Decipher the width of the $X(3872)$ via the QCD sum rules

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In this work, we take the $X(3872)$ as the hidden-charm tetraquark state with both isospin $I = 0$ and $I = 1$ components, then investigate the strong decays $X(3872) \to J/\psi\pi^+\pi^-$, $J/\psi\omega$, $\chi_{c1}\pi^0$, $D^{*0}\bar{D}^0$, and $D^0\bar{D}^0\pi^0$ with the QCD sum rules. We take account of all the Feynman diagrams, and acquire four QCD sum rules based on rigorous quark-hadron duality. We obtain the total decay width about 1 MeV, which is in excellent agreement with the experiment data $\Gamma_X = 1.19 \pm 0.21$ MeV from the PDG, it is the first time to reproduce the tiny width of the $X(3872)$ via the QCD sum rules, which supports assigning the $X(3872)$ as the hidden-charm tetraquark state with the $J^{PC} = 1^{++}$.

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I. INTRODUCTION

In 2003, the Belle Collaboration observed a narrow charmoniumlike state, the $X(3872)$, in the $\pi^+\pi^- J/\psi$ mass spectrum in the exclusive process $B^\pm \to K^\pm\pi^+\pi^- J/\psi$, which has a mass of $3872.0 \pm 0.6 \pm 0.5$ MeV and a width less than 2.3 MeV, the $X(3872)$ lies very near the $D^*\bar{D}$ threshold [1], which stimulated the interpretation in terms of the $D^*\bar{D}$ molecular state [2–10]. At the same time, other interpretations were suggested, such as the tetraquark state [11–16], hybrid state [17], charmonium-molecule mixing state [18–21], charmonium state [22], etc. In fact, the observations of the $X(3872)$ in the pp and $p\bar{p}$ collisions by the CDF, ATLAS, LHCb, and CMS Collaborations disfavor the pure molecule assignment [23–27].

In 2015, the LHCb Collaboration studied the angular correlations in the $B^+ \to X(3872)K^+$ decays with the subprocess $X(3872) \to \rho^0 J/\psi \to \pi^+\pi^-\mu^+\mu^-$ to measure orbital angular momentum contributions and to determine the J^{PC} of the $X(3872)$ to be 1^{++} [28]. The $X(3872)$ state is probably the best known (and most enigmatic) representative of the X, Y, and Z states. One important discriminant between different models is the width of the $X(3872)$. In 2020, the LHCb Collaboration updated the mass and width of the $X(3872)$, and obtained the Breit-Wigner width $\Gamma = 0.96^{+0.19}_{-0.18} \pm 0.21$ MeV [29], or $1.39 \pm 0.24 \pm 0.10$ MeV [30], which indicates nonzero width of the $X(3872)$ and leads to the average width $\Gamma_X = 1.19 \pm 0.21$ MeV listed in [31]. As known, we cannot assign a hadron with the mass alone, and we should study the decays to obtain more robust interpretation. Up to today, only the decays $X(3872) \to J/\psi\pi^+\pi^-$, $J/\psi\omega$, $J/\psi\gamma$, $\psi'\gamma$, $\chi_{c1}\pi^0$, $D^{*0}\bar{D}^0$, and $D^0\bar{D}^0\pi^0$ are established [31].

In the present work, we will focus on the scenario of tetraquark states. In Ref. [16], we take the pseudoscalar, scalar, axialvector, vector, and tensor (anti)diquarks as the basic constituents, and construct the scalar, axialvector, and tensor tetraquark currents to study the mass spectrum of the ground state hidden-charm tetraquark states with the QCD sum rules in a comprehensive way, and observe that the $X(3872)$ can be assigned to be the hidden-charm tetraquark state with the quantum numbers $J^{PC} = 1^{++}$. According to the recent combined data analysis, the decays $X(3872) \to J/\psi\rho \to J/\psi\pi^+\pi^-$ and $X(3872) \to J/\psi\omega \to J/\psi\pi^+\pi^-\pi^0$ have almost the same branching fractions [32], the isospin breaking effects in the decays are large enough and beyond the naive $\rho - \omega$ mixing. In this work, we introduce the isospin breaking effects explicitly and study the decays $X(3872) \to J/\psi\pi^+\pi^-$, $J/\psi\omega$, $\chi_{c1}\pi^0$, $D^{*0}\bar{D}^0$, and $D^0\bar{D}^0\pi^0$ with the QCD sum rules based on rigorous quark-hadron duality, and try to decipher the width of the $X(3872)$.

The article is arranged as follows. We obtain the QCD sum rules for the hadronic coupling constants in Sec. II. In Sec. III, we present numerical results and discussions and Sec. IV is reserved for our conclusion.

II. QCD SUM RULES FOR THE HADRONIC COUPLING CONSTANTS

We write down the three-point correlation functions $\Pi^{1/2}_{\mu\nu\alpha}(p,q)$ and $\Pi^{1/2}_{\mu\alpha}(p,q)$ in the QCD sum rules,

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$$\begin{aligned}
\Pi^{1}_{\mu\nu\alpha}(p,q) &= i^2\int d^4x d^4y e^{ip\cdot x}e^{iq\cdot y}\langle 0|T\{J^{J/\psi}_{\mu}(x)J^{\rho}_{\nu}(y)J^{X\dagger}_{\alpha}(0)\}|0\rangle,\\
\Pi^{2}_{\mu\nu\alpha}(p,q) &= i^2\int d^4x d^4y e^{ip\cdot x}e^{iq\cdot y}\langle 0|T\{J^{J/\psi}_{\mu}(x)J^{\omega}_{\nu}(y)J^{X\dagger}_{\alpha}(0)\}|0\rangle,\\
\Pi^{1}_{\mu\alpha}(p,q) &= i^2\int d^4x d^4y e^{ip\cdot x}e^{iq\cdot y}\langle 0|T\{J^{\chi}_{\mu}(x)J^{\pi}_{5}(y)J^{X\dagger}_{\alpha}(0)\}|0\rangle,\\
\Pi^{2}_{\mu\alpha}(p,q) &= i^2\int d^4x d^4y e^{ip\cdot x}e^{iq\cdot y}\langle 0|T\{J^{D^*}_{\mu}(x)J^{D}_{5}(y)J^{X\dagger}_{\alpha}(0)\}|0\rangle, \qquad (1)
\end{aligned}$$

where the currents

$$\begin{aligned}
J^{J/\psi}_{\mu}(x) &= \bar{c}(x)\gamma_\mu c(x),\\
J^{\chi}_{\mu}(x) &= \bar{c}(x)\gamma_\mu\gamma_5 c(x),\\
J^{D^*}_{\mu}(x) &= \bar{u}(x)\gamma_\mu c(x), \qquad (2)
\end{aligned}$$

