Letter

High-frequency gravitational wave detection via optical frequency modulation

Torsten Bringmann,[1,2] Valerie Domcke,[2] Elina Fuchs,[2,3,4] and Joachim Kopp[2,5]

[1]*Department of Physics, University of Oslo, 0316 Oslo, Norway*
[2]*Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland*
[3]*Institut für Theoretische Physik, Leibniz Universität Hannover, 30167 Hannover, Germany*
[4]*Physikalisch-Technische Bundesanstalt, 38116 Braunschweig, Germany*
[5]*PRISMA+ Cluster of Excellence & Mainz Institute for Theoretical Physics, 55128 Mainz, Germany*

(Received 30 May 2023; accepted 28 August 2023; published 20 September 2023)

High frequency gravitational waves can be detected by observing the frequency modulation they impart on photons. We discuss fundamental limitations to this method related to the fact that it is impossible to construct a perfectly rigid detector. We then propose several novel methods to search for $\mathcal{O}(\mathrm{MHz} - \mathrm{GHz})$ gravitational waves based on the frequency modulation induced in the spectrum of an intense laser beam, by applying optical frequency demodulation techniques, or by using optical atomic clock technology. We find promising sensitivities across a broad frequency range.

DOI: 10.1103/PhysRevD.108.L061303

I. INTRODUCTION

Our Universe is filled with gravitational waves (GWs) which render space and time themselves highly nonstatic. Photons traveling through such an environment are affected by GW-induced spacetime ripples in manifold ways, reminiscent of the way a watercraft is affected by rough seas.

Here we focus on modulations of the photon frequency which arise due to variations of the gravitational field along the photon trajectory and due to boundary conditions imposed by the photon emitter and absorber, such as Doppler shift. The goals of this paper are twofold: first, we discuss the physics underlying GW-induced phton frequency modulation and calculate its magnitude, with a focus on the distinction between detectors composed of free-falling test masses and detectors that are rigid. While we find that in the latter case the sensitivity grows as $\omega_g L$ (where ω_g is the angular frequency of the GW and L is the size of the detector), we demonstrate that this effect is spurious in the limit of large ω_g. In the second part of this work we propose several promising new methods for searching for high frequency GWs, based on experimental methods from quantum optics: (i) detection of sidebands in the spectrum of an intense laser; (ii) optical frequency demodulation to convert frequency shifts into an amplified electrical signal; (iii) an "optical rectifier" to ensure that the detected photons have a nonzero net frequency shift which can be measured using atomic clock techniques.

The impact of GWs on photons has previously been studied in Refs. [1–9], while using optical atomic clock technology to search for GWs has been proposed in Refs. [10–12], albeit for much lower frequencies.

II. PHOTON FREQUENCY SHIFT

We are interested in comparing the frequency of a photon, ω_γ, as measured by two different observers which we will denote source (S) and detector (D), respectively. We define the origin of our coordinate system to be the spacetime point at which the photon is emitted. We assume D is placed on the positive x^1-axis, and that the photon has initial 4-momentum $p^\mu|_{t=0} = (\omega_0, \omega_0, 0, 0)$ in the frame of a free-falling observer. In a frame with metric $g_{\mu\nu}$, an observer moving with four-velocity u^μ will measure a photon frequency $\omega_\gamma = -g_{\mu\nu} p^\mu u^\nu$.

Here we want to investigate possible effects due to tiny space-time perturbations induced by a GW that passes through S and D initially at rest. We write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1}$$

$$p^\mu = (\omega_0, \omega_0, 0, 0) + \delta p^\mu \tag{2}$$

$$u^\mu = (1, 0, 0, 0) + \delta u^\mu, \tag{3}$$

with $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$, and with $h_{\mu\nu}$, δp^μ, and δu^μ denoting $\mathcal{O}(h)$ corrections to the corresponding quantities, where h is the GW amplitude ("strain"). We thus obtain

$$\omega_\gamma = \omega_0(1 + \delta u^0 - \delta u^1 - h_{00} - h_{01}) + \delta p^0 + \mathcal{O}(h^2), \quad (4)$$

where p^0 obeys the geodesic equation

$$\frac{dp^0}{d\lambda} = -\Gamma^0_{\mu\nu} p^\mu p^\nu = -\omega_0^2(\Gamma^0_{00} + 2\Gamma^0_{10} + \Gamma^0_{11}) + \mathcal{O}(h^2). \quad (5)$$

