

## New physics implications of vector boson fusion searches exemplified through the Georgi-Machacek model

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LHC searches for nonstandard scalars in vector boson fusion (VBF) production processes can be particularly efficient in probing scalars belonging to triplet or higher multiplet representations of the Standard Model  $SU(2)_L$  gauge group. They can be especially relevant for models where the additional scalars do not have any tree level couplings to the Standard Model fermions, rendering VBF as their primary production mode at the LHC. In this work we employ the latest LHC data from VBF resonance searches to constrain the properties of nonstandard scalars, taking the Georgi-Machacek model as a prototypical example. We take into account the theoretical constraints on the potential from unitarity and boundedness from below as well as indirect constraints coming from the signal strength measurements of the 125 GeV Higgs boson at the LHC. To facilitate the phenomenological analysis we advocate a convenient reparametrization of the trilinear couplings in the scalar potential. We derive simple correlations among the model parameters corresponding to the decoupling limit of the model. We explicitly demonstrate how a combination of theoretical and phenomenological constraints can push the Georgi-Machacek model towards the decoupling limit. Our analysis suggests that the VBF searches can provide key insights into the composition of the electroweak vacuum expectation value.

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### I. INTRODUCTION

The remarkable success of the Standard Model (SM), culminating in the discovery of the 125 GeV Higgs scalar at the LHC, has fuelled further investigations into the understanding of the precise nature of electroweak symmetry breaking (EWSB). In the  $SU(2)_L \times U(1)_Y$  gauge theory of the SM, the EWSB is driven by an  $SU(2)_L$  scalar doublet in such a way that the electric charge  $Q$ , with  $Q = T_{3L} + \frac{Y}{2}$ , remains conserved [1,2]. A question that naturally arises in this context is whether there exist additional scalar

multiplets of  $SU(2)_L$  beyond the SM (BSM) that contribute to the mechanism of EWSB. If such a scenario is indeed realized in nature, the nonstandard scalars originating from an extended scalar sector are expected to possess trilinear couplings with a pair of massive SM gauge bosons (of the form  $SV_1^\mu V_{2\mu}$ ) with strengths proportional to the vacuum expectation values (VEVs) of the extra multiplets. Thus, a straightforward way to look for such nonstandard scalars at the LHC is via their production in vector boson fusion (VBF) production processes, provided the BSM scenario in question can accommodate sizable trilinear couplings of the scalars to a pair of weak gauge bosons. In other words, the LHC searches for nonstandard scalar resonances in VBF production processes can potentially serve as a powerful tool to pin down any BSM contribution to the process of EWSB.

Nonstandard contribution to the electroweak VEV can arise only from the presence of scalar multiplets transforming nontrivially under the  $SU(2)_L$  part of the SM, since  $SU(2)_L$  singlet scalars, even if charged under the hypercharge gauge group  $U(1)_Y$ , do not participate in EWSB. In the popular multi-Higgs doublet extensions of

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the SM Higgs sector, the tree level trilinear couplings of the nonstandard Higgs bosons to a pair of weak gauge bosons vanish in the well-known “alignment limit” of the model [3,4]. In this limit of multi-Higgs doublet models, the Higgs doublet fields can be rotated to the so-called Higgs basis where the electroweak VEV can be effectively allocated entirely to only one of the Higgs doublets, while the VEVs of all other doublets remain zero [5,6]. The chosen Higgs doublet with nonzero VEV can then give rise to the SM-like Higgs boson observed at the LHC [7,8]. Thus, the nonstandard scalars in the alignment limit do not possess any trilinear coupling to the weak gauge boson pairs. With the current experimental data pointing strongly towards the validity of the alignment limit [9], the possibility of a significantly large trilinear coupling of the nonstandard scalars to a pair of weak vector bosons seems to be thin on the ground. One must note here that the introduction of additional scalar singlets and doublets to the SM field content preserves the custodial  $SU(2)$  symmetry and is therefore safe from the stringent constraints stemming from the precise measurement of electroweak  $\rho$  parameter.

Moving beyond the doublets, the next step will be to introduce an additional triplet scalar field into the SM scalar sector, giving rise to the so-called Higgs triplet model (HTM). Unlike the BSM scenarios with extra singlets and doublets, those involving triplets or higher multiplets of  $SU(2)_L$ , in general, have the tendency to modify the tree level value of the electroweak  $\rho$  parameter. For the HTM, the triplet VEV can significantly alter the tree level value of electroweak  $\rho$  parameter, making the model vulnerable to the constraints coming from precision electroweak measurements. The experimental determination of the electroweak  $\rho$  parameter,  $\rho \approx 1$ , severely restricts the VEV of the neutral component of the triplet to values less than a few GeV [10–12]. This tones down any potential enhancement in the trilinear couplings of the form  $SV_1^\mu V_{2\mu}$  which are directly proportional to the triplet VEVs [13], thereby reducing their observability in the high energy collider experiments. Thus, in order to accommodate a substantially large triplet VEV in a BSM scenario the vacuum state of the model must be designed in such a way that it preserves the custodial symmetry. A prototypical theoretical framework within this class of constructions is provided by the Georgi-Machacek (GM) model [14–16]. In the GM model, the SM scalar sector is extended by introducing two additional triplet scalar fields (one complex and one real) which acquire equal VEVs. This leads to an internal cancellation of the custodial symmetry breaking effects, securing  $\rho = 1$  at tree level. As a consequence of the unbroken custodial symmetry, the common triplet VEVs can now be sizable, allowing for significantly large trilinear couplings of the nonstandard Higgs bosons to the massive gauge boson pairs. Thus, the GM model can serve as the ideal candidate

to illustrate the potential of VBF searches at the LHC in determining the basic ingredients of the electroweak VEV.

Several LHC searches have been designed specifically to look for VBF production of nonstandard scalars in diboson final states [17–21]. In this work, we use the latest results from the full LHC run-II dataset to constrain the properties of additional triplet scalars, within the example framework of GM model. The GM model consists of several nonstandard scalar bosons which can give rise to very distinctive phenomenologies at the colliders. The physical scalar spectrum of the model can be categorized into a number of custodial multiplets, namely, one fiveplet ( $H_5^{++}, H_5^+, H_5^0, H_5^-, H_5^{--}$ ), one triplet ( $H_3^+, H_3^0, H_3^-$ ), and two singlets  $h$  and  $H$ . One important feature of this model is that the members of the custodial fiveplet do not couple to the SM fermions at tree level. Thus, the dominant direct bounds on the common mass of the custodial fiveplet can be obtained from the LHC searches looking for nonstandard bosons in VBF production processes, making these searches tailor-made for this model. In addition to collider data, we also consider the theoretical constraints from perturbative unitarity and boundedness-from-below (BFB) conditions on the potential. A suitable reparametrization of the trilinear couplings of the scalar potential is prescribed, making way for efficient phenomenological analysis. We show that the decoupling limit of the GM model can be expressed in terms of simple correlations among the physical masses, mixing angles and the triplet VEV. We also take into consideration the bounds coming from the measurement of the properties of 125 GeV Higgs boson, which we identify as the lightest  $CP$ -even custodial singlet boson  $h$  present in the GM model. In our analysis, we demonstrate that the VBF searches can provide complementary constraints to the theoretical bounds on the model parameter space. We also give estimates for the potential of the future colliders such as the high luminosity (HL-)LHC, Future Circular colliders etc. to constrain the remaining parameter space further. We explicitly show that the projected limits from the HL-LHC VBF searches for new scalar resonances in combination with the theoretical constraints will practically push the GM model towards the decoupling limit, imposing stringent constraints on the triplet contribution to the electroweak VEV.

Analyses trying to constrain the GM model parameter space from the LHC run-II data have been performed earlier in the literature [22–37]. Our analysis supersedes them by including the full run-II dataset for the diboson resonance searches, some of which were not available during the time of the previous analyses. Apart from the direct collider searches, we employ the latest measurement of the trilinear Higgs self-coupling to provide complementary constraints on the GM model parameter space. Another important feature of our analysis is the simple parametrization of the input variables in terms of physical quantities

resulting in a more direct interpretation of the phenomenological results.

The plan of our paper is as follows. In Sec. II, we briefly review the GM model. In Sec. III, we discuss the theoretical constraints on the model from perturbative unitarity and BFB requirements of the potential. Here we also formulate correlations among the physical parameters corresponding to the decoupling limit of the model. Constraints from the 125 GeV Higgs signal strength measurements are described in Sec. IV A. In Sec. IV B we describe the impact of the direct search constraints from the LHC on the parameter space of the GM model. The prospect for future colliders like the HL-LHC to further constrain this scenario is discussed in Sec. IV C. Finally, we summarize our findings in Sec. V.

