Letter

Two-pole structures as a universal phenomenon dictated by coupled-channel chiral dynamics

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In the past two decades, one of the most puzzling phenomena discovered in hadron physics is that a nominal hadronic state can actually correspond to two poles on the complex energy plane. This phenomenon was first noticed for the $\Lambda(1405)$, then for $K_1(1270)$, and to a lesser extent for $D_0^*(2300)$. In this Letter, we show explicitly how the two-pole structures emerge from the underlying universal chiral dynamics describing the coupled-channel interactions between heavy matter particles and pseudo-Nambu-Goldstone bosons. In particular, the fact that two poles appear in between the two dominant coupled channels can be attributed to the particular form of the leading order chiral potentials of the Weinberg-Tomozawa form. Their line shapes overlap with each other because the degeneracy of the two coupled channels is only broken by explicit chiral symmetry breaking of higher order. We predict that for light-quark (pion) masses heavier than their physical values (e.g., about 200 MeV in the $\Lambda(1405)$ case studied), the lower pole becomes a virtual state, which can be easily verified by future lattice QCD simulations. Furthermore, we anticipate similar two-pole structures in other systems, such as the isopin $1/2$ $\bar{K}\Sigma_c - \pi\Xi_c'$ coupled channel, which await for experimental discoveries.

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Introduction. Hadronic states that decay strongly are referred to as resonances. Experimentally, they often show up as enhancements in the invariant mass distributions of their decay products and are parametrized with the Breit-Wigner formalism. It is well known that close to two-body thresholds, however, the Breit-Wigner parametrization cannot accurately capture all the physics and may misrepresent the true dynamics [1,2]. There are also other scenarios where more careful analyses are needed, one of which is where the enhancements may differ depending on the observing channels. Among the latter, the $\Lambda(1405)$ state has attracted the most attention, and to a lesser extent, the $K_1(1270)$ and $D_0^*(2300)$ states [3].

The $\Lambda(1405)$, with quantum numbers $J^P = 1/2^-$, $I = 0$, and $S = -1$, has remained puzzling in the constituent quark model [4] because it is lighter than its nucleon counterpart $N^*(1535)$ and the mass difference between $\Lambda(1405)$ and its spin-partner $\Lambda(1520)$ with $J^P = 3/2^-$ is much larger than the corresponding splitting in the nucleon sector [5]. On the other hand, the $\Lambda(1405)$ was predicted to be a $\bar{K}N$ bound state even before its experimental discovery [6]. Such a picture received further support in the chiral unitary approaches that combine $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ chiral dynamics and elastic unitarity (see Refs. [7,8] for a more complete list of references). An unexpected finding of the chiral unitary approaches is that the $\Lambda(1405)$ actually corresponds to two dynamically generated poles on the second Riemann sheet of the complex energy plane [9,10], between the thresholds of $\pi\Sigma(1330)$ and $\bar{K}N(1433)$, where the numbers in the brackets are the thresholds of the respective channels in units of MeV. Such a two-pole

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picture[1] has recently been reconfirmed in the unified description of meson-baryon scattering at next-to-next-to-leading order (NNLO) [11]. In the following years, it was shown that the $K_1(1270)$ [12,13] and $D_0^*(2300)$ [14–19] also correspond to two poles,[2] which are needed to explain many relevant experimental data [13,19] or lattice QCD data [22].

The fact that such two-pole structures emerge in three different sectors asks for an explanation. In Ref. [10], it is shown that in the SU(3) flavor symmetry limit, one expects three bound states, one singlet and two degenerate octets. In the physical world where SU(3) symmetry is broken, the singlet develops into the lower pole of the $\Lambda(1405)$, and one octet evolves into the higher pole. Similar observations have been made for $K_1(1270)$ [12] and $D_0^*(2300)$ [17].

