Quantum corrections and the minimal Yukawa sector of $SU(5)$

Ketan M. Patel [1,*] and Saurabh K. Shukla [1,2,†]

[1] *Theoretical Physics Division, Physical Research Laboratory, Navarangpura, Ahmedabad-380009, India*
[2] *Indian Institute of Technology Gandhinagar, Palaj-382055, India*

(Received 1 November 2023; accepted 19 December 2023; published 9 January 2024)

It is well known that the $SU(5)$ grand unified theory, with the standard model quarks and leptons unified in $\bar{5}$ and 10 and the electroweak Higgs doublet residing in five-dimensional representations, leads to the relation $Y_d = Y_e^T$ between the Yukawa couplings of the down-type quarks and the charged leptons. We show that this degeneracy can be lifted in a phenomenologically viable way when quantum corrections to the tree-level matching conditions are taken into account in the presence of one or more copies of gauge singlet fermions. The one-loop threshold corrections arising from heavy leptoquark scalar and vector bosons, already present in the minimal model, and heavy singlet fermions can lead to realistic Yukawa couplings, provided their masses differ by at least 2 orders of magnitude. The latter can also lead to a realistic light neutrino mass spectrum through the type I seesaw mechanism if the color partner of the Higgs stays close to the Planck scale. Most importantly, our findings demonstrate the viability of the simplest Yukawa sector when quantum corrections are considered and sizable threshold effects are present.

DOI: 10.1103/PhysRevD.109.015007

I. INTRODUCTION

After the remarkable realization of the potential unification of the standard model (SM) gauge symmetries into a single gauge symmetry nearly 50 years ago [1–3], it has since become well established that the Yukawa sector of the SM plays a pivotal role in determining the minimal and viable configurations of grand unified theories (GUTs). The latter's potential to partially or completely unite quarks and leptons, in conjunction with the simplest choice of the Higgs field(s) in the Yukawa sector, often results in correlations among the effective SM Yukawa couplings that are inconsistent with observations.

The most glaring and simplest example of the above is the $SU(5)$ GUTs with only five-dimensional (5 and $\bar{5}$) Lorentz scalar(s) in the Yukawa sector in their ordinary (supersymmetric) versions. Both lead to

$$Y_d = Y_e^T, \tag{1}$$

at the scale of the unified symmetry breaking, namely, M_{GUT}, for the down-type quark and charged-lepton Yukawa coupling matrices Y_d and Y_e, respectively. The degeneracy between the two sectors predicted by Eq. (1) is not supported by the GUT-scale extrapolated values of the effective Yukawa couplings determined from the measured masses of the down-type quarks and the charged leptons [4,5]. The largest mismatch arises in the case of nonsupersymmetric theories in which the extrapolation of the SM data implies $y_b/y_\tau \approx 2/3$, $y_s/y_\mu \approx 1/5$, and $y_d/y_e \approx 2$, at $M_{\mathrm{GUT}} = 10^{16}$ GeV.

Deviation from the degeneracy shown in Eq. (1) can be achieved through several means: (a) expanding the scalar sector [6–11], for instance, by introducing a 45-dimensional Higgs field, or (b) incorporating higher-dimensional nonrenormalizable operators [12–18], or (c) introducing vectorlike fermions that mix with the charged leptons and/or down-type quarks residing in the chiral multiplets of $SU(5)$ [19–28]. Each of these approaches alters the tree-level matching condition, Eq. (1), and introduces new couplings. These new couplings can be harnessed to obtain effective Yukawa couplings compatible with the SM.

In this article, we present a rather simple approach to alleviate the degeneracy between charged leptons and down-type quarks. Our method involves incorporating higher-order corrections to the tree-level matching conditions for the Yukawa couplings. Nontrivial implications of such corrections in the context of supersymmetric versions of $SO(10)$ GUTs have been pointed out in [29–31].[1] In the

[*] ketan.hep@gmail.com
[†] saurabhks@prl.res.in

[1] Nevertheless, the degeneracy, as in Eq. (1), is absent in these models even at the tree level due to the presence of multiple scalars containing the SM Higgs doublets.

context of $SU(5)$, we show that the inclusion of such corrections does not necessitate the introduction of new fermions or scalars charged under the $SU(5)$ for modifying the tree-level Yukawa relations. This sets the present proposal apart from the previous ones outlined as (a)–(c) above. Specifically, we demonstrate that by expanding the minimal nonsupersymmetric $SU(5)$ framework to include fermion singlets and accounting for threshold corrections to the Yukawa couplings originating from these singlets, along with the leptoquark scalar and vector components already present in the minimal setup, a fully realistic fermion spectrum can be achieved.

II. YUKAWA RELATIONS AT ONE LOOP

The Yukawa sector of the model is comprised of three generations of $\mathbf{10}$, $\bar{\mathbf{5}}$, and N generations of the gauge singlet $\mathbf{1}$ Weyl fermions and a Lorentz scalar 5_H. The most general renormalizable interactions between these fields can be parametrized as

$$\mathcal{L}_{\rm Y} = \frac{1}{4}(Y_1)_{ij}\mathbf{10}_i^T C\mathbf{10}_j 5_H + \sqrt{2}(Y_2)_{ij}\mathbf{10}_i^T C\bar{\mathbf{5}}_j 5_H^* + (Y_3)_{i\alpha}\bar{\mathbf{5}}_i^T C\mathbf{1}_\alpha 5_H - \frac{1}{2}(M_N)_{\alpha\beta}\mathbf{1}_\alpha^T C\mathbf{1}_\beta + \text{H.c.}, \quad (2)$$

with $i, j = 1, 2, 3$ and $\alpha = 1, \ldots, N$ denotes the generations and C is the usual charge-conjugation matrix. We have suppressed the gauge and Lorentz indices for brevity. The symmetric nature of the first term implies $Y_1 = Y_1^T$. Additionally, M_N is the gauge invariant Majorana mass of the singlet fermions alias the right-handed (RH) neutrinos.