$$\begin{aligned}
J^{\rho}_{\nu}(y) &= \frac{1}{\sqrt{2}}[\bar{u}(y)\gamma_\nu u(y)-\bar{d}(y)\gamma_\nu d(y)],\\
J^{\omega}_{\nu}(y) &= \frac{1}{\sqrt{2}}[\bar{u}(y)\gamma_\nu u(y)+\bar{d}(y)\gamma_\nu d(y)],\\
J^{\pi}_{5}(y) &= \frac{1}{\sqrt{2}}[\bar{u}(y)i\gamma_5 u(y)-\bar{d}(y)i\gamma_5 d(y)],\\
J^{D}_{5}(y) &= \bar{c}(y)i\gamma_5 u(y), \qquad (3)
\end{aligned}$$

$$J^{X}_{\alpha}(0) = \cos\theta J^{u\bar{u}}_{\alpha}(0)+\sin\theta J^{d\bar{d}}_{\alpha}(0), \qquad (4)$$

$$\begin{aligned}
J^{u\bar{u}}_{\alpha}(0) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\Big[u^T_j(0)C\gamma_5 c_k(0)\bar{u}_m(0)\gamma_\alpha C\bar{c}^T_n(0)\\
&\quad - u^T_j(0)C\gamma_\alpha c_k(0)\bar{u}_m(0)\gamma_5 C\bar{c}^T_n(0)\Big],\\
J^{d\bar{d}}_{\mu}(0) &= \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}}\Big[d^T_j(0)C\gamma_5 c_k(0)\bar{d}_m(0)\gamma_\alpha C\bar{c}^T_n(0)\\
&\quad - d^T_j(0)C\gamma_\alpha c_k(0)\bar{d}_m(0)\gamma_5 C\bar{c}^T_n(0)\Big], \qquad (5)
\end{aligned}$$

interpolate the mesons J/ψ, χ_{c1}, D^*, ρ, ω, π, D, and $X(3872)$, respectively. As the decays $X(3872)\to J/\psi\rho\to J/\psi\pi^+\pi^-$ and $X(3872)\to J/\psi\omega\to J/\psi\pi^+\pi^-\pi^0$ have almost the same branching fractions [32], which is beyond the naive expectation of the $\rho-\omega$ mixing, we have to introduce mixing effects in the $X(3872)$ and abandon the obsession that the $X(3872)$ has definite isospin $I=0$. We determine the mixing angle θ by the experimental data via trial and error.

At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the currents into the three-point correlation functions, and isolate the ground state contributions explicitly,

$$\begin{aligned}
\Pi^{1}_{\mu\nu\alpha}(p,q) &= \frac{\lambda_X f_{J/\psi}m_{J/\psi}f_\rho m_\rho G'_{XJ/\psi\rho}}{(m_X^2-p'^2)(m_{J/\psi}^2-p^2)(m_\rho^2-q^2)}i\varepsilon_{\mu\nu\alpha\sigma}q^\sigma p\cdot q+\cdots,\\
&= \Pi_\rho(p'^2,p^2,q^2)i\varepsilon_{\mu\nu\alpha\sigma}q^\sigma p\cdot q+\cdots, \qquad (6)
\end{aligned}$$

$$\begin{aligned}
\Pi^{2}_{\mu\nu\alpha}(p,q) &= \frac{\lambda_X f_{J/\psi}m_{J/\psi}f_\omega m_\omega G'_{XJ/\psi\omega}}{(m_X^2-p'^2)(m_{J/\psi}^2-p^2)(m_\omega^2-q^2)}i\varepsilon_{\mu\nu\alpha\sigma}q^\sigma p\cdot q+\cdots,\\
&= \Pi_\omega(p'^2,p^2,q^2)i\varepsilon_{\mu\nu\alpha\sigma}q^\sigma p\cdot q+\cdots, \qquad (7)
\end{aligned}$$

$$\begin{aligned}
\Pi^{1}_{\mu\alpha}(p,q) &= \frac{\lambda_X f_{\chi_{c1}}m_{\chi_{c1}}\mu_\pi G'_{X\chi\pi}}{(m_X^2-p'^2)(m_{\chi_{c1}}^2-p^2)(m_\pi^2-q^2)}\varepsilon_{\alpha\mu\nu\sigma}p^\nu q^\sigma+\cdots,\\
&= \Pi_\pi(p'^2,p^2,q^2)\varepsilon_{\alpha\mu\nu\sigma}p^\nu q^\sigma+\cdots, \qquad (8)
\end{aligned}$$

$$\begin{aligned}
\Pi^{2}_{\mu\alpha}(p,q) &= \frac{\lambda_X f_{D^*}m_{D^*}\mu_D G'_{XD^*D}}{(m_X^2-p'^2)(m_{D^*}^2-p^2)(m_D^2-q^2)}ip\cdot q g_{\mu\alpha}+\cdots,\\
&= \Pi_D(p'^2,p^2,q^2)ip\cdot q g_{\mu\alpha}+\cdots, \qquad (9)
\end{aligned}$$

where $p' = p + q$, $\mu_\pi = \frac{f_\pi m_\pi^2}{m_u + m_d}$, $\mu_D = \frac{f_D m_D^2}{m_c}$, the hadronic coupling constants $G'_{XJ/\psi\rho}$, $G'_{XJ/\psi\omega}$, $G'_{X\chi\pi}$, and G'_{XD^*D} are defined by

$$\begin{aligned}
\langle J/\psi(p)\rho(q)|X(p')\rangle &= -\varepsilon^{\sigma\alpha\mu\nu} p'_\sigma \zeta_\alpha \xi^*_\mu \varsigma^*_\nu p\cdot q G'_{XJ/\psi\rho},\\
\langle J/\psi(p)\omega(q)|X(p')\rangle &= -\varepsilon^{\sigma\alpha\mu\nu} p'_\sigma \zeta_\alpha \xi^*_\mu \varsigma^*_\nu p\cdot q G'_{XJ/\psi\omega},\\
\langle \chi_{c1}(p)\pi(q)|X(p')\rangle &= i\varepsilon^{\sigma\alpha\mu\nu} p'_\sigma \zeta_\alpha p_\mu \xi^*_\nu G'_{X\chi\pi},\\
\langle D^*(p)D(q)|X(p')\rangle &= -\zeta\cdot\xi^* p\cdot q G'_{XD^*D}, \qquad (10)
\end{aligned}$$

the decay constants λ_X, $f_{J/\psi}$, $f_{\chi_{c1}}$, f_ρ, f_ω, f_{D^*}, f_D, and f_π, are defined by

