Letter

New higher-spin curvatures in flat space

Nicolas Boulanger,* Andrea Campoleoni,† and Simon Pekar‡

Service de Physique de l'Univers, Champs et Gravitation, Université de Mons—UMONS, 20 place du Parc, 7000 Mons, Belgium

(Received 15 June 2023; accepted 13 October 2023; published 28 November 2023)

It was shown that the Lie algebra underlying higher-spin holography admits a contraction including a Poincaré subalgebra in any space-time dimensions. The associated curvatures, however, do not reproduce upon linearization those that are usually employed to formulate the equations of motion of free massless particles in Minkowski space. We show that, despite this mismatch, the new linearized curvatures can also be used to describe massless higher-spin fields. This suggests a new way to build interacting higher-spin gauge theories in Minkowski space that may admit a holographic description.

DOI: 10.1103/PhysRevD.108.L101904

Introduction. The interactions of massless particles of spin greater than two, also known as higher-spin particles, are strongly constrained by several no-go results; see, e.g., [1] for a review. In spite of this, positive results accumulated over the years in an effort motivated, for instance, by the long-held conjecture that string theory might be a broken phase of a higher-spin gauge theory and, more recently, by applications in holography. We refer to [2,3] for reviews on these two research directions and to [4] for a recent status overview of higher-spin theories. In particular, nonlinear equations of motion for massless higher-spin fields on constant-curvature backgrounds were built by Vasiliev and collaborators [5,6]. Later on, these have been conjectured to provide the bulk duals of certain weakly interacting conformal field theories within the AdS/CFT correspondence [7,8]. These developments led to the common lore that higher-spin gauge theories do exist in the presence of a cosmological constant, provided one is ready to accept some unconventional features, like, e.g., an infinite spectrum of fields.

Vasiliev's equations and higher-spin holography rely upon an infinite-dimensional Lie algebra that we shall denote by $\mathfrak{hs}_D$, with D the space-time dimension. This algebra is essentially unique when the dimension of space-time is greater than three [9,10], modulo supersymmetric extensions and Chan-Paton factors [11–13]. For instance, Vasiliev's equations describe the dynamics of massless higher-spin fields with a set of differential forms generalizing the vielbein and spin connection of Cartan's formulation of general relativity. The equations of motion are built by constraining $\mathfrak{hs}_D$-valued curvature two-forms, in analogy with Cartan's approach to gravity where the equations of motion are constraint equations for the curvatures of the isometry algebra of the vacuum. This approach to the interactions of fields of arbitrary spin is referred to as unfolded formulation; see, e.g., [14,15] for a review. In one of the founding papers of the unfolded formulation, it was observed that the algebra $\mathfrak{hs}_4$ admits a contraction containing a Poincaré subalgebra [9]. The result, however, was discarded as a candidate higher-spin algebra in Minkowski space—i.e., as a starting point for developing nonlinear unfolded equations in flat space—because the associated linearized curvatures for spin $s > 2$ do not agree with those of [16], based on which Vasiliev's equations were built [5,6].

This observation was long considered as an additional no-go argument against higher-spin interactions in Minkowski space: no appropriate symmetry algebra seemed to exist, at least for the same spectrum of fields as in Vasiliev's equations. This view was also supported by direct analyses of interactions within Fronsdal's metriclike approach [17] in which a particle of spin s is described starting from a rank-s symmetric tensor, therefore generalizing the metric formulation of linearized gravity. In this setup, various studies pointed out the inconsistency of the non-Abelian, two-derivative, minimal gravitational coupling of Fronsdal's gauge fields in flat space; see, e.g., [1,18], and references therein. As discussed in [1], Weinberg's famous low-energy theorem [19] as well as the generalized Weinberg-Witten theorem of [20] can also be reinterpreted in these terms.

On the other hand, positive results giving non-Abelian interactions of massless higher-spin fields in flat space

*nicolas.boulanger@umons.ac.be
†andrea.campoleoni@umons.ac.be
‡simon.pekar@umons.ac.be
Present address: Centre de Physique Théorique – CPHT, École polytechnique, Unité Mixte de Recherche 7644 du CNRS, Institut Polytechnique de Paris, 91120 Palaiseau Cedex, France.

were also obtained. For instance, in [21,22], a consistent non-Abelian cubic coupling between massless spin-s and spin-2 fields around Minkowski space containing a total of $(2s-2)$ derivatives was obtained in Fronsdal's formulation. It was also shown to induce a consistent deformation of the free gauge algebra satisfying Jacobi identities [22]. More recently, a complete interacting higher-spin gauge theory on four-dimensional flat manifolds with Euclidean or split signature has been built employing a different set of fields [23,24]. The role of these models in flat-space holography also begins to be explored [25–27]. Besides, an analog of the contraction of the algebra $\mathfrak{hs}_4$ discussed in [9] was recently defined in any space-time dimension [28]. The contracted algebra, that we shall denote by $\mathfrak{ihs}_D$, can also be obtained from the Poincaré algebra following a construction close to that relating $\mathfrak{hs}_D$ to the conformal algebra [28,29].