## II. A BRIEF RECAP OF THE GM MODEL

The GM model extends the scalar sector of the SM, consisting of the  $Y = 1$  complex doublet  $\phi \equiv (\phi^+ \phi^0)^\top$ , by adding two  $SU(2)_L$  triplet scalar fields, one real  $\xi \equiv (\xi^+ \xi^0 \xi^-)^\top$  and one complex  $\chi \equiv (\chi^{++} \chi^+ \chi^0)^\top$ , with hypercharges  $Y = 0$  and  $Y = 2$ , respectively [14–16]. The scalar sector of this model is conventionally expressed in terms of a bidoublet  $\Phi$  and a bitriplet  $X$ , defined as

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}, \quad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^- & \xi^0 & \chi^+ \\ \chi^{--} & -\xi^- & \chi^0 \end{pmatrix}. \quad (1)$$

The scalar potential for this model can be written as [27,38]

$$\begin{aligned} V(\Phi, X) = & \frac{\mu_\phi^2}{2} \text{Tr}(\Phi^\dagger \Phi) + \frac{\mu_X^2}{2} \text{Tr}(X^\dagger X) + \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(X^\dagger X) \\ & + \lambda_3 \text{Tr}(X^\dagger X X^\dagger X) + \lambda_4 [\text{Tr}(X^\dagger X)]^2 - \lambda_5 \text{Tr}(\Phi^\dagger \tau_a \Phi \tau_b) \text{Tr}(X^\dagger t_a X t_b) \\ & - M_1 \text{Tr}(\Phi^\dagger \tau_a \Phi \tau_b) (UXU^\dagger)_{ab} - M_2 \text{Tr}(X^\dagger t_a X t_b) (UXU^\dagger)_{ab}, \end{aligned} \quad (2)$$

with  $\tau_a \equiv \sigma_a/2$ , ( $a = 1, 2, 3$ ), where  $\sigma_a$ s are the Pauli matrices and  $t_a$ s are the generators of the triplet representation of  $SU(2)_L$  and are given by

$$t_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (3)$$

The matrix  $U$  appearing in the trilinear terms of Eq. (2) is given by

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 0 & 1 \\ -i & 0 & -i \\ 0 & \sqrt{2} & 0 \end{pmatrix}. \quad (4)$$

After the EWSB, the neutral components of the bidoublet and the bitriplet are expanded around their VEVs as

$$\begin{aligned} \phi^0 &= \frac{1}{\sqrt{2}}(v_d + h_d + i\eta_d), & \xi^0 &= (v_t + h_\xi), \\ \chi^0 &= \left( v_t + \frac{h_\chi + i\eta_\chi}{\sqrt{2}} \right). \end{aligned} \quad (5)$$

The requirement of equal VEVs to the real and the complex triplets ensures that custodial symmetry in the scalar potential remains intact. From the expressions of  $W$  and  $Z$  boson masses, the electroweak VEV can be identified as

$$\sqrt{v_d^2 + 8v_t^2} = v = 246 \text{ GeV}. \quad (6)$$

Thus, there will be two independent minimization conditions for the scalar potential corresponding to the two VEVs of the bidoublet and the bitriplet ( $v_d$  and  $v_t$ ). These can be used to extract the bilinear coefficients of the potential  $\mu_\phi^2$  and  $\mu_X^2$  in terms of  $v_d$  and  $v_t$  as follows:

$$\mu_\phi^2 = -4\lambda_1 v_d^2 - 3(2\lambda_2 - \lambda_5) v_t^2 + \frac{3}{2} M_1 v_t, \quad (7a)$$

$$\mu_X^2 = -(2\lambda_2 - \lambda_5) v_d^2 - 4(\lambda_3 + 3\lambda_4) v_t^2 + \frac{M_1 v_d^2}{4v_t} + 6M_2 v_t. \quad (7b)$$

Now, the bilinear terms in the scalar potential can be diagonalized to obtain the physical Higgs scalars of the model which can be classified according to their transformation properties under the custodial  $SU(2)$  as a quintuplet ( $H_5^{++}, H_5^+, H_5^0, H_5^-, H_5^{--}$ ) with common mass  $m_5$ , a triplet ( $H_3^+, H_3^0, H_3^-$ ) of common mass  $m_3$  and two custodial singlets,  $h$  and  $H$  with masses  $m_h$  and  $m_H$ , respectively. In this article, we refrain ourselves from giving a detailed description of the diagonalization

procedure and we refer the reader to Refs. [38,39]. The mass eigenstates for the charged and neutral scalars are defined below<sup>1</sup>:

$$H_5^{\pm\pm} = \chi^{\pm\pm}, \quad (8a)$$

$$H_5^\pm = \frac{1}{\sqrt{2}}(\chi^\pm - \xi^\pm), \quad (8b)$$

$$H_5^0 = \sqrt{\frac{2}{3}}h_\xi - \sqrt{\frac{1}{3}}h_\chi, \quad (8c)$$

$$H_3^\pm = -\sin\beta\phi^\pm + \frac{\cos\beta}{\sqrt{2}}(\chi^\pm + \xi^\pm), \quad (8c)$$

$$H_3^0 = -\sin\beta\eta_d + \cos\beta\eta_\chi, \quad (8d)$$

$$h = \cos\alpha h_d + \sin\alpha H_5^0, \quad (8e)$$

$$H = -\sin\alpha h_d + \cos\alpha H_5^0, \quad (8f)$$

where

$$H_5^0 = \sqrt{\frac{1}{3}}h_\xi + \sqrt{\frac{2}{3}}h_\chi. \quad (9)$$

The angle  $\alpha$  represents the mixing angle in the neutral Higgs sector while  $\tan\beta$  is defined as

$$\tan\beta = \frac{2\sqrt{2}v_t}{v_d}. \quad (10)$$

It can be observed from Eq. (8) that the members of the custodial fiveplet are composed entirely of  $SU(2)_L$  scalar triplets without any admixture from the doublets. Considering the fact that the SM fermions can couple only to the doublet component, the members of the custodial fiveplet will not have any tree-level coupling to the SM fermions. Thus, the dominant production mode available for these particles at the LHC is via the VBF process, making the VBF searches essential to probe the properties of such particles.

Before closing this section we note from Eq. (2) that there are nine parameters in the GM scalar potential with two bilinears ( $\mu_\phi^2$  and  $\mu_X^2$ ), five quartic couplings ( $\lambda_i$ ,  $i = 1, \dots, 5$ ), and two trilinear couplings ( $M_1$  and  $M_2$ ). Among these, the bilinears can be replaced by the VEVs,  $v_d$  and  $v_t$  using Eq. (7). The five quartic couplings can also be exchanged for the four physical scalar masses,  $m_5$ ,  $m_3$ ,  $m_H$ , and  $m_h$  and the mixing angle,  $\alpha$ . Below, we present the relation between the  $\lambda_i$ s with the physical masses and mixings [27]:

$$\lambda_1 = \frac{1}{8v^2 \cos^2\beta} (m_h^2 \cos^2\alpha + m_H^2 \sin^2\alpha), \quad (11a)$$

$$\lambda_2 = \frac{1}{12v^2 \cos\beta \sin\beta} (\sqrt{6}(m_h^2 - m_H^2) \sin 2\alpha + 12m_3^2 \sin\beta \cos\beta - 3\sqrt{2}v \cos\beta M_1), \quad (11b)$$

$$\lambda_3 = \frac{1}{v^2 \sin^2\beta} (m_5^2 - 3m_3^2 \cos^2\beta + \sqrt{2}v \cos\beta \cot\beta M_1 - 3\sqrt{2}v \sin\beta M_2), \quad (11c)$$

$$\lambda_4 = \frac{1}{6v^2 \sin^2\beta} (2m_H^2 \cos^2\alpha + 2m_h^2 \sin^2\alpha - 2m_5^2 + 6\cos^2\beta m_3^2 - 3\sqrt{2}v \cos\beta \cot\beta M_1 + 9\sqrt{2}v \sin\beta M_2), \quad (11d)$$

$$\lambda_5 = \frac{2m_3^2}{v^2} - \frac{\sqrt{2}M_1}{v \sin\beta}. \quad (11e)$$

We wish to reiterate that in our analysis we consider  $h$  to be the lightest  $CP$ -even scalar corresponding to the Higgs boson discovered at the LHC with mass  $m_h \approx 125$  GeV. In this study we will focus on the VBF production of the nonstandard GM scalars at the LHC.

<sup>1</sup>Note that our convention of  $\alpha$  differs from that of Ref. [38] by a negative sign.

### III. THEORETICAL CONSTRAINTS AND THE DECOUPLING LIMIT

To motivate the benchmark choices for our phenomenological analysis later, it is important to discuss the implications of the theoretical constraints from tree unitarity and BFB [38,40]. We will present our observations in terms of the physical masses and mixings and focus on formulating a systematic method to approach the SM-like limit also known as the decoupling limit [38]. It is quite intuitive that the decoupling limit will be achieved when  $v_t \ll v$  and all the nonstandard scalars are much heavier than the

electroweak scale. Since this has been already discussed in Ref. [38], we will be brief and report only the important relations relevant to our present study. The distinct upshot of our analysis is that the relations we obtain involve only the physical parameters and therefore are quite straightforward to implement into the numerical codes, giving us a greater control over the parameters required for the phenomenological studies.

