Letter

Structure-dependent QED effects in exclusive B decays at subleading power

Claudia Cornella,[1,*] Matthias König,[2,†] and Matthias Neubert[1,3,‡]

[1]*PRISMA+ Cluster of Excellence and MITP, Johannes Gutenberg University, 55099 Mainz, Germany*
[2]*Physik Department T31, Technische Universität München, James-Franck-Strasse 1, 85748 Garching, Germany*
[3]*Department of Physics and LEPP, Cornell University, Ithaca, New York 14853, USA*

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We derive a factorization theorem for the structure-dependent QED effects in the weak exclusive process $B^- \to \mu^- \bar{\nu}_\mu$, i.e., effects probing the internal structure of the B meson. The derivation requires a careful treatment of end-point-divergent convolutions common to subleading-power factorization formulas. We find that the decay amplitude is sensitive to two- and three-particle light-cone distribution amplitudes of the B meson as well as to a new hadronic quantity $F(\mu, \Lambda)$, which generalizes the notion of the B-meson decay constant in the presence of QED effects. This is one of the first derivations of a subleading-power factorization theorem in which the soft functions are nonperturbative hadronic matrix elements.

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Exclusive B-meson decays are powerful probes of the flavor sector and of physics beyond the Standard Model. In order to match the increasing experimental accuracy in several decay channels, a reliable assessment of QED corrections is desirable. In recent years, these have received considerable attention, especially in the context of leptonic and semileptonic B decays. In most cases, QED corrections were treated via the inclusion of soft-photon emissions, under the hypothesis that the leading corrections can be described by photons unable to probe the internal meson structure [1,2]. This assumption is in direct contradiction with the observation that structure-dependent QED corrections constitute an important contribution to the decays $B_{d,s} \to \mu^+\mu^-$ [3,4].

In this work, we present the factorization formula for the exclusive decay $B^- \to \ell^- \bar{\nu}_\ell$ including virtual one-loop QED corrections. This process can be used to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub} and to test lepton-flavor universality, as Belle II can perform accurate measurements of the $\ell = \mu$, τ channels [5]. We focus here on the case $\ell = \mu$. Due to the chirality-suppressed nature of the decay, this process is of next-to-leading power (NLP) in the $1/m_B$ expansion. Factorization formulas at subleading power are plagued by end-point-divergent convolution integrals [6–17], requiring a careful subtraction and rearrangement between different contributions. The refactorization-based subtraction (RBS) scheme introduced in [12,14] for the derivation of the factorization theorem for the Higgs-boson decay $h \to \gamma\gamma$ via b-quark loops provides a method to deal with end-point divergences and establish factorization at NLP. The RBS scheme has also been applied successfully to Higgs production in gluon-gluon fusion [15,18] and to the "off-diagonal gluon thrust" in e^+e^- collisions [16]. Along with [19], the present work applies this approach for the first time in the context of exclusive rare decays of B mesons, where the necessary rearrangements involve objects that are genuinely nonperturbative, giving rise to new types of hadronic matrix elements (see [20] for an application to inclusive B decays). The presence of such quantities is a generic feature of exclusive B-meson decays at NLP.

Below the electroweak scale, the effective weak Lagrangian describing the decay $B^- \to \ell^- \bar{\nu}_\ell$ is given by

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} (\bar{u}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell). \quad (1)$$

When electroweak corrections are neglected $K_{\text{EW}}(\mu) = 1$, and all hadronic effects are encoded in the B-meson matrix element of the quark current,

$$\langle 0|\bar{u}\gamma^\mu\gamma_5 b|B^-\rangle = i m_B f_B v^\mu. \quad (2)$$

Here v^μ denotes the 4-velocity of the B meson and f_B its decay constant. The situation becomes significantly more complicated when QED effects are taken into account. In this case $K_{\text{EW}}(\mu) \neq 1$ [21] and the operator in (1) has a