Here, $\Gamma^\rho_{\mu\nu}$ denote the Christoffel symbols and λ is the affine parameter that parameterizes the photon geodesic, with $\lambda = 0$ corresponding to $t = 0$. At leading order in h, it is sufficient to evaluate the Christoffel symbols at $x^\mu = x^\mu_{\lambda,0} \equiv (\lambda\omega_0, \lambda\omega_0, 0, 0)$. Hence, we find

$$\delta p^0 = -\omega_0^2 \int_0^{\lambda_D} d\lambda' [\Gamma^0_{00} + 2\Gamma^0_{10} + \Gamma^0_{11}]_{x^\mu = x^\mu_{\lambda',0}}, \quad (6)$$

with λ_D being the value of λ at the spacetime point where the photon is detected. Plugging the above expression into Eq. (4), and performing some algebra, we arrive at our master formula for the observed frequency shift,

$$\frac{\omega_\gamma^D - \omega_\gamma^S}{\omega_\gamma^D} = -\frac{\omega_0}{2} \int_0^{\lambda_D} d\lambda' \partial_0 [h_{00} + 2h_{10} + h_{11}]_{x^\mu = x^\mu_{\lambda',0}} + [\delta u^0 - \delta u^1](\lambda_D) - [\delta u^0 - \delta u^1](\lambda_S). \quad (7)$$

Here, ω_γ^S is the frequency with which the photon is emitted by the source S at $t = 0$, and ω_γ^D is its frequency as measured by D. Let us stress that this result is fully general, and in fact valid for any weak gravitational field. The terms in the first line describe the effect of a varying gravitational field along the entire photon trajectory. The terms in the second line describe additional effects due to source and detector motion.

III. FREE-FALLING DETECTORS

We start with the case of S and D being in free fall. (In practice, one only needs to require that they can move freely in the direction of photon propagation.) This situation is most easily described in the transverse–traceless (TT) gauge, defined by the conditions

$$h^{TT}_{\mu 0} = 0, \qquad \partial^i h^{TT}_{ij} = 0, \qquad \eta^{ij} h^{TT}_{ij} = 0. \quad (8)$$

In this gauge, observers initially at rest remain at rest, and hence $\delta u^0 = \delta u^i = 0$ [13]. Also the metric perturbation takes a particularly simple form, with

$$h^{TT}_{11}(x^\mu) = h_+ s_\vartheta^2 \cos[\omega_g(x^0 - c_\vartheta x^1 - s_\vartheta x^3) + \varphi_0] \quad (9)$$

for a plane GW propagating in the x^1–x^3 plane at an angle ϑ from the x^1 axis. Here, φ_0 is the GW phase at $\mathbf{x} = \mathbf{0}$ when the photon is emitted, and $s_\vartheta \equiv \sin\vartheta$ and $c_\vartheta \equiv \cos\vartheta$. The GW strain is denoted by h_+, with the '+' sign indicating a polarization where one of the two quadrupole axes is aligned with the x^2 axis. GWs with the orthogonal ('×') polarization will not affect photons propagating along the x^1 axis. From Eq. (7), we directly obtain

$$\frac{\omega_\gamma^D - \omega_\gamma^S}{\omega_\gamma^D} = h_+ c^2_{\vartheta/2} \{\cos\varphi_0 - \cos[\omega_g L(1 - c_\vartheta) + \varphi_0]\}, \quad (10)$$

consistent with previous results [2,8,12]. Here, $L = x^0_D + \mathcal{O}(h)$ is the coordinate distance between the photon source and detector.

It is instructive to repeat the same calculation in the *proper detector frame*, where spatial coordinates are defined by the distances an observer using a rigid ruler would measure [13,14]. While a source placed at the origin will remain at rest also in this frame, x_D changes with time due to the force exerted by the GW. As expected and explicitly demonstrated in the SM, deriving Eq. (10) in the proper detector frame gives the same result as in the TT frame. However, this calculation reveals an intriguing cancellation between terms coming from the first and second line in Eq. (7). Remarkably, the additional terms that eventually cancel generically grow with $\omega_g L$, that is, if the cancellation could be avoided, they would lead to strongly enhanced experimental sensitivities for $\omega_g \gg 1/L$.

IV. RIGID EXPERIMENTAL SETUPS

Partially motivated by this observation, we next consider a situation where S and D are not free-falling, but are kept at rest in the proper detector frame, i.e., $\delta u^0 = \delta u^i = 0$. This leads to a frequency shift that contains terms growing with $\omega_g L$. In the most favorable case of an incoming GW perpendicular to the laser beam, $\vartheta = \pi/2$, the general result given in the SM simplifies to

$$\frac{\omega_\gamma^D - \omega_\gamma^S}{\omega_\gamma^D} = \frac{h_+}{2} \Big\{ \cos\varphi_0 - \omega_g L \sin(\omega_g L + \varphi_0) + \left(\frac{1}{2}\omega_g^2 L^2 - 1\right) \cos(\omega_g L + \varphi_0) \Big\}. \quad (11)$$

