Letter

Entropy suppression through quantum interference in electric pulses

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The Schwinger process in strong electric fields creates particles and antiparticles that are entangled. The entropy of entanglement between particles and antiparticles has been found to be equal to the statistical Gibbs entropy of the produced system. Here we study the effect of quantum interference in sequences of electric pulses, and show that quantum interference *suppresses* the entanglement entropy of the created quantum state. This is potentially relevant to quantum-enhanced classical communications. Our results can be extended to a wide variety of two-level quantum systems.

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I. INTRODUCTION

Entanglement is the distinctive feature of the quantum world. It is also the core resource quantum computations rely upon to achieve quantum advantage. A key process to harvesting this resource is the generation of entangled states.

The question we address in this work is the following: what is the effect of quantum interference on the entanglement of particles created by a sequence of electric pulses? Specifically, we consider Schwinger pair creation in an electric field [1,2]. The spectrum of produced particles is well studied for a time dependent electric field using a wide range of techniques [3–12]. Quantum interference plays a key role in determining the particle production spectrum, and semiclassical intuition can be used for quantum control to design pulses with desired spectral characteristics [13–16].

Moreover, the resulting quantum state is known to be entangled [17–19], and the entropy of entanglement between the particles and antiparticles is equal to the Gibbs entropy of the produced system [18]. In this work, we show that the effect of quantum interference is to *decrease* the entanglement entropy.

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The underlying physical phenomenon should appear in a number of quantum two-level systems, including ionization of atoms and molecules [20–22], time-dependent tunneling [23], Landau-Zener effect [24–26], driven atomic systems [27,28], chemical reactions [29,30], Hawking radiation [31–33], cosmological particle production [34], heavy ion collisions [35–38], shot noise in tunnel junctions [39,40], and the dynamical Casimir effect [41,42]. The suppression of entanglement entropy has potential practical application in different domains, including quantum-enhanced classical communication [43]. We treat both bosons and fermions as we envisage applications to quantum detectors with both bosonic and fermionic modes. We show that the leading suppression effect is the same.

II. PAIR CREATION, ENTANGLEMENT ENTROPY, AND MULTIPLE PULSES

We consider the phenomenon of pair creation in a background electric field $\vec{E}(t) = (0, 0, E(t))$, with $\vec{A}(t) = (0, 0, A(t))$ the associated gauge potential and $E(t) = -\dot{A}(t)$. Neglecting backreaction on the field, one can solve the Klein-Gordon (respectively Dirac) equation for bosonic (b) [respectively fermionic (f)] fields. A natural formalism is that of Bogoliubov transformations: see, e.g., [6,16], and references therein.

Given some initial creation and annihilation operators $a_k^{b/f}, b_{-k}^{b/f\dagger}$, the Bogoliubov transformation coefficients $\alpha_k^{b/f}(t), \beta_k^{b/f}(t)$ relate them to the time dependent basis $\tilde{a}_k^{b/f}(t), \tilde{b}_{-k}^{b/f\dagger}(t)$

$$\begin{pmatrix} \tilde{a}_k^{b/f}(t) \\ \tilde{b}_{-k}^{b/f\dagger}(t) \end{pmatrix} = \begin{pmatrix} \alpha_k^{b/f}(t) & \pm\beta_k^{b/f*}(t) \\ \beta_k^{b/f}(t) & \alpha_k^{b/f*}(t) \end{pmatrix} \begin{pmatrix} a_k^{b/f} \\ b_{-k}^{b/f\dagger} \end{pmatrix}, \tag{1}$$

with the $+/-$ sign for bosons/fermions. $|\beta_k^{b/f}(t)|^2$ is the density of produced particles with momentum k. The bosonic/fermionic statistics are encoded in the constraint: $|\alpha_k^{b/f}(t)|^2 \mp |\beta_k^{b/f}(t)|^2 = 1$. In particular, the $+$ sign for fermions enforces the Pauli exclusion principle; the number of fermions per mode cannot exceed one.