*claudia.cornella@uni-mainz.de
†matthias.koenig@tum.de
‡matthias.neubert@uni-mainz.de

nontrivial scale dependence, which compensates that of $K_{\rm EW}$, given by [22]

$$\frac{dK_{\rm EW}(\mu)}{d\ln\mu} = Q_\ell Q_u \frac{3\alpha}{2\pi} K_{\rm EW}(\mu). \quad (3)$$

More profoundly, the B-meson decay constant loses its universal meaning and its definition must be generalized, because the flavor-changing quark current is not gauge invariant with respect to QED interactions [4,23]. The simple factorization of the four-fermion operator into a quark and a lepton current, with no interactions between them, no longer holds. While in QCD physical states are color neutral, both the B meson and the charged lepton carry electric charges, and thus electromagnetic interactions inevitably connect the two currents.

In the presence of QED effects, the $B^- \to \ell^- \bar\nu_\ell$ matrix element of the four-fermion operator in (1) is sensitive to six different energy scales. The first four are the scale m_b setting the large mass of the decaying B meson, the intermediate "hard-collinear" scale $\sqrt{m_b\Lambda_{\rm QCD}}$ at which the internal structure of the meson is probed by virtual photons, the scale $\Lambda_{\rm QCD}$ of nonperturbative soft QCD interactions in the meson, and the lepton mass m_ℓ. In order to obtain an infrared (IR) safe observable, it is necessary to define the decay rate for the process $B^- \to \ell^- \bar\nu_\ell(\gamma)$, allowing for the emission of real photons with energies below a resolution scale E_s. The threshold E_s and a related scale $(m_\ell/m_B)E_s$ complete the list of relevant scales. We have analyzed the factorization of these scales using a multistep procedure, in which the effective weak Lagrangian (1) is matched onto two versions of soft-collinear effective theory [24–27]: $\mathcal{L}_{\rm eff} \to$ SCET-1 $\to$ SCET-2. In a final step, the SCET-2 operators are matched onto a low-energy effective theory consisting of products of Wilson lines, which are needed to account for real-photon emissions. Technical details will be presented elsewhere.

In this Letter, we focus on the intricate factorization properties of the decay amplitude above the scale E_s, which is sensitive to virtual photon exchange only. We have established the factorization theorem

$$\mathcal{A}^{\rm virtual}_{B\to\ell\bar\nu} = \sum_j H_j S_j K_j + \sum_i H_i \otimes J_i \otimes S_i \otimes K_i, \quad (4)$$

where the hard functions H_i account for matching corrections at the scale m_b, the jet functions J_i encode matching corrections at the scale $\sqrt{m_b\Lambda_{\rm QCD}}$, and the soft functions S_i are hadronic matrix elements of the B meson defined in heavy-quark effective theory (HQET) [28–31]. The collinear functions K_i describe the leptonic matrix elements, encoding the dependence on the scale m_ℓ. The first set of terms arises from SCET-1 operators containing a soft spectator quark, whereas the second set descends from operators in which the spectator quark is described by a hard-collinear field, carrying a significant fraction of the charged-lepton momentum. The symbol $\otimes$ indicates that the products of component functions must be understood as convolutions, since some of the functions share common momentum variables, over which one must integrate. In SCET-2, interactions between soft and collinear particles can be eliminated at the Lagrangian level using field redefinitions [25,32–34]. The remnants of these interactions appear in the form of soft Wilson lines $Y_n^{(f)}$ for fermion f, which depend on its color and electric charge. The lightlike vector n is aligned with the direction of the muon. Soft-collinear photons, whose momenta are collinear with the muon but whose energy in the B-meson rest frame is smaller than the soft scales $\Lambda_{\rm QCD}$ and E_s by a factor m_ℓ/m_B, also play an important role. Removing their interactions with (massive) collinear and soft particles by field redefinitions gives rise to soft-collinear Wilson lines $C_{v_\ell}^{(\ell)}$ for the muon and $C_{\bar n}^{(u)\dagger} C_{\bar n}^{(b)} \equiv C_{\bar n}^{(B)}$ for the two valence quarks inside the B meson. Here v_ℓ denotes the 4-velocity of the lepton, and the lightlike vector $\bar n$ is aligned with the direction of the neutrino. These soft-collinear Wilson lines are inherited by the effective theory below the scale $\Lambda_{\rm QCD}$. For the purposes of our discussion here, they can simply be ignored.