To begin with, we suggestively reparametrize the trilinear coupling parameters  $M_1$  and  $M_2$  as follows:

$$\Lambda_1^2 = \frac{M_1 v}{\sqrt{2} \sin \beta} \equiv \frac{M_1 v^2}{4 v_t}, \quad (12a)$$

$$\Lambda_2^2 = 3\sqrt{2} v M_2 \sin \beta \equiv 12 v_t M_2. \quad (12b)$$

With these reparametrizations let us now investigate the unitarity conditions. Theoretical constraints from perturbative unitarity put upper bounds on the eigenvalues of the  $2 \rightarrow 2$  scalar scattering amplitude matrix. The eigenvalues can be expressed in terms of certain independent combinations of the scalar quartic couplings, given as [38,40]

$$x_1^\pm = 12\lambda_1 + 14\lambda_3 + 22\lambda_4 \pm \sqrt{(12\lambda_1 - 14\lambda_3 - 22\lambda_4)^2 + 144\lambda_2^2}, \quad (13a)$$

$$x_2^\pm = 4\lambda_1 - 2\lambda_3 + 4\lambda_4 \pm \sqrt{(4\lambda_1 + 2\lambda_3 - 4\lambda_4)^2 + 4\lambda_5^2}, \quad (13b)$$

$$y_1 = 16\lambda_3 + 8\lambda_4, \quad (13c)$$

$$y_2 = 4\lambda_3 + 8\lambda_4, \quad (13d)$$

$$y_3 = 4\lambda_2 - \lambda_5, \quad (13e)$$

$$y_4 = 4\lambda_2 + 2\lambda_5, \quad (13f)$$

$$y_5 = 4\lambda_2 - 4\lambda_5. \quad (13g)$$

The theoretical constraints from perturbative unitarity requires that each of these eigenvalues must obey the condition  $|x_i^\pm|, |y_i| \leq 8\pi$ .

To illustrate the implications of the decoupling limit, we take the example of  $|y_2| \leq 8\pi$  that, in terms of the physical parameters, reduces to

$$\left| \frac{1}{3} [m_5^2 + 2(m_h^2 \sin^2 \alpha + m_H^2 \cos^2 \alpha)] - m_3^2 \cos^2 \beta \right| \leq 2\pi v^2 \sin^2 \beta. \quad (14)$$

In the decoupling limit when  $v_t \ll v$ , i.e.,  $\sin^2 \beta \ll 1$ , the above relation will be extremely constraining and will effectively reduce to the following equality:

$$\frac{1}{3} [m_5^2 + 2(m_h^2 \sin^2 \alpha + m_H^2 \cos^2 \alpha)] - m_3^2 \cos^2 \beta \approx 0. \quad (15)$$

The conditions  $|y_1| \leq 8\pi$  and  $|y_3 - y_5| \leq 16\pi$  will also have similar implications which we do not show explicitly.<sup>2</sup>

Another type of constraint will arise from conditions like  $|y_3| \leq 8\pi$ , which reduces to

$$\left| m_3^2 - \frac{\sqrt{2}}{\sqrt{3}} (m_H^2 - m_h^2) \frac{\sin 2\alpha}{\sin 2\beta} \right| \leq 4\pi v^2. \quad (16)$$

Similar constraints can be obtained from  $|y_4|, |y_5| \leq 8\pi$ . A common feature of all these constraints is the occurrence of the ratio  $(\sin 2\alpha / \sin 2\beta)$  which blows up in the limit  $\sin \beta \ll 1$  and thus jeopardizes the unitarity conditions for  $m_H^2 \gg v^2$ . Therefore, imposition of the unitarity conditions will entail a correlation between  $\sin \alpha$  and  $\sin \beta$  so that constraints like Eq. (16) can be satisfied even for  $v_t \ll v$ . From the example conditions of Eqs. (15) and (16), one may intuitively infer that the unitarity conditions will be trivially satisfied for

$$\sin 2\alpha \approx \sqrt{\frac{3}{2}} \sin 2\beta \quad \text{with} \quad v_t \ll v \quad (17a)$$

$$\text{and} \quad m_H^2 \approx m_3^2 \approx m_5^2 \approx \Lambda_1^2 \gg v^2; \quad \Lambda_2^2 \ll v^2, \quad (17b)$$

Equation (17) defines the decoupling limit in the GM model. Using the definition of  $\tan \beta$  in Eq. (10), we may simplify Eq. (17) as

$$\sin \alpha \approx 2\sqrt{3} \frac{v_t}{v}, \quad (18)$$

which will often be used as benchmark for our phenomenological analysis. A visual confirmation of Eq. (17) is given in Fig. 1 where we see that, for  $v_t \ll v$ , heavy nonstandard scalars beyond the TeV scale would require  $\sin \alpha$  to be strongly correlated to  $v_t$ . Such a correlation is not unique to the GM model and can be found in the usual HTM as well [41].

It should be noted that, in the decoupling limit defined by Eq. (17), the quartic couplings of Eq. (11) take much simpler forms as follows:

$$\lambda_1 \approx \frac{m_h^2}{8v^2}, \quad \lambda_2 \approx \lambda_5 \approx 0. \quad (19)$$

Thus, only some of the quartic coefficients survive in the decoupling limit and  $\lambda_1$  approaches the SM value. Additionally, since the doublet-triplet mixing is also vanishingly small in the decoupling limit, the physical scalar  $h$ , defined in Eq. (8e), will have SM-like couplings and can play the role of the SM-like Higgs boson observed at the

<sup>2</sup>The conditions  $|y_3|, |y_5| \leq 8\pi$  can be combined to obtain  $|y_3 - y_5| \leq 16\pi$  using the triangle inequality.

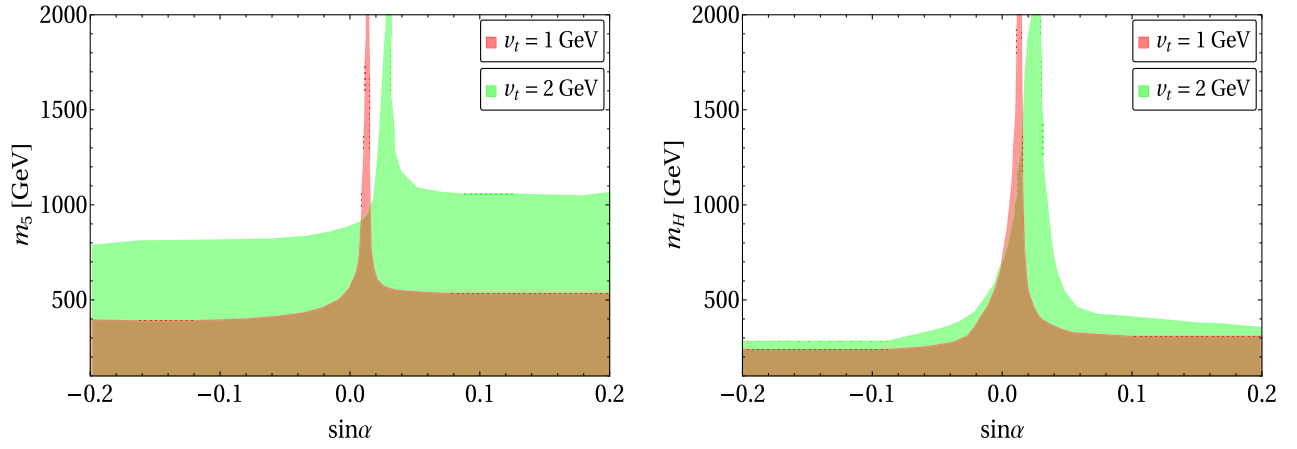


FIG. 1. Allowed regions from the combined constraints of unitarity and BFB in the limit  $v_t \ll v$ . The location of the narrow peak correspond to the decoupling limit defined by Eq. (17).

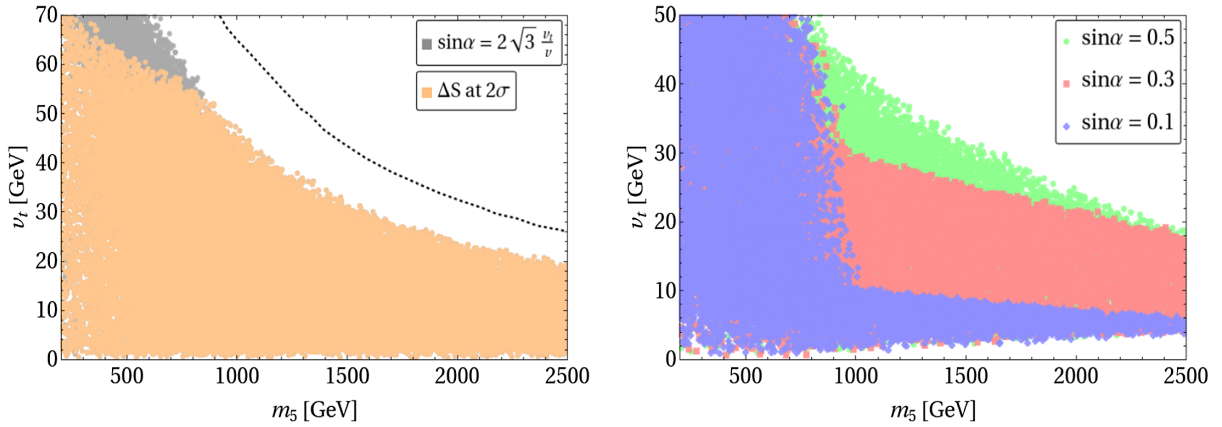


FIG. 2. Points allowed by the theoretical constraints of perturbative unitarity and BFB constraints. In the left panel we have assumed Eq. (18) to correlate  $\sin \alpha$  and  $v_t$ . The dashed black line represents the conservative bound given by Eq. (23). In the right panel different benchmark values of  $\sin \alpha$  are chosen, which are unrelated to  $v_t$ . The different colors in the right panel correspond to different values of  $\sin \alpha$  mentioned in the legends. The gray points in the left panel are excluded when we impose the additional constraint from  $\Delta S$ . As can be seen from the left panel, the constraint from  $\Delta S$  starts to become relevant for  $v_t \gtrsim 50$  GeV. In the right panel,  $\Delta S$  does not impose any additional restriction for the displayed region of parameter space.