These indications naturally lead to reconsider the linearized curvatures of $\mathfrak{ihs}_D$. In this paper, we propose a new system of first-order equations of motion built upon them, that describes the free propagation of massless particles of arbitrary spin on Minkowski space. Our equations follow the same pattern as in the usual unfolded formulation: for any spin s we set to zero all corresponding curvatures but one and impose that the latter is proportional to a tensor that encodes the propagating degrees of freedom of the massless spin-s field and that generalizes to higher spins the Weyl tensor of linearized gravity. We then prove that, even if our curvatures have a nonstandard form, the resulting equations of motion are equivalent to the standard ones [16], that we shall review below.

When expressed in terms of curvatures, the structure of our equations is the same as that of the customary free unfolded equations in anti–de Sitter (AdS) space-time. This strongly suggests the option to deform our linear equations into an interacting theory following the path that led from [9] to [5,6] or, equivalently, the cohomological approach of [30,31]. We defer a detailed analysis to future work, but we wish to stress that this program is expected to provide a model for interacting higher-spin gauge fields propagating in Minkowski space-time, possibly describing a subsector of the unbroken phase of string theory and admitting a holographic description. Indeed, the simplest instance of higher-spin holography within the AdS/CFT correspondence features a free scalar field at the boundary of AdS_D space-time, whose global symmetries are given by the algebra $\mathfrak{hs}_D$ [32,33] (see, e.g., [34] for a review). Similarly, $\mathfrak{ihs}_D$ appears as a subalgebra of the global symmetries of a Carrollian scalar living on null infinity [29], which is defined by the limit of vanishing speed of light of a free scalar living at the boundary of AdS_D. This observation fits within the active field of flat-space holography (see, e.g., [35–40] for an overview of various approaches), where it has been realized that the limit of vanishing cosmological constant in the bulk, gravitational theory, corresponds to a limit of vanishing speed of light in the boundary, conformal field theory. Any nonlinear deformation of our free equations of motion will therefore provide a candidate gravitational dual of the simplest Carrollian field theory, thus fitting within the urgent quest for concrete dual pairs in flat-space holography, that is currently mainly driven by symmetry considerations.

Basics of unfolding. In [16], Lopatin and Vasiliev formulated the equations of motion of a free particle of arbitrary spin s on a space-time of constant curvature in terms of a set of one-forms $\omega^{a(s-1),b(t)}$, with $0\le t\le s-1$, where the shorthand $a(k)$ denotes a set of k symmetrized indices. They satisfy $\eta_{cd}\omega^{a(s-3)cd,b(t)}=0$ and $\omega^{a(s-1),ab(t-1)}=0$, where repeated indices denote a symmetrization with strength one, and correspond to representations of the Lorentz algebra labeled by two-row Young tableaux. For $s=2$, ω^a and $\omega^{a,b}$ correspond, respectively, to the vielbein and the spin connection.

The equations of motion read

$$\nabla\omega^{a(s-1),b(t)}+h_c\wedge\omega^{a(s-1),b(t)c}+\lambda^2c_{s,t}h^{\{b}\wedge\omega^{a(s-1),b(t-1)\}}=0,\qquad 0\le t\le s-2, \tag{1a}$$

$$\nabla\omega^{a(s-1),b(s-1)}+\lambda^2c_{s,s-1}h^{\{b}\wedge\omega^{a(s-1),b(s-2)\}}=h_c\wedge h_dC^{a(s-1)c,b(s-1)d}, \tag{1b}$$

where $\nabla=\mathrm{d}x^\mu\nabla_\mu$ denotes the Lorentz-covariant derivative of the (A)dS background, with $\nabla^2V^a=-\sigma\lambda^2h^a\wedge h_bV^b$ in terms of the vielbein h^a of the background. The cosmological constant is $\Lambda=-\sigma\lambda^2\frac{(D-1)(D-2)}{2}$, where $\sigma=\pm1$. The coefficients $c_{s,t}$ are fixed by requiring the invariance of these equations under

$$\delta\omega^{a(s-1),b(t)}=\nabla\epsilon^{a(s-1),b(t)}+h_c\epsilon^{a(s-1),b(t)c}+\lambda^2c_{s,t}h^{\{b}\epsilon^{a(s-1),b(t-1)\}}, \tag{2}$$

