Electroweak phase transition with radiative symmetry breaking in a type-II seesaw model with an inert doublet

Shilpa Jangid[1,*] and Hiroshi Okada[2,†]

[1]*Asia Pacific Center for Theoretical Physics (APCTP)—Headquarters San 31, Hyoja-dong, Nam-gu, Pohang 790-784, South Korea*

[2]*Department of Physics, Kyushu University, 744 Motooka, Nishi-ku, Fukuoka 819-0395, Japan*

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We consider the type-II seesaw model extended with another Higgs doublet, which is odd under the Z_2 symmetry. We look for the possibility of triggering the electroweak symmetry breaking via radiative effects. The Higgs mass parameter changes sign from being positive at higher-energy scales to negative at lower-energy scales in the presence of the TeV scalar triplet. The Planck scale perturbativity is demanded and the electroweak phase transition is studied using two-loop β-functions. The maximum allowed values for the interaction quartic coupling of the second doublet field and the triplet field with the Higgs field are $\lambda_3 = 0.15$ and $\lambda_{\Phi_{1\Delta}} = 0.50$, respectively. Considering these EW values, the first-order phase transition, i.e., $\phi_+(T_c)/T_c \sim 0.6$ is satisfied only for vanishing doublet and triplet bare mass parameters, $m_{\Phi_2} = 0.0$ GeV and $m_\Delta = 0.0$ GeV. The small nonzero induced vacuum expectation value for the scalar triplet also generates the neutrino mass, and the lightest stable neutral particle from the inert doublet satisfies the dark matter constraints for the chosen parameter space. The impact of the thermal corrections on the stability of the electroweak vacuum is also studied, and the current experimental values of the Higgs mass and the top mass lie in the stable region both at the zero temperature and the finite temperature.

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I. INTRODUCTION

The ATLAS [1] and CMS [2] experiments at the Large Hadron Collider (LHC) made the momentous finding of the Higgs boson predicted by the Standard Model (SM), which resulted in a triumphant moment for particle physicists. This significant finding demonstrated that the SM is the most effective theory for describing all fundamental interactions in particle physics. The SM physics up to accessible energies was really validated by this discovery, together with the electroweak precision evidence. Despite this, the evidence of the existence of nonzero neutrino mass and the predictions for a cold dark matter (DM) candidate by PLANCK [3] and WMAP [4,5] explained the shortcomings of the SM. The possibility of nonzero mass for neutrinos is explained by different seesaw mechanisms, out of which, the light neutrino masses are generated by the existence of heavy Majorona masses in the extension of SM with right-handed neutrinos (RHNs), i.e., type-I seesaw [6–9]. The other possibility is that the extension to a heavy scalar triplet and the small induced vacuum expectation value (VEV) will generate the small neutrino mass, i.e., type-II seesaw [10–14]. Secondly, the simplest way to get a DM candidate is to extend the SM to scalar multiplets with a discrete Z_2 symmetry for inert doublet models [15–17]. This Z_2 symmetry can also be a remnant symmetry of higher gauge groups, i.e., $SO(10)$ grand unified theories (GUTs) [18–26]. These higher gauge groups provide interesting physics at higher energies; unification of SM gauge couplings is one of them.

The possibility of the Higgs boson being a part of larger multiplets in various extensions breaks some higher symmetries. For example, in supersymmetric scenarios, for the extension to a $U(1)_{B-L}$ symmetry, the breaking scale of $U(1)_{B-L}$ can be brought down to the supersymmetric scale by choosing parameters in an appropriate way [27,28]. This high-scale symmetry breaking demands all physical Higgs bosons, including the SM Higgs, have positive mass-squared values at higher energies. There is a way to trigger electroweak symmetry breaking (EWSB) by radiative effects in supersymmetric scenarios [29–34]. In such cases, the Higgs mass parameter, which is positive at higher

*shilpa.jangid@apctp.org
†okada.hiroshi@phys.kyushu-u.ac.jp

energies, can turn negative at lower energies via radiative loop corrections and lead to EWSB. The idea of radiative electroweak symmetry breaking in various nonsupersymmetry scalar extensions, i.e., the type-II seesaw with TeV scalar triplet and dark matter models (DM) in singlet and doublet extensions of the SM, has been studied in detail [35].

Another missing thing in the SM is that the electroweak phase transition (EWPT) from the symmetric phase to the broken phase is a smooth crossover [36] for the Higgs boson mass with more than 80 GeV [37–39], which is not at all consistent with the measured Higgs boson mass of 125.5 GeV [40]. This is another motivation for the extension of the SM with additional scalar degrees of freedom, minimally with singlet [41,42], doublet [43] or triplet [42] in the $SU(2)$ representation. The strongly first-order phase transition in the type-II seesaw has already been discussed in detail [44,45]. As clear from the various SM extensions to the singlet and the triplet, the EWPT is favored by lighter masses [42,46]. Hence, it is interesting to look for the strongly first-order EWPT in accordance with the radiative EWSB. One possibility is to introduce two different multiplets; one of them can be chosen to be of higher mass to provide the radiative symmetry breaking, while the masses for the second multiplet can be considered small enough to accommodate the strongly first-order phase transition. Either of the multiplets can be considered odd under the Z_2 symmetry in order to accommodate the cold DM candidate. Hence, we consider the extension of the SM with TeV scalar triplet, which also serves the purpose of generating the neutrino mass with a tiny nonzero induced VEV, i.e., type-II seesaw, and a second $SU(2)$ Higgs doublet, which is considered to be odd under the Z_2 symmetry. The lightest stable neutral particle would be the cold DM candidate. The extension of SM with singlet is also a possibility, but doublet is chosen because of the interesting phenomenology with restricted regions of the mass range satisfying the DM constraints.

Lastly, the theoretical incompatibility of the SM with the stability of the electroweak (EW) vacuum at zero temperature [47,48] leads us to another motivation for extension to scalar degrees of freedom. The additional scalar degrees of freedom from the inert doublet and TeV scalar triplet gives positive contribution to the effective quartic coupling and enhances the stability of the EW vacuum. The electroweak vacuum stability from a cosmological perspective has already been studied in detail [49–58]. The Higgs field may be forced to tunnel down to the true vacuum because of the quantum fluctuations during inflation. This situation can happen even before the inflation ends, but this is not realized if the reheating temperature is sufficiently large after inflation. Hence, the Universe is characterized by sufficiently high value of the temperature after the end of inflation [50]. As a consequence, the thermal corrections effect the stability of the electroweak vacuum [59–61] triggering the tunneling between the false vacuum to the true vacuum of the potential [62–65] and cannot be neglected. Since, we are considering the finite-temperature corrections to the effective potential, it is interesting to investigate the status for the stability of the EW vacuum at both zero temperature and the finite temperature.

The detailed outline of the article is as follows. The model description with EWSB and the mass expressions is given in Sec. II. The detailed description for the different possibilities of radiative EWSB is given in Sec. III. We check for the allowed parameter space from radiative symmetry breaking with strongly first-order phase transition in Sec. IV for the values allowed from the Planck scale perturbativity using two-loop β-functions. The stability of the EW vacuum at zero temperature and the finite temperature are studied in Sec. V. The detailed two-loop expressions for the quartic couplings and the gauge couplings are given in Appendix.

II. MODEL SETUP

The minimal SM is augmented with a $SU(2)$ Higgs triplet of $Y = 1$, and a $SU(2)$ Higgs doublet of $Y = 1/2$. The second doublet is considered to be odd under the Z_2 symmetry [named the inert doublet (ID)] while the other fields are considered to be even under the Z_2 symmetry. The minimal type-II seesaw is considered to provide the radiative EWSB for the TeV mass scale of scalar triplet. The motivation for such a triplet comes from left-right symmetric theories, which can be perceived either at low energy or can be realized in GUTs or E_6. The nonzero VEV of the Higgs triplet also provides the neutrino masses, and the lightest stable Z_2 odd particle from the inert doublet becomes the cold DM candidate. The field definition for the Higgs multiplets is given as follows:

$$Z_2\colon\ \Phi_1 \to \Phi_1, \quad \Delta \to \Delta, \quad \Phi_2 \to -\Phi_2, \tag{2.1}$$

where

$$\Phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v_h + \rho_1 + iG^0) \end{pmatrix},$$
$$\Delta = \frac{1}{\sqrt{2}} \begin{pmatrix} \delta^+ & \sqrt{2}\delta^{++} \\ (v_\Delta + \delta^0 + i\eta^0) & -\delta^+ \end{pmatrix},$$
$$\Phi_2 = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(H^0 + iA^0) \end{pmatrix},$$

where Φ_2, being odd under the Z_2 symmetry, does not take part in the EWSB, and the mass eigenstates will be the same as the gauge eigenstates. The scalar potential for the minimal type-II seesaw scenario with additional inert doublet is as follows:

$$\begin{aligned}V(\Phi_1,\Delta,\Phi_2) = {}& m_{\Phi_1}^2(\Phi_1^\dagger\Phi_1) + \lambda_{\Phi 1}(\Phi_1^\dagger\Phi_1)^2 + m_{\Phi_2}^2(\Phi_2^\dagger\Phi_2) + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + m_\Delta^2\mathrm{Tr}(\Delta^\dagger\Delta) + [\mu_1(\Phi_1^T i\sigma_2\Delta^\dagger\Phi_1) + \mathrm{H.c.}]\\ &+ \mu_2[\Phi_2^T i\sigma_2\Delta^\dagger\Phi_2] + \lambda_{\Delta 1}(\mathrm{Tr}(\Delta^\dagger\Delta))^2 + \lambda_{\Delta 2}\mathrm{Tr}(\Delta^\dagger\Delta)^2 + \lambda_{\Phi_1\Delta}(\Phi_1^\dagger\Phi_1)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_{\Phi_1\Delta 1}\Phi_1^\dagger\Delta\Delta^\dagger\Phi_1\\ &+ \lambda_3\Phi_1^\dagger\Phi_1\Phi_2^\dagger\Phi_2 + \lambda_4\Phi_1^\dagger\Phi_2\Phi_2^\dagger\Phi_1 + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + \mathrm{H.c.}] + \lambda_{\Phi_2\Delta}(\Phi_2^\dagger\Phi_2)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_{\Phi_2\Delta 1}(\Phi_2^\dagger\Delta\Delta^\dagger\Phi_2),\end{aligned} \tag{2.2}$$

where Tr is the trace over the 2×2 matrix, and σ_2 is the second Pauli spin matrix. The triplet VEV is restricted by the ρ parameter, $\rho = \frac{v_h^2+2v_\Delta^2}{v_h^2+4v_\Delta^2} \simeq 1 - \frac{2v_\Delta^2}{v_h^2}$, where the experimental measured value $\rho = 1.00038^{+0.00020}_{-0.00020}$ and $\sqrt{v_h^2+v_\Delta^2} \approx 246.0$ GeV restrict the triplet VEV $0 \lesssim v_\Delta/\mathrm{GeV} \lesssim 2.56$ implying that $v_\Delta \ll v_h$. The $\lambda_{\Phi_1\Delta}$ and $\lambda_{\Phi_1\Delta 1}$ terms allow for the mixing between the SM Higgs and the scalar triplet for nonzero VEV for the triplet Higgs. Rewriting the scalar potential given in Eq. (2.2) in terms of the VEVs and using the minimization conditions, the masses for the new scalar particles after spontaneous symmetry breaking are computed as follows [66]:

$$m_{\Phi_1}^2 = \frac{1}{2}(2\lambda_{\Phi_1}v_h^2 + (\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})v_\Delta^2 - 2\sqrt{2}\mu_1 v_\Delta), \tag{2.3}$$

$$m_\Delta^2 = \frac{-(\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})v_h^2 v_\Delta - 2(\lambda_{\Delta 1}+\lambda_{\Delta 2})v_\Delta^3 + \sqrt{2}\mu_1 v_h^2}{2v_\Delta}. \tag{2.4}$$

The doubly charged Higgs mass can be computed directly by collecting the coefficients for $\delta^{++}\delta^{--}$ in the scalar potential, and the mass expression for the doubly charged boson comes out to be

$$m_{H^{\pm\pm}}^2 = \frac{\mu_1 v_h^2}{\sqrt{2}v_\Delta} - \frac{1}{2}\lambda_{\Phi_1\Delta 1}v_h^2 - \lambda_{\Delta 2}v_\Delta^2. \tag{2.5}$$

The mass-squared mixing matrix for the singly charged Higgs is given as

$$\mathcal{M}_\pm^2 = \left(\sqrt{2}\mu_1 - \frac{\lambda_{\Phi_1\Delta 1}v_\Delta}{2}\right)\begin{pmatrix} v_\Delta & -v_h/\sqrt{2}\\ -v_h/\sqrt{2} & v_h^2/2v_\Delta\end{pmatrix}.$$

This matrix can be diagonalized using an orthogonal matrix, and one of the eigenvalues will be zero, which corresponds to the charged Goldstone degrees of freedom, and the nonzero eigenvalue would be the mass for the charged Higgs and is given by

$$\mathcal{R}_{\beta^\pm} = \begin{pmatrix}\cos\beta_\pm & \sin\beta_\pm\\ -\sin\beta_\pm & \cos\beta_\pm\end{pmatrix},$$

$$m_{H^\pm}^2 = \frac{(v_h^2+2v_\Delta^2)(2\sqrt{2}\mu_1 - \lambda_{\Phi_1\Delta 1}v_\Delta)}{4v_\Delta}.$$

In the similar way, the mixing matrix for the neutral CP-odd and CP-even scalars are given as

$$\mathcal{M}_{CP_{\mathrm{even}}}^2 = \begin{pmatrix} A & B\\ B & C\end{pmatrix}, \quad \mathcal{M}_{CP_{\mathrm{odd}}}^2 = \sqrt{2}\mu_1\begin{pmatrix} 2v_\Delta & -v_h\\ v_h & v_h^2/2v_\Delta\end{pmatrix},$$

where

$$A = 2\lambda_{\Phi_1}v_h^2, \qquad B = v_h((\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})v_\Delta - \sqrt{2}\mu_1), \qquad C = \frac{\sqrt{2}\mu_1 v_h^2 + 4(\lambda_{\Delta 1}+\lambda_{\Delta 2})v_\Delta^3}{2v_\Delta}. \tag{2.6}$$

These matrices can be diagonalized by another rotation matrix, and for CP-odd mixing matrix, one of the eigenvalues would be zero, corresponding to neutral Goldstone degree of freedom, and the nonzero eigenvalue would be the mass for the neutral pseudoscaler. The CP-even mixing matrix will give two physical eigenstates h and H, where the lighter one h is recognized as the SM Higgs boson, and the corresponding expressions are given as

$$\mathcal{R}_{\beta^0} = \begin{pmatrix}\cos\beta_0 & \sin\beta_0\\ -\sin\beta_0 & \cos\beta_0\end{pmatrix}, \qquad \mathcal{R}_\alpha = \begin{pmatrix}\cos\alpha_0 & \sin\alpha_0\\ -\sin\alpha_0 & \cos\alpha_0\end{pmatrix},$$

$$m_A^2 = \frac{\mu_1(v_h^2+4v_\Delta^2)}{\sqrt{2}v_\Delta}, \tag{2.7}$$

$$m_h^2 = \frac{1}{2}\left[A + C - \sqrt{(A-C)^2+4B^2}\right], \tag{2.8}$$

$$m_H^2 = \frac{1}{2}\left[A + C + \sqrt{(A-C)^2 + 4B^2}\right]. \tag{2.9}$$

Since the second doublet is inert and does not take part in the electroweak symmetry breaking, there would be no mixing, and the mass eigenstates would be the same as the gauge eigenstates. The physical masses for the second SU(2) Higgs doublet would be in terms of the Higgs VEV and the triplet VEV, and the corresponding mass expressions are as follows:

$$m_{\phi^\pm}^2 = m_{\Phi_2}^2 + \frac{1}{2}\lambda_3 v_h^2 + \frac{1}{2}\lambda_{\Phi_2\Delta} v_\Delta^2 \simeq m_{\Phi_2}^2 + \frac{1}{2}\lambda_3 v_h^2, \tag{2.10}$$

$$m_{H^0}^2 = m_{\Phi_2}^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5) + \frac{1}{2}(\lambda_{\Phi_2\Delta} + \lambda_{\Phi_2\Delta 1}) v_\Delta^2 - \sqrt{2}\mu_2 v_\Delta \simeq m_{\Phi_2}^2 + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5) v_h^2, \tag{2.11}$$

$$m_{A^0}^2 = m_{\Phi_2}^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5) + \frac{1}{2}(\lambda_{\Phi_2\Delta} + \lambda_{\Phi_2\Delta 1}) v_\Delta^2 + \sqrt{2}\mu_2 v_\Delta \simeq m_{\Phi_2}^2 + \frac{1}{2}(\lambda_3 + \lambda_4 - \lambda_5) v_h^2. \tag{2.12}$$

The SU(2) Higgs triplet generates Majorona mass terms for the neutrinos through the Yukawa interaction, are given as

$$\mathcal{L}_Y = -\frac{(y_\nu)_{\alpha\beta}}{\sqrt{2}} l_i^T C i\sigma_2 \Delta l_\beta + \text{H.c.}, \tag{2.13}$$

where α and β are the flavor indices for the leptons. The neutrino masses are generated by the small nonzero induced triplet VEV, $v_\Delta = \frac{\mu_1 v_h^2}{\sqrt{2} m_\Delta^2} \ll v_h$ as[1]

$$(m_\nu)_{\alpha\beta} \simeq \sqrt{2} v_\Delta (y_\nu)_{\alpha\beta}. \tag{2.14}$$

In the case of type-II seesaw, it is possible to acheive tiny neutrino masses even with $\mathcal{O}(1)$ neutrino Yukawa couplings because of the small VEV for the Higgs triplet. This will be true simultaneously for light triplet scalars, even below TeV. In this article, we explore the possibility of radiative EWSB, which demands the additional scalar to be below a few TeV. The detailed analysis for the radiative EWSB is discussed in the next section.

