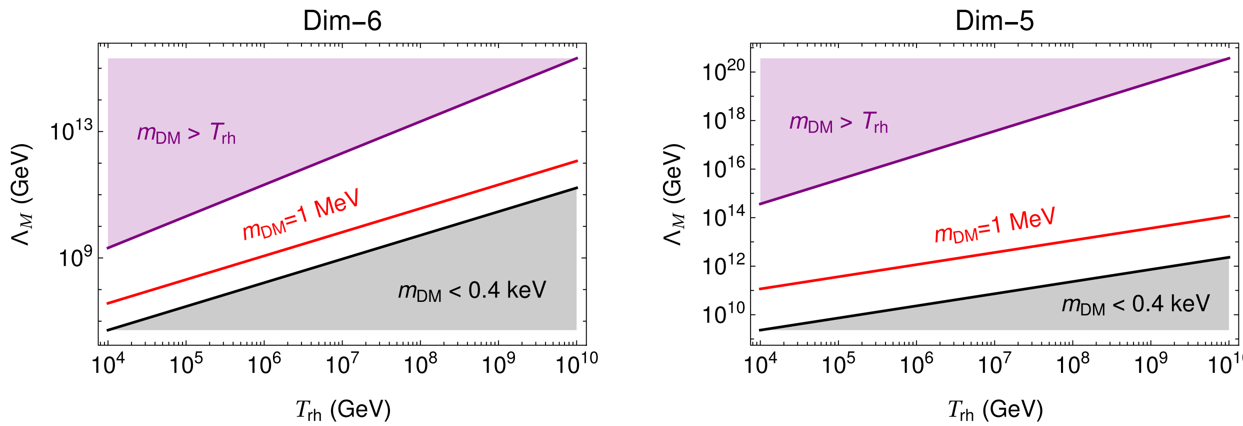

FIG. 2. Suitable values of $T_{\rm rh}$ and Λ_M for obtaining the observed DM relic density via ultraviolet freeze-in in the SYM model for $N_c = 3$. The left panel corresponds to the dim-6 operator, while the right one to the dim-5 operator. For different values of N_c, $m_{\rm DM}$ is rescaled to keep $N_c\, m_{\rm DM}$ fixed. The preferred region lies between the red line and the lower edge of the lilac region.

$$\Omega_{\rm DM} h^2 \simeq \begin{cases} 0.134 \times 10^{21} N_c \frac{T_{\rm rh} m_{\rm DM}}{\Lambda_M^2} & \text{(dim-5)}, \\ 0.185 \times 10^{21} N_c \frac{T_{\rm rh}^3 m_{\rm DM}}{\Lambda_M^4} & \text{(dim-6)}. \end{cases} \tag{12}$$

After repopulation by freeze-in, the dark sector undergoes a nontrivial thermal history, characterized by thermalization via self-interactions and the confinement phase transition [36]. We checked that a more accurate treatment of these effects leads to results similar to our naive estimate. By matching Eq. (12) to the measured value, we can relate the required DM mass to the reheating temperature and mediation scale Λ_M, as shown in Fig. 2. The region in lilac is excluded as the required DM mass is larger than $T_{\rm rh}$. In the gray region, the dark sector is produced thermally in the early Universe instead of via freeze-in, corresponding to the model-independent bound $m_{\rm DM} \gtrsim 0.4$ keV [33]. Finally, we highlighted by a red line the region where $m_{\rm DM} = 1$ MeV, as below this line the DM is too light to allow effective structure formation [28,29]. This leaves an allowed band in the parameter space, with DM masses between the MeV and the reheating scale. Finally, we checked that the ~~SUSY~~ scale in the hidden sector remains stable under radiative corrections within the relevant parameter space in Fig. 2. A contribution to $m_{\rm HS}$ comes at loop level, giving in both cases

$$\delta m_{\rm HS} \approx \frac{\lambda_{\rm MSSM}^2}{16\pi^2} \frac{m_{\rm VS}^3}{\Lambda_M^2}, \tag{13}$$

for a generic MSSM coupling $\lambda_{\rm MSSM}$. For $\lambda_{\rm MSSM} \sim 1$ and $m_{\rm VS} \sim 1$ TeV, $\delta m_{\rm HS} \lesssim 1$ eV as long as $\Lambda_M \gtrsim 10^8$ GeV.