LHC [7,8]. Also comparing Eqs. (12a) and (17b) we see that a small triplet VEV is intimately connected to large nonstandard scalar masses, which carries the reminiscence of a type II seesaw mechanism.

Another interesting point to note from Eqs. (12) and (17) is that the trilinear coupling  $M_1$  plays a crucial role in ensuring safe decoupling of the nonstandard scalars. Therefore, variants of the GM model without the trilinear couplings  $M_1$  and  $M_2$ , do not have a decoupling limit and therefore can be ruled out rather easily [27].

Next we display in Fig. 2 the points that pass the combined constraints arising from unitarity and BFB. We exhibit results for the cases when Eq. (18) is satisfied as well as when  $\sin \alpha$  and  $v_t$  are unrelated. If  $\sin \alpha$  and  $v_t$  are unrelated then, as expected, the decoupling limit cannot be achieved and as a result we see in the right panel of Fig. 2

that a substantial area in the low  $v_t$  region is excluded. We would like to comment here that our result agrees with Ref. [38] with regards to the exclusion from the perturbative unitarity and BFB requirements. The forbidden region in the upper right corners of Fig. 2 can be qualitatively understood from the trilinear couplings of the Higgs bosons with a pair of  $W$  bosons. Let us express these couplings as follows<sup>3</sup>:

$$\begin{aligned} \mathcal{L}_{WWS}^{\text{tri}} = & gM_W W_\mu^+ W^{\mu-} (\kappa_W^h h + \kappa_W^H H + \kappa_W^{H_5} H_5^0) \\ & + gM_W \frac{\kappa_2}{2} (W_\mu^+ W^{\mu+} H_5^{--} + \text{H.c.}), \end{aligned} \quad (20)$$

<sup>3</sup>Note that  $H_3^0$  being a pseudoscalar does not possess coupling of the form  $H_3^0 W_\mu^+ W^{\mu-}$ .

where  $S$  represents a generic scalar,  $g$  denotes the  $SU(2)_L$  gauge coupling strength and the  $\kappa$ s are given as

$$\kappa_W^h = \left( \sqrt{\frac{8}{3}} \sin \alpha \sin \beta + \cos \alpha \cos \beta \right), \quad (21a)$$

$$\kappa_W^H = \left( \sqrt{\frac{8}{3}} \cos \alpha \sin \beta - \sin \alpha \cos \beta \right), \quad (21b)$$

$$\kappa_W^{H_5} = \frac{\sin \beta}{\sqrt{3}}, \quad (21c)$$

$$\kappa_2 = \sqrt{2} \sin \beta. \quad (21d)$$

It is instructive to verify that these coupling modifiers obey the unitarity sum rule [42]

$$(\kappa_W^h)^2 + (\kappa_W^H)^2 + (\kappa_W^{H_5})^2 = 1 + (\kappa_2)^2. \quad (22)$$

The Lee-Quigg-Thacker bound on the Higgs boson masses [43], in this context, should thus read

$$(\kappa_W^h)^2 m_h^2 + (\kappa_W^H)^2 m_H^2 + \left[ (\kappa_W^{H_5})^2 + \frac{1}{2} (\kappa_2)^2 \right] m_3^2 \leq 4\pi v^2. \quad (23)$$

This inequality gives rise to a conservative bound from perturbative unitarity which can qualitatively explain the forbidden region in the upper-right corner of Fig. 2. We show this bound as a black dashed line in the left panel of Fig. 2.

To make the notion of decoupling more explicit, we calculate the trilinear couplings of the SM-like Higgs,  $h$ , with a pair of charged scalars in the limit of Eq. (17). In particular, the factors that control the contributions of the charged scalar loops in decays like  $h \rightarrow \gamma\gamma$  and  $h \rightarrow Z\gamma$  are given by [44]

$$\kappa_{3+} \equiv \frac{v}{2m_3^2} g_{hH_3^+H_3^-} \approx -\frac{1}{m_3^2} (m_3^2 - \Lambda_1^2 + m_h^2), \quad (24a)$$

$$\kappa_{5+} \equiv \frac{v}{2m_5^2} g_{hH_5^+H_5^-} \approx -\frac{1}{m_5^2} (2m_5^2 - 3m_3^2 + \Lambda_1^2 - \Lambda_2^2 + m_h^2), \quad (24b)$$

$$\begin{aligned} \kappa_{5++} &\equiv \frac{v}{2m_5^2} g_{hH_5^{++}H_5^{--}} \\ &\approx -\frac{1}{m_5^2} (2m_5^2 - 3m_3^2 + \Lambda_1^2 - \Lambda_2^2 + m_h^2), \end{aligned} \quad (24c)$$

where Eq. (17a) has been assumed. Clearly, when Eq. (17b) is also imposed, we will have  $\kappa_{3+}, \kappa_{5+}, \kappa_{5++} \approx 0$  implying that the charged scalars are decoupled from the

loop-induced Higgs decays in the limit of Eq. (17), as expected. In this context it should be emphasized that the expressions of Eq. (24) crucially depend on how the limit  $\sin \alpha \rightarrow 0$  is approached. For example, instead of Eq. (17a), if we first apply  $\sin \alpha \approx 0$  independent of  $v_t$ , and then take  $v_t \ll v$ , then we would obtain

$$\kappa_{3+} \approx -1, \quad (25a)$$

$$\kappa_{5+} \approx -\frac{1}{m_5^2} (3m_3^2 - 2\Lambda_1^2), \quad (25b)$$

$$\kappa_{5++} \approx -\frac{1}{m_5^2} (3m_3^2 - 2\Lambda_1^2), \quad (25c)$$

which do not lead to proper decoupling of the heavy charged scalars. Therefore, to ensure safe decoupling of the nonstandard scalars one must approach  $\sin \alpha \rightarrow 0$  limit in the way dictated by Eq. (17a).

Here we wish to clarify the distinction between the notions of ‘‘alignment’’ and ‘‘decoupling.’’ ‘‘Alignment’’ refers to the limit when the lighter neutral Higgs boson ( $h$ ) originates entirely from the  $SU(2)_L$  doublet (i.e.,  $\sin \alpha = 0$ ) making fermionic couplings of  $h$  to be SM-like.<sup>4</sup> Additionally, in this limit  $v_t \ll v_d$  so that the  $SU(2)_L$  triplet Higgses have negligible couplings of the form  $SV_1^\mu V_{2\mu}$ , making the trilinear  $hV_1^\mu V_{2\mu}$  couplings to be SM-like as well. However, one of the crucial observations of our paper is that merely making the tree level couplings of  $h$  to be SM-like does not guarantee the decoupling of heavy scalars, as has been emphasized through our Eqs. (23) and (24). For proper decoupling, we should approach  $\sin \alpha \rightarrow 0$  and  $\sin \beta \rightarrow 0$  in a correlated manner. Such discussions of alignment vs decoupling are quite widespread for doublet extensions [44,45]. In this work we have performed a similar analysis for the triplet extensions, which has remained somewhat less explored in the literature so far.

We also wish to add that the discussion made in this section highlights an underemphasized fact that, in the scenario of superheavy nonstandard scalars (much heavier than the EW scale) with only the SM-like scalar at the EW scale, the perturbative unitarity constraints automatically push us towards the decoupling limit (in a spirit similar to Ref. [46]). Such an aspect of perturbative unitarity has been discussed earlier for multi-Higgs-doublet models [6,44,47] and the HTM [41]. Here we explicitly demonstrate the connection between perturbative unitarity and the

<sup>4</sup>Putting  $\sin \alpha = 0$  in Eqs. (11b) and (11e), we obtain  $4(2\lambda_2 - \lambda_5)v_t = M_1$ . Note that, such a relation can also be inferred by demanding the off-diagonal element of the  $2 \times 2$  mass matrix in the  $h_d - H_3^0$  basis [38] to be zero. However, if  $\sin \alpha = 0$  limit is approached in this way, it will lead to alignment without decoupling.

decoupling limit for the GM model as well. We believe that this section will help to view the perturbative unitarity constraints in a new light and serve as a motivation for the parametrization that we advocate.