III. RADIATIVE SYMMETRY BREAKING

The radiative EWSB is achieved when running of the Higgs mass parameter, $m_{\Phi_1}^2$ changes sign from positive to negative while evolving from high energy to low energies. The renormalization group flow for the Higgs mass parameter is proportional to itself as given in Eq. (3.1) [67,68], which makes it impossible to turn negative at any energy scale. The magnitude of the Higgs mass parameter is $m_{\Phi_1}^2 = -(88.0\ \text{GeV})^2$. In order to make this negative, the contribution of the additional scalar goes like m_{multi}^2 and is constrained to be not much heavier than TeV even if the interaction couplings are not very weak,

$$\beta_{m_{\Phi_1}^2}^{\text{SM}} = \frac{1}{16\pi^2}\left[6\lambda_{\Phi_1} - \frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 + 2\text{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + 3Y_e^\dagger Y_e)\right] m_{\Phi_1}^2. \tag{3.1}$$

The presence of a TeV scale triplet scalar in type-II seesaw can aid in radiative symmetry breaking (RSB), and the masses for the second SU(2) doublet can be assumed to be small. The expressions for the one-loop running of the mass parameters of this model are given as follows:

$$\beta_{m_{\Phi_1}^2} = \frac{1}{(16\pi)^2}\left[\left(-\frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda_{\Phi_1} + 2T\right)m_{\Phi_1}^2 + (4\lambda_3 + 2\lambda_4)m_{\Phi_2}^2 + (6\lambda_{\Phi_1\Delta} + 3\lambda_{\Phi_1\Delta 1})m_\Delta^2 + 12\mu_1^2\right], \tag{3.2}$$

$$\beta_{m_{\Phi_2}^2} = \frac{1}{(16\pi)^2}\left[\left(-\frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 + 12\lambda_2\right)m_{\Phi_2}^2 + (4\lambda_3 + 2\lambda_4)m_{\Phi_1}^2 + (6\lambda_{\Phi_2\Delta} + 3\lambda_{\Phi_2\Delta 1})m_\Delta^2 + 12\mu_2^2\right], \tag{3.3}$$

[1]It is assumed that $m_\Delta^2 > 0$ implying that v_Δ is only induced after Higgs VEV, v_h, is generated. In the case where μ_1 is so small that $\mu_1 v_h^2 \ll m_\Delta^2$, the triplet scalar decouples. After integrating out the heavy scalar triplet, the Yukawa-Lagrangian in Eq. (2.13) generates an effective dimension-5 Weinberg operator to generate the neutrino mass.

$$\beta_{m_\Delta^2} = \frac{1}{(16\pi)^2}\left[\left(-\frac{18}{5}g_1^2 - 12g_2^2 + 16\lambda_{\Delta 1} + 12\lambda_{\Delta 2} + 4\mathrm{Tr}(Y_N Y_N^\dagger)\right)m_\Delta^2 + (4\lambda_{\Phi_1\Delta} + 2\lambda_{\Phi_1\Delta 1})m_{\Phi_1}^2 \right. \tag{3.4}$$

$$\left. +(4\lambda_{\Phi_2\Delta} + 2\lambda_{\Phi_2\Delta 1})m_{\Phi_2}^2 + 4\mu_1^2 + 4\mu_2^2\right], \tag{3.5}$$

$$\beta_{\mu_1} = \frac{1}{(16\pi)^2}\left[\left(-\frac{27}{10}g_1^2 - \frac{21}{2}g_2^2 + 4\lambda_{\Phi_1} + 4\lambda_{\Phi_1\Delta} + 6\lambda_{\Phi_1\Delta 1} + 2T + 2\mathrm{Tr}(Y_N Y_N^\dagger)\right)\mu_1 + 4\lambda_5\mu_2\right], \tag{3.6}$$

$$\beta_{\mu_2} = \frac{1}{(16\pi)^2}\left[\left(-\frac{27}{10}g_1^2 - \frac{21}{2}g_2^2 + 4\lambda_2 + 4\lambda_{\Phi_2\Delta} + 6\lambda_{\Phi_2\Delta 1} + 2\mathrm{Tr}(Y_N Y_N^\dagger)\right)\mu_2 + 4\lambda_5\mu_1\right], \tag{3.7}$$

where

$$T = \mathrm{Tr}[3\mathrm{Tr}(Y_d Y_d^\dagger) + \mathrm{Tr}(Y_e Y_e^\dagger) + 3\mathrm{Tr}(Y_u Y_u^\dagger)]. \tag{3.8}$$

The full two-loop β-functions for the quartic couplings are computed using SARAH [69] and are given in Appendix.

The following terms are crucial for changing the sign of the Higgs mass parameter; $m_{\Phi 1}$ can be $(4\lambda_3 + 2\lambda_4)m_{\Phi_2}^2$ and $(6\lambda_{\Phi_1\Delta} + 3\lambda_{\Phi_1\Delta 1})m_\Delta^2$ or either it can be a combination of both from Eq. (3.2). Three different values for the mass parameters $m_{\Phi_2}^2$ and m_Δ^2 are considered in Fig. 1, i.e., $m_{\Phi_2} = 223.6, m_\Delta = 173.2$ GeV, $m_{\Phi_2} = 300.0,$

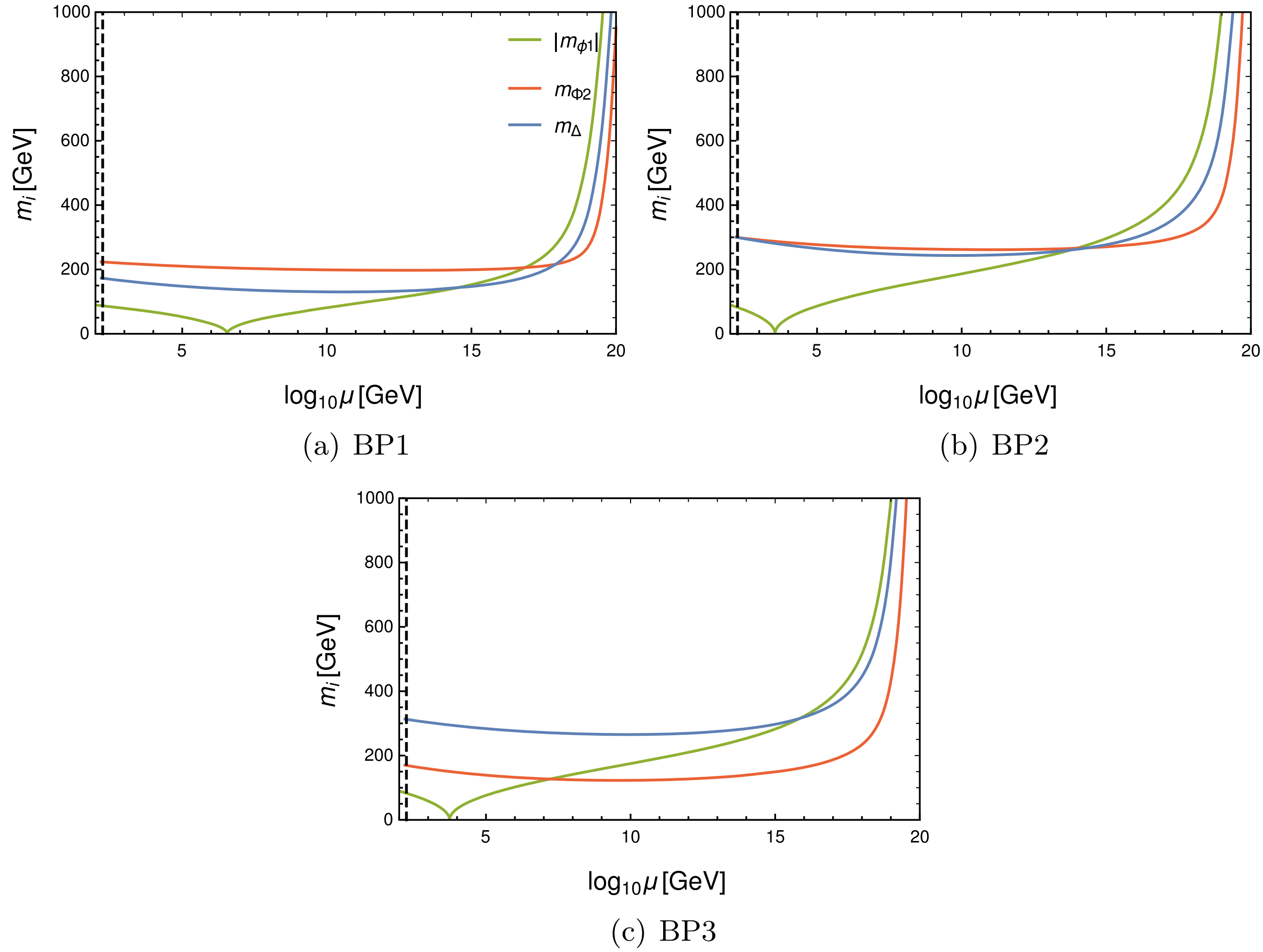


FIG. 1. Variation of the bare mass parameter for the SM Higgs doublet, inert Higgs doublet and the SU(2) triplet with the energy scale in GeV. The black dashed line corresponds to the scale equal to the mass of the lightest particle in the theory. The values of the different parameters needed in computing the radiative electroweak symmetry breaking are given in Table I for three different benchmark points.

TABLE I. The values relevant for the running of the bare mass parameters are provided for three benchmark points. Λ denotes the scale corresponding to the mass of the lightest particle in the theory. The values of the quartic couplings chosen are allowed from the Planck-scale perturbativity.

	BP1	BP2	BP3
$m^2_{\Phi 1}(\Lambda)/(\mathrm{GeV})^2$	$-(85.83)^2$	$-(85.83)^2$	$-(85.83)^2$
$m^2_{\Phi 2}(\Lambda)/(\mathrm{GeV})^2$	223.6^2	300.0^2	173.3^2
$m^2_{\Delta}(\Lambda)/(\mathrm{GeV})^2$	173.2^2	300.0^2	316.0^2
λ_3	0.15	0.15	0.15
λ_4	0.00	0.00	0.00
$\lambda_{\Phi_1\Delta}$	0.50	0.50	0.50
$\lambda_{\Phi_1\Delta 1}$	0.00	0.00	0.00

$m_\Delta = 300.0$ GeV, and $m_{\Phi_2} = 173.3, m_\Delta = 316.0$ in Figs. 1(a), 1(b) and 1(c), respectively.

The variation for the running mass parameters from Planck scale down to weak scale, i.e., $m_{\Phi 1}, m_{\Phi 2}$, and m_Δ is given in Fig. 1. The variation for the SM Higgs mass parameter, the additional SU(2) Higgs doublet mass parameter, and the triplet bare-mass parameter are denoted by the green, red, and the blue curves, respectively. The dotted black vertical line corresponds to the scale chosen equal to the mass of the lightest particle in the theory. The values of quartic couplings $\lambda_3, \lambda_4, \lambda_{\Phi_1\Delta}$, and $\lambda_{\Phi_1\Delta 1}$ are chosen to be equal for three different benchmark points (BPs) to check the effect of different mass ranges for the bare-mass parameters, and the corresponding values are also given in Table I. These values of quartic couplings are the maximum allowed values at the EW scale for the Planck scale perturbativity. The Higgs mass parameter changes sign from positive to negative, i.e., ($m_{\Phi 1}$ touches the horizontal line) at a particular energy scale depending on contributions from other bare-mass parameters and the quartic couplings, i.e., $10^{6.5}$ GeV, $10^{3.6}$ GeV, and $10^{3.8}$ GeV for BP1, BP2, and BP3 in Table I, respectively. The radiative EWSB is triggered at these energy scales. The effect of triplet mass parameter is more crucial in triggering the radiative EWSB, since the value of quartic coupling $\lambda_{\Phi_1\Delta}$ is more compared to the interaction quartic coupling for the second doublet, i.e., λ_3. This higher value of the quartic coupling in multiplication with the bare-mass parameter of the triplet enhances the positive contribution even more from the triplet. The scale for which the radiative symmetry breaking is triggered is reduced for more and more positive effect, as can be seen from Figs. 1(a)–(c). The mass parameter for the second doublet and the triplet are positive for all energy scales up to Planck scale, implying that the symmetry remains unbroken (since the triplet VEV is very small and induced only after the Higgs VEV is generated). The mass parameter for the second doublet splits into $m^\pm_\phi, m_{H^0}$, and m_{A^0}, when the electroweak symmetry is broken.

The variation of the quartic couplings relevant for the radiative EWSB and the Higgs quartic coupling using two-loop β-functions is given in Fig. 2. The Planck-scale perturbativity is achieved for all the quartic couplings (including those which are not plotted here). The running of the Higgs quartic coupling encounters a discontinuity at the scale Λ; above this energy scale, the full set of two-loop renormalization group equations (RGEs) is used for the running of all the parameters of the theory. The values chosen at the EW scale for the quartic couplings are same as given in Table I.

The quartic couplings are chosen in terms of maximizing the value of the interaction coupling at the EW scale,

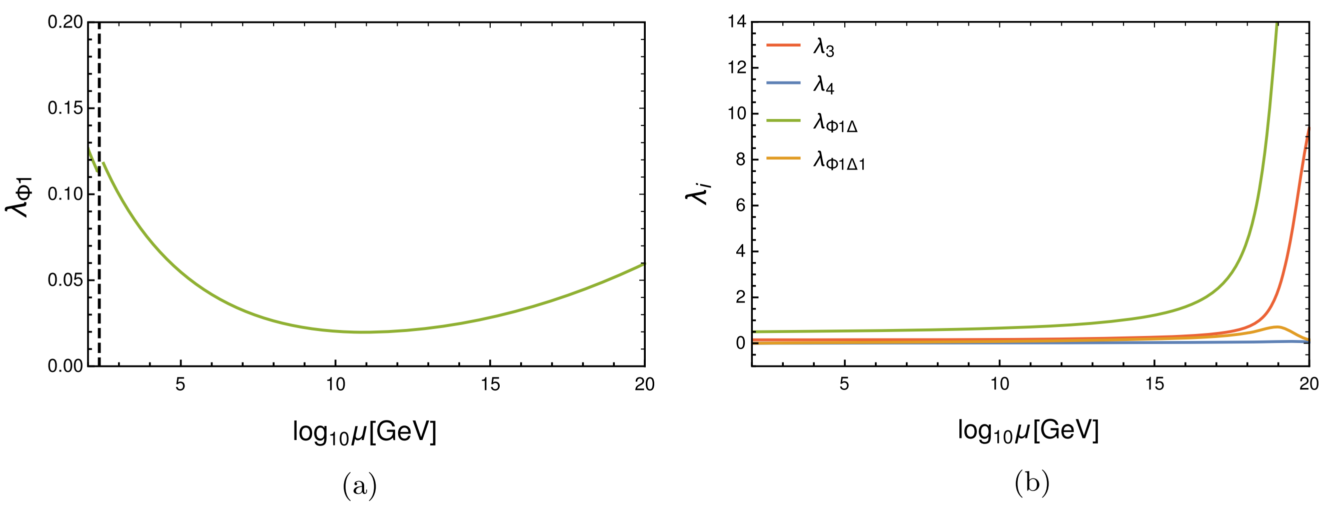


FIG. 2. (a) Variation of the interaction quartic couplings with the energy scale using two-loop β-functions. The Higgs quartic coupling variation is plotted in Fig. 2(b), where the dashed black line corresponds to the scale equal to the mass of the lightest particle in the theory (similar for BP2 and BP3).

which was possible more for the triplet case, not the inert doublet. This higher value of the interaction coupling would be very crucial in enhancing the phase transition dynamics. It would also be interesting to see how this interplay between the triplet mass and the inert doublet mass effects the strength of the phase transition. Hence, after discussing the radiative EWSB, the next section is devoted to EWPT using the RG-improved potential.

IV. ELECTROWEAK PHASE TRANSITION

The EWPT from the symmetric phase to the broken phase is accomplished by studying the finite-temperature effective potential in terms of background fields. The tree-level potential in Eq. (2.2) includes contributions from one-loop Coleman-Weinberg effective potential, i.e., $V^{\mathrm{CW}}_{1-\mathrm{loop}}(\tilde{m}_i^2)$, and one-loop potential at finite temperature, i.e., $V^{T\neq 0}_{1-\mathrm{loop}}(\tilde{m}_i^2)$. The full one-loop finite-temperature effective potential is given as follows:

$$V_1(T) = V_0^{\mathrm{eff}} + V^{\mathrm{CW}}_{1-\mathrm{loop}}(\tilde{m}_i^2) + V^{T\neq 0}_{1-\mathrm{loop}}(\tilde{m}_i^2), \tag{4.1}$$

where V_0^{eff} is the tree-level potential in Eq. (2.2) rewritten in terms of background fields.

The one-loop effective potential at zero and finite temperature is given as follows [70];

$$V^{\mathrm{CW}}_{1-\mathrm{loop}} = \frac{1}{(64\pi)^2}\sum_{i=B,F}(-1)^{F_i} n_i \widehat{m_i}^4\left[\log\left(\frac{\tilde{m}_i^2}{\mu^2}\right) - k_i\right], \tag{4.2}$$

$$V^{T\neq 0}_{1-\mathrm{loop}} = \frac{T^4}{(2\pi)^2}\sum_{i=B,F}(-1)^{F_i} n_i J_{B/F}\left(\frac{\tilde{m}_i^2}{T^2}\right), \tag{4.3}$$

where $\tilde{m}_i^2 = \tilde{m}_i^2(h_1,h_2,h_3;T) = \hat{m}_i^2(h_1,h_2,h_3) + \Pi_i T^2$ are the thermally corrected field dependent mass expressions, and the h_1, h_2, and h_3 are the background fields for the SM Higgs doublet, additional triplet, and the inert doublet, respectively. The field-dependent mass expressions and the corresponding Debye corrections are given as

$$\mathcal{M}^2_{CP_{\mathrm{even}}} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix},$$

$$\begin{aligned}
A_{11} &= \frac{(\lambda_3+\lambda_4+\lambda_5)h_3^2}{2} + 3\lambda_{\Phi_1}h_1^2 - \lambda_{\Phi_1}v_h^2 + \frac{(\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})h_2^2}{2} - \frac{(\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})v_\Delta^2}{2} - \sqrt{2}\mu_1 h_2 + \sqrt{2}\mu_1 v_\Delta, \\
A_{12} &= A_{21} = (\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})h_1h_2 - \sqrt{2}\mu_1 h_1, \qquad A_{13} = A_{31} = (\lambda_3+\lambda_4+\lambda_5)h_1h_3, \\
A_{22} &= \frac{(\lambda_{\Phi_2\Delta}+\lambda_{\Phi_2\Delta 1})h_3^2}{2} + \frac{(\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})h_1^2}{2} - \frac{(\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})v_h^2}{2} + 3(\lambda_{\Delta 1}+\lambda_{\Delta 2})h_2^2 - (\lambda_{\Delta 1}+\lambda_{\Delta 2})v_\Delta^2 + \frac{\mu_1 v_h^2}{\sqrt{2}v_\Delta}, \\
A_{33} &= m_{\Phi_2}^2 + 3\lambda_2 h_3^2 + \frac{(\lambda_3+\lambda_4+\lambda_5)h_1^2}{2} + \frac{(\lambda_{\Phi_2\Delta}+\lambda_{\Phi_2\Delta 1})h_2^2}{2} - \sqrt{2}\mu_2 h_2, \\
A_{23} &= A_{32} = (\lambda_{\Phi_2\Delta}+\lambda_{\Phi_2\Delta 1})h_2h_3 - \sqrt{2}\mu_2 h_3.
\end{aligned} \tag{4.4}$$

