Letter

Vacuum stability, fixed points, and phases of QED_3 at large N_f

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We consider three-dimensional quantum electrodynamics in the presence of a Chern-Simons term at level k and N_f flavors, in the limit of large N_f and k with k/N_f fixed. We consider either bosonic or fermionic matter fields, with and without quartic terms at criticality: the resulting theories are critical and tricritical bosonic QED_3, Gross-Neveu, and fermionic QED_3. For all such theories we compute the effective potentials and the β functions of classically marginal couplings, at the leading order in the large N_f limit and to all orders in k/N_f and in the couplings. We determine the renormalization group fixed points and discuss the quantum stability of the corresponding vacua. While critical bosonic and fermionic QED_3 are always stable conformal field theories, we find that tricritical bosonic and Gross-Neveu QED_3 exist as stable conformal field theories only for specific values of k/N_f. Finally, we discuss the phase diagrams of these theories as a function of their relevant deformations.

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I. INTRODUCTION

Three-dimensional quantum electrodynamics (QEDs) with either bosonic or fermionic degrees of freedom are among the simplest and yet very rich examples of gauge theories. At high energies, these theories are defined in terms of a three-dimensional $U(1)$ Maxwell field a with gauge coupling e^2 (of mass dimension one) and a Chern-Simons term at level k, coupled to N_f flavors of charged bosons or fermions. The Lagrangian is (we will always work in Euclidean signature)

$$\mathcal{L} = \frac{1}{2e^2} da \wedge \star da + \frac{ik}{4\pi} a \wedge da + \mathcal{L}_{\mathrm{matter}}. \qquad (1)$$

In absence of charged matter, if $k = 0$ the theory is dual to a compact scalar and it confines in the sense of [1,2], whereas if $k \neq 0$ it flows to a pure $U(1)_k$ Chern-Simons theory. With charged matter, the low-energy behavior of QED theories for small N_f has not been rigorously established, and this question remains an open problem which has been studied with a variety of approaches (see e.g. Refs. [3–28]), including lattice simulations (see e.g. Refs. [29–35]) and conformal bootstrap (see e.g. [27,36–44] and references therein). When N_f is large enough, these theories are generically expected to flow to interacting conformal field theories (CFTs) at low energies. This expectation is corroborated by the existence of fixed points of the renormalization group (RG) flow, which can be found in the limit of large N_f. However, as we will discuss in this work, some of these fixed points happen to lie in a region of instability of the theory.[1]

In order to clarify this issue, we will compute the effective potentials $\mathcal{V}_{\mathrm{eff}}$ for a few instances of such quantum electrodynamics (to be defined below): two bosonic, called "tricritical" and "critical" QED_3, and two fermionic, called "Gross-Neveu (GN)" and "fermionic" QED_3. While critical and fermionic QED_3 do not admit any marginal deformations, tricritical and Gross-Neveu QED_3 admit classically marginal couplings. Requiring that the quantum vacuum is stable constrains such couplings. We will perform these computations by working at the leading order in the $1/N_f$ expansion, with $\Lambda = e^2 N_f$ and $\kappa = k/N_f$ held fixed. The low-energy limit is reached by taking $\Lambda \to \infty$ while retaining the dimensionless parameter κ, which does not run. The gauge interactions do not enter $\mathcal{V}_{\mathrm{eff}}$ at the leading order in the large-N_f limit, nevertheless the parameter κ will play an important role in the following analysis.

[1]In this paper by "stability" we refer to the condition that the potential has a stable minimum, not to the RG stability of the fixed point under deformations by some coupling.

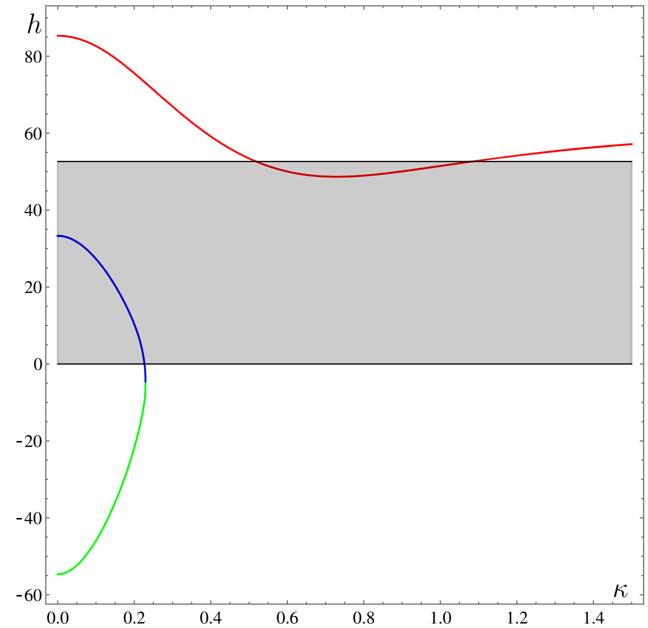

FIG. 1. Fixed points of β_h as a function of κ for tricritical QED_3. Stable vacua must lie inside the gray region, see (8).