where the gauge parameters $\epsilon^{a(s-1),b(t)}$ are Lorentz-irreducible tensors as the one-forms. In Eqs. (1) and (2), braces denote the projection on the Lorentz-irreducible tensor representation carried by $\omega^{a(s-1),b(t)}$. The precise form of both $c_{s,t}$ and the projectors will not be relevant in the ensuing discussion where we focus on the $\lambda\to0$ limit. The zero-form $C^{a(s),b(s)}$

entering (1b) is a gauge-invariant and Lorentz-irreducible tensor interpreted as the spin-s Weyl tensor; see, e.g., [14,15]. For $s = 2$, Eqs. (1) are the linearized vacuum Einstein equations, where the vanishing of the Ricci tensor is reformulated by equating the Riemann curvature with the Weyl tensor. In the following we shall refer to the limit $\lambda \to 0$ of Eqs. (1) as the Lopatin-Vasiliev equations in flat space.

For $s = 3$, the latter read

$$\mathrm{d}e^{ab} + h_c \wedge \omega^{ab,c} = 0, \tag{3a}$$

$$\mathrm{d}\omega^{ab,c} + h_d \wedge X^{ab,cd} = 0, \tag{3b}$$

$$\mathrm{d}X^{ab,cd} = h_e \wedge h_f C^{abe,cdf}, \tag{3c}$$

where we renamed the fields $\omega^{ab} \to e^{ab}$ and $\omega^{ab,cd} \to X^{ab,cd}$ and we chose Cartesian coordinates so that the background vielbein reads $h_\mu{}^a = \delta_\mu{}^a$ and the background spin connection vanishes. Equations (3) are invariant under

$$\delta e^{ab} = \mathrm{d}\xi^{ab} + h_c \lambda^{ab,c}, \tag{4a}$$

$$\delta\omega^{ab,c} = \mathrm{d}\lambda^{ab,c} + h_d \rho^{ab,cd}, \tag{4b}$$

$$\delta X^{ab,cd} = \mathrm{d}\rho^{ab,cd}. \tag{4c}$$

To prove that Eqs. (3) propagate the degrees of freedom of a spin-three particle, one can first use the parameter $\lambda^{ab,c}$ to gauge away the corresponding component of $h^{\mu c} e_\mu{}^{ab}$, so as to recover a Fronsdal field $\varphi_{\mu\nu\rho} = h_{(\mu}{}^a h_\nu{}^b e_{\rho)ab}$. The constraint (3a) then allows one to express $\omega^{ab,c}$ in terms of the first derivatives of $\varphi_{\mu\nu\rho}$, except for a pure-gauge component which is gauged away using $\rho^{ab,cd}$. The constraint (3b) plays a double role: some of its irreducible components only involve $\omega^{ab,c}$ and impose Fronsdal's equation on $\varphi_{\mu\nu\rho}$, while the others express $X^{ab,cd}$ in terms of the first derivatives of $\omega^{ab,c}$ and, eventually, in terms of two derivatives of $\varphi_{\mu\nu\rho}$. The same mechanism applies for any value of the spin; see, e.g., [41–43] for more details.

The first step to introduce interactions in the unfolded approach is then to look for non-Abelian algebras whose curvature two-forms reproduce the lhs of (1) upon linearization around the gravitational background. The latter requirement fixes the commutators of all generators with the Poincaré or (A)dS subalgebra, and one can check if non-Abelian algebras reproducing these commutators exist, e.g., by solving the Jacobi identities. Following this strategy, in [9] it was shown that the algebra $\mathfrak{hs}_4$ provides the unique solution to this problem in AdS$_4$, while no solution was found in Minkowski space. The full $\mathfrak{hs}_4$-valued curvatures then constituted the basis of 4D Vasiliev's nonlinear equations [5].

Higher-spin extension of the Poincaré algebra. The higher-spin algebra $\mathfrak{ihs}_D$ can be obtained as an İnönü-Wigner contraction of the algebra $\mathfrak{hs}_D$ [28]. As previously mentioned, the latter can be built by solving the Jacobi identities with the initial data provided by the Lopatin-Vasiliev equations. In modern terms, it corresponds to the universal enveloping algebra of $\mathfrak{so}(2, D-1)$ evaluated on Dirac's singleton module [33,44–46]. In correspondence with the spectrum of gauge fields, its generators can be collected in irreducible and traceless tensors $M_{a(s),b(t)}$ with $s \geq 0$ and $0 \leq t \leq s$, that have the same Lorentz symmetries as the one-forms $\omega^{a(s),b(t)}$ presented above. Their commutators take the form

$$\left[M_{a(s_1),b(t_1)}, M_{c(s_2),d(t_2)}\right] \propto \sum_{s_3=|s_1-s_2|+1}^{s_1+s_2-1} \sum_{t_3=0}^{s_3} M_{e(s_3),f(t_3)} \tag{5}$$

with $(s_1+s_2+s_3) \bmod 2 = 1$ and $(t_1+t_2+t_3) \bmod 2 = 1$. For $s = 1$ one recovers the $\mathfrak{so}(2, D-1)$ conformal algebra and explicit structure constants can be found in [46].