The SM quarks and leptons residing in the $SU(5)$ multiplets are identified as $\mathbf{10}^{ab} = \frac{1}{\sqrt{2}}\epsilon^{abc}u_c^C$, $\mathbf{10}^{am} = -\frac{1}{\sqrt{2}}q^{am}$, $\mathbf{10}^{mn} = -\frac{1}{\sqrt{2}}\epsilon^{mn}e^C$, $\bar{\mathbf{5}}_a = d_a^C$, $\bar{\mathbf{5}}_m = \epsilon_{mn}l^n$, and $\mathbf{1} = \nu^C$, where a, b, and c denote the color while m and n are $SU(2)$ indices. For the scalar, we define a color triplet $T^a \equiv 5_H^a$ and an electroweak doublet $h^m \equiv 5_H^m$ [32]. Decompositions of Eq. (2) then lead to the following Yukawa interactions with the color triplet and Higgs:

$$-\mathcal{L}_{\rm Y}^{(T)} = (Y_1)_{ij}\left(u_i^{CT}Ce_j^C + \frac{1}{2}q_i^T Cq_j\right)T - (Y_3)_{i\alpha}d_i^{CT}C\nu_\alpha^C T - (Y_2)_{ij}(u_i^{CT}Cd_j^C + q_i^T Cl_j)T^* + \text{H.c.} \quad (3)$$

and

$$-\mathcal{L}_{\rm Y}^{(h)} = (Y_1)_{ij}q_i^T Cu_j^C\tilde{h} + (Y_2)_{ij}q_i^T Cd_j^C h^* + (Y_3)_{i\alpha}l_i^T C\nu_\alpha^C\tilde{h} + (Y_2^T)_{ij}l_i^T Ce_j^C h^* + \text{H.c.}, \quad (4)$$

where $\tilde{h} = \epsilon h$ and we have suppressed the $SU(3)$ and $SU(2)$ contractions. Matching of $\mathcal{L}_{\rm Y}^{(h)}$ with the SM Yukawa Lagrangian at tree level leads to $Y_u = Y_1$ and $Y_d = Y_e^T = Y_2$ at the renormalization scale $\mu = M_{\rm GUT}$.

For the matching at one loop, the Yukawa couplings receive two types of contributions. The first arises from the vertex corrections involving the color triplet or the leptoquark gauge boson in the loop. The interaction of the latter with the SM fermions originates from the unified gauge interaction, and it is given by [4,5]

$$-\mathcal{L}_{\rm G}^{(X)} = \frac{g}{\sqrt{2}}\bar{X}_\mu\left(\overline{d^C}_i\bar{\sigma}^\mu l_i - \bar{q}_i\bar{\sigma}^\mu u_i^C - \overline{e^C}_i\bar{\sigma}^\mu q_i\right) + \text{H.c.}, \quad (5)$$

where X transforms as $(3, 2, -5/6)$ under the SM gauge symmetry. The second type of contribution to the Yukawa threshold correction is due to wave function renormalization of fermions and scalar involving at least one of the heavy fields in the loop.

The one-loop corrected matching condition for the Yukawa couplings at a renormalization scale μ is given by

$$Y_f = Y_f^0\left(1 - \frac{K_h}{2}\right) + \delta Y_f - \frac{1}{2}(K_f^T Y_f^0 + Y_f^0 K_{f^C}), \quad (6)$$

where $f = u$, d, e, ν. The details of the derivation of the above expression are outlined in Appendix A. In Eq. (6), δY_f are the finite parts of one-loop corrections to the Yukawa vertex Y_f, while $K_{f,f^C,h}$ are the finite parts of the wave function renormalization diagrams involving heavy particles in the loops evaluated in the $\overline{\rm MS}$ scheme. Y_f^0 denotes the tree-level Yukawa coupling matrix. As mentioned earlier,

$$Y_u^0 = Y_1, \qquad Y_d^0 = Y_2, \qquad Y_e^0 = Y_2^T, \qquad Y_\nu^0 = Y_3, \quad (7)$$

at $\mu = M_{\rm GUT}$.

Next, we compute δY_f using the interaction terms given in Eqs. (3)–(5) and assuming massive color triplet scalar T, vector leptoquark X, and N generations of the RH neutrinos ν_α^C. We find

$$
\begin{aligned}
(\delta Y_u)_{ij} &= 4g^2 (Y_1)_{ij} f[M_X^2, 0] + (Y_1 Y_2^* Y_2^T + Y_2 Y_2^\dagger Y_1^T)_{ij} f[M_T^2, 0], \\
(\delta Y_d)_{ij} &= 2g^2 (Y_2)_{ij} f[M_X^2, 0] + (Y_1 Y_1^* Y_2)_{ij} f[M_T^2, 0] + \sum_\alpha (Y_2 Y_3^*)_{i\alpha} (Y_3^T)_{\alpha j} f[M_T^2, M_{N_\alpha}^2], \\
(\delta Y_e)_{ij} &= 6g^2 (Y_2^T)_{ij} f[M_X^2, 0] + 3(Y_2^T Y_1^* Y_1)_{ij} f[M_T^2, 0], \\
(\delta Y_\nu)_{i\alpha} &= 3(Y_2^T Y_2^* Y_3)_{i\alpha} f[M_T^2, 0],
\end{aligned}
\tag{8}
$$

at the scale μ. Here, M_{N_α} is the mass of ν_α^C and $f[m_1^2, m_2^2]$ is a loop integration factor, and it is given in Eq. (B1) in Appendix B. It can be noticed that, other than the overall color factor, δY_d and δY_e differ by the contribution from the heavy RH neutrinos. Because of the tree-level Yukawa couplings between d_i^C, ν_α^C, and T in Eq. (3), the Y_d gets threshold correction from the RH neutrinos and color triplet scalar. It is noteworthy that the corrections δY_f vanish in the supersymmetric version of the model [33], due to the perturbative nonrenormalization theorem for the supersymmetric field theories [34,35].