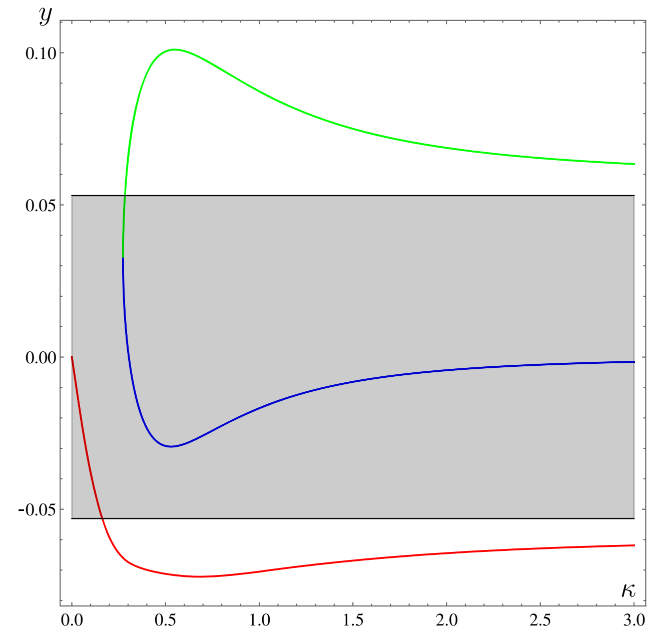

FIG. 2. Fixed points of β_y as a function of κ for Gross-Neveu QED_3. Stable vacua must lie inside the gray region, see Eq. (18).

Next, we will compute the β functions of classically marginal couplings of tricritical and Gross-Neveu QEDs, at leading order in the $1/N_f$ expansion and to all orders in κ and in the couplings. Details of the RG computations will be presented elsewhere [45]. The zeroes of the β functions determine families of RG fixed points parametrized by κ.

We finally impose on those fixed points the constraint of vacuum stability from $\mathcal{V}_{\text{eff}}$. The result of this analysis is shown in Figs. 1 and 2, which summarize the main results of this paper. In the Supplemental Material [46] we discuss the large-N_f phase diagrams of all four QEDs in presence of relevant deformations.

A general takeaway of our analysis is the importance of a joint study of the zeroes of the β function *and* of the stability of the potential when looking for perturbative fixed points. Another example where this joint analysis is important are Banks-Zaks fixed points in 4D gauge theories, see e.g. [47].

II. BOSONIC THEORIES

A. Tricritical QED

The first theory we consider is that of N_f massless complex scalars ϕ^m ($m = 1, \ldots, N_f$) coupled to $U(1)_k$ and with the quartic coupling tuned to zero. The matter Lagrangian is

$$\mathcal{L}_{\text{matter}} = (\mathcal{D}_\mu\phi^m)^\dagger(\mathcal{D}_\mu\phi^m) + \frac{h}{N_f^2}(\phi^{\dagger m}\phi^m)^3. \quad (2)$$

Here $\mathcal{D}_\mu = \partial_\mu + ia_\mu$ denotes the covariant derivative and h is held fixed in the large-N_f limit. The continuous part of the global symmetry is $SU(N_f) \times U(1)_{\text{m}}$, where the first factor is a flavor symmetry and the second one is the magnetic symmetry of the gauge field. For $\kappa = 0$ the theory further enjoys parity symmetry. In the $\kappa \to \infty$ limit the gauge field decouples and the theory describes $2N_f$ real scalars with a sextic interaction, restricted to the $U(1)$-invariant sector.