IV. LOW ENERGY DYNAMICS OF THE HIDDEN SYM

At low energy, where the DM mass is dynamically generated, DM interactions are described by an effective field theory where the SYM gluons and gluinos are confined into massive bound states. The original construction by Veneziano and Yankeliowicz (VY) only included one superfield S, corresponding to the supersymmetric gluino-ball states [37]. Later the model was generalized to include glue-ball states in terms of a chiral superfield χ (gVY). The Lagrangian for the model reads [38,39]:

$$\begin{aligned} \mathcal{L}_{\rm gVY} = {} & \frac{9N_c^2}{\alpha} \int d^4\theta (S^\dagger S)^{\frac{1}{3}} (1 + \gamma \chi\chi^\dagger) \\ & + \frac{2N_c}{3} \int d^2\theta \left[S \left(\log \left(\frac{S}{\Lambda^3} \right)^{N_c} - N_c \right) \right. \\ & \left. - N_c S \log \left(-e \frac{\chi}{N_c} \log \chi^{N_c} \right) \right] + \text{H.c.} \end{aligned} \tag{14}$$

Besides the number of colors N_c and the confinement scale Λ, the interactions depend on two parameters: α and γ (e is the Euler's number). The former, mainly controls the overall strength of the couplings. The latter, instead, crucially controls the spectrum of the theory, which can be obtained from the action in Eq. (14) [39] (see the appendix for further details). After diagonalization, the two mass eigenvalues can be written as

$$m_{L/H} = \alpha \Lambda \mu_{L/H}(\gamma), \tag{15}$$

where $\alpha\Lambda$ sets the scale, while the mass ratio and the mixing between the light (L) and heavy (H) supersymmetric multiplets only depend on the parameter γ. For $\gamma \to 0$, the glue-ball states decouple and the light state consists of pure gluino-balls (as in the original VY model) with mass $m_L \equiv m_S = 2/3\, \alpha\Lambda$, while for larger γ a mixing is always in place. In principle, γ is not a free parameter as it is fully determined by the dynamics of the SYM interactions. Large-N_c arguments support that the gluino-balls should be lightest [39], pointing toward small $\gamma \lesssim 1$. On the

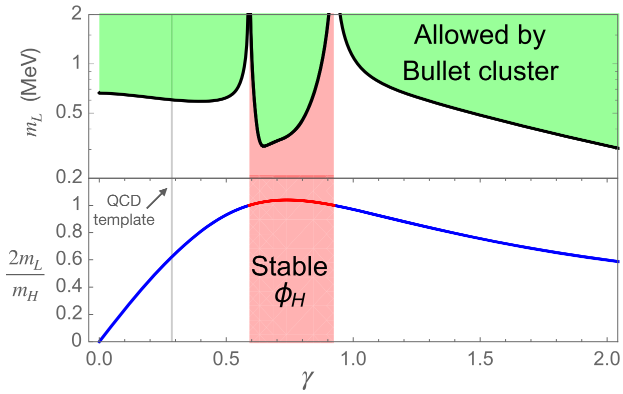

FIG. 3. Lower limit on $m_{\rm DM} = m_L$ stemming from the bullet cluster bound on the self-scattering cross section, as a function of γ and for $\alpha = 1$ and $N_c = 3$ (top panel). The bound is compared to the ratio of the mass eigenvalues (bottom panel). The red band corresponds to a two-component DM case, where the bullet bound should be considered as a conservative estimate. The vertical line marks the QCD-inspired benchmark value of γ.

other hand, perturbative arguments suggest lightest glueballs [40,41]. Lattice results are available for $N_c = 2$ [42,43] and $N_c = 3$ [44], however unable to resolve the question yet [45]. Note that ~~SUSY~~ effects can be included in the form of a gaugino mass [46,47], however as $m_{\rm HS} \sim$ eV, this effect can be neglected for our work.

The ratio of the two masses only depends on γ, as shown in the bottom panel of Fig. 3. Only within the range $0.59 \lesssim \gamma \lesssim 0.92$ the heavy state cannot decay into two light ones, hence the model would predict a two-component DM model. This region is highlighted in red in the figure. For all other values of γ, the DM is constituted purely of the lightest states. One-flavor QCD can be used to estimate the mass ratio, as suggested in Ref. [39], giving the value of $\gamma_{\rm QCD} \sim 0.29$, which we could consider as a benchmark value for γ.