In the similar way, the CP-odd mixing matrix in terms of the background fields is computed as

$$\mathcal{M}^2_{CP_{\mathrm{odd}}} = \begin{pmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{pmatrix},$$

$$\begin{aligned}
B_{11} &= \frac{(\lambda_3+\lambda_4-\lambda_5)h_3^2}{2} + \lambda_{\Phi_1}h_1^2 - \lambda_{\Phi_1}v_h^2 + \frac{(\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})h_2^2}{2} - \frac{(\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})v_\Delta^2}{2} + \sqrt{2}\mu_1 h_2 + \sqrt{2}\mu_1 v_\Delta, \\
B_{12} &= B_{21} = -\sqrt{2}\mu_1 h_1, \qquad B_{13} = B_{31} = 2\lambda_5 h_1h_3, \qquad B_{23} = B_{32} = -\sqrt{2}\mu_2 h_3, \\
B_{22} &= \frac{(\lambda_{\Phi_2\Delta}+\lambda_{\Phi_2\Delta 1})h_3^2}{2} + \frac{(\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})h_1^2}{2} - \frac{(\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})v_h^2}{2} + (\lambda_{\Delta 1}+\lambda_{\Delta 2})h_2^2 - (\lambda_{\Delta 1}+\lambda_{\Delta 2})v_\Delta^2 + \frac{\mu_1 v_h^2}{\sqrt{2}v_\Delta}, \\
B_{33} &= m_{\Phi_2}^2 + \lambda_2 h_3^2 + \frac{(\lambda_3+\lambda_4-\lambda_5)h_1^2}{2} + \frac{(\lambda_{\Phi_2\Delta}+\lambda_{\Phi_2\Delta 1})h_2^2}{2} + \sqrt{2}\mu_2 h_2.
\end{aligned} \tag{4.5}$$

The field dependent charged mixing matrix is computed as follows:

$$\mathcal{M}_{\pm}^2 = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix},$$

$$C_{11} = \frac{\lambda_3 h_3^2}{2} + \lambda_{\Phi_1} h_1^2 - \lambda_{\Phi_1} v_h^2 + \frac{\lambda_{\Phi_1\Delta} h_2^2}{2} - \frac{(\lambda_{\Phi_1\Delta} + \lambda_{\Phi_1\Delta 1}) v_\Delta^2}{2} + \sqrt{2}\mu_1 v_\Delta,$$

$$C_{12} = C_{21} = \frac{\lambda_{\Phi_1\Delta 1}}{2} h_1 h_2 - \sqrt{2}\mu_1 h_1, \qquad C_{13} = C_{31} = \frac{\lambda_4}{2} h_1 h_3 + \lambda_5 h_1 h_3, \qquad C_{23} = C_{32} = \frac{\lambda_{\Phi_2\Delta 1}}{2} h_2 h_3 - \sqrt{2}\mu_2 h_3,$$

$$C_{22} = \frac{(2\lambda_{\Phi_2\Delta} + \lambda_{\Phi_2\Delta 1}) h_3^2}{2} + \frac{(2\lambda_{\Phi_1\Delta} + \lambda_{\Phi_1\Delta 1}) h_1^2}{2} - (\lambda_{\Phi_1\Delta} + \lambda_{\Phi_1\Delta 1}) v_h^2 + (2\lambda_{\Delta 1} + \lambda_{\Delta 2}) h_2^2 - 2(\lambda_{\Delta 1} + \lambda_{\Delta 2}) v_\Delta^2 + \frac{\sqrt{2}\mu_1 v_h^2}{v_\Delta},$$

$$C_{33} = m_{\Phi_2}^2 + \lambda_2 h_3^2 + \frac{\lambda_3 h_1^2}{2} + \frac{\lambda_{\Phi_2\Delta} h_2^2}{2}, \tag{4.6}$$

and

$$\hat{m}_{G^0}^2 = \lambda_h h_1^2 - m_h^2, \qquad \hat{m}_W^2 = \frac{g_2^2}{4} h_1^2,$$

$$\hat{m}_Z^2 = \frac{g_2^2 + g_1^2}{4} h_1^2, \qquad \hat{m}_t^2 = \frac{y_t^2}{2} h_1^2,$$

where $\hat{m}_{G_0}, \hat{m}_W^2, \hat{m}_Z^2$, and $\hat{m}_t^2$ are the masses for the Goldstone bosons, the gauge bosons, and the top quark, respectively. The thermal corrections for the masses belonging to the same multiplet would be the same, and the corresponding expressions for the Debye coefficients are given as

$$\Pi_h = \left(\frac{g_1^2 + 3g_2^2}{16} + \frac{\lambda_{\Phi_1}}{2} + \frac{y_t^2}{4} + \frac{2\lambda_3 + \lambda_4}{12} + \frac{2\lambda_{\Phi_1\Delta} + \lambda_{\Phi_1\Delta 1}}{6}\right) T^2,$$

$$\Pi_{G^0} = \left(\frac{g_1^2 + 3g_2^2}{16} + \frac{\lambda_{\Phi_1}}{2} + \frac{y_t^2}{4} + \frac{2\lambda_3 + \lambda_4}{12} + \frac{2\lambda_{\Phi_1\Delta} + \lambda_{\Phi_1\Delta 1}}{6}\right) T^2,$$

$$\Pi_\Delta = \left(\frac{2\lambda_{\Phi_1\Delta} + \lambda_{\Phi_1\Delta 1}}{12} + \frac{2\lambda_{\Phi_2\Delta} + \lambda_{\Phi_2\Delta 1}}{12} + \frac{(5\lambda_{\Delta 1} + 3\lambda_{\Delta 2})}{6}\right) T^2,$$

$$\Pi_{\Phi_2} = \left(\frac{\lambda_2}{2} + \frac{2\lambda_3 + \lambda_4}{12} + + \frac{2\lambda_{\Phi_2\Delta} + \lambda_{\Phi_2\Delta 1}}{6}\right) T^2,$$

$$\Pi_{W_L} = \frac{7}{3} g_2^2 T^2,$$

$$\Pi_{W_T} = \Pi_{Z_T} = \Pi_{\gamma_T} = 0,$$

$$\tilde{m}_{Z_L}^2 = \frac{1}{2}\hat{m}_Z^2 + \frac{7}{6}(g_1^2 + g_2^2) T^2 + \delta,$$

$$\tilde{m}_{\gamma_L}^2 = \frac{1}{2}\hat{m}_Z^2 + \frac{7}{6}(g_1^2 + g_2^2) T^2 - \delta, \tag{4.7}$$

where $\{H, A, H^{\pm\pm}, H^{\pm}\} \in \Delta$ and $\{H^0, A^0, \phi^{\pm}\} \in \Phi_2$ as subscripts in Eq. (4.7) consist of all the degrees of freedom for the additional triplet and the doublet. $\Pi_h, \Pi_{G^0}, \Pi_\Delta$, Π_{Φ_2}, Π_{W_L}, and Π_{W_T} are the thermal corrections for the SM-Higgs boson, Goldstone boson, triplet, inert doublet, longitudinal, and the transverse component of the W boson, respectively. The thermal corrections for all the particles belonging to the same multiplet are same. $\tilde{m}_{Z_L}^2$ and $\tilde{m}_{\gamma_L}^2$ are the Debye masses for the longitudinal and transverse component of Z boson, and δ is computed as follows:

$$\delta^2 = \left(\frac{1}{2}\hat{m}_z^2 + \frac{7}{6}(g_1^2 + g_2^2) T^2\right)^2 - \frac{7}{18} g_1^2 g_2^2 T^2 (3v_h^2 + 14T^2). \tag{4.8}$$

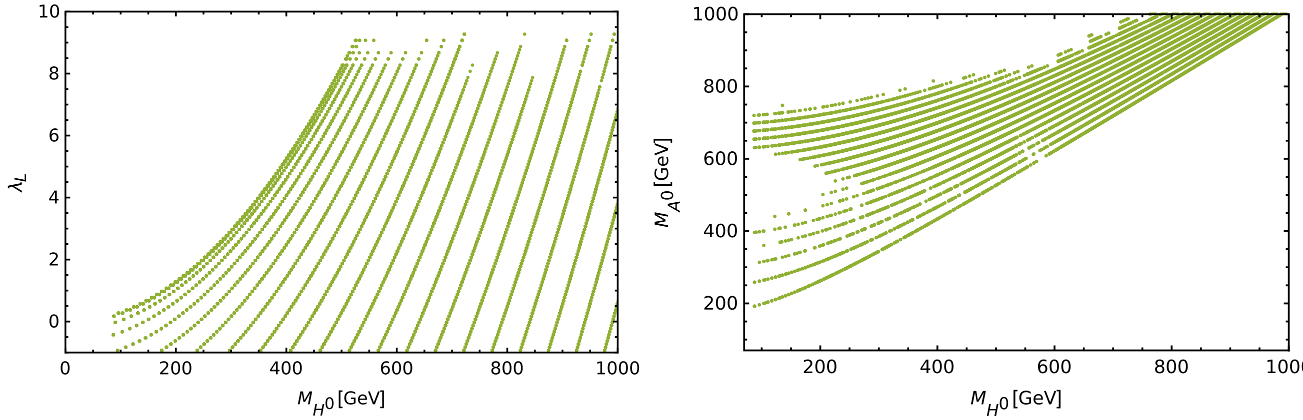
After computing the masses in terms of background fields and the corresponding thermal corrections, the input parameters to be fed at the EW scale for two-loop β-functions are given in Table II for computing the phase-transition dynamics.

The one-loop Coleman-Weinberg potential has scale dependence in Eq. (4.2), and if the values chosen at the EW scale change a lot at higher scales, then it would be interesting to see the dynamics of the phase transition with the running couplings [43]. The variation of the interaction coupling $\lambda_L = (\lambda_3 + \lambda_4 + \lambda_5)$ with the DM mass in GeV is shown in Fig. 3(a). The running of these parameters is considered until $\mu = Q = 246$ GeV, and the corresponding values of these parameters at the scale $Q = 246.0$ GeV are used as input for the EWPT. The values of λ_L are quite high but these values are perturbative until the considered energy

TABLE II. Initial values chosen at the EW scale for the RG evolution of the different SM parameters.

g_1	g_2	g_3	y_t	λ_{Φ_1}
0.46256[a]	0.64779	1.1666	0.93690	0.12604

[a]In this paper, we have used $SU(5)$ normalization for g_1 since SARAH inherently use this convention. However, to achieve results involving usual g_1 coupling, one has to replace g_1 by $\sqrt{\frac{5}{3}}g_1$ would become 0.358297.



(a) $Q = 246$ GeV (b) $Q = 246$ GeV

FIG. 3. (a) Variation of the DM mass in GeV with the interaction coupling $\lambda_L = \lambda_3 + \lambda_4 + \lambda_5$, and (b) depicts the mass splitting between the DM mass M_{H^0} and M_{A^0} in GeV. The scattered points correspond to the strongly first-order phase transition and $Q = 246$ GeV is the scale at which the running value of the couplings is considered for the strongly first-order phase transition.

scale of $Q = 246$ GeV. Such high values of λ_L will allow a strongly first-order phase transition for all possible values of the DM mass under consideration. The green points are all those points that strongly indicate a strongly first-order phase transition, i.e., $\frac{\phi_+(T_c)}{T_c} \gtrsim 1.0$ [71,72] where $\frac{\phi_+(T_c)}{T_c} = \frac{\sqrt{h_1^2(T_c)+h_2^2(T_c)+(h_3(T_c)-h_3^h(T_c))^2}}{T_c}$ (in case of multiplets [73,74], when the false vacuum is $(0.0, 0.0, h_3^h)$ instead of (0.0,0.0,0.0). The mass splitting between the DM mass M_{H^0} and M_{A^0} is given in Fig. 3(b). The values of the other quartic couplings, $\lambda_i \in \{\lambda_{\Phi_{1\Delta}}, \lambda_{\Phi_{1\Delta1}}, \lambda_{\Phi_{2\Delta}}, \lambda_{\Phi_{2\Delta1}}, \lambda_{\Delta1}, \lambda_{\Delta2}\}$ are chosen to be 0.01 and 0.1, and the solutions are the same for both the values.

The type-II seesaw extended with the inert doublet has a larger positive contribution to the running of couplings because of the more number of degrees of freedom. In that case, very low values for the quartic couplings would be allowed for achieving the Planck-scale perturbativity, which would not be consistent with the strongly first-order phase transition. The same is also true for achieving perturbativity at any higher scale compared to $Q = 246$ GeV. Hence, it would be interesting to see if the mass bounds triggering the radiative EWSB are also consistent with the strongly first-order phase transition and perturbative unitarity until that particular scale.

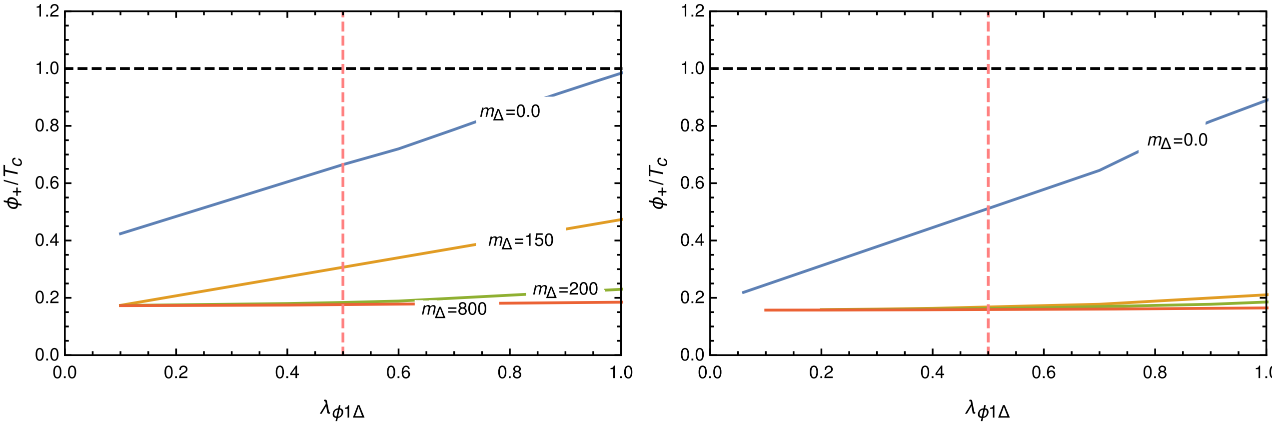
For perturbative unitarity up to the Planck scale, we show the maximum allowed values for the quartic couplings at the EW scale, which are crucial for the strongly first-order phase transition are given in Table III. Figure 4 shows the variation of the interaction quartic coupling for the Higgs field and the triplet scalar field with the strength of phase transition, $\frac{\phi_+(T_c)}{T_c}$. The black-dashed horizontal line corresponds to the criteria for a strongly first-order phase transition. The dashed pink line corresponds to the maximum allowed value of the interaction quartic coupling at the EW scale in Figs. 4(a) and 4(b). The strongly first-order phase transition is not achieved for any of the triplet masses, as shown in Figs. 4(a) and 4(b) for two different values of the mass parameter of the inert doublet. The values at the EW scale are small enough and the strongly first order phase transition is not achieved for any of the doublet or the triplet masses. Still, the first-order phase transition is satisfied, i.e., $\frac{\phi_+(T_c)}{T_c} \gtrsim 0.6$ for the vanishing doublet and triplet bare mass parameters, m_{Φ_2} and $m_\Delta = 0.0$ GeV [Figs. 4(a) and 4(b)].

The allowed values of the quartic couplings at the EW scale which gives the Planck scale perturbativity will satisfy the first-order phase transition but this much lower masses cannot trigger the electroweak symmetry breaking radiatively.

Figure 5(a) corresponds to the variation of the mass parameters of the theory with the energy scale. The values of the mass parameters are chosen to be $m_\Delta(\Lambda)$ and $m_{22}(\Lambda) = 100.0$ GeV, and Λ corresponds to the scale equal to the mass of the lightest particle in the theory as discussed in Sec. III. The electroweak symmetry breaking is triggered around $\sim 10^{6.80}$ GeV and the inert-doublet mass parameter remains positive for all higher scales indicating

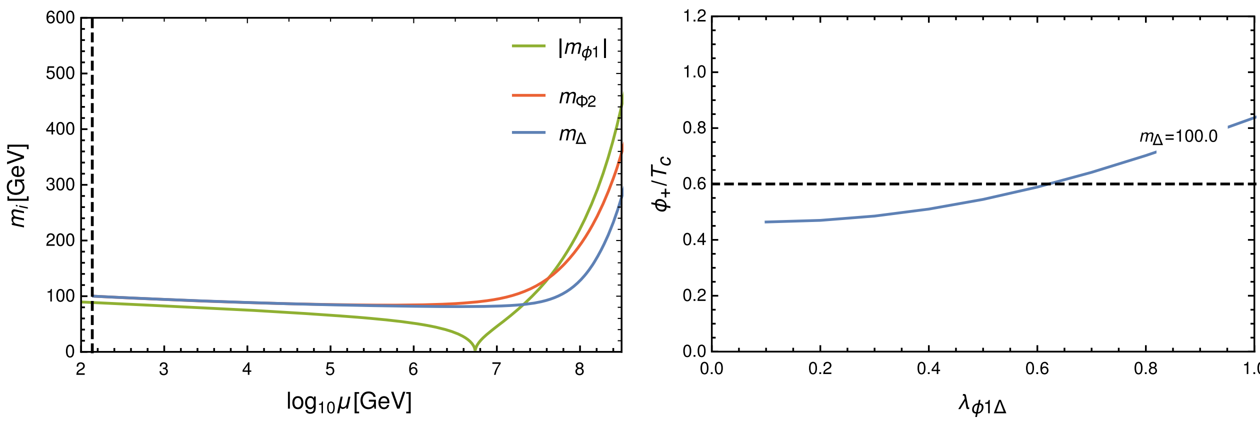
TABLE III. The maximum allowed values of the quartic couplings from the Planck-scale perturbativity at the EW scale.

Q (GeV)	λ_{Φ_1}	λ_2	λ_3	λ_4	λ_5	$\lambda_{\Phi_{1\Delta}}$	$\lambda_{\Phi_{1\Delta1}}$	$\lambda_{\Phi_{2\Delta}}$	$\lambda_{\Phi_{2\Delta1}}$	$\lambda_{\Delta1}$	$\lambda_{\Delta2}$
EW	0.1264	0.01	0.15	0.00	−0.01	0.50	0.00	0.00	−1.00	0.001	0.001



(a) Q =EW, $m_{\Phi_2} = 0.0$ GeV (b) Q=EW, $m_{\Phi_2} = 500.0$ GeV

FIG. 4. Variation of the strength of phase transition $\frac{\phi_+(T_c)}{T_c}$ with the interaction quartic coupling for SM Higgs and the triplet with different values of the mass parameter of the triplet. The inert doublet mass parameter is chosen as 0.0 GeV and 500.0 GeV in (a) and (b). (a) and (b) correspond to the phase transition for the maximum allowed values of the quartic couplings at the EW scale that gives Planck scale perturbativity as given in Table III.



