Letter

Precise prediction for the mass of the *W* boson in gauged U(1) extensions of the standard model

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We present the one-loop radiative corrections to the muon decay in $\mathrm{U}(1)_z$ extensions of the standard model. We compute the mass of the *W* boson using those corrections and compare it to an approximation of the complete one-loop prediction implemented in automated computational tools. We point out that the truncation of the complete formulas become unreliable if the mass of the Z' boson, corresponding to the new $\mathrm{U}(1)_z$ gauge group, is larger than about 1 TeV.

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The measurements for extracting the mass of the *W* boson at hadron colliders [1–4] are steadily improving, and its precision is approaching the per myriad level [5,6]. The caveat is that the last two results differ quite significantly, and there is vigorous research activity to find the origin of this disagreement. The current combined world average of the Particle Data Group [7] $M_W = (80377 \pm 12)$ MeV does not include the CDF 2022 result.

On the theoretical side, the SM prediction has also reached a similar precision for M_W by including the one-, two-, and leading three-loop quantum corrections in perturbation theory [8,9]. These experimental and theoretical advances have elevated M_W to a prime precision parameter of the standard model (SM), which any extension of the SM must respect. Currently, there is a 2σ discrepancy between the theoretical prediction of the SM and the world average for M_W, which does not warrant any new physics effect, but one at least expects that any physics beyond the standard model (BSM) should not worsen the agreement between the measured value and the theoretical prediction. Thus we assume that the potential new physics contributions to M_W must lie within the difference of the experimental value and the SM prediction and the corrections stemming from new physics must be determined with similar precision as the SM value.

The mass of the *W* boson can best inferred from the muon decay width. In this Letter, we compute the complete one-loop radiative corrections to the muon decay process, hence to M_W, for the first time in a specific class of extensions of the SM. We consider models where the SM gauge group supplemented by an additional $\mathrm{U}(1)_z$ gauge symmetry and the particle spectrum includes a complex scalar field χ that is neutral under the standard model gauge interactions, but contributes to the mass of the neutral gauge bosons through spontaneous symmetry breaking (SSB). U(1) extensions of the SM are popular, in spite of being relatively simple, for they can explain a multitude of BSM phenomena [10–17].

The specific example we have in mind is the superweak extension of the standard model (SWSM) [18], although different charge assignments are also possible, and our formulas do not depend on the choice explicitly. The SWSM contains also three generations of SM sterile right-handed neutrinos that are clearly necessary for the cancellation of gauge and gravity anomalies and to explain the origin of neutrino masses. We do not include their effect here to simplify the parameter dependence in the numerical analysis, but it can be seamlessly integrated into our complete one-loop prediction.

The Lagrangian of the scalar fields contains a potential energy with quadratic and quartic terms such that the nonvanishing vacuum expectation value (VEV) v of the Brout-Englert-Higgs (BEH) field breaks the usual $\mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ symmetry, while the VEV w of the χ breaks the $\mathrm{U}(1)_z$ symmetry via SSB.

In addition to the appearance of the massive scalar bosons, the SSB generates mass terms also for the gauge bosons. The Lagrangian containing the vector boson (VB) mass terms is

$$\mathcal{L}_\mathrm{M}^\mathrm{VB} = \frac{v^2}{2}\left[\frac{g_\mathrm{L}^2}{2} W_\mu^+ W^{-\mu} + \frac{g_z^2}{2}\tan^2\beta\, B'_\mu B'^\mu + \frac{1}{4}\left(g_y B_\mu + (g_z - g_{yz})B'_\mu - g_\mathrm{L} W_\mu^3\right)^2\right], \quad (1)$$