Our first goal is to compute the effective potential of (2), at the leading order in the $1/N_f$ expansion and exactly in κ and h. Following the standard strategy [48], we rewrite the sextic interaction in (2) in terms of two auxiliary fields σ and ρ

$$\sigma\left(\frac{\phi^{\dagger m}\phi^m}{\sqrt{N_f}} - \rho\right) + \frac{h}{\sqrt{N_f}}\rho^3. \quad (3)$$

Here σ is a Lagrange multiplier, while ρ is identified with the composite operator $\phi^{\dagger m}\phi^m/\sqrt{N_f}$. Then, we let $\phi^m = \sqrt{N_f}v^m + \delta\phi^m$, $\rho = \sqrt{N_f}\eta + \delta\rho$, and $\sigma = \sqrt{N_f}\Sigma + \delta\sigma$, being v^m, η, and Σ vacuum expectation values (VEV) that scale as $\mathcal{O}(N_f^0)$. Finally we path integrate over the fluctuations $\delta\phi^m$, $\delta\rho$, and $\delta\sigma$ to get (in dimensional regularization)

$$\mathcal{V}_{\text{eff}}(v^2, \Sigma, \eta) = N_f\left(\Sigma v^2 - \Sigma\eta + h\eta^3 - \frac{1}{6\pi}\Sigma^{\frac{3}{2}}\right), \quad (4)$$

where $v^2 = v^m v^m \geq 0$ (without loss of generality we take $v^m = v\delta^{m1}$), and we require $\Sigma \geq 0$. The derivatives of the potential with respect to v, Σ, and η are

$$2\Sigma v = 0, \quad v^2 - \eta - \frac{\sqrt{\Sigma}}{4\pi} = 0, \quad -\Sigma + 3h\eta^2 = 0. \tag{5}$$

We refer to these as "gap equations" for v, Σ, and η, respectively. The only stationary point is at $(\Sigma, v, \eta) = P^* \equiv (0,0,0)$, as it should since the problem has no scale. Note that if we restrict to the gap equations for v and Σ, we find two classes of solutions:

$$v \neq 0, \quad \Sigma = 0, \qquad \eta = v^2 > 0, \quad \text{(Higgsed)},$$
$$v = 0, \quad \Sigma \geq 0, \quad \eta = -\frac{\sqrt{\Sigma}}{4\pi} \leq 0, \quad \text{(unHiggsed)}. \tag{6}$$

We now discuss quantum stability. Unlike ρ, σ does not correspond on shell to a physical operator of the theory. Hence we do not study the stability as a function of its expectation value Σ, but rather we "integrate out Σ" by plugging its gap equation back into (4) to find

$$\mathcal{V}_{\text{eff}}(v^2, \eta) = N_f \left(\frac{16\pi^2}{3} (v^2 - \eta)^3 + h\eta^3 \right), \quad \eta \leq v^2, \tag{7}$$

where the rightmost condition comes from $\Sigma \geq 0$. There is a global minimum if[2]

$$0 < h < \frac{16\pi^2}{3}. \tag{8}$$

Note that, while classically $\eta \geq 0$ (and so $h \geq 0$), quantum mechanically η can be negative, but consequently Eq. (8) must hold. In other words, bosonic self-interactions are repulsive and tend to destabilize the vacuum. In the Supplemental Material [46], we discuss the effective potential and the phases of the theory in presence of a massive and a quartic deformation.

As a consistency check, note that the determinant of the Hessian of (4) restricted to the Higgsed and unHiggsed directions (6) reads, respectively,

$$\det H_{\text{H}} = -24h\eta^2, \qquad \det H_{\text{uH}} = (6h - 32\pi^2)\eta^2, \tag{9}$$

which are both negative everywhere (except at P^*) precisely when (8) is satisfied. This has to be the case: a stable quantum vacuum P^* for this theory should be at the same time a minimum of $\mathcal{V}_{\text{eff}}$ in two directions and a maximum in the third direction. Indeed, while in the path integrals for $\delta\phi^m$ and $\delta\rho$ the integration contours run along the real axis, in the one for $\delta\sigma$ it runs along the imaginary axis. Correspondingly, the Hessian of $\mathcal{V}_{\text{eff}}$ must have two positive and one negative eigenvalues in a neighborhood of P^*.

Our next task will be to verify whether (8) is satisfied at the RG fixed points of tricritical QED_3. The β function for h, at the leading order in the $1/N_f$ expansion and to all orders in κ and h reads [45,50]

$$\beta_h(h, \kappa) = \frac{1}{\pi^2 N_f} \left(-\frac{9}{256} h^3 + \frac{9}{4} h^2 + \frac{128\pi^2(\pi^2 - 128\kappa^2)}{(\pi^2 + 64\kappa^2)^2} h - \frac{16384\pi^4(\pi^2 - 192\kappa^2)}{3(\pi^2 + 64\kappa^2)^3} \right). \tag{10}$$

Being an even function of κ, we can assume $\kappa > 0$ while solving for its zeros.

