Letter

Evidence for web of dualities from monopole operators

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We give evidence for the web of 3d bosonization dualities in conformal field theories (CFTs) by computing monopole operator scaling dimensions in $(2+1)$-dimensional quantum electrodynamics (QED3) with Chern-Simons level k and N complex bosons in a large-N, k expansion. We first consider the $k=0$ case, where we show that scaling dimensions previously computed to subleading order in $1/N$ can be extrapolated to $N=1$ and matched to $O(2)$ Wilson-Fisher CFT scaling dimensions with around 5% error, which is evidence for particle-vortex duality. We then generalize the subleading calculation to large N, k and fixed k/N, extrapolate to $N=k=1$, and consider monopole operators that are conjectured to be dual to nondegenerate scalar operators in a theory of a single Dirac fermion. We find matches typically with 1% error or less, which is strong evidence of this so-called "seed" duality that implies a web of 3d bosonization dualities among CFTs.

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I. INTRODUCTION

IR duality is when two quantum field theories that are completely different at short distances (the UV), nonetheless flow to the same conformal field theory (CFT) at long distances (the IR). While duality is common in two spacetime dimensions, in 3d this phenomenon is much more rare. For many years, the only experimentally relevant example in 3d was particle/vortex duality [1,2], which conjectures that the $O(2)$ Wilson-Fisher fixed point is dual to 3d quantum electrodynamics (QED3) with $N=1$ complex bosonic field and $k=0$ Chern-Simons level, the so-called Abelian Higgs model. Recently, new dualities were conjectured between QED3 with various Chern-Simons levels and matter content [3–5], and these dualities were shown to be part of a so-called web of dualities that generically relates CFTs with fermionic matter to bosonic matter [6,7], and is thus an example of 3d bosonization. This duality web can be derived from a conjectured "seed" duality, which relates QED3 with $N=1$ boson and $k=1$ to the free theory of a single complex two-component fermion. The central idea is that the Chern-Simons term effectively attaches flux (a magnetic instanton or monopole) to the boson, leading to an additional Berry phase and Fermi statistics for the boson-flux composite.

QED3 at finite N and k is strongly coupled in the IR, which makes it hard to verify these dualities. While the conjectured dualities satisfy kinematic consistency checks such as 't Hooft anomalies [7], we would ideally like to check local dynamical observables such as critical exponents (i.e. scaling dimensions). When $k=0$, the theory can be modeled on the lattice [8,9], which found evidence for particle/vortex duality by comparing the lowest scaling dimensions Δ_q^O from $O(2)$ lattice studies [10,11] of operators with charge q under the $U(1)\cong O(2)$ global symmetry, to lattice estimates Δ_q^Q of the dual operator scaling dimensions in QED3 [12]:

$$\Delta_0^O = 1.511, \qquad \Delta_{\frac{1}{2}}^O = .5191, \qquad \Delta_1^O = 1.236,$$
$$\Delta_{\frac{3}{2}}^O = 2.109, \qquad \Delta_0^Q = 1.508, \qquad \Delta_{\frac{1}{2}}^Q = .48,$$
$$\Delta_1^Q = 1.23, \qquad \Delta_{\frac{3}{2}}^Q = 2.15. \tag{1}$$

Lattice methods have more difficulty when $k \neq 0$ due to the sign problem, however, so it has been difficult to numerically verify the seed duality for 3d bosonization. Instead, the duality has been motivated by an uncontrolled flow [13,14] from more well-established supersymmetric dualities [15], as well as an extrapolation to $N_C = 1$ of 3d bosonization for quantum chromodynamics (QCD3) at large colors N_C and k [16], which has been checked at leading order in $1/N_C$ starting with [17–19].

Here we give evidence for both particle/vortex duality, and the seed bosonization duality by computing the scaling dimension of monopole operators in scalar QED3, and matching these to the dimensions of the operators in the dual theories. We will do this by considering QED3 in the limit of large-N scalars, and also large k and fixed $\kappa \equiv k/N$ for the bosonization case, where monopole operator scaling dimensions can be computed in a $1/N$ expansion [20,21]. For $k = 0$, the scaling dimensions have already been computed to subleading order in [22], so we will simply extrapolate these results to $N = 1$ and compare to scaling dimensions of the critical $O(2)$ model as computed from the conformal bootstrap [23,24] and lattice [11]. For nonzero k, we will extend the leading order calculation in [25] to subleading order for general κ, extrapolate to $N = \kappa = 1$, and compare to scaling dimensions of nondegenerate scalar operators in the free fermion theory [26]. In all cases, we find that our perturbative calculation matches the conjectured dualities with a relative error of just a few percent, as shown in Tables I and II.