(a) $m_{22} = 100.0$ GeV (b) $m_{22} = 100.0$ GeV

FIG. 5. (a) Variation of the mass parameters with the scale for triplet and the inert doublet mass parameters both as 100.0 GeV. (b) Variation of the strength of phase transition $\frac{\phi_+(T_c)}{T_c}$ with the interaction quartic coupling for SM Higgs and the triplet. The triplet mass parameter and the doublet mass parameter are both chosen as $m_\Delta = 100.0$ GeV and $m_{22} = 100.0$ GeV, respectively. The black dashed line corresponds to the criteria for the first-order phase transition.

that the Z_2 symmetry remains intact. The values of the relevant quartic couplings are chosen to be $\lambda_3 = 0.9$ and $\lambda_{\Phi_{1\Delta}} = 0.9$ as given in Table IV, which also satisfies the stability conditions since the interaction coupling λ_3 and $\lambda_{\phi 1\Delta}$ are quite large as given in [16,17,75]. The same values are used for the variation of interaction quartic coupling $\lambda_{\Phi_{1\Delta}}$ with the strength of phase transition in Fig. 5(b). These are the minimum possible values for the mass parameters and the quartic couplings which satisfy both the first-order phase transition and radiative electroweak symmetry breaking. These bounds also satisfies the constraints from the direct detection in the DM region from 70 GeV to 1 TeV suggests $\mathcal{O}(0.01) \lesssim |\lambda_L| \lesssim \mathcal{O}(0.1)$ [76].

After discussing the EWPT, it is important to check for the stability of the EW vacuum by considering the running of all couplings till Planck scale. Since the phase transition dynamics includes thermal corrections, the thermal corrections also effect the tunneling probability from the false

TABLE IV. Benchmark points allowed for both radiative symmetry breaking and first-order phase transition.

Q (GeV)	λ_{Φ_1}	λ_2	λ_3	λ_4	λ_5	$\lambda_{\Phi_{1\Delta}}$	$\lambda_{\Phi_{1\Delta 1}}$	$\lambda_{\Phi_{2\Delta}}$	$\lambda_{\Phi_{2\Delta 1}}$	$\lambda_{\Delta 1}$	$\lambda_{\Delta 2}$
EW	0.1264	0.01	0.9	0.00	−0.01	0.90	0.00	0.00	−1.00	0.001	0.001

vacuum to the true vacuum. Hence, in the next section, we study stability of the EW vacuum both at the zero temperature, and the finite temperature.

V. FINITE-TEMPERATURE STABILITY

The electroweak vacuum stability until Planck scale is the theoretical drawback of the Standard Model within the top mass uncertainity. The tunneling probability from the electroweak minima to the second minima existing at the higher-field values is triggered by the thermal corrections to the masses of the theory. The decay lifetime of the electroweak vacuum is computed by considering the bounce solution to the classical equation of motion for the classical potential, $V(h_1) = \frac{\lambda_1}{4} h_1^4$ [51]. The zero-temperature expression for the equation of motion is given as follows [62,63]:

$$\frac{d^2 h_1}{dr^2} + \frac{3}{r}\frac{dh_1}{dr} = \frac{dV(h_1)}{dh_1}, \quad \lim_{r\to\infty} h_1(r) = 0, \quad \left.\frac{dh_1}{dr}\right|_{r=0} = 0, \tag{5.1}$$

with $r = |\vec{r}|$. The corresponding euclidean action for this $O(4)$ spherically symmetric solution is as follows:

$$S_E[h_1(r)] = 2\pi^2 \int_0^\infty dr r^3 \left[\frac{1}{2}\left(\frac{dh_1}{dr}\right)^2 + V(h_1)\right], \tag{5.2}$$

and the bounce solutions to the classical equation of motion in Eq. (5.3) come out to be

$$h_{1B}(r) = \frac{8}{|\lambda_1|}\frac{R}{R^2 + r^2}, \qquad S_E[h_{1b}(r)] = \frac{8\pi^2}{3|\lambda_1|}, \tag{5.3}$$

where R is the arbitrary scale characterizing the size of the bounce $(0 < R < \infty)$. This arbitrary parameter appears in Eq. (5.3), since the approximation that the potential considered above is scale invariant indicating that there is an infinite set of bounce solutions which lead to the same value for action. Addition of the quantum corrections break this scale invariance of the tree-level potential. This implies that the bounce solutions with different values of R which used to give same action at the semiclassical level now gives a one-loop action as $S[h_{1b}(r)] \sim \frac{8\pi^2}{(3|\lambda(1/R)|)}$. Considering the effective potential as $V_{\rm eff} = \frac{\lambda_{\rm eff}}{4} h_1^4$, with $\lambda_{\rm eff}$ is the effective quartic coupling including contributions from all the particles which are coupled to the Higgs field, the bounce solution in Eq. (5.3) now becomes,

$$h_{1B}(r) = \frac{8}{|\lambda_{\rm eff}(\mu)|}\frac{R}{R^2 + r^2}, \quad S_E[h_{1b}(r)] = \frac{8\pi^2}{3|\lambda_{\rm eff}(\mu)|}. \tag{5.4}$$

Now, there is just one particular value of R which saturates the path integral, and defined as R_M. Considering the running of effective quartic coupling $\lambda_{\rm eff}$, an instability is encountered when $\lambda_{\rm eff}(\mu)$ hits zero, and then goes to negative values. The scale μ here is the renormalization scale, ($R_M \sim 1/\mu$ is the size of the bounce) which minimizes the action, i.e., $\beta_{\lambda_{\rm eff}}(\mu) = 0$. After computing the scale μ, it is easy to plot the profile of the bounce using Eq. (5.4). The field h_1 and the four-dimensional Euclidean distance r are both scaled using the Planck mass $M_P = 1.22 \times 10^{19}$ GeV, where $\lambda_{\rm eff}$ is the effective potential including contribution from all the particles which are coupled to the Higgs field and is given by

$$\lambda_{\rm eff}(h,\mu) \simeq \underbrace{\lambda_h(\mu)}_{\text{tree-level}} + \underbrace{\frac{1}{16\pi^2}\sum_{\substack{i=W^\pm,Z,t,\\ h,G^\pm,G^0}} n_i \kappa_i^2 \left[\log\frac{\kappa_i h^2}{\mu^2} - c_i\right]}_{\text{Contribution from SM}} + \underbrace{\frac{1}{16\pi^2}\sum_{\substack{i=\phi^\pm,\\ H^0,A^0}} n_i \kappa_i^2 \left[\log\frac{\kappa_i h^2}{\mu^2} - c_i\right]}_{\text{Contribution from inert doublet}} + \underbrace{\frac{1}{16\pi^2}\sum_{\substack{i=H^{\pm\pm},A,\\ H,H^\pm}} n_i \kappa_i^2 \left[\log\frac{\kappa_i h^2}{\mu^2} - c_i\right]}_{\text{Contribution from Type-II seesaw}}, \tag{5.5}$$

Since, we have studied the electroweak phase transition from the symmetric phase to the broken phase using the finite-temperature analysis, it would be interesting to see the effect of thermal corrections on the stability of the vacuum. In that case we use the full one-loop thermally corrected effective potential as given in Eq. (4.1).

The one-loop thermally corrected effective potential can be rewritten as the contribution of $V^{T\neq0}_{1-\rm loop}$ and $V^{T\neq0}_{\rm ring}$ as follows:

$$V^{T\neq0}_{1-\rm loop} = \sum_{i=W,G^0,Z,h,\Delta,\Phi_2} \frac{n_i T^4}{2\pi^2} J_B\left(\frac{\hat{m}_i^2}{T^2}\right) + \frac{n_t T^4}{2\pi^2} J_F\left(\frac{\hat{m}_t^2}{T^2}\right), \tag{5.6}$$

$$V^{T\neq0}_{\rm ring} = \sum_{i=W_L,G^0,Z_L,\gamma_L,h,\Delta,\Phi_2} \frac{n_i T^4}{12\pi}\left\{\left[\frac{\hat{m}_i^2}{T^2}\right]^{3/2} - \left[\frac{\mathcal{M}_i^2}{T^2}\right]^{3/2}\right\}, \tag{5.7}$$

where $\hat{m}_i^2$ are the masses in terms of the background filed and $n_i's$ are the degrees of freedom. One important thing to note is that the thermal corrections for the gauge bosons are nonzero only for the longitudinal degrees of freedom and there are no thermal corrections for the fermionic degrees of freedom. The expressions for the spline functions $J_{B,F}$ are defined as

$$J_{B,F}(x^2) = \int_0^\infty dy y^2 \log\left(1 \mp e^{-\sqrt{y^2+x^2}}\right). \tag{5.8}$$

The zero-temperature expression for the bounce given in Eq. (5.1) is also modified at finite-temperature and is given as follows:

$$\frac{d^2h_1}{dr^2}+\frac{3}{r}\frac{dh_1}{dr}=\frac{dV_{\rm eff}^{T\neq0}(h_1)}{dh_1},\quad \lim_{r\to\infty}h_1(r)=0,\quad \left.\frac{dh_1}{dr}\right|_{r=0}=0, \tag{5.9}$$

with $r=|\vec{r}|$. The corresponding Euclidean action for this $O(3)$ spherically symmetric solution is as follows:

$$S_E[h_1(r)]=4\pi\int_0^\infty drr^2\left[\frac{1}{2}\left(\frac{dh_1}{dr}\right)^2+V_{\rm eff}^{T\neq0}(h_1)\right]. \tag{5.10}$$

The effective potential can now be rewritten in high-temperature limit as follows:

$$V_{\rm eff}^{T\neq0}\simeq\frac{\lambda_{\rm eff}}{4}h_1^4+\frac{1}{2}\eta^2h_1^2T^2+{\rm const}, \tag{5.11}$$

where the constant terms are neglected which are independent of the field h_1. The coefficient for the field and the temperature dependence is given as

$$\begin{aligned}\eta^2=&\frac{1}{12}\left(\frac{3}{4}g_1^2+\frac{9}{4}g_2^2+3y_t^2+6\lambda_{\Phi_1}+2\lambda_3+\lambda_4\right)-\frac{\sqrt{2}}{32\pi}(g_1^3+3g_2^3)-\frac{1}{16\sqrt{3}\pi}(2\lambda_3+\lambda_4)\sqrt{6\lambda_2+(2\lambda_3+\lambda_4)+4\lambda_{\Phi_2\Delta}+2\lambda_{\Phi_2\Delta1}}\\&-\frac{\sqrt{3}}{16\pi}\lambda_{\Phi_1}\sqrt{3g_1^2+9g_2^2+24\lambda_{\Phi_1}+12y_t^2+8\lambda_3+4\lambda_4+16\lambda_{\Phi_1\Delta}+8\lambda_{\Phi_1\Delta1}}\\&-\frac{\sqrt{3}}{16\pi}\lambda_{\Phi_1\Delta}\sqrt{2\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta1}+2\lambda_{\Phi_2\Delta}+\lambda_{\Phi_2\Delta1}+(10\lambda_{\Delta1}+6\lambda_{\Delta2})},\end{aligned} \tag{5.12}$$

where the first term comes from the finite-temperature one-loop potential in Eq. (5.6) and the second and the third term comes from resummation in Eq. (5.7). Using the expression for the effective potential in Eq. (5.11), the bounce solution at finite temperature can be immediately computed as $S_3[h_{1B}(r)]\simeq-(6.015)\pi\eta/\lambda_{\rm eff}T$. It is important to note that all the couplings in the effective quartic coupling and η are scale dependent and the running values are considered for all these parameters.

After computing the action, it is easy to evaluate the differential decay probability of the nucleating bubble at a particular temperature T as follows:

$$\frac{dP}{d\ln T}\simeq\Gamma(T)\frac{M_P}{T^2}\left(\frac{\tau_UT_0}{T}\right)^3, \tag{5.13}$$

with $\Gamma(T)\simeq T^4\{\frac{S_3[h_{1B}(r)]}{2\pi T}\}^{3/2}e^{-S_3[h_{1B}(r)]/T}$ is the vacuum decay rate per unit volume at a fixed temperature, T. τ_U is the age of the Universe and $T_0\simeq2.35\times10^{-4}$ eV. This differential-decay probability is valid only in a radiation-dominated Universe.

The total integrated probability can be computed by integrating the differential decay probability as follows:

$$P(T_{\rm cut-off})=\int_0^{T_{\rm cut-off}}\frac{dP(T')}{dT'}dT', \tag{5.14}$$

with $T_{\rm cut-off}$ is the maximum cutoff on the temperature which is decided by the validity until the cutoff scale is Λ when the bounce $h_{1B}(0)$ is computed at different temperatures.

The bounds on the tunneling probability can distinguish the regions of stability, metastability, and instability. In case where the second minima is higher than the EW minima, there is no possibility of tunneling, and this condition is defined as stable with criteria $\lambda_{\rm eff}>0$. When the second minima lies below the EW minima, if the tunneling probability is greater than the age of the Universe, then the condition is defined as metastability. For the case, when the tunneling probability is less than the age of the Universe, then the region is unstable. The stable, metastable, and the unstable regions are denoted by green, yellow, and red colors in Fig. 6. The black contours correspond to 1σ, 2σ, and 3σ

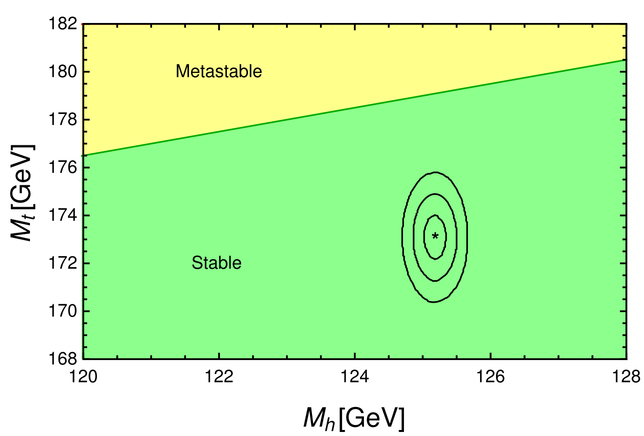


FIG. 6. Phase-space plot for the Higgs mass and the top mass in GeV at zero temperature. The green and yellow regions correspond to the stable and the metastable regions, respectively. The black contours denotes the 1σ, 2σ, and 3σ uncertainities and the black dot at the center corresponds to the current measured values of the Higgs mass and the top mass.

uncertainities, and the black dot at the center defines the experimentally measured values of the Higgs mass and the top mass. The current measured values of the Higgs mass and the top-quark mass lie in the stable region because of the positive contribution from more number of scalar degrees of freedom both at the zero temperature and the finite temperature.

VI. CONCLUSION

The extension of the Standard Model with a scalar triplet and a second SU(2) Higgs doublet is considered with an additional Z_2 symmetry. The breaking of electroweak symmetry by radiative effects demands the mass of additional scalars to be below few TeV but, the strongly first-order phase transition which is crucial for explaining the baryon asymmetry of the Universe, demands lower masses. The interaction quartic coupling of the additional multiplet with the Higgs field plays a very significant role in the strength of the phase transition. The higher the value of this quartic coupling, the higher the allowed mass from the strongly first-order phase transition would be. Hence, we have considered the extension of the type-II seesaw with another SU(2) Higgs doublet where the masses of the inert doublet are chosen to be small enough to enhance the strength of the phase transition. In this way, it becomes possible to accommodate the additional doublet and the triplet scalar which provides the radiative electroweak symmetry breaking alongside the strongly first-order phase transition.

The type-II seesaw extended to inert doublet has more number of degrees of freedom which restricts the interaction coupling λ_3 for inert doublet to be ~ 0.15 and for the triplet scalar $\lambda_{\Phi_{1\Delta}}$ to be ~ 0.5 from Planck-scale perturbativity. Hence, the effect on the scale where the electroweak symmetry breaking is triggered is more from the triplet in comparison to the doublet. Also, the maximum allowed values for the interaction couplings 0.15 and 0.5 for the doublet and the triplet are not enough to provide the strongly first-order phase transition. It can only provide the first-order phase transition $\phi_+(T_c)/T_c \gtrsim 0.6$ for doublet and triplet bare-mass parameter to be 0.0 GeV. Therefore, we have looked for the possibility for accommodating both radiative-electroweak symmetry breaking and the first-order phase transition. The electroweak symmetry breaking is triggered around the scale $10^{6.80}$ GeV for $m_\Delta(\Lambda)$ and $m_{22}(\Lambda) = 100.0$ GeV and the first-order phase transition is also satisfied, i.e., $\frac{\phi_+(T_c)}{T_c} \gtrsim 0.6$.

The stability of the electroweak vacuum is also studied both at the zero temperature and the finite temperature. The running of the couplings is considered until the Planck scale for this analysis using two-loop β-functions. The positive contribution coming from the scalar degrees of freedom is too large and the measured values for the Higgs mass and the top mass lies in the stable region both at the zero temperature and the finite temperature.

The second doublet is considered to be odd under the Z_2 symmetry and the lightest stable neutral particle M_{H^0} becomes the cold DM candidate. The DM-relic density bound is satisfied in different mass ranges in the case of the inert doublet. Still, the DM constraints from DM relic in a low-mass regime $M_{H^0} \lesssim m_h/2$ and in the resonant or funnel region; $M_{H^0} \sim m_h/2$ will also give the first-order phase transition and accommodate the radiative electroweak symmetry breaking.