The time evolution of the system can directly be rewritten in terms of the Bogoliubov coefficients, whose time evolution is that of a two-level system [5,6]. Extracting suitable phases, $c_{\alpha,k}^{b/f} = e^{-i\int^t \mathrm{d}\tau \mathcal{E}(\tau)} \alpha_k^{b/f}$, $c_{\beta,k}^{b/f} = e^{i\int^t \mathrm{d}\tau \mathcal{E}(\tau)} \beta^{b/f}$, the evolution equations become [15,16]

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} c_{\alpha,k}^{b/f} \\ c_{\beta,k}^{b/f} \end{pmatrix} = \begin{pmatrix} -i\mathcal{E}(t) & \Omega^{b/f}(t) \\ \pm\Omega^{b/f}(t) & i\mathcal{E}(t) \end{pmatrix} \begin{pmatrix} c_{\alpha,k}^{b/f} \\ c_{\beta,k}^{b/f} \end{pmatrix}. \quad (2)$$

Here $\mathcal{E}(t) = \sqrt{m^2 + |\vec{k}_\perp|^2 + (k_\parallel - A(t))^2}$ is the dispersion relation with $\vec{k}_\perp$ the momentum perpendicular to $\vec{E}(t)$ and $k_\parallel$ the longitudinal component. The frequencies $\Omega^{b/f}(t)$ are read from the Klein-Gordon and Dirac equations and differ for bosons $\Omega^b(t) = E(t)(k_\parallel - A(t))/(2\mathcal{E}(t)^2)$, and fermions $\Omega^f(t) = E(t)\sqrt{m^2 + k_\perp}/(2\mathcal{E}(t)^2)$. The pair production spectrum is exponentially suppressed in $k_\perp$ for linearly polarized fields, so we focus for the rest of this work on the case $k_\perp = 0, k_\parallel = k$.

The presence of the strong electric field effectively reduces the physics to one dimension, creating a distinction between particles whose momentum is aligned with the electric field ("left movers") and particles whose momentum is antialigned with the electric field ("right movers"). An informative characterization of the entanglement present in the produced quantum state is obtained by computing the entanglement entropy between left and right movers, by tracing out particles with momentum opposite to the electric field [17,18]:

$$S = -\int \frac{\mathrm{d}k}{2\pi}[|\alpha_k|^2 \log(|\alpha_k|^2) + |\beta_k|^2 \log(|\beta_k|^2)] \quad (3)$$

We will use the notation S_α and S_β for the first and second terms in (3). Note that this left-right entanglement entropy has been shown to be equal to the statistical Gibbs entropy of the produced pairs [18].

In this work, we focus on specific time sequences of pulses. We contrast symmetric configurations, where all the pulses have an electric field with the same sign, with antisymmetric configurations, where pulses have electric fields of alternating signs. Our analysis applies to very general temporal shapes of each individual pulse, but for definiteness we choose sequences of Sauter pulses, the pulse shape analyzed in [18]. The basic physics can be seen in the 2-pulse configurations: $A^A(t) = E\tau(1 + \tanh(\frac{1}{\tau}(t - \frac{T}{2})) - \tanh(\frac{1}{\tau}(t + \frac{T}{2})))$, $A^S(t) = -E\tau(\tanh(\frac{1}{\tau}(t - \frac{T}{2})) + \tanh(\frac{1}{\tau}(t + \frac{T}{2})))$, $E^{A/S}(t) = -\dot{A}^{A/S}(t)$.

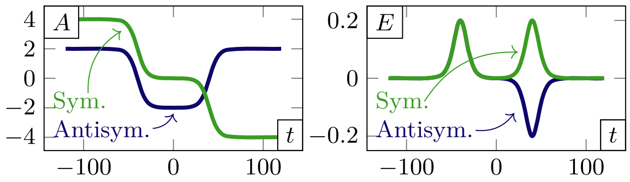

FIG. 1. (Anti)symmetric configurations for two pulses, $m = 1, E = 0.2, \tau = 10, T = 80$ (dimensionful quantities are expressed in units of m). Left: vector potential. Right: electric field.

