Letter

Universal aspects of holographic quantum critical transport with self-duality

Ángel J. Murcia[1,*] and Dmitri Sorokin[1,2,†]

[1]INFN, Sezione di Padova, Via Marzolo 8, 35131 Padova, Italy
[2]Dipartimento di Fisica ed Astronomia "Galileo Galilei," Università degli Studi di Padova, Via Marzolo 8, 35131 Padova, Italy

(Received 6 April 2023; accepted 17 July 2023; published 10 August 2023)

We prove several universal properties of charge transport in generic conformal field theories holographic to nonminimal extensions of four-dimensional Einstein-Maxwell theory with exact electromagnetic duality invariance. First, we explicitly verify that the conductivity of these theories at zero momentum is a universal frequency-independent constant. Then, we derive the analytical expressions for the conductivities at nonzero momentum in any holographic duality-invariant theory for large frequencies and in the limit of small frequencies and momenta. Next, in the absence of terms that couple covariant derivatives of the curvature to gauge field strengths, two universal features are proven. On the one hand, it is shown that for a general-relativity neutral black-hole background the conductivities at any frequency and momentum are independent of the choice of duality-invariant theory, thus coinciding with those in the Einstein-Maxwell case. On the other hand, if higher-curvature terms affect the gravitational background, the conductivities get modified, but the contributions from nonminimal couplings of the gauge field to gravity are subleading. We illustrate this feature with an example.

DOI: 10.1103/PhysRevD.108.L041901

I. INTRODUCTION

The AdS/CFT correspondence [1–3] has become a powerful and fruitful tool for the study of strongly coupled systems in the vicinity of quantum critical points, leading to the development of the so-called AdS/condensed matter (AdS/CMT) duality [4–9]. Among other aspects, it has been possible to identify a variety of holographic models which exhibit properties characteristic of condensed-matter systems, such as superfluidity, superconductivity or (quantum) Hall conductivity [10–15]. Further scrutiny of such interesting features have turned AdS/CMT into a highly active topic of research—see e.g. [16–22].

On the other hand, the potential of higher-order theories of gravity to unveil generic aspects of conformal field theories (CFTs) has become evident in recent years. Apart from being able to capture finite N and finite coupling effects within the canonical holographic correspondence between type IIB string theory and $\mathcal{N} = 4$ super-Yang-Mills theory [23–25], higher-order gravities make it possible to explore holographic CFTs whose correlators take the most generic form allowed by conformal symmetry [26–30] or to identify new universal relations that hold for arbitrary CFTs [31–39]. These features have motivated the study of (charge) transport properties of holographic duals of higher-order gravities, observing novel and intriguing phenomena in the shear viscosity to entropy density ratio [40–42], holographic superconductivity [43–45] and (electrical) conductivities [46–50].

In this work we explore various aspects of charge transport in CFTs holographic to duality-invariant theories of electrodynamics with nonminimal couplings to gravity. Duality invariance is a symmetry of the equations of motion of Einstein-Maxwell theory in vacuum, so it is justified to consider higher-order modifications that respect this symmetry. Explicit examples of duality-invariant theories are known to exist both with minimal couplings (see e.g. [51] for a review) and with nonminimal couplings to gravity [52,53].

In particular, we study CFTs holographic to duality-invariant theories whose equilibrium state is characterized by vanishing expectation values of all global charges (i.e. systems without chemical potentials). We begin by explicitly checking that the conductivity at zero momentum is a frequency-independent universal constant. Then we are able to derive the explicit expressions for the conductivities in any holographic duality-invariant theory in the regimes of large frequencies and for sufficiently small frequencies and momenta. The latter depend on both the gravitational

*angel.murcia@pd.infn.it
†dmitri.sorokin@pd.infn.it

background selected and the duality-invariant theory under study, so that conductivities at nonzero momentum will generically differ with respect to their Einstein-Maxwell values.

Nonetheless, when dealing with CFTs holographic to duality-invariant theories which do not couple covariant derivatives of the curvature to gauge field strengths, it turns out that conductivities do possess two remarkable universal features. First, whenever a general-relativity black hole background is considered, the conductivities for any frequency and momentum are the same for all such holographic theories. Second, if the black hole background is modified by higher-curvature terms, the conductivities get corrected, but the contributions coming from nonminimal couplings between the curvature and the gauge field are subleading. We corroborate this feature with an explicit example.

II. DUALITY-INVARIANT BULK SETUP

Let us consider generic nonminimal extensions of Einstein-Maxwell theory in four dimensions described by the following action:

$$I = \kappa_N \int \mathrm{d}^4x\sqrt{|g|}\left[R + \frac{6}{L^2} - \chi^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \mathcal{L}^{\mathrm{high}}_{\mathrm{grav}}\right], \quad (1)$$

where $\chi^{\mu\nu\rho\sigma}$ depends solely on the metric and the curvature, $\kappa_N = (16\pi G)^{-1}$ and

$$\mathcal{L}^{\mathrm{high}}_{\mathrm{grav}} = L^2\sum_i \alpha_i^{(2)}\mathcal{R}_i^{(2)} + L^4\sum_i \alpha_i^{(3)}\mathcal{R}_i^{(3)} + \cdots, \quad (2)$$

where $\mathcal{R}_i^{(n)}$ stands for curvature invariants of nth order—the index i denoting every such inequivalent term—L is the cosmological-constant length scale and $\alpha_i^{(n)}$ are dimensionless couplings characterizing the theory. Equation (1) may also be interpreted as a generic effective action [54] obtained by adding to Einstein-Maxwell theory all possible terms quadratic in $F_{\mu\nu}$ which are compatible with diffeomorphism and gauge invariance. The reason why it suffices for our purposes to work at $\mathcal{O}(F^2)$ will become apparent afterwards.