Let us first inspect the zeros of β_h for $\kappa \to \infty$. In this limit (10) reproduces the known results [51] for the β function of the sextic coupling in the free ungauged $O(N)$ model.[3] It has a double zero at $h^* = 0$ (free CFT) and a single zero at $h^* = 64$. At the free point $(\phi^\dagger\phi)^3$ has a dimension of exactly 3, while being marginally irrelevant (relevant) for $h > 0$ ($h < 0$). At the single zero it is relevant, since $\partial_h \beta_h(h^* = 64, \infty) < 0$. Note that $h^* = 64$ is outside the window (8) of vacuum stability, while $h^* = 0$ is at the boundary of the window and the corresponding theory has a stable vacuum since it is free.

For $\kappa = 0$ we find a total of three fixed points: a first one at $h^* = 32(\sqrt{17} - 1)/3$, where $(\phi^\dagger\phi)^3$ is irrelevant, and two more with relevant $(\phi^\dagger\phi)^3$ at $h^* = 256/3$ and at $h^* = -32(\sqrt{17} + 1)/3$. Only the former lies within the stability region (8), and we identify it with tricritical QED_3 at large N_f and $k = 0$. Besides these special values of κ, we can solve numerically for the zeros of β_h and plot them as functions of κ, as shown in Fig. 1. We see that there are three families of solutions. A family of zeros with relevant $(\phi^\dagger\phi)^3$ (depicted in red in the figure) that exists for any value of $\kappa \geq 0$, and two families of zeros with $(\phi^\dagger\phi)^3$ irrelevant/relevant (depicted in blue/green, respectively) that exist for $\kappa \leq \kappa_0 \simeq 0.229$, above which they annihilate and move to the complex plane, until they reappear at $(\kappa, h) = (\infty, 0)$. In the figure, the region highlighted in gray corresponds to the values of h inside (8). The blue curve, which is "tricritical," is inside the stability region for κ between $\kappa = 0$ [where $h^* = 32(\sqrt{17} - 1)/3$] and $\kappa = \pi/8\sqrt{3} \simeq 0.227$ (where $h^* = 0$). The red curve, which is "tetracritical," is inside the stability region in the interval $0.518 \lesssim \kappa \lesssim 1.082$. Outside these values of κ there is no CFT with a stable vacuum.

[2] This result is consistent with Eq. (1.10) in [49], where the $\kappa \to \infty$ limit of the stability bound can be recovered as the limit $\lambda_B \to 0$. We find perfect agreement using the dictionary $4\pi^2 \lambda_B^2 x_6 = h$. A discussion on the stability of the multicritical point in the large N bosonic vector model can also be found in Sec. 10.2 of Ref. [48]. Our stability bound agrees with Eq. (10.22) therein.

[3] In particular, our result matches Eq. (2a) of [51], using the dictionary $N = 2N_f$ and $8\pi^2\lambda = 3hN_f^2$.

B. Critical QED

The second theory we consider is that of the N_f massless complex scalars, with quartic coupling λ tuned at its nontrivial critical point, i.e. $\lambda = \infty$:

$$\mathcal{L}_{\text{matter}} = (\mathcal{D}_\mu \phi^m)^\dagger (\mathcal{D}_\mu \phi^m) + \frac{\sigma}{\sqrt{N_f}} \phi^{\dagger m} \phi^m, \quad (11)$$

with $\mathcal{D}_\mu = \partial_\mu + i a_\mu$ and σ is the Hubbard-Stratonovich (HS) field with scaling dimension $2 + \mathcal{O}(1/N_f)$. We did not include a sextic term in the Lagrangian, as from the equation of motion of σ we get that $\phi^{\dagger m}\phi^m = 0$. As in the case of tricritical QED_3, the continuous global symmetry is $SU(N_f) \times U(1)_{\mathrm{m}}$ and the theory further enjoys parity symmetry when $\kappa = 0$. In the limit $\kappa \to \infty$ the gauge field decouples and theory becomes the critical $O(2N_f)$ vector model restricted to the $U(1)$-singlet sector. As it turns out, at the leading order in the $1/N_f$ expansion, critical QED_3 exists as a stable CFT for any value of κ. One can verify this by computing the effective potential $\mathcal{V}_{\text{eff}}$ of the theory at large N_f. Following a similar strategy as in the case of tricritical QED_3, we write $\phi^m = \sqrt{N_f} v^m + \delta\phi^m$ and $\sigma = \sqrt{N_f}\Sigma + \delta\sigma$, and we path integrate over the fluctuations $\delta\phi^m$ and $\delta\sigma$ to get