These 2-pulse configurations are illustrated in Fig. 1. The antisymmetric configuration, with an alternating electric field, is in blue, and the symmetric one is in green. E is the electric field amplitude, τ the duration of a single pulse, and T the separation between two consecutive pulses. The corresponding N-pulse configurations are given in the Supplemental Material [44]: see Eqs. (16)–(17).

For notational convenience, we further define $n_N^{b/f,A/S}(k) \equiv |\beta_k^{b/f,A/S}|^2$, the bosonic/fermionic number density per mode of particles created by the antisymmetric (A) or symmetric (S) configuration of N pulses. We consider the semiclassical limit $E \ll m^2, E\tau \gg m$, in which pair creation occurs by nonperturbative tunneling from the Dirac sea, and we study the quantum interference effects by focussing on well-separated pulses: $\tau \ll T$. In this situation, the particle number is small and can be computed in the semiclassical approximation [3,5,14–16]. The solutions localize around complex "turning points" t_p, defined by $\mathcal{E}(t_p) = 0$, where the phase $\int^t \mathcal{E}$ is approximately stationary. Thus, for a single pulse [3–5]

$$n_1(k) \approx \exp\left(-2\left|\int_{t_0}^{t_0^*} \mathrm{d}t\mathcal{E}(t)\right|\right), \quad (4)$$

with t_0 the turning point closest to the real axis. For example, the case $A(t) = -E\tau \tanh(\frac{1}{\tau}(t - T))$ gives $t_0|_T = T + \tau\mathrm{arctanh}(\frac{-(k-im)}{E\tau})$.

For *sequences* of multiple pulses, interference effects can arise, which can be understood semiclassically [15]. The situation is very different for the antisymmetric (A) or symmetric (S) pulse sequences. Numerically computed particle momentum spectra for the case $N = 2$ are shown in Figs. 2 and 3. For the antisymmetric configuration, interferences are strong and the spectrum is highly oscillatory, whereas for the symmetric configuration, there is no interference for well-separated pulses. The bosonic case is qualitatively similar, up to the expected Maslov phase difference for the antisymmetric configuration.

These interference effects in the particle spectra are well described semiclassically [15]. The particle number is given by the modulus squared of a sum of amplitude contributions A_p from the different turning points: $n(k) \approx |\sum_p A_p(k)|^2$. In the symmetric case and for large enough separation T, the dominant turning points are

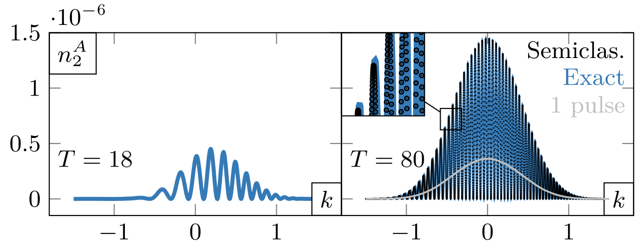

FIG. 2. Spectra of produced fermionic particles for two pulses in the antisymmetric configuration. Left: overlapping pulses. Right: well-separated pulses. The interference pattern is clear and the semiclassical expression works well for separated pulses. The single pulse spectrum is shown in gray for scale.