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where $\tan\beta = w/v$, $g_{\rm L}$, g_y, and g_z are the $\mathrm{SU}(2)_{\rm L}$, $\mathrm{U}(1)_Y$, and $\mathrm{U}(1)_z$ couplings, while the mixing coupling g_{yz} parametrizes the kinetic mixing between the B_μ and B'_μ fields [19]. The fields $W^\pm_\mu = (W^1_\mu \pm \mathrm{i}W^2_\mu)/\sqrt{2}$ are the charged, while the neutral gauge eigenstates are B_μ, B'_μ [belonging to the $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_z$ symmetries], and W^3_μ. The latter fields are related to the neutral mass eigenstates A_μ, Z_μ, and Z'_μ via two rotations,

$$\begin{pmatrix} B_\mu \\ W^3_\mu \\ B'_\mu \end{pmatrix} = \begin{pmatrix} \mathrm{c}_W & -\mathrm{s}_W & 0 \\ \mathrm{s}_W & \mathrm{c}_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}_Z & -\mathrm{s}_Z \\ 0 & \mathrm{s}_Z & \mathrm{c}_Z \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}, \tag{2}$$

where we introduced the abbreviations $\mathrm{c}_X = \cos\theta_X$ and $\mathrm{s}_X = \sin\theta_X$ for mixing angles. The Weinberg angle $\theta_{\rm W}$ is defined as

$$\mathrm{s}_W = \frac{g_y}{g_{Z^0}}, \qquad \text{with the abbreviation } g^2_{Z^0} = g^2_y + g^2_{\rm L}, \tag{3}$$

so $e = g_{\rm L}\mathrm{s}_W$, where $g_{\rm L}$ is the SU(2) gauge coupling and e is the elementary charge. The $Z - Z'$ mixing angle $\theta_Z \in (-\pi/4, \pi/4)$ is defined implicitly in terms of effective couplings

$$\kappa = \frac{2z_\phi g_z - g_{yz}}{g_{Z^0}} \quad \text{and} \quad \tau = \frac{2g_z}{g_{Z^0}}\tan\beta \tag{4}$$

as

$$\tan(2\theta_Z) = -\frac{2\kappa}{1-\kappa^2-\tau^2}, \tag{5}$$

with z_ϕ being the z charge of the BEH scalar. Then the masses of the gauge bosons are

$$M_W = \frac{1}{2}g_{\rm L}v, \qquad M_Z = \frac{M_W}{\mathrm{c}_W}\sqrt{R(\mathrm{c}_Z, \mathrm{s}_Z)},$$
$$M_{Z'} = \frac{M_W}{\mathrm{c}_W}\sqrt{R(\mathrm{s}_Z, -\mathrm{c}_Z)}, \tag{6}$$

with $R(x,y) = (x-\kappa y)^2 + (\tau y)^2$. The free parameters from the extended gauge sector are either the Lagrangian couplings (g_z, g_{yz}) or the effective couplings (κ, τ). The latter can be expressed as functions of experimentally more accessible parameters $M_{Z'}$ and θ_Z as

$$\kappa = -\mathrm{c}_Z\mathrm{s}_Z\frac{M^2_Z - M^2_{Z'}}{\mathrm{c}^2_Z M^2_Z + \mathrm{s}^2_Z M^2_{Z'}}, \qquad \tau = \frac{M_Z M_{Z'}}{\mathrm{c}^2_Z M^2_Z + \mathrm{s}^2_Z M^2_{Z'}}. \tag{7}$$

The well-known SM relationship between the W and Z boson masses is modified at tree level to

$$\frac{M^2_W}{\mathrm{c}^2_W} = \mathrm{c}_Z{}^2 M^2_Z + \mathrm{s}^2_Z M^2_{Z'}. \tag{8}$$

This formula coincides with the one obtained later in Ref. [20] using sum rules for tree-level unitarity in U(1) extensions.