The appearance of a hard-collinear scale between m_b and $\Lambda_{\rm QCD}$ is an important feature of the factorization formula. Electromagnetic radiation with virtuality $q^2 \sim m_b\Lambda_{\rm QCD}$ emitted from the muon can recoil against the meson and probe its internal structure. This effect arises from the interactions between soft and collinear particles [35–37], which in SCET-1 are mediated by the exchange of a virtual photon between the muon and the soft spectator quark in the B meson, as illustrated in Fig. 1. After matching onto SCET-2 this gives rise to nonlocal operators, whose component fields have lightlike separation. Their matrix elements define the B-meson light-cone distribution amplitudes (LCDAs) [38–41]. From a systematic analysis of the operators contributing to the decay at $\mathcal{O}(1/m_b)$, we find that the amplitude is sensitive to a hadronic parameter F generalizing the concept of the B-meson decay constant, as well as to two- and three-particle LCDAs of the B meson.

A natural definition of the parameter F would be in terms of the B-meson matrix element of the operator

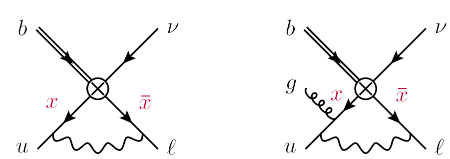

FIG. 1. SCET-1 loop diagrams generating structure-dependent QED corrections. The up-quark and muon leaving the weak-interaction operator carry fractions x and $\bar x = 1 - x$ of the large component $\bar n \cdot p_\ell$ of the muon momentum. The resulting contributions involve convolutions with a two-particle (left) and three-particle (right) LCDA of the B meson.

$$O_A = \bar{n}_\mu \bar{u}_s \gamma^\mu P_L h_v Y_n^{(\ell)\dagger}, \qquad (5)$$

where u_s denotes a soft quark field, and h_v is the effective b-quark field in HQET. The factor $\bar{n}_\mu$ appears in the evaluation of the leptonic matrix element. The Wilson line arises from the decoupling of soft photons from the muon. It ensures that the operator is gauge invariant under both QCD and QED. In the presence of QED corrections, the anomalous dimension of O_A exhibits a sensitivity to IR regulators, which must be removed with a suitable subtraction [4,23]. Following these authors, one can thus define F as the matching coefficient of the B-meson matrix element of O_A onto a Wilson-line operator in a low-energy effective theory for very soft photons (with $E_\gamma \ll \Lambda_{\rm QCD}$), which see the B meson as a pointlike particle,

$$\langle 0|O_A|B^-\rangle = -\frac{i}{2}\sqrt{m_B} F(\mu) \langle 0|Y_v^{(B)} Y_n^{(\ell)\dagger}|0\rangle. \qquad (6)$$

However, an unusual aspect of this definition is that the renormalization of the "local" (with regard to the quark fields) operator O_A requires the nonlocal operator

$$O_B(\omega) = \int \frac{dt}{2\pi} e^{i\omega t} \bar{u}_s(tn)[tn,0] \not{\bar{n}} P_L h_v(0) Y_n^{(\ell)\dagger}(0) \qquad (7)$$

as a counterterm. Here the quark fields are separated by a lightlike distance, and $[tn, 0]$ denotes a soft Wilson-line segment connecting them.

There exists another problem with the factorization formula (4), as some of the convolution integrals suffer from end-point divergences. This is a common feature of NLP factorization theorems. Neglecting corrections of $\mathcal{O}(\alpha\alpha_s)$, the divergent convolutions are those involving the hard and jet functions. These divergences are troublesome, because they give rise to $1/\epsilon$ poles that cannot be removed by renormalizing the hard and jet functions individually, and hence break the desired factorization of scales. Interestingly, we find that removing the end-point divergences using the RBS scheme [12,14] allows us to redefine the soft operator O_A in such a way that it no longer mixes with the nonlocal operator $O_B(\omega)$.