$$\langle 0|J^X_\mu(0)|X(p')\rangle = \lambda_X \zeta_\mu, \qquad (11)$$

$$\begin{aligned}
\langle 0|J^{J/\psi}_\mu(0)|J/\psi(p)\rangle &= f_{J/\psi} m_{J/\psi} \xi_\mu,\\
\langle 0|J^{\chi}_\mu(0)|\chi_{c1}(p)\rangle &= f_{\chi_{c1}} m_{\chi_{c1}} \xi_\mu,\\
\langle 0|J^{D^*}_\mu(0)|D^*(p)\rangle &= f_{D^*} m_{D^*} \xi_\mu, \qquad (12)
\end{aligned}$$

$$\begin{aligned}
\langle 0|J^{\rho}_\mu(0)|\rho(q)\rangle &= f_\rho m_\rho \varsigma_\mu,\\
\langle 0|J^{\omega}_\mu(0)|\omega(q)\rangle &= f_\omega m_\omega \varsigma_\mu, \qquad (13)
\end{aligned}$$

$$\begin{aligned}
\langle 0|J^{D}_5(0)|D(q)\rangle &= \frac{f_D m_D^2}{m_c},\\
\langle 0|J^{\pi}_5(0)|\pi(q)\rangle &= \frac{f_\pi m_\pi^2}{m_u + m_d}, \qquad (14)
\end{aligned}$$

the ζ_μ, ξ_μ and ς_μ are polarization vectors of the axialvector or vector mesons.

We choose the components $\Pi_\rho(p'^2, p^2, q^2)$, $\Pi_\omega(p'^2, p^2, q^2)$, $\Pi_\pi(p'^2, p^2, q^2)$, and $\Pi_D(p'^2, p^2, q^2)$ to study the hadronic coupling constants $G'_{XJ/\psi\rho}$, $G'_{XJ/\psi\omega}$, $G'_{X\chi\pi}$, and G'_{XD^*D}, respectively. In Ref. [14], the tensor structures $p_\mu \varepsilon_{\nu\alpha\sigma\tau} p^\sigma q^\tau$ and $\varepsilon_{\mu\nu\alpha\sigma} q^\sigma$, which differ from the structure $\varepsilon_{\mu\nu\alpha\sigma} q^\sigma p\cdot q$ in Eqs. (6) and (7) greatly, are chosen to study the hadronic coupling constants $G_{XJ/\psi\rho}$ and $G_{XJ/\psi\omega}$. The $G_{XJ/\psi\rho}$ and $G_{XJ/\psi\omega}$ defined in Ref. [14] (also Ref. [33]) and the $G'_{XJ/\psi\rho}$ and $G'_{XJ/\psi\omega}$ defined in this work have the relations, $G_{XJ/\psi\rho} = p\cdot q G'_{XJ/\psi\rho}$ and $G_{XJ/\psi\omega} = p\cdot q G'_{XJ/\psi\omega}$, although the two definitions are both reasonable, they lead to quite different QCD sum rules. It is not odd that different predictions may be obtained. Then, we acquire the hadronic spectral densities $\rho_H(s', s, u)$ through triple dispersion relation,

$$\Pi_H(p'^2, p^2, q^2) = \int_{\Delta_s'^2}^\infty ds' \int_{\Delta_s^2}^\infty ds \int_{\Delta_u^2}^\infty du \frac{\rho_H(s', s, u)}{(s' - p'^2)(s - p^2)(u - q^2)}, \qquad (15)$$

where the $\Delta_s'^2$, Δ_s^2, and Δ_u^2 are thresholds, we add the subscript H to stand for the components $\Pi_\rho(p'^2, p^2, q^2)$, $\Pi_\omega(p'^2, p^2, q^2)$, $\Pi_\pi(p'^2, p^2, q^2)$, and $\Pi_D(p'^2, p^2, q^2)$ at the hadron side.

As far as the operator product expansion is concerned, we calculate the vacuum condensates up to dimension 5, and obtain the QCD spectral densities through the double-dispersion relation,

$$\Pi_{\rm QCD}(p'^2, p^2, q^2) = \int_{\Delta_s^2}^\infty ds \int_{\Delta_u^2}^\infty du \frac{\rho_{\rm QCD}(p'^2, s, u)}{(s - p^2)(u - q^2)}, \qquad (16)$$

as

$$\lim_{\epsilon\to 0} \frac{{\rm Im}\Pi_{\rm QCD}(s' + i\epsilon, p^2, q^2)}{\pi} = 0. \qquad (17)$$

In calculations, we neglect the gluon condensates due to their tiny contributions [34,35]. We accomplish the integral over ds' firstly at the hadron side, then match the hadron side with the QCD side below the continuum thresholds s_0 and u_0 to obtain rigorous quark-hadron duality [34,35],

$$\begin{aligned}
&\int_{\Delta_s^2}^{s_0} ds \int_{\Delta_u^2}^{u_0} du \left[\int_{\Delta_s'^2}^\infty ds' \frac{\rho_H(s', s, u)}{(s' - p'^2)(s - p^2)(u - q^2)}\right]\\
&= \int_{\Delta_s^2}^{s_0} ds \int_{\Delta_u^2}^{u_0} du \frac{\rho_{\rm QCD}(s, u)}{(s - p^2)(u - q^2)}. \qquad (18)
\end{aligned}$$

In the following, we write down the hadron representation explicitly,

$$\begin{aligned}
\Pi_\rho(p'^2, p^2, q^2) &= \frac{\lambda_X f_{J/\psi} m_{J/\psi} f_\rho m_\rho G'_{XJ/\psi\rho}}{(m_X^2 - p'^2)(m_{J/\psi}^2 - p^2)(m_\rho^2 - q^2)} + \frac{C'_\rho}{(m_{J/\psi}^2 - p^2)(m_\rho^2 - q^2)},\\
\Pi_\omega(p'^2, p^2, q^2) &= \frac{\lambda_X f_{J/\psi} m_{J/\psi} f_\omega m_\omega G'_{XJ/\psi\omega}}{(m_X^2 - p'^2)(m_{J/\psi}^2 - p^2)(m_\omega^2 - q^2)} + \frac{C'_\omega}{(m_{J/\psi}^2 - p^2)(m_\omega^2 - q^2)},\\
\Pi_\pi(p'^2, p^2, q^2) &= \frac{\lambda_X f_{\chi_{c1}} m_{\chi_{c1}} \mu_\pi G'_{X\chi\pi}}{(m_X^2 - p'^2)(m_{\chi_{c1}}^2 - p^2)(m_\pi^2 - q^2)} + \frac{C'_\pi}{(m_{\chi_{c1}}^2 - p^2)(m_\pi^2 - q^2)},\\
\Pi_D(p'^2, p^2, q^2) &= \frac{\lambda_X f_{D^*} m_{D^*} \mu_D G'_{XD^*D}}{(m_X^2 - p'^2)(m_{D^*}^2 - p^2)(m_D^2 - q^2)} + \frac{C'_D}{(m_{D^*}^2 - p^2)(m_D^2 - q^2)}, \qquad (19)
\end{aligned}$$