The generators $M_{a(s),b(t)}$ with $s-t$ even thus form a subalgebra and one can rescale the others as

$$M_{a(s),b(s-2n-1)} \to \varepsilon^{-1} M_{a(s),b(s-2n-1)} \quad \forall\ s, n \in \mathbb{N}. \tag{6}$$

In the limit $\varepsilon \to 0$ all commutators involving only generators with $s-t$ odd vanish, while the others remain untouched. The $\mathfrak{so}(2, D-1)$ subalgebra contracts into a $\mathfrak{iso}(1, D-1)$ subalgebra and one obtains a non-Abelian higher-spin extension of the Poincaré algebra. We chose to present this algebra as a contraction of the AdS higher-spin algebra $\mathfrak{hs}_D$, although one can also build it as a quotient of the universal enveloping algebra of $\mathfrak{iso}(1, D-1)$ [28,29]. One can also prove that, in a generic space-time dimension $D > 3$, $\mathfrak{ihs}_D$ is the only algebra with the same set of generators as $\mathfrak{hs}_D$ that can be built with this procedure [28]. When $D = 3$, the option to build sensible higher-spin algebras in Minkowski space from contractions of the AdS ones was already observed in [47] (see also [28,48–50]), and Vasiliev-like equations of motion were proposed in [51].

New equations of motion in Minkowski space. We now consider a one-form taking values in the Lie algebra $\mathfrak{ihs}_D$:

$$A = \sum_{s=0}^{\infty} \sum_{t=0}^{s} \omega^{a(s),b(t)} M_{a(s),b(t)} \tag{7}$$

and its Yang-Mills curvature:

$$\mathrm{d}A + A \wedge A = \sum_{s=0}^{\infty} \sum_{t=0}^{s} F^{a(s),b(t)} M_{a(s),b(t)}. \tag{8}$$

We wish to build equations of motion describing free massless particles using the linearization of the curvatures

$F^{a(s),b(t)}$ around the Minkowski background, linearization that we shall denote as $\bar{F}^{a(s),b(t)}$. We thus split the vielbein as $\omega^a = h^a + e^a$ and, for simplicity, we choose again Cartesian coordinates. The following discussion can be extended to arbitrary coordinates by introducing a flat background Lorentz connection, but working in Cartesian coordinates makes some arguments more transparent.

The linearized curvatures only depend on the commutators between the higher-spin generators $M_{a(s),b(t)}$ and those of the Poincaré subalgebra. With our choice of coordinates, they read

$$\bar{F}^{a(s),b(t)} = \mathrm{d}\omega^{a(s),b(t)} \qquad \text{for } s-t \text{ even}, \tag{9a}$$

$$\begin{aligned}\bar{F}^{a(s),b(t)} = \mathrm{d}\omega^{a(s),b(t)} &+ h^{\{b} \wedge \omega^{a(s),b(t-1)\}} \\ &+ h_c \wedge \omega^{a(s),b(t)c} \qquad \text{for } s-t \text{ odd},\end{aligned} \tag{9b}$$

where braces denote a two-row Young projection together with a traceless projection as explained after Eq. (2). Notice that the second term on the rhs of (9b) is absent for $t=0$ when this value is allowed by the parity condition. The curvature with $t=s$, instead, always fits in the class (9a). The linearized curvatures (9) are invariant under the gauge transformations

$$\delta\omega^{a(s),b(t)} = \mathrm{d}\epsilon^{a(s),b(t)} \qquad \text{for } s-t \text{ even}, \tag{10a}$$

$$\begin{aligned}\delta\omega^{a(s),b(t)} = \mathrm{d}\epsilon^{a(s),b(t)} &+ h^{\{b}\epsilon^{a(s),b(t-1)\}} \\ &+ h_c\epsilon^{a(s),b(t)c} \qquad \text{for } s-t \text{ odd}.\end{aligned} \tag{10b}$$

To describe a particle with spin s we propose to impose the equations of motion

$$\bar{F}^{a(s-1),b(t)} = 0, \qquad 0 \le t \le s-2, \tag{11a}$$

$$\bar{F}^{a(s-1),b(s-1)} = h_c \wedge h_d C^{a(s-1)c,b(s-1)d}, \tag{11b}$$

where $C^{a(s),b(s)}$ is a gauge-invariant and Lorentz-irreducible tensor. In the following, we prove that they describe the free propagation of a massless particle of spin s by showing that they are equivalent to the Lopatin-Vasiliev equations on Minkowski space.