The mass of the W boson with small theoretical uncertainty can be extracted from the muon decay width. The order $O(\alpha)$ corrections to the muon decay process in the SM can be summarized by properly modifying the tree-level relation among the Fermi coupling G_F, the fine structure constant α, and the mass of the W boson, which was first derived in Ref. [21],

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M^2_W(1-\mathrm{c}^2_W)}[1-\Delta r]^{-1}, \tag{9}$$

where the parameter Δr collects the radiative corrections that enter electroweak precision observables, as well as being used to express $M_W - M_Z$ interdependence. In a $\mathrm{U}(1)_z$ extension, the relation among the masses of the gauge bosons including the radiative corrections follows from Eqs. (8) and (9) as

$$M^2_W = \frac{\mathrm{c}^2_Z M^2_Z + \mathrm{s}^2_Z M^2_{Z'}}{2} \times \left(1 + \sqrt{1 - \frac{4\pi\alpha/(\sqrt{2}G_{\rm F})}{\mathrm{c}^2_Z M^2_Z + \mathrm{s}^2_Z M^2_{Z'}}\frac{1}{1-\Delta r}}\right). \tag{10}$$

We can classify the quantum corrections to the tree-level amplitude for the muon decay process into three categories: (i) the renormalization of the SU(2) gauge coupling $g_{\rm L}$ collected into the counterterm $\delta g_{\rm L}$, (ii) loop corrections to the W boson propagator, and (iii) contribution of the vertex and box loop diagrams $\delta_{\rm BV}$ to the muon decay. In our notation, we split a generic bare coupling $g^{(0)}$ (the superscript referring to the order of the perturbative expansion) into the renormalized coupling g and a counterterm δg: $g^{(0)} = g + \delta g$. We decompose the VEVs and masses similarly, for example, $v^{(0)} = v + \delta v$ and $M^{(0)} = M_W + \delta M_W$.

The Lagrangian (1) containing the renormalized gauge boson masses can be recast as

$$\begin{aligned} \mathcal{L}^{\rm VB}_{\rm M} = {} & (M^2_W + \delta M^2_W)W^{+(0)}_\mu W^{-(0),\mu} \\ & + \frac{1}{2}(M^2_Z + \delta M^2_Z)Z^{(0)}_\mu Z^{(0),\mu} \\ & + \frac{1}{2}(M^2_{Z'} + \delta M^2_{Z'})Z'^{(0)}_\mu Z'^{(0),\mu} \\ & + (\delta M^2_{ZA}Z^{(0)}{}_\mu + \delta M^2_{Z'A}Z'^{(0)}{}_\mu)A^{0,\mu} \\ & + \delta M^2_{ZZ'}Z^{(0)}{}_\mu Z'^{(0)\mu}, \end{aligned} \tag{11}$$

where M_W, M_Z, and $M_{Z'}$ are given in Eq. (6), and the counterterms can be written symbolically as

$$\delta M_x^2 = \sum_{i=y,L,z,yz} c_{x,i}\delta g_i + c_{x,v}\delta v + c_{x,w}\delta w, \tag{12}$$

where $x = W, Z, Z', ZA, Z'A, ZZ'$. The coefficients $c_{x,i}$ are functions of the renormalized couplings g_y, $g_{\rm L}$, g_z, and g_{yz} and VEVs are v, w. In a similar way as done in the SM, we can eliminate δv in favor of the category (i) corrections,

$$\delta g_{\rm L} = \frac{\delta e}{{\rm s}_W} - \frac{e{\rm c}_W^2}{2M_W^2{\rm s}_W^3}\left[{\rm c}_W^2\left({\rm c}_Z^2\delta M_Z^2 + {\rm s}_Z^2\delta M_{Z'}^2 - 2(M_Z^2 - m_{Z'}^2)({\rm s}_Z\delta{\rm s}_Z)\right) - \delta M_W^2\right], \tag{13}$$

where

$$\frac{2\delta e}{e} = -\left.\frac{\partial\Pi_{AA}(k^2)}{\partial k^2}\right|_{k^2=0} - 2\frac{{\rm s}_W}{{\rm c}_W}\left({\rm c}_Z\frac{\Pi_{ZA}(0)}{M_Z^2} - {\rm s}_Z\frac{\Pi_{Z'A}(0)}{M_{Z'}^2}\right), \tag{14}$$

and

$$\frac{\delta{\rm s}_Z}{{\rm c}_Z} = \frac{{\rm c}_Z^2\Pi_{ZZ'}(M_Z^2) + {\rm s}_Z^2\Pi_{ZZ'}(M_{Z'}^2)}{M_Z^2 - M_{Z'}^2} - \frac{1}{2}{\rm s}_Z{\rm c}_Z\left(\left.\frac{\partial\Pi_{ZZ}(k^2)}{\partial k^2}\right|_{k^2=M_Z^2} - \left.\frac{\partial\Pi_{Z'Z'}(k^2)}{\partial k^2}\right|_{k^2=M_{Z'}^2}\right). \tag{15}$$