At face value this would imply detection prospects that are even enhanced as $\sim(\omega_g L)^2$ at high GW frequencies. To the experienced reader, this may sound too good to be true. And indeed, constructing a perfectly rigid ruler on such scales turns out to be impossible. To show this, we model the material separating the photon source and detector as a chain of harmonic oscillators in the x^1-direction. Writing the displacement of an oscillator at x^1 from its rest position as ξ, the oscillator equation for such a system is [13]

$$\ddot{\xi} - \frac{\omega_0^2 L^2}{\pi^2} \xi'' + \gamma\dot{\xi} = \frac{1}{2} x^1 \ddot{h}^{TT}_{11}, \quad (12)$$

where $\ddot{\xi} \equiv d^2\xi/dt^2$, $\xi'' \equiv d^2\xi/d(x^1)^2$, ω_0 is the resonance frequency of the fundamental mode of the system, γ is a damping coefficient, and we have again used $\vartheta = \pi/2$. Note that Eq. (12) is the equation of motion of an oscillator in the proper detector frame, even though the metric perturbation is the much simpler one from the TT frame. In the limit $\omega_g \gg \omega_0, \gamma$, the last two terms on the left hand side become negligible, since $\xi \propto hL$, $\dot{\xi} \propto \omega_g hL$, $\ddot{\xi} \propto \omega_g^2 hL$, $\xi'' \propto h/L$, and $\ddot{h}_{11}^{TT} \sim \omega_g^2 h$. In this limit, the equation therefore becomes identical to that of a free-falling test mass in the PD frame (see SM), with correction terms suppressed by ω_0^2/ω_g^2. At large ω_g, the two extremities of our "rigid" setup are thus responding to the GW just like free-falling test masses would, which is quite the opposite of what one would expect from a rigid system. (In the opposite limit $\omega_g \ll \omega_0$, on the other hand, we recover the expected behavior of a rigid ruler [13].)

The fact that Eq. (11) cannot be naïvely applied for $\omega_g L \gtrsim v_s$, where v_s is the velocity of sound in the detector, is by itself an important observation for the construction of high frequency GW detectors and one of the main results of this letter. Our explicit calculations above assume a very simple setup, i.e., a source and detector separated by some material. Similar arguments will however also apply to more complex systems, such as electronic equipment used to generate "static" electromagnetic fields in electromagnetic GW detectors [15,16]. Moreover, even though using the idealized setup of a rigid ruler, our discussion illustrates the general importance of the boundary conditions in Eq. (7). Suitable choices of material and suspension thus have the potential of influencing the sensitivity of high frequency GW searches, and need to be studied carefully on a case-by-case basis.

V. FREQUENCY MODULATION OF A LASER BEAM

We now consider a continuous flux of photons in a laser beam. In Eqs. (10) and (11), the photon frequency is then modulated by the phase $\varphi_0(t) = \omega_g t$ of the GW at the time of photon emission, where (without loss of generality) we have set the phase to zero at $t = 0$. As we observe the photons arrive at the detector over some finite time interval, φ_0 oscillates with frequency ω_g. For a photon coherence length $\gg 1/\omega_g$, this leads to sidebands at $\omega_\gamma^\pm \equiv \omega_\gamma^S \pm \omega_g$ in the spectrum. Quantitatively, the emitted photon wave takes the form $A(t,0) = A_\gamma \cos(\omega_\gamma^S t + \phi_\gamma)$, with amplitude A_γ and phase ϕ_γ. After propagation, this becomes

$$\begin{aligned}\frac{A(t,L)}{A_\gamma} &= \cos\left(\int_0^t \omega_\gamma^D(t')dt' + \phi_\gamma'\right) = \cos(\omega_\gamma^S t + \phi_\gamma') \\ &+ \frac{h_+}{4}\frac{\omega_\gamma^S}{\omega_g}[\widetilde{\sin}(\omega_\gamma^S, -\omega_g L) + \widetilde{\sin}(\omega_\gamma^S, \omega_g L) + \widetilde{\sin}(\omega_\gamma^-) \\ &- \widetilde{\sin}(\omega_\gamma^-, \omega_g L) - \widetilde{\sin}(\omega_\gamma^+) + \widetilde{\sin}(\omega_\gamma^+, \omega_g L)], \qquad (13)\end{aligned}$$

where we have introduced $\widetilde{\sin}(\omega, \varphi) \equiv \sin(\omega t + \phi_\gamma' + \varphi)$, denoting with ϕ_γ' the photon phase at $t = 0$ and $x = L$. For simplicity, we have assumed here and in the following that source and detector are freely falling, as in Eqs. (10) and (11), and that $\vartheta = \pi/2$. The first term on the right-hand side of Eqs. (13) is the carrier wave, i.e., the unperturbed photon signal. The first two terms in the second line describe tiny corrections to the amplitude of the carrier wave; these are irrelevant in practice. The remaining four terms generate the sidebands. In the following, we discuss three different ways that may allow the detection of such a signal.