Alternatively, one can obtain Eqs. (11) by rescaling $\omega^{a(s-1),b(s-2n)} \to \varepsilon\omega^{a(s-1),b(s-2n)}$ in Eqs. (1) and sending $\varepsilon \to 0$ while keeping their dependence on the cosmological constant fixed. The latter can then be absorbed in a redefinition of the connections $\omega^{a(s-1),b(s-2n-1)}$. If one instead keeps ε fixed while sending the cosmological constant to zero, one obtains the Lopatin-Vasiliev equations in flat space.

Equivalence with the Lopatin-Vasiliev equations. To prove that Eqs. (11) are equivalent to the Lopatin-Vasiliev equations [16] and, therefore, to the Fronsdal equation in Minkowski space [17], we begin with the instructive spin-three example. We then extend the proof to any spin.

The spin-three example: For $s=3$, Eqs. (11) read

$$\bar{F}^{ab} \coloneqq \mathrm{d}e^{ab} = 0, \tag{12a}$$

$$\bar{F}^{ab,c} \coloneqq \mathrm{d}\omega^{ab,c} + h^{\{c} \wedge e^{ab\}} + h_d \wedge X^{ab,cd} = 0, \tag{12b}$$

$$\bar{F}^{ab,cd} \coloneqq \mathrm{d}X^{ab,cd} = h_e \wedge h_f C^{abe,cdf}, \tag{12c}$$

where, for clarity, we renamed the fields as in (3). These equations are invariant under

$$\delta e^{ab} = \mathrm{d}\xi^{ab}, \tag{13a}$$

$$\delta\omega^{ab,c} = \mathrm{d}\lambda^{ab,c} + h^{\{c}\xi^{ab\}} + h_d\rho^{ab,cd}, \tag{13b}$$

$$\delta X^{ab,cd} = \mathrm{d}\rho^{ab,cd}. \tag{13c}$$

Thanks to the Poincaré lemma, Eq. (12a) implies that e^{ab} is pure gauge. We can thus set it to zero using the gauge symmetry generated by ξ^{ab}. In this gauge, Eqs. (12b) and (12c) take the same form as Eqs. (3b) and (3c). To show that these two equations suffice to describe a massless particle, notice that, in the gauge $e^{ab}=0$, Eq. (12b) implies

$$h_c \wedge \bar{F}^{ab,c} = h_c \wedge \mathrm{d}\omega^{ab,c} = -\mathrm{d}(h_c \wedge \omega^{ab,c}) = 0. \tag{14}$$

This is the case because $X^{ab,cd}$ is symmetric in the last two indices and with our choice for the background vielbein, $\mathrm{d}h^a = 0$. The Poincaré lemma then allows one to introduce the one-form $\tilde{e}^{ab}$ as

$$-h_c \wedge \omega^{ab,c} = \mathrm{d}\tilde{e}^{ab}. \tag{15}$$

This relation is valid for all $\omega^{ab,c}$, in particular for a pure-gauge infinitesimal configuration $\delta\omega^{ab,c}$ for which we denote the corresponding rhs of the previous equation by $\mathrm{d}\delta\tilde{e}^{ab}$. The configuration $\delta\tilde{e}^{ab}$ is then seen to be identically equal to

$$\delta\tilde{e}^{ab} = \mathrm{d}\tilde{\xi}^{ab} + h_c\lambda^{ab,c} \tag{16}$$

(recall that we used ξ^{ab} to fix the gauge $e^{ab}=0$, so that $\tilde{\xi}^{ab}$ is a new gauge parameter that does not affect $\omega^{ab,c}$). The fields $\tilde{e}^{ab}$, $\omega^{ab,c}$ and $X^{ab,cd}$ then manifestly satisfy Eq. (3).

Arbitrary spin: For an arbitrary value s of the spin, from Eq. (9a) one finds that $\bar{F}^{a(s-1),b(s-2n-1)} = 0$ implies that $\omega^{a(s-1),b(s-2n-1)}$ can be set to zero for $n \ge 1$ with a gauge transformation (10a), thanks to the Poincaré lemma. In this gauge, most of the other torsionlike equations become closure conditions too, so that the fields $\omega^{a(s-1),b(s-2n)}$ for $n \geq 2$ can also be eliminated by gauge fixing. Eventually, one is left with

$$\bar{F}^{a(s-1),b(s-2)} := \mathrm{d}\omega^{a(s-1),b(s-2)} + h_c \wedge \omega^{a(s-1),b(s-2)c} = 0, \tag{17a}$$

$$\bar{F}^{a(s-1),b(s-1)} := \mathrm{d}\omega^{a(s-1),b(s-1)} = h_c \wedge h_d C^{a(s-1)c,b(s-1)d}. \tag{17b}$$

