Letter

Rotating black holes in semiclassical gravity

Pedro G. S. Fernandes

School of Physics and Astronomy, University of Nottingham, University Park, Nottingham, NG7 2RD, United Kingdom

(Received 22 May 2023; accepted 1 September 2023; published 25 September 2023)

We present analytic stationary and axially symmetric black hole solutions to the semiclassical Einstein equations that are sourced by the trace anomaly. We also find evidence that the same spacetime geometry satisfies the field equations of a subset of Horndeski theories featuring a conformally coupled scalar field. We explore various properties of these solutions and determine the domain of existence of black holes. These black holes display distinctive features, such as noncircularity, a non-spherically-symmetric event horizon, and violations of the Kerr bound.

DOI: 10.1103/PhysRevD.108.L061502

I. INTRODUCTION

The trace anomaly [1,2] is a quantum-level phenomenon related to the breaking of the conformal symmetry of a conformally invariant classical theory. It occurs due to one-loop quantum corrections that result in a renormalized stress-energy tensor with expectation value $\langle T_{\mu\nu}\rangle$ and a nonzero trace. This is a general feature of quantum theories in gravitational fields, on the same footing [3–8] as the chiral anomaly in QCD, which led to the successful prediction of the decay rate of the neutral pion into two photons [9,10]. The anomalous trace is only dependent on the local curvature of spacetime and is not affected by the quantum state of the quantum fields. In a four-dimensional spacetime, it can be expressed as

$$g^{\mu\nu}\langle T_{\mu\nu}\rangle = \frac{\beta}{2}C^2 - \frac{\alpha}{2}\mathcal{G}, \tag{1}$$

where $C^2 = \frac{1}{3}R^2 - 2R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the square of the Weyl tensor, $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet scalar, and β and α are coupling constants. The Gauss-Bonnet and Weyl contributions to the anomaly are named type-A and type-B anomalies [11], respectively.

Unlike most other modifications of general relativity, incorporating the contributions from the trace anomaly into the low-energy effective field theory of gravity is essential, and it is expected to result in macroscopic effects (see the discussion in Refs. [6–8]). To account for the effects of the trace anomaly on the spacetime geometry, a semiclassical approach can be used. This involves treating spacetime classically while considering the backreaction of quantum fields. The semiclassical Einstein equations take the form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\langle T_{\mu\nu}\rangle, \tag{2}$$

where contributions from other matter sources are neglected.

Investigating the backreaction of quantum fields is often challenging, primarily because the expectation value of the renormalized stress-energy tensor $\langle T_{\mu\nu}\rangle$ is typically unknown. Exceptions exist, such as in the case of a homogeneous and isotropic spacetime, where the trace anomaly (1) completely determines the renormalized stress-energy tensor [12–14]. However, in a static and spherically symmetric system, the renormalized stress-energy tensor can only be determined up to an arbitrary function of position [15–18]. To overcome this difficulty, Ref. [19] imposed an additional condition: that the geometry should depend solely on one free function. In light of the field equations (2), this assumption is equivalent to imposing an additional equation of state on the stress-energy tensor. By doing so and considering only the type-A anomaly ($\beta = 0$), Ref. [19] was able to fully determine the renormalized stress-energy tensor. This approach led to the successful derivation of analytic static and spherically symmetric black hole solutions to the semiclassical Einstein equations (2). Interestingly, the black hole solutions obtained in Ref. [19] exhibit a logarithmic correction to their entropy. This is consistent with the expectation that the leading-order quantum corrections to black hole entropy are logarithmic [20,21]. Similar quantum corrections were also studied, e.g., in Refs. [22–26], and in the scope of the recently formulated 4D Einstein-Gauss-Bonnet theory [27–31].

In this paper, we will tackle the challenge of studying the backreaction of quantum fields sourcing the trace anomaly in a stationary and axially-symmetric setting. To achieve this, we will adopt a Kerr-Schild ansatz [32] for the metric, which will allow us to have just one free function in the geometry. By doing so, we can fully determine the renormalized stress-energy tensor and derive exact analytic stationary and axially symmetric black hole solutions to semiclassical Einstein equations (2). These solutions present quantum-corrected alternatives to the Kerr black hole [33]. Throughout this paper, we will use units where $8\pi G = c = 1$.

II. SETUP

We are interested in stationary and axially symmetric spacetimes. The simplest choice for this class of metrics follows from a Kerr-Schild ansatz [32,34,35]:

$$ds^2 = ds^2_{\rm flat} + H(\mathbf{x})(l_\mu dx^\mu)^2, \tag{3}$$

where $ds^2_{\rm flat}$ is the line element of Minkowski spacetime, $H(\mathbf{x})$ is a scalar function, and l^μ is the tangent vector to a shear-free and geodesic null congruence. In ingoing Kerr-like coordinates $x^\mu = (v, r, \theta, \varphi)$, the line element we will employ reads

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2r\mathcal{M}(r,\theta)}{\Sigma}\right)(dv - a\sin^2\theta d\varphi)^2 \\ &+ 2(dv - a\sin^2\theta d\varphi)(dr - a\sin^2\theta d\varphi) \\ &+ \Sigma(d\theta^2 + \sin^2\theta d\varphi^2), \end{aligned} \tag{4}$$

where we have taken $H(\mathbf{x}) = 2r\mathcal{M}(r,\theta)/\Sigma$ in Eq. (3), $\Sigma = r^2 + a^2\cos^2\theta$, and where $\mathcal{M}(r,\theta)$ is the mass function defining the spacetime. When we set $\mathcal{M}(r,\theta) = M$, where M is a constant that represents the ADM mass, we recover the Kerr metric [33], which is the stationary and axially symmetric black hole solution to the vacuum Einstein equations.[1]