where we introduce the parameters $C'_{\rho/\omega/\pi/D}$ to stand for all the contributions concerning the higher resonances in the s' channel,

$$\begin{aligned} C'_\rho &= \int_{s'_0}^\infty ds' \frac{\tilde{\rho}_\rho(s', m_{J/\psi}^2, m_\rho^2)}{s'-p'^2}, \\ C'_\omega &= \int_{s'_0}^\infty ds' \frac{\tilde{\rho}_\omega(s', m_{J/\psi}^2, m_\omega^2)}{s'-p'^2}, \\ C'_\pi &= \int_{s'_0}^\infty ds' \frac{\tilde{\rho}_\pi(s', m_{\chi_{c1}}^2, m_\pi^2)}{s'-p'^2}, \\ C'_D &= \int_{s'_0}^\infty ds' \frac{\tilde{\rho}_D(s', m_{D^*}^2, m_D^2)}{s'-p'^2}, \end{aligned} \tag{20}$$

where the densities $\rho_H(s', s, u) = \tilde{\rho}_\rho(s', s, u)\delta(s - m_{J/\psi}^2) \times \delta(u - m_\rho^2)$, $\tilde{\rho}_\omega(s', s, u)\delta(s - m_{J/\psi}^2)\delta(u - m_\omega^2)$, $\tilde{\rho}_\pi(s', s, u) \times \delta(s - m_{\chi_{c1}}^2)\delta(u - m_\pi^2)$, and $\tilde{\rho}_D(s', s, u)\delta(s - m_{D^*}^2)\delta(u - m_D^2)$, respectively. The densities $\tilde{\rho}_\rho(s', m_{J/\psi}^2, m_\rho^2)$, $\tilde{\rho}_\omega(s', m_{J/\psi}^2, m_\omega^2)$, $\tilde{\rho}_\pi(s', m_{\chi_{c1}}^2, m_\pi^2)$, and $\tilde{\rho}_D(s', m_{D^*}^2, m_D^2)$ are complex and we have no knowledge about the higher-resonant states, as the spectrum is vague. We take the unknown functions $C'_{\rho/\omega/\pi/D}$ as free parameters and adjust the suitable values to obtain flat Borel platforms for the hadronic coupling constants $G'_{XJ/\psi\rho}$, $G'_{XJ/\psi\omega}$, $G'_{X\chi\pi}$, and G'_{XD^*D}, respectively [34,35].

In Ref. [14] (also Ref. [33]), Navarra and Nielsen approximate the hadron side of the correlation functions as

$$\begin{aligned} \Pi_\rho(p'^2, p^2, q^2) &= \frac{\lambda_X f_{J/\psi} m_{J/\psi} f_\rho m_\rho G_{XJ/\psi\rho}}{(m_X^2 - p'^2)(m_{J/\psi}^2 - p^2)(m_\rho^2 - q^2)} + \frac{B_\rho}{(s'_0 - p'^2)(m_\rho^2 - q^2)}, \\ \Pi_\omega(p'^2, p^2, q^2) &= \frac{\lambda_X f_{J/\psi} m_{J/\psi} f_\omega m_\omega G_{XJ/\psi\omega}}{(m_X^2 - p'^2)(m_{J/\psi}^2 - p^2)(m_\omega^2 - q^2)} + \frac{B_\omega}{(s'_0 - p'^2)(m_\omega^2 - q^2)}, \end{aligned} \tag{21}$$

then only match them with the QCD side below the continuum threshold s_0, where the $B_{\rho/\omega}$ stand for the pole-continuum transitions, and we have changed their notations (symbols) into the present form for convenience. Although Navarra and Nielsen take account of the continuum contributions by introducing a parameter s'_0 in the s' channel phenomenologically, they neglect the continuum contributions in the u channel at the hadron side by hand. It is the shortcoming of that work. While in this work, we match the hadron side with the QCD side below the continuum thresholds, s_0 and u_0, to obtain rigorous quark-hadron duality, and we take account of the continuum contributions in the s' channel.

We set $p'^2 = p^2$ in the correlation functions $\Pi_H(p'^2, p^2, q^2)$, and perform double Borel transform in regard to $P^2 = -p^2$ and $Q^2 = -q^2$, respectively, then we set $T_1^2 = T_2^2 = T^2$ to obtain four QCD sum rules,

$$\begin{aligned} &\frac{\lambda_{XJ/\psi\rho} G'_{XJ/\psi\rho}}{m_X^2 - m_{J/\psi}^2} \left[\exp\left(-\frac{m_{J/\psi}^2}{T^2}\right) - \exp\left(-\frac{m_X^2}{T^2}\right)\right] \exp\left(-\frac{m_\rho^2}{T^2}\right) + C'_\rho \exp\left(-\frac{m_{J/\psi}^2 + m_\rho^2}{T^2}\right) \\ &= \frac{\cos\theta - \sin\theta}{\sqrt{2}} \frac{m_c}{16\sqrt{2}\pi^4} \int_{4m_c^2}^{s_{J/\psi}^0} ds \int_0^{s_\rho^0} du \sqrt{1 - \frac{4m_c^2}{s}} \exp\left(-\frac{s+u}{T^2}\right), \end{aligned} \tag{22}$$

$$\begin{aligned} &\frac{\lambda_{XJ/\psi\omega} G'_{XJ/\psi\omega}}{m_X^2 - m_{J/\psi}^2} \left[\exp\left(-\frac{m_{J/\psi}^2}{T^2}\right) - \exp\left(-\frac{m_X^2}{T^2}\right)\right] \exp\left(-\frac{m_\omega^2}{T^2}\right) + C'_\omega \exp\left(-\frac{m_{J/\psi}^2 + m_\omega^2}{T^2}\right) \\ &= \frac{\cos\theta + \sin\theta}{\sqrt{2}} \frac{m_c}{16\sqrt{2}\pi^4} \int_{4m_c^2}^{s_{J/\psi}^0} ds \int_0^{s_\omega^0} du \sqrt{1 - \frac{4m_c^2}{s}} \exp\left(-\frac{s+u}{T^2}\right), \end{aligned} \tag{23}$$

$$\begin{aligned} &\frac{\lambda_{X\chi\pi} G'_{X\chi\pi}}{m_X^2 - m_{\chi_{c1}}^2} \left[\exp\left(-\frac{m_{\chi_{c1}}^2}{T^2}\right) - \exp\left(-\frac{m_X^2}{T^2}\right)\right] \exp\left(-\frac{m_\rho^2}{T^2}\right) + C'_\pi \exp\left(-\frac{m_{\chi_{c1}}^2 + m_\pi^2}{T^2}\right) \\ &= \frac{\cos\theta - \sin\theta}{\sqrt{2}} \frac{m_c \langle\bar{q} g_s \sigma G q\rangle}{16\sqrt{2}\pi^4} \int_{4m_c^2}^{s_{\chi_{c1}}^0} ds \frac{s - 3m_c^2}{s\sqrt{s(s - 4m_c^2)}} \exp\left(-\frac{s}{T^2}\right), \end{aligned} \tag{24}$$