In Eq. (15), Π is $(-{\rm i})$ times the transverse part of self-energy graphs. The one-loop charge renormalization counterterm δe is exactly equal to the one-loop charge renormalization expression in the SM because the formula in the parentheses is independent of θ_Z. In the counterterm $\delta{\rm s}_Z$, the $Z-Z'$ mixing self-energy $\Pi_{ZZ'}(M_{Z'}^2)$ and the derivatives $\partial_{k^2}\Pi_{ZZ}(k^2)$, $\partial_{k^2}\Pi_{Z'Z'}(k^2)$ appear as completely new contributions of the extended gauge sector through the renormalization of the $Z-Z'$ mixing angle θ_Z. We shall present the detailed derivation of our formulas for $\delta g_{\rm L}$, δe, and $\delta{\rm s}_Z$ elsewhere [22].

Our main result is the complete one-loop prediction in ${\rm U}(1)_z$ extensions of the SM to the parameter Δr defined in Eq. (9). In the on shell renormalization scheme,

$$\Delta r_{\rm BSM} = \frac{{\rm Re}\Pi_{WW}(M_W^2) - \Pi_{WW}(0)}{M_W^2} + \delta_{\rm BV} + \frac{2\delta e}{e} + \frac{{\rm c}_W^2}{{\rm s}_W^2M_W^2}\left[{\rm c}_W^2{\rm Re}\Pi_{ZZ}(M_Z^2) - {\rm Re}\Pi_{WW}(M_W^2)\right] - {\rm s}_Z^2\frac{{\rm c}_W^2}{{\rm s}_W^2}\frac{{\rm c}_W^2}{M_W^2}\left[{\rm Re}\Pi_{ZZ}(M_Z^2) - {\rm Re}\Pi_{Z'Z'}(M_{Z'}^2) + 2(M_Z^2 - M_{Z'}^2)\frac{\delta{\rm s}_Z}{{\rm s}_Z}\right], \tag{16}$$

where the first two lines are only formally the same as in the SM as they also include the BSM contributions. The one-loop self energies $\Pi_{WW}(k^2)$, $\Pi_{ZZ}(k^2)$ [category (ii) corrections], and the box and vertex contribution $\delta_{\rm BV}$ (third category) have to be evaluated analogously to the SM, but with the inclusion of the BSM couplings and fields. We used the projection method of Ref. [23] to compute $\delta_{\rm BV}$. The same applies to the last two terms where the new feature is that ${\rm c}_W^2$ and ${\rm s}_W^2 = 1 - {\rm c}_W^2$ has to be evaluated according to Eq. (8).

The expression (16) must be finite and gauge independent as it collects the complete one-loop radiative corrections to the muon decay process. We checked explicitly that the ϵ poles cancel in Δr, and it is independent of the gauge parameters ξ_i ($i = A, W, Z, Z'$).