The expectation value of the renormalized stress-energy tensor should be compatible with the metric (4) and its symmetries, and it must reduce to the anisotropic stress-energy tensor employed in Ref. [19] in the static limit. Moreover, it is important to note that the trace anomaly in Eq. (1) is insensitive to traceless contributions to the stress-energy tensor. The most general nontrivial ansatz that satisfies these constraints is [42]

$$\langle T_{\mu\nu}\rangle = 2(\rho + p_t)l_{(\mu}n_{\nu)} + p_t g_{\mu\nu} + \mu l_\mu l_\nu - 4\mathrm{Re}\{\omega l_{(\mu}\bar{m}_{\nu)}\}, \tag{5}$$

where l^μ, n^μ, and m^μ form a complex null tetrad $e_{\hat{\mu}}{}^\mu = (l^\mu, n^\mu, m^\mu, \bar{m}^\mu)$ (given in Appendix A) and obey the usual relations $l_\mu n^\mu = -m_\mu \bar{m}^\mu = -1$, with all other inner products vanishing. The quantities ρ, p_t, μ, and ω are all functions of the coordinates r and θ, and the brackets denote symmetrization in the usual way. The energy density and transverse pressure of the quantum fields are given by ρ and p_t, respectively, and both are Lorentz invariant. On the other hand, μ and ω are not Lorentz invariant and are associated with a traceless contribution. Stress-energy tensors of a similar form to Eq. (5) have been previously used to construct rotating radiating black holes [42–44], and their matter content interpreted as a combination of an anisotropic fluid and a null string fluid [45,46].

The trace of the semiclassical Einstein equations (2) together with the trace anomaly (1) yields a useful relation that depends solely on the geometry

$$R + \frac{\beta}{2}C^2 - \frac{\alpha}{2}\mathcal{G} = 0. \tag{6}$$

III. SOLVING THE SEMICLASSICAL EINSTEIN EQUATIONS

To obtain exact analytical solutions, we will set $\beta = 0$ for now, focusing on the type-A anomaly. The nonvanishing β case will be addressed later. We have five unknowns—namely, ρ, p_t, μ, ω, and $\mathcal{M}$—and four independent equations provided by the semiclassical Einstein equations (2) [shown in Appendix A for the metric ansatz (4)] and the covariant conservation of the stress-energy tensor, $\nabla_\mu\langle T^{\mu\nu}\rangle = 0$. One additional equation is needed to close the system, which can be chosen as either the trace anomaly (1) or the trace equation (6). To expedite the solution-finding process, we will take a shortcut by solving the trace equation (6) and then determining the stress-energy tensor quantities from the semiclassical Einstein equations. Note that we can only take this shortcut because our "equation of state," the trace anomaly, is such that the trace of the renormalized stress-energy tensor depends only on the metric. An alternative approach, which is equally valid, involves fully determining the stress-energy tensor using its covariant conservation and the trace anomaly (1), and subsequently employing it in the semiclassical Einstein equations to obtain a solution, as is done, e.g., in Refs. [12,19]. Both approaches lead to the same outcome.

For our spacetime ansatz (4), it can be shown that the following relations hold:

$$\Sigma R = 2\partial_r^2(r\mathcal{M}), \qquad \Sigma\mathcal{G} = 8\partial_r^2\left(\frac{r^2\mathcal{M}^2\xi}{\Sigma^3}\right), \tag{7}$$

where $\xi = r^2 - 3a^2\cos^2\theta$. To solve the trace equation (6), we use these expressions for the Ricci

[1] See also, e.g., Refs. [35–41] for other examples of black hole solutions of Kerr-Schild form in modified theories of gravity.

and Gauss-Bonnet scalars, obtaining the following equivalent condition:

$$\partial_r^2\left(r\mathcal{M}-2\alpha\frac{r^2\mathcal{M}^2\xi}{\Sigma^3}\right)=0. \tag{8}$$

This equation can trivially be solved by integrating twice and solving algebraically for $\mathcal{M}$, resulting in the general solution

$$\mathcal{M}(r,\theta)=\frac{2(M-\frac{q}{2r})}{1\pm\sqrt{1-\frac{8\alpha r\xi}{\Sigma^3}(M-\frac{q}{2r})}}, \tag{9}$$

where $M\equiv M(\theta)$ and $q\equiv q(\theta)$ are integration constants. Using the Einstein equations (2), we fully determine the renormalized stress-energy tensor by obtaining the profiles for ρ, p_t, μ, and ω.

IV. NATURE OF THE STRESS-ENERGY TENSOR

To better understand the physical origins of the stress-energy tensor (5) that sources the semiclassical Einstein equations with the type-A trace anomaly (1), one possible approach is to consider an appropriate effective action that captures the trace anomaly in a gravitational theory [6,26,38,47–50]. Such an effective action can be written as $S=\int d^4x\mathcal{L}$, where the Lagrangian density $\mathcal{L}$ takes the form [38,50]

$$\frac{\mathcal{L}}{\sqrt{-g}}=\frac{R}{2}-\frac{1}{2}(\partial\phi)^2-\frac{\phi^2}{12}R+\frac{\alpha}{2}\left[\ln(\phi)\mathcal{G}-\frac{4G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}{\phi^2}\right.$$
$$\left.-\frac{4\Box\phi(\partial\phi)^2}{\phi^3}+\frac{2(\partial\phi)^4}{\phi^4}\right], \tag{10}$$

modulo other conformally invariant terms. Observe that the Einstein-Hilbert term is supplemented with a conformally coupled scalar field [38,51–53]. It can be shown that the on-shell trace of the stress-energy tensor of the scalar gives the type-A trace anomaly (1) [38]. The theory described by the above action belongs to the Horndeski class [38,54] and is intimately connected to the well-defined scalar-tensor formulations of the 4D Einstein-Gauss-Bonnet class of theories [27–31].