$$\mathcal{V}_{\text{eff}}(v^2, \Sigma) = N_f \left(\Sigma v^2 - \frac{1}{6\pi} \Sigma^{\frac{3}{2}} \right), \quad (12)$$

with again the condition $\Sigma \geq 0$. The gap equations for v and Σ read, respectively,

$$2\Sigma v = 0, \qquad v^2 - \frac{\sqrt{\Sigma}}{4\pi} = 0, \quad (13)$$

which imply that the vacuum is $(v, \Sigma) = P^* \equiv (0,0)$. In order for P^* to be a stable vacuum, in the neighborhood of P^* one eigenvalue of the Hessian is positive and one is negative. A necessary condition is therefore

$$\det H = -\frac{\sqrt{\Sigma}}{4\pi} - 4v^2 < 0, \quad (14)$$

which is satisfied everywhere (except at most P^*). Since the eigenvalues never change sign, this condition turns out to be also sufficient and we conclude that the vacuum of critical QED_3 is always stable for any value of κ.

III. FERMIONIC THEORIES

A. Gross-Neveu QED

The third theory we consider is that of N_f massless Dirac fermions ψ^m ($m = 1, \ldots, N_f$), coupled to $U(1)_k$ and with the quartic coupling g at the UV fixed point, i.e. $g = \infty$:

$$\mathcal{L}_{\text{matter}} = \bar{\psi}^m \not{\mathcal{D}} \psi^m + \frac{\sigma}{\sqrt{N_f}} \bar{\psi}^m \psi^m + \frac{y}{\sqrt{N_f}} \sigma^3, \quad (15)$$

where $\mathcal{D}_\mu = \partial_\mu + i a_\mu$ and σ is the HS field with scaling dimension $1 + \mathcal{O}(1/N_f)$, whose equation of motion imposes $\bar{\psi}^m \psi^m = 0$. We have included in the Lagrangian the classically marginal coupling y (held fixed in the large N_f limit), which is generated for any finite $\kappa \neq 0$. Similarly to the bosonic case, the continuous global symmetry is $SU(N_f) \times U(1)_{\mathrm{m}}$, and theory also enjoys parity symmetry if $\kappa = y = 0$ and N_f is even. In the limit $\kappa \to \infty$ we recover the $O(2N_f)$ Gross-Neveu model restricted to the $U(1)$-singlet sector.

To study the vacuum stability of this theory we shall compute $\mathcal{V}_{\text{eff}}$ to leading order in the $1/N_f$ expansion and to all orders in κ and y. We write $\sigma = \sqrt{N_f}\Sigma + \delta\sigma$, being Σ a VEV that scales as $\mathcal{O}(N_f^0)$, and we path integrate over ψ^m and $\delta\sigma$. Importantly, in contrast to the bosonic case, for fermionic vector models the HS field has to be path integrated along the real axis in order to get a convergent path integral. All in all we get

$$\mathcal{V}_{\text{eff}}(\Sigma) = N_f \left(y\Sigma^3 + \frac{1}{6\pi} |\Sigma|^3 \right). \quad (16)$$

In this equation the first term is the classical contribution, whereas the second term comes from quantum fluctuations. Due to the real path-integration contour of σ, the stability of the vacuum at $\Sigma = 0$ now requires that

$$\det H = 6 \left(y \,\mathrm{sign}(\Sigma) + \frac{1}{6\pi} \right) |\Sigma| > 0, \quad (17)$$

and therefore we get the stability bound[4]

$$|y| < \frac{1}{6\pi}. \quad (18)$$

Again, this condition implies that the local minimum is also a global one, since the second derivative never changes its sign. As is turns out, a cubic scalar potential for Σ—which would be classically unbounded from below since Σ can have both signs, corresponding to the sign of the effective fermion mass—is allowed by quantum corrections as long as y lies within the region (18). This is due to the fact that fermionic self-interactions are attractive and tend to stabilize the vacuum, as opposite to what happens in the bosonic case. In the Supplemental Material [46], we discuss the effective potential and the phases of the theory in presence of a massive and a quartic deformation.