Let $\mathcal{T}_{\mu\nu}$ be a traceless and symmetric tensor constructed from contractions of the curvature tensor and its covariant derivatives, and let b_n be the coefficients appearing in the Taylor series $\sqrt{1+x^2} = 1 + \sum_{n=1}^{\infty} b_n x^{2n}$. If we take the tensor $\chi_{\mu\nu}{}^{\rho\sigma}$ in (1) to be [55]

$$\begin{aligned}\chi_{\mu\nu}{}^{\rho\sigma} &= \delta_{[\mu}{}^{[\rho}\delta_{\nu]}{}^{\sigma]} + \Theta_{\mu\nu}{}^{\rho\sigma} + \sum_{n=1}^{\infty} b_n \Theta^{2n}{}_{\mu\nu}{}^{\rho\sigma},\\ \Theta^{2n}{}_{\mu\nu}{}^{\rho\sigma} &= \Theta_{\mu\nu}{}^{\alpha_1\alpha_2}\Theta_{\alpha_1\alpha_2}{}^{\alpha_3\alpha_4}\cdots\Theta_{\alpha_{4n-3}\alpha_{4n-2}}{}^{\rho\sigma},\\ \Theta_{\mu\nu}{}^{\rho\sigma} &= \mathcal{T}_{[\mu}{}^{[\rho}\delta_{\nu]}{}^{\sigma]},\end{aligned} \quad (3)$$

then the action (1) describes the most general exactly duality-invariant theory of electrodynamics with nonminimal couplings to gravity and having at most quadratic terms in the Maxwell field strength [53]. More concretely, this means that the set of equations formed by the equations of motion of (1) and by the Bianchi identity for $F_{\mu\nu}$ is invariant under rigid U(1) rotations of the complex tensor $F'_{\mu\nu} + iH'_{\mu\nu} = e^{i\alpha}(F_{\mu\nu} + iH_{\mu\nu})$, where

$$\star H_{\mu\nu} = \frac{1}{2}\frac{\delta I}{\delta F^{\mu\nu}} = -\chi_{\mu\nu}{}^{\rho\sigma}F_{\rho\sigma}. \quad (4)$$

In the case of Maxwell theory minimally coupled to a higher-curvature gravity, $\chi_{\mu\nu}{}^{\rho\sigma} = \delta_{[\mu}{}^{[\rho}\delta_{\nu]}{}^{\sigma]}$ and $H_{\mu\nu}$ is just the Hodge dual of $F_{\mu\nu}$.

With the goal of studying thermal CFTs in flat Minkowski space, we consider gravitational backgrounds of (1) (i.e. we take $F_{\mu\nu} = 0$) which correspond to AdS black holes with a planar horizon—usually called (AdS) black branes in the literature:

$$\begin{aligned}\mathrm{d}s^2 &= \frac{r_0^2}{\tilde{L}^2u^2}(-N^2(u)f(u)\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2)\\ &\quad + \frac{\tilde{L}^2}{u^2 f(u)}\mathrm{d}u^2,\end{aligned} \quad (5)$$

where r_0 is a constant of dimension of length, $\tilde{L}$ denotes the AdS length scale, generically differing from the cosmological-constant scale L because of the higher-order corrections [27,56,57], and

$$N = 1 + \tilde{N}, \qquad f = 1 - u^3 + \tilde{f}, \quad (6)$$

where $\tilde{N}$ and $\tilde{f}$ are u-dependent functions encoding the higher-order corrections with respect to the General-Relativity (GR) solution ($\tilde{N} = \tilde{f} = 0$) such that $\lim_{u\to 0}\tilde{f} = \lim_{u\to 0}\tilde{N} = 0$. In this coordinate system the AdS boundary is located at $u = 0$, while the horizon (which we assume to exist) is at $u = u_h$. The black hole temperature is

$$T = -\frac{r_0 f'(u_h)}{4\pi\tilde{L}^2}. \quad (7)$$

III. RETARDED CORRELATORS FROM THE AdS/CFT CORRESPONDENCE

We are interested in computing the retarded two-point current correlator C_{ab} of CFTs at finite temperature which are holographically dual to exactly duality-invariant theories quadratic in $F_{\mu\nu}$.

In a generic three-dimensional QFT with a current J_a ($a = t,\ x,\ y$), the retarded current-current correlator in momentum space $p^a = (\omega, \mathbf{k})$ is given by

$$C_{ab}(p) = -i\int d^3x e^{-ip_ax^a}\Theta_H(t)\langle[J_a(x), J_b(0)]\rangle, \quad (8)$$

where $x^a = (t, x, y)$ are boundary coordinates and $\Theta_H(t)$ is the Heaviside step function.