The field equations resulting from the action (10) are highly complex and challenging to solve in a stationary and axially symmetric setting. They are presented in Appendix B. However, we have been able to obtain a "simple solution" to these field equations by considering the metric ansatz (4). As discussed earlier, the mass function in Eq. (9) solves the trace condition of the theory, $R-\frac{\alpha}{2}\mathcal{G}=0$. The remaining field equations are then automatically identically satisfied by choosing a constant but nonzero scalar field $\phi=\pm\sqrt{6}$, for which the theory becomes pathological. Namely, the Einstein equations are identically satisfied and reduce to $G_{\mu\nu}=G_{\mu\nu}$. Hairy black hole solutions supported by a constant conformally coupled scalar field have been previously studied, e.g., in Ref. [55]. A strong coupling issue arises for the scalar field at this particular constant value [52], which leads to a divergent effective Newton's constant. Nevertheless, it is plausible that for the class of theories in Eq. (10) and Ref. [38] alternative scalar profiles exist, resulting in the same geometry, similar to what has been observed in the static and spherically symmetric case [38] and the slowly rotating case [56,57]. To bolster this assertion, we have undertaken the task of solving the field equations for the class of theories as presented in Ref. [38]. This was done order by order in a large r expansion, where we assumed a scalar field following the behavior $\phi(r,\theta)=\sum_{n=1}^{\infty}c_n(\theta)/r^n$. During our investigation, we consistently found suitable nonsingular coefficients $c_n(\theta)$ that satisfied all the field equations up to the orders we examined. Furthermore, this perturbative solution aligns with the known scalar field profiles in the static and slowly rotating limits [38,56,57]. Although this result does not guarantee the existence of a scalar field profile consistent with our geometry in a nonperturbative manner, it provides further evidence supporting the existence of alternative scalar profiles that maintain the desired geometric properties. With the proposed asymptotic expansion for the scalar field profile, no strong coupling issue is expected to arise. This is because the scalar field is nonconstant according to this expansion, avoiding a divergence in the effective Newton's constant everywhere. Additionally, the terms containing derivatives of the scalar field in the stress-energy tensor and scalar field equation provide nontrivial contributions. This differs from the problematic case of a constant scalar field, where the Einstein equations reduce to the pathological form $G_{\mu\nu}=G_{\mu\nu}$, indicating a strong coupling issue. By instead employing a scalar field profile following the suggested asymptotic expansion, we can circumvent the strong coupling pathologies that emerge for a constant scalar field scenario. This toy model serves as an illustrative example that can help clarify the nature of the matter/energy content of the stress-energy tensor, given its unconventional form.

The fulfillment of energy conditions [58] is crucial for the viability of classical solutions to the Einstein equations. However, there is a growing body of evidence, derived from both experimental and theoretical sources, suggesting that quantum effects are likely to lead to the widespread violation of some or potentially all of these conditions (see Refs. [58–64] and references therein, as well as Refs. [65,66] for explicit examples of energy condition violations due to the trace anomaly). Due to the purely quantum nature of $\langle T_{\mu\nu}\rangle$, it is anticipated that our system will exhibit violations of at least some energy conditions. In fact, a straightforward analysis demonstrates that violations, e.g., of the weak energy condition, are already generically present in static solutions.[2] A detailed analysis of these violations in the rotating case can be done using the results of Ref. [42], but further exploration of these considerations is reserved for future investigations.

V. PROPERTIES OF THE SOLUTION

The mass function in Eq. (9) defines the spacetime, and it contains two integration constants, $M(\theta)$ and $q(\theta)$, and two different branches of solutions (as indicated by the $\pm$ sign in the denominator). Imposing the boundary condition that the mass function should approach a constant at infinity, which is interpreted as the ADM mass, leads us to choose the solution with the plus sign as the physical one, and constant M.

The integration constant $q(\theta)$ can be understood through the falloff behavior of the energy density and transversal pressure near spatial infinity, which is given by $\rho \approx p_t \approx q(\theta)/r^4 + \mathcal{O}(r^{-5})$. This falloff is characteristic of a conformal field theory with $U(1)$ squared conserved charge proportional to q. Therefore, q is interpreted as an artifact of considering all possible suitable traceless contributions to the stress-energy tensor and plays a role similar to that of an electric charge [19]. For simplicity, we set $q = 0$ from now on. The same conditions on the integration constants could be obtained by demanding that the Kerr metric is recovered as α approaches zero.

From the asymptotic behavior of the metric component $g_{v\phi}$, we can determine the angular momentum (per unit mass), which is given by a. This solution is algebraically special—namely, Petrov type II in the Petrov classification scheme [34,67]. Moreover, this solution does not satisfy the circularity conditions [68,69] in general[3] because of the angular dependency of the function $\mathcal{M}$ [70]. Other examples of noncircular black hole solutions can be found, e.g., in Refs. [71–73], in the context of scalar and vector-tensor theories.

Although the solution of Ref. [19] can be obtained in the limit where a approaches zero, it is not possible to derive the solution (9) directly by applying the Newman-Janis algorithm [74] to the static solution in Ref. [19]. This algorithm is known to fail in modified theories of gravity [75,76], and in our case, it results in a geometry that violates the trace condition (6). Our findings demonstrate the importance of not blindly applying the Newman-Janis algorithm. Additionally, in the limit of slow rotation, we also recover the slowly rotating black holes of 4D Einstein-Gauss-Bonnet gravity [56,57].

The physical singularities of the spacetime (9) can be analyzed by computing, for example, the Ricci scalar in Eq. (8). We can observe the presence of two singularities. The first one is the usual ring singularity, located at $\Sigma = 0$—i.e., $r = 0$ and $\theta = \pi/2$. The second singularity is introduced by Gauss-Bonnet quantum effects and is located where the quantity inside the square root in the mass function (9) vanishes:

$$1 - \frac{8\alpha r\xi}{\Sigma^3} M = 0, \tag{11}$$

where r is to be evaluated at $r_s(\theta)$, the location of the singularity. This condition has to be solved numerically, except for $\theta = \pi/2$, where we obtain $r_s(\pi/2) = 2(M\alpha)^{1/3}$. It is worth noting that finite-radius singularities are a common feature of theories containing Gauss-Bonnet terms, and they have been extensively studied, for instance, in Ref. [77].

To confirm that the solution (9) describes a black hole spacetime, we need to investigate the existence of the event and Killing horizons. In vacuum general relativity, the rigidity theorem holds [78], and both types of horizons coincide. However, this is not necessarily the case for the solution (9). The coordinate location of the event horizon r_H depends on the coordinate θ and is given by the solution to the differential equation [71,79,80]

$$[\partial_\theta r_H(\theta)]^2 + \Delta|_{r=r_H(\theta)} = 0, \tag{12}$$

where

$$\Delta = r^2 + a^2 - 2r\mathcal{M}(r,\theta). \tag{13}$$

The location of the Killing horizon is given by the solution to the equation $\Delta = 0$ [79,80].

