Double distributions and pseudodistributions

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We describe the approach to lattice extraction of generalized parton distributions (GPDs) that is based on the use of the double distribution (DD) formalism within the pseudodistribution framework. The advantage of using DDs is that GPDs obtained in this way have the mandatory polynomiality property, a nontrivial correlation between x and ξ dependences of GPDs. Another advantage of using DDs is that the D-term appears as an independent entity in the DD formalism rather than a part of GPDs H and E. We relate the ξ dependence of GPDs to the width of the α profiles of the corresponding DDs and discuss strategies for fitting lattice-extracted pseudodistributions by DDs. The approach described in the present paper may be used in ongoing and future lattice extractions of GPDs.

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I. INTRODUCTION

Generalized parton distributions (GPDs) [1–6] (for reviews see Refs. [7–9]) are a major object of study at the future Electron-Ion Collider and existing facilities at Jefferson Laboratory and CERN. They provide detailed information about hadronic structure. Being functions $H(x, \xi, t)$ of three kinematic variables (while there are other GPDs: $E, \tilde{H}, \tilde{E}$, etc., we will use H as a generic notation), they combine properties of usual parton distributions $f(x)$, hadronic form factors $F(t)$, and in the central region $|x| < \xi$, of the distribution amplitudes $\varphi(x/\xi)$.

However, this multidimensional nature of GPDs highly complicates their extraction from experimental data. In particular, deeply virtual Compton scattering (DVCS), which is the main tool for obtaining information about GPDs, gives information about GPDs on the lines $x = \pm\xi$ or through the Compton form factors that are x integrals of GPDs with the $1/(x - \xi)$ weight.

More complicated processes, like double DVCS or recently proposed single diffractive hard exclusive photoproduction [10], may provide information about GPDs off the $x = \pm\xi$ diagonals. The study of such processes is in its early stage.

During the last decade, starting with the pioneering paper of Ji [11] that introduced the quasidistribution approach (see also Ref. [12] for "lattice cross sections" approach), strong efforts have been made to calculate parton distributions on the lattice (for reviews, see Refs. [13–16]). In particular, matching conditions for GPDs in the quasidistribution approach were discussed in Refs. [17–19]. For a review of recent lattice calculations of GPDs, see Refs. [20,21].

In our paper [22], general aspects of lattice QCD extraction of GPDs have been discussed in the framework of the pseudodistribution approach [23,24]. The advantage of lattice calculations is that matrix elements $M(\nu, \xi, t)$ ["Ioffe-time distributions" (ITDs)] of nonlocal operators measured on the lattice are related to Fourier transforms of GPDs $H(x, \xi, t)$, which may be inverted using various techniques to produce GPDs as functions of x for fixed values of skewness ξ and invariant momentum transfer t.

An important property of GPDs is "polynomiality" [7], which states that x^N moment of $H(x, \xi, t)$ must be a polynomial of ξ of not larger than $(N + 1)$th power. This nontrivial correlation between x and ξ dependences of $H(x, \xi, t)$ is automatically satisfied when GPDs are obtained from double distributions $F(\beta, \alpha, t)$ [1,3,4,25,26].

The goal of the present work is to outline the approach of lattice extraction of double distributions from lattice calculations. The paper organized as follows. To make it self-contained, in Sec. II we formulate the definitions of usual (light-cone) GPDs and double distributions (DDs) and discuss their relationship. Some basic properties of GPDs are discussed in Sec. III. There we also introduce Ioffe-time distributions. Pseudodistributions, as generalizations of the ITDs onto correlators off the light cone, are introduced in Sec. IV. Some strategies for fitting lattice-extracted pseudodistributions by DDs are discussed in Sec. V. Finally, in Sec. VI, we summarize our discussion.

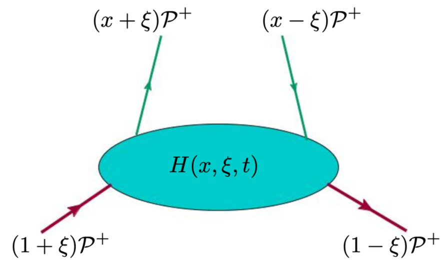

FIG. 1. Flux of the momentum plus components in terms of GPD variables.

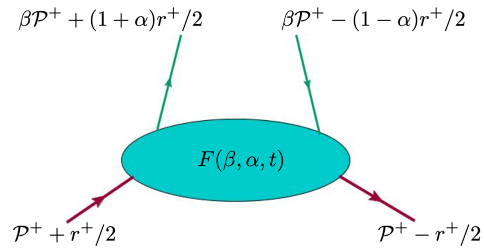

FIG. 2. Flux of the momentum plus components in terms of DD variables.

II. GPDs AND DDs

A. Definition of GPD

In the GPD description of a nonforward kinematics proposed by Ji [2], the plus components of the initial p and final p' hadron momenta are given by $(1+\xi)\mathcal{P}^+$ and $(1-\xi)\mathcal{P}^+$, respectively, with $\mathcal{P}$ being the average momentum $\mathcal{P}=(p+p')/2$, while the partons have $(x+\xi)\mathcal{P}^+$ and $(x-\xi)\mathcal{P}^+$ as the plus components of their momenta, see Fig. 1.

For the pion, one may define the light-cone GPDs $H(x,\xi,t;\mu^2)$ [1,2,6] by

$$\langle p'|\mathcal{O}^\lambda(z)|p\rangle = 2\mathcal{P}^\lambda \int_{-1}^{1} dx e^{-ix(\mathcal{P}z)} H\big(x,\xi,t;\mu^2\big), \quad (2.1)$$

where $\mathcal{O}^\lambda(z)=\bar\psi(-z/2)\gamma^\lambda\hat W(-z/2,z/2)\psi(z/2)$ is the quark bilocal operator with $\hat W(-z/2,z/2;A)$ being the Wilson line in the fundamental representation, the coordinate z has only the z_- light-cone component, and $\gamma^\lambda=\gamma^+$. The matrix element is singular on the light cone, so one should use some regularization for it specified by a scale μ. For brevity, we will skip reference to μ^2 in what follows.