VI. DIRECT OBSERVATION OF SIDEBANDS

For large ω_g, the sidebands in Eq. (13) are separated by a relatively large frequency gap from the carrier frequency ω_γ^S. However, their *intensity* is suppressed by h_+^2. (Experimental attempts to detect the interference term, linear in h_+, would have to deal with an overwhelming background of photons from the main carrier line. Heterodyne detection schemes, modulating the carrier line with a beat frequency such that the frequency difference between this beat frequency and the GW frequency becomes tractable for readout, may provide an interesting possibility to overcome this challenge and are left for future work.) Detecting such faint sidebands requires a powerful photon source that is highly monochromatic, complemented by a very efficient optical filter system that removes the carrier frequency after the photons have propagated to the detector.

In this respect, optical cavities or techniques from fiber optics may offer a promising avenue towards tabletop high frequency GW detectors. Let us consider a filter of width $\Delta\lambda$ which suppresses the main carrier frequency by $\alpha_T \ll 1$ while ensuring an $\mathcal{O}(1)$ transmission at the location of the sideband. In Fig. 1 we consider filter efficiencies of $\alpha_T = 10^{-10}..10^{-20}$ and a bandwidth of $\Delta\lambda \simeq 100$ kHz, which may e.g., be achieved by employing optical cavities tuned to the sideband frequency [17], or potentially also by stacking multiple fiber Bragg gratings [18,19]. We will neglect propagation effects induced by the GW in this filtering system, noting that they can be suppressed by choosing a suitable geometry (e.g., parallel to the incoming GW).

We will further assume that the sensitivity to a gravitational wave signal is only limited by the requirement to find a sufficiently large number s of signal photons in the side bands. From Eq. (13), we have $s \simeq (P\tau/\omega_\gamma^S) \times h_+^2(\omega_\gamma^S/\omega_g)^2 \ \min(1, \omega_g^2 L^2)$, where P is the laser power and τ is the signal duration or measurement time, whichever is shorter. The sensitivity curves we show as orange lines in Fig. 1 assume a mW laser emitting at a wave length of 1500 nm. They are based on requiring s to be larger than the square root of the number of spillover photons from the carrier mode, $n_{\mathrm{s.o.}} \simeq \alpha_T P\tau/\omega_\gamma^S$ plus the number of photons due to thermal noise in the optical filter system,

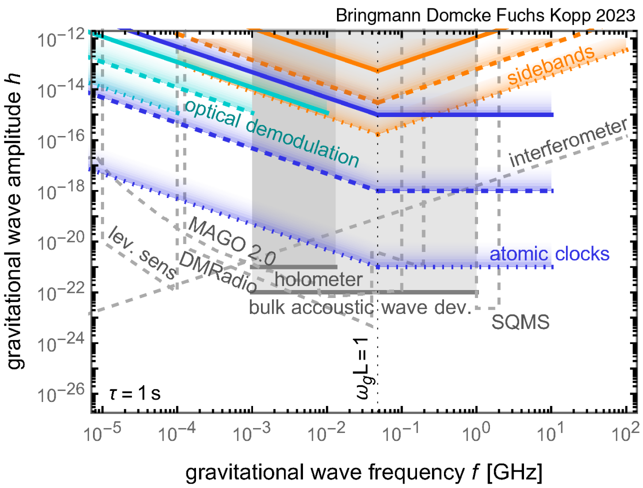

FIG. 1. Sensitivity estimates for the three novel high frequency GW detection methods proposed here (colored lines). For each proposal we show the sensitivity under conservative (solid), realistic (dashed), and optimistic (dotted) assumptions for the achievable experimental sensitivity, in particular $(\alpha_T, \alpha_{th}) = \{(10^{-10}, 10^{-15}), (10^{-15}, 10^{-17}), (10^{-20}, 10^{-19})\}$, $\sigma = 10$, 1, 0.1 MHz and $\delta = 10^{-15}, 10^{-18}, 10^{-21}$. In all cases we have set $\tau = 1$ s, $L = 1$ m, $\omega_\gamma^S/2\pi = 2 \times 10^{14}$ Hz and $P =$ mW. The gray shaded regions indicate other existing (solid) and proposed (dashed) experiments, in particular interferometers (extrapolated from Ref. [22]), levitated sensors [23], axion haloscopes such as DMRadio GUT [16], the holometer experiment [24], bulk acoustic wave devices [25] and microwave cavities like SQMs [15] and MAGO 2.0 [26].

$n_{th} \simeq p^{th}(\omega_\gamma^S + \omega_g, \alpha_{th}) P\tau/\omega_\gamma^S$. Here, we take the thermal noise spectrum $p^{th}(\omega, \alpha_{th})$ to be a Lorentzian centered at ω and with relative width α_{th}. Optical fiber links with noise levels below $\alpha_{th} \simeq 10^{-17}$ have been described in Refs. [20,21].