The supersymmetric Lagrangian in Eq. (14) contains self-interactions which are mainly controlled by α and γ. The DM self scattering is bound by the Bullet cluster observation, providing a generic upper limit $\frac{\sigma_{\rm DM}}{m_{\rm DM}} \leq 2\ {\rm cm}^2 \cdot {\rm g}^{-1}$ [48], where $\sigma_{\rm DM}$ contains all $2 \to 2$ self-scattering at low velocity. In our model, it is sufficient to consider the self-scattering of the lightest mass eigenstate, φ_L. Hence, we computed all the scattering cross sections of the scalar components, $\sigma(\varphi_L \varphi_L^\dagger)$, $\sigma(\varphi_L \varphi_L)$ and $\sigma(\varphi_L^\dagger \varphi_L^\dagger)$ to all allowed final states. Assuming that the DM halo has an equal distribution of all DM components, $\sigma_{\rm DM}$ is replaced by the average of the cross sections, as detailed in the appendix. The final cross section has the form

$$\sigma_{\rm DM} = \frac{\alpha^6}{N_c^4} \frac{|\tilde{\mathcal{A}}(\gamma)|^2}{128\pi\, m_L^2}, \tag{16}$$

where the effective amplitude in the numerator is a pure function of γ, depending on the nontrivial mixing between the two states in the gVY model. Hence, the Bullet cluster observation imposes a lower limit on m_L, which is shown in the top panel of Fig. 3 for $\alpha = 1$ and $N_c = 3$, as a function of γ. Due to the scaling of the cross section, the limit for other parameters can be obtained by keeping $N_c^{4/3} m_L/\alpha^2$ constant. The features appearing at the edge of the red band are due to the resonant contribution of the heavy state, when $m_H \sim 2m_L$, enhancing the cross section.

SYM also suffers from the presence of domain walls due to the N_c degenerate minima [49]: their surface tension, proportional to $\sigma \propto N_c^2 \Lambda^3 \sim (N_c^2/\alpha^3) m_{\rm DM}^3$, is bound to be below the MeV scale [50], in order not to dominate the total energy of the Universe. An alternative solution would be to lift the degeneracy of the minima by adding a constant term to the superpotential: as a consequence, the domain walls become unstable and source gravitational waves at late times [51,52]. If the constant term is related to the gravitino mass, in order to cancel the contribution to the cosmological constant [53], the peak frequency of the gravitational waves can be directly related to the gaugino mass. In our case, for large reheating temperatures and $m_{3/2} \lesssim 1$ eV, the predicted frequencies are below 10^{-2} Hz and well within the reach of the LISA experiment [51].

Hence, a nontrivial limit on the DM mass is imposed by the sizeable self-interactions of the lightest DM state and by domain walls, which further reduce the available parameter space in Fig. 2 and limits the DM mass to be around the MeV. Heavier resonances could also contribute to the self-scattering, however this effect crucially depends on the mass spectrum. One-flavor QCD on the lattice suggests the presence of relatively light spin-1 resonances [54]. SYM has also been considered as a model for inflation [55], however requiring a too large composite scale to also provide a good DM candidate.

V. CONCLUSION AND OUTLOOK

The general proposal depicted in Fig. 1 has been illustrated with a simple $\mathcal{N} = 1$ SYM theory. This scenario can support a variety of other hidden supersymmetric sectors to generate a sDM candidate, hence providing a phenomenological application to many different theories. For instance, by extending the supersymmetry to the maximal type in four dimensions, $\mathcal{N} = 4$ [56], the theory can feature a superconformal phase [57]. Furthermore, this theory is believed to be solvable and a lot of theoretical studies are ongoing [58], including applications of the AdS/CFT duality [59,60]. The case of conformal thermal DM has been studied in general in Ref. [61], finding a good description of the DM relic density for masses around the MeV. Another interesting class of theories yields calculable low energy interactions by using duality properties: the prime examples are $\mathcal{N} = 2$ Seiberg-Witten theories [62,63]. Furthermore, SQCD has been considered as a source of self-interacting DM [20] to solve puzzles in structure formation. All these theories imbue the dark sector with clear predictive power, which has not been fully exploited for the DM phenomenology, yet.