The invariant momentum transfer is given by $t=(p-p')^2$. In principle, the rhs of Eq. (2.1) has also the r^λ term, where $r=p-p'$ is the momentum transfer. However, the GPD convention is to write $r^+=2\xi\mathcal{P}^+$, where ξ is the skewness variable, and the two terms are combined in one GPD $H(x,\xi,t)$.

A similar definition holds for nucleons,

$$\langle p',s'|\mathcal{O}^+(z)|p,s\rangle = \int_{-1}^{1} dx e^{-ix\mathcal{P}^+z_-}\Big[\big(\bar u'\gamma^+u\big)H(x,\xi,t) - \frac{1}{2M}\big(\bar u' i\sigma^{+\mu}r_\mu u\big)E(x,\xi,t)\Big], \quad (2.2)$$

where $\bar u'\equiv \bar u(p',s')$ and $u\equiv u(p,s)$ are the nucleon spinors, while $H(x,\xi,t)$ and $E(x,\xi,t)$ are the nucleon GPDs.

One may rewrite these definitions in a more covariant form that uses Lorentz invariants $(\mathcal{P}z)$ and (rz) only. For the pion, we have

$$\langle p_2|z_\lambda\mathcal{O}^\lambda(z)|p_1\rangle|_{z^2=0} = 2(\mathcal{P}z)\int_{-1}^{1} dx e^{-ix(\mathcal{P}z)} H(x,\xi,t;\mu^2)\Big|_{z^2=0}. \quad (2.3)$$

For nucleons, we have two GPDs,

$$\langle p',s'|z_\lambda\mathcal{O}^\lambda(z)|p,s\rangle|_{z^2=0} = \int_{-1}^{1} dx e^{-ix(\mathcal{P}z)}\Big\{(\bar u'\not{z}u)H(x,\xi,t) - \frac{1}{2M}(\bar u' i\sigma^{zr}u)E(x,\xi,t)\Big\}_{z^2=0}. \quad (2.4)$$

B. Double distribution description

An alternative approach to describe nonforward matrix elements is based on DD formalism [1,3,4,25,26]. Its guiding idea is to treat P^+ and r^+ as independent variables and organize the plus-momentum flux as a "superposition" of P^+ and r^+ momentum flows.

The parton momentum in this picture is written as $k^+=\beta\mathcal{P}^+ + (1+\alpha)r^+/2$, i.e., as a sum of the component $\beta\mathcal{P}^+$ due to the average hadron momentum P (flowing in the s channel) and the component $(1+\alpha)r^+/2$ due to the t-channel momentum r, see Fig. 2.

Thus, the α dependence of the DD $F(\beta,\alpha)$ describes the distribution of the momentum transfer r^+ between the initial and final quarks in fractions $(1+\alpha)/2$ and $(1-\alpha)/2$. One may expect that it has a shape similar to those of parton distribution amplitudes, e.g., with maximum at $\alpha=0$ (equal sharing of r^+) and vanishing at kinematical boundaries. These are located at $\alpha=\pm(1-|\beta|)$, since the support region for DDs is $|\alpha|+|\beta|\le 1$ [26].

1. *Pion*

In terms of DDs, the matrix element (2.3) is written as [1,3,26,27]

$$\begin{aligned}\langle \mathcal{P}-r/2|z_\lambda \mathcal{O}^\lambda(z)|\mathcal{P}+r/2\rangle_{z^2=0} &= \int_\Omega d\alpha d\beta e^{-i\beta(\mathcal{P}z)-i\alpha(rz)/2} \\ &\quad\times \{2(\mathcal{P}z)F(\beta,\alpha,t)+(rz)G(\beta,\alpha,t)\}\Big|_{z^2=0}, \end{aligned} \tag{2.5}$$

where Ω is the DD support region, i.e., a rhombus in the $(\alpha\beta)$ plane defined by $|\alpha|+|\beta|\leq 1$. The time reversal invariance requires that $F(\beta,\alpha,t)$ is an even function of α, while $G(\beta,\alpha,t)$ is odd in α.

Expanding $e^{-i\beta(\mathcal{P}z)-i\alpha(rz)/2}$ in powers of $(\mathcal{P}z)$ and (rz), one observes that the generic term $(\mathcal{P}z)^{N-k}(rz)^k$ may be obtained both from F and G parts [28], with two exceptions. Namely, one cannot obtain the $(\mathcal{P}z)^N$ term from the G part, and one cannot obtain the $(rz)^N$ term from the F part. The usual convention is to absorb all the $(\mathcal{P}z)^{N-k}(rz)^k$ terms with $k<N$ into the F function, leaving the $(rz)^N$ terms in the G function [27]. As a result, the G part would not depend on $(\mathcal{P}z)$, and one can write

$$\begin{aligned}\langle \mathcal{P}-r/2|z_\lambda \mathcal{O}^\lambda(z)|\mathcal{P}+r/2\rangle_{z^2=0} &= \Big\{2(\mathcal{P}z)\int_\Omega d\alpha d\beta e^{-i\beta(\mathcal{P}z)-i\alpha(rz)/2}F(\beta,\alpha,t) \\ &\quad + (rz)\int_{-1}^{1} d\alpha e^{-i\alpha(rz)/2}D(\alpha,t)\Big\}\Big|_{z^2=0}, \end{aligned} \tag{2.6}$$

where $D(\alpha,t)$ is the D-term function introduced in Ref. [27]. It is odd in α.

Comparing GPD and DD parametrizations (2.3) and (2.6), we get the relation between the pion GPD and DD [1,6,27]

$$\begin{aligned}H(x,\xi,t) &= \int_\Omega d\alpha d\beta \delta(x-\beta-\alpha\xi)F(\beta,\alpha,t) \\ &\quad + \mathrm{sgn}(\xi)D(x/\xi,t;\mu^2) \\ &\equiv H_{DD}+D. \end{aligned} \tag{2.7}$$

As noticed in Ref. [28], the $(\alpha\beta)$ integral above, i.e., the "DD part" $H_{DD}(x,\xi,t)$, may be treated as the Radon transform of F.