$$
\begin{aligned}
&\frac{\lambda_{XD^*D}G'_{XD^*D}}{m_X^2-m_{D^*}^2}\left[\exp\left(-\frac{m_{D^*}^2}{T^2}\right)-\exp\left(-\frac{m_X^2}{T^2}\right)\right]\exp\left(-\frac{m_D^2}{T^2}\right)+C'_D\exp\left(-\frac{m_{D^*}^2+m_D^2}{T^2}\right)\\
&=\cos\theta\frac{3m_c^2}{64\sqrt{2}\pi^4}\int_{m_c^2}^{s_{D^*}^0}ds\int_{m_c^2}^{s_D^0}du\left(1-\frac{m_c^2}{s}\right)^2\left(1-\frac{m_c^2}{u}\right)^2\exp\left(-\frac{s+u}{T^2}\right)\\
&\quad-\cos\theta\frac{m_c\langle\bar{q}q\rangle}{8\sqrt{2}\pi^2}\int_{m_c^2}^{s_{D^*}^0}ds\left(1-\frac{m_c^2}{s}\right)^2\exp\left(-\frac{s+m_c^2}{T^2}\right)\\
&\quad-\cos\theta\frac{m_c\langle\bar{q}q\rangle}{8\sqrt{2}\pi^2}\int_{m_c^2}^{s_D^0}du\left(1-\frac{m_c^2}{u}\right)^2\exp\left(-\frac{u+m_c^2}{T^2}\right)\\
&\quad+\cos\theta\frac{m_c\langle\bar{q}g_s\sigma Gq\rangle}{32\sqrt{2}\pi^2T^2}\left(1+\frac{m_c^2}{T^2}\right)\int_{m_c^2}^{s_{D^*}^0}ds\left(1-\frac{m_c^2}{s}\right)^2\exp\left(-\frac{s+m_c^2}{T^2}\right)\\
&\quad+\cos\theta\frac{m_c\langle\bar{q}g_s\sigma Gq\rangle}{96\sqrt{2}\pi^2T^2}\left(1+\frac{3m_c^2}{T^2}\right)\int_{m_c^2}^{s_D^0}du\left(1-\frac{m_c^2}{u}\right)^2\exp\left(-\frac{u+m_c^2}{T^2}\right)\\
&\quad-\cos\theta\frac{m_c\langle\bar{q}g_s\sigma Gq\rangle}{32\sqrt{2}\pi^2}\int_{m_c^2}^{s_{D^*}^0}ds\frac{1}{s}\left(1-\frac{m_c^2}{s}\right)\exp\left(-\frac{s+m_c^2}{T^2}\right)\\
&\quad-\cos\theta\frac{m_c^3\langle\bar{q}g_s\sigma Gq\rangle}{96\sqrt{2}\pi^2}\int_{m_c^2}^{s_{D^*}^0}ds\frac{1}{s^2}\exp\left(-\frac{s+m_c^2}{T^2}\right)\\
&\quad-\cos\theta\frac{m_c\langle\bar{q}g_s\sigma Gq\rangle}{32\sqrt{2}\pi^2}\int_{m_c^2}^{s_D^0}du\frac{1}{u}\left(1-\frac{m_c^2}{u}\right)^2\exp\left(-\frac{u+m_c^2}{T^2}\right)\\
&\quad+\cos\theta\frac{m_c^3\langle\bar{q}g_s\sigma Gq\rangle}{96\sqrt{2}\pi^2}\int_{m_c^2}^{s_D^0}du\frac{1}{u^2}\exp\left(-\frac{u+m_c^2}{T^2}\right),
\end{aligned}
\tag{25}
$$

where $\lambda_{XJ/\psi\rho}=\lambda_X f_{J/\psi}m_{J/\psi}f_\rho m_\rho$, $\lambda_{XJ/\psi\omega}=\lambda_X f_{J/\psi}m_{J/\psi}f_\omega m_\omega$, $\lambda_{X\chi\pi}=\lambda_X f_{\chi_{c1}}m_{\chi_{c1}}\mu_\pi$, and $\lambda_{XD^*D}=\lambda_X f_{D^*}m_{D^*}\mu_D$.

In Ref. [14] (also Ref. [33]), Navarra and Nielsen set $p'^2=p^2$ in the correlation functions $\Pi_H(p'^2,p^2,q^2)$, perform single Borel transform in regard to $P^2=-p^2$, and take the $Q^2=-q^2$ as a free parameter to parametrize the off shellness of the hadronic coupling constants $G_{XJ/\psi\rho}$ and $G_{XJ/\psi\omega}$, which are fitted into some functions of Q^2, then extract them to the physical points $q^2=m_{\rho/\omega}^2$, and finally too large partial decay widths are obtained. The schemes are quite different, we should not be surprised that the predictions in Ref. [14] and in this work are also quite different.

In calculations, we factorize out the mixing angle θ in Eqs. (22)–(25) so as to facilitate determining the mixing effects, and redefine the hadronic coupling constants G and free parameters C,

$$
\begin{aligned}
G'_{XJ/\psi\rho}&=G_{XJ/\psi\rho}\frac{\cos\theta-\sin\theta}{\sqrt{2}},\\
G'_{XJ/\psi\omega}&=G_{XJ/\psi\omega}\frac{\cos\theta+\sin\theta}{\sqrt{2}},\\
G'_{X\chi\pi}&=G_{X\chi\pi}\frac{\cos\theta-\sin\theta}{\sqrt{2}},\\
G'_{XD^*D}&=G_{XD^*D}\cos\theta,
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
C'_\rho&=C_\rho\frac{\cos\theta-\sin\theta}{\sqrt{2}},\\
C'_\omega&=C_\omega\frac{\cos\theta+\sin\theta}{\sqrt{2}},\\
C'_\pi&=C_\pi\frac{\cos\theta-\sin\theta}{\sqrt{2}},\\
C'_D&=C_D\cos\theta,
\end{aligned}
\tag{27}
$$

then it is easy to study the dependence on the mixing angle θ.