The RBS scheme offers a systematic procedure for dealing with end-point divergences. In a first step, they are removed by performing plus-type subtractions of the integrand, i.e.,

$$\begin{aligned} H_i \otimes J_i &\equiv \int_0^1 dx\, H_i(m_b, x) J_i(m_b\omega, x) \\ &\to \int_0^1 dx [H_i(m_b, x) J_i(m_b\omega, x) \\ &\quad - \theta(\lambda - x) [\![H_i(m_b, x)]\!] [\![J_i(m_b\omega, x)]\!]], \end{aligned} \qquad (8)$$

where x is a shared longitudinal momentum fraction defined in Fig. 1, and ω denotes the $n \cdot p_u$ component of the soft spectator momentum, which the jet and soft functions share. (In some cases there can be more than one such variable.) The singular limit is $x \to 0$, corresponding to the region in which the virtual spectator quark becomes soft. The double brackets indicate that one needs to retain only the leading singular terms in the expressions for the hard and jet functions. More accurately, when $x = \mathcal{O}(\Lambda_{\rm QCD}/m_b)$ the quark and photon propagators in the loop become soft and should no longer be described using hard-collinear fields. We introduce a parameter $0 < \lambda < 1$ to subtract these contributions (see also [16]). By construction, this subtraction removes the end-point divergence, but the subtraction term must be added back in a consistent way. This is done using exact, D-dimensional refactorization conditions [12–14], which govern the structure of the component functions in the singular limits. In our case, these conditions read

$$\begin{aligned} [\![H_i(m_b, x)]\!] &= H_i'(m_b) S_i'(\omega'), \\ [\![J_i(m_b\omega, x)]\!] &= m_b S_i''(\omega, \omega'), \end{aligned} \qquad (9)$$

where H_i' are new hard functions, while S_i', S_i'' are new soft functions, which depend on the variable $\omega' \equiv x m_b$. The term that needs to be added back thus takes the form of a hard matching coefficient times a soft function,

$$\begin{aligned} &\int d\omega \int_0^\lambda dx [\![H_i(m_b, x)]\!] [\![J_i(m_b\omega, x)]\!] S_i(\omega) \\ &\quad = -H_i' \int d\omega \int_\Lambda^\infty d\omega' \hat{S}_i(\omega, \omega'), \end{aligned} \qquad (10)$$

where $\Lambda = \lambda m_b$. In the last step we have defined $\hat{S}_i = S_i S_i' S_i''$ and added a scaleless integral, which vanishes in dimensional regularization. After adding back this term, it can be combined with other terms of similar form.

Let us illustrate this procedure for the soft operators relevant for the subtraction of end-point divergences in our problem. These are the local operator O_A in (5) and the associated nonlocal operator $O_B(\omega)$ in (7). There is also a third operator giving rise to a three-particle LCDA, which we omit here for simplicity, but we include its effect in our final result (22) below. (The SCET-1 and SCET-2 operator bases needed to establish the factorization theorem are of course much larger. They can be found by building all gauge- and boost-invariant operators of mass dimension 6 and the correct power counting [32,33]. The additional operators do not give rise to end-point divergences, however.) The contribution of these two operators to the decay amplitude can be written as

$$\mathcal{A}_{B\to\ell\bar{\nu}}^{(A,B)} = -\frac{4G_F}{\sqrt{2}} K_{\rm EW} V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell) \bar{u}(p_\ell) P_L v(p_\nu) \cdot \left[H_A(m_b) S_A + \int d\omega \int_0^1 dx\, H_B(m_b, x) J_B(m_b\omega, x) S_B(\omega) \right], \tag{11}$$