TABLE I. Scaling dimensions $\Delta_{q,0} = N\Delta_{q,0}^{(0)} + \Delta_{q,0}^{(1)} + O(1/N)$ for charge q scalar monopole operators in QED3 with N scalars and $k = 0$ in a large-N expansion [22,28] extrapolated to $N = 1$, compared to values of the dual operators in the critical $O(2)$ model as computed from the numerical bootstrap ($q \leq 2$) and lattice ($q > 2$), along with the relative errors from the comparison.

q	$\Delta_{q,0}^{(0)}$	$\Delta_{q,0}^{(1)}$	$N = 1$	$O(2)$	Error (%)
1/2	0.12459	0.38147	0.50609	0.519130434	2.5
1	0.31110	0.87452	1.1856	1.23648971	4.1
3/2	0.54407	1.4646	2.0087	2.1086(3)	4.7
2	0.81579	2.1388	2.9546	3.11535(73)	5.2
5/2	1.1214	2.8879	4.0093	4.265(6)	5.8
3	1.4575	3.7053	5.1628	5.509(7)	6.3
7/2	1.8217	4.5857	6.4074	6.841(8)	6.3
4	2.2118	5.5249	7.7367	8.278(9)	6.5
9/2	2.6263	6.5194	9.1458	9.796(9)	6.6
5	3.0638	7.5665	10.630	11.399(10)	6.7

TABLE II. Scaling dimensions $\Delta_{q,1} = N\Delta_{q,1}^{(0)} + \Delta_{q,1}^{(1)} + O(1/N)$ for charge q monopole operators in QED3 with N scalars and $k/N = 1$ in a large-N, k expansion extrapolated to $N = k = 1$, compared to values of the dual operators in the free fermion CFT, along with the relative errors from the comparison. We expect the comparison to be most precise when the operator is a unique scalar $q = 1, 3, 6, \ldots$, as italicized.

q	$\Delta_{q,1}^{(0)}$	$\Delta_{q,1}^{(1)}$	$N = 1$	Fermion	Error (%)
1/2	1	−0.2789	0.7211	1	28
1	*2.5833*	−0.6312	*1.952*	*2*	*2.4*
3/2	4.5873	−1.052	3.535	4	15
2	6.9380	−1.534	5.404	6	9.9
5/2	9.5904	−2.070	7.521	8	6.0
3	*12.514*	−2.655	*9.859*	*10*	*1.4*
6	*34.727*	−7.032	*27.70*	*28*	*1.1*
10	*74.141*	−14.71	*59.43*	*60*	*0.95*
15	*135.67*	−26.64	*109.03*	*110*	*0.88*
21	*224.23*	−43.76	*180.5*	*182*	*0.84*

The rest of this letter is organized as follows. We first introduce monopole operators and discuss our new calculation of their scaling dimension at large N, k. We then review how these operators are expected to map to the dual theories, and compare our new results. We end with a discussion of our results. Technical details are discussed in the Supplemental Material [29].

II. MONOPOLES AT LARGE *N*, *k*

Monopole operators are defined in three-dimensional Abelian gauge theories as local operators that are charged under the topological global symmetry $U(1)_{\mathrm{top}}$ [21,30], whose conserved current and charge are

$$j^{\mu}_{\mathrm{top}} = \frac{1}{8\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho}, \qquad q = \frac{1}{4\pi}\int_{\Sigma} F, \tag{2}$$

where $F_{\nu\rho} \equiv \partial_\nu A_\rho - \partial_\rho A_\nu$ is the gauge field strength with spacetime index $\mu = 1, 2, 3$, Σ is a closed two-dimensional surface, and j^{μ}_{top} is conserved due to the Bianchi identity. In the normalization (2), the charge q is restricted by Dirac quantization to take the values $q \in \mathbb{Z}/2$. As in [21,22,25,31–39], we will compute the scaling dimension of the lowest dimension monopole operators using the state-operator correspondence, which identifies the scaling dimensions of monopole operators of charge q with the energies of states in the Hilbert space on $S^2 \times \mathbb{R}$ with $4\pi q$ magnetic flux through the sphere [21]. The ground state energy on $S^2 \times \mathbb{R}$ can then be computed in the large-N and large-k limit using a saddle point expansion. When $k \neq 0$, the Chern-Simons term induces a gauge charge proportional to q, so that the naive $S^2 \times \mathbb{R}$ vacuum must be dressed by charged-matter modes. Following [25], we can enforce this dressing by computing the small temperature

$T \equiv \beta^{-1}$ limit of the thermal free energy on $S^2 \times S^1_\beta$, where the saddle point value of the holonomy of the gauge field on S^1_β acts like a chemical potential for the matter fields. This dressing will make the monopole transform in a nontrivial representation under the $SU(N)$ flavor symmetry with a nonzero spin for the $SO(3)$ rotation symmetry.