In conclusion, our general proposal opens up a new avenue for the constructions of models that may explain the presence of dark matter in the universe. Supersymmetry imbues the dark sector with calculability and predictive power, also motivating a detailed study of supersymmetric theories on the lattice. In the supersymmetric dark matter scenario, both spins are always present with degenerate masses. This could lead to observable features in cosmology and astrophysics. For instance, Bose-Einstein condensation may occur in high density regions due to the presence of bosons degenerate with the fermionic partners [64,65]. Bose-Einstein condensate have also been considered as the source for DM itself [66]. Furthermore, if the sDM candidates can be effectively captured by neutron stars, its presence could be revealed in neutron star merger events [67]. The spin of the DM particles may give characteristic modifications to the neutron star elasticity, hence giving testable effects on the multi-messenger signals coming from the merger [68]. Finally, an indirect signature may stem from cosmological gravitational waves, produced during the confinement phase transition or by domain walls, with frequencies determined by the DM properties. In this work we have explored a simple coupling between the visible and hidden sectors, however other possibilities remain to be explored, providing potential links between the supersymmetric dark sector with inflation and baryogenesis.

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APPENDIX: THE SYM MODEL IN THE CONFINED PHASE

Below the confinement scale Λ, the $\mathcal{N}=1$ SYM model is described by a generalized Veneziano-Yankielowicz (gVY) supersymmetric Lagrangian [38,39]. It contains the following Kähler potential:

$$K(\tilde{S},\tilde{S}^\dagger,\chi,\chi^\dagger) = \frac{9N_c^2\Lambda^2}{\alpha}(\tilde{S}^\dagger\tilde{S})^{\frac{1}{3}}(1+\gamma\chi^\dagger\chi), \tag{A1}$$

and a superpotential

$$W(\tilde{S},\chi) = \frac{2N_c\Lambda^3}{3}\tilde{S}\left[\log\tilde{S}^{N_c} - N_c - N_c\log\left(-e\frac{\chi}{N_c}\log\chi^{N_c}\right)\right], \tag{A2}$$

where we have defined a dimensionless gluino-ball field $\tilde{S}=S/\Lambda^3$. The superpotential implies the presence of a supersymmetric vacuum, characterized by

$$\frac{\partial W}{\partial\tilde{S}} = 0, \qquad \frac{\partial W}{\partial\chi} = 0, \tag{A3}$$

whose solutions describe the N_c vacua of SYM theories [38]:

$$\chi_0 = \frac{1}{e}\exp\left(-2\pi i\frac{k}{N_c}\right), \qquad \tilde{S}_0 = \exp\left(-2\pi i\frac{k}{N_c}\right), \quad \text{where } k=0,\ldots N_c-1. \tag{A4}$$

In the following, we will consider the vacuum with $k=0$.

For our purposes, it suffices to study the spectrum and interactions of the scalar components, as the properties of the fermionic partners are tied by supersymmetry. Expanding the superfield Lagrangian, one obtains the standard formula

$$\mathcal{L}_{\text{scalars}} = \partial_\mu\varphi^l g_{lm}\partial^\mu\varphi^{\dagger,m} - \frac{\partial W}{\partial\varphi^l}g^{lm}\frac{\partial W^\dagger}{\partial\varphi^{\dagger,m}}, \tag{A5}$$

where $m,l=\tilde{S},\chi$ labels the two superfields and

$$g_{lm} = \frac{\partial^2 K}{\partial\varphi^l\partial\varphi^{\dagger,m}}, \qquad g^{l,m} = (g^{-1}_{l,m})^T, \tag{A6}$$

are the Kähler metric and its inverse, respectively. To obtain the mass eigenstates, it suffices to expand the above Lagrangian, diagonalize and normalize the kinetic term, and finally diagonalize the resulting mass term, as described in Ref. [38]. Finally, one obtains the following masses

$$m_{L,H} = \alpha\Lambda\mu_{L,H}(\gamma), \tag{A7}$$

where the dimensionless functions $\mu_{L,H}$ only depend on γ. The numerical values are shown in Fig. 4. In the limit $\gamma\to 0$, the glueball field χ decouples (it becomes an auxiliary field with no kinetic term), and the only remaining state (gluino-ball) has mass