For later use, we also give the expression of the coupling modifier for the trilinear Higgs self-coupling, which reads

$$\begin{aligned} \kappa_\lambda \equiv \frac{\lambda_{hhh}}{(\lambda_{hhh})^{\text{SM}}} &= \cos^3 \alpha \sec \beta + \frac{2\sqrt{2}}{\sqrt{3}} \sin^3 \alpha \csc \beta + \frac{2\Lambda_1^2}{m_h^2} \sin^2 \alpha \cos \beta \left( \cos \alpha - \frac{\sqrt{2}}{\sqrt{3}} \sin \alpha \cot \beta \right) \\ &+ \frac{\sqrt{2}}{3\sqrt{3}} \frac{\Lambda_2^2}{m_h^2} \sin^3 \alpha \csc \beta. \end{aligned} \quad (26)$$

One can easily check that  $\kappa_\lambda = 1$  in the limit of Eq. (17). Moving away from the decoupling limit, however, the deviation in  $\lambda_{hhh}$  can be significantly large depending on the parameter combinations. Preliminary measurements of  $\kappa_\lambda$  can already put important constraints on the model parameter space, as will be discussed in more detail in Sec. IV B.

In passing, we recall that the oblique  $S$  parameter [48] has been known to put important constraints on the parameter space of the GM model [24,49,50]. The new physics contribution to the  $S$  parameter in the GM model is given by [50],<sup>5</sup>

$$\begin{aligned} \Delta S &\equiv S^{\text{GM}} - S^{\text{SM}}, \\ &\approx \frac{s_W^2 c_W^2}{\pi e^2} \left\{ -\frac{e^2}{12s_W^2 c_W^2} (\ln m_3^2 + 5 \ln m_5^2) + 2|g_{ZH H_3^0}|^2 f_1(m_h, m_3) \right. \\ &+ 2|g_{ZH H_3^0}|^2 f_1(m_H, m_3) + 2(|g_{ZH_5^0 H_3^0}|^2 + 2|g_{ZH_5^+ H_3^-}|^2) f_1(m_5, m_3) \\ &+ |g_{ZZH}|^2 \left[ \frac{f_1(M_Z, m_h)}{2M_Z^2} - f_3(M_Z, m_h) \right] - |g_{ZZh}^{\text{SM}}|^2 \left[ \frac{f_1(M_Z, m_h^{\text{SM}})}{2M_Z^2} - f_3(M_Z, m_h^{\text{SM}}) \right] \\ &+ |g_{ZZH}|^2 \left[ \frac{f_1(M_Z, m_H)}{2M_Z^2} - f_3(M_Z, m_H) \right] + |g_{ZZH_5^0}|^2 \left[ \frac{f_1(M_Z, m_5)}{2M_Z^2} - f_3(M_Z, m_5) \right] \\ &\left. + 2|g_{ZW^+ H_5^-}|^2 \left[ \frac{f_1(M_W, m_5)}{2M_W^2} - f_3(M_W, m_5) \right] \right\}, \end{aligned} \quad (27)$$

where  $e$  stands for the electric charge,  $s_W(c_W)$  is the sine (cosine) of the weak mixing angle,  $M_Z$  is the  $Z$ -boson mass and  $g_{XYZ}$  denotes the coupling among the  $X$ ,  $Y$ , and  $Z$  particles excluding the Lorentz factor. For explicit expressions of these couplings, we refer the reader to Refs. [38,50] where a sign difference in the definition of  $\sin \alpha$  needs to be taken into account. The quantity  $m_h^{\text{SM}}$  is the reference value of the SM Higgs boson mass for which the fit value of the  $S$  parameter is obtained. The  $f_1$  and  $f_3$  functions are given as

$$f_1(m_X, m_Y) = \begin{cases} \frac{1}{36(m_X^2 - m_Y^2)^3} [5(m_Y^6 - m_X^6) + 27(m_X^4 m_Y^2 - m_X^2 m_Y^4)] \\ + 12(m_X^6 - 3m_X^4 m_Y^2) \ln m_X + 12(3m_X^2 m_Y^4 - m_Y^6) \ln m_Y, & \text{for } m_X \neq m_Y \\ \frac{1}{6} \ln m_X^2, & \text{for } m_X = m_Y \end{cases} \quad (28)$$

and

$$f_3(m_X, m_Y) = \begin{cases} \frac{m_X^4 - m_Y^4 + 2m_X^2 m_Y^2 (\ln m_Y^2 - \ln m_X^2)}{2(m_X^2 - m_Y^2)^3}, & \text{for } m_X \neq m_Y \\ \frac{1}{6m_X^2}, & \text{for } m_X = m_Y \end{cases}. \quad (29)$$

At first glance the expression in Eq. (27) might seem strange because of the appearance of dimensionful quantities inside logarithms. However, the expression is meaningful because  $\Delta S$  remains unaffected if all the mass-dimensionful quantities are multiplied by a common scale factor. This happens as a result of the following relation satisfied by the couplings

<sup>5</sup>It should be noted that this expression of  $\Delta S$  relies on the assumption that the new physics scale is much larger than  $M_Z$ . Although it is possible to define the oblique parameters without this assumption [51,52], the corresponding expressions for the GM model do not seem to be available in the literature. Keeping this in mind, the results that follow from Eq. (27) should be interpreted with caution.



$$\begin{aligned}
& -\frac{e^2}{s_W^2 c_W^2} + \frac{2}{3} \{ |g_{ZhhH_3^0}|^2 + |g_{ZHH_3^0}|^2 + |g_{ZH_3^0 H_3^0}|^2 + 2|g_{ZH_3^+ H_3^-}|^2 \} + \frac{1}{3M_W^2} |g_{ZW^+ H_3^-}|^2 \\
& + \frac{1}{6M_Z^2} \{ |g_{ZZh}|^2 + |g_{ZZH}|^2 + |g_{ZZH_3^0}|^2 - |g_{ZZh}^{\text{SM}}|^2 \} = 0.
\end{aligned} \tag{30}$$

For our numerical analysis, the fit value of  $\Delta S$  has been taken to be [53]

$$\Delta S = -0.01 \pm 0.07. \tag{31}$$

The impact of the  $\Delta S$  constraints can be seen in the left panel of Fig. 2 where we show the points excluded at  $2\sigma$  in gray. It can be seen that the  $\Delta S$  constraints start becoming important for  $v_t \gtrsim 50$  GeV. This observation holds even when the benchmarks of  $\sin \alpha$  are chosen independently of  $v_t$ . Therefore, we do not explicitly show the excluded region in the right panel of Fig. 2. As we will see in Secs. IV A and IV B, such large values of  $v_t$  are already excluded by the collider constraints from the LHC.

#### IV. EXPERIMENTAL CONSTRAINTS

In this section we will describe the relevant experimental constraints on the parameter space of the GM model. First, we will discuss the constraints coming from the measurement of the 125 GeV Higgs signal strengths. After outlining the parameter space satisfying the theoretical constraints as described in Sec. III and the Higgs signal strength constraints, we will derive the most relevant direct search bounds from the LHC on the remaining parameter region. The future prospects for the HL-LHC will also be discussed.<sup>6</sup>

##### A. Constraint from Higgs signal strengths

The physical Higgs state  $h$  with mass  $m_h \approx 125$  GeV in our scenario arises from the mixing between two custodial singlets. This mixing will cause the couplings of  $h$  with the massive fermions and gauge bosons to deviate from their corresponding SM values. Such deviations are tightly constrained by the precision measurements of the Higgs boson couplings at the LHC. Both the ATLAS [9] and CMS [54] collaborations have studied various production and decay modes of the 125 GeV Higgs boson, thereby giving limits on the signal strength observables defined as

$$\mu_j^i = \frac{\sigma^i}{(\sigma^i)_{\text{SM}}} \times \frac{\text{BR}_j}{(\text{BR}_j)_{\text{SM}}}, \tag{32}$$

<sup>6</sup>The indirect constraints from flavor data (especially from  $b \rightarrow sy$ ) start becoming relevant for  $v_t \gtrsim 40$  GeV for all  $m_3$  and  $m_5$  [50]. As we shall see, this region of parameter space is excluded by the direct LHC searches for nonstandard scalars.

where  $\sigma^i$  represents the production cross section in the  $i$ th mode and  $\text{BR}_j$  denotes the branching ratio into the final state  $j$ .

We depict our results in Fig. 3 which shows the range of allowed values for  $v_t$  with respect to  $\sin \alpha$ , as obtained after applying Higgs signal strength constraints. We show the regions allowed by the signal strengths for gluon-gluon fusion and VBF production modes as red and blue shaded regions, respectively. We consider constraints from the  $f\bar{f}$  and  $VV$  final states,  $f$  and  $V$  being the generic symbols for massive SM fermions and vector bosons, respectively. Additionally, we also take into account the constraint from the  $\gamma\gamma$  final state. The allowed parameter space thus corresponds to the common region covered by the red and blue shaded zones. The bottom line is that the Higgs signal strength data restricts  $v_t$  to a finite region, putting clearly defined upper and lower limits on it for a fixed  $\sin \alpha$ .