Specifically, we will assume that the expectation values of all global conserved charges vanish in the equilibrium state, which is equivalent to exploring systems with no chemical potential [46,58]. In such a case, the correlator C_{ab} can be derived holographically via studying linear perturbations A_μ which solve the classical equations of motion around a neutral black brane background (5). To this aim, we impose the gauge $A_u = 0$ and decompose the remaining nonvanishing components in momentum space:

$$A_a(t, u, \mathbf{x}) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}} A_a(u, \omega, \mathbf{k}). \tag{9}$$

Working in momentum space and taking the spatial momentum vector to be $\mathbf{k} = (k, 0)$, the equations of motion for A_a in a duality-invariant theory given by Eqs. (1) and (3) can be expressed in the following compact form [59]:

$$\mathcal{S}\mathbf{A}'' - \mathcal{S}'\mathbf{A}' + \frac{\tilde{L}^4}{r_0^2}\mathcal{S}^2\left(\omega^2\mathcal{S} - \frac{k^2}{\mathcal{B}}\right)\mathbf{A} = 0, \tag{10}$$

$$\omega\mathcal{S}\mathcal{B}A_t' + kA_x' = 0, \tag{11}$$

where prime denotes derivative with respect to u, $\mathbf{A} = (\mathcal{B}A_t', A_y)$ and where $\mathcal{B}$ and $\mathcal{S}$ are identified after evaluation of $\mathcal{T}_{\mu\nu}$ on the black brane background (5) as follows:

$$\mathcal{T}_\mu{}^\nu|_{N,f} = 2(\theta + \varphi)\delta_\mu{}^t\delta_t{}^\nu + 2(\theta - \varphi)\delta_\mu{}^u\delta_u{}^\nu - \theta\delta_\mu{}^\nu, \tag{12}$$

$$\theta = \frac{N^2\mathcal{B}^2 - 1}{2N\mathcal{B}}, \qquad \varphi = \frac{f^2N^2\mathcal{S}^2 - 1}{2fN\mathcal{S}}, \tag{13}$$

where we used that (12) represents the most general form for a symmetric and traceless tensor built from the curvature of (5) and its covariant derivatives (see the Supplemental Material [60]). The reason for the equations of motion of A_μ to take such a compact form is due to duality invariance [58].

Now, applying the holographic prescriptions originally presented in [61,62], it is explained in the Supplemental Material [60] that C_{ab} can be obtained as follows:

$$C_{ab} = -\frac{4r_0\kappa_N}{\tilde{L}^2}M'_{ab}\Big|_{u=0}, \tag{14}$$

where M_{ab} is defined by [63] the relation $A_a = M_a{}^bA_b(0)$, with $A_b(0) = A_b|_{u=0}$. We impose infalling boundary conditions at the horizon for all components of A_a. This implies, on account of Eqs. (10) and (11), that both $\mathcal{B}A_t'$ and $A_x'/\mathcal{S}$ are proportional to A_y. Then, transforming these equations into an explicit second-order differential system for A_a (see Ref. [59]) and adapting the computations presented in [58], one may identify $M'_{ab}|_{u=0}$ and obtain the nonvanishing components of C_{ab} from (14):

$$\frac{C_{tt}}{k^2} = \frac{C_{xx}}{\omega^2} = -\frac{C_{tx}}{k\omega} = -\frac{C_{xt}}{k\omega} = \frac{4\tilde{L}^2\kappa_N}{r_0}\frac{A_y(0)}{A_y'(0)}, \tag{15}$$

$$C_{yy} = -\frac{4r_0\kappa_N}{\tilde{L}^2}\frac{A_y'(0)}{A_y(0)}. \tag{16}$$

Having at our disposal the correlator C_{ab}, one may compute the so-called longitudinal and transverse self-energies $K^L(\omega, \mathbf{k})$ and $K^T(\omega, \mathbf{k})$, defined as [58]

$$C_{xx} = -\frac{\omega^2}{\sqrt{k^2 - \omega^2}}K^L, \qquad C_{yy} = \sqrt{k^2 - \omega^2}K^T. \tag{17}$$

Comparing (15) with (16), we find that the product of K^L and K^T is the following universal constant for all frequencies and momenta:

$$K^L(\omega, \mathbf{k})K^T(\omega, \mathbf{k}) = 16\kappa_N^2. \tag{18}$$

The above relation holds for all CFTs holographic to duality-invariant theories, since its derivation just requires to know their form up to quadratic order in the vector field, captured by the theories defined by Eqs. (1) and (3). It matches [64] with the result obtained in [46] in the particular case of a background (5) with $N = 1$ after requiring duality invariance.

IV. CONDUCTIVITY OF HOLOGRAPHIC DUALITY-INVARIANT THEORIES

Following the usual holographic prescriptions, a gauge vector field on the bulk couples to a current on the boundary CFT. We are interested in studying the subsequent longitudinal and transverse conductivities σ_x and σ_y for the holographic theories defined by Eqs. (1) and (3). According to the Kubo formula [65], they are given by

$$\sigma_j(\omega, k) = -\mathrm{Im}\left(\frac{C_{jj}}{\omega}\right), \qquad j = x, y. \tag{19}$$

Particularly simple is the computation of the conductivities at zero momentum $k = 0$. In this case, spatial rotational invariance ensures that $K^L(\omega, 0) = K^T(\omega, 0)$ and $\sigma_x(\omega, 0) = \sigma_y(\omega, 0)$. Using (18) and the expression for C_{yy} given in (16), one obtains [66]

$$\sigma_x(\omega, 0) = \sigma_y(\omega, 0) = 4\kappa_N. \tag{20}$$

Therefore, the conductivity at zero momentum in any CFT holographic to a duality-invariant theory is a universal constant, independent of the frequency. Evidently, this is a consequence of the universal relation (18), as remarked in [46,58]. If duality symmetry is absent, Eq. (20) may not necessarily hold—see Ref. [46].