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APPENDIX: TWO-LOOP β-FUNCTIONS FOR DIMENSIONLESS COUPLINGS

1. Scalar quartic couplings

$$\begin{aligned}
\beta_{\lambda_{\Phi_1}} = \frac{1}{16\pi^2}\Big[&+\frac{27}{200}g_1^4+\frac{9}{20}g_1^2g_2^2+\frac{9}{8}g_2^4+2\lambda_3^2+2\lambda_3\lambda_4+\lambda_4^2-\frac{9}{5}g_1^2\lambda_{\Phi_1}-9g_2^2\lambda_{\Phi_1}+24\lambda_{\Phi_1}+3\lambda_{\Phi_1\Delta}^2+3\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}+\frac{5}{4}\lambda_{\Phi_1\Delta 1}^2 \\
&+4\lambda_5^2+12\lambda\mathrm{Tr}(Y_dY_d^\dagger)+4\lambda\mathrm{Tr}(Y_eY_e^\dagger)+12\lambda\mathrm{Tr}(Y_uY_u^\dagger)-6\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger)-2\mathrm{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger)-6\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger)\Big] \\
&+\frac{1}{(16\pi^2)^2}\Big[-\frac{4293}{2000}g_1^6-\frac{1971}{400}g_1^4g_2^2-\frac{359}{80}g_1^2g_2^4+\frac{235}{16}g_2^6+\frac{9}{10}g_1^4\lambda_3+\frac{15}{2}g_2^4\lambda_3+\frac{12}{5}g_1^2\lambda_3^2+12g_2^2\lambda_3^2-8\lambda_3^3+\frac{9}{20}g_1^4\lambda_4 \\
&+\frac{3}{2}g_1^2g_2^2\lambda_4+\frac{15}{4}g_2^4\lambda_4+\frac{12}{5}g_1^2\lambda_3\lambda_4+12g_2^2\lambda_3\lambda_4-12\lambda_3^2\lambda_4+\frac{6}{5}g_1^2\lambda_4^2+3g_2^2\lambda_4^2-16\lambda_3\lambda_4^2-6\lambda_4^3+\frac{2349}{200}g_1^4\lambda_{\Phi_1}+\frac{117}{20}g_1^2g_2^2\lambda_{\Phi_1}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{37}{8}g_2^4\lambda_{\Phi_1}-20\lambda_3^2\lambda_{\Phi_1}-20\lambda_3\lambda_4\lambda_{\Phi_1}-12\lambda_4^2\lambda_{\Phi_1}+\frac{108}{5}g_1^2\lambda_{\Phi_1}^2+108g_2^2\lambda_{\Phi_1}^2-312\lambda_{\Phi_1}^3+\frac{27}{5}g_1^4\lambda_{\Phi_1\Delta}+30g_2^4\lambda_{\Phi_1\Delta}\\
&+\frac{72}{5}g_1^2\lambda_{\Phi_1\Delta}^2+48g_2^2\lambda_{\Phi_1\Delta}^2-30\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}^2-12\lambda_{\Phi_1\Delta}^3+\frac{27}{10}g_1^4\lambda_{\Phi_1\Delta1}+6g_1^2g_2^2\lambda_{\Phi_1\Delta1}+15g_2^4\lambda_{\Phi_1\Delta1}+\frac{72}{5}g_1^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta1}\\
&+48g_2^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta1}-30\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta1}-18\lambda_{\Phi_1\Delta}^2\lambda_{\Phi_1\Delta1}+6g_1^2\lambda_{\Phi_1\Delta1}^2+17g_2^2\lambda_{\Phi_1\Delta1}^2-\frac{29}{2}\lambda_{\Phi_1}\lambda_{\Phi_1\Delta1}^2-19\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta1}^2\\
&-\frac{13}{2}\lambda_{\Phi_1\Delta1}^3-\frac{4}{5}(100\lambda_3+110\lambda_4+3g_1^2+70\lambda_{\Phi_1})\lambda_5^2+\frac{33}{10}g_1^2g_2^2\mathrm{Tr}(Y_eY_e^\dagger)-\frac{3}{4}g_2^4\mathrm{Tr}(Y_eY_e^\dagger)+\frac{15}{2}g_1^2\lambda_{\Phi_1}\mathrm{Tr}(Y_eY_e^\dagger)\\
&+\frac{1}{20}(-5(64\lambda_{\Phi_1}(-5g_3^2+9\lambda_{\Phi_1})-90g_2^2\lambda_{\Phi_1}+9g_2^4)+9g_1^4+g_1^2(50\lambda_{\Phi_1}+54g_2^2))\mathrm{Tr}(Y_dY_d^\dagger)-\frac{9}{4}g_1^4\mathrm{Tr}(Y_eY_e^\dagger)\\
&+\frac{15}{2}g_2^2\lambda_{\Phi_1}\mathrm{Tr}(Y_eY_e^\dagger)-48\lambda_{\Phi_1}^2\mathrm{Tr}(Y_eY_e^\dagger)-12\lambda_{\Phi_1\Delta}^2\mathrm{Tr}(Y_NY_N^\dagger)-12\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta1}\mathrm{Tr}(Y_NY_N^\dagger)-5\lambda_{\Phi_1\Delta1}^2\mathrm{Tr}(Y_NY_N^\dagger)\\
&-\frac{171}{100}g_1^4\mathrm{Tr}(Y_uY_u^\dagger)+\frac{63}{10}g_1^2g_2^2\mathrm{Tr}(Y_uY_u^\dagger)-\frac{9}{4}g_2^4\mathrm{Tr}(Y_uY_u^\dagger)+\frac{17}{2}g_1^2\lambda_{\Phi_1}\mathrm{Tr}(Y_uY_u^\dagger)+\frac{45}{2}g_2^2\lambda_{\Phi_1}\mathrm{Tr}(Y_uY_u^\dagger)+80g_3^2\lambda_{\Phi_1}\mathrm{Tr}(Y_uY_u^\dagger)\\
&-144\lambda_{\Phi_1}^2\mathrm{Tr}(Y_uY_u^\dagger)+\frac{4}{5}g_1^2\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger)-32g_3^2\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger)-3\lambda_{\Phi_1}\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger)-42\lambda_{\Phi_1}\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger)\\
&-\frac{12}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger)-\lambda_{\Phi_1}\mathrm{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger)-18\lambda_{\Phi_1}\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)-\frac{8}{5}g_1^2\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger)-32g_3^2\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger)\\
&-3\lambda_{\Phi_1}\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger)+30\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger Y_dY_d^\dagger)-12\mathrm{Tr}(Y_dY_d^\dagger Y_dY_u^\dagger Y_uY_d^\dagger)+6\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger Y_dY_d^\dagger)\\
&+10\mathrm{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger Y_eY_e^\dagger)+6\mathrm{Tr}(Y_eY_e^\dagger Y_eY_N^\dagger Y_NY_e^\dagger)+6\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger Y_eY_e^\dagger)+8\mathrm{Tr}(Y_eY_N^\dagger Y_e^TY_e^*Y_NY_e^\dagger)\\
&-6\mathrm{Tr}(Y_dY_u^\dagger Y_uY_u^\dagger Y_uY_d^\dagger)+30\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger Y_uY_u^\dagger)\Big],
\end{aligned}
$$

$$
\begin{aligned}
\beta_{\lambda_2}=&\frac{1}{16\pi^2}\Big[24\lambda_2^2+2\lambda_3^2+2\lambda_3\lambda_4+3\lambda_{\Phi_2\Delta}^2+3\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}+4\lambda_5^2-9g_2^2\lambda_2+\frac{27}{200}g_1^4+\frac{5}{4}\lambda_{\Phi_2\Delta1}^2+\frac{9}{20}g_1^2(-4\lambda_2+g_2^2)+\frac{9}{8}g_2^4+\lambda_4^2\Big]\\
&+\frac{1}{(16\pi^2)^2}\Big[-\frac{4293}{2000}g_1^6-\frac{1971}{400}g_1^4g_2^2-\frac{359}{80}g_1^2g_2^4+\frac{235}{16}g_2^6+\frac{2349}{200}g_1^4\lambda_2+\frac{117}{20}g_1^2g_2^2\lambda_2+\frac{37}{8}g_2^4\lambda_2+\frac{108}{5}g_1^2\lambda_2^2+108g_2^2\lambda_2^2-312\lambda_2^3\\
&+\frac{9}{10}g_1^4\lambda_3+\frac{15}{2}g_2^4\lambda_3+\frac{12}{5}g_1^2\lambda_3^2+12g_2^2\lambda_3^2-20\lambda_2\lambda_3^2-8\lambda_3^3+\frac{9}{20}g_1^4\lambda_4+\frac{3}{2}g_1^2g_2^2\lambda_4+\frac{15}{4}g_2^4\lambda_4+\frac{12}{5}g_1^2\lambda_3\lambda_4+12g_2^2\lambda_3\lambda_4\\
&-20\lambda_2\lambda_3\lambda_4-12\lambda_3^2\lambda_4+\frac{6}{5}g_1^2\lambda_4^2+3g_2^2\lambda_4^2-12\lambda_2\lambda_4^2-16\lambda_3\lambda_4^2-6\lambda_4^3+\frac{27}{5}g_1^4\lambda_{\Phi_2\Delta}+30g_2^4\lambda_{\Phi_2\Delta}+\frac{72}{5}g_1^2\lambda_{\Phi_2\Delta}^2+48g_2^2\lambda_{\Phi_2\Delta}^2\\
&-30\lambda_2\lambda_{\Phi_2\Delta}^2-12\lambda_{\Phi_2\Delta}^3+\frac{27}{10}g_1^4\lambda_{\Phi_2\Delta1}+6g_1^2g_2^2\lambda_{\Phi_2\Delta1}+15g_2^4\lambda_{\Phi_2\Delta1}+\frac{72}{5}g_1^2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}+48g_2^2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}-30\lambda_2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}\\
&-18\lambda_{\Phi_2\Delta}^2\lambda_{\Phi_2\Delta1}+6g_1^2\lambda_{\Phi_2\Delta1}^2+17g_2^2\lambda_{\Phi_2\Delta1}^2-\frac{29}{2}\lambda_2\lambda_{\Phi_2\Delta1}^2-19\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}^2-\frac{13}{2}\lambda_{\Phi_2\Delta1}^3-6(2\lambda_3^2+2\lambda_3\lambda_4+\lambda_4^2)\mathrm{Tr}(Y_dY_d^\dagger)\\
&-4\lambda_3^2\mathrm{Tr}(Y_eY_e^\dagger)-4\lambda_3\lambda_4\mathrm{Tr}(Y_eY_e^\dagger)-2\lambda_4^2\mathrm{Tr}(Y_eY_e^\dagger)-12\lambda_{\Phi_2\Delta}^2\mathrm{Tr}(Y_NY_N^\dagger)-12\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}\mathrm{Tr}(Y_NY_N^\dagger)-5\lambda_{\Phi_2\Delta1}^2\mathrm{Tr}(Y_NY_N^\dagger)\\
&-12\lambda_3^2\mathrm{Tr}(Y_uY_u^\dagger)-12\lambda_3\lambda_4\mathrm{Tr}(Y_uY_u^\dagger)-6\lambda_4^2\mathrm{Tr}(Y_uY_u^\dagger)-\frac{4}{5}\lambda_5^2(100\lambda_3+10\mathrm{Tr}(Y_eY_e^\dagger)+110\lambda_4+30\mathrm{Tr}(Y_dY_d^\dagger)\\
&+30\mathrm{Tr}(Y_uY_u^\dagger)+3g_1^2+70\lambda_2)\Big],
\end{aligned}
$$

$$
\begin{aligned}
\beta_{\lambda_3} = {} & \frac{1}{16\pi^2}\Big[+\frac{27}{100}g_1^4 - \frac{9}{10}g_1^2g_2^2 + \frac{9}{4}g_2^4 - \frac{9}{5}g_1^2\lambda_3 - 9g_2^2\lambda_3 + 12\lambda_2\lambda_3 + 4\lambda_3^2 + 4\lambda_2\lambda_4 + 2\lambda_4^2 + 12\lambda_3\lambda_{\Phi_1} + 4\lambda_4\lambda_{\Phi_1} + 6\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta} \\
& + 3\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta} + 3\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1} + \frac{1}{2}\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} + 8\lambda_5^2 + 6\lambda_3\mathrm{Tr}(Y_dY_d^\dagger) + 2\lambda_3\mathrm{Tr}(Y_eY_e^\dagger) + 6\lambda_3\mathrm{Tr}(Y_uY_u^\dagger)\Big] \\
& + \frac{1}{(16\pi^2)^2}\Big[-\frac{4293}{1000}g_1^6 + \frac{1161}{200}g_1^4g_2^2 + \frac{89}{40}g_1^2g_2^4 + \frac{235}{8}g_2^6 + \frac{27}{10}g_1^4\lambda_2 - 3g_1^2g_2^2\lambda_2 + \frac{45}{2}g_2^4\lambda_2 + \frac{2169}{200}g_1^4\lambda_3 + \frac{33}{20}g_1^2g_2^2\lambda_3 - \frac{23}{8}g_2^4\lambda_3 \\
& + \frac{72}{5}g_1^2\lambda_2\lambda_3 + 72g_2^2\lambda_2\lambda_3 - 60\lambda_2^2\lambda_3 + \frac{6}{5}g_1^2\lambda_3^2 + 6g_2^2\lambda_3^2 - 72\lambda_2\lambda_3^2 - 12\lambda_3^3 + \frac{9}{10}g_1^4\lambda_4 - \frac{9}{5}g_1^2g_2^2\lambda_4 + \frac{15}{2}g_2^4\lambda_4 + \frac{24}{5}g_1^2\lambda_2\lambda_4 \\
& + 36g_2^2\lambda_2\lambda_4 - 16\lambda_2^2\lambda_4 - 12g_2^2\lambda_3\lambda_4 - 32\lambda_2\lambda_3\lambda_4 - 4\lambda_3^2\lambda_4 - \frac{6}{5}g_1^2\lambda_4^2 + 6g_2^2\lambda_4^2 - 28\lambda_2\lambda_4^2 - 16\lambda_3\lambda_4^2 - 12\lambda_4^3 + \frac{27}{10}g_1^4\lambda_{\Phi_1} \\
& - 3g_1^2g_2^2\lambda_{\Phi_1} + \frac{45}{2}g_2^4\lambda_{\Phi_1} + \frac{72}{5}g_1^2\lambda_3\lambda_{\Phi_1} + 72g_2^2\lambda_3\lambda_{\Phi_1} - 72\lambda_3^2\lambda_{\Phi_1} + \frac{24}{5}g_1^2\lambda_4\lambda_{\Phi_1} + 36g_2^2\lambda_4\lambda_{\Phi_1} - 32\lambda_3\lambda_4\lambda_{\Phi_1} - 28\lambda_4^2\lambda_{\Phi_1} \\
& - 60\lambda_3\lambda_{\Phi_1}^2 - 16\lambda_4\lambda_{\Phi_1}^2 + \frac{27}{5}g_1^4\lambda_{\Phi_1\Delta} + 30g_2^4\lambda_{\Phi_1\Delta} - 3\lambda_3\lambda_{\Phi_1\Delta}^2 + \frac{27}{10}g_1^4\lambda_{\Phi_1\Delta 1} - 6g_1^2g_2^2\lambda_{\Phi_1\Delta 1} + 15g_2^4\lambda_{\Phi_1\Delta 1} - 3\lambda_3\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1} \\
& - \frac{9}{4}\lambda_3\lambda_{\Phi_1\Delta 1}^2 - 2\lambda_4\lambda_{\Phi_1\Delta 1}^2 + \frac{27}{5}g_1^4\lambda_{\Phi_2\Delta} + 30g_2^4\lambda_{\Phi_2\Delta} + \frac{144}{5}g_1^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta} + 96g_2^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta} - 24\lambda_3\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta} - 12\lambda_{\Phi_1\Delta}^2\lambda_{\Phi_2\Delta} \\
& + \frac{72}{5}g_1^2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta} + 48g_2^2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta} - 12\lambda_3\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta} - 12\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta} - 9\lambda_{\Phi_1\Delta 1}^2\lambda_{\Phi_2\Delta} - 3\lambda_3\lambda_{\Phi_2\Delta}^2 - 12\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta}^2 \\
& - 6\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta}^2 + \frac{27}{10}g_1^4\lambda_{\Phi_2\Delta 1} - 6g_1^2g_2^2\lambda_{\Phi_2\Delta 1} + 15g_2^4\lambda_{\Phi_2\Delta 1} + \frac{72}{5}g_1^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1} + 48g_2^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1} - 12\lambda_3\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1} \\
& - 6\lambda_{\Phi_1\Delta}^2\lambda_{\Phi_2\Delta 1} + \frac{12}{5}g_1^2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} + 14g_2^2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} - 2\lambda_3\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} - 4\lambda_4\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} - 2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} \\
& - \frac{5}{2}\lambda_{\Phi_1\Delta 1}^2\lambda_{\Phi_2\Delta 1} - 3\lambda_3\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1} - 12\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1} - 2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1} - \frac{9}{4}\lambda_3\lambda_{\Phi_2\Delta 1}^2 - 2\lambda_4\lambda_{\Phi_2\Delta 1}^2 - 9\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1}^2 \\
& - \frac{5}{2}\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}^2 + \frac{1}{20}(-5(-45g_2^2\lambda_3 + 8(-20g_3^2\lambda_3 + 36\lambda_3\lambda_{\Phi_1} + 3\lambda_4(4\lambda_{\Phi_1} + \lambda_4) + 6\lambda_3^2) + 9g_2^4) + 9g_1^4 \\
& + g_1^2(25\lambda_3 - 54g_2^2))\mathrm{Tr}(Y_dY_d^\dagger) - \frac{9}{4}g_1^4\mathrm{Tr}(Y_eY_e^\dagger) - \frac{33}{10}g_1^2g_2^2\mathrm{Tr}(Y_eY_e^\dagger) - \frac{3}{4}g_2^4\mathrm{Tr}(Y_eY_e^\dagger) + \frac{15}{4}g_1^2\lambda_3\mathrm{Tr}(Y_eY_e^\dagger) \\
& + \frac{15}{4}g_2^2\lambda_3\mathrm{Tr}(Y_eY_e^\dagger) - 4\lambda_3^2\mathrm{Tr}(Y_eY_e^\dagger) - 2\lambda_4^2\mathrm{Tr}(Y_eY_e^\dagger) - 24\lambda_3\lambda_{\Phi_1}\mathrm{Tr}(Y_eY_e^\dagger) - 8\lambda_4\lambda_{\Phi_1}\mathrm{Tr}(Y_eY_e^\dagger) - 24\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta}\mathrm{Tr}(Y_NY_N^\dagger) \\
& - 12\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta}\mathrm{Tr}(Y_NY_N^\dagger) - 12\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger) - 2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger) - \frac{63}{10}g_1^2g_2^2\mathrm{Tr}(Y_uY_u^\dagger) - \frac{9}{4}g_2^4\mathrm{Tr}(Y_uY_u^\dagger) \\
& + \frac{8}{5}\lambda_5^2(-110\lambda_4 - 15\mathrm{Tr}(Y_dY_d^\dagger) - 15\mathrm{Tr}(Y_uY_u^\dagger) - 45\lambda_3 - 5\mathrm{Tr}(Y_eY_e^\dagger) + 6g_1^2 - 90\lambda_2 - 90\lambda_{\Phi_1}) - \frac{171}{100}g_1^4\mathrm{Tr}(Y_uY_u^\dagger) \\
& + \frac{17}{4}g_1^2\lambda_3\mathrm{Tr}(Y_uY_u^\dagger) + \frac{45}{4}g_2^2\lambda_3\mathrm{Tr}(Y_uY_u^\dagger) + 40g_3^2\lambda_3\mathrm{Tr}(Y_uY_u^\dagger) - 12\lambda_3^2\mathrm{Tr}(Y_uY_u^\dagger) - 6\lambda_4^2\mathrm{Tr}(Y_uY_u^\dagger) - 72\lambda_3\lambda_{\Phi_1}\mathrm{Tr}(Y_uY_u^\dagger) \\
& - 24\lambda_4\lambda_{\Phi_1}\mathrm{Tr}(Y_uY_u^\dagger) - \frac{27}{2}\lambda_3\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger) - 21\lambda_3\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger) - 24\lambda_4\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger) - \frac{9}{2}\lambda_3\mathrm{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger) \\
& - 9\lambda_3\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger) - \frac{27}{2}\lambda_3\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger)\Big],
\end{aligned}
$$