III. NUMERICAL RESULTS AND DISCUSSIONS

We take the conventional vacuum condensates, $\langle\bar{q}q\rangle=-(0.24\pm0.01\ \text{GeV})^3$, $\langle\bar{q}g_s\sigma Gq\rangle=m_0^2\langle\bar{q}q\rangle$, and $m_0^2=(0.8\pm0.1)\ \text{GeV}^2$ at the energy scale $\mu=1$ GeV [36–38], and take the $\overline{\text{MS}}$ mass $m_c(m_c)=(1.275\pm0.025)$ GeV from the PDG [31]. We set $m_u=m_d=0$ and take account of the energy-scale dependence from renormalization group equation,

$$\langle\bar{q}q\rangle(\mu)=\langle\bar{q}q\rangle(1\text{ GeV})\left[\frac{\alpha_s(1\text{ GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}},$$

$$\langle\bar{q}g_s\sigma Gq\rangle(\mu)=\langle\bar{q}g_s\sigma Gq\rangle(1\text{ GeV})\left[\frac{\alpha_s(1\text{ GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}},$$

$$m_c(\mu)=m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}},$$

$$\alpha_s(\mu)=\frac{1}{b_0t}\left[1-\frac{b_1}{b_0^2}\frac{\log t}{t}+\frac{b_1^2(\log^2t-\log t-1)+b_0b_2}{b_0^4t^2}\right], \tag{28}$$

where $t=\log\frac{\mu^2}{\Lambda^2_{\rm QCD}}$, $b_0=\frac{33-2n_f}{12\pi}$, $b_1=\frac{153-19n_f}{24\pi^2}$, $b_2=\frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda_{\rm QCD}=210$, 292, and 332 MeV for the flavors $n_f=5$, 4, and 3, respectively [31,39], and we choose $n_f=4$, and evolve all the input parameters to the energy scale $\mu=1$ GeV.

At the hadron side, we take $m_{\pi^\pm}=0.13957$ GeV, $m_{\pi^0}=0.13498$ GeV, $m_{J/\psi}=3.0969$ GeV, $m_{\chi_{c1}}=3.51067$ GeV, $m_\rho=0.77526$ GeV, $m_\omega=0.78266$ GeV, $f_\pi=0.130$ GeV from the PDG [31], $m_{D^*}=2.01$ GeV, $m_D=1.87$ GeV, $f_{D^*}=263$ MeV, $f_D=208$ MeV, $s^0_{D^*}=6.4$ GeV^2, $s^0_D=6.2$ GeV^2 [40], $f_{J/\psi}=0.418$ GeV [41], $f_{\chi_{c1}}=0.338$ GeV [42], $f_\rho=0.215$ GeV, $f_\omega=f_\rho$, $\sqrt{s^0_\rho}=1.2$ GeV, $s^0_\omega=s^0_\rho$ [43], $m_X=3.91$ GeV, $\lambda_X=2.10\times10^{-2}$ GeV^5 [16] from the QCD sum rules, and $f_\pi m_\pi^2/(m_u+m_d)=-2\langle\bar{q}q\rangle/f_\pi$ from the Gell-Mann-Oakes-Renner relation.

In calculations, we fit the free parameters as $C_\rho=0.000250(T^2-1.5\text{ GeV}^2)$ GeV^3, $C_\omega=0.000245(T^2-1.5\text{ GeV}^2)$ GeV^3, $C_\pi=0$, and $C_D=0.0000725(T^2-2.1\text{ GeV}^2)$ GeV^4 to obtain the Borel windows $T^2_\rho=(2.3-3.3)$ GeV^2, $T^2_\omega=(2.3-3.3)$ GeV^2, $T^2_\pi=(3.6$–$4.6)$ GeV^2, and $T^2_D=(4.0-5.0)$ GeV^2, where the subscripts ρ, ω, π, and D denote the corresponding channels. We obtain uniform enough flat platforms $T^2_{\rm max}-T^2_{\rm min}=1$ GeV^2, where max and min denote the maximum and minimum, respectively. In Fig. 1, we plot the hadronic coupling constants $G_{XJ/\psi\rho}$, $G_{XJ/\psi\omega}$, $G_{X\chi\pi}$, and G_{XD^*D} with variations of the Borel parameters at large intervals. In the Borel windows, very flat platforms appear; indeed, it is reliable to extract the hadron coupling constants.

Now, we estimate the uncertainties in the following ways. For example, the uncertainties of an input parameter ξ, $\xi=\bar{\xi}+\delta\xi$, result in the uncertainties $\lambda_Xf_{J/\psi}f_\rho G_{XJ/\psi\rho}=\bar{\lambda}_X\bar{f}_{J/\psi}\bar{f}_\rho\bar{G}_{XJ/\psi\rho}+\delta\lambda_Xf_{J/\psi}f_\rho G_{XJ/\psi\rho}$, $C_\rho=\bar{C}_\rho+\delta C_\rho$,

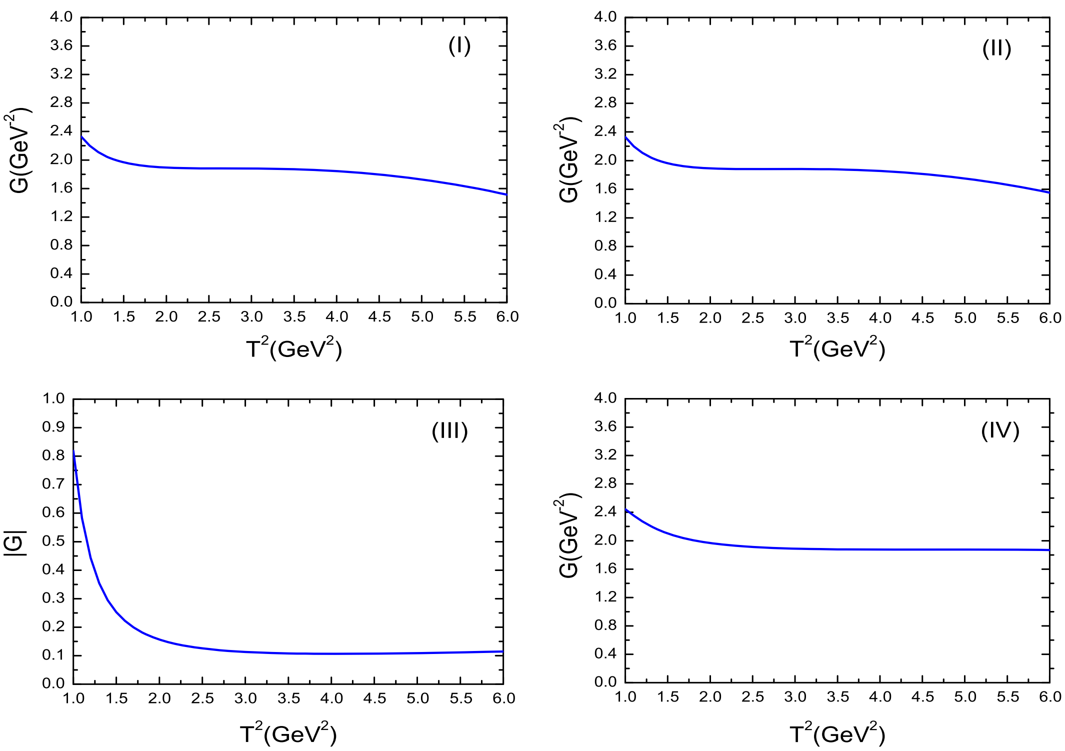

FIG. 1. The central values of the hadronic coupling constants with variations of the Borel parameters T^2, where the (I), (II), (III), and (IV) denote the $G_{XJ/\psi\rho}$, $G_{XJ/\psi\omega}$, $G_{X\chi\pi}$, and G_{XD^*D}, respectively.