It is the purpose of the present Letter to explicitly demonstrate how the two-pole structures emerge from the underlying coupled-channel chiral dynamics and the pseudo Nambu-Goldstone nature of the pseudoscalar mesons. In particular, we would like to answer the following three questions. (1) Does the off-diagonal coupling between the two dominant channels play a decisive role? (2) How does explicit chiral symmetry breaking generate the two-pole structures? (3) Is the energy dependence of the Weinberg-Tomozawa (WT) potential relevant?

Formalism. In this work, we focus on the two poles of $\Lambda(1405)$ and $K_1(1270)$ and highlight their common origins.[3] We first spell out the LO chiral Lagrangians describing the pseudoscalar-baryon (PB) and pseudoscalar-vector (PV) interactions,[4] from which one can derive potentials V of the WT type responsible for the dynamical generation of $\Lambda(1405)$ and $K_1(1270)$, and highlight their common feature. Then we briefly review the chiral unitary approaches.

For the PB interaction describing the scattering of a pseudoscalar meson off a ground-state octet baryon, the LO chiral Lagrangian has the following form [10]:

$$\mathcal{L}_{PB}^{\mathrm{WT}} = \frac{1}{4f^2}\mathrm{Tr}(\bar{\mathcal{B}} i\gamma^\mu[\Phi\partial_\mu\Phi - \partial_\mu\Phi\Phi, \mathcal{B}]), \tag{1}$$

from which one can obtain the potential in the c.m. frame

$$V_{ij} = -\frac{C_{ij}}{4f^2}(2\sqrt{s} - M_i - M_j) = -\frac{C_{ij}}{4f^2}(E_i + E_j), \tag{2}$$

where the subscripts i and j represent the incoming and outgoing channels in isospin basis, M is the mass of the baryon and E is the energy of the pseudoscalar meson. C_{ij} are the corresponding Clebsch-Gordan coefficients. Note that we have neglected the three momentum of the baryon in comparison with its mass and numerically verified that such an approximation has no impact on our discussion.

Likewise for the PV interaction, the LO chiral Lagrangian [12,38] is

$$\mathcal{L}_{\mathrm{PV}}^{\mathrm{WT}} = -\frac{1}{4f^2}\mathrm{Tr}([\mathcal{V}^\mu, \partial^\nu\mathcal{V}_\mu][\Phi, \partial_\nu\Phi]), \tag{3}$$

from which one can obtain the following potential projected onto S wave,

$$V_{ij}(s) = -\epsilon^i\cdot\epsilon^j\frac{C_{ij}}{8f^2}\bigg[3s - (M_i^2 + m_i^2 + M_j^2 + m_j^2) - \frac{1}{s}(M_i^2 - m_i^2)(M_j^2 - m_j^2)\bigg]. \tag{4}$$

Note that $M_{i,j}$ are masses of vector mesons and $m_{i,j}$ are those of pseudoscalar mesons. Close to threshold, considering the light masses of the pseudoscalar mesons as well as the chiral limit of $M_i = M_j \equiv M$, Eq. (4) can be simplified to

$$V_{ij}(s) = -\epsilon^i\cdot\epsilon^j\frac{C_{ij}}{8f^2}4M(E_i + E_j), \tag{5}$$

which is the same as Eq. (2) up to the scalar product of polarization vectors, trivial dimensional factors, and Clebsch-Gordan coefficients.

In the chiral unitary approaches [7], the unitarized amplitude reads

$$T = (1 - VG)^{-1}V, \tag{6}$$

where G is a diagonal matrix with elements $G_{kk} \equiv G_k(\sqrt{s})$. The loop function $G_k(\sqrt{s})$ of channel k is logarithmically divergent and can be regulated either in the dimensional regularization scheme or the cutoff scheme. In the dimensional regularization scheme, a subtraction constant is introduced, while in the cutoff scheme one needs a cutoff. In practice, the subtraction constants or cutoff values are determined by fitting to the scattering data but should be of natural size in order for the chiral unitary approaches to make sense. For details, see, e.g., Refs. [7,12].