where $S_A = -\frac{i}{2}\sqrt{m_B}F$, $S_B(\omega) = -\frac{i}{2}\sqrt{m_B}F\phi_-^B(\omega)$, and $H_{A,B} = 1 + \mathcal{O}(\alpha_s, \alpha)$. Here $\phi_-^B(\omega)$ is one of the twist-3 LCDAs of the B meson [38], which is normalized to unity. Starting at one-loop order H_B contains logarithmic singularities at $x = 0$ ($\sim x^{-n\epsilon}$ in dimensional regularization). The collinear functions for the two contributions are equal and normalized so that $K_A = K_B = 1 + \mathcal{O}(\alpha)$. At one-loop order, the (bare) jet function is given by

$$J_B(m_b\omega, x) = -Q_\ell Q_u \frac{\alpha}{2\pi} \frac{e^{\epsilon\gamma_E}\Gamma(\epsilon)}{1-\epsilon} \left(\frac{1}{x} + 1 - 2\epsilon \right) \cdot \left(\frac{\mu^2}{m_b\omega x(1-x)} \right)^\epsilon. \tag{12}$$

The refactorization conditions for H_B and J_B read

$$\begin{aligned} [\![H_B(m_b, x)]\!] &= H_A(m_b) S_B'(\omega'), \\ [\![J_B(m_b\omega, x)]\!] &= m_b S_B''(\omega, \omega'), \end{aligned} \tag{13}$$

where $\omega' = x m_b$, $S_B' = 1 + \mathcal{O}(\alpha_s, \alpha)$, and

$$S_B''(\omega, \omega') = -Q_\ell Q_u \frac{\alpha}{2\pi} \frac{e^{\epsilon\gamma_E}\Gamma(\epsilon)}{1-\epsilon} \frac{1}{\omega'} \left(\frac{\mu^2}{\omega\omega'} \right)^\epsilon. \tag{14}$$

We find that at one-loop order the subtraction term in (10) is given by

$$H_A S_A Q_\ell Q_u \frac{\alpha}{2\pi} \frac{e^{\epsilon\gamma_E}\Gamma(\epsilon)}{1-\epsilon} \int_0^\infty d\omega\, \phi_-^B(\omega) \int_\Lambda^\infty \frac{d\omega'}{\omega'} \left(\frac{\mu^2}{\omega\omega'} \right)^\epsilon. \tag{15}$$

We consistently neglect terms of $\mathcal{O}(\alpha^2)$ and thus do not include QED corrections to the LCDA. The presence of the hard function H_A in this result suggests that we should combine it with the contribution of the operator O_A. In all previous applications of the RBS scheme, the soft functions were perturbatively calculable, and the effect of the subtraction terms could be worked out order by order in perturbation theory. In the present case, we apply the refactorization conditions for the first time in a nonperturbative context, where the soft functions are hadronic matrix elements, which cannot be calculated using short-distance methods. Adding the subtraction term (including the three-particle contribution neglected above) has the effect of replacing the operator O_A by

$$O_A \to O_A^{(\Lambda)} = \bar{u}_s \not{n} P_L h_v \theta(i\bar{n}\cdot D_s - \Lambda) Y_n^{(\ell)\dagger}, \tag{16}$$

where the covariant derivative in the θ-function acts on the leptonic Wilson line. Generalizing (6), we now define the hadronic parameter F as

$$\langle 0|O_A^{(\Lambda)}|B^-\rangle = -\frac{i}{2}\sqrt{m_B} F(\mu, \Lambda) \langle 0|Y_v^{(B)} Y_n^{(\ell)\dagger}|0\rangle. \tag{17}$$

We find that the presence of the θ-function in (16) removes the mixing with the nonlocal operator $O_B(\omega)$. At one-loop order, the anomalous dimension defined via $dF(\mu, \Lambda)/d\ln\mu = -\gamma_F F(\mu, \Lambda)$ is given by

$$\gamma_F = -C_F \frac{3\alpha_s}{4\pi} + \frac{3\alpha}{4\pi} \left(Q_\ell^2 - Q_b^2 + \frac{2}{3} Q_\ell Q_u \ln\frac{\Lambda^2}{\mu^2} \right). \tag{18}$$