##### B. Direct search constraints from the LHC

As mentioned in the introduction, the presence of a rich variety of nonstandard scalars makes the GM model subject to various constraints coming from the nonstandard Higgs boson searches at the LHC. In this study we focus mostly on the potential impact of the bounds coming from the VBF production of the custodial fiveplet charged and neutral scalars. We also discuss direct search bounds on the custodial triplet and singlet nonstandard Higgs bosons, namely  $H_3^\pm$ ,  $H_3^0$ , and  $H$ . In the following, we briefly

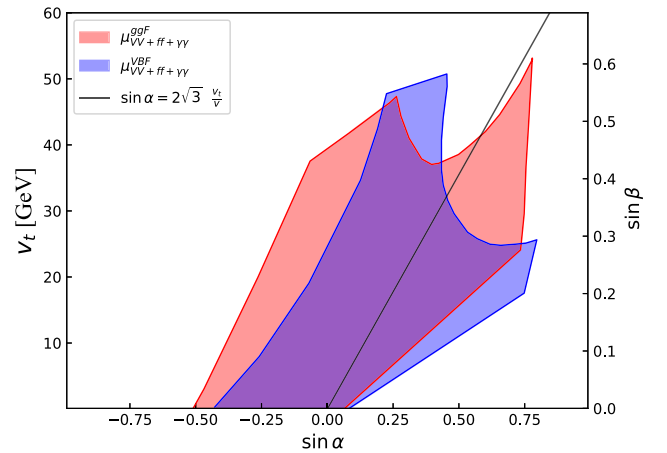


FIG. 3. Allowed parameter space from Higgs signal strength measurements at 95% confidence level (CL) [9,54]. For a fixed  $\sin \alpha$ ,  $v_t$  is restricted to a finite region, with clear upper and lower limits.

describe the most important direct searches considered by us in this analysis.

- (1) The ATLAS collaboration has performed searches for the VBF production of a neutral heavy resonance  $S_0$  decaying to  $Z$ -boson pairs in the leptonic final states [18]. The result is interpreted as a model-independent upper bound on the production cross section times branching ratio ( $\sigma_{\text{VBF}} \times \text{BR}(S_0 \rightarrow ZZ)$ ) for the VBF production process as a function of the resonance mass. This data can be effective in constraining the neutral member of the custodial fiveplet  $H_5^0$  as well as the custodial singlet  $H$  in the GM model.
- (2) Searches for a charged Higgs boson have been performed in VBF production mode and its subsequent decay into  $W^\pm Z$  modes [20] and the null results have been translated into exclusion bounds on the signal cross section as a function of the charged Higgs boson mass. This bound can be crucial to constrain the properties of the charged Higgs state  $H_5^\pm$ .
- (3) The CMS collaboration has looked for the VBF production of a doubly charged scalar in like-sign  $WW$  final states, producing model independent bounds on the corresponding signal strength as a function of doubly charged Higgs mass [55]. Additionally, the Drell-Yan production of a pair of doubly charged Higgs bosons with decays to  $WW$  pairs has been investigated by the ATLAS collaboration [19]. We employ these data to constrain the properties of the  $H_5^{\pm\pm}$  particle.

Apart from the direct LHC searches, we also discuss here the constraints coming from 125 GeV Higgs boson trilinear self-coupling measurements at the LHC [54,56–58].

Additionally, we checked our parameter space against the constraints on the quartic gauge-Higgs coupling modifier  $\kappa_{2V}$  [59] from di-Higgs production process [58]. This provides a much weaker bound compared to other constraints, excluding only  $\sin \alpha \gtrsim 0.81$ . Therefore, we do not explicitly show this constraint in the figures that follow.

We simulate the production of the nonstandard Higgs bosons via the VBF process and their subsequent decays using MadGraph-v-3.4.0 [60]. For this, we implement our model in FeynRules [61,62] which generates UFO files to be used by MadGraph5\_aMC@NLO. The direct search limits are interpreted separately for the two mass hierarchies  $m_5 < m_3$  and  $m_5 > m_3$ .

In Fig. 4 we show the combined effect of applying the bounds from theoretical constraints, Higgs signal strength data and the direct search limits from the LHC in the  $m_5 - v_t$  plane for the mass hierarchy  $m_5 < m_3$ . The gray shaded region corresponds to parameter space excluded from theoretical constraints of unitarity and BFB, and the Higgs signal strength data. The collider bounds are then determined for the surviving parameter points. The blue and purple shaded regions are excluded from direct search constraints on  $H_5^0$  in the  $ZZ$  decay mode [18] and  $H_5^\pm$  in the  $W^\pm Z$  final states [20], respectively. The exclusion limit from the VBF production of  $H_5^{\pm\pm}$  decaying to two like-sign  $W$  bosons [55] is shown as a red shaded region. In Fig. 4, we depict our results for two benchmark values of  $\sin \alpha$ . Larger values of  $\sin \alpha$ , independent of the value of  $v_t$ , correspond to farther departure from the decoupling limit. Therefore, such  $\sin \alpha$  values are expected to receive more stringent constraints from the LHC searches in general. As can be observed from the right panel of Fig. 4, the allowed parameter space is squeezed to a narrow region around the

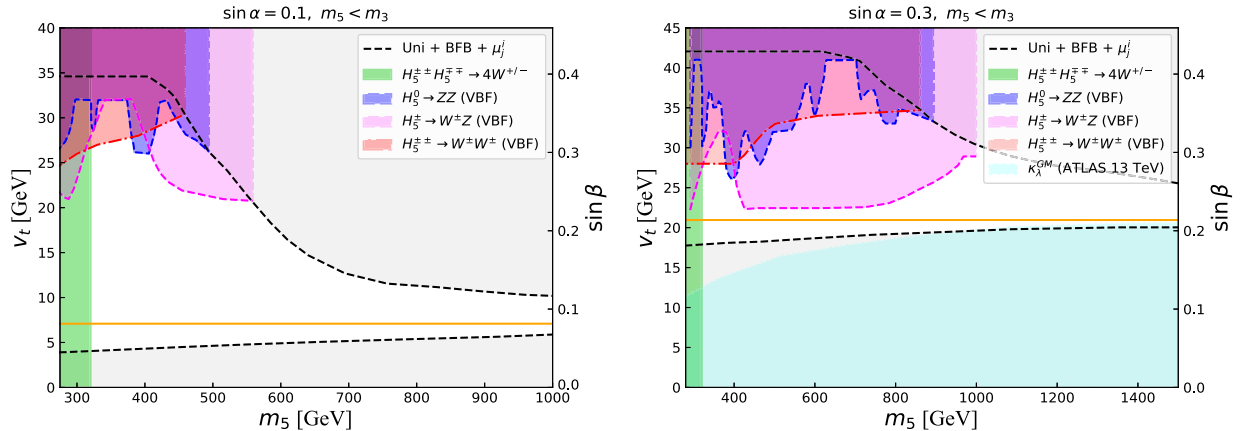


FIG. 4. Combined theoretical and experimental constraints on the  $m_5 - v_t$  parameter plane for  $m_5 < m_3$  hierarchy with  $\sin \alpha = 0.1$  (left) and  $\sin \alpha = 0.3$  (right). The excluded regions are shaded with various colors explained in the text. The orange solid line corresponds to the correlation  $\sin \alpha = 2\sqrt{3}v_t/v$ . Beyond the green band, the VBF searches dominate over Drell-Yan search channel and put nontrivial constraints on the parameter space.

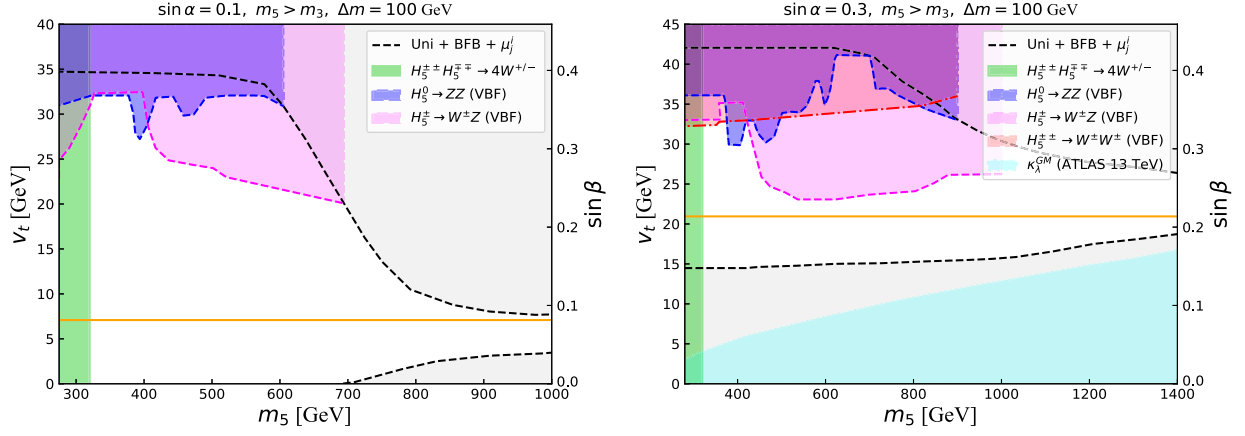


FIG. 5. Combined theoretical and experimental constraints on  $m_5 - v_t$  parameter plane for  $m_5 > m_3$  hierarchy with  $\Delta m = m_5 - m_3 = 100$  GeV for  $\sin \alpha = 0.1$  (left) and  $\sin \alpha = 0.3$  (right). The excluded regions are shaded with various colors explained in the text. The orange solid line corresponds to the correlation  $\sin \alpha = 2\sqrt{3}v_t/v$ . The Drell-Yan search channels lose sensitivity beyond the green shaded region where VBF constraints can still provide nontrivial constraints on the parameter space.