For nonzero momentum k, the longitudinal and transverse conductivities σ_x and σ_y are no longer the same and possess an explicit frequency dependence, as already observed in Einstein-Maxwell theory [58]. Although an exact analytical expression for the conductivities at any frequency and momentum in an arbitrary duality-invariant theory seems currently out of reach (it remains elusive even in the Einstein-Maxwell case), it is in fact possible to obtain explicit results in certain limits. For large frequencies, straightforward application of the WKB approximation shows that

$$\sigma_x(\omega, k) = 4\kappa_N \frac{\omega}{\sqrt{\omega^2 - k^2}}, \qquad \omega^2 \gg k^2, \tag{21}$$

$$\sigma_y(\omega, k) = 4\kappa_N \frac{\sqrt{\omega^2 - k^2}}{\omega}, \qquad \omega^2 \gg k^2. \tag{22}$$

Therefore, the behavior of conductivities for large frequencies is theory independent. Besides, we note that they tend to the universal value (20) as $\omega \to \infty$. On the other hand, in the limit of sufficiently small frequencies and momenta $\omega, k \ll r_0/\tilde{L}^2$, one may generalize the results in [62] for the retarded correlators to obtain

$$\sigma_x(\omega, k) = \frac{4\kappa_N \omega^2}{\omega^2 + D^2 k^4}, \qquad \omega, k \ll \frac{r_0}{\tilde{L}^2} \tag{23}$$

$$\sigma_y(\omega, k) = \frac{4\kappa_N}{1 + ck^2}, \qquad \omega, k \ll \frac{r_0}{\tilde{L}^2}, \tag{24}$$

where we have implicitly defined

$$D = \frac{\tilde{L}^2}{r_0} \int_0^{u_h} \frac{\mathrm{d}z}{\mathcal{B}(z)}, \quad c = \frac{2\tilde{L}^4}{r_0^2} \int_0^{u_h} \mathrm{d}z \mathcal{S}(z) \int_z^{u_h} \frac{\mathrm{d}w}{\mathcal{B}(w)}. \tag{25}$$

These expressions show that the conductivities for nonzero momenta will generically depend on both the gravitational background and the particular choice of duality-invariant theory (since this is the case for small frequencies and momenta). Also, a closer look at Eqs. (21) and (23) reveals that the longitudinal conductivity undergoes a hydrodynamic-to-collisionless crossover [67] as we go from small to large frequencies. This is signaled by the fact that Eq. (23) possesses a pole at $\omega = -iDk^2$ (which is precisely the dispersion relation of diffusion modes in the heat equation), while (21) presents a pole at $\omega = k$ (which is the dispersion relation for free particles). Moreover, as a consistency check of our results, we have verified that the expression for the diffusion constant that can be derived from the membrane paradigm [41,68,69] coincides with our formula (25) for D.

Away from the small/large frequency regimes, it appears to be challenging to obtain specific formulas for the conductivities. Despite that, a universal statement regarding their form in generic duality-invariant theories can be made by noticing that every traceless and symmetric tensor $\mathcal{T}_{\mu\nu}$ built from algebraic combinations (i.e. with no covariant derivatives) of the curvature of (5) vanishes identically when evaluated on the GR AdS black brane solution:

$$\mathcal{T}_{\mu\nu}|_{N=1, f=1-u^3} = 0. \tag{26}$$

The explicit proof of this result is given in the Supplemental Material [60]. Therefore, the retarded correlators C_{ab} and the conductivities $\sigma_x(\omega, k)$ and $\sigma_y(\omega, k)$ will coincide with those of Einstein-Maxwell theory for any duality-invariant theory with no covariant derivatives of the curvature in $\mathcal{T}_{\mu\nu}$ as long as the GR AdS black brane background is considered. It is important to note that taking the background to be that of GR does not imply that $\mathcal{L}_{\mathrm{grav}}^{\mathrm{high}} = 0$ in (1). Indeed, there exist myriads of higher-order gravities which do not correct the GR AdS black brane solution [e.g. the well-known $f(R)$ gravities [70,71]]. Therefore, one may interpret duality invariance as a very powerful tool to constrain observables to have a simple and fixed expression: that of Einstein-Maxwell theory.