We begin by writing the conformally invariant action of QED3 with N complex scalars ϕ^i on $S^2 \times S^1_\beta$ as [40]

$$\mathcal{S} = \int d^3x \left[\sqrt{g}\left(|(\nabla_\mu - iA_\mu)\phi^i|^2 + \left(\frac{1}{4} + i\lambda\right)|\phi^i|^2\right) - \frac{ik}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right], \quad (3)$$

where g is the determinant of the metric, λ is a Hubbard-Stratonovich field, and $i = 1, \ldots, N$. We are interested in computing the thermal free energy $F_{q,\kappa}$ in the presence of a magnetic flux $\int dA = 4\pi q$ through S^2. We can integrate out the matter fields in the path integral on this background to get

$$e^{-\beta F_{q,\kappa}} = \int DA \exp\left[-N\mathrm{tr}\log\left[\frac{1}{4} + i\lambda - (\nabla_\mu - iA_\mu)^2\right] + iN\int d^3x\left(\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right)\right], \quad (4)$$

where $\kappa = k/N$. We now expand A_μ and λ around a saddle point by taking

$$A_\mu = \mathcal{A}_\mu + a_\mu, \qquad i\lambda = \mu + i\sigma, \quad (5)$$

where a_μ and σ are fluctuations around a background $A_\mu = \mathcal{A}_\mu$ and $i\lambda = \mu$ that satisfy

$$\left.\frac{\delta F_{q,\kappa}[A_\mu, \lambda]}{\delta A_\mu}\right|_{\sigma=a_\mu=0} = \left.\frac{\delta F_{q,\kappa}[A_\mu, \lambda]}{\delta\lambda}\right|_{\sigma=a_\mu=0} = 0. \quad (6)$$

On $S^2 \times S^1_\beta$ with magnetic flux $4\pi q$, the most general such background is μ constant and $\mathcal{A}^q_\mu$ given by

$$\mathcal{A}_\tau = -i\alpha, \qquad \mathcal{F}_{\theta\phi}d\theta \wedge d\phi = q\sin\theta d\theta \wedge d\phi, \quad (7)$$

where $\alpha = i\beta^{-1}\int_{S^1_\beta} A$ is a real constant called the holonomy of the gauge field.

Since the integrand in (4) is proportional to N, the thermal free energy $F_{q,\kappa}$ can then be expanded at large N as

$$F_{q,\kappa} = NF^{(0)}_{q,\kappa} + F^{(1)}_{q,\kappa} + \frac{1}{N}F^{(2)}_{q,\kappa} + \ldots, \quad (8)$$

where $F^{(0)}_{q,\kappa}$ comes from evaluating $F_{q,\kappa}$ at the saddle point and $F^{(1)}_{q,\kappa}$ comes from the functional determinant of the quantum fluctuations around the saddle point. The scaling dimension $\Delta_{q,\kappa}$ is then obtained from the zero temperature limit as

$$\Delta_{q,\kappa} = N\Delta^{(0)}_{q,\kappa} + \Delta^{(1)}_{q,\kappa} + \ldots, \qquad \Delta^{(n)}_{q,\kappa} \equiv \lim_{\beta\to\infty} F^{(n)}_{q,\kappa}. \quad (9)$$

At leading order there are many degenerate monopoles when $k \neq 0$, due to the different ways of dressing the bare monopole, which can be detected from the $O(\beta^{-1})$ terms in $F^{(0)}_{q,\kappa}$. This leads to degeneracy breaking contributions to $\Delta^{(1)}_{q,\kappa}$ that were shown [25] to depend on spin, but whose explicit form was not worked out in general.