2. *Nucleon*

In the nucleon case, we have the following representation [1,3,26]:

$$\begin{aligned}\langle \mathcal{P}-r/2,s'|z_\lambda \mathcal{O}^\lambda(z))|\mathcal{P}+r/2,s\rangle_{z^2=0} &= \int_\Omega d\alpha d\beta e^{-i\beta(\mathcal{P}z)-i\alpha(rz)/2} \\ &\quad\times\left[(\bar{u}'\not{z}u)h(\beta,\alpha,t)-\frac{1}{2M}(\bar{u}'i\sigma^{zr}u)e(\beta,\alpha,t)\right] \\ &\quad + (rz)\frac{(\bar{u}'u)}{M}\int_{-1}^{1} d\alpha e^{-i\alpha(rz)/2}D(\alpha,t). \end{aligned} \tag{2.8}$$

Here, $h(\beta,\alpha,t)$ and $e(\beta,\alpha,t)$ are even functions of α, while $D(\alpha)$ is odd. Using Gordon decomposition

$$\frac{\mathcal{P}^\lambda}{M}\bar{u}'u = \frac{1}{2M}\bar{u}'i\sigma^{\lambda r}u + \bar{u}'\gamma^\lambda u, \tag{2.9}$$

and comparing (2.8) with the GPD representation (2.4), gives the relation between the nucleon GPDs, DDs, and D-term [29]. Namely, we have

$$\begin{aligned}H(x,\xi,t) &= \int_\Omega d\alpha d\beta \delta(x-\beta-\alpha\xi)h(\beta,\alpha,t) \\ &\quad + \mathrm{sgn}(\xi)D(x/\xi,t) \\ &\equiv H_{DD}+D, \end{aligned} \tag{2.10}$$

for $H(x,\xi,t)$, and

$$\begin{aligned}E(x,\xi,t) &= \int_\Omega d\alpha d\beta \delta(x-\beta-\alpha\xi)e(\beta,\alpha,t) \\ &\quad - \mathrm{sgn}(\xi)D(x/\xi,t) \\ &\equiv E_{DD}-D, \end{aligned} \tag{2.11}$$

for $E(x,\xi,t)$.

Again, we may talk about the DD parts $H_{DD}(x,\xi,t)$ and $E_{DD}(x,\xi,t)$ of the corresponding GPDs. Note that the D-term cancels in the sum $H(x,\xi,t)+E(x,\xi,t)\equiv A(x,\xi,t)$. So, $A(x,\xi,t)$ is built purely from the DD $a(\beta,\alpha,t)\equiv h(\beta,\alpha,t)+e(\beta,\alpha,t)$.

C. Fixed parity cases

Usually we are interested in the functions corresponding to operators

$$\mathcal{O}^\lambda_\pm(z) = \frac{1}{2}\left[\mathcal{O}^\lambda(z)\pm\mathcal{O}^\lambda(-z,A)\right] \tag{2.12}$$

that are symmetric or antisymmetric with respect to the inversion of z. These combinations appear when we consider "nonsinglet" $q-\bar{q}$ or "singlet" $q+\bar{q}$ parton distributions, respectively. Since the D-term contribution [without the overall (rz) factor] is odd in z, it appears in the singlet case only. However, the $H+E$ sum does not contain the D-term even in the singlet case.

In fact, it is sufficient to consider the matrix element of the original $\mathcal{O}^{\lambda}(z)$ operator. The real part of this matrix element is even in z, while its imaginary part is odd in z.

III. SOME PROPERTIES OF GPDs and DDs

A. DD parts of GPDs

In this section, we consider the relations between the DDs and the DD parts of GPDs that they generate, thus ignoring for a while the D-term contributions to GPDs. The D-term will be discussed later in the paper. For definiteness, we will have in mind relations between the DD part of the pion GPD and its DD. All the relations are equally applicable to the DD parts of the nucleon GPDs.

B. $\xi = 0$ limit

Taking $\xi = 0$, we have

$$H(x, \xi = 0, t) = \int_{-1}^{1} d\beta \delta(x-\beta) \int_{-1+|\beta|}^{1-|\beta|} d\alpha F(\beta, \alpha, t) = \int_{-1+|x|}^{1-|x|} d\alpha F(x, \alpha, t) \equiv f(x, t). \quad (3.1)$$

This means that integrating $F(\beta, \alpha, t)$ over vertical lines $\beta = x$ (see Fig. 3) gives the $\xi = 0$ ("nonskewed") GPD $\mathcal{H}(x, \xi = 0, t)$, which we will also denote as $f(x, t)$. It is the simplest GPD, which was called "nonforward parton density" in the paper [30], where it was introduced. It differs from the forward parton distribution function (PDF) $f(x)$ by the presence of t dependence and satisfies $f(x, t = 0) = f(x)$.

C. Polynomiality

The DD representation automatically produces a GPD satisfying the polynomiality property. Indeed,

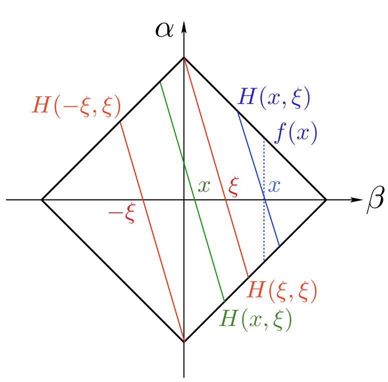

FIG. 3. DD support rhombus and integration lines producing the DD parts of $H(\xi, \xi)$, $H(-\xi, \xi)$, $H(x, \xi = 0) = f(x)$, and $H(x, \xi)$ in ($|x| > \xi$) and ($|x| < \xi$) regions.

$$\int_{-1}^{1} dx x^n H_{DD}(x, \xi, t) = \int_{-1}^{1} dx x^n \int_{\Omega} d\alpha d\beta \delta(x - \beta - \alpha\xi) F(\beta, \alpha, t) = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} \xi^k \int_{\Omega} d\alpha d\beta \beta^{n-k} \alpha^k F(\beta, \alpha, t), \quad (3.2)$$

i.e., the nth x moment of $H_{DD}(x, \xi, t)$ is a polynomial in ξ of the order not exceeding n.