If higher-curvature terms correct the GR solution, the retarded correlator, and hence the associated conductivities, will generically differ from [72] those of Einstein-Maxwell theory. In such a case, the subsequent charge transport will no longer be independent of the choice of duality-invariant theory. However, in spite of this lack of universality, if $\mathcal{T}_{\mu\nu}$ contains no covariant derivatives of the curvature it turns out that duality invariance forces corrections with respect to the case $\mathcal{T}_{\mu\nu} = 0$ to be highly suppressed, since Eq. (26) implies that $\mathcal{T}_{\mu\nu}|_{N,f}$ is subleading with respect to the leading-order corrections in the gravitational background. Therefore, corrections associated to the specific choice of $\mathcal{T}_{\mu\nu}$ (without covariant derivatives of the curvature) are subleading with respect to those arising from the choice of the gravitational background (or equivalently, of $\mathcal{L}_{\mathrm{grav}}^{\mathrm{high}}$). In particular, this means that the corrections to the conductivities with respect to Maxwell theory on top of a gravitational background modified by higher-curvature terms are extremely small.

V. HOLOGRAPHIC CONDUCTIVITIES IN AN EXPLICIT EXAMPLE

Now we illustrate the previous aspects with the simplest nontrivial choices for $\mathcal{T}_{\mu\nu}$ and $\mathcal{L}_{\mathrm{grav}}^{\mathrm{high}}$. Regarding $\mathcal{T}_{\mu\nu}$, this corresponds to

$$\mathcal{T}_{\mu\nu} = \lambda L^2 \hat{R}_{\mu\nu}, \tag{27}$$

where $\hat{R}_{\mu\nu}$ denotes the traceless part of the Ricci tensor and λ is a dimensionless coupling. Demanding $\mathcal{T}_{\mu\nu}$ to respect the weak gravity conjecture [73] and causality [46], we find that the acceptable range for λ is $0 \geq \lambda \gtrsim -0.50105$ (see the Supplemental Material [60]) for the specific choice of $\mathcal{L}_{\mathrm{grav}}^{\mathrm{high}}$ we are about to make.

To pick a suitable $\mathcal{L}_{\mathrm{grav}}^{\mathrm{high}}$ one needs to consider gravitational theories which contain at least terms of cubic order in the curvature, since quadratic terms do not correct the (four-dimensional) GR AdS black brane solution. Among this class of theories, there is a unique subset admitting black brane solutions (5) with $N(u) = 1$ and second-order equation for f. All such theories are equivalent on *Ansätze* of the form (5), so it is enough to select a convenient representative. We will choose it to be

Einsteinian cubic gravity (ECG) [56,74], whose higher order terms have the following form:

$$-8\mathcal{L}_{\text{grav}}^{\text{high}} = \mu L^4[12R_{\mu}{}^{\rho}{}_{\nu}{}^{\sigma}R_{\rho}{}^{\gamma}{}_{\sigma}{}^{\delta}R_{\gamma}{}^{\mu}{}_{\delta}{}^{\nu} + 8R_{\mu\nu}R^{\nu\rho}R_{\rho}{}^{\mu} + R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\gamma\delta}R_{\gamma\delta}{}^{\mu\nu} - 12R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma}], \tag{28}$$

where μ is a dimensionless coupling. The equation of motion for $f(u)$, though second order, is too complicated to be solved analytically for generic μ, so we will resort to numeric methods (details are given in the Supplemental Material [60]). We will pick μ to be within the range $0 < \mu < 4/27$, since this ensures the existence of both a unique stable vacuum and black hole solutions [29].

In Fig. 1 we present the longitudinal and transverse conductivities we get for Einstein-Maxwell theory and for the choices (27) and (28). We have set $\mu = 1/10$, $L^2k/r_0 = 1$ and $\lambda = 0, -1/2, -1/4$, since the qualitative behavior of the conductivities turns out to replicate for any $0 < \mu < 4/27$ (approaching of course the Einstein-Maxwell case as $\mu \to 0$) and k [approaching the constant universal value (20) as $k \to 0$]. By direct inspection of the graphs we check that corrections associated to the specific choice of λ—i.e. of $\mathcal{T}_{\mu\nu}$—are clearly subleading with respect to those arising from the choice of the gravitational background (characterized, in this case, by the parameter μ).