$$\beta_{\lambda_4} = \frac{1}{16\pi^2}\left[+\frac{9}{5}g_1^2g_2^2 - \frac{9}{5}g_1^2\lambda_4 - 9g_2^2\lambda_4 + 4\lambda_2\lambda_4 + 8\lambda_3\lambda_4 + 4\lambda_4^2 + 4\lambda_4\lambda_{\Phi_1} + 2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} + 32\lambda_5^2 + 6\lambda_4\mathrm{Tr}(Y_dY_d^\dagger)\right.$$
$$\left. + 2\lambda_4\mathrm{Tr}(Y_eY_e^\dagger) + 6\lambda_4\mathrm{Tr}(Y_uY_u^\dagger)\right]$$
$$+ \frac{1}{(16\pi^2)^2}\left[-\frac{783}{50}g_1^4g_2^2 - \frac{56}{5}g_1^2g_2^4 + 6g_1^2g_2^2\lambda_2 + \frac{6}{5}g_1^2g_2^2\lambda_3 + \frac{1809}{200}g_1^4\lambda_4 + \frac{153}{20}g_1^2g_2^2\lambda_4 - \frac{143}{8}g_2^4\lambda_4 + \frac{24}{5}g_1^2\lambda_2\lambda_4 - 28\lambda_2^2\lambda_4\right.$$
$$+ \frac{12}{5}g_1^2\lambda_3\lambda_4 + 36g_2^2\lambda_3\lambda_4 - 80\lambda_2\lambda_3\lambda_4 - 28\lambda_3^2\lambda_4 + \frac{24}{5}g_1^2\lambda_4^2 + 18g_2^2\lambda_4^2 - 40\lambda_2\lambda_4^2 - 28\lambda_3\lambda_4^2 + 6g_1^2g_2^2\lambda_{\Phi_1} + \frac{24}{5}g_1^2\lambda_4\lambda$$
$$- 40\lambda_4^2\lambda_{\Phi_1} - 28\lambda_4\lambda_{\Phi_1}^2 - 3\lambda_4\lambda_{\Phi_1\Delta}^2 + 12g_1^2g_2^2\lambda_{\Phi_1\Delta 1} - 3\lambda_4\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1} + \frac{7}{4}\lambda_4\lambda_{\Phi_1\Delta 1}^2 - 24\lambda_4\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta} - 12\lambda_4\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta}$$
$$- 3\lambda_4\lambda_{\Phi_2\Delta}^2 + 12g_1^2g_2^2\lambda_{\Phi_2\Delta 1} - 12\lambda_4\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1} + \frac{48}{5}g_1^2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} + 20g_2^2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} - 8\lambda_3\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}$$
$$- 8\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} - 4\lambda_{\Phi_1\Delta 1}^2\lambda_{\Phi_2\Delta 1} - 3\lambda_4\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1} - 8\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1} + \frac{7}{4}\lambda_4\lambda_{\Phi_2\Delta 1}^2 - 4\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}^2 + \frac{1}{20}(225g_2^2\lambda_4$$
$$- 80\lambda_4(-10g_3^2 + 3\lambda_4 + 6\lambda_3 + 6\lambda_{\Phi_1}) + g_1^2(108g_2^2 + 25\lambda_4))\mathrm{Tr}(Y_dY_d^\dagger) + \frac{33}{5}g_1^2g_2^2\mathrm{Tr}(Y_eY_e^\dagger) + \frac{15}{4}g_1^2\lambda_4\mathrm{Tr}(Y_eY_e^\dagger)$$
$$+ \frac{15}{4}g_2^2\lambda_4\mathrm{Tr}(Y_eY_e^\dagger) - 8\lambda_3\lambda_4\mathrm{Tr}(Y_eY_e^\dagger) - 4\lambda_4^2\mathrm{Tr}(Y_eY_e^\dagger) - 8\lambda_4\lambda_{\Phi_1}\mathrm{Tr}(Y_eY_e^\dagger) - 8\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger)$$
$$+ \frac{8}{5}\lambda_5^2(-120\lambda_2 - 120\lambda_3 - 120\lambda_{\Phi_1} + 135g_2^2 - 20\mathrm{Tr}(Y_eY_e^\dagger) + 24g_1^2 - 60\mathrm{Tr}(Y_dY_d^\dagger) - 60\mathrm{Tr}(Y_uY_u^\dagger) - 65\lambda_4)$$
$$+ \frac{63}{5}g_1^2g_2^2\mathrm{Tr}(Y_uY_u^\dagger) + \frac{17}{4}g_1^2\lambda_4\mathrm{Tr}(Y_uY_u^\dagger) + \frac{45}{4}g_2^2\lambda_4\mathrm{Tr}(Y_uY_u^\dagger) + 40g_3^2\lambda_4\mathrm{Tr}(Y_uY_u^\dagger) - 24\lambda_3\lambda_4\mathrm{Tr}(Y_uY_u^\dagger) - 12\lambda_4^2\mathrm{Tr}(Y_uY_u^\dagger)$$
$$- 24\lambda_4\lambda_{\Phi_1}\mathrm{Tr}(Y_uY_u^\dagger) - \frac{27}{2}\lambda_4\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger) + 27\lambda_4\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger) - 10\lambda_4\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1} - 80\lambda_3\lambda_4\lambda_{\Phi_1}$$
$$\left.-\frac{9}{2}\lambda_4\mathrm{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger) - 9\lambda_4\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger) - \frac{27}{2}\lambda_4\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger)\right].$$

$$\beta_{\lambda_5} = \frac{1}{16\pi^2}\left[12\lambda_4\lambda_5 + 2\lambda_5\mathrm{Tr}(Y_eY_e^\dagger) + 4\lambda_2\lambda_5 + 4\lambda_5\lambda_{\Phi_1} + 6\lambda_5\mathrm{Tr}(Y_dY_d^\dagger) + 6\lambda_5\mathrm{Tr}(Y_uY_u^\dagger) + 8\lambda_3\lambda_5 - 9g_2^2\lambda_5 - \frac{9}{5}g_1^2\lambda_5\right]$$
$$+ \frac{1}{(16\pi^2)^2}\left[+\frac{1809}{200}g_1^4\lambda_5 + \frac{57}{20}g_1^2g_2^2\lambda_5 - \frac{143}{8}g_2^4\lambda_5 - \frac{12}{5}g_1^2\lambda_2\lambda_5 - 28\lambda_2^2\lambda_5 + \frac{48}{5}g_1^2\lambda_3\lambda_5 + 36g_2^2\lambda_3\lambda_5 - 80\lambda_2\lambda_3\lambda_5 - 28\lambda_3^2\lambda_5\right.$$
$$+ \frac{72}{5}g_1^2\lambda_4\lambda_5 + 72g_2^2\lambda_4\lambda_5 - 88\lambda_2\lambda_4\lambda_5 - 76\lambda_3\lambda_4\lambda_5 - 32\lambda_4^2\lambda_5 - \frac{12}{5}g_1^2\lambda_5\lambda_{\Phi_1} - 80\lambda_3\lambda_5\lambda_{\Phi_1} - 88\lambda_4\lambda_5\lambda_{\Phi_1} - 28\lambda_5\lambda_{\Phi_1}^2 - 3\lambda_5\lambda_{\Phi_1\Delta}^2$$
$$- 3\lambda_5\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1} - \frac{1}{4}\lambda_5\lambda_{\Phi_1\Delta 1}^2 - 24\lambda_5\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta} - 12\lambda_5\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta} - 3\lambda_5\lambda_{\Phi_2\Delta}^2 - 12\lambda_5\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1} - 14\lambda_5\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}$$
$$- 3\lambda_5\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1} - \frac{1}{4}\lambda_5\lambda_{\Phi_2\Delta 1}^2 + 24\lambda_5^3 - \frac{1}{40}(-5(32\lambda_5(10g_3^2 - 6\lambda_3 - 6\lambda_{\Phi_1} - 9\lambda_4) + 90g_2^2\lambda_5 + 9g_2^4) + 9g_1^4 + g_1^2(-50\lambda_5$$
$$+ 54g_2^2))\mathrm{Tr}(Y_dY_d^\dagger) + \frac{9}{8}g_1^4\mathrm{Tr}(Y_eY_e^\dagger) - \frac{33}{20}g_1^2g_2^2\mathrm{Tr}(Y_eY_e^\dagger) + \frac{3}{8}g_2^4\mathrm{Tr}(Y_eY_e^\dagger) + \frac{15}{4}g_1^2\lambda_5\mathrm{Tr}(Y_eY_e^\dagger) + \frac{15}{4}g_2^2\lambda_5\mathrm{Tr}(Y_eY_e^\dagger)$$
$$- 8\lambda_3\lambda_5\mathrm{Tr}(Y_eY_e^\dagger) - 12\lambda_4\lambda_5\mathrm{Tr}(Y_eY_e^\dagger) - 8\lambda_5\lambda_{\Phi_1}\mathrm{Tr}(Y_eY_e^\dagger) - \frac{171}{200}g_1^4\mathrm{Tr}(Y_uY_u^\dagger) + \frac{63}{20}g_1^2g_2^2\mathrm{Tr}(Y_uY_u^\dagger) - \frac{9}{8}g_2^4\mathrm{Tr}(Y_uY_u^\dagger)$$
$$+ \frac{17}{4}g_1^2\lambda_5\mathrm{Tr}(Y_uY_u^\dagger) + \frac{45}{4}g_2^2\lambda_5\mathrm{Tr}(Y_uY_u^\dagger) + 40g_3^2\lambda_5\mathrm{Tr}(Y_uY_u^\dagger) - 24\lambda_3\lambda_5\mathrm{Tr}(Y_uY_u^\dagger) - 36\lambda_4\lambda_5\mathrm{Tr}(Y_uY_u^\dagger) - 24\lambda_5\lambda_{\Phi_1}\mathrm{Tr}(Y_uY_u^\dagger)$$
$$\left.+ \frac{9}{2}\lambda_5\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger) + 3\lambda_5\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger) + \frac{3}{2}\lambda_5\mathrm{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger) - 9\lambda_5\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger) - \frac{15}{2}\lambda_5\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger)\right],$$

$$
\begin{aligned}
\beta_{\lambda_{\Delta 1}} = & \frac{1}{16\pi^2}\bigg[15g_2^4 - 24g_2^2\lambda_{\Delta 1} + 24\lambda_{\Delta 1}\lambda_{\Delta 2} + 28\lambda_{\Delta 1}^2 + 2\lambda_{\Phi_1\Delta}^2 + 2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1} + 2\lambda_{\Phi_2\Delta}^2 + 2\lambda_{\Phi_2\Delta}\lambda_{\Phi_1\Delta 1} + 6\lambda_{\Delta 2}^2 + 8\lambda_{\Delta 1}\mathrm{Tr}(Y_N Y_N^\dagger) \\
& - \frac{36}{5}g_1^2(g_2^2 + \lambda_{\Delta 1}) + \frac{54}{25}g_1^4\bigg] \\
& + \frac{1}{(16\pi^2)^2}\bigg[-\frac{5508}{125}g_1^6 + \frac{1296}{25}g_1^4 g_2^2 + \frac{224}{5}g_1^2 g_2^4 - \frac{542}{3}g_2^6 + \frac{18}{5}g_1^4\lambda_{\Phi_1\Delta} + 20g_2^4\lambda_{\Phi_1\Delta} + \frac{12}{5}g_1^2\lambda_{\Phi_1\Delta}^2 + 12g_2^2\lambda_{\Phi_1\Delta}^2 - 8\lambda_{\Phi_1\Delta}^3 \\
& + \frac{9}{5}g_1^4\lambda_{\Phi_1\Delta 1} - 6g_1^2 g_2^2\lambda_{\Phi_1\Delta 1} + 10g_2^4\lambda_{\Phi_1\Delta 1} + \frac{12}{5}g_1^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1} + 12g_2^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1} - 12\lambda_{\Phi_1\Delta}^2\lambda_{\Phi_1\Delta 1} + 3g_2^2\lambda_{\Phi_1\Delta 1}^2 - 6\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}^2 \\
& - \lambda_{\Phi_1\Delta 1}^3 + \frac{18}{5}g_1^4\lambda_{\Phi_2\Delta} + 20g_2^4\lambda_{\Phi_2\Delta} + \frac{12}{5}g_1^2\lambda_{\Phi_2\Delta}^2 + 12g_2^2\lambda_{\Phi_2\Delta}^2 - 8\lambda_{\Phi_2\Delta}^3 + \frac{9}{5}g_1^4\lambda_{\Phi_2\Delta 1} - 6g_1^2 g_2^2\lambda_{\Phi_2\Delta 1} + 10g_2^4\lambda_{\Phi_2\Delta 1} \\
& + \frac{12}{5}g_1^2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1} + 12g_2^2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1} - 12\lambda_{\Phi_2\Delta}^2\lambda_{\Phi_2\Delta 1} + 3g_2^2\lambda_{\Phi_2\Delta 1}^2 - 6\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1}^2 - \lambda_{\Phi_2\Delta 1}^3 + \frac{2466}{25}g_1^4\lambda_{\Delta 1} - \frac{168}{5}g_1^2 g_2^2\lambda_{\Delta 1} \\
& + \frac{760}{3}g_2^4\lambda_{\Delta 1} - 20\lambda_{\Phi_1\Delta}^2\lambda_{\Delta 1} - 20\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1} - 3\lambda_{\Phi_1\Delta 1}^2\lambda_{\Delta 1} - 20\lambda_{\Phi_2\Delta}^2\lambda_{\Delta 1} - 20\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1}\lambda_{\Delta 1} - 3\lambda_{\Phi_2\Delta 1}^2\lambda_{\Delta 1} + \frac{528}{5}g_1^2\lambda_{\Delta 1}^2 \\
& + 352g_2^2\lambda_{\Delta 1}^2 - 384\lambda_{\Delta 1}^3 + \frac{216}{5}g_1^4\lambda_{\Delta 2} - \frac{432}{5}g_1^2 g_2^2\lambda_{\Delta 2} + 168g_2^4\lambda_{\Delta 2} - 4\lambda_{\Phi_1\Delta 1}^2\lambda_{\Delta 2} - 4\lambda_{\Phi_2\Delta 1}^2\lambda_{\Delta 2} + \frac{576}{5}g_1^2\lambda_{\Delta 1}\lambda_{\Delta 2} \\
& + 384g_2^2\lambda_{\Delta 1}\lambda_{\Delta 2} - 528\lambda_{\Delta 1}^2\lambda_{\Delta 2} + \frac{72}{5}g_1^2\lambda_{\Delta_2}^2 + 120g_2^2\lambda_{\Delta 2}^2 - 284\lambda_{\Delta 1}\lambda_{\Delta 2}^2 - 96\lambda_{\Delta 2}^3 - 12\lambda_{\Phi_1\Delta}(\lambda_{\Phi_1\Delta} + \lambda_{\Phi_1\Delta 1})\mathrm{Tr}(Y_d Y_d^\dagger) \\
& - 4\lambda_{\Phi_1\Delta}(\lambda_{\Phi_1\Delta} + \lambda_{\Phi_1\Delta 1})\mathrm{Tr}(Y_e Y_e^\dagger) + \frac{144}{25}g_1^4\mathrm{Tr}(Y_N Y_N^\dagger) - \frac{96}{5}g_1^2 g_2^2\mathrm{Tr}(Y_N Y_N^\dagger) + 52g_2^4\mathrm{Tr}(Y_N Y_N^\dagger) + 6g_1^2\lambda_{\Delta 1}\mathrm{Tr}(Y_N Y_N^\dagger) \\
& + 30g_2^2\lambda_{\Delta 1}\mathrm{Tr}(Y_N Y_N^\dagger) - 112\lambda_{\Delta 1}^2\mathrm{Tr}(Y_N Y_N^\dagger) - 96\lambda_{\Delta 1}\lambda_{\Delta 2}\mathrm{Tr}(Y_N Y_N^\dagger) - 24\lambda_{\Delta 2}^2\mathrm{Tr}(Y_N Y_N^\dagger) - 12\lambda_{\Phi_1\Delta}^2\mathrm{Tr}(Y_u Y_u^\dagger) \\
& - 12\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_u Y_u^\dagger) - 12\lambda_{\Delta 1}\mathrm{Tr}(Y_e Y_N^\dagger Y_N Y_e^\dagger) - 72\lambda_{\Delta 1}\mathrm{Tr}(Y_N Y_N^\dagger Y_N Y_N^\dagger) + 32\lambda_{\Delta 2}\mathrm{Tr}(Y_N Y_N^\dagger Y_N Y_N^\dagger) \\
& + 64\mathrm{Tr}(Y_N Y_N^\dagger Y_N Y_N^\dagger Y_N Y_N^\dagger)\bigg],
\end{aligned}
$$