The computations of the finite parts of wave function renormalization for the light fermions and scalar at one loop, involving at least one heavy fields in the loop, lead to

$$
\begin{aligned}
(K_q)_{ij} &= 3g^2 \delta_{ij} h[M_X^2, 0] - \frac{1}{2}(Y_1^* Y_1^T + 2Y_2^* Y_2^T)_{ij} h[M_T^2, 0], \\
(K_{u^C})_{ij} &= 4g^2 \delta_{ij} h[M_X^2, 0] - (Y_1^* Y_1^T + 2Y_2^* Y_2^T)_{ij} h[M_T^2, 0], \\
(K_{d^C})_{ij} &= 2g^2 \delta_{ij} h[M_X^2, 0] - 2(Y_2^\dagger Y_2)_{ij} h[M_T^2, 0] - \sum_\alpha (Y_3^*)_{i\alpha} (Y_3^T)_{\alpha j} h[M_T^2, M_{N_\alpha}^2], \\
(K_l)_{ij} &= 3g^2 \delta_{ij} h[M_X^2, 0] - 3(Y_2^\dagger Y_2)_{ij} h[M_T^2, 0], \\
(K_{e^C})_{ij} &= 6g^2 \delta_{ij} h[M_X^2, 0] - 3(Y_1^\dagger Y_1)_{ij} h[M_T^2, 0], \\
(K_{\nu^C})_{\alpha\beta} &= -3(Y_3^\dagger Y_3)_{\alpha\beta} h[M_T^2, 0], \\
K_h &= \frac{g^2}{2}(f[M_X^2, M_T^2] + g[M_X^2, M_T^2]),
\end{aligned}
\tag{9}
$$

at the scale μ. The loop integration factors are defined in Appendix B. Again, only K_{d^C} receives a contribution from the singlet fermions. As we show in the next sections, these contributions from singlet fermions are crucial for uplifting degeneracy between the charged lepton and down-type quarks.

III. DEVIATION FROM $Y_d = Y_e^T$

It is seen from Eqs. (6), (8), and (9) that the one-loop corrections break the degeneracy between Y_e and Y_d. Explicitly, we obtain at the GUT scale

$$
\begin{aligned}
(Y_d - Y_e^T)_{ij} = &-2g^2 (Y_2)_{ij} (2f[M_X^2, 0] - h[M_X^2, 0]) \\
&- (Y_1 Y_1^* Y_2)_{ij} \left(f[M_T^2, 0] + \frac{5}{8} h[M_T^2, 0] \right) \\
&+ \sum_\alpha (Y_2 Y_3^*)_{i\alpha} (Y_3)_{j\alpha} \left(f[M_T^2, M_{N_\alpha}^2] \right. \\
&\left. + \frac{1}{2} h[M_T^2, M_{N_\alpha}^2] \right).
\end{aligned}
\tag{10}
$$

The above is the main result of this paper. It is noteworthy that Eq. (10) not only suggests $Y_d \neq Y_e^T$, but also implies that the difference between the two matrices is calculable in terms of the masses of the heavy scalar, gauge boson, and RH neutrinos and their couplings. The latter also determines the masses of other fermions and, hence, can be severely constrained as we discuss in the next section.

Before assessing the viability of Eq. (10) in reproducing the complete and realistic fermion mass spectrum, we investigate its role for the third-generation Yukawa couplings, namely, y_b and y_τ, through a simplified analysis. Considering only one RH neutrino with $M_{N_1} = M_N$ and only the third generation, one finds from Eq. (10)

$$
\begin{aligned}
\frac{y_b}{y_\tau} \simeq 1 &- 2g^2 (2f[M_X^2, 0] - h[M_X^2, 0]) \\
&- 2y_t^2 \left(f[M_T^2, 0] + \frac{5}{8} h[M_T^2, 0] \right) \\
&+ y_\nu^2 \left(f[M_T^2, M_N^2] + \frac{1}{2} h[M_T^2, M_N^2] \right),
\end{aligned}
\tag{11}
$$

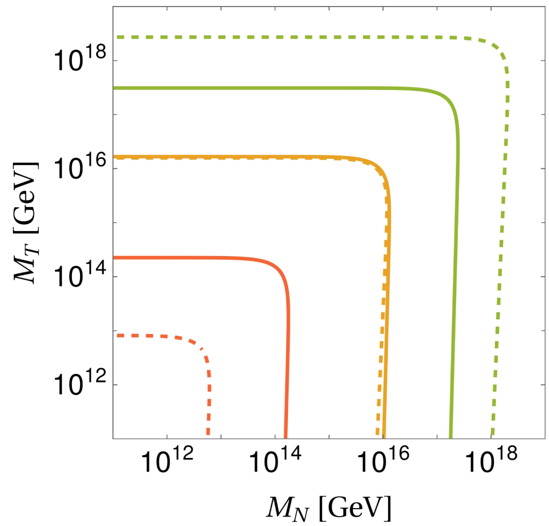

FIG. 1. Contours of $y_b/y_\tau = 3/2$ (red lines), $y_b/y_\tau = 1$ (orange lines), and $y_b/y_\tau = 2/3$ (green lines) drawn using Eq. (11) for $y_t = 0.427$, $g = 0.53$, and $\mu = M_X = 10^{16}$ GeV and for $y_\nu = \sqrt{4\pi}$ (solid lines) and $y_\nu = 2.7$ (dashed lines).

at the GUT scale. Here, y_t is the top-quark Yukawa coupling and $y_\nu = (Y_3)_{31}$. For some sample values of y_t, y_ν, and $\mu = M_X = 10^{16}$ GeV, the contours corresponding to different values of the ratio y_b/y_τ on the $M_T - M_N$ plane are displayed in Fig. 1.

The GUT-scale extrapolation of the observed fermion mass data requires $y_b/y_\tau \approx 2/3$. As can be seen from Fig. 1, this can be achieved only if either M_T or M_N is larger than $\mu = M_X$ by at least 1–2 orders of magnitude. Moreover, y_ν is also required to be large. For $g, y_t < 1$, it is the third term in Eq. (11) which is required to dominantly contribute to uplift the degeneracy between y_b and y_τ, and, hence, the largest possible value of y_ν is preferred. $M_T \gg M_{\rm GUT}$ or $M_N \gg M_{\rm GUT}$ along with large y_ν are needed to overcome the loop suppression factor of $1/(16\pi)^2$. This simple picture provides a clear and qualitative understanding of the favorable mass scales of the color triplet scalar and RH neutrino, and it also holds more or less when the full three-generation fermion spectrum is considered as we show in the next section.