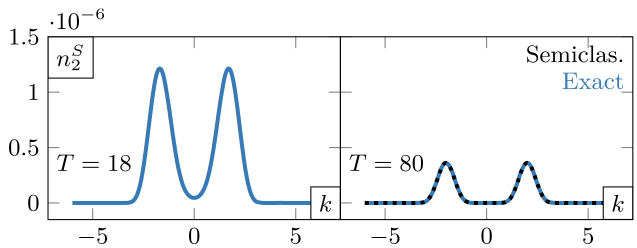

FIG. 3. Spectra of produced fermionic particles for two pulses in the symmetric configuration. Left: overlapping pulses. Right: well-separated pulses. The semiclassical approximation works in the latter case and is shown on top of the exact solution. Note that the semiclassical solution is simply the addition of two single pulse spectrum centered around different modes.

independent, for any given k. The resulting spectrum is therefore effectively a sum of single pulse spectra

$$n_N^{b/f,S}(k) \approx \sum_{l=1}^{N} n_1|_{(l)}(k). \tag{5}$$

The situation is very different in the antisymmetric case. For any given k, N turning points contribute with equal strength but different phases, leading to coherent interference. The resulting particle spectrum is well approximated by a Fabry-Perot-like form [15]

$$n_N^{f,A}(k) = \begin{cases} n_1(k)\frac{\sin^2(N\phi_k)}{\cos^2(\phi_k)}, & N \text{ even} \\ n_1(k)\frac{\cos^2(N\phi_k)}{\cos^2(\phi_k)}, & N \text{ odd} \end{cases} \tag{6}$$

$$n_N^{b,A}(k) = n_1(k)\frac{\sin^2(N\phi_k)}{\sin^2(\phi_k)}. \tag{7}$$

In (6)–(7), ϕ_k is the semiclassical phase difference between two turning point pairs $t_\pm$

$$\phi_k = \int_{\mathrm{Re}(t_-)}^{\mathrm{Re}(t_+)} \mathrm{d}t\sqrt{m^2 + (k - A^A(t))^2}. \tag{8}$$

The constant phase difference between the bosonic and fermionic case results from the behavior of the effective potential around the turning point. It is linear in the fermionic case and quadratic in the bosonic case, leading to different Maslov indices [14,15,46,47]. Thus Eqs. (6) and (7) can be rewritten as

$$n_N^{b/f,A}(k) = \left(N + 2\sum_{n=1}^{N-1}(\pm 1)^n (N-n)\cos(2n\phi_k)\right) n_1(k) \tag{9}$$

with ϕ_k defined in Eq. (8). The $+/-$ sign for bosons/fermions is from the constant phase in Eq. (8).

III. ENTROPY SUPPRESSION

We now analyze the effect of quantum interference on the entanglement/Gibbs entropy (3) of the produced particles. Recall that $|\beta_k|^2 = n(k)$, and $|\alpha_k|^2 = 1 \pm |\beta_k|^2$ for bosons/fermions, so the entropy can be computed directly from the particle spectra. The different interference effects for the symmetric/antisymmetric pulse sequences is most pronounced for well-separated pulses (recall Figs. 2 and 3). The numerical results for the ratio of entropy to particle number are shown in Fig. 4 for the symmetric and antisymmetric configurations of two pulses, as a function of the temporal pulse separation T. The solid lines show the result of solving numerically the two-level system equations (2) using `DifferentialEquations.jl` [48]. We observe that for the symmetric pulse sequence the entropy per particle number asymptotes to a value that matches with that for a single pulse, while for the antisymmetric pulse sequence this asymptotic entropy/number value is lower. In other words, *the presence of quantum interference results in a decrease of the entanglement/Gibbs entropy*. Corresponding results are shown in

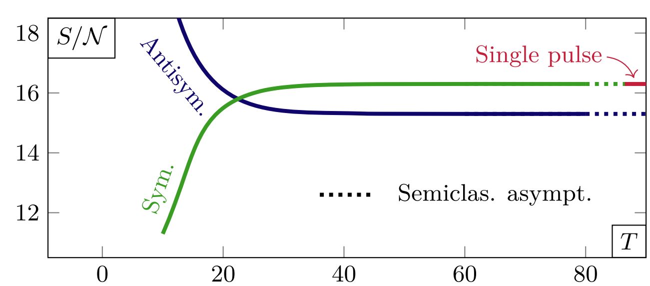

FIG. 4. Entropy to particle number ratio in the fermionic case, $m = 1, E = 0.2, \tau = 10$ (dimensionful quantities are expressed in units of m), for antisymmetric and symmetric configurations of two pulses. The symmetric configuration asymptotes to the single pulse ratio. The presence of interference reduces the entropy per particle and the antisymmetric configuration asymptotes to a lower ratio, correctly predicted by the semiclassical expression (14). The bosonic case leads to qualitatively similar results and is not shown.