The couplings of a resonance/bound state to its constituents can be obtained from the residues of the corresponding pole on the complex energy plane, i.e.,

$$g_i g_j = \lim_{\sqrt{s}\to z_R}(\sqrt{s} - z_R)T_{ij}(\sqrt{s}), \tag{7}$$

where $z_R \equiv m_R - i\Gamma_R/2$ is the pole position.

[1] A proper definition of two-pole structures is needed for clarifying this mysterious phenomenon. In this work, two-pole structures refer to the fact that two dynamically generated states, one resonant and one bound (with respect to the most strongly coupled channels), are located close to each other between two coupled channels and have a mass difference smaller than the sum of their widths. As a result, the two poles overlap in the invariant mass distribution of their decay products, which creates the impression that there is only one state.

[2] In two recent works, it was shown that the $\Xi(1890)$ [20] and $b_1(1235)$ [21] also correspond to two poles. The former is governed by the same chiral dynamics highlighted in the present work, while the latter is generated by a more complicated coupled-channel interaction.

[3] We leave out the $D_0^*(2300)$ to the Supplemental Material [23].

[4] As the two-pole structure persists up to higher chiral orders [11,34–37], we stick to the leading order to demonstrate the chiral dynamics at play and its universality in the present work.

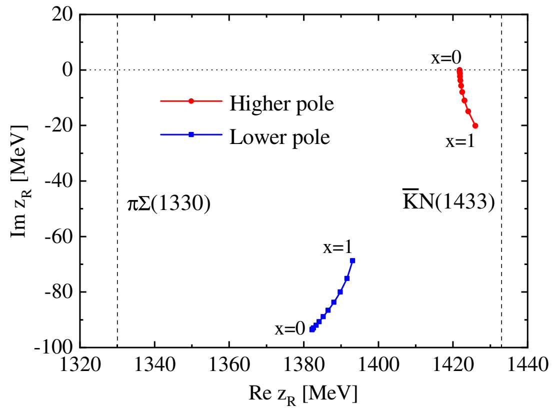

FIG. 1. Evolution of the two poles of $\Lambda(1405)$ as a function of the off-diagonal potential $x \times V_{\bar{K}N-\pi\Sigma}$ with $0 \leq x \leq 1$. Every point on the lines is taken in steps of $x = 0.1$.

Coupled channel effects. We first focus on the $\Lambda(1405)$ state. In the isospin 0 and strangeness -1 meson-baryon system, the $\bar{K}N$ and $\pi\Sigma$ channels play the most important role around the 1400 MeV region [10,39]. With the following subtraction constants $a_{\bar{K}N} = -1.95$ and $a_{\pi\Sigma} = -1.92$, we find two poles on the complex energy plane, i.e., $W_H = 1426.0 - 20.1i$ MeV and $W_L = 1393.1 - 68.7i$ MeV, consistent with the LO [10], NLO [34–36], and NNLO results [11]. One might naively expect that the two poles are linked to the coupling between the $\bar{K}N$ and $\pi\Sigma$ channels. This is actually not the case. To demonstrate this, we decrease the coupling between $\bar{K}N$ and $\pi\Sigma$ by multiplying a factor $0 \leq x \leq 1$ to the off-diagonal matrix elements of the WT potential and obtain the evolution of the two poles shown in Fig. 1. Two things are noteworthy. First, even in the limit of complete decoupling, i.e., $x = 0$, the two poles still appear in between the $\bar{K}N$ and $\pi\Sigma$ thresholds, but the imaginary part of the higher pole approaches zero while the imaginary part of the lower pole becomes larger. Second, the coupling between the two channels not only pushes the two poles higher, but also allows the higher pole to decay into the $\pi\Sigma$ channel and as a result develops a finite width. Nevertheless the most important issue to note is that the coupling between the two channels is not the driving factor for the existence of two dynamically generated states in between the two relevant channels. On the other hand, it does play a role in the development of the two-pole structure, because otherwise the higher pole will not manifest itself in the invariant mass distribution of the lower $\pi\Sigma$ channel. We note that Ref. [40] has used a similar approach, the so-called zero coupling limit [39], to study the pole contents of various unitarized chiral approaches.