Note that, since $F(\beta, \alpha, t)$ is even in α, the summation over k involves even k only, i.e., (3.1) is in fact an expansion in powers of ξ^2.

D. Ioffe-time distributions

By Lorentz invariance, the matrix element (2.3) defining GPD is a function of the scalars $(pz) \equiv -\nu_1$ and $(p'z) \equiv -\nu_2$, two Ioffe-time parameters, so we may write

$$\langle p_2 | \bar{\psi}(-z/2) \not{z} \psi(z/2) | p_1 \rangle = 2(\mathcal{P}z) I(\nu_1, \nu_2, t), \quad (3.3)$$

where $I(\nu_1, \nu_2, t)$ is the double Ioffe-time distribution. Since $z = z_-$ is assumed, only the values of the plus components of momenta are essential in the scalar products $(p_1 z)$ and $(p_2 z)$. The skewness variable ξ in this case is given by

$$\xi = \frac{\nu_1 - \nu_2}{\nu_1 + \nu_2} \equiv \frac{\nu_1 - \nu_2}{2\nu}. \quad (3.4)$$

We have introduced here the variable $\nu = (\nu_1 + \nu_2)/2$. Treating ν and ξ as independent variables, we define the generalized Ioffe-time distribution (GITD) as

$$I(\nu_1, \nu_2, t) = \mathcal{I}(\nu, \xi, t). \quad (3.5)$$

According to (2.1), it is a Fourier transform of the GPD,

$$\mathcal{I}(\nu, \xi, t) = \int_{-1}^{1} dx e^{ix\nu} H(x, \xi, t). \quad (3.6)$$

Using Eq. (2.5), we can write the DD part of GITD in terms of DD,

$$\mathcal{I}_{DD}(\nu, \xi, t) = \int_{-1}^{1} d\beta e^{i\nu\beta} \int_{-1+|\beta|}^{1-|\beta|} d\alpha e^{i\nu\alpha\xi} F(\beta, \alpha, t). \quad (3.7)$$

E. DD profile and ξ dependence

If $F(\beta, \alpha, t)$ has an infinitely narrow profile in the α direction, i.e., if $F(\beta, \alpha, t) = f(\beta, t)\delta(\alpha)$, then the ξ dependence disappears, and we deal with the simplest GPD $f(x, t)$. A nontrivial dependence on the skewness ξ is obtained if the DD has a nonzero-width profile in the α direction.

Using the DD representation (3.7) for the GITD and expanding $e^{i\nu\alpha\xi}$ into the Taylor series, we get the following expansion in powers of ξ^2:

$$\mathcal{I}_{DD}(\nu,\xi,t) = \int_{-1}^{1} d\beta e^{i\nu\beta} \int_{-1+|\beta|}^{1-|\beta|} d\alpha F(\beta,\alpha,t) - \frac{\xi^2\nu^2}{2} \times \int_{-1}^{1} d\beta e^{i\nu\beta} \int_{-1+|\beta|}^{1-|\beta|} d\alpha \alpha^2 F(\beta,\alpha,t) + \cdots \tag{3.8}$$

[odd powers of ξ do not appear because $F(\beta,\alpha,t)$ is even in α]. By analogy with (3.1), we will use the notation $f_2(\beta,t)$ for the second α moment of $F(\beta,\alpha,t)$,

$$\int_{-1+|\beta|}^{1-|\beta|} d\alpha \alpha^2 F(\beta,\alpha,t) \equiv f_2(\beta,t). \tag{3.9}$$

As a result, we write

$$\begin{aligned}\mathcal{I}_{DD}(\nu,\xi,t) &= \int_{-1}^{1} d\beta e^{i\nu\beta} \left\{ f(\beta,t) - \frac{\xi^2\nu^2}{2} f_2(\beta,t) \right\} + \mathcal{O}(\xi^4) \\ &\equiv \mathcal{I}_0(\nu,t) - \frac{\xi^2\nu^2}{2}\mathcal{I}_2(\nu,t) + \mathcal{O}(\xi^4).\end{aligned} \tag{3.10}$$

IV. PSEUDODISTRIBUTIONS

A. Definitions

On the lattice, we choose $z = z_3$, and introduce pseudo-GPDs $\mathcal{H}(x,\xi,t;z_3^2)$ [and also $\mathcal{E}(x,\xi,t;z_3^2)$ in the nucleon case].

The two Ioffe-time parameters are given now by $\nu_1 = p^{(3)} z_3 \equiv P_1 z_3$ and $\nu_1 = p'^{(3)} z_3 \equiv P_2 z_3$. In terms of momenta $P_{1,2}$, the skewness ξ is given by

$$\xi = \frac{P_1 - P_2}{P_1 + P_2}. \tag{4.1}$$

The pseudo-GITD will be denoted as $\mathcal{M}(\nu,\xi,t;z_3^2)$, e.g., the inverse transformation for $\mathcal{H}$ is written as

$$\mathcal{H}(x,\xi,t;z_3^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu e^{-ix\nu} \mathcal{M}(\nu,\xi,t;z_3^2). \tag{4.2}$$

Similarly, to denote pseudo-DDs, we will just add z_3^2 to their arguments.