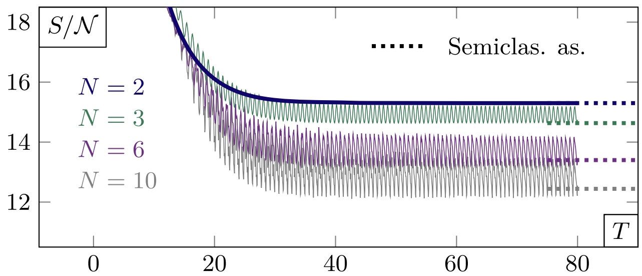

FIG. 5. Entropy to the particle number ratio in the fermionic case, $m = 1, E = 0.2, \tau = 10$ (dimensionful quantities are expressed in units of m). Antisymmetric configuration of $N = 2$, 3, 6, 10 pulses. The asymptotes are correctly predicted by the semiclassical expression (15). The presence of increasingly stronger oscillations as a function of N is an indication of interferences between multiple turning points, an effect not included in the semiclassical spectrum (6). The bosonic case leads to qualitatively similar results and is not shown.

Fig. 5 for the antisymmetric configuration with further sequences of pulses.

The underlying physics of these entropy suppression effects can be explained using semiclassical arguments. We start by discussing the total number of produced particles: $\mathcal{N}_N^{b/f,A/S} \coloneqq \langle n_N^{b/f,A/S} \rangle = \int \frac{\mathrm{d}k}{2\pi} n_N^{b/f,A/S}(k)$. In the limit of large time separation, we find that

$$\mathcal{N}^{b/f,A/S} \approx N \mathcal{N}_1^{b/f}. \tag{10}$$

This is easily seen in the symmetric case (recall (5) and Fig. 3), but is more generally true because the semiclassical phase becomes a rapidly varying function of k, leading to an incoherent sum of densities. Therefore, all integrals of the type $\int \mathrm{d}k f(k) e^{i\phi_k}$ are asymptotically exponentially suppressed. Physically, the modes are redistributed due to quantum interference between separated pulses, but the total number simply scales with N [15].

Since quantum interference has a dramatic effect on the particle spectra (recall Figs. 2 and 3), which enter the definition (3) of the entanglement entropy, it is natural to ask how quantum interference affects the entanglement entropy. We show here that the entanglement entropy has an even more interesting dependence on the interference than does the total particle number (10). The symmetric configuration is the simplest, as there is no quantum interference at large separation, so all powers of the spectral density just scale with the number of pulses N. Therefore

$$S_N^{b/f,S} \approx N S_1^{b/f}. \tag{11}$$

Combined with (10), this explains the fact that in Fig. 4 we see that for the symmetric configuration the entropy to particle number ratio tends to that of a single pulse.

For the antisymmetric configuration, we focus first on $N = 2$ (as in Fig. 4) and then generalize to N alternating sign pulses. For the $N = 2$ antisymmetric pulse case, the particle spectra (6)–(7) are: $n_2^{f,A}(k) = 4n_1(k)\sin^2(\phi_k)$ and $n_2^{b,A}(k) = 4n_1(k)\cos^2(\phi_k)$. Integrating over k, we find that expectations of powers of the density scale in a universal way in the semiclassical limit:

$$\langle (n_2^{b/f,A})^p \rangle \sim \frac{(2p)!}{(p!)^2} \langle (n_1^{b/f})^p \rangle. \tag{12}$$