To make numerical predictions, we adapted our findings to the $\overline{\rm MS}$ renormalization scheme, employed frequently. We used the computational algorithm of Ref. [24] where the prediction for the pole mass of the W is expressed as $M_W^2 = M_{W,{\rm SM}}^2(1 + \Delta_W)$. In this equation, we use the fit formula in Eq. (45) of Ref. [9] for the standard model value $M_{W,{\rm SM}}^2$, while the correction term is written in terms of $\overline{\rm MS}$ renormalized parameters, denoted here by a hat,

$$\Delta_W = \frac{\hat{s}_{\rm W}^2}{\hat{c}_{\rm W}^2 - \hat{s}_{\rm W}^2}\left(\frac{\hat{c}_{\rm W}^2}{\hat{s}_{\rm W}^2}\Delta\hat{\rho} + \Delta\hat{r}_W^{(1)}\right), \tag{17}$$

where $\hat{c}_{\rm W}^2$ and $\hat{s}_{\rm W}^2$ are the SM values computed as in Ref. [9]. As mentioned, the renormalization constant for the electric charge at one loop in the ${\rm U}(1)_z$ extensions is exactly the same as in the SM, hence our formula for Δ_W does not contain the last term of Eq. (5) in Ref. [24].

The term $\Delta\hat{\rho}$ is the difference of the full BSM and the SM predictions for $\hat{\rho}$, with formal expansion in perturbation theory at one-loop accuracy as $\Delta\hat{\rho} = \Delta\hat{\rho}^{(0)} + \Delta\hat{\rho}^{(1)}$, where at tree level

$$\Delta\hat{\rho}^{(0)} = \left(\frac{M_{Z'}^2}{M_Z^2} - 1\right){\rm s}_Z^2. \tag{18}$$

Denoting the one-loop contributions in the full BSM by $\Delta\hat{\rho}_{\rm BSM}^{(1)}$, we have

$$\Delta\rho_{\rm BSM}^{(1)} = \frac{1}{M_W^2}\Bigg\{{\rm Re}\Pi_{WW}(M_W^2) - \hat{c}_{\rm W}^2\Bigg[{\rm c}_Z^2\left({\rm Re}\Pi_{ZZ}(M_Z^2) - 2{\rm s}_Z{\rm c}_Z\Pi_{ZZ'}(M_Z^2)\right) + {\rm s}_Z{}^2\left({\rm Re}\Pi_{Z'Z'}(M_{Z'}^2) - 2{\rm s}_Z{\rm c}_Z\Pi_{ZZ'}(M_{Z'}^2)\right) + {\rm s}_Z^2{\rm c}_Z^2(M_Z^2 - M_{Z'}^2) \times \left.\left(\frac{\partial\Pi_{ZZ}(k^2)}{\partial k^2} - \frac{\partial\Pi_{Z'Z'}(k^2)}{\partial k^2}\right)\right|_{k^2=M_{Z'}^2}\Bigg]\Bigg\}, \tag{19}$$

so the difference to the SM is

$$\Delta\hat{\rho}^{(1)} = \Delta\hat{\rho}^{(1)}_{\rm BSM} - \Delta\hat{\rho}^{(1)}_{\rm SM}. \tag{20}$$

The term $\Delta\hat{r}^{(1)}_W$ collects the one-loop diagrammatic corrections to the muon decay process,

$$\Delta\hat{r}^{(1)}_{W,\rm BSM} = \frac{\mathrm{Re}\Pi_{WW}(M_W^2) - \Pi_{WW}(0)}{M_W^2} + \delta_{\rm BV}, \tag{21}$$

so the difference to the SM is

$$\Delta\hat{r}^{(1)}_W = \Delta\hat{r}^{(1)}_{W,\rm BSM} - \Delta\hat{r}^{(1)}_{W,\rm SM}, \tag{22}$$

where the subtracted term is formally the same as formula (21), but computed with SM degrees of freedom.

The mass M_W can also be computed with automated programs once the model and the input parameters are defined, see for instance, SARAH/SPheno [25–28] and FlexibleSusy [24,29]. However, the predictions for M_W in U(1) extensions provided by these programs employ approximate one-loop BSM corrections for $\Delta\hat{\rho}^{(1)}_{\rm BSM}$ in $\mathrm{U}(1)_z$ type extensions. Our goal here is to check and explore by a numerical analysis the validity of these approximations depending on the values of the input parameters and to point out that such automated computations can lead to significantly different prediction than ours.