B. Contaminations

On the lattice, we have $z^2 \neq 0$, and we need to add extra z-dependent structures to the original twist-two parametrization,

$$\begin{aligned} z_\lambda M^\lambda &\equiv \langle \mathcal{P} - r/2, s' | z_\lambda \mathcal{O}^\lambda(z) | \mathcal{P} + r/2, s \rangle \\ &= \int_\Omega d\alpha d\beta e^{-i\beta(\mathcal{P}z) - i\alpha(rz)/2} \Big\{ (\bar{u}' \not{z} u) h(\beta,\alpha,t) \\ &\quad - \frac{1}{2M}(\bar{u}' i\sigma^{zr} u) e(\beta,\alpha,t) + \frac{\bar{u}'u}{M}(rz)\delta(\beta) D(\alpha,t) \Big\} \\ &\equiv (\bar{u}'\not{z}u) H_{DD} - \frac{1}{2M}(\bar{u}' i\sigma^{zr} u) E_{DD} + (rz)\frac{\bar{u}'u}{M} D, \end{aligned} \tag{4.3}$$

where $z^2 = 0$ and the ITDs H_{DD}, E_{DD}, and D are functions of ν, ξ, and t.

A classification of additional tensor structures that appear in parametrizations of M^λ off the light cone was done in Ref. [31], where eight independent structures have been identified.

However, there is some subtlety related to the fact that, for lattice extractions, we need to parametrize the "non-contracted" matrix element M^λ. In this case, the index λ in local operators $\bar{\psi}\gamma^\lambda (zD)^N \psi$ is not symmetrized with the indices $D^{\mu_1} \ldots D^{\mu_N}$ in covariant derivatives, which is necessary for building twist-two local operators.

A way to perform symmetrization for bilocal operators was indicated in Ref. [32]. Further studies of parametrizations for matrix elements with an open index have been done in Refs. [33–38]. An important observation made there is that M^λ *should* contain terms that vanish when contracted with z_λ, such as $r^\lambda(\mathcal{P}z) - \mathcal{P}^\lambda(rz)$. One can see that $r^\lambda - \mathcal{P}^\lambda(rz)/(\mathcal{P}z) \equiv \Delta_\perp^\lambda$ is the part of the momentum transfer r that is transverse to z.

As shown in these papers, one should add Wandzura-Wilczek-type (WW) terms [39] to parametrizations of GPDs to secure electromagnetic gauge invariance of the DVCS amplitude [40] with $\mathcal{O}(\Delta_\perp)$ accuracy. While the WW terms are "kinematical twist-three" contributions built from twist-two GPDs, one cannot exclude nonperturbative (dynamical) twist-three terms accompanied by the $\Delta_\perp^\lambda$ factor.

Among additional structures listed in Ref. [31], one can see the structure $(\bar{u}' i\sigma^{\lambda z} u)$ that also vanishes when multiplied by z_λ and thus should be treated as a "higher-twist" term.

On the other hand, two other additional structures, $(\bar{u}' i\sigma^{zr}\mathcal{P}^\lambda u)$ and $(\bar{u}' i\sigma^{zr} r^\lambda u)$, after contraction with z_λ, produce the same "twist-two" structure $\sim\sigma^{zr}$ that accompanies the E_{DD} contribution. In this sense, the invariant amplitudes accompanying these structures have a twist-two component.

Note, however, that combinations $\sigma^{zr}\mathcal{P}^\lambda - \sigma^{\lambda r}(\mathcal{P}z)$ and $\sigma^{zr} r^\lambda - \sigma^{\lambda r}(rz)$ vanish after contraction with z_λ. So, we propose to use these "subtracted" forms in building the basis of additional terms, rather than just $\sigma^{zr}\mathcal{P}^\lambda$ and $\sigma^{zr} r^\lambda$. Since the subtracted structures do not contribute to the twist-two parametrization (4.3), the DDs associated with them should be classified as higher-twist ones.

For this reason, we construct a parametrization for M^λ in which twist-two and higher-twist terms are explicitly separated,

$$M^\lambda = (\bar{u}'\gamma^\lambda u)H_{DD} - \frac{1}{2M}(\bar{u}' i\sigma^{\lambda r} u)E_{DD} + r^\lambda \frac{\bar{u}'u}{M} D + \left[r^\lambda(\mathcal{P}z) - \mathcal{P}^\lambda(rz)\right]\frac{\bar{u}'u}{M} Y - \frac{1}{M}\left[(\bar{u}' i\sigma^{zr} u)\mathcal{P}^\lambda - (\bar{u}' i\sigma^{\lambda r} u)(\mathcal{P}z)\right]X_1 \\ - \frac{1}{M}\left[(\bar{u}' i\sigma^{zr} u)r^\lambda - (\bar{u}' i\sigma^{\lambda r} u)(rz)\right]X_2 + (\bar{u}' i\sigma^{\lambda z} u)MX_3 + i(\bar{u}'u)Mz^\lambda Z_1 - (\bar{u}' i\sigma^{zr} u)Mz^\lambda Z_2. \quad (4.4)$$

We have here Y and X_i terms whose contribution vanishes when contracted with z_λ, and Z_i terms that produce the z^2 factor after contraction with z_λ.

Formally, the $r^\lambda(\mathcal{P}z) - \mathcal{P}^\lambda(rz)$ combination does not contain new structures that are independent from those present in the twist-two line. However, the corresponding invariant amplitude, which is denoted as Y, is generated by a new DD. This higher-twist DD is different from the twist-two DDs $h(\beta,\alpha)$, $e(\beta,\alpha)$ and the D-term, which are also associated with the $\sim\bar{u}'u$ structures.