VII. OPTICAL DEMODULATION

A GW modulates the frequency of a propagating photon in the same way an FM radio transmitter modulates the frequency of an FM carrier, for which advanced demodulation techniques exist. Inspired by Refs. [27,28], we propose here to split the incoming photon beam and filter it through two slightly detuned optical cavities (alternatively, a setup using fiber Bragg gratings could be envisioned). The system is adjusted such that the carrier frequency lies exactly between the transmission peaks of the two cavities, and the two filtered components interfere destructively in a photon detector. A slight frequency shift due to the modulation will then enhance the signal in one cavity with respect to the other, disrupting the destructive interference and thus creating a nonzero signal in the detector. This signal is converted into a voltage whose evolution with time carries the information originally encoded in the optical wave.

The main challenge is the tiny $\mathcal{O}(h)$ amplitude of the frequency modulation. Notably, the relative width of the carrier mode should be smaller than h—in which case the Heisenberg uncertainty principle dictates coherence times much longer than $2\pi/\omega_g$. Another relevant consideration is laser shot noise. The degree to which destructive interference between the two filtered beams can be realized is subject to Poisson fluctuations in the intensity of each mode. Assuming transmission profiles shifted by half of the profile width σ to either side of the carrier frequency, we estimate the number of signal photons after interference as $s \simeq h_+ P\tau \min(\omega_g L, 1)/\sigma$. Notably, the number of signal photons now scales linearly and no longer quadratically with the GW amplitude. Requiring s to be larger than the square root of the shot noise, $n_{sh} \simeq P\tau/\omega_\gamma^S$, yields the sensitivity estimates shown in Fig. 1 (cyan lines). In the plot, we have considered values of σ between 10 MHz and 0.1 MHz. In this case thermal noise is subdominant as long as $\alpha_{th}\omega_\gamma^S \ll \sigma$. Note that small σ implies long integration times (high finesse); for $\omega_g \gtrsim \sigma$, the photons' retention time inside the cavity would be $> 1/\omega_g$, and the signal would average to zero, as indicated by the endpoint of the cyan lines in Fig. 1.

VIII. ATOMIC CLOCK TECHNIQUES

The most powerful methods for detecting tiny frequency shifts in optical signals have been developed in the context of optical atomic clocks [29,30], allowing e.g., for gravitational redshift measurements over $\mathcal{O}$(mm) distances [31]. The techniques of Refs. [10–12], however, based on optical lattice clocks in a space-based GW detector, are not directly applicable for our purposes: achieving the desired frequency resolution would require extremely narrow optical lines, and hence long integration times beyond several seconds. The effect of a high frequency GW would average out over such time intervals.

Here, we propose to use an "optical rectifier" to prevent averaging. In the simplest case, this means blocking the optical signal during half of each GW period using a shutter. The net frequency shift of the photons will then be nonzero, $\Delta\omega_\gamma \sim h_+ \omega_\gamma^S \min(1, \omega_g L)$, and can be detected for instance by comparing the shifted photon frequency to an atomic reference transition using Ramsey spectroscopy [12]. By splitting off part of the beam before the optical rectifier, the laser can be locked to the atomic transition such that a passing GW appears as de-tuning of the beam passing the optical rectifier. Such an approach seemingly requires prior knowledge of the GW frequency and phase, which is of course not possible. However, if the GW signal features a broad, or time-varying, frequency spectrum, being sensitive in only a very narrow frequency interval is sufficient to make a detection. We propose to operate at least two detectors in parallel, with the phases of their optical rectifiers offset by $\pi/2$, such that at least one of them can observe a nonzero signal.

The resulting GW sensitivity shown in Fig. 1 (blue lines) is based on a sensitivity to frequency shifts of $\Delta\omega_\gamma/\omega_\gamma^S \simeq \delta \cdot (1\ \mathrm{sec}/\tau)^{1/2}$, assuming that the measurement is limited by statistical uncertainty. Here, as before, $\Delta\omega_\gamma$ is given by Eq. (10). Accuracies of $\delta \simeq \{3 \times 10^{-18}, 9.7 \times 10^{-18}\}$ have been achieved with optical clock techniques in Refs. [31,32], respectively. With precision doubling roughly every year [30,33], significant future improvements seem possible. ^{229}Th nuclear clocks [34,35] are expected to reach even better precision. The method outlined here is limited at low frequencies by the stability of the laser and at high frequencies by the shutter speed, but we expect it to be feasible within the frequency range shown in Fig. 1.

IX. COMPARISON WITH OTHER LIMITS

Gray lines in Fig. 1 show existing (solid, shaded) and projected (dashed) limits on high frequency GWs from the literature. We caution that these serve to guide the eye only, due to different search strategies and assumptions on the signal duration. Concretely, for Refs. [22–25] quoting bounds or projected sensitivities in terms of a power spectral density, $(S_n)^{1/2}$, we show the limit on the amplitude of coherent signals obtained as $h < (S_n/\tau)^{1/2}$ whereas for Refs. [15,16,26] quoting bounds on the dimensionless gravitational wave amplitude based on an observation time T_{obs}, we rescale the limits as $(T_{\mathrm{obs}}/\tau)^{1/4}$, assuming the coherence of the signal to be limited only by the signal duration. For recasting [16] we have assumed an observation time per frequency bin T_{obs} of 1 s, for [15] we work with the values quoted in those references. Finally, for [26] we show the limits on the GW amplitude as quoted therein since with the proposed scanning strategy, the sensitivity is limited by the measurement time per frequency and not the signal duration in most of the parameter space of interest. In Fig. 1 we adopted a common signal duration of $\tau = 1\ \mathrm{sec}$ (in the SM we show, for comparison, the case of $\tau = 10^5\ \mathrm{sec}$). See Ref. [36] for details.