Furthermore, the contributions from $S_{N,\alpha}^{b/f,A}$, the first term in (3), are subdominant in the semiclassical limit. Therefore, we find that

$$S_2^{b/f,A} \approx S_{\beta,2}^{b/f,A} = \int \frac{\mathrm{d}k}{2\pi} \frac{\mathrm{d}}{\mathrm{d}p} (n_2^{b/f,A}(k))^p \Big|_{p=1} \tag{13}$$

$$\approx 2S_1^{b/f} - 2\mathcal{N}_1^{b/f}. \tag{14}$$

Combined with (10), this explains the numerical observation in Fig. 4 that for the antisymmetric 2-pulse configuration the entropy to particle number ratio is approximately reduced by 1 due to quantum interference. This suppression can partially be attributed to a reduction of the phase space, but this is not the full explanation, and in particular it does not explain the magnitude of the suppression effect. In the antisymmetric case, the momentum distribution is centered around a single mode (unimodal) while it is bimodal in the antisymmetric case. The entropy reduction due to this difference can be assessed by computing the entropy to number particle ratio of twice the single-particle spectrum, centered around a single mode. We find (for the same parameters as in Fig. 4) $\mathcal{S}_{2N_1}/2\mathcal{N}_1 = 15.61$, which is larger than $\mathcal{S}_2/\mathcal{N}_2 = 15.29$.

An analogous interference argument (see the Supplemental Material [44]) for the N-pulse antisymmetric configuration yields a universal leading suppression

$$S_N^{b/f,A} \approx N S_1^{b/f} - 2\mathcal{N}_1^{b/f}(1 - N + N H_{N-1}), \tag{15}$$

Here $H_N = \sum_{n=1}^N \frac{1}{n}$ is the Nth harmonic number. This explains the behavior of the exact numerical results in Fig. 5. The increasingly large oscillations are due to the interference beyond neighboring turning points, not accounted for in the semiclassical spectrum (6). Recall that the difference between the effect on the particle spectra under the influence of a symmetric or antisymmetric pulse sequence is attributed to quantum interference (Figs. 2 and 3) [15]. Similarly, the contrast between the entanglement entropy result for the symmetric pulse sequence (11) and for the antisymmetric pulse sequence (15) shows that this is a quantum interference effect related to the statistics of the produced particles. In particular, notice that for the far-separated antisymmetric pulse configuration the entanglement entropy (15) is always *less* than for the symmetric configuration (11): quantum interference leads to entropy suppression.

IV. DISCUSSION

We have shown that the effect of quantum interference on the process of Schwinger pair creation is to *reduce* the entropy of the produced quantum state. This remarkable result directly generalizes to other two-level systems, as no specific assumptions beyond the validity of the semi-classical approximation were made about the type of the process or shape of the incoming signal.

An interesting theoretical avenue is to study this effect in dynamical systems. In particular, linking quantum interference to entanglement spreading may provide valuable insights into the study of information scrambling, see Ref. [49] for a review. A similarly exciting prospect is to study the effect of backreaction using quantum computations in $1+1$ directions.

Another more direct and concrete outlook is to utilize this effect in the context of information transmission. Classical information enhanced by quantum detection is an active field of research, see Ref. [43] for a review. Considering the produced quantum excitations as the state of a quantum receiver, our result suggests that classical signals with the same entropy generate quantum states with different entropies. Note that the leading effect is independent of the quantum statistics of the produced particles. This can be recast as the fact that the mutual information $\mathcal{I}_N^{A/S}$ [50–52] of the antisymmetric configuration is larger than the symmetric configuration, $\mathcal{I}_N^A - \mathcal{I}_N^S \sim 2\mathcal{N}^1(1 - N + NH_{N-1}) > 0$ by Eq. (15), opening up the possibility of improved efficiencies. Similar prospects are offered by optimized pulse shapes [53]. The effect on the entropy of (quasi)-periodic driving [54–58], known to produce spectra (sometimes exponentially) more localized than our antisymmetric configuration, as well as the link between entropy suppression and localization, are equally interesting. We leave these for future work.

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