We investigate the predictions for M_W at fixed renormalization scale $\mu = M_Z$ in the $\overline{\rm MS}$ scheme in two approximations: (i) one includes the complete set of one-loop radiative corrections and two-loop SM corrections computed by us and (ii) a truncation when the one-loop radiative corrections to $\hat{\rho}$ are formally the same as in the SM,

$$\Delta\rho^{(1)}_{\rm BSM} = \frac{1}{M_W^2}\left[\mathrm{Re}\Pi_{WW}(M_W^2) - \hat{c}^2_{\rm W}\mathrm{Re}\Pi_{ZZ}(M_Z^2)\right], \tag{23}$$

but with self-energies evaluated in the BSM extension, which is the $\mathrm{U}(1)_z$ extension in our work. Case (ii) is implemented in automated high energy physics tools such as SARAH/SPheno [25–28] and FlexibleSusy [24,29].

We use the set of input parameters

$$\begin{gathered} G_{\rm F} = 1.1663787\times10^{-5}\ \mathrm{GeV}^{-2}, \qquad M_Z = 91.1876\ \mathrm{GeV}, \\ M_H = 125.25\ \mathrm{GeV}, \qquad m_t = 172.83\ \mathrm{GeV}, \qquad m_b = 4.18\ \mathrm{GeV}, \\ \alpha_s(M_Z) = 0.1179, \qquad \alpha = (137.036)^{-1}, \qquad \Delta\alpha^{(5)}_{\rm had} = 0.02760 \end{gathered} \tag{24}$$

taken from [7]. In particular, the value of the top quark pole mass m_t is presented in the Quark Masses subsection of Chap. 10 and the numerical value for $\Delta\alpha^{(5)}_{\rm had}$ is the average of the results presented in Table 10.1 of Ref. [7].

Once the parameters in (24) are set, the prediction for M_W at fixed μ depends on five free parameters $M_{Z'}, \mathrm{s}_Z, M_S, \mathrm{s}_S$, and $\tan\beta$ where M_S is the mass of the scalar particle appearing after SSB of the complex scalar field χ and s_S is the scalar mixing angle. The SM is recovered in the limit of vanishing massive neutral gauge boson and scalar mixings, $\mathrm{s}_Z = \mathrm{s}_S = 0$, which produces our reference SM predictions in agreement with the literature,

$$M_{W,\rm SM} = 80.353\ \mathrm{GeV}, \qquad \hat{s}^2_{W,\rm SM}(M_Z) = 0.2313, \qquad \hat{\alpha}^{-1}_{\rm SM}(M_Z) = 127.952,$$

once the decoupling of the top quark [30] is applied.

The extension of the SM gauge sector affects the vector and axial vector (V-A) couplings of the Z boson and introduces the Z' boson, which also interacts with fermions through its own V-A couplings. The exact form of these couplings depend on the z-charge assignment of the new $\mathrm{U}(1)_z$ gauge group. In order to present numerical values for our predictions, we select the SWSM where the z charges are fixed as $z_Q = 1/6$, $z_U = 7/6$, and $z_\phi = z_U - z_Q$ [18].

We compare the predictions of the two cases in order to explore the validity of the approximations applied in (ii). We present our findings as benchmark points expressed as the differences $\Delta M_W = M_W - M_{W,\rm SM}$, sampled from different regions of the parameter space spanned by $M_{Z'}, \mathrm{s}_Z, M_S, \mathrm{s}_S$, and $\tan\beta$. In general, the mixing angle $\theta_{\rm Z}$ severely affects the predictions for electroweak observables. We present our benchmark points depending on the ratios $M_{Z'}/M_Z$ and M_S/M_h being smaller or larger than 1 (see Supplemental Material [31]).