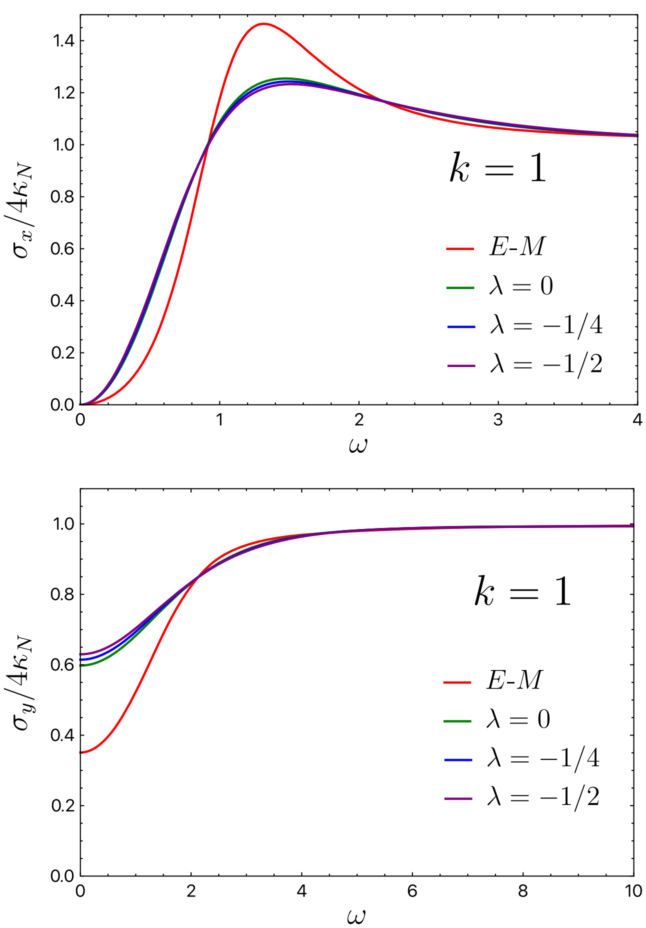

FIG. 1. Longitudinal (above) and transverse (below) conductivities in units of $L^2/r_0 = 1$ for Einstein-Maxwell (*E-M*) theory and for an Einsteinian cubic gravity (ECG) background. We have picked $\mu = 1/10$, $k = 1$ and several values of λ.

VI. FINAL COMMENTS

We have examined various universal aspects of the holographic quantum critical transport associated to duality-invariant theories. In the first place, we have explicitly checked that the conductivity at zero momentum is a universal constant for all these theories. Next we have obtained the expressions for the conductivities in the limit of large frequencies and for small frequencies and momenta in every CFT holographic to a duality-invariant theory. From their form in this latter regime, we have concluded that conductivities at nonzero momentum generically depend on both the gravitational background and the theory under study.

Despite that, we have proven that the conductivities in CFTs associated to duality-invariant theories which do not couple covariant derivatives of the curvature to gauge field strengths display two universal features. First, we have shown that, as long as a GR background is chosen, conductivities are universal and equal to those of Einstein-Maxwell theory for any frequency and momentum. Second, when the gravitational background is corrected by higher-curvature terms, we have proven that conductivities get modified in such a way that contributions from nonminimal couplings of the gauge field to gravity are subleading.

In another vein, there are several directions that would be interesting to address. First, one could study other correlators of CFTs holographic to duality-invariant theories. For instance, consider the Euclidean correlators $\langle J_a J_b\rangle_E$ and $\langle T_{ab}J_cJ_d\rangle_E$ at zero temperature, where T_{ab} is the stress-energy tensor. Conformal symmetry fixes the form of such correlators as follows [75,76]:

$$\langle J_a(x_1)J_b(x_2)\rangle_E = \frac{C_J}{|x_{12}|^4}\mathcal{I}_{ab}, \tag{29}$$

$$\langle T_{ab}(x_1)J_c(x_2)J_d(x_3)\rangle_E = \frac{f_{abcd}(C_J, a_2)}{|x_{12}|^3|x_{13}|^3|x_{23}|}, \tag{30}$$

where $\mathcal{I}_{ab}$ and $f_{abcd}(C_J, a_2)$ are fixed tensorial structures, $x_{mn} = x_m - x_n$, C_J is the current central charge and a_2 is a parameter that controls, together with C_J, the energy flux measured at infinity after the insertion of a current operator [77]. The holographic expressions for C_J and a_2 for the most general effective four-derivative theory were presented in [30,77]. Applying their results to the choice (27) and $\mathcal{L}_{\text{grav}}^{\text{high}} = 0$, one finds $a_2 = 0$ and that C_J takes its Einstein-Maxwell value, so the Euclidean correlators at zero temperature are not modified to the fourth-derivative order.

This is another manifestation of the strength of duality invariance to constrain the form of correlators to be those of Einstein-Maxwell.

Second, it would be intriguing to extend our results to systems with chemical potentials (i.e. with nonvanishing expectation values of global conserved charges in the equilibrium state). This is carried out by considering linear fluctuations of the vector field on top of a fixed charged gravitational background with a nonzero background electromagnetic field, as in [78–84].

Finally, one could also examine higher-point correlators. Indeed, it is natural to wonder what constraints duality invariance could impose on generic (current) n-point correlators. This would require the construction of (all) duality-invariant nonminimal extensions of Einstein-Maxwell theory of arbitrary order in $F_{\mu\nu}$, which remains as an outstanding open problem.