$$\delta\lambda_X f_{J/\psi} f_\rho G_{XJ/\psi\rho} = \bar{\lambda}_X \bar{f}_{J/\psi} \bar{f}_\rho \bar{G}_{XJ/\psi\rho} \left(\frac{\delta f_{J/\psi}}{\bar{f}_{J/\psi}} + \frac{\delta f_\rho}{\bar{f}_\rho} + \frac{\delta\lambda_X}{\bar{\lambda}_X} + \frac{\delta G_{XJ/\psi\rho}}{\bar{G}_{XJ/\psi\rho}} \right), \quad (29)$$

we can set $\delta C_\rho = 0$ and $\frac{\delta f_{J/\psi}}{\bar{f}_{J/\psi}} = \frac{\delta f_\rho}{\bar{f}_\rho} = \frac{\delta\lambda_X}{\bar{\lambda}_X} = \frac{\delta G_{XJ/\psi\rho}}{\bar{G}_{XJ/\psi\rho}}$ approximately.

Finally, we obtain the values of the hadronic coupling constants,

$$\begin{aligned} G_{XJ/\psi\rho} &= 1.88^{+0.11}_{-0.10}\ \text{GeV}^{-2}, \\ G_{XJ/\psi\omega} &= 1.88^{+0.11}_{-0.10}\ \text{GeV}^{-2}, \\ |G_{X\chi\pi}| &= 0.11, \\ G_{XD^*D} &= 1.875^{+0.064}_{-0.064}\ \text{GeV}^{-2}, \end{aligned} \quad (30)$$

where we only present the central value of the $G_{X\chi\pi}$ due to the tiny partial decay width of the $X(3872) \to \chi_{c1}\pi^0$.

Now we take the hadron masses $m_X = 3.87165$ GeV, $m_{D^{*0}} = 2.00685$ GeV, $m_{D^0} = 1.86484$ GeV and $m_{J/\psi} = 3.09690$ GeV from the PDG to calculate the partial decay widths [31]. As the $X(3872)$ lies near the thresholds of the final states $J/\psi\rho$, $J/\psi\omega$, and $D^*\bar{D}$, we should take account of the finite width effects of the ρ, ω, and D^* mesons, due to the decay cascades,

$$\begin{aligned} &X(3872) \to J/\psi\rho^0 \to J/\psi\pi^+\pi^-, \\ &X(3872) \to J/\psi\omega \to J/\psi\pi^+\pi^-\pi^0, \\ &X(3872) \to D^{*0}\bar{D}^0 \to D^0\bar{D}^0\pi^0. \end{aligned} \quad (31)$$

Then we obtain the partial decay widths via trial and error, as there is an additional parameter θ, $\theta = 0.12\pi = 21.6°$. The partial widths are listed in the following:

$$\begin{aligned} \Gamma(X \to J/\psi\pi\pi) &= \frac{1}{24\pi^2 m_X^2} \int_{\Delta_{2\pi}^2}^{(m_X - m_{J/\psi})^2} ds |T_\rho|^2 \frac{m_\rho \Gamma_\rho p(m_X, m_{J/\psi}, \sqrt{s})}{(s - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2}, \\ &= 0.132^{+0.016}_{-0.013}\ \text{MeV}, \end{aligned} \quad (32)$$

$$\begin{aligned} \Gamma(X \to J/\psi\pi\pi\pi) &= \frac{1}{24\pi^2 m_X^2} \int_{\Delta_{3\pi}^2}^{(m_X - m_{J/\psi})^2} ds |T_\omega|^2 \frac{m_\rho \Gamma_\rho p(m_X, m_{J/\psi}, \sqrt{s}) \text{Br}}{(s - m_\omega^2)^2 + m_\omega^2 \Gamma_\omega^2}, \\ &= 0.129^{+0.016}_{-0.013}\ \text{MeV}, \end{aligned} \quad (33)$$

$$\begin{aligned} \Gamma(X \to \chi_{c1}\pi) &= \frac{|T_\pi|^2 p(m_X, m_{\chi_{c1}}, m_\pi)}{24\pi m_X^2}, \\ &= 0.0016\ \text{MeV}, \end{aligned} \quad (34)$$

$$\begin{aligned} \Gamma(X \to D^0\bar{D}^0\pi^0) &= \frac{1}{24\pi^2 m_X^2} \int_{(m_D + m_\pi)^2}^{(m_X - m_D)^2} ds |T_D|^2 \frac{m_{D^*} \Gamma_{D^*} p(m_X, m_D, \sqrt{s})}{(s - m_{D^*}^2)^2 + m_{D^*}^2 \Gamma_{D^*}^2}, \\ &= 2.262^{+0.157}_{-0.152}\ \text{MeV} \quad \text{for } \Gamma_{D^{*0}} = 2.0\ \text{MeV}, \\ &= 1.795^{+0.124}_{-0.121}\ \text{MeV} \quad \text{for } \Gamma_{D^{*0}} = 1.0\ \text{MeV}, \\ &= 1.326^{+0.092}_{-0.089}\ \text{MeV} \quad \text{for } \Gamma_{D^{*0}} = 0.5\ \text{MeV}, \\ &= 0.485^{+0.034}_{-0.032}\ \text{MeV} \quad \text{for } \Gamma_{D^{*0}} = 0.1\ \text{MeV}, \end{aligned} \quad (35)$$

where

$$|T_\rho|^2 = G'^2_{XJ/\psi\rho} \frac{(m_X^2 - m_{J/\psi}^2 - s)^2}{4} \left[\frac{(m_X^2 - s)^2}{2m_{J/\psi}^2} + \frac{(m_X^2 - m_{J/\psi}^2)^2}{2s} + 4m_X^2 - \frac{m_{J/\psi}^2 + s}{2} \right],$$

$$|T_\omega|^2 = G'^2_{XJ/\psi\omega} \frac{(m_X^2 - m_{J/\psi}^2 - s)^2}{4} \left[\frac{(m_X^2 - s)^2}{2m_{J/\psi}^2} + \frac{(m_X^2 - m_{J/\psi}^2)^2}{2s} + 4m_X^2 - \frac{m_{J/\psi}^2 + s}{2} \right],$$