Explicit chiral symmetry breaking. In the following, we explicitly show that it is the underlying chiral dynamics that is responsible for the emergence of the two-pole structure. According to Eq. (2), the diagonal WT interaction is proportional to the energy of the pseudoscalar meson. For the $\bar{K}N$ and $\pi\Sigma$ channels of our interest, they read[5]

$$V_{\bar{K}N-\bar{K}N}(\sqrt{s}) = -\frac{6}{4f^2}E_{\bar{K}} = -\frac{6}{4f^2}\sqrt{m_{\bar{K}}^2 + q_{\bar{K}}^2},$$
$$V_{\pi\Sigma-\pi\Sigma}(\sqrt{s}) = -\frac{8}{4f^2}E_{\pi} = -\frac{8}{4f^2}\sqrt{m_{\pi}^2 + q_{\pi}^2}. \quad (8)$$

Due to the explicit chiral symmetry breaking, the mass of the kaon is much larger than that of the pion. As a result, close to threshold, the $\bar{K}N$ interaction is stronger than the $\pi\Sigma$ one, which leads to a $\bar{K}N$ bound state. In addition, the energy dependence and the small pion mass together enhance the q^2 term of the $\pi\Sigma$ interaction and therefore are responsible for the existence of a $\pi\Sigma$ resonance.[6] We stress that the role of explicit symmetry breaking can be appreciated by studying the pole trajectories as a function of the light-quark (pion) mass.

As the pion mass changes, masses of the baryons and the kaon also vary. We adopt the covariant baryon chiral perturbation theory to describe their light-quark mass dependence. Up to $\mathcal{O}(p^2)$, the octet baryon masses read

$$M_B(m_\pi) = M_0 + M_B^{(2)} = M_0 + \sum_{\phi=\pi,K} \xi_{B,\phi} m_\phi^2, \quad (9)$$

where M_0 is the chiral limit baryon mass and $\xi_{B,\phi}$ are the relevant coefficients that contain three low-energy constants, which are fitted to the lattice QCD data of the PACS-CS collaboration [43] in Ref. [44], where one can also find the pion mass dependence of the kaon.

The trajectories of the two poles of $\Lambda(1405)$ are shown in Fig. 2. The evolution of the higher pole is simple. As the pion mass increases, both its real and imaginary parts decrease. This indicates that the effective $\bar{K}N$ interaction and coupling to $\pi\Sigma$ both decrease as the pion (kaon) mass increases. Note that as the pion mass increases, the two thresholds increase as well. On the other hand, the trajectory of the lower pole is more complicated and highly nontrivial. As the pion mass increases, it first becomes a virtual state from a resonant state for a pion mass of about 200 MeV. For a pion mass of about 300 MeV, it becomes a bound state and remains so up to the pion mass of 500 MeV. The evolution of the lower pole clearly demonstrates the chiral dynamics underlying the two-pole structure of $\Lambda(1405)$.

[5]As we have shown that the coupling between $\bar{K}N$ and $\pi\Sigma$ does not play a decisive role in generating the two-pole structure, the following discussion should be understood in the single-channel approximation.

[6]The different role played by the heavy kaon and the light pion has been noted previously in other contexts, see, e.g., Refs. [41,42].