ACKNOWLEDGMENTS

We would like to thank Pablo A. Cano for very useful and enlightening discussions. Á. J. M. was supported by a postdoctoral fellowship from the INFN, Bando 23590. D. S. acknowledges partial support of the Spanish MICINN/FEDER Grant No. PID2021-125700NB-C21 and the Basque Government Grant No. IT-979-16.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[3] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[4] S. A. Hartnoll, Classical Quantum Gravity **26**, 224002 (2009).
[5] S. Sachdev, Lect. Notes Phys. **828**, 273 (2011).
[6] J. Zaanen, Y.-W. Sun, Y. Liu, and K. Schalm, *Holographic Duality in Condensed Matter Physics* (Cambridge University Press, Cambridge, England, 2015).
[7] S. A. Hartnoll, A. Lucas, and S. Sachdev, arXiv:1612.07324.
[8] M. Baggioli, Gravity, holography and applications to condensed matter, Ph.D. thesis, Barcelona, Autonoma University, 2016, arXiv:1610.02681.
[9] H. Nastase, *String Theory Methods for Condensed Matter Physics* (Cambridge University Press, Cambridge, England, 2017).
[10] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Phys. Rev. Lett. **101**, 031601 (2008).
[11] E. Keski-Vakkuri and P. Kraus, J. High Energy Phys. 09 (2008) 130.
[12] C. P. Herzog, P. K. Kovtun, and D. T. Son, Phys. Rev. D **79**, 066002 (2009).
[13] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, J. High Energy Phys. 12 (2008) 015.
[14] C. P. Herzog, J. Phys. A **42**, 343001 (2009).
[15] O. Bergman, N. Jokela, G. Lifschytz, and M. Lippert, J. High Energy Phys. 10 (**2010**) 063.
[16] S. A. Hartnoll, G. T. Horowitz, J. Kruthoff, and J. E. Santos, SciPost Phys. **10**, 009 (2021).
[17] D. Arean, M. Baggioli, S. Grieninger, and K. Landsteiner, J. High Energy Phys. 11 (**2021**) 206.
[18] M. Ammon, D. Arean, M. Baggioli, S. Gray, and S. Grieninger, J. High Energy Phys. 03 (2022) 015.
[19] L. Sword and D. Vegh, J. High Energy Phys. 04 (2022) 135.
[20] A. Donos and P. Kailidis, J. High Energy Phys. 11 (2022) 053.
[21] M. Flory, S. Grieninger, and S. Morales-Tejera, arXiv: 2209.09251.
[22] A. Donos and P. Kailidis, J. High Energy Phys. 12 (2022) 028.
[23] A. Buchel, J. T. Liu, and A. O. Starinets, Nucl. Phys. B **707**, 56 (2005).
[24] P. Benincasa and A. Buchel, J. High Energy Phys. 01 (2006) 103.
[25] R. C. Myers, M. F. Paulos, and A. Sinha, Phys. Rev. D **79**, 041901 (2009).
[26] J. de Boer, M. Kulaxizi, and A. Parnachev, J. High Energy Phys. 03 (2010) 087.
[27] A. Buchel, J. Escobedo, R. C. Myers, M. F. Paulos, A. Sinha, and M. Smolkin, J. High Energy Phys. 03 (2010) 111.
[28] R. C. Myers, M. F. Paulos, and A. Sinha, J. High Energy Phys. 08 (2010) 035.
[29] P. Bueno, P. A. Cano, and A. Ruipérez, J. High Energy Phys. 03 (2018) 150.
[30] P. A. Cano, Á. J. Murcia, A. Rivadulla Sánchez, and X. Zhang, J. High Energy Phys. 07 (2022) 010.
[31] R. C. Myers and A. Sinha, Phys. Rev. D **82**, 046006 (2010).
[32] E. Perlmutter, J. High Energy Phys. 03 (2014) 117.
[33] M. Mezei, Phys. Rev. D **91**, 045038 (2015).
[34] P. Bueno, R. C. Myers, and W. Witczak-Krempa, Phys. Rev. Lett. **115**, 021602 (2015).
[35] C.-S. Chu and R.-X. Miao, J. High Energy Phys. 12 (2016) 036.
[36] Y.-Z. Li, H. Lü, and Z.-F. Mai, J. High Energy Phys. 10 (2018) 063.
[37] P. Bueno, P. A. Cano, R. A. Hennigar, and R. B. Mann, Phys. Rev. Lett. **122**, 071602 (2019).
[38] P. Bueno, P. A. Cano, Á. Murcia, and A. Rivadulla Sánchez, Phys. Rev. Lett. **129**, 021601 (2022).
[39] S. Baiguera, L. Bianchi, S. Chapman, and D. A. Galante, J. High Energy Phys. 06 (**2022**) 068.
[40] Y. Kats and P. Petrov, J. High Energy Phys. 01 (2009) 044.
[41] M. Brigante, H. Liu, R. C. Myers, S. Shenker, and S. Yaida, Phys. Rev. D **77**, 126006 (2008).
[42] X.-H. Ge, Y. Matsuo, F.-W. Shu, S.-J. Sin, and T. Tsukioka, J. High Energy Phys. 10 (2008) 009.