$$|T_\pi|^2 = G'^2_{X\chi\pi} \frac{\lambda(m_X^2, m_{\chi_{c1}}^2, m_\pi^2)}{2},$$

$$|T_D|^2 = G'^2_{XD^*D}\frac{(m_X^2-m_D^2-s)^2}{4}\left[3+\frac{\lambda(m_X^2,s,m_D^2)}{4m_X^2 s}\right],$$

$$G'_{XJ/\psi\rho} \to \tilde{G}'_{XJ/\psi\rho} = G'_{XJ/\psi\rho}\exp\left(-\frac{m_\rho^2-s}{s-\Delta_{2\pi}^2}\frac{m_\rho^4}{\Delta_{2\pi}^4}\right),$$

$$G'_{XJ/\psi\omega} \to \tilde{G}'_{XJ/\psi\omega} = G'_{XJ/\psi\omega}\exp\left(-\frac{m_\omega^2-s}{s-\Delta_{3\pi}^2}\frac{m_\omega^6}{\Delta_{3\pi}^6}\right), \quad (36)$$

where $\Delta_{2\pi}=m_{\pi^+}+m_{\pi^-}$, $\Delta_{3\pi}=m_{\pi^+}+m_{\pi^-}+m_{\pi^0}$, $\mathrm{Br}(\omega\to\pi\pi\pi)=0.892$ [31], $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$, and $p(A,B,C)=\frac{\sqrt{[A^2-(B+C)^2][A^2-(B-C)^2]}}{2A}$. The hadronic coupling constants from the QCD sum rules in Eqs. (22)–(25) are physical quantities under zero width approximation. The physical widths from the PDG are $\Gamma_\rho=147.4$ MeV, $\Gamma_\omega=8.68$ MeV, $\Gamma_{\chi_{c1}}=0.88$ MeV, and $\Gamma_{D^{*0}}<2.1$ MeV, respectively, we introduce exponential form factors $\exp\left(-\frac{m_\rho^2-s}{s-\Delta_{2\pi}^2}\frac{m_\rho^4}{\Delta_{2\pi}^4}\right)$ and $\exp\left(-\frac{m_\omega^2-s}{s-\Delta_{3\pi}^2}\frac{m_\omega^6}{\Delta_{3\pi}^6}\right)$ to parametrize the off shell effects due to the $J/\psi\rho$ and $J/\psi\omega$ thresholds, as the $X(3872)$ lies near the $J/\psi\rho$ and $J/\psi\omega$ thresholds. At the mass-shell $s=m_\rho^2$ and m_ω^2, they reduce to 1 to match with the zero width approximation in the QCD sum rules. At the thresholds, $s=\Delta_{2\pi}^2$ and $\Delta_{3\pi}^2$, the available phase spaces are very small, the decays $\rho\to\pi\pi$ and $\omega\to\pi\pi\pi$ only take place through the lower tails, which can be taken as some intermediate sates with the same quantum numbers as the ρ and ω except for the masses, and are greatly suppressed. On the other hand, since the "off shell" effects on the hadronic coupling constants are considerable, we need to introduce some form factors to parametrize them.

The width $\Gamma_{D^{*\pm}}=0.0834\pm0.0018$ MeV from the PDG [31], if we take the approximation $\Gamma_{D^{*0}}\approx\Gamma_{D^{*\pm}}\approx0.1$ MeV, then $\Gamma(X\to D^{*0}\bar{D}^0)+\Gamma(X\to\bar{D}^{*0}D^0)=0.970^{+0.068}_{-0.064}$ MeV from Eq. (35), which is in excellent agreement with the branching fraction $(52.4^{+25.3}_{-14.3})\%$ from the combined data analysis [32] and the total width $\Gamma_X=1.19\pm0.21$ MeV from the PDG [31]. On the other hand, if we take the masses of the $X(3872)$, $D^\pm$, D^0, $\pi^\pm$, and π^0 from the PDG [31], the decays $X(3872)\to D^{*+}\bar{D}^-\to\bar{D}^+\bar{D}^-\pi^0$ and $X(3872)\to D^{*+}\bar{D}^-\to\bar{D}^0\bar{D}^-\pi^+$ cannot take place due to the negative phase space.

The partial widths $\Gamma(X\to J/\psi\pi\pi)=0.132^{+0.016}_{-0.013}$ MeV and $\Gamma(X\to J/\psi\pi\pi\pi)=0.129^{+0.016}_{-0.013}$ MeV from Eqs. (32) and (33) are in excellent agreement with the branching fractions $\mathrm{Br}(X\to J/\psi\pi\pi)=(4.1^{+1.9}_{-1.1})\%$ and $\mathrm{Br}(X\to J/\psi\omega)=(4.4^{+2.3}_{-1.3})\%$ from the combined data analysis [32], so the mixing angle $\theta=21.6°$, which is compatible with the values $\theta=20.0°$ [11] and $\theta=23.5°$ [14]. The significant difference is that we take account of all the Feynman diagrams and take rigorous quark-hadron duality, while in Ref. [14], only the connected diagrams are taken into account to obtain small partial decay widths.

The partial decay width $\Gamma(X\to\chi_{c1}\pi)=0.0016$ MeV from Eq. (34) is much smaller than the branching fraction $(3.6^{+2.2}_{-1.6})\%$ from the combined data analysis [32] or the branching fraction $(3.4\pm1.6)\%$ from the PDG [31], more precise measurement is still needed.

All in all, in this work, we reproduce the small width of the $X(3872)$ via the QCD sum rules for the first time.

IV. CONCLUSION

In this work, we take the $X(3872)$ as the hidden-charm tetraquark state with both isospin $I=0$ and $I=1$ components, then investigate the hadronic coupling constants $G'_{XJ/\psi\rho}$, $G'_{XJ/\psi\omega}$, $G'_{X\chi\pi}$, and G'_{XD^*D} with the QCD sum rules in details. We select the optimal tensor structures and take account of all the Feynman diagrams, then acquire four QCD sum rules based on the rigorous quark-hadron duality. After careful calculations, we obtain the hadronic coupling constants, then determine the mixing angle via trial and error, and obtain the partial decay widths for the $X(3872)\to J/\psi\pi^+\pi^-$, $J/\psi\omega$, $\chi_{c1}\pi^0$, $D^{*0}\bar{D}^0$, and $D^0\bar{D}^0\pi^0$. The total width is about 1 MeV, which is in excellent agreement with the experiment data $\Gamma_X=1.19\pm0.21$ MeV from the PDG, it is the first time to reproduce the small width of the $X(3872)$ via the QCD sum rules. The present calculations support assigning the $X(3872)$ as the mixed hidden-charm tetraquark state with the quantum numbers $J^{PC}=1^{++}$.

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