It is noteworthy that the RH neutrino through its coupling with the lepton doublet generates a contribution to the light neutrino mass through the usual type I seesaw mechanism [36–39]. It is obtained as $m_\nu = v^2 y_\nu^2/M_N$. If this contribution is required to generate the atmospheric neutrino mass scale, then one finds

$$M_N = 7.6 \times 10^{16}\ {\rm GeV}\left(\frac{y_\nu}{\sqrt{4\pi}}\right)^2\left(\frac{0.05\ {\rm eV}}{m_\nu}\right). \quad (12)$$

Since M_N cannot be much larger than $M_{\rm GUT}$ in this case, phenomenologically viable y_b/y_τ can be achieved only if $M_T > M_{\rm GUT}$. Conversely, when considering perturbative values of y_ν and a situation where M_N greatly surpasses $M_{\rm GUT}$, the RH neutrino's contribution to the light neutrino mass is rather negligible. This inadequacy to reproduce a viable atmospheric neutrino mass scale necessitates the inclusion of an additional source of neutrino masses. We also provide an example of this in the next section.

IV. VIABILITY TEST AND RESULTS

To establish if the Y_u, Y_d, and Y_e are evaluated from Eqs. (6), (8), and (9) can reproduce the realistic values of the SM Yukawa couplings and the quark mixing (CKM) matrix, we carry out the χ^2 optimization. Focusing on the minimal setup, we first consider only one RH neutrino with mass $M_{N_1} \equiv M_N$ as mentioned in the previous section. The χ^2 function (see, for example, [40,41] for the definition and optimization procedure) includes nine diagonal charged fermion Yukawa couplings and four CKM parameters. For the input values of these parameters at the GUT scale, we evolve the SM Yukawa couplings from $\mu = M_t$ (M_t being the top pole mass) to $\mu = M_{\rm GUT} = 10^{16}$ GeV using the two-loop renormalization group equations (RGEs) in the $\overline{\rm MS}$ scheme following the procedure outlined in [41]. The two-loop SM RG equations have been computed using the PyR@TE 3 package [42]. The values of the SM Yukawa and gauge couplings at $\mu = M_t$ are taken from [43]. The RGE extrapolated values at the GUT scale are listed as $O_{\rm exp}$ in Table I. For the standard deviations, we use $\pm 30\%$ in the light quark Yukawa couplings $(y_{u,d,s})$ and $\pm 10\%$ in the rest of the observables as considered in the previous fits [41].

Using the freedom to choose a basis in Eq. (2), we set Y_1 diagonal and real. The RH neutrino mass matrix M_N in the general N flavor case can also be chosen real and diagonal simultaneously. $Y_{2,3}$ are complex in this basis. Using Eqs. (6), (8), and (9), we then compute the matrices $Y_{u,d,e}$ and diagonalize them to obtain the nine diagonal Yukawa couplings and quark mixing parameters. These quantities are fitted to the extrapolated data at $\mu = M_{\rm GUT}$ by minimizing the χ^2 function. We set $M_X = M_{\rm GUT}$ and $g = 0.53$, which is an approximate value of the RGE evolved SM gauge couplings at $\mu = 10^{16}$ GeV. Fixing M_T and M_N to some values, we then minimize the χ^2 along with a constraint $|(Y_{1,2,3})_{ij}| < \sqrt{4\pi}$ on all the input Yukawa couplings to ensure that they are within the perturbative limits [44]. We repeat this procedure for several values of M_T and M_N. The obtained distribution of the minimized χ^2 ($\equiv \chi^2_{\rm min}$) is displayed in Fig. 2.

Note that without one-loop corrections, i.e., with $Y_d = Y_e^T$, the obtained value of $\chi^2_{\rm min}$ is 53. Therefore, values of $\chi^2_{\rm min} < 53$ show improvements due to quantum corrected matching conditions in the model. In particular, for $\chi^2_{\rm min} < 9$, it is ensured that no observable is more than $\pm 3\sigma$ away from its central value and, therefore, can be

TABLE I. The benchmark best-fit solutions obtained for three example cases as discussed in the text. $O_{\rm exp}$ denote the extrapolated values of the underlying observables at $\mu = 10^{16}$ GeV. The reproduced values through χ^2 minimization are listed under $O_{\rm th}$, and corresponding pulls are given for each solution. The optimized values of the masses of leptoquark scalar and RH neutrinos are listed at the bottom of the table.

		Solution I		Solution II		Solution III	
Observable	$O_{\rm exp}$	$O_{\rm th}$	Pull	$O_{\rm th}$	Pull	$O_{\rm th}$	Pull
y_u	2.81×10^{-6}	2.92×10^{-6}	0	2.81×10^{-6}	0	2.81×10^{-6}	0
y_c	1.42×10^{-3}	1.42×10^{-3}	0	1.42×10^{-3}	0	1.42×10^{-3}	0
y_t	4.27×10^{-1}	4.35×10^{-1}	0.2	4.30×10^{-1}	0.1	4.28×10^{-1}	~0
y_d	6.14×10^{-6}	3.60×10^{-6}	−1.2	2.85×10^{-6}	−1.8	2.91×10^{-6}	−1.8
y_s	1.25×10^{-4}	1.26×10^{-4}	~0	1.24×10^{-4}	~0	1.25×10^{-4}	~0
y_b	5.80×10^{-3}	5.77×10^{-3}	~0	6.09×10^{-3}	0.5	5.79×10^{-3}	~0
y_e	2.75×10^{-6}	2.76×10^{-6}	0.2	2.81×10^{-6}	0.2	2.82×10^{-6}	0.3
y_μ	5.72×10^{-4}	5.71×10^{-4}	~0	5.65×10^{-4}	−0.1	5.71×10^{-4}	~0
y_τ	9.68×10^{-3}	9.83×10^{-3}	0.2	9.06×10^{-3}	−0.6	9.70×10^{-3}	~0
$\lvert V_{us}\rvert$	0.2286	0.2303	0.1	0.2292	~0	0.2291	~0
$\lvert V_{cb}\rvert$	0.0457	0.0461	0.1	0.0458	~0	0.0458	~0
$\lvert V_{ub}\rvert$	0.0042	0.0043	~0	0.0042	~0	0.0042	~0
$\sin\delta_{\rm CKM}$	0.78	0.78	0	0.78	0	0.78	~0
$\Delta m^2_{\rm sol}[{\rm eV}^2]$	7.41×10^{-5}	⋯	⋯	7.53×10^{-5}	~0	7.51×10^{-5}	~0
$\Delta m^2_{\rm atm}[{\rm eV}^2]$	2.511×10^{-3}	⋯	⋯	2.586×10^{-3}	~0	2.572×10^{-3}	~0
$\sin^2\theta_{12}$	0.303	⋯	⋯	0.303	~0	0.303	~0
$\sin^2\theta_{23}$	0.572	⋯	⋯	0.558	−0.2	0.571	~0
$\sin^2\theta_{13}$	0.02203	⋯	⋯	0.02194	~0	0.02201	~0
$\delta_{\rm MNS}[°]$	197	⋯	⋯	192	−0.2	197	~0
$\chi^2_{\rm min}$			2.0		4.0		3.1
M_T [GeV]		10^{12}		7.7×10^{17}		10^{13}	
M_{N_1} [GeV]		4.7×10^{18}		4.1×10^{16}		5.6×10^{12}	
M_{N_2} [GeV]		⋯		2.3×10^{12}		6.9×10^{17}	
M_{N_3} [GeV]		⋯		⋯		1.2×10^{13}	