X. CONCLUSIONS

We revisited the frequency modulation of photons in a GW background, pointing out fundamental limits to detecting this effect and proposing three novel experimental setups which promise highly competitive sensitivities to high frequency GWs. We stress that the methods outlined here are in no way expected to be exhaustive of the possibilities of searching for GW induced frequency shifts in optical systems. In fact, our work aims to trigger more in-depth studies of these and related ideas. To further aid this development, we summarize in the SM some of our "failed attempts" of using electromagnetic precision experiments in this context, hoping that the lessons learned from these considerations might be instructive.

ACKNOWLEDGMENTS

It is a pleasure to thank Wolfram Ratzinger for very illuminating discussions on the Doppler shift of photons in a GW background, Johannes Skaar for educating us on Bragg filters and fiber optics in general, Jun Ye for innumerable crucial insights into the physics of atomic clocks, Fritz Wagner for sharing his expertise on the Mössbauer effect, Klemens Hammerer for useful explanations on sideband detection with cavities, Tom Melia, Piet Schmidt, and Tadahiro Takahashi for illuminating discussions on optical clock comparisons, Lingze Duan for important insights on optical demodulation, Camilo Garcia Cely, Sebastian Ellis, Sung Mook Lee, and Nick Rodd for their insights on comparing high frequency GW sensitivities as well as on the quirks of the proper detector frame, and Clara Murgui for her explanations around optomechanical cavities. We moreover thank Camilo Garcia Cely, Lingze Duan, Klemens Hammerer, and Tadahiro Takahashi for valuable comments on the manuscript. E. F. acknowledges support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy—EXC-2123 "QuantumFrontiers"—390837967.

[1] W. J. Kaufmann, Redshift fluctuations arising from gravitational waves, Nature (London) **227**, 157 (1970).
[2] F. B. Estabrook and H. D. Wahlquist, Response of Doppler spacecraft tracking to gravitational radiation., Gen. Relativ. Gravit. **6**, 439 (1975).
[3] M. Tinto and J. W. Armstrong, Spacecraft Doppler tracking as a narrow band detector of gravitational radiation, Phys. Rev. D **58**, 042002 (1998).
[4] S. M. Kopeikin, G. Schaefer, C. R. Gwinn, and T. M. Eubanks, Astrometric and timing effects of gravitational waves from localized sources, Phys. Rev. D **59**, 084023 (1999).
[5] M. Tinto, F. B. Estabrook, and J. W. Armstrong, Time delay interferometry for LISA, Phys. Rev. D **65**, 082003 (2002).
[6] S. Siparov, A two-level atom in the field of a gravitational wave—on the possibility of parametric resonance, Astron. Astrophys. **416**, 815 (2004).
[7] J. W. Armstrong, Low-frequency gravitational wave searches using spacecraft Doppler tracking, Living Rev. Relativity **9**, 1 (2006).