To solve the differential equation (12), we used a pseudospectral method (see, e.g., Refs. [81,82]) by expanding $r_H(\theta)$ in a spectral series of even cosines, taking into account the symmetries of the problem, $\partial_\theta r_H(0) = \partial_\theta r_H(\pi/2) = 0$. We verified the existence of regular event horizons for the solution (9) in a domain, confirming that it is a black hole spacetime. The coordinate locations of the event and Killing horizons both depend on the angular coordinate θ, thereby deviating from spherical symmetry, and they coincide at the poles and equator. We remark that this angular dependence is a consequence of noncircularity. As an example, in Fig. 1, we plot the locations of the event horizon, the Killing horizon, and that of the curvature singularity in Eq. (11) for a solution with $a/M = 0.85$ and $\alpha/M^2 = 2$, where we observe that the singularity is hidden inside the event horizon.

In Fig. 2, the shaded region shows the domain of existence of black holes. For values $-1 \leq \alpha/M^2 \lesssim 6.2754$, the domain is bounded by extremal black holes. As we approach the blue line for positive (negative) couplings, the event and inner horizons, the two roots of Δ, overlap at the poles (equator). However, for higher values

[2]In the static case, in an orthonormal frame, the stress-energy tensor assumes a simple diagonal form, $\langle T_{\hat{\mu}\hat{\nu}}\rangle = \mathrm{diag}(\rho, -\rho, p_t, p_t)$.

[3]This is true for nonvanishing α and a. If either is zero, the solution is Petrov type D and obeys the circularity conditions.

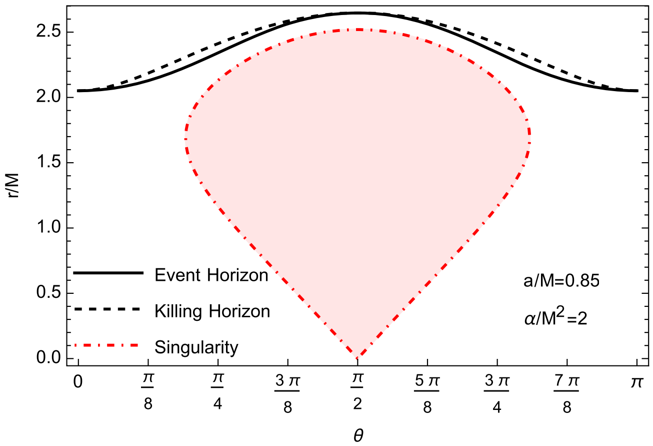

FIG. 1. Profile for the event (black solid line) and Killing (black dashed line) horizons' coordinate locations as a function of the angular coordinate θ for a black hole with $a/M = 0.85$ and $\alpha/M^2 = 2$. The red dot-dashed line indicates the location of the curvature singularity in Eq. (11). The singularity is hidden by the event horizon.

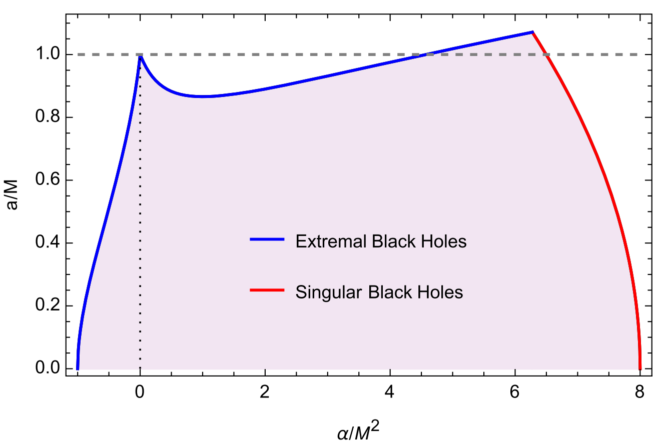

FIG. 2. Domain of existence for black hole solutions in the $(\alpha/M^2, a/M)$ plane. The shaded region represents the parameter space in which black hole solutions exist. The boundary of this region is formed by extremal (blue) and singular black holes (red).

of α/M^2, we observe the overlap of the location of the event horizon and the curvature singularity (11) at the equator as we approach the red line, resulting in a singular solution. We find violations of the Kerr bound, $a/M \leq 1$, for values $4.5698 \lesssim \alpha/M^2 \lesssim 6.4976$, with a maximum value of $a/M \approx 1.07109$ for $\alpha/M^2 \approx 6.2754$. This is the point at which the extremal and singular black hole branches come together. This kind of behavior was also observed in previous studies of black holes in Einstein-dilaton-Gauss-Bonnet gravity [83].

The region where the normalized timelike Killing vector field at infinity ∂_v becomes null is known as the ergoregion. In the spacetime described by Eq. (4), ergoregions are located at points where the condition $g_{vv} = 0$ is satisfied. We have confirmed that ergoregions are present in the entire domain of existence of black holes with the usual S^2 topology.

VI. ADDRESSING THE FULL ANOMALY

When β is nonvanishing, the Weyl tensor introduces nonlinearities into the system and closed-form solutions are not known, even in the static and spherically symmetric case [19,23]. To address the full anomaly, we take $\beta = k\alpha$ for some proportionality constant $k \sim \mathcal{O}(1)$ and solve the trace condition (6) perturbatively in powers of α/M^2. There are two special cases to consider: when $k = 0$, we recover the results and solution presented in previous sections; when $k = 1$, all terms proportional to the square of the Riemann tensor cancel in Eq. (6), leaving us with a trace condition depending solely on the Ricci tensor and scalar. In this case, it follows that the (Ricci-flat) Kerr solution $\mathcal{M}(r,\theta) = M$ solves the field equations for a vanishing stress-energy tensor. For general k, we can consider perturbative solutions in an expansion in α/M^2 away from the Kerr solution[4]

$$\mathcal{M}(r,\theta) = M + \sum_{n=1}^{\infty} \left(\frac{\alpha}{M^2}\right)^n \tilde{\mathcal{M}}_n(r,\theta). \tag{14}$$

By solving the trace condition (6) order by order, we obtain the following perturbative solution to order $(\alpha/M^2)^2$:

$$\tilde{\mathcal{M}}_1 = \frac{2(1-k)M^4 r\xi}{\Sigma^3}, \quad \tilde{\mathcal{M}}_2 = \frac{8(1-k)^2 M^7 r^2 \xi^2}{\Sigma^6}. \tag{15}$$

The perturbative solutions retain many of the characteristics of the solution (9), such as a function $\mathcal{M}$ that is dependent on both r and θ, leading to noncircularity and an event horizon that is not spherically symmetric.