The first equation implies

$$h_c \wedge \bar{F}^{a(s-1),b(s-3)c} = -\mathrm{d}(h_c \wedge \omega^{a(s-1),b(s-3)c}) = 0 \tag{18}$$

and, thanks to the Poincaré lemma,

$$-h_c \wedge \omega^{a(s-1),b(s-3)c} = \mathrm{d}\tilde{\omega}^{a(s-1),b(s-3)}. \tag{19}$$

The procedure can be iterated to obtain

$$\begin{aligned} &-\mathrm{d}(h_c \wedge \tilde{\omega}^{a(s-1),b(s-k)c}) = 0 \\ &\Rightarrow -h_c \wedge \tilde{\omega}^{a(s-1),b(s-k)c} = \mathrm{d}\tilde{\omega}^{a(s-1),b(s-k)} \end{aligned} \tag{20}$$

for all $4 \leq k \leq s$, thus reconstructing the full Lopatin-Vasiliev equations on Minkowski space, i.e., the $\lambda \to 0$ limit of Eqs. (1) written in Cartesian coordinates.

Discussion. We proposed new first-order equations of motion for free massless particles of arbitrary spin in Minkowski space, built upon the linearized curvatures of the flat-space higher-spin algebra $\mathfrak{ihs}_D$ introduced in [28,29]. The set of fields that we use in our equations is the same as in Lopatin-Vasiliev's equations on (A)dS background [16], that are the free equations of motion on top of which Vasiliev's unfolded formulation for interacting higher-spin fields is constructed. Nevertheless, we stress that the precise expressions of our equations differ from the zero cosmological constant limit of those in [16]. In spite of this difference, that *a priori* could prevent one from eliminating some auxiliary fields, we showed that our equations propagate the degrees of freedom of a massless field in Minkowski space-time of dimension $D \geq 4$. This is so because all fields $\omega^{a(s-1),b(t)}$ with $t \leq s-3$ are actually pure gauge, while the field equations involving $\omega^{a(s-1),b(s-1)}$, which encode the degrees of freedom via the Weyl tensor, take the same form in both systems of equations.

As a result, the nonlinear curvatures of the non-Abelian, flat-space higher-spin algebra $\mathfrak{ihs}_D$ of [28] can be considered as the building blocks to construct an interacting higher-spin gauge theory in Minkowski space in the unfolded formalism, along the purely algebraic lines of [5,6] or of its reformulation in [30,31]. The fact that the flat-space higher-spin algebra $\mathfrak{ihs}_D$ possesses an Abelian ideal, contrary to the AdS algebra $\mathfrak{hs}_D$, suggests even more freedom in introducing interactions via the cohomological approach of [30,31].

Another remark supporting the proposal to build a nonlinear theory based on the algebra $\mathfrak{ihs}_D$ is that such a theory should include the $(2s-2)$-derivative coupling of a massless spin-s field to gravity of [21,52]. In [21] it was shown that this cubic vertex induces a non-Abelian deformation of the free gauge algebra, leading to a contribution proportional to the translation generator P_a in the commutator $[M^{a(s-1),b(s-1)}, M^{c(s-1),d(s-2)}]$. As explained below Eq. (6), the latter commutator is unaffected by the contraction leading from $\mathfrak{hs}_D$ to $\mathfrak{ihs}_D$ and does contain a contribution proportional to P_a. Moreover, as discussed in [21], the $(2s-2)$-derivative vertex possesses the highest number of derivatives among those that constitute the Fradkin-Vasiliev gravitational coupling in AdS [53], which is included in the unfolded formulation. Only this top vertex survives the flat limit, coinciding with the high-energy limit, thereby evading the low-energy no-go results [19,20].

Finally, let us stress that a non-linear theory based on the algebra $\mathfrak{ihs}_D$ is also expected to play the role of holographic dual of a Carrollian scalar at null infinity, thanks to the matching of their symmetries that we discussed in the introduction. Symmetry matching is a necessary condition to establish a dual holographic pair and we plan to further investigate the holographic dictionary between these two models, along the lines of, e.g., [39,40,54], in future work. See also [55] for similar bulk and boundary realizations of higher-spin symmetries in $D = 3$.

Acknowledgments. We thank E. Skvortsov for discussions. A. C. and S. P. are, respectively, a research associate and a FRIA grantee of the Fonds de la Recherche Scientifique - FNRS. This work was supported by FNRS under Grants No. FC.36447, No. F.4503.20, and No. T.0022.19. S. P. acknowledges the support of the SofinaBoël Fund for Education and Talent, the Foundation of the École polytechnique and the Alexander Friedmann Fund for cosmology. A. C. and S. P. thank the École polytechnique, and S. P. thanks the University of Edinburgh for hospitality.