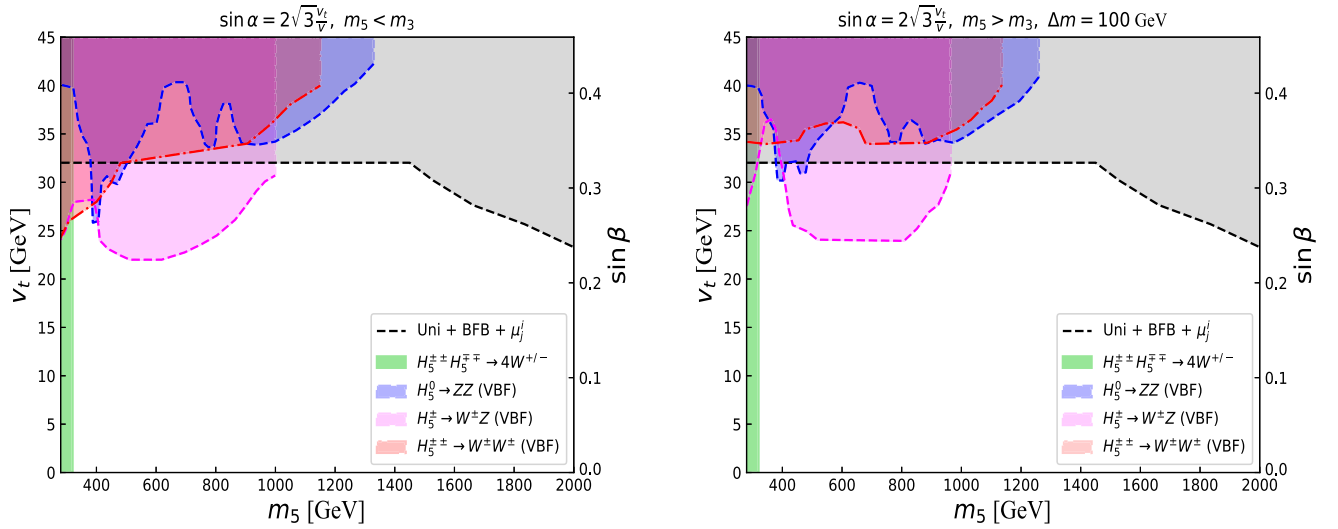


FIG. 6. Combined theoretical and LHC constraints on  $m_5 - v_t$  parameter plane for  $\sin \alpha$  and  $v_t$  correlated as in Eq. (18). The plots correspond to two hierarchies,  $m_5 < m_3$  (left) and  $m_5 > m_3$  with  $(m_5 - m_3) = 100$  GeV (right). The excluded regions are shaded with colors detailed in the text. The Drell-Yan search channels lose sensitivity beyond the green band where VBF constraints can still provide effective constraints on the parameter space.

orange horizontal line, reinforcing the strong correlation between  $\sin \alpha$  and  $v_t$  as dictated by Eq. (18).<sup>7</sup> We also show the exclusion limit from the searches for Drell-Yan production of  $H_5^{\pm\pm}$  decaying to  $4W$  final state [19] as a green shaded region. However, the corresponding constraint is much weaker than the others, with masses up to 320 GeV excluded independent of the value of  $v_t$ . The limit from the associated Drell-Yan production of  $H_5^{\pm\pm}H_5^{\mp\mp}$  [19] is found to be even weaker. Beyond the green band, where the Drell-Yan search becomes ineffective, the VBF searches can still

<sup>7</sup>In Ref. [20] the bound on the cross section has been reported for nonstandard masses up to 1 TeV. This is why the purple exclusion contours in Figs. 4–6 do not extend beyond 1 TeV.

put important constraints on the parameter space. This is a common feature which can be seen in Figs. 5 and 6 as well.

Thus, one can observe from the Fig. 4 that, while the theoretical constraints can most effectively put bounds on the model parameter space towards the high  $m_5$  region, complementary constraints can be provided by the LHC VBF searches for the smaller values of  $m_5$  as well. With the advent of future colliders like the HL-LHC, the collider constraints are expected to cut down deeper into the allowed parameter space towards lower  $v_t$  region [63]. Thus, in the absence of any definitive signal of new scalars in the future, triplet contribution to the electroweak VEV will become severely constrained.

In addition to direct collider bounds, we also take into account the constraint coming from the measurement of 125 GeV Higgs self-coupling [56]. We evaluate  $\kappa_\lambda$  according to Eq. (26). The parameter space excluded from the  $\kappa_\lambda$  constraint is shown as a cyan shaded region. For  $\sin\alpha = 0.1$  in the left panel of Fig. 4, the current measurement of  $\kappa_\lambda$  does not impose any additional constraint on the parameter space. On the other hand, only a marginal additional bound is obtained in the high  $m_5$  region for  $\sin\alpha = 0.3$ , as can be seen from the cyan shaded region in the right panel of Fig. 4. We would like to mention that  $\kappa_\lambda$  constraint becomes increasingly more important for larger values of  $\sin\alpha (> 0.3)$  and together with the bounds from unitarity and BFB, Higgs signal strength data and direct LHC search limits, it can provide crucial constraints on the large  $\sin\alpha$  parameter space.

We do not explicitly show the limits from  $H_5^0 \rightarrow WW$  in the plots. The bound presented for  $H_5^0 \rightarrow WW$  via VBF production is much weaker than that of  $ZZ$  [21]. Furthermore, we have checked that this particular decay mode has half of the branching fraction to that of  $H_5^0 \rightarrow ZZ$ . Thus, the bounds for  $H_5^0 \rightarrow WW$  mode provides no additional constraint on our parameter space.

In Fig. 5 we show our allowed parameter region for mass hierarchy  $m_5 > m_3$  for two benchmark values of  $\sin\alpha$ . In this case we present results for  $\Delta m = m_5 - m_3 = 100$  GeV, which plays a crucial role in determining the branching ratios of the decaying particles. For  $m_5 < m_3$  scenario considered earlier, the only possible decay modes for  $H_5^\pm$  are those involving a pair of gauge bosons in the final state. However, here the decay modes of  $H_5^\pm$  become more diversified. Additional decays into  $H_5^\pm \rightarrow H_3^0 W^\pm$ ,  $H_5^\pm \rightarrow H_3^\pm Z$  final states are now kinematically accessible, which decreases the signal strengths for  $H_5^\pm \rightarrow W^\pm Z$  mode, weakening the corresponding bounds. Similarly, the presence of  $H_5^0 \rightarrow H_3^0 Z$ ,  $H_3^\pm W^\mp$  modes results in the relaxation of the experimental constraint on  $H_5^0$ . Our choice of  $\Delta m = 100$  GeV serves as an illustrative benchmark to showcase the relaxation of the collider constraints brought in by the opening up of the additional decay modes.

From Figs. 4 and 5, one may observe a slight variation of the cyan region with  $m_5$ , as well as with the hierarchy being considered. This may seem counterintuitive by looking at Eq. (26), which shows no apparent functional dependence of  $\kappa_\lambda$  on  $m_5$ . Such dependence of  $\kappa_\lambda$  on  $m_5$  is an indirect effect generated by the perturbative unitarity and BFB conditions which correlates the parameters  $\Lambda_1$  and  $\Lambda_2$  with values of  $m_5$  and  $m_3$ .

In principle, the ATLAS data [18] should also translate as a lower limit on  $m_H$  as a function of  $v_t$ . Although the  $H_5 ZZ$  and  $HZZ$  couplings are of similar magnitudes, the width-over-mass ratio for  $H$  tends to become large (above 1%) in a significant region of parameter space, not respecting the narrow-width approximation. On the other

hand, the  $\text{BR}(H \rightarrow ZZ)$  gets suppressed in this case because of the presence of various other decays including di-Higgs and fermionic modes that were not present for  $H_5^0$ . We have explicitly checked that the effective cross section  $\sigma(pp \xrightarrow{\text{VBF}} H \rightarrow ZZ)$  lies well below the ATLAS sensitivity reach. The limits on  $H$  from  $H \rightarrow hh$  searches can be effective in constraining parameter spaces with  $\sin\beta \gtrsim 0.4$  [29].<sup>8</sup> In our scans, this region lies inside the parameter space already excluded from other complementary constraints.

For the  $m_3 < m_5$  hierarchy, the bounds on the charged Higgs boson  $H_3^\pm$  must also be taken into account. Out of the two pairs of charged Higgs bosons  $H_3^\pm$  and  $H_5^\pm$  of the GM model, only the custodial triplet  $H_3^\pm$  can couple to the SM fermions through its mixing with the doublet. Thus,  $H_3^\pm$  is likely to receive constraints from the charged Higgs boson searches performed at the LHC. The ATLAS collaboration has published search results for the production of charged Higgs bosons decaying to  $t\bar{b}$  final state [64]. The corresponding analysis by the CMS collaboration, however, gives a much weaker bound [65]. In GM model the dominant production mode for  $H_3^\pm$  is in association with a  $t\bar{b}$  pair. There are two possible decay modes for  $H_3^\pm$  that dominates its total decay width.  $\text{BR}(H_3^\pm \rightarrow t\bar{b})$  is dominant for smaller values of  $m_3$ . However,  $\text{BR}(H_3^\pm \rightarrow W^\pm h)$  soon takes over, once it is kinematically allowed [28]. This relaxes the direct search bounds on  $m_3$  from the LHC. In fact, we have explicitly checked that our parameter space of interest lies below the sensitivity region of ATLAS [64] in the  $t\bar{b}$  final state.