Of course, using Gordon decomposition

$$\frac{\mathcal{P}^\lambda}{M}\bar{u}'u = \frac{1}{2M}\bar{u}' i\sigma^{\lambda r} u + \bar{u}'\gamma^\lambda u, \quad (4.5)$$

we can rewrite (4.4) in a form explicitly having just eight structures,

$$M^\lambda = (\bar{u}'\gamma^\lambda u)\left[H_{DD} - (rz)Y\right] + r^\lambda \frac{\bar{u}'u}{M}\left[D + (\mathcal{P}z)Y\right] \\ - \frac{1}{2M}(\bar{u}' i\sigma^{\lambda r} u)\left[E_{DD} + (rz)Y\right] - \frac{1}{M}\left[(\bar{u}' i\sigma^{zr} u)\mathcal{P}^\lambda \right. \\ \left. - (\bar{u}' i\sigma^{\lambda r} u)(\mathcal{P}z)\right]X_1 - \frac{1}{M}\left[(\bar{u}' i\sigma^{zr} u)r^\lambda \right. \\ \left. - (\bar{u}' i\sigma^{\lambda r} u)(rz)\right]X_2 + (\bar{u}' i\sigma^{\lambda z} u)MX_3 \\ + i(\bar{u}'u)Mz^\lambda Z_1 - (\bar{u}' i\sigma^{zr} u)Mz^\lambda Z_2, \quad (4.6)$$

like in Ref. [31]. To establish a direct correspondence, we note that Ref. [31] uses a basis in which $(\bar{u}'\gamma^\lambda u)$ is substituted by two other structures that appear in the Gordon decomposition (4.5). Also, all the terms containing $(\bar{u}' i\sigma^{\lambda r} u)$ are combined in one contribution. Using this basis, we have

$$M^\lambda = \frac{\mathcal{P}^\lambda}{M}(\bar{u}'u)\left[H_{DD} - (rz)Y\right] + r^\lambda \frac{\bar{u}'u}{M}\left[D + (\mathcal{P}z)Y\right] \\ - \frac{1}{2M}(\bar{u}' i\sigma^{\lambda r} u)\left[H_{DD} + E_{DD} + 2(\mathcal{P}z)X_1 + 2(rz)X_2\right] \\ + (\bar{u}' i\sigma^{\lambda z} u)MX_3 + i(\bar{u}'u)Mz^\lambda Z_1 \\ - \frac{(\bar{u}' i\sigma^{zr} u)}{M}\left[\mathcal{P}^\lambda X_1 + r^\lambda X_2 + z^\lambda M^2 Z_2\right]. \quad (4.7)$$

Comparing Eq. (4.7) with the coefficients A_i in Eq. (35) of Ref. [31], we establish the correspondence $A_1 = [H_{DD} - (rz)Y]$, $A_2 = iZ_1$, $-A_3 = D + (\mathcal{P}z)Y$, $A_4 = -X_3$, $A_5 = (H_{DD} + E_{DD})/2 + (\mathcal{P}z)X_1 + (rz)X_2$, $A_6 = X_1$, $A_7 = Z_2$, and $A_8 = -X_2$.

The main difference is that H_{DD} and D contributions in Eq. (4.7) come with the contamination from the Y function, the 9th higher-twist ITD. Also, the $(\bar{u}' i\sigma^{\lambda r} u)$ structure is accompanied by a factor in which the Y term is absent, but there are contaminations from X_1 and X_2 contributions.

V. FITTING PSEUDODISTRIBUTIONS

A. Nonforward parton pseudodensity $f(\beta,t,z_3^2)$

Taking $\xi = 0$ we have

$$\mathcal{M}(\nu, \xi = 0, t; z_3^2) = \int_{-1}^{1} d\beta e^{i\nu\beta} f(\beta, t, z_3^2), \quad (5.1)$$

where $\nu = P_1 z_3 = P_2 z_3$. An important point is that $\xi = 0$ may be achieved for different pairs of equal initial and final momenta $P_1 = P_2 \equiv P$. One should check that the lattice gives the same curve for different P's, up to evolution-type dependence on z_3^2.

One can use relation (5.1) to fit $f(\beta, t, z_3^2)$. First, taking $t = 0$, we fit the forward pseudodistribution $f(\beta, z_3^2)$, just as a pseudo-PDF. After that, one can vary t, by changing the transverse components $\Delta_\perp^{1,2}$, for several fixed ν. In this way, one can study what kind of dependence on t we have (dipole, monopole, etc.) and how it changes with ν.

B. α^2-moment function $f_2(\beta,t,z_3^2)$

The next step is to check if the ξ dependence of the lattice data for $\mathcal{M}(\nu, \xi, t; z_3^2)$ agrees with the form

$$\mathcal{M}\left(\nu, \xi, t; z_3^2\right) = \mathcal{M}\left(\nu, \xi = 0, t; z_3^2\right) - \frac{\xi^2\nu^2}{2}\mathcal{M}_2\left(\nu, t; z_3^2\right) \\ + \mathcal{O}(\xi^4), \quad (5.2)$$

and extract $f_2(\beta, t, z_3^2)$ using

$$\mathcal{M}_2\left(\nu, \xi, t; z_3^2\right) = \int_{-1}^{1} d\beta e^{i\nu\beta} f_2\left(\beta, t; z_3^2\right). \quad (5.3)$$

The α dependence of the DD $F(\beta,\alpha)$ describes the distribution of the momentum transfer $r = P_1 - P_2$ between the initial and final quarks. It is expected that it has a shape similar to those of parton distribution amplitudes.

C. Factorized DD ansatz

A nonzero-width profile of DD in the α direction may be modeled by using the factorized DD ansatz [25,26],

$$F_N(\beta,\alpha,t) = f(\beta,t)\frac{\Gamma(N+\frac{3}{2})}{\sqrt{\pi}\Gamma(N+1)}\frac{[(1-|\beta|)^2-\alpha^2]^N}{(1-|\beta|)^{2N+1}}, \quad (5.4)$$

with N being the parameter governing the width of the α profile of the model DD $F_N(\beta,\alpha,t)$. The α integral of $F_N(\beta,\alpha,t)$ gives the nonforward parton density $f(\beta,t)$.

The $[(1-|\beta|)^2-\alpha^2]$ factor reflects the support properties of the DD, which vanishes if $|\beta|+|\alpha|>1$. The ansatz also complies with the requirement that $F(\beta,\alpha)$ should be an even function of α.