[1] X. Bekaert, N. Boulanger, and P. Sundell, How higher-spin gravity surpasses the spin two barrier: No-go theorems versus yes-go examples, Rev. Mod. Phys. **84**, 987 (2012).
[2] A. Sagnotti, Notes on strings and higher spins, J. Phys. A **46**, 214006 (2013).
[3] S. Giombi, Higher spin—CFT duality, in *New Frontiers in Fields and Strings* (World Scientific, Singapore, 2017), pp. 137–214, https://www.worldscientific.com/doi/abs/10.1142/9789813149441_0003.
[4] X. Bekaert, N. Boulanger, A. Campoleoni, M. Chiodaroli, D. Francia, M. Grigoriev *et al.*, Snowmass White Paper: Higher spin gravity and higher spin symmetry, in *Snowmass 2021* (2022), arXiv:2205.01567.
[5] M. A. Vasiliev, Consistent equation for interacting gauge fields of all spins in $(3+1)$-dimensions, Phys. Lett. B **243**, 378 (1990).
[6] M. A. Vasiliev, Nonlinear equations for symmetric massless higher spin fields in (A)dS(d), Phys. Lett. B **567**, 139 (2003).
[7] E. Sezgin and P. Sundell, Massless higher spins and holography, Nucl. Phys. **B644**, 303 (2002).
[8] I. R. Klebanov and A. M. Polyakov, AdS dual of the critical O(N) vector model, Phys. Lett. B **550**, 213 (2002).
[9] E. S. Fradkin and M. A. Vasiliev, Candidate to the role of higher spin symmetry, Ann. Phys. (N.Y.) **177**, 63 (1987).
[10] N. Boulanger, D. Ponomarev, E. D. Skvortsov, and M. Taronna, On the uniqueness of higher-spin symmetries in AdS and CFT, Int. J. Mod. Phys. A **28**, 1350162 (2013).
[11] S. E. Konstein and M. A. Vasiliev, Extended higher spin superalgebras and their massless representations, Nucl. Phys. **B331**, 475 (1990).
[12] M. A. Vasiliev, Higher spin superalgebras in any dimension and their representations, J. High Energy Phys. 12 (2004) 046.
[13] E. Sezgin and P. Sundell, Supersymmetric higher spin theories, J. Phys. A **46**, 214022 (2013).
[14] X. Bekaert, S. Cnockaert, C. Iazeolla, and M. A. Vasiliev, Nonlinear higher spin theories in various dimensions, in *Proceedings of the 1st Solvay Workshop on Higher Spin Gauge Theories* (2004), pp. 132–197, arXiv:hep-th/0503128.
[15] V. E. Didenko and E. D. Skvortsov, Elements of Vasiliev theory, arXiv:1401.2975.
[16] V. E. Lopatin and M. A. Vasiliev, Free Massless Bosonic Fields of Arbitrary Spin in d-dimensional De Sitter Space, Mod. Phys. Lett. A **03**, 257 (1988).
[17] C. Fronsdal, Massless fields with integer spin, Phys. Rev. D **18**, 3624 (1978).
[18] E. Joung and M. Taronna, Cubic-interaction-induced deformations of higher-spin symmetries, J. High Energy Phys. 03 (2014) 103.
[19] S. Weinberg, Photons and gravitons in s-matrix theory: Derivation of charge conservation and equality of gravitational and inertial mass, Phys. Rev. **135**, B1049 (1964).
[20] M. Porrati, Universal limits on massless high-spin particles, Phys. Rev. D **78**, 065016 (2008).
[21] N. Boulanger, S. Leclercq, and P. Sundell, On the uniqueness of minimal coupling in higher-spin gauge theory, J. High Energy Phys. 08 (2008) 056.
[22] X. Bekaert, N. Boulanger, and S. Leclercq, Strong obstruction of the Berends-Burgers-van Dam spin-3 vertex, J. Phys. A **43**, 185401 (2010).
[23] D. Ponomarev and E. D. Skvortsov, Light-Front Higher-Spin Theories in Flat Space, J. Phys. A **50**, 095401 (2017).
[24] K. Krasnov, E. Skvortsov, and T. Tran, Actions for self-dual higher spin gravities, J. High Energy Phys. 08 (2021) 076.
[25] L. Ren, M. Spradlin, A. Yelleshpur Srikant, and A. Volovich, On effective field theories with celestial duals, J. High Energy Phys. 08 (2022) 251.
[26] D. Ponomarev, Towards higher-spin holography in flat space, J. High Energy Phys. 01 (2023) 084.
[27] R. Monteiro, From Moyal deformations to chiral higher-spin theories and to celestial algebras, J. High Energy Phys. 03 (2023) 062.
[28] A. Campoleoni and S. Pekar, Carrollian and Galilean conformal higher-spin algebras in any dimensions, J. High Energy Phys. 02 (2022) 150.
[29] X. Bekaert, A. Campoleoni, and S. Pekar, Carrollian conformal scalar as flat-space singleton, Phys. Lett. B **838**, 137734 (2023).
[30] A. A. Sharapov and E. D. Skvortsov, Formal higher-spin theories and Kontsevich–Shoikhet–Tsygan formality, Nucl. Phys. **B921**, 538 (2017).
[31] A. Sharapov and E. Skvortsov, Formal higher spin gravities, Nucl. Phys. **B941**, 838 (2019).
[32] A. G. Nikitin, Generalized killing tensors of arbitrary rank and order, Ukrainian Mathematical Journal **43**, 734 (1991).
[33] M. G. Eastwood, Higher symmetries of the Laplacian, Ann. Math. **161**, 1645 (2005).
[34] X. Bekaert, E. Joung, and J. Mourad, Comments on higher-spin holography, Fortschr. Phys. **60**, 882 (2012).
[35] J. de Boer and S. N. Solodukhin, A holographic reduction of Minkowski space-time, Nucl. Phys. **B665**, 545 (2003).
[36] G. Arcioni and C. Dappiaggi, Holography in asymptotically flat space-times and the BMS group, Classical Quantum Gravity **21**, 5655 (2004).
[37] L. Ciambelli, C. Marteau, A. C. Petkou, P. M. Petropoulos, and K. Siampos, Flat holography and Carrollian fluids, J. High Energy Phys. 07 (2018) 165.
[38] S. Pasterski, M. Pate, and A.-M. Raclariu, Celestial holography, in *Snowmass 2021* (2022), arXiv:2111.11392.
[39] L. Donnay, A. Fiorucci, Y. Herfray, and R. Ruzziconi, Carrollian perspective on celestial holography, Phys. Rev. Lett. **129**, 071602 (2022).
[40] A. Bagchi, S. Banerjee, R. Basu, and S. Dutta, Scattering amplitudes: Celestial and Carrollian, Phys. Rev. Lett. **128**, 241601 (2022).
[41] E. D. Skvortsov, Mixed-symmetry massless fields in Minkowski space unfolded, J. High Energy Phys. 07 (2008) 004.
[42] E. D. Skvortsov, Frame-like actions for massless mixed-symmetry fields in Minkowski space, Nucl. Phys. **B808**, 569 (2009).
[43] N. Boulanger, C. Iazeolla, and P. Sundell, Unfolding mixed-symmetry fields in AdS and the BMV conjecture: I. General formalism, J. High Energy Phys. 07 (2009) 013.
[44] C. Iazeolla and P. Sundell, A fiber approach to harmonic analysis of unfolded higher-spin field equations, J. High Energy Phys. 10 (2008) 022.
[45] N. Boulanger and E. D. Skvortsov, Higher-spin algebras and cubic interactions for simple mixed-symmetry fields in AdS spacetime, J. High Energy Phys. 09 (2011) 063.