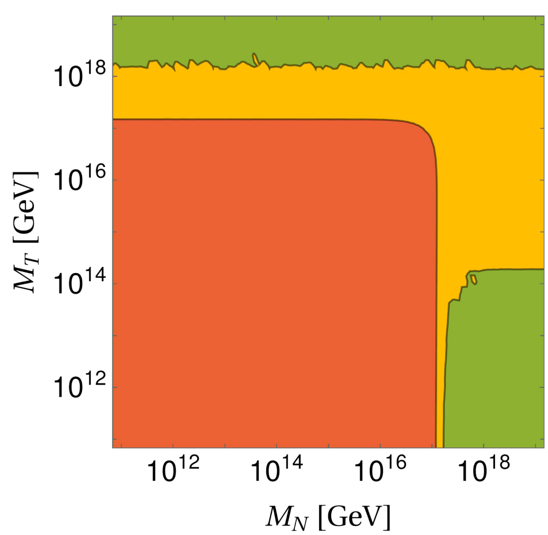

FIG. 2. The distribution of minimized χ^2 for different values of M_T and M_N. The green, yellow, and red regions correspond to $\chi^2_{\rm min} \leq 3$, $3 < \chi^2_{\rm min} \leq 9$, and $\chi^2_{\rm min} > 9$, respectively. For the fits, we set $\mu = M_X = 10^{16}$ GeV and $g = 0.53$ and impose $|(Y_{1,2,3})_{ij}| < \sqrt{4\pi}$.

considered to lead to viable charged fermion mass spectrum and the quark mixing.

As can be seen from Fig. 2, a very good fit of the entire charged fermion mass spectrum and the quark mixing parameters can be obtained if M_T or $M_N \geq 10^{17.2}$ GeV. These results are in very good agreement with the limits on M_T and M_N obtained for $y_b/y_\tau \lesssim 2/3$ in a simplified case discussed earlier and shown in Fig. 1.

The three-generation χ^2 analysis also reveals that all the underlying 13 observables can be fitted within their $\pm 1\sigma$ range (corresponding to $\chi^2_{\rm min} \leq 3$) provided (i) $M_T \leq 10^{14.5}$ GeV and $M_N \geq 10^{17.2}$ GeV or (ii) $M_T \geq 10^{18.2}$ GeV. While the second leads to M_T alarmingly close to the Planck scale, making the doublet-triplet splitting problem [45–47] more severe, the possibility (i) is conceptually allowed and technically a safe choice. Since M_N is a scale independent of $M_{\rm GUT}$ in the present framework, the large hierarchy between them is permitted. Also, M_T can be significantly smaller than $M_{\rm GUT}$, provided it satisfies the proton lifetime limit, $M_T \gtrsim 10^{11}$ GeV [48]. We list explicitly one benchmark solution from region (i) which is displayed as solution I in Table I. The fitted values of the corresponding input parameters are given in Appendix C.

Although the RH neutrino is introduced to reproduce the viable charged fermion mass spectrum, its mass and couplings are not constrained from the requirement of the light neutrino masses and mixing parameters. To account for both the solar and atmospheric neutrino mass scales, one needs at least two RH neutrinos in the minimal realization. The light neutrino masses are then generated through the usual type I seesaw mechanism:

$$M_\nu = -v^2 Y_\nu M_N^{-1} Y_\nu^T. \tag{13}$$

Here, M_ν is a 3×3 light neutrino mass matrix, while M_N is a 2×2 heavy neutrino mass matrix. Y_ν is a 3×2 matrix which can be computed using Eqs. (6), (8), and (9). The above leads to one massless light neutrino.

We extend the χ^2 function to include the solar and atmospheric squared mass differences, three mixing angles, and a Dirac CP phase to assess if Eq. (13) along with Eqs. (6), (8), and (9) can provide a realistic spectrum of quarks and leptons. For the input values of neutrino observables, we use the results of the latest fit from [49] and set $\pm 10\%$ uncertainty as earlier. The RGE effects in neutrino data are neglected, as they are known to be small [50–53] and within the set uncertainty for normal hierarchy in the neutrino masses which is the case considered here.

The result of χ^2 minimization for this case is shown in Table I as solution II, and the optimized values of parameters are listed in Appendix C. As can be seen, we find very good agreement with all the fermion masses and mixing parameters with $\chi^2_{\rm min} = 4$. The resulting values of M_{N_1} and M_{N_2} are smaller than $M_{\rm GUT}$, which requires $M_T > 10^{17.2}$ GeV as anticipated from Fig. 2.

As a simple extension of the possibilities discussed above, it is straightforward to anticipate a case in which there are more than two RH neutrinos present. At least one of them is strongly coupled with the SM fermions and has a mass greater than $M_{\rm GUT}$. It leads to the required threshold corrections for a viable charged fermion spectrum; however, its contribution to the neutrino masses is subdominant. The other RH neutrinos have sub-GUT-scale masses and can lead to a realistic light neutrino spectrum without significantly altering the threshold corrections. This scenario is exemplified by solution III in Table I. In this case, N_3, with $M_{N_2} > M_{\rm GUT}$, couples to the SM leptons with large couplings and gives the required threshold corrections to the down-type quark sector. Notably, in this context, it is evident that the color triplet scalar need not approach Planck-scale values to fulfil its role.