A light (heavy) Z' boson with $M_{Z'} < M_Z$ ($M_{Z'} > M_Z$) contributes a negative (positive) shift to the mass of the W boson. For light new physics (Table I), $M_{Z'}/M_Z \ll 1$, both cases provide a good approximation. For heavy new physics (Table II), however, when $M_{Z'}/M_Z \gg 1$, the approximation of case (ii) may become unreliable and the difference from the complete prediction (i) can surpass the size of the typical experimental uncertainty of about 10 MeV. The advantage of the computational algorithm of Ref. [24] is that it removes the nondecoupling logarithmic contributions from the one-loop formula. Those logarithms can become potentially large and are canceled in perturbation theory only if the two-loop BSM contributions are also computed. The cancellation of those large logarithms can be seen in Fig. 1 where we present the dependence of our prediction for M_W on the renormalization scale μ. The solid

TABLE I. Predictions for $\Delta M_W = M_W - M_{W,\mathrm{SM}}$ in MeV units at parameter values $M_{Z'} = 50$ MeV, $s_Z = 0.005$, and $s_S = 0.1$.

s_Z	5×10^{-4}			
	M_S			
	0.5 TeV		5 TeV	
$\tan\beta$	(i)	(ii)	(i)	(ii)
0.1	−1	−1	−2	−2
1	−1	−1	−2	−2
10	−1	−1	−2	−2

TABLE II. Predictions for $\Delta M_W = M_W - M_{W,\mathrm{SM}}$ in MeV units at parameter values $M_{Z'} = 5$ TeV and $s_S = 0.1$.

s_Z	5×10^{-4}				7×10^{-4}			
	M_S							
	0.5 TeV		5 TeV		0.5 TeV		5 TeV	
$\tan\beta$	(i)	(ii)	(i)	(ii)	(i)	(ii)	(i)	(ii)
10	37	10	35	13	75	29	73	36
20	39	34	35	34	81	76	74	79
30	40	38	35	37	83	85	75	85

line corresponds to case (i) and the dotted one to case (ii). The input values are the same as in Table II with $\tan\beta = 10$ and $M_S = 500$ GeV.

We also compare the two predictions by showing the 2σ allowed band $|M_W^{\mathrm{exp}} - M_W| < 2\sigma$, where M_W is our theoretical prediction in the $\mathrm{U}(1)_z$ extension, M_W^{exp} is the experimentally measured value, and $\sigma = \sqrt{\sigma_{\mathrm{exp}}^2 + \sigma_{\mathrm{theo}}^2 + \sigma_{\mathrm{param}}^2}$, with σ_{exp} being the uncertainty of M_W^{exp}, σ_{theo} being the theoretical, and σ_{param} being the parametric uncertainty of our prediction M_W. The theoretical uncertainty is estimated in Ref. [9] to be $\sigma_{\mathrm{theo}} = 4$ MeV, while we estimate the parametric uncertainty with the input values presented in Eq. (24) to be $\sigma_{\mathrm{theo}} = 8$ MeV. Figure 2 shows the 2σ allowed bands obtained with the PDG world average [7] for M_W^{exp}. We see that the approximation (ii) leads to different allowed regions for a heavy Z' whose extent depends on the values of the free parameters. This warns us that one has to be careful when using the automated computations for the radiative corrections to the mass of the W boson.

In summary, we computed the one-loop corrections Δr to the muon decay in a general $\mathrm{U}(1)_z$ extension of the standard model. We did not make any assumptions for the parameters of the model or employing any truncations beyond the systematic perturbation theory. We presented the complete expression for Δr in the on-shell renormalization scheme and also in the $\overline{\mathrm{MS}}$ scheme. We found that, not only additional loops appear in the transverse W and Z boson self-energies $\Pi_{WW}(p^2)$, $\Pi_{ZZ}(p^2)$, and in the box and vertex corrections δ_{BV}, but the Z' boson self-energy $\Pi_{Z'Z'}(p^2)$ and the wave function renormalizations Z_{ZZ},