It is interesting to note that in the static limit of the perturbative solution (15), a direct application of the first law of thermodynamics [19] shows that to leading order in the coupling, the black hole entropy $\mathcal{S}$ obeys

$$\mathcal{S} = \frac{A_H}{4} - 2\pi\alpha(1-k)\ln\left(\frac{A_H}{A_0}\right), \tag{16}$$

where A_H is the event horizon area, and A_0 is a (squared) length scale to be set by the ultraviolet complete theory. This means that the leading-order logarithmic corrections to the black hole entropy are a general prediction of trace anomaly quantum corrections and are not limited to the $\beta = 0$ case [19].

[4]An expansion in powers of $(1-k)$ away from the Kerr metric could also be considered.

VII. DISCUSSION

The stationary and axially symmetric solution (9) found in this paper describes a spinning black hole that incorporates trace anomaly quantum corrections. This solution possesses distinctive characteristics, including a non-spherically-symmetric event horizon and violations of the Kerr bound. It provides an alternative to the traditional Kerr black hole geometry and serves as an analytical black hole solution within the framework of scalar Gauss-Bonnet gravity [83–89], broadly defined. These unique features make it a subject worthy of further investigation and study. Suggested follow-up work includes studying the quasinormal modes, light rings, innermost stable circular orbit, and black hole mechanics/thermodynamics in depth. Regarding the latter, it is straightforward to verify that in the static limit, the surface gravity remains constant on the event horizon and that the first law of black hole mechanics, $dM = Td\mathcal{S}$, holds. However, a more meticulous analysis is necessary for the rotating case due to the angular dependence of the horizons on the coordinate θ resulting from noncircularity, and the fact that the Killing and event horizons do not coincide (except at the poles and equator). The potential departure from the standard laws of black hole mechanics would originate solely from quantum mechanical aspects, as the Kerr metric is recovered when we approach the classical limit ($\alpha \to 0$).

Additionally, it would be important to study the image features and shadow of this black hole geometry in light of observations by the Event Horizon Telescope Collaboration [90,91]. The noncircularity of the spacetime is anticipated to give rise to interesting image features [68,80,92]. Another direction for new analytic solutions would be to generalize this solution to include the effects of a cosmological constant [24].

ACKNOWLEDGMENTS

P. F. is supported by a Research Leadership Award, Grant No. RL-2016-028, from the Leverhulme Trust. P. F. thanks Tiago França for valuable discussions that led to this project, as well as Clare Burrage and Astrid Eichhorn for their insightful feedback and comments on a previous version of the manuscript.

APPENDIX A: COMPLEX NULL TETRAD AND THE EINSTEIN EQUATIONS

We define the complex null tetrad $e_{\hat{\mu}}{}^{\mu} = (l^\mu, n^\mu, m^\mu, \bar{m}^\mu)$, where

$$l^\mu = \{0,1,0,0\}, \qquad n^\mu = -\frac{1}{\Sigma}\{(r^2+a^2), \Delta/2, 0, a\}, \qquad m^\mu = \frac{1}{\sqrt{2}\sigma}\{ia\sin\theta, 0, -1, i/\sin\theta\}, \tag{A1}$$

with the overline denoting complex conjugation, $\sigma = r + ia\cos\theta$, and Δ is defined in Eq. (13). In the null basis (A1), the nonvanishing components of the stress-energy tensor take the simple form

$$\langle T_{\hat{v}\hat{r}}\rangle = \rho, \qquad \langle T_{\hat{r}\hat{r}}\rangle = \mu, \qquad \langle T_{\hat{r}\hat{\theta}}\rangle = \overline{\langle T_{\hat{r}\hat{\varphi}}\rangle} = \omega, \qquad \langle T_{\hat{\theta}\hat{\varphi}}\rangle = p_t, \tag{A2}$$

and the nontrivial components of the Einstein tensor are

$$\begin{aligned} G_{\hat{v}\hat{r}} &= \frac{2r^2\partial_r\mathcal{M}}{\Sigma^2}, \qquad G_{\hat{r}\hat{r}} = -\frac{r(\cot\theta\partial_\theta + \partial_\theta^2)\mathcal{M}}{\Sigma^2}, \\ G_{\hat{r}\hat{\theta}} &= \overline{G_{\hat{r}\hat{\varphi}}} = \frac{(-\sigma\partial_\theta + r\bar{\sigma}\partial_r\partial_\theta)\mathcal{M}}{\sqrt{2}\Sigma^2}, \qquad G_{\hat{\theta}\hat{\varphi}} = -\frac{(2a^2\cos^2\theta\partial_r + r\Sigma\partial_r^2)\mathcal{M}}{\Sigma^2}. \end{aligned} \tag{A3}$$

APPENDIX B: FIELD EQUATIONS FOR THE CONFORMALLY COUPLED SCALAR FIELD THEORY

The conformally coupled scalar field theory in Eq. (10) has field equations given by (2) with [38]

$$\begin{aligned} \langle T_{\mu\nu}\rangle = \alpha\Big[&2G_{\mu\nu}(\partial\varphi)^2 - 4{}^*R^*_{\mu\alpha\nu\beta}(\nabla^\alpha\varphi\nabla^\beta\varphi - \nabla^\beta\nabla^\alpha\varphi) + 4(\nabla_\alpha\varphi\nabla_\mu\varphi - \nabla_\alpha\nabla_\mu\varphi)(\nabla^\alpha\varphi\nabla_\nu\varphi - \nabla^\alpha\nabla_\nu\varphi) \\ &+ 4(\nabla_\mu\varphi\nabla_\nu\varphi - \nabla_\nu\nabla_\mu\varphi)\Box\varphi + g_{\mu\nu}\big(2(\Box\varphi)^2 - (\partial\varphi)^4 + 2\nabla_\beta\nabla_\alpha\varphi(2\nabla^\alpha\varphi\nabla^\beta\varphi - \nabla^\beta\nabla^\alpha\varphi)\big)\Big] \\ &+ \frac{1}{6}e^{2\varphi}\Big[G_{\mu\nu} + 2\nabla_\mu\varphi\nabla_\nu\varphi - 2\nabla_\mu\nabla_\nu\varphi + g_{\mu\nu}\big(2\Box\varphi + (\partial\varphi)^2\big)\Big], \end{aligned} \tag{B1}$$