For $f(\beta,t)$ one can also take a factorized form $f(\beta,t)=f(\beta)F(t)$, where $f(\beta)$ is the forward PDF, and $F(t)$ is some form factor. Combining (3.7) and (5.4) gives

$$\mathcal{M}_N(\nu,\xi,t;z_3^2) = \int_{-1}^{1} d\beta e^{i\nu\beta} f\left(\beta,t;z_3^2\right) \times \int_{-1+|\beta|}^{1-|\beta|} d\alpha e^{i\nu\alpha\xi}\frac{\Gamma(N+\frac{3}{2})}{\sqrt{\pi}\Gamma(N+1)} \times \frac{[(1-|\beta|)^2-\alpha^2]^N}{(1-|\beta|)^{2N+1}}. \quad (5.5)$$

The integral over α can be taken as

$$\begin{aligned} A_N(\beta) &= \int_{-1+|\beta|}^{1-|\beta|} d\alpha e^{i\nu\alpha\xi}\frac{[(1-|\beta|)^2-\alpha^2]^N}{(1-|\beta|)^{2N+1}} \\ &= \int_{-1}^{1} d\eta e^{i\nu\xi(1-|\beta|)\eta}(1-\eta^2)^N \\ &= {}_0\tilde{F}_1\left(;N+\frac{3}{2};-\frac{\nu^2\xi^2(1-|\beta|)^2}{4}\right)\sqrt{\pi}\Gamma(N+1), \end{aligned} \quad (5.6)$$

where

$${}_0\tilde{F}_1(;b;z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(b+k)k!} \quad (5.7)$$

is a hypergeometric function.

So, we have a model for pseudo-GITD,

$$\begin{aligned} &\mathcal{M}_N(\nu,\xi,t;z_3^2) \\ &= \int_{-1}^{1} d\beta e^{i\nu\beta} f(\beta,t)\,{}_0\tilde{F}_1\left(;N+\frac{3}{2};-\frac{\nu^2\xi^2(1-|\beta|)^2}{4}\right) \\ &\times\Gamma\left(N+\frac{3}{2}\right), \end{aligned} \quad (5.8)$$

or, expanding in ξ,

$$\mathcal{M}_N(\nu,\xi,t;z_3^2) = \int_{-1}^{1} d\beta e^{i\nu\beta} f(\beta,t) \times\sum_{k=0}^{\infty}\left(-\frac{\nu^2\xi^2(1-|\beta|)^2}{4}\right)^k \times\frac{\Gamma(N+\frac{3}{2})}{k!\Gamma(N+3/2+k)}. \quad (5.9)$$

This expansion may be also obtained by taking the Taylor series of $e^{i\nu\xi(1-|\beta|)\eta}$ in Eq. (5.6) and integrating over η.

D. Check of polynomiality

Getting GPDs from a DD representation guarantees that the resulting GPD has the polynomiality property. Still, we can double check this. Note that the x^n moment $\mathcal{M}^{(n)}(\xi,t;z_3^2)$ of a pseudo-GPD $\mathcal{H}(x,\xi,t;z_3^2)$ is proportional to the coefficient accompanying ν^n in the Taylor expansion,

$$\mathcal{M}(\nu,\xi,t;z_3^2) = \sum_{N=0}^{\infty}\frac{i^n\nu^n}{n!}\mathcal{M}^{(n)}(\xi,t;z_3^2). \quad (5.10)$$

Now, from

$$\mathcal{M}_N(\nu,\xi,t;z_3^2) = \int_{-1}^{1} d\beta\sum_{m=0}^{\infty}\frac{(i\nu\beta)^m}{m!}f(\beta,t;z_3^2) \times\sum_{k=0}^{\infty}\left(-\frac{\nu^2\xi^2(1-|\beta|)^2}{4}\right)^k \times\frac{\Gamma(N+\frac{3}{2})}{k!\Gamma(N+3/2+k)}, \quad (5.11)$$

we see that "n" in $\mathcal{M}_N^{(n)}$ corresponds here to $n=m+2k$. On the other hand, $\mathcal{M}_N^{(n)}(\xi,t;z_3^2)$ is a polynomial in ξ of order $2k$, which is equal to or smaller than n since $m\geq 0$.

E. Fitting α-profile width

After fixing $f(\beta,t;z_3^2)$ that gives the profile of the DD in the β direction, we may quantify what kind of profile it has in the α direction. The presence of a nontrivial profile is indicated by the presence of ξ dependence. Using the first terms of the series for ${}_0\tilde{F}_1(;b;z)$,

$$\begin{aligned} \Gamma(b)\,{}_0\tilde{F}_1(;b;z) &= \sum_{k=0}^{\infty}\frac{z^k\Gamma(b)}{\Gamma(b+k)k!} \\ &= 1+\frac{z}{b}+\frac{z^2}{2b(b+1)}+\cdots, \end{aligned} \quad (5.12)$$

we write (5.9) as

$$\mathcal{M}(\nu,\xi,t;z_3^2;N) = \int_{-1}^{1} d\beta e^{i\nu\beta} f(\beta,t)\left\{1 - \frac{\nu^2\xi^2(1-|\beta|)^2}{4(N+3/2)} + \left(\frac{\nu^2\xi^2(1-|\beta|)^2}{4}\right)^2 \times \frac{1}{2(N+3/2)(N+5/2)} + \cdots\right\}. \quad (5.13)$$

In Eq. (5.13), ξ appears through the combination $\xi\nu = (\nu_1 - \nu_2)/2$. On the lattice, we have $\nu_1 = P_1 z_3$, $\nu_2 = P_2 z_3$. Hence, the presence of a nontrivial profile should be reflected by the dependence of the data on the difference $P_1 - P_2$ for a fixed sum $P_1 + P_2$. The first correction in Eq. (5.13) is given by

$$\delta\mathcal{M}(\nu,\xi,t;z_3^2;N) = -\int_{-1}^{1} d\beta e^{i\nu\beta} f(\beta,t;z_3^2)(1-|\beta|)^2 \times \frac{\xi^2\nu^2}{4(N+3/2)}. \quad (5.14)$$

Using this expression, one can try to determine the profile parameter N. This task probably will not be easy, since the correction looks rather small due to a small overall factor $\sim\xi^2/4$.