[8] J. a. C. Lobato, I. S. Matos, L. T. Santana, R. R. R. Reis, and M. O. Calvão, Influence of gravitational waves upon light in the Minkowski background: From null geodesics to interferometry, Phys. Rev. D **104**, 024024 (2021).
[9] S. Weinberg, *Cosmology* (Oxford University Press, New York, 2008), see in particular discussion of ISW effect, Eq. (7.1.29).
[10] A. Loeb and D. Maoz, Using atomic clocks to detect gravitational waves, arXiv:1501.00996.
[11] A. C. Vutha, Optical frequency standards for gravitational wave detection using satellite Doppler velocimetry, New J. Phys. **17**, 063030 (2015).
[12] S. Kolkowitz, I. Pikovski, N. Langellier, M. D. Lukin, R. L. Walsworth, and J. Ye, Gravitational wave detection with optical lattice atomic clocks, Phys. Rev. D **94**, 124043 (2016).
[13] M. Maggiore, *Gravitational Waves. Vol. 1: Theory and Experiments*, Oxford Master Series in Physics (Oxford University Press, New York, 2007).
[14] M. Rakhmanov, Fermi-normal, optical, and wave-synchronous coordinates for spacetime with a plane gravitational wave, Classical Quantum Gravity **31**, 085006 (2014).
[15] A. Berlin, D. Blas, R. Tito D'Agnolo, S. A. R. Ellis, R. Harnik, Y. Kahn, and J. Schütte-Engel, Detecting high-frequency gravitational waves with microwave cavities, Phys. Rev. D **105**, 116011 (2022).
[16] V. Domcke, C. Garcia-Cely, and N. L. Rodd, Novel Search for High-Frequency Gravitational Waves with Low-Mass Axion Haloscopes, Phys. Rev. Lett. **129**, 041101 (2022).
[17] D. G. Matei, T. Legero, S. Häfner, C. Grebing, R. Weyrich *et al.*, 1.5 μm Lasers with Sub-10 mHz Linewidth, Phys. Rev. Lett. **118**, 263202 (2017).
[18] J. Canning, Fibre gratings and devices for sensors and lasers, Laser Photonics Rev. **2**, 275 (2008).
[19] J. Chen, B. Liu, and H. Zhang, Review of fiber bragg grating sensor technology, Front. Optoelectron. China **4**, 204 (2011).
[20] T. Akatsuka, H. Ono, K. Hayashida, K. Araki, M. Takamoto, and H. Katori, 30-km-long optical fiber link at 1397 nm for frequency comparison between distant strontium optical lattice clocks, Jpn. J. Appl. Phys. **53**, 032801 (2014).
[21] M. Schioppo *et al.*, Comparing ultrastable lasers at 7×10^{-17} fractional frequency instability through a 2220 km optical fibre network, Nat. Commun. **13**, 212 (2022).
[22] B. P. Abbott *et al.* (KAGRA, LIGO Scientific, and Virgo, Collaborations), Prospects for observing and localizing gravitational-wave transients with Advanced LIGO, Advanced Virgo and KAGRA, Living Rev. Relativity **21**, 3 (2018).
[23] N. Aggarwal, G. P. Winstone, M. Teo, M. Baryakhtar, S. L. Larson, V. Kalogera, and A. A. Geraci, Searching for New Physics with a Levitated-Sensor-Based Gravitational-Wave Detector, Phys. Rev. Lett. **128**, 111101 (2022).
[24] A. S. Chou *et al.* (Holometer Collaboration), MHz gravitational wave constraints with decameter Michelson interferometers, Phys. Rev. D **95**, 063002 (2017).
[25] M. Goryachev and M. E. Tobar, Gravitational wave detection with high frequency phonon trapping acoustic cavities, Phys. Rev. D **90**, 102005 (2014).
[26] A. Berlin, D. Blas, R. T. D'Agnolo, S. A. R. Ellis, R. Harnik *et al.*, MAGO2.0: Electromagnetic cavities as mechanical bars for gravitational waves, arXiv:2303.01518.
[27] D. Barot and L. Duan, A novel frequency-modulation (fm) demodulator for microwave photonic links based on polarization-maintaining fiber bragg grating, in *Optical Fiber Communication Conference (OFC) 2020, San Diego, CA, USA* (Optica Publishing Group, 2020), p. M1C.7, https://opg.optica.org/conference.cfm?meetingid=5&yr=2020.
[28] D. Barot, R. Zhou, and L. Duan, Optical phase/frequency demodulation using polarization-maintaining fiber Bragg gratings, J. Lightwave Technol. **40**, 7307 (2022).
[29] A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, Optical atomic clocks, Rev. Mod. Phys. **87**, 637 (2015).
[30] M. S. Safronova, D. Budker, D. DeMille, D. F. J. Kimball, A. Derevianko, and C. W. Clark, Search for new physics with atoms and molecules, Rev. Mod. Phys. **90**, 025008 (2018).
[31] T. Bothwell, C. J. Kennedy, A. Aeppli, D. Kedar, J. M. Robinson, Eric Oelker, A. Staron, and J. Ye, Resolving the gravitational redshift within a millimeter atomic sample, Nature (London) **602**, 420 (2022).
[32] X. Zheng, J. Dolde, V. Lochab, B. N. Merriman, H. Li, and S. Kolkowitz, Differential clock comparisons with a multiplexed optical lattice clock, Nature (London) **602**, 425 (2022).
[33] E. F. Arias, D. Matsakis, T. J. Quinn, and P. Tavella, The 50th anniversary of the atomic second, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **65**, 898 (2018).
[34] C. J. Campbell, A. G. Radnaev, A. Kuzmich, V. A. Dzuba, V. V. Flambaum, and A. Derevianko, Single-Ion Nuclear Clock for Metrology at the 19th Decimal Place, Phys. Rev. Lett. **108**, 120802 (2012).
[35] E. Peik, T. Schumm, M. S. Safronova, A. Pálffy, J. Weitenberg, and P. G. Thirolf, Nuclear clocks for testing fundamental physics, Quantum Sci. Technol. **6**, 034002 (2021).
[36] N. Aggarwal *et al.*, Challenges and opportunities of gravitational-wave searches at MHz to GHz frequencies, Living Rev. Relativity **24**, 4 (2021).
[37] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevD.108.L061303 for further technical details to supplement the information in the main text; additionally, we discuss possible GW sources at high frequencies, and we provide an overview of concepts and ideas that are *not* suitable for the construction of high-frequency GW detectors, which includes Refs. [38–61].
[38] J. Aasi *et al.* (LIGO Scientific Collaboration), Advanced LIGO, Classical Quantum Gravity **32**, 074001 (2015).
[39] F. Acernese *et al.* (Virgo Collaboration), Advanced Virgo: A second-generation interferometric gravitational wave detector, Classical Quantum Gravity **32**, 024001 (2015).
[40] T. Akutsu *et al.* (KAGRA Collaboration), Overview of KAGRA: Detector design and construction history, Prog. Theor. Exp. Phys. **2021**, 05A101 (2021).
[41] J. Casalderrey-Solana, D. Mateos, and M. Sanchez-Garitaonandia, Mega-hertz gravitational waves from neutron star mergers, arXiv:2210.03171.