where, for simplicity, we used the auxiliary field $\varphi = \ln\phi$, and ${}^*R^*_{\mu\alpha\nu\beta}$ is the double-dual of the Riemann tensor. The scalar field equation resulting from the action (10) is given by

$$-\frac{\alpha}{2}\left[\mathcal{G}-8R^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi+8G^{\mu\nu}\nabla_\mu\nabla_\nu\varphi+8\Box\varphi(\partial\varphi)^2-8(\nabla_\mu\nabla_\nu\varphi)^2+8(\Box\varphi)^2+16(\nabla_\mu\varphi\nabla_\nu\varphi)(\nabla^\mu\nabla^\nu\varphi)\right]$$
$$+\frac{1}{6}e^{2\varphi}\left[R-6\Box\varphi-6(\partial\varphi)^2\right]=0. \tag{B2}$$

Using the equation of motion for the scalar field, we can confirm that $g^{\mu\nu}\langle T_{\mu\nu}\rangle=-\frac{\alpha}{2}\mathcal{G}$. We can verify that $\phi=\pm\sqrt{6}$ solves the Einstein equations (2). The equation of motion for the scalar field then simplifies to $R-\frac{\alpha}{2}\mathcal{G}=0$, which can be solved by using the metric (4) and the mass function presented in Eq. (9).

[1] D. M. Capper and M. J. Duff, Trace anomalies in dimensional regularization, Nuovo Cimento Soc. Ital. Fis. **23A**, 173 (1974).
[2] M. J. Duff, Twenty years of the Weyl anomaly, Classical Quantum Gravity **11**, 1387 (1994).
[3] R. Balbinot, A. Fabbri, and I. L. Shapiro, Anomaly Induced Effective Actions and Hawking Radiation, Phys. Rev. Lett. **83**, 1494 (1999).
[4] R. Balbinot, A. Fabbri, and I. L. Shapiro, Vacuum polarization in Schwarzschild space-time by anomaly induced effective actions, Nucl. Phys. **B559**, 301 (1999).
[5] P. R. Anderson, E. Mottola, and R. Vaulin, Stress tensor from the trace anomaly in Reissner-Nordstrom spacetimes, Phys. Rev. D **76**, 124028 (2007).
[6] E. Mottola, The effective theory of gravity and dynamical vacuum energy, J. High Energy Phys. 11 (2022) 037.
[7] E. Mottola and R. Vaulin, Macroscopic effects of the quantum trace anomaly, Phys. Rev. D **74**, 064004 (2006).
[8] E. Mottola, Scalar gravitational waves in the effective theory of gravity, J. High Energy Phys. 07 (2017) 043; 09 (2017) 107(E).
[9] S. L. Adler, Axial-vector vertex in spinor electrodynamics, Phys. Rev. **177**, 2426 (1969).
[10] J. S. Bell and R. Jackiw, A PCAC puzzle: $\pi^0\to\gamma\gamma$ in the σ model, Nuovo Cimento A **60**, 47 (1969).
[11] S. Deser and A. Schwimmer, Geometric classification of conformal anomalies in arbitrary dimensions, Phys. Lett. B **309**, 279 (1993).
[12] S. W. Hawking, T. Hertog, and H. S. Reall, Trace anomaly driven inflation, Phys. Rev. D **63**, 083504 (2001).
[13] A. A. Starobinsky, A new type of isotropic cosmological models without singularity, Phys. Lett. **91B**, 99 (1980).
[14] C. P. Herzog and K.-W. Huang, Stress tensors from trace anomalies in conformal field theories, Phys. Rev. D **87**, 081901 (2013).
[15] S. M. Christensen and S. A. Fulling, Trace anomalies and the Hawking effect, Phys. Rev. D **15**, 2088 (1977).
[16] P.-M. Ho, H. Kawai, Y. Matsuo, and Y. Yokokura, Back reaction of 4D conformal fields on static geometry, J. High Energy Phys. 11 (2018) 056.
[17] J. Abedi and H. Arfaei, Obstruction of black hole singularity by quantum field theory effects, J. High Energy Phys. 03 (2016) 135.
[18] P. Beltrán-Palau, A. del Río, and J. Navarro-Salas, Quantum corrections to the Schwarzschild metric from vacuum polarization, Phys. Rev. D **107**, 085023 (2023).
[19] R.-G. Cai, L.-M. Cao, and N. Ohta, Black holes in gravity with conformal anomaly and logarithmic term in black hole entropy, J. High Energy Phys. 04 (2010) 082.
[20] D. N. Page, Hawking radiation and black hole thermodynamics, New J. Phys. **7**, 203 (2005).
[21] A. Sen, Logarithmic corrections to Schwarzschild and other non-extremal black hole entropy in different dimensions, J. High Energy Phys. 04 (2013) 156.
[22] Y. Tomozawa, Quantum corrections to gravity, arXiv:1107.1424.
[23] G. Cognola, R. Myrzakulov, L. Sebastiani, and S. Zerbini, Einstein gravity with Gauss-Bonnet entropic corrections, Phys. Rev. D **88**, 024006 (2013).
[24] R.-G. Cai, Thermodynamics of conformal anomaly corrected black holes in AdS space, Phys. Lett. B **733**, 183 (2014).
[25] M. Calzá, A. Casalino, and L. Sebastiani, Local solutions of general relativity in the presence of the trace anomaly, Phys. Dark Universe **37**, 101066 (2022).
[26] S. Tsujikawa, Black holes in a new gravitational theory with trace anomalies, Phys. Lett. B **843**, 138054 (2023).
[27] D. Glavan and C. Lin, Einstein-Gauss-Bonnet Gravity in Four-Dimensional Spacetime, Phys. Rev. Lett. **124**, 081301 (2020).
[28] H. Lu and Y. Pang, Horndeski gravity as $D\to 4$ limit of Gauss-Bonnet, Phys. Lett. B **809**, 135717 (2020).
[29] P. G. S. Fernandes, P. Carrilho, T. Clifton, and D. J. Mulryne, Derivation of regularized field equations for the Einstein-Gauss-Bonnet theory in four dimensions, Phys. Rev. D **102**, 024025 (2020).
[30] R. A. Hennigar, D. Kubizňák, R. B. Mann, and C. Pollack, On taking the $D\to 4$ limit of Gauss-Bonnet gravity: Theory and solutions, J. High Energy Phys. 07 (2020) 027.
[31] P. G. S. Fernandes, P. Carrilho, T. Clifton, and D. J. Mulryne, The 4D Einstein-Gauss-Bonnet theory of gravity: A review, Classical Quantum Gravity **39**, 063001 (2022).
[32] R. P. Kerr and A. Schild, Republication of: A new class of vacuum solutions of the Einstein field equations, Gen. Relativ. Gravit. **41**, 2485 (2009).
[33] R. P. Kerr, Gravitational Field of a Spinning Mass as an Example of Algebraically Special Metrics, Phys. Rev. Lett. **11**, 237 (1963).