The pseudoscalar  $H_3^0$  may in principle be subjected to the bounds coming from the LHC searches for a  $CP$ -odd neutral scalar decaying into  $Zh$  final state [66]. However, these searches target the production of the  $CP$ -odd state either in gluon-gluon fusion process or in association with  $b$ -quark pairs. Such production modes for  $H_3^0$  suffer a  $\mathcal{O}(\sin\beta)$  suppression in our scenario because of the doublet-triplet mixing, making the bound considerably weak. We have checked that the signal yield for this process stays below the limit for all  $\sin\alpha$  and  $v_t \lesssim 45$  GeV.

In Fig. 6, we explain our results assuming the correlation between  $\sin\alpha$  and  $v_t$  defined in Eq. (18) to identify the decoupling limit of the model. Compared to Figs. 4 and 5, a significantly larger parameter space is now allowed by the theoretical constraints of unitarity and BFB, especially in the low  $v_t$  region. The current measurement of  $\kappa_\lambda$  does not impose any additional constraints on the parameter space. Once again, one can observe that stronger constraints from the direct collider searches in future will drive  $v_t$  to lower

<sup>8</sup>The custodial symmetry forbids couplings of the form  $H_5^0 hh$  and  $H_3^0 hh$ . Also,  $H_3^0$  being a pseudoscalar provides an additional reason not to give rise to the  $H_3^0 hh$  coupling.

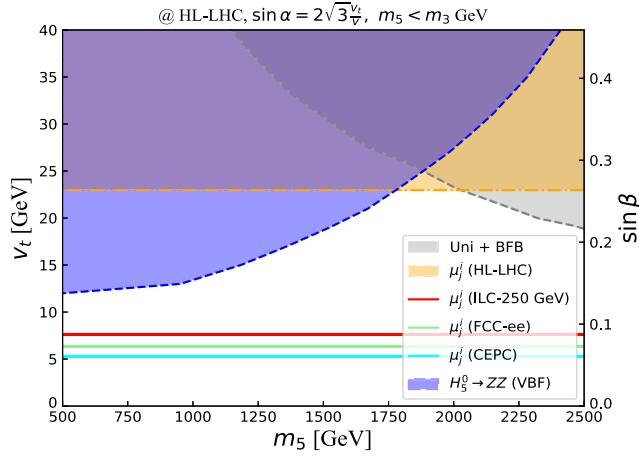


FIG. 7. The expected sensitivity for the HL-LHC assuming the correlation  $\sin \alpha = 2\sqrt{3}v_t/v$ . The blue shaded area represents the excluded regions from HL-LHC projected limit at  $\sqrt{s} = 13$  TeV with integrated luminosity of  $3000 \text{ fb}^{-1}$  considering the VBF production of  $H_5^0$  and its decay to  $ZZ$  [67]. The orange shaded region corresponds to the expected exclusion reach from the HL-LHC measurements of the 125 GeV Higgs signal strength. The corresponding limits from the FCC-ee, CEPC, and International Linear Collider are also shown. The region excluded from the unitarity and BFB constraints is shaded gray. The combined limits from HL-LHC and theoretical constraints will restrict  $v_t$  to lower values, driving it closer to the decoupling limit.

values, thereby constantly pushing us towards the decoupling limit.

### C. Future prospects: HL-LHC projected limits

Here, we estimate the potential of the upcoming HL-LHC to probe the parameter space of the GM model to a greater extent. In Fig. 7 we present our results assuming the correlation  $\sin \alpha = 2\sqrt{3}v_t/v$  for the  $m_5 < m_3$  mass hierarchy. The projected sensitivity of HL-LHC searches for the VBF production of a BSM scalar resonance decaying to  $ZZ$  mode [67] is used to put constraints on the  $m_5 - v_t$  parameter plane. The corresponding exclusion is shown as a blue shaded region. We also show the exclusion from the unitarity and BFB constraints as a gray shaded region. The projected exclusion limit from the HL-LHC measurements of Higgs signal strength is shown as the orange shaded region, assuming the central values of the signal strengths to be consistent with the corresponding SM expectations. Similar limits from the planned Higgs factory experiments [68] like the Future Circular Collider (FCC-ee), the Circular Electron-Positron Collider (CEPC) and the International Linear Collider are also shown in the plot. For the direct searches, one can see almost an order of magnitude improvement in the exclusion bounds compared to the current data, pushing the parameter space down towards lower  $v_t$  and hence towards the

decoupling limit. On the other hand, even with the projected sensitivity of the HL-LHC to  $\kappa_\lambda$ , no additional constraints can be obtained on the allowed parameter space for correlated  $\sin \alpha$  and  $v_t$ . The future Higgs factories can be much more effective in this regard, restricting  $v_t$  below 5 GeV. The exclusion contours for the opposite mass hierarchy turn out to be very similar to the one presented here.

## V. SUMMARY

In this article, we studied the implications of the nonstandard scalar (neutral as well as charged) searches in the VBF production channel at the LHC, taking the GM model as an illustrative example. The trilinear coupling of a nonstandard scalar with a pair of massive SM gauge bosons should come out to be proportional to the VEV of the electroweak multiplets from which the nonstandard scalar is primarily derived. However, as outlined in the introduction, for models comprising of only  $SU(2)_L$  doublets in the scalar sector, the Higgs signal strength measurements force us towards the alignment limit when the SM-like Higgs scalar arises from a particular  $SU(2)_L$  doublet in the Higgs basis [5,6], which consumes the entire electroweak VEV. Thus, for these multi-Higgs-doublet models, the trilinear couplings mentioned above become extremely suppressed near the alignment limit. Consequently, the VBF searches for the nonstandard scalars will not be very effective in constraining these multi-Higgs-doublet scenarios.

For models featuring higher  $SU(2)_L$  multiplet scalars (in addition to the SM Higgs doublet), the tree level value of the electroweak  $\rho$  parameter can get significantly modified. Therefore, the VEV of the additional multiplets may get severely constrained from the precision electroweak measurements. The HTM is a prime example of this scenario when the triplet VEV can, at the most, be of  $\mathcal{O}(1 \text{ GeV})$ . As a result, the trilinear couplings will still be too small to be impactful in the VBF searches for the nonstandard scalars. One may accommodate larger VEVs for the higher  $SU(2)_L$  multiplets either by restoring the custodial symmetry [69] or by arranging accidental cancellations [70]. The GM model constitutes an example of the first category where the VEV of the  $SU(2)_L$  triplets can be substantially large while still maintaining  $\rho = 1$  at the tree level. Thus, it serves as a well-motivated example where the VBF searches can put important constraints on the parameter space especially on the triplet VEV as a function of the common mass of the members of the custodial fiveplet.

In our study of the GM model, we have first analyzed the theoretical constraints from unitarity and BFB. Here we observe that, for low values of the triplet VEV, we need to have a correlation between  $\sin \alpha$  and  $v_t$  to allow for very heavy nonstandard scalars decoupled from the electroweak scales. The correlation  $\sin \alpha \simeq 2\sqrt{3}v_t/v$  grows stronger as the ratio  $v_t/v$  becomes smaller. Such a feature seems to have been underemphasized in the literature. Because of this correlation, we systematically approach the SM limit

first by imposing  $\sin\alpha = 2\sqrt{3}v_t/v$  and then requiring  $v_t \ll v$ . We also take into account the constraints coming from the 125 GeV Higgs signal strength measurements.

Next, we study the phenomenological constraints arising from the VBF searches for the nonstandard scalars. Since the members of the custodial fiveplet are derived solely from the component fields of the  $SU(2)_L$  triplets [see Eq. (8)], they do not couple to the SM fermions at all and therefore can be exclusively probed via the VBF channels. We have primarily considered the direct searches in the  $pp \xrightarrow{\text{VBF}} S \rightarrow V_1 V_2$  channel. We have explored benchmark scenarios with uncorrelated  $\sin\alpha$  and  $v_t$  as well as with  $\sin\alpha = 2\sqrt{3}v_t/v$ . We have found that the LHC constraints can be complementary to the theoretical constraints from unitarity and BFB to constrain the triplet VEV as a function of the custodial fiveplet mass. For  $m_5 < m_3$ , the upper bound on the triplet VEV can be as strong as  $v_t \lesssim 25$  GeV. The relaxation of this bound has also been demonstrated

for the opposite hierarchy,  $m_5 > m_3$ . With the improved sensitivity projected for the HL-LHC, more stringent constraints on the triplet VEV are expected from the VBF searches, implying that the combination of the direct HL-LHC collider limits and the theoretical constraints will enforce the decoupling limit of the GM model if no deviation from the SM is found in future. Our analysis

thus goes to show that the VBF searches for nonstandard scalars can be really useful in restricting the nondoublet contributions to the electroweak VEV and thereby providing valuable intuitions into the constructional aspects of new BSM scenarios. This observation, in turn, underscores the fact that the null results for the BSM searches at the LHC are a lot more than just upper bounds on cross sections as they can be translated into practical insights regarding the anatomy of EWSB.

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