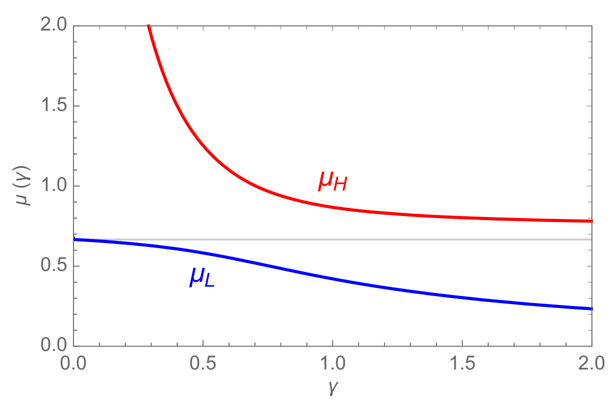

FIG. 4. Function of γ defining the mass eigenvalues of the two states in the gVY model. The horizontal line corresponds to the pure gluino-ball mass, achieved for $\gamma=0$, as $\mu_L(0)=2/3$.

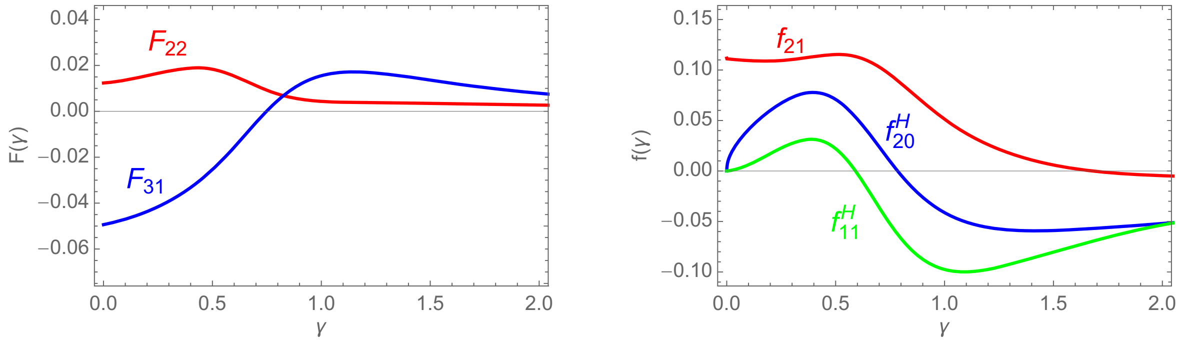

FIG. 5. Functions of γ defining the quartic (left) and trilinear (right) couplings of the scalar components, relevant for the self-scattering of the lightest state.

$$m_S = \frac{2}{3}\alpha\Lambda. \tag{A8}$$

To estimate the self-interactions of the DM candidate, we focus on the self-scattering of the scalar mode of the lightest mass eigenstate. Here, we will assume that the DM halo is equally populated by the various components, including their antiparticles, and that the cross sections involving fermions are related to those with pure scalars by supersymmetry. Hence, the effective DM cross section will be an average of all the possible scalar self-scattering:

$$\sigma_{\mathrm{DM}} = \frac{2\sigma(\varphi_L\varphi_L^\dagger) + \sigma(\varphi_L\varphi_L) + \sigma(\varphi_L^\dagger\varphi_L^\dagger)}{4} = \frac{\sigma(\varphi_L\varphi_L^\dagger) + \sigma(\varphi_L\varphi_L)}{2}, \tag{A9}$$

where we include all possible final states, and we use the identity $\sigma(\varphi_L\varphi_L) = \sigma(\varphi_L^\dagger\varphi_L^\dagger)$. The potential generates both quartic and trilinear interactions. The ones relevant for our purpose can be parametrized as

$$\mathcal{L} \supset C_{31}(\varphi_L^3\varphi_L^\dagger + \mathrm{H.c.}) + C_{22}\varphi_L^2(\varphi_L^\dagger)^2 + m_L(c_{21}\varphi_L^2\varphi_L^\dagger + \mathrm{H.c.}) + m_H(c_{20}^H\varphi_L^2\varphi_H^\dagger + c_{11}^H\varphi_L\varphi_L^\dagger\varphi_H^\dagger + \mathrm{H.c.}). \tag{A10}$$