V. CONCLUSION

This article demonstrates that the seemingly unviable relationship $Y_d = Y_e^T$, predicted by the simplest and most minimal Yukawa sector of nonsupersymmetric $SU(5)$ GUT, can be rendered viable when accounting for one-loop corrections to the tree-level matching conditions. This is accomplished by introducing one or more copies of fermion singlets. While they do not alter the tree-level matching conditions at the scale of unification, they can yield significant corrections at the one-loop level through their direct Yukawa interactions with the down-type quarks and the color triplet scalar. Sizable nondegeneracy among the singlet fermions, color triplet scalar, and leptoquark vector can, thus, impart large enough threshold corrections ensuring the compatibility of the minimal Yukawa sector with the effective SM description.

Our quantitative analysis reveals that achieving a realistic spectrum for the charged fermion Yukawa couplings and quark mixing necessitates either a significantly larger mass for the color triplet scalar ($M_T \gg M_X$) or vastly higher masses for the RH neutrinos ($M_{N_\alpha} \gg M_X$), under the assumption that the mass of the leptoquark gauge boson (M_X) defines the unification scale. The latter possibility is disfavored if the same fermion singlets are expected to generate a viable light neutrino spectrum through the conventional type I seesaw mechanism. Nonetheless, the scenario of $M_{N_\alpha} \gg M_X$ remains a plausible option if neutrinos acquire their masses through other means. This also includes the type I seesaw mechanism with additional copies of RH neutrinos with sub-GUT-scale masses and comparatively smaller couplings with the SM leptons.

It is noteworthy that the inclusion of quantum corrections can substantially alter the conclusions regarding the minimal Yukawa sector within the framework of an underlying grand unified theory. These findings provide motivation for conducting analogous investigations in the context of supersymmetric variants of $SU(5)$,[2] as well as both the ordinary and supersymmetric versions of $SO(10)$ GUTs, which feature more diverse particle spectra for threshold corrections and, simultaneously, more stringent symmetries that engage in intricate interplays.

ACKNOWLEDGMENTS

We acknowledge illuminating discussions with Charanjit Singh Aulakh and Anjan S. Joshipura. This work is partially supported under the Mathematical Research Impact Centric Support (MATRICS) project (MTR/2021/000049) funded by the Science & Engineering Research Board (SERB), Department of Science and Technology (DST), Government of India. K. M. P. also acknowledges support from the ICTP through the Associates Program (2023–2028) where part of this work was carried out.

[2]Radiative corrections to $Y_d = Y_e^T$ arising from the superpartners of the SM fields in the loop have been considered in [54,55]. This, however, requires nonminimal supersymmetry-breaking trilinear terms.

APPENDIX A: GENERAL FORMULA OF ONE-LOOP MATCHING

In this appendix, we show an explicit derivation of the one-loop matching relation, Eq. (6), for the Yukawa coupling matrices. Computation of Yukawa thresholds is carried out for the first time in [19,33,56,57] following the analogous procedure developed for the gauge couplings in [58,59]. We closely follow [33] and outline the treatment for the Yukawa couplings for completeness.

Consider chiral fermions ψ_i, χ_i, and a scalar ϕ with the following gauge and Yukawa interactions in the full theory:

$$\mathcal{L} = i\bar{\psi}_i \not{D} \psi_i + i\bar{\chi}_i \not{D} \chi_i + D_\mu \phi^\dagger D^\mu \phi - \{Y_{ij} \psi_i^T C \chi_j \phi + \text{H.c.}\}. \tag{A1}$$

Let ψ_i, χ_i, and ϕ decompose in the light fields, namely, ψ_{li}, χ_{li}, and ϕ_l and the heavy fields ψ_{hi}, χ_{hi}, and ϕ_h, respectively. Integrating out the heavy fields, the effective Lagrangian of the light fields is given by

$$\begin{aligned}\mathcal{L}_{\text{eff}} = {}& i\bar{\psi}_{li} \not{D} (Z_\psi)_{ij} \psi_{lj} + i\bar{\chi}_{li} \not{D} (Z_\psi)_{ij} \chi_{lj} + D_\mu \phi_l^\dagger Z_\phi D^\mu \phi_l \\ & - \{\tilde{Y}_{ij} \psi_{li}^T C \chi_{lj} \phi_l + \text{H.c.}\} + \cdots,\end{aligned} \tag{A2}$$

where $\cdots$ denotes the nonrenormalizable operators induced because of the integrated-out fields. The Z parameters can be parametrized as

$$(Z_{\psi,\chi})_{ij} = \delta_{ij} + (K_{\psi,\chi})_{ij}, \qquad Z_\phi = 1 + K_\phi, \tag{A3}$$

where $K_{\psi,\chi,\phi}$ can be evaluated using the wave function renormalization of the corresponding light field at one loop involving at least one heavy field in the loop. Similarly, $\tilde{Y}$ in Eq. (A2) can be written as

$$\tilde{Y} = Y + \delta Y, \tag{A4}$$

where δY is the one-loop Yukawa vertex correction with heavy fields in the loop.

Canonical normalization of the kinetic terms requires field redefinitions. To achieve this, we define

$$\begin{aligned}&\psi_{li} = (U_\psi \tilde{Z}_\psi^{-1/2} U_\psi^\dagger)_{ij} \tilde{\psi}_{lj}, \qquad \chi_{li} = (U_\chi \tilde{Z}_\chi^{-1/2} U_\chi^\dagger)_{ij} \tilde{\chi}_{lj}, \\ &\phi_l = Z_\phi^{-1/2} \tilde{\phi}_l.\end{aligned} \tag{A5}$$

Here, $\tilde{Z}_{\psi,\chi} = U_{\psi,\chi}^\dagger Z_{\psi,\chi} U_{\psi,\chi}$ are diagonal matrices. Substitution of the above in Eq. (A2) leads to canonically normalized kinetic terms for the light fermions $\tilde{\psi}_{li}$ and $\tilde{\chi}_{li}$ and the scalar $\tilde{\phi}_l$ as can be verified easily. Furthermore, the effective Yukawa couplings in the new basis can be determined by substituting Eq. (A5) in the last term in Eq. (A2). We then find

$$\mathcal{L}_{\text{eff}} \supset (Y_{\text{eff}})_{ij} \tilde{\psi}_{li}^T C \tilde{\chi}_{lj} \phi_l + \text{H.c.}, \tag{A6}$$

with

$$Y_{\text{eff}} = U_\psi^* \tilde{Z}_\psi^{-1/2} U_\psi^T \tilde{Y} U_\chi \tilde{Z}_\chi^{-1/2} U_\chi^\dagger Z_\phi^{-1/2}. \tag{A7}$$