[34] H. Stephani, D. Kramer, M. A. H. MacCallum, C. Hoenselaers, and E. Herlt, *Exact Solutions of Einstein's Field Equations*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2003).
[35] E. Babichev, C. Charmousis, A. Cisterna, and M. Hassaine, Regular black holes via the Kerr-Schild construction in DHOST theories, J. Cosmol. Astropart. Phys. 06 (2020) 049.
[36] O. Baake, C. Charmousis, M. Hassaine, and M. San Juan, Regular black holes and gravitational particle-like solutions in generic DHOST theories, J. Cosmol. Astropart. Phys. 06 (2021) 021.
[37] P. Beltracchi and P. Gondolo, Physical interpretation of Newman-Janis rotating systems: I. A unique family of Kerr-Schild systems, Phys. Rev. D **104**, 124066 (2021).
[38] P. G. S. Fernandes, Gravity with a generalized conformal scalar field: Theory and solutions, Phys. Rev. D **103**, 104065 (2021).
[39] E. Babichev, C. Charmousis, and A. Lehébel, Asymptotically flat black holes in Horndeski theory and beyond, J. Cosmol. Astropart. Phys. 04 (2017) 027.
[40] C. Charmousis, M. Crisostomi, R. Gregory, and N. Stergioulas, Rotating black holes in higher order gravity, Phys. Rev. D **100**, 084020 (2019).
[41] D. Psaltis, D. Perrodin, K. R. Dienes, and I. Mocioiu, Kerr Black Holes are Not Unique to General Relativity, Phys. Rev. Lett. **100**, 091101 (2008).
[42] N. Ibohal, Rotating metrics admitting nonperfect fluids in general relativity, Gen. Relativ. Gravit. **37**, 19 (2005).
[43] M. Carmeli and M. Kaye, Gravitational field of a radiating rotating body, Ann. Phys. (Amsterdam) **103**, 97 (1977).
[44] M. Murenbeeld and J. R. Trollope, Slowly rotating radiating sphere and a Kerr-Vaidya metric, Phys. Rev. D **1**, 3220 (1970).
[45] M. Gurses and F. Gursey, Lorentz covariant treatment of the Kerr-Schild metric, J. Math. Phys. (N.Y.) **16**, 2385 (1975).
[46] B. P. Brassel, S. D. Maharaj, and R. Goswami, Higher-dimensional inhomogeneous composite fluids: Energy conditions, Prog. Theor. Exp. Phys. **2021**, 103E01 (2021).
[47] R. J. Riegert, A nonlocal action for the trace anomaly, Phys. Lett. **134B**, 56 (1984).
[48] Z. Komargodski and A. Schwimmer, On renormalization group flows in four dimensions, J. High Energy Phys. 12 (2011) 099.
[49] P. O. Mazur and E. Mottola, Weyl cohomology and the effective action for conformal anomalies, Phys. Rev. D **64**, 104022 (2001).
[50] G. Gabadadze, A new gravitational action for the trace anomaly, Phys. Lett. B **843**, 138031 (2023).
[51] E. Ayón-Beato and M. Hassaine, Non-Noetherian conformal scalar fields, arXiv:2305.09806.
[52] E. Babichev, C. Charmousis, M. Hassaine, and N. Lecoeur, Selecting Horndeski theories without apparent symmetries and their black hole solutions, Phys. Rev. D **108**, 024019 (2023).
[53] E. Babichev, C. Charmousis, M. Hassaine, and N. Lecoeur, Conformally coupled scalar in Lovelock theory, Phys. Rev. D **107**, 084050 (2023).
[54] G. W. Horndeski, Second-order scalar-tensor field equations in a four-dimensional space, Int. J. Theor. Phys. **10**, 363 (1974).
[55] M. Astorino, C-metric with a conformally coupled scalar field in a magnetic universe, Phys. Rev. D **88**, 104027 (2013).
[56] C. Charmousis, A. Lehébel, E. Smyrniotis, and N. Stergioulas, Astrophysical constraints on compact objects in 4D Einstein-Gauss-Bonnet gravity, J. Cosmol. Astropart. Phys. 02 (2022) 033.
[57] M. Gammon and R. Mann, Slowly rotating black holes in 4D Gauss-Bonnet gravity, arXiv:2210.01909.
[58] P. Martin-Moruno and M. Visser, Classical and semi-classical energy conditions, Fundam. Theor. Phys. **189**, 193 (2017).
[59] H. B. G. Casimir, On the attraction between two perfectly conducting plates, Indagat. Math **10**, 261 (1948).
[60] P. Hajicek, Origin of Hawking radiation, Phys. Rev. D **36**, 1065 (1987).
[61] M. Visser, General relativistic energy conditions: The Hubble expansion in the epoch of galaxy formation, Phys. Rev. D **56**, 7578 (1997).
[62] A. Burinskii, E. Elizalde, S. R. Hildebrandt, and G. Magli, Regular sources of the Kerr-Schild class for rotating and nonrotating black hole solutions, Phys. Rev. D **65**, 064039 (2002).
[63] M. Visser, Gravitational vacuum polarization, in *Proceedings of the 8th Marcel Grossmann Meeting on Recent Developments in Theoretical and Experimental General Relativity, Gravitation and Relativistic Field Theories (MG 8)* (1997), pp. 842–844, arXiv:gr-qc/9710034.
[64] J. Santiago, S. Schuster, and M. Visser, Generic warp drives violate the null energy condition, Phys. Rev. D **105**, 064038 (2022).
[65] M. Visser, Scale anomalies imply violation of the averaged null energy condition, Phys. Lett. B **349**, 443 (1995).
[66] R. Wald and U. Yurtsever, General proof of the averaged null energy condition for a massless scalar field in two-dimensional curved spacetime, Phys. Rev. D **44**, 403 (1991).
[67] A. Z. Petrov, The Classification of spaces defining gravitational fields, Gen. Relativ. Gravit. **32**, 1661 (2000).
[68] H. Delaporte, A. Eichhorn, and A. Held, Parameterizations of black-hole spacetimes beyond circularity, Classical Quantum Gravity **39**, 134002 (2022).
[69] Y. Xie, J. Zhang, H. O. Silva, C. de Rham, H. Witek, and N. Yunes, Square Peg in a Circular Hole: Choosing the Right Ansatz for Isolated Black Holes in Generic Gravitational Theories, Phys. Rev. Lett. **126**, 241104 (2021).
[70] E. Ayón-Beato, M. Hassaïne, and D. Higuita-Borja, Role of symmetries in the Kerr-Schild derivation of the Kerr black hole, Phys. Rev. D **94**, 064073 (2016); **96**, 049902(A) (2017).
[71] T. Anson, E. Babichev, C. Charmousis, and M. Hassaine, Disforming the Kerr metric, J. High Energy Phys. 01 (2021) 018.
[72] J. Ben Achour, H. Liu, H. Motohashi, S. Mukohyama, and K. Noui, On rotating black holes in DHOST theories, J. Cosmol. Astropart. Phys. 11 (2020) 001.
[73] M. Minamitsuji, Disformal transformation of stationary and axisymmetric solutions in modified gravity, Phys. Rev. D **102**, 124017 (2020).
[74] E. T. Newman and A. I. Janis, Note on the Kerr spinning particle metric, J. Math. Phys. (N.Y.) **6**, 915 (1965).