[4]This result is consistent with Ref. [49], where the $\kappa \to \infty$ limit of the stability bound can be recovered as a particular case of their Eq. (1.10), namely in the limit $|\lambda_F| = 1 - |\lambda_B| \to 0$. We find perfect agreement using the dictionary $x_6/\lambda_F = -16\pi y$.

Next, we shall check whether (18) is satisfied at the RG fixed points of Gross-Neveu QED_3. For the β function for y, to leading order in the $1/N_f$ expansion and to all orders in κ and y we find [45]

$$\beta_y(y,\kappa) = \frac{32}{3\pi^2 N_f}\left(-864y^3 + \frac{3(4096\kappa^4 + 640\pi^2\kappa^2 - 3\pi^4)}{(\pi^2+64\kappa^2)^2}y + \frac{4\pi^3(320\kappa^2 - 3\pi^2)\kappa}{(\pi^2+64\kappa^2)^3}\right). \tag{19}$$

Since the zeros of β_y are odd functions of κ, we can assume $\kappa > 0$ without loss of generality.

When $\kappa = 0$, the only real zero of (19) is $y^* = 0$, which lies in the middle of the stability region of Eq. (18). At this point σ^3 is relevant, since $\partial_y\beta_y(y^* = 0, 0) < 0$, so tuning of y is needed.

When $\kappa = \infty$ the β function agrees with the findings of Ref. [49] for the ungauged Gross-Neveu model.[5] This β function has a fixed point at $y^* = 0$ with σ^3 irrelevant, and two (parity-related) zeros at $y^* = \pm\sqrt{2}/24$ with σ^3 relevant. Only the fixed point at $y^* = 0$ lies within the vacuum stability bound, and corresponds to the usual Gross-Neveu CFT. The zeros of β_y as functions of κ are presented in Fig. 2. The first family of fixed points (the red curve in the figure) has a relevant σ^3 and exists for any value of κ, interpolating from $(\kappa, y) = (0, 0)$ to $(\kappa, y) = (\infty, -\sqrt{2}/24)$. Along this curve, the fixed points are stable only for $\kappa \lesssim 0.162$. There are two more families of fixed points, the blue and green curves. These exist for $\kappa \geq \kappa_1 \simeq 0.273$, below which they annihilate and become complex conjugate. For $\kappa \to \infty$ they reach the values $y = 0$ (blue) and $y = +\sqrt{2}/24$ (green). The fixed points along the blue curve, where σ^3 is seen to be irrelevant, always lie within the region of vacuum stability. The fixed points on the green curve, where σ^3 is seen to be relevant, have a stable vacuum only if $\kappa_1 \lesssim \kappa \lesssim 0.283$. For $0.162 \lesssim \kappa \lesssim \kappa_1$ there is a "blind spot" with no stable CFT.

B. Fermionic QED

Finally, we consider N_f massless Dirac fermions with no quartic fermionic self-interaction:

$$\mathcal{L}_{\mathrm{matter}} = \bar{\psi}^m \not{\mathcal{D}} \psi^m, \tag{20}$$

where $\mathcal{D}_\mu = \partial_\mu + ia_\mu$. At large N_f this theory leads to a stable CFT for any κ: the effective potential does not depend on gauge interactions and the stability analysis is analogous to that for free fermions.

IV. DISCUSSION

In this work, we have studied the fixed points of either bosonic or fermionic three-dimensional large-N_f QEDs coupled to a Chern-Simons term at level k, with fixed $\kappa = k/N_f$. For each of these theories we computed the effective potential, as well as the β functions of the classically marginal couplings, at the leading order in the $1/N_f$ expansion and to all orders in κ and in the couplings. As a byproduct we obtained the following anomalous dimensions for critical/GN QEDs:

$$\gamma_\sigma = -\frac{16}{3\pi^2 N_f}\frac{9\pi^4 - 896\pi^2\kappa^2 + 4096\kappa^4}{(\pi^2+64\kappa^2)^2}, \tag{21}$$

and for tricritical/fermionic QEDs:

$$\gamma_{\bar{\psi}\psi} = \gamma_{\phi^\dagger\phi} = \frac{128}{3N_f}\frac{\pi^2 - 128\kappa^2}{(\pi^2+64\kappa^2)^2}. \tag{22}$$

For tricritical bosonic (GN) QED our analysis shows that a CFT with a stable vacuum can only exist when κ lies in the gray region of Fig. 1 (2). There is no such restriction for critical bosonic or fermionic QEDs.

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[5] In particular, our result matches Eq. (3.65) of Ref. [49], using the dictionary $N_F = N_f$ and $\lambda_6^F = -16\pi y$.

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