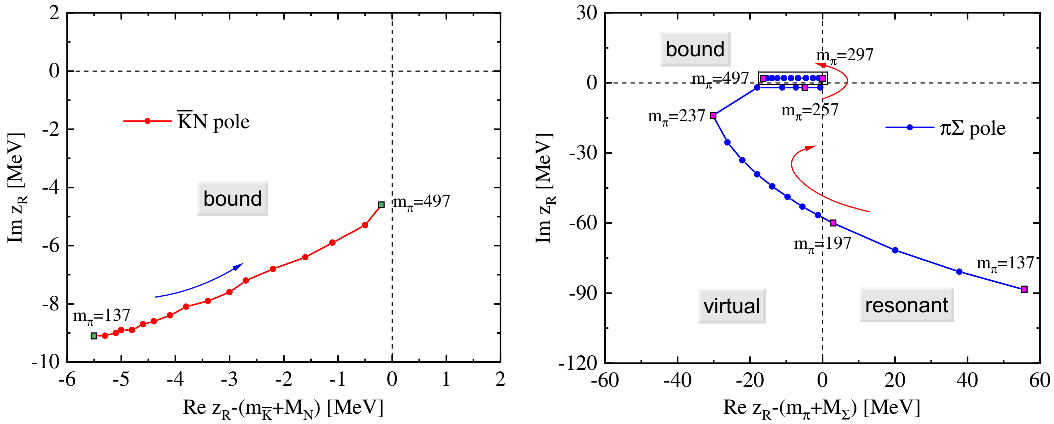

FIG. 2. Trajectories of the two poles of $\Lambda(1405)$ as functions of the pion mass m_π from 137 to 497 MeV. Critical masses are labeled by solid squares, between which the points are equally spaced.

To check how the energy dependence of the chiral potential affects the two-pole structure,[7] we replace $E_i + E_j$ of the chiral potential of Eq. (2) with $m_i + m_j$. With the original subtraction constants, we obtain only one pole at $1413.3 - 13.2i$ MeV, corresponding to a $\bar{K}N$ bound state. We checked that switching off the off-diagonal interaction affects little our conclusion. As the pion mass is much smaller than the kaon mass, the attraction of the $\pi\Sigma$ single channel is weaker than that of the $\bar{K}N$ single channel, and thus cannot support a bound state. Of course if we increase the strength of the attractive potential, we can obtain two bound states but not a bound state and a resonant state, and as a result there is no two-pole structure any longer.

$K_1(1270)$. From the above study, one immediately realizes that if one replaces the matter particles (the ground-state baryons) with the ground-state vector mesons, one may also expect the existence of a two-pole structure. This is indeed the case as shown in Refs. [12,13], where $K_1(1270)$ is found to correspond to two poles. The most relevant channels are $K^*\pi(1030)$ and $\rho K(1271)$. In Ref. [13], it was shown that with $\mu = 900$ MeV, $a(\mu) = -1.85$, and $f = 115$ MeV, where μ is the renormalization scale, $a(\mu)$ is the common subtraction constant, and f is the pion decay constant, one finds two poles located at $W_H = 1269.3 - 1.9i$ MeV and $W_L = 1198.1 - 125.2i$ MeV below the ρK and above the $K^*\pi$ thresholds. Eliminating the three higher channels, we can find almost the same two poles located at $W_H = 1269.5 - 12.0i$ MeV and $W_L = 1198.5 - 123.2i$ MeV by adjusting slightly the subtraction constants as $a_{K^*\pi} = -2.21$, $a_{\rho K} = -2.44$.

Further two-pole structures. In principle, because of the universality of chiral dynamics discussed in this work, one can expect more such two-pole structures in other systems composed of a pair of heavy matter particles and pseudoscalar mesons, such as the singly charmed baryon sector [46]. Using the criteria proposed in this work, one can identify the two channels $\bar{K}\Sigma_c(2949)$ and $\pi\Xi_c'(2714)$ that has the potential of generating a two-pole structure.

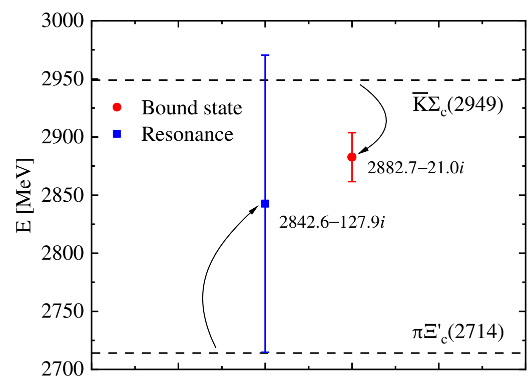

FIG. 3. Two poles of the $\bar{K}\Sigma_c - \pi\Xi_c'$ system. The dominant channels in relation to the two states are denoted by the arrows. The vertical bars are the widths corresponding to twice of the imaginary parts of the pole positions.