[75] D. Hansen and N. Yunes, Applicability of the Newman-Janis algorithm to black hole solutions of modified gravity theories, Phys. Rev. D **88**, 104020 (2013).

[76] A. Kamenshchik and P. Petriakova, Regular rotating black hole: To Kerr or not to Kerr?, Phys. Rev. D **107**, 124020 (2023).

[77] P. G. S. Fernandes, D. J. Mulryne, and J. F. M. Delgado, Exploring the small mass limit of stationary black holes in theories with Gauss–Bonnet terms, Classical Quantum Gravity **39**, 235015 (2022).

[78] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2011).

[79] T. Johannsen, Systematic study of event horizons and pathologies of parametrically deformed Kerr spacetimes, Phys. Rev. D **87**, 124017 (2013).

[80] A. Eichhorn and A. Held, From a locality-principle for new physics to image features of regular spinning black holes with disks, J. Cosmol. Astropart. Phys. 05 (2021) 073.

[81] O. J. C. Dias, J. E. Santos, and B. Way, Numerical methods for finding stationary gravitational solutions, Classical Quantum Gravity **33**, 133001 (2016).

[82] P. G. S. Fernandes and D. J. Mulryne, A new approach and code for spinning black holes in modified gravity, Classical Quantum Gravity **40**, 165001 (2023).

[83] B. Kleihaus, J. Kunz, S. Mojica, and E. Radu, Spinning black holes in Einstein-Gauss-Bonnet-dilaton theory: Nonperturbative solutions, Phys. Rev. D **93**, 044047 (2016).

[84] D. D. Doneva and S. S. Yazadjiev, New Gauss-Bonnet Black Holes with Curvature-Induced Scalarization in Extended Scalar-Tensor Theories, Phys. Rev. Lett. **120**, 131103 (2018).

[85] H. O. Silva, J. Sakstein, L. Gualtieri, T. P. Sotiriou, and E. Berti, Spontaneous Scalarization of Black Holes and Compact Stars from a Gauss-Bonnet Coupling, Phys. Rev. Lett. **120**, 131104 (2018).

[86] G. Antoniou, A. Bakopoulos, and P. Kanti, Evasion of No-Hair Theorems and Novel Black-Hole Solutions in Gauss-Bonnet Theories, Phys. Rev. Lett. **120**, 131102 (2018).

[87] C. A. R. Herdeiro, E. Radu, H. O. Silva, T. P. Sotiriou, and N. Yunes, Spin-Induced Scalarized Black Holes, Phys. Rev. Lett. **126**, 011103 (2021).

[88] E. Berti, L. G. Collodel, B. Kleihaus, and J. Kunz, Spin-Induced Black-Hole Scalarization in Einstein-scalar-Gauss-Bonnet Theory, Phys. Rev. Lett. **126**, 011104 (2021).

[89] P. V. P. Cunha, C. A. R. Herdeiro, and E. Radu, Spontaneously Scalarized Kerr Black Holes in Extended Scalar-Tensor–Gauss-Bonnet Gravity, Phys. Rev. Lett. **123**, 011101 (2019).

[90] K. Akiyama *et al.* (Event Horizon Telescope Collaboration), First M87 event horizon telescope results: I. The shadow of the supermassive black hole, Astrophys. J. Lett. **875**, L1 (2019).

[91] K. Akiyama *et al.* (Event Horizon Telescope Collaboration), First Sagittarius A* event horizon telescope results: I. The shadow of the supermassive black hole in the center of the Milky Way, Astrophys. J. Lett. **930**, L12 (2022).

[92] A. Eichhorn and A. Held, Image features of spinning regular black holes based on a locality principle, Eur. Phys. J. C **81**, 933 (2021).