[46] E. Joung and K. Mkrtchyan, Notes on higher-spin algebras: Minimal representations and structure constants, J. High Energy Phys. 05 (2014) 103.
[47] M. P. Blencowe, A consistent interacting massless higher spin field theory in $D = (2 + 1)$, Classical Quantum Gravity **6**, 443 (1989).
[48] H. Afshar, A. Bagchi, R. Fareghbal, D. Grumiller, and J. Rosseel, Spin-3 gravity in three-dimensional flat space, Phys. Rev. Lett. **111**, 121603 (2013).
[49] H. A. Gonzalez, J. Matulich, M. Pino, and R. Troncoso, Asymptotically flat spacetimes in three-dimensional higher spin gravity, J. High Energy Phys. 09 (2013) 016.
[50] M. Ammon, D. Grumiller, S. Prohazka, M. Riegler, and R. Wutte, Higher-spin flat space cosmologies with soft hair, J. High Energy Phys. 05 (2017) 031.
[51] M. Ammon, M. Pannier, and M. Riegler, Scalar fields in 3D asymptotically flat higher-spin gravity, J. Phys. A **54**, 105401 (2021).
[52] R. R. Metsaev, Cubic interaction vertices of massive and massless higher spin fields, Nucl. Phys. **B759**, 147 (2006).
[53] E. S. Fradkin and M. A. Vasiliev, On the gravitational interaction of massless higher spin fields, Phys. Lett. B **189**, 89 (1987).
[54] A. Saha, Carrollian approach to $1 + 3$D flat holography, J. High Energy Phys. 06 (2023) 051.
[55] D. Grumiller and W. Merbis, Near horizon dynamics of three dimensional black holes, SciPost Phys. **8**, 010 (2020).