It is also possible to control the dependence on the cutoff Λ using perturbation theory. At one-loop order, we find

$$\frac{d\ln F}{d\ln\Lambda} = Q_\ell Q_u \frac{\alpha}{2\pi} \left[\int_0^\infty d\omega \phi_-^B(\omega) \ln\frac{\omega\Lambda}{\mu^2} - 1 + \ldots \right], \tag{19}$$

where the dots stand for a contribution involving the three-particle LCDA.

When the subtraction term is combined with the original contribution of the operator O_A, we obtain from (11)

$$\begin{aligned} \mathcal{A}_{B\to\ell\bar{\nu}}^{(A,B)} = -\frac{4G_F}{\sqrt{2}} K_{\rm EW} V_{ub} \frac{m_\ell}{m_b} S_A^{(\Lambda)} K_A(m_\ell) \bar{u}(p_\ell) P_L v(p_\nu) \cdot \bigg[H_A(m_b) + \int_0^\infty d\omega \phi_-^B(\omega) \int_0^1 dx [H_B(m_b, x) J_B(m_b\omega, x) \\ - \theta(\lambda - x) [\![H_B(m_b, x)]\!]\, [\![J_B(m_b\omega, x)]\!] \bigg]. \end{aligned} \tag{20}$$

The subtracted convolution and the soft function $S_A^{(\Lambda)} = -\frac{i}{2}\sqrt{m_B}F(\mu, \Lambda)$ depend on the cutoff Λ, and there is no choice for which both objects depend only on their natural scales. Following [12], we choose $\Lambda = m_b$ and hence $\lambda = 1$ to eliminate the second scale from the subtracted convolution, at the expense of introducing the scale m_b in the definition of F in (17). The translation of $F(\mu, m_b)$ to $F(\mu, \Lambda)$ with a different choice of Λ can be obtained by solving the evolution equation (19).

We are now ready to present our main result. We find that the $B^- \to \mu^- \bar{\nu}_\mu$ decay amplitude including virtual QED corrections is given by

$$\mathcal{A}^{\text{virtual}}_{B^-\to\ell\bar{\nu}} = i\sqrt{2} G_F K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} \bar{u}(p_\ell) P_L v(p_\nu) \cdot \sqrt{m_B} F(\mu, m_b) [\mathcal{M}_{2p}(\mu) + \mathcal{M}_{3p}(\mu)], \tag{21}$$

where the two terms in the second line probe the two- and three-particle Fock states of the B meson. After renormalizing the four-fermion operator in (1), the muon mass, and the parameter F in the $\overline{\text{MS}}$ scheme, and performing the integrations over x, we obtain at one-loop order

$$\begin{aligned}
\mathcal{M}_{2p}(\mu) &= 1 + \frac{C_F\alpha_s}{4\pi}\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right] + \frac{\alpha}{4\pi}\left\{Q_b^2\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right] - Q_\ell Q_b\left[\frac{1}{2}\ln^2\frac{m_b^2}{\mu^2} + 2\ln\frac{m_b^2}{\mu^2} - 3\ln\frac{m_\ell^2}{\mu^2} + 1 + \frac{5\pi^2}{12}\right]\right. \\
&\quad \left. + 2Q_\ell Q_u \int_0^\infty d\omega\, \phi_-^B(\omega) \ln\frac{m_b\omega}{\mu^2} + Q_\ell^2\left[\frac{1}{\epsilon_{\text{IR}}}\left(\ln\frac{m_B^2}{m_\ell^2} - 2\right) + \frac{1}{2}\ln^2\frac{m_\ell^2}{\mu^2} - \frac{1}{2}\ln\frac{m_\ell^2}{\mu^2} + 2 + \frac{5\pi^2}{12}\right]\right\}, \\
\mathcal{M}_{3p}(\mu) &= \frac{\alpha}{\pi} Q_\ell Q_u \int_0^\infty d\omega \int_0^\infty d\omega_g\, \phi_{3g}^B(\omega,\omega_g)\left[\frac{1}{\omega_g}\ln\left(1 + \frac{\omega_g}{\omega}\right) - \frac{1}{\omega + \omega_g}\right].
\end{aligned} \tag{22}$$

In the virtual amplitude there remain IR divergences, which cancel against the IR divergences from real-photon emission in the process $B^- \to \ell^- \bar{\nu}_\ell(\gamma)$.