$$
\begin{aligned}
\beta_{\lambda_{\Delta 2}} = & \frac{1}{16\pi^2}\bigg[-16\mathrm{Tr}(Y_N Y_N^\dagger Y_N Y_N^\dagger) + 18\lambda_{\Delta 2}^2 - 24g_2^2\lambda_{\Delta 2} + 24\lambda_{\Delta 1}\lambda_{\Delta 2} - 6g_2^4 + 8\lambda_{\Delta 2}\mathrm{Tr}(Y_N Y_N^\dagger) - \frac{36}{5}g_1^2\lambda_{\Delta 2} + \frac{72}{5}g_1^2 g_2^2 + \lambda_{\Phi_1\Delta 1}^2 \\
& + \lambda_{\Phi_2\Delta 1}^2\bigg] \\
& + \frac{1}{(16\pi^2)^2}\bigg[-\frac{4752}{25}g_1^4 g_2^2 - \frac{1168}{5}g_1^2 g_2^4 + \frac{476}{3}g_2^6 + 12g_1^2 g_2^2\lambda_{\Phi_1\Delta 1} + \frac{6}{5}g_1^2\lambda_{\Phi_1\Delta 1}^2 - 8\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}^2 - 4\lambda_{\Phi_1\Delta 1}^3 + 12g_1^2 g_2^2\lambda_{\Phi_2\Delta 1} \\
& + \frac{6}{5}g_1^2\lambda_{\Phi_2\Delta 1}^2 - 8\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1}^2 - 4\lambda_{\Phi_2\Delta 1}^3 + \frac{576}{5}g_1^2 g_2^2\lambda_{\Delta 1} - 48g_2^4\lambda_{\Delta 1} - 8\lambda_{\Phi_1\Delta 1}^2\lambda_{\Delta 1} - 8\lambda_{\Phi_2\Delta 1}^2\lambda_{\Delta 1} + \frac{1026}{25}g_1^4\lambda_{\Delta 2} + 216g_1^2 g_2^2\lambda_{\Delta 2} \\
& - \frac{80}{3}g_2^4\lambda_{\Delta 2} - 20\lambda_{\Phi_1\Delta}^2\lambda_{\Delta 2} - 20\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2} - 7\lambda_{\Phi_1\Delta}^2\lambda_{\Delta 2} - 20\lambda_{\Phi_2\Delta}^2\lambda_{\Delta 2} - 20\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1}\lambda_{\Delta 2} - 7\lambda_{\Phi_2\Delta 1}^2\lambda_{\Delta 2} \\
& + \frac{288}{5}g_1^2\lambda_{\Delta 1}\lambda_{\Delta 2} + 192g_2^2\lambda_{\Delta 1}\lambda_{\Delta 2} - 448\lambda_{\Delta 1}^2\lambda_{\Delta 2} + 72g_1^2\lambda_{\Delta 2}^2 + 144g_2^2\lambda_{\Delta 2}^2 - 672\lambda_{\Delta 1}\lambda_{\Delta 2}^2 - 228\lambda_{\Delta 2}^3 - 6\lambda_{\Phi_1\Delta 1}^2\mathrm{Tr}(Y_d Y_d^\dagger) \\
& - 2\lambda_{\Phi_1\Delta 1}^2\mathrm{Tr}(Y_e Y_e^\dagger) + \frac{192}{5}g_1^2 g_2^2\mathrm{Tr}(Y_N Y_N^\dagger) - 40g_2^4\mathrm{Tr}(Y_N Y_N^\dagger) + 6g_1^2\lambda_{\Delta 2}\mathrm{Tr}(Y_N Y_N^\dagger) + 30g_2^2\lambda_{\Delta 2}\mathrm{Tr}(Y_N Y_N^\dagger) \\
& - 96\lambda_{\Delta 1}\lambda_{\Delta 2}\mathrm{Tr}(Y_N Y_N^\dagger) - 72\lambda_{\Delta 2}^2\mathrm{Tr}(Y_N Y_N^\dagger) - 6\lambda_{\Phi_1\Delta 1}^2\mathrm{Tr}(Y_u Y_u^\dagger) - 12\lambda_{\Delta 2}\mathrm{Tr}(Y_e Y_N^\dagger Y_N Y_e^\dagger) + \frac{48}{5}g_1^2\mathrm{Tr}(Y_N Y_N^\dagger Y_N Y_N^\dagger) \\
& + 16g_2^2\mathrm{Tr}(Y_N Y_N^\dagger Y_N Y_N^\dagger) + 64\lambda_{\Delta 1}\mathrm{Tr}(Y_N Y_N^\dagger Y_N Y_N^\dagger) - 40\lambda_{\Delta 2}\mathrm{Tr}(Y_N Y_N^\dagger Y_N Y_N^\dagger) + 32\mathrm{Tr}(Y_e Y_N^\dagger Y_N Y_N^\dagger Y_N Y_e^\dagger) \\
& + 256\mathrm{Tr}(Y_N Y_N^\dagger Y_N Y_N^\dagger Y_N Y_N^\dagger)\bigg],
\end{aligned}
$$

$$
\begin{aligned}
\beta_{\lambda_{\Phi_1\Delta}} =& \frac{1}{16\pi^2}\Big[+\frac{27}{25}g_1^4-\frac{18}{5}g_1^2g_2^2+6g_2^4-\frac{9}{2}g_1^2\lambda_{\Phi_1\Delta}-\frac{33}{2}g_2^2\lambda_{\Phi_1\Delta}+12\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}+4\lambda_{\Phi_1\Delta}^2+4\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}+\lambda_{\Phi_1\Delta 1}^2+4\lambda_3\lambda_{\Phi_2\Delta}\\
&+2\lambda_4\lambda_{\Phi_2\Delta}+2\lambda_3\lambda_{\Phi_2\Delta 1}+16\lambda_{\Phi_1\Delta}\lambda_{\Delta 1}+6\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}+12\lambda_{\Phi_1\Delta}\lambda_{\Delta 2}+2\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}+6\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_dY_d^\dagger)+2\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_eY_e^\dagger)\\
&+4\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_NY_N^\dagger)+6\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_uY_u^\dagger)-16\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)\Big]\\
&+\frac{1}{(16\pi^2)^2}\Big[-\frac{9801}{500}g_1^6+\frac{2457}{100}g_1^4g_2^2+\frac{112}{5}g_1^2g_2^4+\frac{245}{6}g_2^6+\frac{18}{5}g_1^4\lambda_3+20g_2^4\lambda_3+\frac{9}{5}g_1^4\lambda_4-6g_1^2g_2^2\lambda_4+10g_2^4\lambda_4+\frac{54}{5}g_1^4\lambda_{\Phi_1}\\
&-12g_1^2g_2^2\lambda_{\Phi_1}+60g_2^4\lambda_{\Phi_1}+\frac{18693}{400}g_1^4\lambda_{\Phi_1\Delta}+\frac{429}{40}g_1^2g_2^2\lambda_{\Phi_1\Delta}+\frac{3287}{48}g_2^4\lambda_{\Phi_1\Delta}-2\lambda_3^2\lambda_{\Phi_1\Delta}-2\lambda_3\lambda_4\lambda_{\Phi_1\Delta}-2\lambda_4^2\lambda_{\Phi_1\Delta}\\
&+\frac{72}{5}g_1^2\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}+72g_2^2\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}-60\lambda_{\Phi_1}^2\lambda_{\Phi_1\Delta}+3g_1^2\lambda_{\Phi_1\Delta}^2+11g_2^2\lambda_{\Phi_1\Delta}^2-72\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}^2-13\lambda_{\Phi_1\Delta}^3+\frac{45}{4}g_1^4\lambda_{\Phi_1\Delta 1}\\
&-\frac{114}{5}g_1^2g_2^2\lambda_{\Phi_1\Delta 1}+\frac{185}{4}g_2^4\lambda_{\Phi_1\Delta 1}-2\lambda_4^2\lambda_{\Phi_1\Delta 1}+\frac{24}{5}g_1^2\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}+36g_2^2\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}-16\lambda_{\Phi_1}^2\lambda_{\Phi_1\Delta 1}-12g_2^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\\
&-32\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}-5\lambda_{\Phi_1\Delta}^2\lambda_{\Phi_1\Delta 1}-\frac{21}{20}g_1^2\lambda_{\Phi_1\Delta 1}^2+\frac{11}{4}g_2^2\lambda_{\Phi_1\Delta 1}^2-18\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}^2-\frac{39}{4}\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}^2-\frac{11}{2}\lambda_{\Phi_1\Delta 1}^3+\frac{9}{10}g_1^4\lambda_{\Phi_2\Delta}\\
&+\frac{15}{2}g_2^4\lambda_{\Phi_2\Delta}+\frac{24}{5}g_1^2\lambda_3\lambda_{\Phi_2\Delta}+24g_2^2\lambda_3\lambda_{\Phi_2\Delta}-8\lambda_3^2\lambda_{\Phi_2\Delta}+\frac{12}{5}g_1^2\lambda_4\lambda_{\Phi_2\Delta}+12g_2^2\lambda_4\lambda_{\Phi_2\Delta}-8\lambda_3\lambda_4\lambda_{\Phi_2\Delta}-8\lambda_4^2\lambda_{\Phi_2\Delta}\\
&-16\lambda_3\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta}-8\lambda_4\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta}-8\lambda_3\lambda_{\Phi_2\Delta}^2-4\lambda_4\lambda_{\Phi_2\Delta}^2-2\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta}^2+\frac{9}{20}g_1^4\lambda_{\Phi_2\Delta 1}-\frac{3}{2}g_1^2g_2^2\lambda_{\Phi_2\Delta 1}\\
&+\frac{15}{4}g_2^4\lambda_{\Phi_2\Delta 1}+\frac{12}{5}g_1^2\lambda_3\lambda_{\Phi_2\Delta 1}+12g_2^2\lambda_3\lambda_{\Phi_2\Delta 1}-4\lambda_3^2\lambda_{\Phi_2\Delta 1}+6g_2^2\lambda_4\lambda_{\Phi_2\Delta 1}-2\lambda_4^2\lambda_{\Phi_2\Delta 1}-8\lambda_3\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1}-2\lambda_4\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}\\
&-8\lambda_3\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1}-2\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1}-6\lambda_3\lambda_{\Phi_2\Delta 1}^2-\lambda_4\lambda_{\Phi_2\Delta 1}^2-\frac{3}{2}\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1}^2-\frac{1}{2}\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}^2+\frac{72}{5}g_1^4\lambda_{\Delta 1}-12g_1^2g_2^2\lambda_{\Delta 1}\\
&+80g_2^4\lambda_{\Delta 1}+\frac{384}{5}g_1^2\lambda_{\Phi_1\Delta}\lambda_{\Delta 1}+256g_2^2\lambda_{\Phi_1\Delta}\lambda_{\Delta 1}-96\lambda_{\Phi_1\Delta}^2\lambda_{\Delta 1}+\frac{144}{5}g_1^2\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}+108g_2^2\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}-48\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}\\
&-24\lambda_{\Phi_1\Delta 1}^2\lambda_{\Delta 1}-80\lambda_{\Phi_1\Delta}\lambda_{\Delta 1}^2-24\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}^2+\frac{54}{5}g_1^4\lambda_{\Delta 2}-24g_1^2g_2^2\lambda_{\Delta 2}+60g_2^4\lambda_{\Delta 2}+\frac{288}{5}g_1^2\lambda_{\Phi_1\Delta}\lambda_{\Delta 2}+192g_2^2\lambda_{\Phi_1\Delta}\lambda_{\Delta 2}\\
&-72\lambda_{\Phi_1\Delta}^2\lambda_{\Delta 2}+\frac{48}{5}g_1^2\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}+56g_2^2\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}-16\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}-18\lambda_{\Phi_1\Delta 1}^2\lambda_{\Delta 2}-120\lambda_{\Phi_1\Delta}\lambda_{\Delta 1}\lambda_{\Delta 2}-16\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}\lambda_{\Delta 2}\\
&-70\lambda_{\Phi_1\Delta}\lambda_{\Delta 2}^2-16\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}^2-4(2(6\lambda_{\Phi_2\Delta}+\lambda_{\Phi_1\Delta 1}+\lambda_{\Phi_2\Delta 1})+3\lambda_{\Phi_1\Delta})\lambda_5^2+\frac{1}{20}(-120g_2^4+225g_2^2\lambda_{\Phi_1\Delta}+36g_1^4\\
&-60(4\lambda_{\Phi_1\Delta}^2+8\lambda_{\Phi_1}(3\lambda_{\Phi_1\Delta}+\lambda_{\Phi_1\Delta 1})+\lambda_{\Phi_1\Delta 1}^2)+800g_3^2\lambda_{\Phi_1\Delta}+g_1^2(-216g_2^2+25\lambda_{\Phi_1\Delta}))\mathrm{Tr}(Y_dY_d^\dagger)-9g_1^4\mathrm{Tr}(Y_eY_e^\dagger)\\
&-\frac{66}{5}g_1^2g_2^2\mathrm{Tr}(Y_eY_e^\dagger)-2g_2^4\mathrm{Tr}(Y_eY_e^\dagger)+\frac{15}{4}g_1^2\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_eY_e^\dagger)+\frac{15}{4}g_2^2\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_eY_e^\dagger)-24\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_eY_e^\dagger)\\
&-4\lambda_{\Phi_1\Delta}^2\mathrm{Tr}(Y_eY_e^\dagger)-8\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_eY_e^\dagger)-\lambda_{\Phi_1\Delta 1}^2\mathrm{Tr}(Y_eY_e^\dagger)+\frac{36}{25}g_1^4\mathrm{Tr}(Y_NY_N^\dagger)-\frac{24}{5}g_1^2g_2^2\mathrm{Tr}(Y_NY_N^\dagger)+2g_2^4\mathrm{Tr}(Y_NY_N^\dagger)\\
&+3g_1^2\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_NY_N^\dagger)+15g_2^2\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_NY_N^\dagger)-8\lambda_{\Phi_1\Delta}^2\mathrm{Tr}(Y_NY_N^\dagger)-2\lambda_{\Phi_1\Delta 1}^2\mathrm{Tr}(Y_NY_N^\dagger)-64\lambda_{\Phi_1\Delta}\lambda_{\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger)\\
&-24\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger)-48\lambda_{\Phi_1\Delta}\lambda_{\Delta 2}\mathrm{Tr}(Y_NY_N^\dagger)-8\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}\mathrm{Tr}(Y_NY_N^\dagger)-\frac{171}{25}g_1^4\mathrm{Tr}(Y_uY_u^\dagger)-\frac{126}{5}g_1^2g_2^2\mathrm{Tr}(Y_uY_u^\dagger)\\
&-6g_2^4\mathrm{Tr}(Y_uY_u^\dagger)+\frac{17}{4}g_1^2\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_uY_u^\dagger)+\frac{45}{4}g_2^2\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_uY_u^\dagger)+40g_3^2\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_uY_u^\dagger)-72\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_uY_u^\dagger)\\
&-12\lambda_{\Phi_1\Delta}^2\mathrm{Tr}(Y_uY_u^\dagger)-24\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_uY_u^\dagger)-3\lambda_{\Phi_1\Delta 1}^2\mathrm{Tr}(Y_uY_u^\dagger)-\frac{27}{2}\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger)-21\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger)\\
&-24\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger)-\frac{9}{2}\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger)-\frac{24}{5}g_1^2\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)+8g_2^2\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)\\
&+17\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)-36\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_NY_N^\dagger Y_NY_N^\dagger)-\frac{27}{2}\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger)+18\mathrm{Tr}(Y_eY_e^\dagger Y_eY_N^\dagger Y_NY_e^\dagger)\\
&+30\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger Y_eY_e^\dagger)+208\mathrm{Tr}(Y_eY_N^\dagger Y_NY_N^\dagger Y_NY_e^\dagger)+24\mathrm{Tr}(Y_eY_N^\dagger Y_e^TY_e^*Y_NY_e^\dagger)\Big],
\end{aligned}
$$

$$\beta_{\lambda_{\Phi_1\Delta 1}}=\frac{1}{16\pi^2}\Big[+\frac{36}{5}g_1^2g_2^2-\frac{9}{2}g_1^2\lambda_{\Phi_1\Delta 1}-\frac{33}{2}g_2^2\lambda_{\Phi_1\Delta 1}+4\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}+8\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}+4\lambda_{\Phi_1\Delta 1}^2+2\lambda_4\lambda_{\Phi_2\Delta 1}+4\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}$$
$$+8\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}+6\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_dY_d^\dagger)+2\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_eY_e^\dagger)+4\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger)+6\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_uY_u^\dagger)+16\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)\Big]$$
$$+\frac{1}{(16\pi^2)^2}\Big[-\frac{1971}{25}g_1^4g_2^2-\frac{449}{5}g_1^2g_2^4+12g_1^2g_2^2\lambda_4+24g_1^2g_2^2\lambda_{\Phi_1}+\frac{24}{5}g_1^2g_2^2\lambda_{\Phi_1\Delta}+\frac{9693}{400}g_1^4\lambda_{\Phi_1\Delta 1}+\frac{2541}{40}g_1^2g_2^2\lambda_{\Phi_1\Delta 1}$$
$$-\frac{1153}{48}g_2^4\lambda_{\Phi_1\Delta 1}-2\lambda_3^2\lambda_{\Phi_1\Delta 1}-2\lambda_3\lambda_4\lambda_{\Phi_1\Delta 1}+2\lambda_4^2\lambda_{\Phi_1\Delta 1}+\frac{24}{5}g_1^2\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}-28\lambda^2\lambda_{\Phi_1\Delta 1}+6g_1^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}+46g_2^2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}$$
$$-80\lambda_{\Phi_1}\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}-29\lambda_{\Phi_1\Delta}^2\lambda_{\Phi_1\Delta 1}+\frac{33}{5}g_1^2\lambda_{\Phi_1\Delta 1}^2+23g_2^2\lambda_{\Phi_1\Delta 1}^2-40\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}^2-29\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}^2-\frac{27}{4}\lambda_{\Phi_1\Delta 1}^3$$
$$-16\lambda_3\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta}-8\lambda_4\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta}-2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta}^2+3g_1^2g_2^2\lambda_{\Phi_2\Delta 1}+\frac{12}{5}g_1^2\lambda_4\lambda_{\Phi_2\Delta 1}-8\lambda_3\lambda_4\lambda_{\Phi_2\Delta 1}-4\lambda_4^2\lambda_{\Phi_2\Delta 1}$$
$$-8\lambda_4\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta 1}-8\lambda_3\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}-8\lambda_4\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}-8\lambda_4\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1}-2\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta 1}-4\lambda_4\lambda_{\Phi_2\Delta 1}^2-\frac{1}{2}\lambda_{\Phi_1\Delta 1}\lambda_{\Phi_2\Delta 1}^2$$
$$+24g_1^2g_2^2\lambda_{\Delta 1}+\frac{96}{5}g_1^2\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}+40g_2^2\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}-96\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}-48\lambda_{\Phi_1\Delta 1}^2\lambda_{\Delta 1}-32\lambda_{\Phi_1\Delta 1}\lambda_T^2+48g_1^2g_2^2\lambda_{\Delta 2}$$
$$+\frac{192}{5}g_1^2\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}+80g_2^2\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}-112\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}-56\lambda_{\Phi_1\Delta 1}^2\lambda_{\Delta 2}-88\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}\lambda_{\Delta 2}-38\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}^2$$
$$+4(-8\lambda_{\Phi_2\Delta 1}+\lambda_{\Phi_1\Delta 1})\lambda_5^2+\frac{1}{20}(225g_2^2\lambda_{\Phi_1\Delta 1}-80\lambda_{\Phi_1\Delta 1}(-10g_3^2+3\lambda_{\Phi_1\Delta 1}+6\lambda_{\Phi_1}+6\lambda_{\Phi_1\Delta})$$
$$+g_1^2(25\lambda_{\Phi_1\Delta 1}+432g_2^2))\mathrm{Tr}(Y_dY_d^\dagger)+\frac{132}{5}g_1^2g_2^2\mathrm{Tr}(Y_eY_e^\dagger)+\frac{15}{4}g_1^2\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_eY_e^\dagger)+\frac{15}{4}g_2^2\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_eY_e^\dagger)$$
$$-8\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_eY_e^\dagger)-8\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_eY_e^\dagger)-4\lambda_{\Phi_1\Delta 1}^2\mathrm{Tr}(Y_eY_e^\dagger)+\frac{48}{5}g_1^2g_2^2\mathrm{Tr}(Y_NY_N^\dagger)+3g_1^2\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger)$$
$$+15g_2^2\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger)-16\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger)-8\lambda_{\Phi_1\Delta 1}^2\mathrm{Tr}(Y_NY_N^\dagger)-16\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger)$$
$$-32\lambda_{\Phi_1\Delta 1}\lambda_{\Delta 2}\mathrm{Tr}(Y_NY_N^\dagger)+\frac{252}{5}g_1^2g_2^2\mathrm{Tr}(Y_uY_u^\dagger)+\frac{17}{4}g_1^2\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_uY_u^\dagger)+\frac{45}{4}g_2^2\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_uY_u^\dagger)+40g_3^2\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_uY_u^\dagger)$$
$$-24\lambda_{\Phi_1}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_uY_u^\dagger)-24\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_uY_u^\dagger)-12\lambda_{\Phi_1\Delta 1}^2\mathrm{Tr}(Y_uY_u^\dagger)-\frac{27}{2}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger)$$
$$+27\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger)-\frac{9}{2}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_eY_e^\dagger Y_eY_e^\dagger)+\frac{24}{5}g_1^2\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)-8g_2^2\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)$$
$$-32\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)-31\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)-36\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_NY_N^\dagger Y_NY_N^\dagger)-\frac{27}{2}\lambda_{\Phi_1\Delta 1}\mathrm{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger)$$
$$-14\mathrm{Tr}(Y_eY_e^\dagger Y_eY_N^\dagger Y_NY_e^\dagger)-18\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger Y_eY_e^\dagger)-176\mathrm{Tr}(Y_eY_N^\dagger Y_NY_N^\dagger Y_NY_e^\dagger)-24\mathrm{Tr}(Y_eY_N^\dagger Y_e^TY_e^*Y_NY_e^\dagger)\Big],$$