We may also estimate the extra suppression due to the $(1-|\beta|)^2$ factor in the integrand of (5.14). For a simple illustration, take $f(\beta,t) = 4(1-|\beta|)^3$. In this case,

$$\int_{-1}^{1} d\beta e^{i\nu\beta} f(\beta,t) = \frac{48}{\nu^4}\left(\cos(\nu) - 1 + \frac{\nu^2}{2}\right) = 2 - \frac{\nu^2}{15} + \frac{\nu^4}{840} + O(\nu^5), \quad (5.15)$$

while

$$\int_{-1}^{1} d\beta e^{i\nu\beta} f(\beta,t)(1-|\beta|)^2 = \frac{960}{\nu^6}\left(-\cos(\nu) + 1 - \frac{\nu^2}{2} + \frac{\nu^4}{24}\right) = \frac{4}{3} - \frac{\nu^2}{42} + \frac{\nu^4}{3780} + O(\nu^5). \quad (5.16)$$

Thus, the additional suppression is by about $2/3$ for small ν, i.e., not very strong.

F. *D*-term

When we take the z-odd part $\mathcal{O}^\lambda_-$ of the operator $\mathcal{O}^\lambda(z)$, its parametrization contains a nonzero D-term. In GPD description, it appears in a mixture with H_{DD} (and also E_{DD} in the nucleon case) GPDs. However, using all possible helicity states for nucleons and various values of λ, one can construct a sufficient number of linearly independent relations. To separate the DDs that appear in the parametrization of Eq. (4.6) one can use, e.g., the singular value decomposition technique. Unfortunately, as seen from Eq. (4.6), the D-term obtained in this way comes together with the Y contamination.

Another way is to eliminate H_{DD}, E_{DD}, etc. contributions from the matrix element of $\mathcal{O}^\lambda_-$ by taking kinematics in which $(\mathcal{P}z) = 0$. As a result, α-even DD $h(\beta,\alpha)$ will be integrated with the α-odd function $\sin(\alpha(rz))$, etc., so that we will have

$$\langle \mathcal{P} - r/2, s'|\mathcal{O}^\lambda_-(z)|\mathcal{P} + r/2, s\rangle|_{(\mathcal{P}z)=0} = r^\lambda \frac{(\bar{u}'u)}{M}\int_{-1}^{1} d\alpha e^{-i\alpha(rz)/2} D(\alpha,t) + (\bar{u}'u)Mz^\lambda \int_\Omega d\alpha d\beta z_1(\beta,\alpha,t)\cos(\alpha(rz)/2). \quad (5.17)$$

On the lattice, choosing $z = z_3$, we can arrange $(\mathcal{P}z) = 0$, i.e., $\mathcal{P}_3 = 0$, by taking p_1 and p_2 with opposite components in the z direction, namely, $p = (E_1, \mathbf{p}_{1T}, P)$ and $p' = (E_2, \mathbf{p}_{2T}, -P)$. Introducing the relevant Ioffe time $\nu_D \equiv -(rz)/2 \Rightarrow Pz_3$, we deal with the ITD

$$\mathcal{I}_D(\nu_D,t) = \int_{-1}^{1} d\alpha e^{i\alpha\nu_D} D(\alpha,t). \quad (5.18)$$

However, if we choose $\lambda = 0$, we get $r^0 = E_1 - E_2$ as the accompanying factor. It vanishes for purely longitudinal momenta $p = (E, \mathbf{0}_T, P)$, $p' = (E_2, \mathbf{0}_T, -P)$ and remains rather small when one takes nonequal transverse momenta $\mathbf{p}_{1T}, \mathbf{p}_{2T}$.

Another choice is to take $\lambda = 3$. In this case, we have $\sim z_3$ contamination,

$$\frac{1}{i}\langle (E_2, \mathbf{p}_{2T}, -P)|\mathcal{O}^3_-(z)|(E_1, \mathbf{p}_{1T}, P)\rangle = 2P\int_{-1}^{1} d\alpha \sin(\nu_D\alpha) D(\alpha,t) + z^{(3)} M^2 \int_{-1}^{1} d\alpha \cos(\nu_D\alpha) Z_1(\alpha,t), \quad (5.19)$$

where the "Z-term" function $Z_1(\alpha,t)$ is even in α. Multiplying by $z_\lambda = z_3$, we have

$$i\langle (E_2, \mathbf{p}_{2T}, -P)|z_\lambda \mathcal{O}^\lambda_-(z)|(E_1, \mathbf{p}_{1T}, P)\rangle = \nu_D \int_{-1}^{1} d\alpha \sin(\nu_D\alpha) D(\alpha,t) + \frac{\nu_D^2}{4P^2}\int_{-1}^{1} d\alpha \cos(\nu_D\alpha) Z(\alpha,t) = \nu_D \mathcal{I}_D(\nu_D,t) + \frac{\nu_D^2}{4P^2}\mathcal{I}_Z(\nu_D,t). \quad (5.20)$$

As we see, for a fixed ν, the contamination term decreases with P. In principle, one may try to extract $\mathcal{I}_D(\nu_D,t)$ by fitting the P dependence of the matrix element.

VI. SUMMARY

In the present paper, we have outlined the approach of lattice extraction of GPDs based on a combined use of the double distribution formalism and pseudo-PDF framework. The use of DDs guarantees that GPDs obtained from them have the required polynomiality property that imposes a nontrivial correlation between x and ξ dependences of GPDs.

We have introduced Ioffe-time distributions, writing these directly in terms of DDs, and generalized them onto correlators off the light cone. An important advantage of using DDs is that the D-term appears then as an independent quantity rather than a nonseparable part of GPDs H and E.

We have discussed the relation of the ξ dependence of GPDs with the width of the α profiles of the corresponding DDs and proposed strategies for fitting lattice-extracted pseudodistributions by DDs. The approach described in the present paper is already used in ongoing lattice extractions of GPDs by the HadStruc Collaboration.

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