Using the definitions Eq. (A3), one can express

$$\tilde{Z}_{\psi,\chi}^{-1/2} = (\mathbf{1} + \tilde{K}_{\psi,\chi})^{-1/2} \simeq \mathbf{1} - \frac{1}{2}\tilde{K}_{\psi,\chi}, \tag{A8}$$

where $\tilde{K}_{\psi,\chi} = U_{\psi,\chi}^\dagger K_{\psi,\chi} U_{\psi,\chi}$ are diagonal and real matrices with $(\tilde{K}_{\psi,\chi})_{ii} < 1$. Similarly, $Z_\phi^{-1/2} \simeq 1 - \frac{1}{2}K_\phi$. Substituting these in Eq. (A7) and keeping only the leading-order terms in δY and K, we find

$$Y_{\text{eff}} = Y\left(1 - \frac{1}{2}K_\phi\right) + \delta Y - \frac{1}{2}K_\psi^T Y - \frac{1}{2} Y K_\chi. \tag{A9}$$

The above can be used as a one-loop corrected matching condition at a renormalization scale μ by replacing δY and $K_{\psi,\chi,\phi}$ by their finite parts defined in the $\overline{\text{MS}}$ scheme. Equation (A9) then provides the one-loop corrected expression for the effective Yukawa couplings in terms of the original Yukawa couplings of the full theory and the leading corrections arising from the heavy particles.

APPENDIX B: LOOP INTEGRATION FACTORS

The loop integration factors appearing in Eqs. (8) and (9) are given by

$$f[m_1^2, m_2^2] = -\frac{1}{16\pi^2}\left(\frac{m_1^2 \log\frac{m_1^2}{\mu^2} - m_2^2 \log\frac{m_2^2}{\mu^2}}{m_1^2 - m_2^2} - 1\right), \tag{B1}$$

$$h[m_1^2, m_2^2] = \frac{1}{16\pi^2}\left(\frac{1}{2}\log\frac{m_1^2}{\mu^2} + \frac{\frac{1}{2}r^2 \log r - \frac{3}{4}r^2 + r - \frac{1}{4}}{(1-r)^2}\right), \tag{B2}$$

and

$$g[m_1^2, m_2^2] = \frac{1}{16\pi^2} \frac{\frac{r^3}{6} - r^2 + \frac{r}{2} + r\log r + \frac{1}{3}}{(1-r)^3}, \tag{B3}$$

where $r = m_2^2/m_1^2$ in the last two equations.

APPENDIX C: EXAMPLE NUMERICAL SOLUTIONS

In this appendix, we give the fitted values of the Yukawa coupling matrices $Y_{1,2,3}$ corresponding to the benchmark solutions I, II, and III, as listed in Table I, obtained at $\mu = 10^{16}$ GeV. For solution I, we find

$$Y_1 = \begin{pmatrix} 2.79\times10^{-6} & 0 & 0 \\ 0 & 1.41\times10^{-3} & 0 \\ 0 & 0 & 4.36\times10^{-1} \end{pmatrix}, \qquad Y_3 = \begin{pmatrix} 6.63\times10^{-2} \\ -3.46 \\ 2.93 \end{pmatrix},$$

$$Y_2 = \begin{pmatrix} -4.24\times10^{-6} - i1.07\times10^{-5} & -1.84\times10^{-4} + i4.81\times10^{-5} & 1.24\times10^{-4} - i4.0\times10^{-5} \\ 8.98\times10^{-5} + i7.71\times10^{-5} & 2.76\times10^{-4} - i7.98\times10^{-5} & -3.57\times10^{-4} + i3.84\times10^{-4} \\ -7.58\times10^{-4} - i7.53\times10^{-4} & 8.60\times10^{-3} - i1.45\times10^{-3} & -1.15\times10^{-3} - i2.55\times10^{-3} \end{pmatrix}. \tag{C1}$$

In the case of solution II, we get

$$Y_1 = \begin{pmatrix} 2.78\times10^{-6} & 0 & 0 \\ 0 & 1.40\times10^{-3} & 0 \\ 0 & 0 & -4.24\times10^{-1} \end{pmatrix}, \qquad Y_3 = \begin{pmatrix} 2.96\times10^{-1} & -2.43\times10^{-2} \\ -3.44 & 8.92\times10^{-3} \\ -3.50 & -4.25\times10^{-2} \end{pmatrix},$$

$$Y_2 = \begin{pmatrix} -1.54\times10^{-6} + i4.92\times10^{-6} & -2.65\times10^{-5} - i1.05\times10^{-4} & 1.39\times10^{-5} - i1.20\times10^{-4} \\ -2.45\times10^{-5} - i3.34\times10^{-5} & 9.05\times10^{-4} + i2.73\times10^{-5} & 5.95\times10^{-4} - i1.33\times10^{-4} \\ 3.31\times10^{-4} + i4.24\times10^{-4} & -6.19\times10^{-3} + i4.74\times10^{-3} & 2.15\times10^{-3} + i3.98\times10^{-3} \end{pmatrix}. \tag{C2}$$

Similarly, for solution III, we find

$$Y_1 = \begin{pmatrix} 2.79\times10^{-6} & 0 & 0 \\ 0 & 1.41\times10^{-3} & 0 \\ 0 & 0 & -4.28\times10^{-1} \end{pmatrix}, \qquad Y_3 = \begin{pmatrix} 3.40\times10^{-2} & 9.83\times10^{-2} & 1.98\times10^{-2} \\ -4.23\times10^{-3} & 3.50 & -2.64\times10^{-1} \\ -5.33\times10^{-2} & -3.44 & 9.50\times10^{-2} \end{pmatrix},$$

$$Y_2 = \begin{pmatrix} 6.10\times10^{-6} - i6.22\times10^{-7} & 4.33\times10^{-5} - i1.62\times10^{-4} & -3.93\times10^{-5} + i1.27\times10^{-4} \\ 1.26\times10^{-5} + i4.40\times10^{-5} & 8.54\times10^{-4} - i4.19\times10^{-4} & -5.30\times10^{-4} + i5.20\times10^{-4} \\ 2.26\times10^{-4} - i4.77\times10^{-4} & -7.03\times10^{-3} + i9.19\times10^{-4} & 8.49\times10^{-4} - i5.74\times10^{-3} \end{pmatrix}. \tag{C3}$$

For all the solutions, we have used $g = 0.53$ as the value of unified coupling at the GUT scale and $M_X = 10^{16}$ GeV. The optimized values of the masses of the color triplet scalar and gauge singlet fermions are listed in Table I in the case of each solution.