$$\beta_{\lambda_{\Phi_2\Delta}}=\frac{1}{16\pi^2}\Big[12\lambda_2\lambda_{\Phi_2\Delta}+12\lambda_{\Phi_2\Delta}\lambda_{\Delta 2}+16\lambda_{\Phi_2\Delta}\lambda_{\Delta 1}+2\lambda_3\lambda_{\Phi_1\Delta 1}+2\lambda_4\lambda_{\Phi_1\Delta}+2\lambda_{\Phi_2\Delta 1}\lambda_{\Delta 2}+4\lambda_2\lambda_{\Phi_2\Delta 1}+4\lambda_3\lambda_{\Phi_1\Delta}+4\lambda_{\Phi_2\Delta}^2$$
$$+4\lambda_{\Phi_2\Delta}\mathrm{Tr}(Y_NY_N^\dagger)+6g_2^4+6\lambda_{\Phi_2\Delta 1}\lambda_{\Delta 1}+\frac{27}{25}g_1^4-\frac{33}{2}g_2^2\lambda_{\Phi_2\Delta}-\frac{9}{10}g_1^2(4g_2^2+5\lambda_{\Phi_2\Delta})+\lambda_{\Phi_2\Delta 1}^2\Big]$$
$$+\frac{1}{(16\pi^2)^2}\Big[-\frac{9801}{500}g_1^6+\frac{2457}{100}g_1^4g_2^2+\frac{112}{5}g_1^2g_2^4+\frac{245}{6}g_2^6+\frac{54}{5}g_1^4\lambda_2-12g_1^2g_2^2\lambda_2+60g_2^4\lambda_2+\frac{18}{5}g_1^4\lambda_3+20g_2^4\lambda_3+\frac{9}{5}g_1^4\lambda_4$$
$$-6g_1^2g_2^2\lambda_4+10g_2^4\lambda_4+\frac{9}{10}g_1^4\lambda_{\Phi_1\Delta}+\frac{15}{2}g_2^4\lambda_{\Phi_1\Delta}+\frac{24}{5}g_1^2\lambda_3\lambda_{\Phi_1\Delta}+24g_2^2\lambda_3\lambda_{\Phi_1\Delta}-8\lambda_3^2\lambda_{\Phi_1\Delta}+\frac{12}{5}g_1^2\lambda_4\lambda_{\Phi_1\Delta}+12g_2^2\lambda_4\lambda_{\Phi_1\Delta}$$
$$-8\lambda_3\lambda_4\lambda_{\Phi_1\Delta}-8\lambda_4^2\lambda_{\Phi_1\Delta}-8\lambda_3\lambda_{\Phi_1\Delta}^2-4\lambda_4\lambda_{\Phi_1\Delta}^2+\frac{9}{20}g_1^4\lambda_{\Phi_1\Delta 1}-\frac{3}{2}g_1^2g_2^2\lambda_{\Phi_1\Delta 1}+\frac{15}{4}g_2^4\lambda_{\Phi_1\Delta 1}+\frac{12}{5}g_1^2\lambda_3\lambda_{\Phi_1\Delta 1}$$

$$
\begin{aligned}
&-4\lambda_3^2\lambda_{\Phi_1\Delta1}+6g_2^2\lambda_4\lambda_{\Phi_1\Delta1}-2\lambda_4^2\lambda_{\Phi_1\Delta1}-8\lambda_3\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta1}-6\lambda_3\lambda_{\Phi_1\Delta1}^2-\lambda_4\lambda_{\Phi_1\Delta1}^2+\frac{18693}{400}g_1^4\lambda_{\Phi_2\Delta}+\frac{429}{40}g_1^2g_2^2\lambda_{\Phi_2\Delta}\\
&+\frac{3287}{48}g_2^4\lambda_{\Phi_2\Delta}+\frac{72}{5}g_1^2\lambda_2\lambda_{\Phi_2\Delta}+72g_2^2\lambda_2\lambda_{\Phi_2\Delta}-60\lambda_2^2\lambda_{\Phi_2\Delta}-2\lambda_3^2\lambda_{\Phi_2\Delta}-2\lambda_3\lambda_4\lambda_{\Phi_2\Delta}-2\lambda_4^2\lambda_{\Phi_2\Delta}-16\lambda_3\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta}\\
&-8\lambda_4\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta}-2\lambda_{\Phi_1\Delta}^2\lambda_{\Phi_2\Delta}-8\lambda_3\lambda_{\Phi_1\Delta1}\lambda_{\Phi_2\Delta}-2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta1}\lambda_{\Phi_2\Delta}-\frac{3}{2}\lambda_{\Phi_1\Delta1}^2\lambda_{\Phi_2\Delta}+3g_1^2\lambda_{\Phi_2\Delta}^2+11g_2^2\lambda_{\Phi_2\Delta}^2-72\lambda_2\lambda_{\Phi_2\Delta}^2\\
&-13\lambda_{\Phi_2\Delta}^3+\frac{45}{4}g_1^4\lambda_{\Phi_2\Delta1}-\frac{114}{5}g_1^2g_2^2\lambda_{\Phi_2\Delta1}+\frac{185}{4}g_2^4\lambda_{\Phi_2\Delta1}+\frac{24}{5}g_1^2\lambda_2\lambda_{\Phi_2\Delta1}+36g_2^2\lambda_2\lambda_{\Phi_2\Delta1}-16\lambda_2^2\lambda_{\Phi_2\Delta1}-2\lambda_4^2\lambda_{\Phi_2\Delta1}\\
&-2\lambda_4\lambda_{\Phi_1\Delta1}\lambda_{\Phi_2\Delta1}-\frac{1}{2}\lambda_{\Phi_1\Delta1}^2\lambda_{\Phi_2\Delta1}-12g_2^2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}-32\lambda_2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}-5\lambda_{\Phi_2\Delta}^2\lambda_{\Phi_2\Delta1}-\frac{21}{20}g_1^2\lambda_{\Phi_2\Delta1}^2+\frac{11}{4}g_2^2\lambda_{\Phi_2\Delta1}^2\\
&-18\lambda_2\lambda_{\Phi_2\Delta1}^2-\frac{39}{4}\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}^2-\frac{11}{2}\lambda_{\Phi_2\Delta1}^3+\frac{72}{5}g_1^4\lambda_{\Delta1}-12g_1^2g_2^2\lambda_{\Delta1}+80g_2^4\lambda_{\Delta1}+\frac{384}{5}g_1^2\lambda_{\Phi_2\Delta}\lambda_{\Delta1}+256g_2^2\lambda_{\Phi_2\Delta}\lambda_{\Delta1}\\
&-96\lambda_{\Phi_2\Delta}^2\lambda_{\Delta1}+\frac{144}{5}g_1^2\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}+108g_2^2\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}-48\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}-24\lambda_{\Phi_2\Delta1}^2\lambda_{\Delta1}-80\lambda_{\Phi_2\Delta}\lambda_T^2-24\lambda_{\Phi_2\Delta1}\lambda_T^2\\
&+\frac{54}{5}g_1^4\lambda_{\Delta2}-24g_1^2g_2^2\lambda_{\Delta2}+60g_2^4\lambda_{\Delta2}+\frac{288}{5}g_1^2\lambda_{\Phi_2\Delta}\lambda_{\Delta2}+192g_2^2\lambda_{\Phi_2\Delta}\lambda_{\Delta2}-72\lambda_{\Phi_2\Delta}^2\lambda_{\Delta2}+\frac{48}{5}g_1^2\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}\\
&+56g_2^2\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}-16\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}-18\lambda_{\Phi_2\Delta1}^2\lambda_{\Delta2}-120\lambda_{\Phi_2\Delta}\lambda_{\Delta1}\lambda_{\Delta2}-16\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}\lambda_{\Delta2}-70\lambda_{\Phi_2\Delta}\lambda_{\Delta2}^2-16\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}^2\\
&-4(12\lambda_{\Phi_1\Delta}+2\lambda_{\Phi_1\Delta1}+2\lambda_{\Phi_2\Delta1}+3\lambda_{\Phi_2\Delta})\lambda_5^2-12(2\lambda_3\lambda_{\Phi_1\Delta}+\lambda_3\lambda_{\Phi_1\Delta1}+\lambda_4\lambda_{\Phi_1\Delta})\mathrm{Tr}(Y_dY_d^\dagger)-8\lambda_3\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_eY_e^\dagger)\\
&-4\lambda_4\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_eY_e^\dagger)-4\lambda_3\lambda_{\Phi_1\Delta1}\mathrm{Tr}(Y_eY_e^\dagger)+\frac{36}{25}g_1^4\mathrm{Tr}(Y_NY_N^\dagger)-\frac{24}{5}g_1^2g_2^2\mathrm{Tr}(Y_NY_N^\dagger)+2g_2^4\mathrm{Tr}(Y_NY_N^\dagger)\\
&+15g_2^2\lambda_{\Phi_2\Delta}\mathrm{Tr}(Y_NY_N^\dagger)-8\lambda_{\Phi_2\Delta}^2\mathrm{Tr}(Y_NY_N^\dagger)-2\lambda_{\Phi_2\Delta1}^2\mathrm{Tr}(Y_NY_N^\dagger)-64\lambda_{\Phi_2\Delta}\lambda_{\Delta1}\mathrm{Tr}(Y_NY_N^\dagger)-24\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}\mathrm{Tr}(Y_NY_N^\dagger)\\
&-48\lambda_{\Phi_2\Delta}\lambda_{\Delta2}\mathrm{Tr}(Y_NY_N^\dagger)-8\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}\mathrm{Tr}(Y_NY_N^\dagger)-24\lambda_3\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_uY_u^\dagger)-12\lambda_4\lambda_{\Phi_1\Delta}\mathrm{Tr}(Y_uY_u^\dagger)-12\lambda_3\lambda_{\Phi_1\Delta1}\mathrm{Tr}(Y_uY_u^\dagger)\\
&+12g_2^2\lambda_3\lambda_{\Phi_1\Delta1}+3g_1^2\lambda_{\Phi_2\Delta}\mathrm{Tr}(Y_NY_N^\dagger)-6\lambda_{\Phi_2\Delta}\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)-36\lambda_{\Phi_2\Delta}\mathrm{Tr}(Y_NY_N^\dagger Y_NY_N^\dagger)\Bigg],
\end{aligned}
$$

$$
\begin{aligned}
\beta_{\lambda_{\Phi_2\Delta1}}=\frac{1}{16\pi^2}\Bigg[&2\lambda_4\lambda_{\Phi_1\Delta1}+4\lambda_2\lambda_{\Phi_2\Delta1}+4\lambda_{\Phi_2\Delta}2^2+4\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}+4\lambda_{\Phi_2\Delta1}\mathrm{Tr}(Y_NY_N^\dagger)+8\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}+8\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}\\
&-\frac{33}{2}g_2^2\lambda_{\Phi_2\Delta1}+\frac{9}{10}g_1^2(-5\lambda_{\Phi_2\Delta1}+8g_2^2)\Bigg]\\
&+\frac{1}{(16\pi^2)^2}\Bigg[-\frac{1971}{25}g_1^4g_2^2-\frac{449}{5}g_1^2g_2^4+24g_1^2g_2^2\lambda_2+12g_1^2g_2^2\lambda_4+3g_1^2g_2^2\lambda_{\Phi_1\Delta1}+\frac{12}{5}g_1^2\lambda_4\lambda_{\Phi_1\Delta1}-8\lambda_3\lambda_4\lambda_{\Phi_1\Delta1}-4\lambda_4^2\lambda_{\Phi_1\Delta1}\\
&-8\lambda_4\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta1}-4\lambda_4\lambda_{\Phi_1\Delta}2^2+\frac{24}{5}g_1^2g_2^2\lambda_{\Phi_2\Delta}-8\lambda_4\lambda_{\Phi_1\Delta1}\lambda_{\Phi_2\Delta}+\frac{9693}{400}g_1^4\lambda_{\Phi_2\Delta1}+\frac{2541}{40}g_1^2g_2^2\lambda_{\Phi_2\Delta1}-\frac{1153}{48}g_2^4\lambda_{\Phi_2\Delta1}\\
&+\frac{24}{5}g_1^2\lambda_2\lambda_{\Phi_2\Delta1}-28\lambda_2^2\lambda_{\Phi_2\Delta1}-2\lambda_3^2\lambda_{\Phi_2\Delta1}-2\lambda_3\lambda_4\lambda_{\Phi_2\Delta1}+2\lambda_4^2\lambda_{\Phi_2\Delta1}-16\lambda_3\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta1}-8\lambda_4\lambda_{\Phi_1\Delta}\lambda_{\Phi_2\Delta1}-2\lambda_{\Phi_1\Delta}^2\lambda_{\Phi_2\Delta1}\\
&-8\lambda_3\lambda_{\Phi_1\Delta1}\lambda_{\Phi_2\Delta1}-8\lambda_4\lambda_{\Phi_1\Delta1}\lambda_{\Phi_2\Delta1}-2\lambda_{\Phi_1\Delta}\lambda_{\Phi_1\Delta1}\lambda_{\Phi_2\Delta1}-\frac{1}{2}\lambda_{\Phi_1\Delta1}^2\lambda_{\Phi_2\Delta1}+6g_1^2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}+46g_2^2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}\\
&-80\lambda_2\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}-29\lambda_{\Phi_2\Delta}^2\lambda_{\Phi_2\Delta1}+\frac{33}{5}g_1^2\lambda_{\Phi_2\Delta1}^2+23g_2^2\lambda_{\Phi_2\Delta1}^2-40\lambda_2\lambda_{\Phi_2\Delta1}^2-29\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}^2-\frac{27}{4}\lambda_{\Phi_2\Delta1}^3\\
&+24g_1^2g_2^2\lambda_{\Delta1}+\frac{96}{5}g_1^2\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}+40g_2^2\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}-96\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}-48\lambda_{\Phi_2\Delta1}^2\lambda_{\Delta1}-32\lambda_{\Phi_2\Delta1}\lambda_T^2+48g_1^2g_2^2\lambda_{\Delta2}\\
&+\frac{192}{5}g_1^2\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}+80g_2^2\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}-112\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}-56\lambda_{\Phi_2\Delta1}^2\lambda_{\Delta2}-88\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}\lambda_{\Delta2}-38\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}^2\\
&+4(-8\lambda_{\Phi_1\Delta1}+\lambda_{\Phi_2\Delta}2)\lambda_5^2-12\lambda_4\lambda_{\Phi_1\Delta1}\mathrm{Tr}(Y_dY_d^\dagger)-4\lambda_4\lambda_{\Phi_1\Delta1}\mathrm{Tr}(Y_eY_e^\dagger)+\frac{48}{5}g_1^2g_2^2\mathrm{Tr}(Y_NY_N^\dagger)+3g_1^2\lambda_{\Phi_2\Delta1}\mathrm{Tr}(Y_NY_N^\dagger)\\
&+15g_2^2\lambda_{\Phi_2\Delta1}\mathrm{Tr}(Y_NY_N^\dagger)-16\lambda_{\Phi_2\Delta}\lambda_{\Phi_2\Delta1}\mathrm{Tr}(Y_NY_N^\dagger)-8\lambda_{\Phi_2\Delta}2^2\mathrm{Tr}(Y_NY_N^\dagger)-16\lambda_{\Phi_2\Delta1}\lambda_{\Delta1}\mathrm{Tr}(Y_NY_N^\dagger)\\
&-32\lambda_{\Phi_2\Delta1}\lambda_{\Delta2}\mathrm{Tr}(Y_NY_N^\dagger)-12\lambda_4\lambda_{\Phi_1\Delta1}\mathrm{Tr}(Y_uY_u^\dagger)-6\lambda_{\Phi_2\Delta1}\mathrm{Tr}(Y_eY_N^\dagger Y_NY_e^\dagger)-36\lambda_{\Phi_2\Delta1}\mathrm{Tr}(Y_NY_N^\dagger Y_NY_N^\dagger)\Bigg].
\end{aligned}
$$

2. Gauge couplings

$$\beta_{g_1} = \frac{1}{16\pi^2}\left[\frac{24}{5}g_1^3\right] + \frac{1}{(16\pi^2)^2}\left[\frac{1}{50}g_1^3(-25\mathrm{Tr}(Y_dY_d^\dagger) + 424g_1^2 + 440g_3^2 - 75\mathrm{Tr}(Y_eY_e^\dagger) - 85\mathrm{Tr}(Y_uY_u^\dagger) + 900g_2^2 - 90\mathrm{Tr}(Y_NY_N^\dagger))\right].$$

$$\beta_{g_2} = \frac{1}{16\pi^2}\left[-\frac{7}{3}g_2^3\right] + \frac{1}{(16\pi^2)^2}\left[\frac{1}{6}g_2^3(160g_2^2 - 18\mathrm{Tr}(Y_NY_N^\dagger) + 36g_1^2 - 3\mathrm{Tr}(Y_eY_e^\dagger) + 72g_3^2 - 9\mathrm{Tr}(Y_dY_d^\dagger) - 9\mathrm{Tr}(Y_uY_u^\dagger))\right].$$

$$\beta_{g_3} = \frac{1}{16\pi^2}[-7g_3^3] + \frac{1}{(16\pi^2)^2}\left[-\frac{1}{10}g_3^3(-11g_1^2 + 20\mathrm{Tr}(Y_dY_d^\dagger) + 20\mathrm{Tr}(Y_uY_u^\dagger) + 260g_3^2 - 45g_2^2)\right].$$

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