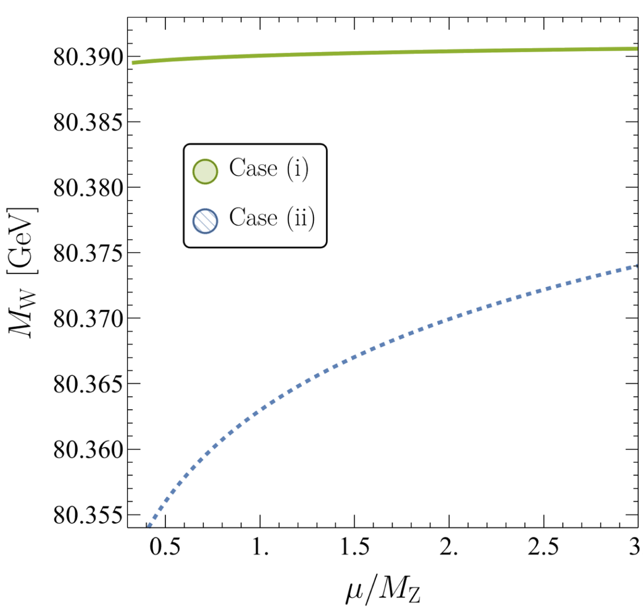

FIG. 1. The dependence of our prediction for M_W on the renormalization scale μ in case (i) (solid green curve) and (ii) (dotted blue curve).

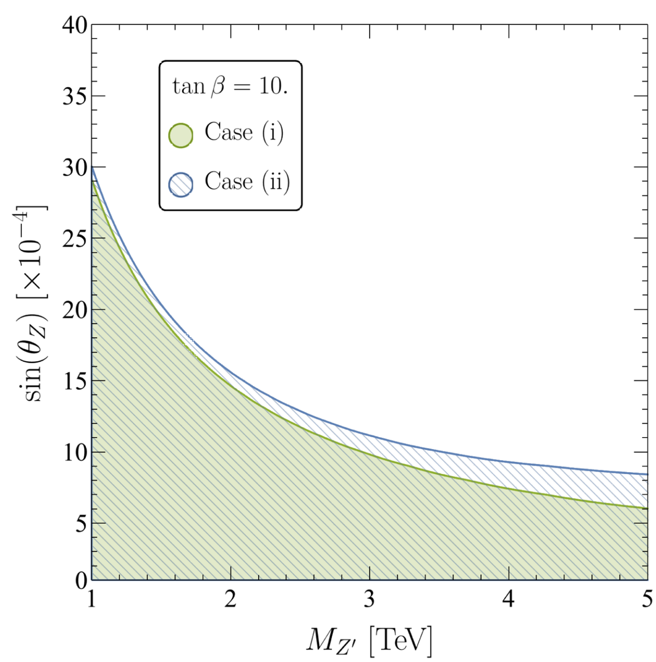

FIG. 2. Comparison of the predictions by plotting the regions allowed by the requirement $|M_W^{\mathrm{exp}} - M_W| < 2\sigma$ for the fixed values $s_S = 0.1$ and $M_S = 500$ GeV in the $M_{Z'} - s_Z$ plane for large values of $M_{Z'}$.

$Z_{Z'Z'}$, and $Z_{ZZ'}$ also contribute to Δr. The new terms appear in the renormalization of θ_W in the on shell scheme or in the $\hat{\rho}$ parameter in the $\overline{\mathrm{MS}}$ scheme.

The high energy physics tools that can automatically compute the radiative corrections to M_W in BSM models neglect several terms from the complete expression in Eq. (19) for $\mathrm{U}(1)_z$ type BSM extensions. We pointed out using a specific example model, the SWSM [18], that the effect of the neglected terms can affect significantly the prediction. In qualitatively different regions of the parameter space spanned by the free input parameters, we selected benchmark points for small $Z-Z'$ mixing s_Z. We found that neglecting Z_{ZZ}, $Z_{Z'Z'}$, and $\Pi_{Z'Z'}(p^2)$ produces small $\mathcal{O}(\mathrm{MeV})$ numerical differences for $\tan\beta \geq 1$ in the region where the mass of the new neutral gauge boson is much lighter then the Z boson, but for $M_{Z'} \gg M_Z$ the use of the complete expression employed in case (i) is more appropriate.

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