[1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
[2] H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93**, 193 (1975).
[3] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); **11**, 703(E) (1975).
[4] A. J. Buras, J. R. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B135**, 66 (1978).
[5] P. Langacker, Phys. Rep. **72**, 185 (1981).
[6] H. Georgi and C. Jarlskog, Phys. Lett. **86B**, 297 (1979).
[7] R. Barbieri, D. V. Nanopoulos, and D. Wyler, Phys. Lett. **103B**, 433 (1981).
[8] A. Giveon, L. J. Hall, and U. Sarid, Phys. Lett. B **271**, 138 (1991).
[9] I. Dorsner and P. Fileviez Perez, Phys. Lett. B **642**, 248 (2006).
[10] P. Fileviez Perez, Phys. Lett. B **654**, 189 (2007).
[11] T. Goto, S. Mishima, and T. Shindou, Phys. Rev. D **108**, 095012 (2023).
[12] J. R. Ellis and M. K. Gaillard, Phys. Lett. **88B**, 315 (1979).
[13] V. S. Berezinsky and A. Y. Smirnov, Phys. Lett. **140B**, 49 (1984).
[14] G. Altarelli, F. Feruglio, and I. Masina, J. High Energy Phys. 11 (2000) 040.
[15] D. Emmanuel-Costa and S. Wiesenfeldt, Nucl. Phys. **B661**, 62 (2003).
[16] I. Dorsner, P. Fileviez Perez, and G. Rodrigo, Phys. Rev. D **75**, 125007 (2007).

[17] S. Antusch and K. Hinze, Nucl. Phys. **B976**, 115719 (2022).
[18] S. Antusch, K. Hinze, and S. Saad, Nucl. Phys. **B986**, 116049 (2023).
[19] R. Hempfling, Phys. Rev. D **49**, 6168 (1994).
[20] Q. Shafi and Z. Tavartkiladze, Phys. Lett. B **451**, 129 (1999).
[21] S. M. Barr and I. Dorsner, Phys. Lett. B **566**, 125 (2003).
[22] N. Oshimo, Phys. Rev. D **80**, 075011 (2009).
[23] K. S. Babu, B. Bajc, and Z. Tavartkiladze, Phys. Rev. D **86**, 075005 (2012).
[24] I. Dorsner, S. Fajfer, and I. Mustac, Phys. Rev. D **89**, 115004 (2014).
[25] P. Fileviez Pérez, A. Gross, and C. Murgui, Phys. Rev. D **98**, 035032 (2018).
[26] S. Antusch, K. Hinze, S. Saad, and J. Steiner, arXiv:2308.11705.
[27] S. Antusch, K. Hinze, and S. Saad, Nucl. Phys. **B991**, 116195 (2023).
[28] S. Antusch, K. Hinze, and S. Saad, Phys. Rev. D **108**, 095010 (2023).
[29] C. S. Aulakh and S. K. Garg, arXiv:hep-ph/0612021.
[30] C. S. Aulakh and S. K. Garg, Nucl. Phys. **B857**, 101 (2012).
[31] C. S. Aulakh, I. Garg, and C. K. Khosa, Nucl. Phys. **B882**, 397 (2014).
[32] K. M. Patel and S. K. Shukla, J. High Energy Phys. 08 (2022) 042.
[33] B. D. Wright, arXiv:hep-ph/9404217.
[34] M. T. Grisaru, W. Siegel, and M. Rocek, Nucl. Phys. **B159**, 429 (1979).
[35] N. Seiberg, Phys. Lett. B **318**, 469 (1993).
[36] P. Minkowski, Phys. Lett. **67B**, 421 (1977).
[37] T. Yanagida, Conf. Proc. C **7902131**, 95 (1979).
[38] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
[39] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).
[40] A. S. Joshipura and K. M. Patel, Phys. Rev. D **83**, 095002 (2011).
[41] V. S. Mummidi and K. M. Patel, J. High Energy Phys. 12 (2021) 042.
[42] L. Sartore and I. Schienbein, Comput. Phys. Commun. **261**, 107819 (2021).
[43] V. Suryanarayana Mummidi and K. M. Patel, Phys. Rev. D **101**, 115008 (2020).
[44] L. Allwicher, P. Arnan, D. Barducci, and M. Nardecchia, J. High Energy Phys. 10 (2021) 129.
[45] A. Masiero, D. V. Nanopoulos, K. Tamvakis, and T. Yanagida, Phys. Lett. **115B**, 380 (1982).
[46] B. Grinstein, Nucl. Phys. **B206**, 387 (1982).
[47] K. S. Babu, I. Gogoladze, and Z. Tavartkiladze, Phys. Lett. B **650**, 49 (2007).
[48] I. Dorsner, Phys. Rev. D **86**, 055009 (2012).
[49] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, J. High Energy Phys. 09 (2020) 178.
[50] P. H. Chankowski and Z. Pluciennik, Phys. Lett. B **316**, 312 (1993).
[51] K. S. Babu, C. N. Leung, and J. T. Pantaleone, Phys. Lett. B **319**, 191 (1993).
[52] S. Antusch, J. Kersten, M. Lindner, M. Ratz, and M. A. Schmidt, J. High Energy Phys. 03 (2005) 024.
[53] J. w. Mei, Phys. Rev. D **71**, 073012 (2005).
[54] J. L. Diaz-Cruz, H. Murayama, and A. Pierce, Phys. Rev. D **65**, 075011 (2002).
[55] T. Enkhbat, arXiv:0909.5597.
[56] J. Oliensis and M. Fischler, Phys. Rev. D **28**, 194 (1983).
[57] G. L. Kane, C. F. Kolda, L. Roszkowski, and J. D. Wells, Phys. Rev. D **49**, 6173 (1994).
[58] S. Weinberg, Phys. Lett. **91**, 51 (1980).
[59] L. J. Hall, Nucl. Phys. **B178**, 75 (1981).