An explicit calculation shows that the couplings C_x and $c_x^{(H)}$ can be written as

$$C_x = \frac{\alpha^3}{N_c^2}F_x(\gamma), \qquad c_x = \sqrt{\frac{\alpha^3}{N_c^2}}f_x(\gamma), \qquad c_x^H = \sqrt{\frac{\alpha^3}{N_c^2}}f_x^H(\gamma), \tag{A11}$$

where F, f and f^H are functions of γ only. The values of these functions are shown numerically in Fig. 5. The amplitudes for the $\varphi_L\varphi_L^\dagger$ scattering processes at zero velocity are given by

$$i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L^\dagger) = \frac{\alpha^3}{N_c^2}\left[4F_{22} + \frac{20}{3}f_{21}^2 + 4(f_{20}^H)^2 + 4(f_{11}^H)^2\left(1 - \frac{1}{4\zeta - 1}\right)\right], \tag{A12}$$

$$i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L) = \frac{\alpha^3}{N_c^2}\left[6F_{31} + \frac{20}{3}f_{21}^2 + 2f_{20}^Hf_{11}^H\left(2 - \frac{1}{4\zeta - 1}\right)\right], \tag{A13}$$

$$i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L^\dagger\varphi_L^\dagger) = i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L); \tag{A14}$$

where $\zeta = m_L^2/m_H^2$, being a pure function of γ. For the $\varphi_L\varphi_L$ scattering, we have

$$i\mathcal{A}(\varphi_L\varphi_L \to \varphi_L\varphi_L) = \frac{\alpha^3}{N_c^2}\left[4F_{22} + \frac{20}{3}f_{21}^2 - 4(f_{20}^H)^2\left(\frac{1}{4\zeta - 1}\right) + 8(f_{11}^H)^2\right], \tag{A15}$$

$$i\mathcal{A}(\varphi_L\varphi_L \to \varphi_L\varphi_L^\dagger) = i\mathcal{A}(\varphi_L\varphi_L^\dagger \to \varphi_L\varphi_L). \tag{A16}$$

Finally,

$$\sigma(\varphi_L\varphi_L^\dagger) = \sum_f \frac{|\mathcal{A}(\varphi_L\varphi_L^\dagger \to f)|^2}{128\pi m_L^2},$$
$$\sigma(\varphi_L\varphi_L) = \sum_f \frac{|\mathcal{A}(\varphi_L\varphi_L \to f)|^2}{128\pi m_L^2}, \tag{A17}$$

where the sum runs over all allowed final states. As mentioned above, the average of these cross sections is used to estimate the bound on m_L from the Bullet cluster. The two cross sections are plotted in Fig. 6 as a function of γ and in units of the mass. While $\sigma(\varphi_L\varphi_L)$ clearly shows the presence of the two resonances in the s-channel, where $m_H = 2m_L$, one resonance is missing for $\sigma(\varphi_L\varphi_L^\dagger)$. This is due to the fact that, in the latter, the s-channel is proportional to the coupling c_{11}^H, which vanished at the resonance, as shown in Fig. 5.

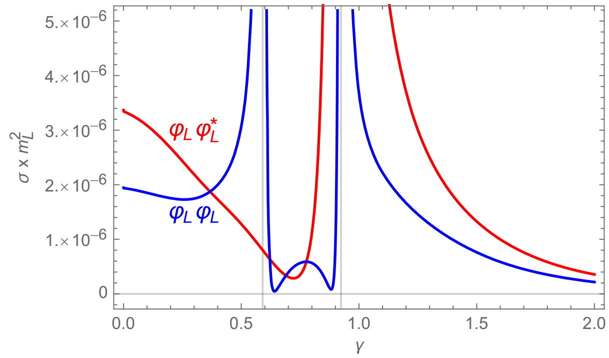

FIG. 6. Cross sections in units of the mass for the two relevant processes, as a function of γ. The vertical lines highlight the values of γ for which $m_H = 2m_L$.

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