When corrections of $\mathcal{O}(\alpha\alpha_s)$ are included, the integrals over the LCDA $\phi_-^B(\omega)$ in (20) and (22) no longer converge at infinity [38,39], indicating another occurrence of an end-point divergence. One then needs to refactorize these integrals in the region where $\Lambda_{\text{QCD}} \ll \omega \ll m_b$, using techniques developed in [42,43]. Since these corrections are bound to be very small numerically, this issue will be discussed in detail elsewhere.

The three-particle LCDAs of the B meson have been studied in [40,41]. Our function $\phi_{3g}^B(\omega, \omega_g)$ is related to the functions defined in these references by

$$\phi_{3g}^B(\omega, \omega_g) = \frac{1}{\omega_g}[\psi_A(\omega, \omega_g) - \psi_V(\omega, \omega_g)], \tag{23}$$

where the momentum variables ω and ω_g refer to the spectator quark and the gluon, respectively. For small values of these parameters one finds the asymptotic behavior $\phi_{3g}^B(\omega, \omega_g) \propto \omega\omega_g$ [41], showing that the convolution integral in the three-particle term is convergent.

The above expressions show the structure-dependent nature of the QED corrections. The appearance of the two- and three-particle LCDAs highlights the fact that hard-collinear photons are energetic enough to probe the internal structure of the B meson. Various phenomenological models for the LCDAs have been proposed in the literature [38,40,41] and could be used to obtain an estimate of these effects. The terms sensitive to the quark electric charges in (22) are missed in a theory in which the B meson is treated as a pointlike particle. It is evident that the one-loop QCD corrections contain large single and double logarithms, which can be resummed using renormalization-group equations in SCET.

In the absence of QED corrections, we have

$$\sqrt{m_B} f_B^{\text{QCD}} = \left[1 - C_F \frac{\alpha_s(m_b)}{2\pi}\right] F(m_b, m_b)|_{\alpha\to 0} \tag{24}$$

up to power corrections of $\mathcal{O}(1/m_b)$. The parameter f_B^{QCD} can be computed with high precision using lattice QCD [44]. While the QED correction included in the definition of F is expected to be small, being governed by α, its value is sensitive to nonperturbative dynamics and difficult to estimate. Due to the presence of the lightlike Wilson line in (17), it appears challenging to compute F on a Euclidean lattice.

To summarize, we have derived the first QCD + QED factorization formula for a NLP observable using SCET methods. Focusing on the virtual QED corrections to the exclusive $B^- \to \mu^- \bar{\nu}_\mu$ decay amplitude, our main goal was to separate perturbative QED corrections from nonperturbative ones, which are sensitive to hadronic dynamics. This is of great importance for future precision determinations of the CKM matrix element V_{ub}, because the new hadronic parameter $F(\mu, \Lambda)$ and the B-meson LCDAs introduce significant hadronic uncertainties in the analysis of QED corrections. Our derivations have required a careful handling of end-point-divergent convolutions, which we have treated in the RBS scheme [12,14]. While this scheme has previously been applied to other observables, our case is special in that applying refactorization in a nonperturbative context requires a modification of the relevant hadronic matrix elements. This leads to the introduction of the θ-function in (16) and thus to a novel class of soft operators. The result (22) is valid for $\ell = \mu$ only, and its generalization to other lepton flavors will be discussed elsewhere. The approach presented here provides a framework for future studies of structure-dependent QED corrections to other rare exclusive B decays at NLP.

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