[43] R. Gregory, S. Kanno, and J. Soda, J. High Energy Phys. 10 (2009) 010.
[44] Q. Pan and B. Wang, Phys. Lett. B **693**, 159 (2010).
[45] J. D. Edelstein, N. Grandi, and A. Rivadulla Sánchez, J. High Energy Phys. 05 (**2022**) 188.
[46] R. C. Myers, S. Sachdev, and A. Singh, Phys. Rev. D **83**, 066017 (2011).
[47] S. S. Pal, Phys. Rev. D **84**, 126009 (2011).
[48] W. Witczak-Krempa and S. Sachdev, Phys. Rev. B **86**, 235115 (2012).
[49] W. Witczak-Krempa, Phys. Rev. B **89**, 161114 (2014).
[50] Y. Ling, P. Liu, J.-P. Wu, and Z. Zhou, Phys. Lett. B **766**, 41 (2017).
[51] D. P. Sorokin, Fortschr. Phys. **70**, 2200092 (2022).
[52] P. A. Cano and Á. Murcia, J. High Energy Phys. 08 (2021) 042.
[53] P. A. Cano and Á. Murcia, Phys. Rev. D **104**, L101501 (2021).
[54] S. Weinberg, *The Quantum Theory of Fields. Vol. 1: Foundations* (Cambridge University Press, Cambridge, England, 2005).
[55] Observe that ${\chi_{\mu\nu}}^{\rho\sigma}$ can be compactly expressed as ${\chi_{\mu\nu}}^{\rho\sigma} = {\Theta_{\mu\nu}}^{\rho\sigma} + \sqrt{\delta_{[\mu}{}^{[\rho}\delta_{\nu]}{}^{\sigma]} + {\Theta^2_{\mu\nu}}^{\rho\sigma}}$, as in [53].
[56] P. Bueno and P. A. Cano, Phys. Rev. D **94**, 104005 (2016).
[57] P. Bueno, P. A. Cano, V. S. Min, and M. R. Visser, Phys. Rev. D **95**, 044010 (2017).
[58] C. P. Herzog, P. Kovtun, S. Sachdev, and D. T. Son, Phys. Rev. D **75**, 085020 (2007).
[59] Expressed in this way, it might appear that the equations of motion are of third order in derivatives. However, they can be seen to be equivalent to the (second-order) equations $\nabla_\mu(\chi^{\mu\nu\rho\sigma}F_{\rho\sigma}) = 0$ in the gauge $A_u = 0$ by appropriate manipulations which allow to integrate them into second-order equations, uniquely fixed after requiring $A_\mu = 0$ to be a solution.
[60] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevD.108.L041901. Four directions are addressed. First, two algebraic properties of all traceless and symmetric two tensors on black-brane backgrounds are proven. Then, our holographic prescription for the retarded correlators is put forward and justified. Third, Einsteinian cubic gravity is reviewed, highlighting the main properties of interest in relation to the present manuscript. Finally, it is analyzed when the theory defined by (27) and (28) satisfies the Weak Gravity Conjecture and causality.
[61] D. T. Son and A. O. Starinets, J. High Energy Phys. 09 (2002) 042.
[62] G. Policastro, D. T. Son, and A. O. Starinets, J. High Energy Phys. 09 (2002) 043.
[63] Boundary indices are lowered and raised with the Minkowski metric η^{ab}.
[64] The match is up to a constant on the right-hand side of (18), which corresponds to the different conventions used for the electromagnetic duality transformation.
[65] R. Kubo, J. Phys. Soc. Jpn. **12**, 570 (1957).
[66] We have chosen the sign for $K^L(\omega, 0) = K^T(\omega, 0)$ that guarantees a positive spectral function, defined by $-2\mathrm{Im}(\sqrt{k^2 - \omega^2}K^T)$. The same result could also be derived by direct resolution of (10) for $k = 0$.
[67] This hydrodynamic-to-collisionless crossover manifests in the longitudinal conductivity and not in the transverse one since the longitudinal correlator C_{xx} is related via Eq. (15) to the density-density correlator C_{tt}, which is a clear probe of hydrodynamic behavior [58].
[68] P. Kovtun, D. T. Son, and A. O. Starinets, J. High Energy Phys. 10 (2003) 064.
[69] N. Iqbal and H. Liu, Phys. Rev. D **79**, 025023 (2009).
[70] H. A. Buchdahl, Mon. Not. R. Astron. Soc. **150**, 1 (1970).
[71] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010).
[72] If one insists on working with the GR solution, the lowest-order choice for $\mathcal{T}_{\mu\nu}$ which is nonzero on the AdS black brane solution of GR is the traceless part of $\nabla_\mu W_{\alpha\beta\rho\sigma}\nabla_\nu W^{\alpha\beta\rho\sigma}$, where $W_{\alpha\beta\rho\sigma}$ stands for the Weyl tensor. This term would in principle modify the conductivities of the dual CFT. Nevertheless, such a term is of order $\mathcal{O}(L^8)$ and there are other higher-curvature terms of less order that already modify the gravitational background.
[73] N. Arkani-Hamed, L. Motl, A. Nicolis, and C. Vafa, J. High Energy Phys. 06 (2007) 060.
[74] P. Bueno, P. A. Cano, J. Moreno, and Á. Murcia, J. High Energy Phys. 11 (2019) 062.
[75] H. Osborn and A. C. Petkou, Ann. Phys. (N.Y.) **231**, 311 (1994).
[76] J. Erdmenger and H. Osborn, Nucl. Phys. B **483**, 431 (1997).
[77] D. M. Hofman and J. Maldacena, J. High Energy Phys. 05 (2008) 012.
[78] S. A. Hartnoll and P. Kovtun, Phys. Rev. D **76**, 066001 (2007).
[79] S. A. Hartnoll, P. K. Kovtun, M. Muller, and S. Sachdev, Phys. Rev. B **76**, 144502 (2007).
[80] K. Goldstein, S. Kachru, S. Prakash, and S. P. Trivedi, J. High Energy Phys. 08 (**2010**) 078.
[81] M. Blake and A. Donos, Phys. Rev. Lett. **114**, 021601 (2015).
[82] X. Guo, P. Wang, and H. Yang, Phys. Rev. D **98**, 026021 (2018).
[83] P. Wang, H. Wu, and H. Yang, Eur. Phys. J. C **79**, 6 (2019).
[84] B. Kiczek, M. Rogatko, and K. I. Wysokinski, Phys. Rev. D **104**, 086022 (2021).