[42] A. Ito and J. Soda, A formalism for magnon gravitational wave detectors, Eur. Phys. J. C **80**, 545 (2020).

[43] L. Parker, One electron atom as a probe of space-time curvature, Phys. Rev. D **22**, 1922 (1980).

[44] T. K. Leen, L. Parker, and L. O. Pimentel, Remote quantum mechanical detection of gravitational radiation, Gen. Relativ. Gravit. **15**, 761 (1983).

[45] U. Fischer, Transition probabilities for a Rydberg atom in the field of a gravitational wave, Classical Quantum Gravity **11**, 463 (1994).

[46] F. Pinto, Rydberg atoms as gravitational wave antennas, Gen. Relativ. Gravit. **27**, 9 (1995).

[47] P. W. Graham, J. M. Hogan, M. A. Kasevich, S. Rajendran, and R. W. Romani (MAGIS Collaboration), Mid-band gravitational wave detection with precision atomic sensors, arXiv:1711.02225.

[48] Y. A. El-Neaj *et al.* (AEDGE Collaboration), AEDGE: Atomic experiment for dark matter and gravity exploration in space, Eur. Phys. J. Quantum Technol. **7**, 6 (2020).

[49] L. Badurina *et al.*, AION: An atom interferometer observatory and network, J. Cosmol. Astropart. Phys. 05 (2020) 011.

[50] I. Alonso *et al.*, Cold atoms in space: Community workshop summary and proposed road-map, Eur. Phys. J. Quantum Technol. **9**, 30 (2022).

[51] R. L. Mössbauer, Kernresonanzfluoreszenz von Gammastrahlung in Ir^{191}, Z. Phys. **151**, 124 (1958).

[52] H. Frauenfelder, *The Mössbauer Effect* (W. A. Benjamin Inc., New York, 1962).

[53] M. D. Dyar, D. G. Agresti, M. W. Schaefer, C. A. Grant, and E. C. Sklute, Mössbauer spectroscopy of earth and planetary materials, Annu. Rev. Earth Planet Sci. **34**, 83 (2006).

[54] R. Coussement, G. S'heeren, M. van den Bergh, and P. Boolchand, Nuclear resonant absorption in long-lived isomeric transitions, Phys. Rev. B **45**, 9755 (1992).

[55] R. Coussement, M. van den Bergh, G. S'heeren, and P. Boolchand, Mössbauer absorption on ^{109m}Ag, fake or reality?, Hyperfine Interact. **71**, 1487 (1992).

[56] W. Wildner and U. Gonser, A new attempt to observe the resonance in ag109, J. Phys. (Paris), Colloq. **40**, C2–47 (1979).

[57] G. R. Hoy and R. D. Taylor, Some Moessbauer effect considerations in gamma-ray laser development, J. Quant. Spectrosc. Radiat. Transfer **40**, 763 (1988).

[58] G. R. Hoy, S. Rezaie-Serej, and R. D. Taylor, The Mössbauer effect in ^{109}Ag revisited, Hyperfine Interact. **58**, 2513 (1990).

[59] V. G. Alpatov, Y. D. Bayukov, A. V. Davydov, Y. N. Isaev, G. R. Kartashov, M. M. Korotkov, and V. V. Migachev, Initial studies of the gamma resonance of the ^{109m}Ag isomer with a gravitational gamma spectrometer, Phys. At. Nucl. **71**, 1156 (2008).

[60] Y. D. Bayukov, A. V. Davydov, Y. N. Isaev, G. R. Kartashov, M. M. Korotkov *et al.*, Observation of the gamma resonance of a long-lived ^{109m}Ag isomer using a gravitational gamma-ray spectrometer, Sov. J. Exp. Theor. Phys. Lett. **90**, 499 (2009).

[61] B. Abi *et al.* (Muon g-2 Collaboration), Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm, Phys. Rev. Lett. **126**, 141801 (2021).