With a common cutoff of $\Lambda = 800$ MeV in the cutoff regularization scheme and using the WT potential similar to Eq. (2),[8] we find two poles in the isospin $1/2$ channel, located at $W_H = 2882.7 - 21.0i$ MeV and $W_L = 2842.6 - 127.9i$ MeV. The higher pole couples strongly to the $\bar{K}\Sigma_c$ channel, while the lower pole couples more to the $\pi\Xi_c'$ channel, as shown in Fig. 3. With the unitary amplitudes $\bar{K}\Sigma_c \to \pi\Xi_c'$ and $\pi\Xi_c' \to \pi\Xi_c'$ and following Ref. [46], we can construct the $\pi\Xi_c'$ invariant mass distributions shown in Fig. 4. Note that the line shape of the $\bar{K}\Sigma_c \to \pi\Xi_c'$

[7]The relation between the energy dependence of the WT potential and the simultaneous appearance of a bound state and a resonant state has been noted in, e.g., Ref. [45].

[8]It is hard to directly compare with Ref. [47], because there the extra coupled channels of charmed mesons and ground-state baryons are considered, which are not constrained by the same chiral dynamics studied in this work.

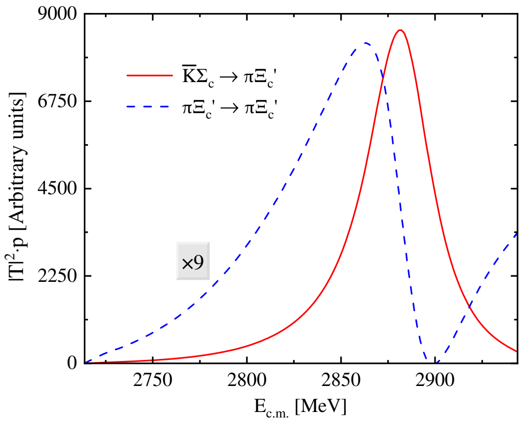

FIG. 4. Invariant mass distributions of $\pi\Xi_c'$ (in arbitrary units) as functions of the c.m. energy, $|T_{\bar{K}\Sigma_c\to\pi\Xi_c'}|^2 p_\pi$ (red solid) and $9\times|T_{\pi\Xi_c'\to\pi\Xi_c'}|^2 p_\pi$ (blue dashed), where p_π is the 3-momentum of the pion in the c.m. frame of the final states.

amplitude overlaps much with that of the $\pi\Xi_c' \to \pi\Xi_c'$ one, but they peak at slightly different positions and have different widths. These can be qualitatively explained by the fact that the process $\bar{K}\Sigma_c \to \pi\Xi_c'$ receives more contribution from the higher pole, while the process $\pi\Xi_c' \to \pi\Xi_c'$ couples more to the lower pole. We need to stress that although the two-pole structure is tied to the underlying chiral dynamics, the regularization, i.e., the cutoff in the present case, plays a relevant role. We therefore encourage further theoretical and experimental studies of the states predicted.

Conclusion and outlook. We have examined the origin of the mysterious two-pole structures and attributed this fascinating phenomenon to the underlying chiral dynamics. First, chiral symmetry strongly constrains the interactions of a matter particle with a pseudoscalar meson, which are often referred to as the Weinberg-Tomozawa potentials. Second, the pseudo Nambu-Goldstone boson nature of π, K, and η are responsible for the generation of two nearby poles: one bound and one resonant. Furthermore, the explicit chiral and SU(3) flavor symmetry breaking dictates that the two relevant coupled channels are close to each other such that the line shapes of the two states overlap and thus create the impression that there is only one state. We anticipate more such two-pole structures in other systems governed by the same chiral dynamics and encourage dedicated experimental and lattice QCD studies to verify the chiral dynamics underlying such phenomena. Last, we stress that flavor symmetry also plays a relevant role here, as it dictates the relative coupling strengths between different channels.

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