The leading-order $F^{(0)}_{q,\kappa}$ was computed in [25] for general k, q by fixing μ and α from the saddle point equations (6), setting A_μ and λ in (4) to their saddle point values (7), and then doing the resulting mode expansion. We review the details of this calculation in the Supplemental Material [29], and give some of the resulting $\Delta^{(0)}_{q,\kappa}$ in Tables I and II. The subleading $F^{(1)}_{q,\kappa}$ comes from expanding (4) to quadratic order in the fluctuations a_μ and σ around the saddle point values to get the Gaussian integral

$$\exp(-\beta F^{(1)}_{q,\kappa}) = \int Da D\sigma \exp\left[-\frac{N}{2}\int d^3x d^3x' \times \sqrt{g}\sqrt{g'}(a_\mu(x)K^{\mu\nu}_q(x,x')a_\nu(x') + \sigma(x)K^{\sigma\sigma}_q(x,x')\sigma(x') + 2\sigma(x)K^{\sigma\nu}_q(x,x')a_\nu(x'))\right], \quad (10)$$

where the kernels are expressed by expectation values of the matter fields for the saddle point values of A_μ and λ. These kernels can be computed in terms of the thermal Green's function $\langle\phi^i(x)\phi^*_j(x')\rangle = \delta^i_j G^q(x,x')$, which was computed for general q, κ in [25]. We give explicit expressions in the Supplemental Material [29], where we explain how to use these kernels to compute the small temperature expansion of $F^{(1)}_{q,\kappa}$, which yields the subleading $\Delta^{(1)}_{q,\kappa}$. For $\kappa = 0$, this calculation was performed in [22,28], and we list some of their results in Table I. For $\kappa \neq 0$, we have additional parity-breaking contributions to the matter kernels, which makes the calculation much more challenging. When $\kappa = 1$ and $q = 1/2$, it was shown in [27] that the Green's function simplifies, so that matter kernels could be computed in closed form and used to compute the scaling dimension. In the Supplemental Material [29], we extend this calculation to general q, κ using an algorithmic approach, which yields the scaling dimensions in Table II.

III. DUALITY COMPARISON

We will now extrapolate the large-N monopole scaling dimensions to $N=1$ and compare to the conjectured dual theories. For $k=0$, we expect the monopole operators of charge q to be dual to the lowest dimension scalar operators of charge q in the critical $O(2)$ Wilson-Fisher CFT, where $U(1)_{\text{top}}$ is identified with the $O(2)$ flavor symmetry. The scaling dimensions of operators with $q=1/2, 1, 3/2, 2$ have been determined using the conformal bootstrap [23,24], while higher values of q were determined with less accuracy using lattice methods [10,11]. We compare these values in Table I [42], and find that the monopole scaling dimensions match their expected duals with just a few percent relative error, which gradually grows with q. It is remarkable that the contribution of the quantum correction $\Delta^{(1)}_{q,0}$ exceeds the leading saddle-point one by a factor of more than 2. This extends the previous lattice evidence for the singlet [8] and $q=1/2, 1, 3/2$ [9] monopole scaling dimensions as reviewed in (1). Monopoles with $q=1/2$ and higher N were also successfully matched to lattice calculations in antiferromagnets with $\mathrm{SU}(N)$ symmetry that can be described by an effective CP^{N-1} gauge theory as in Eq. (3) with $k=0$ [22], so the subleading computation seems accurate for general N. Note that all our large-N estimates are strictly below the estimates from other methods, which is also true for the boson-fermion duality that we now discuss.

We next consider the extrapolation of the large-N, k monopole scaling dimensions to $N=k=1$, where the theory is conjectured to be dual to a single free fermion ψ_α with spinor index $\alpha=1,2$. Monopoles of charge q should be dual to the lowest dimension operator formed by $2q$ fermions, where we identify $U(1)_{\text{top}}$ with the $U(1)$ flavor symmetry of the complex fermion [43]. Due to the antisymmetry of the fermions and the equations of motion, the lowest operator must sometimes include derivatives, and so there will be multiple such operator with different spins for different contractions of the indices. For instance, while the lowest $q=1/2$ operator is the spin half ψ_α, and the lowest $q=1$ is the spin zero $\psi_{[\alpha}\psi_{\beta]}$, already at $q=2$ the lowest dimension operators are $\psi_{\alpha_1}\psi_{\alpha_2}\partial\!\!\!/_{\alpha_3\alpha_4}\psi_{\alpha_5}\partial\!\!\!/_{\alpha_6\alpha_7}\psi_{\alpha_8}$, where the two ways of contracting the indices give spin zero or two. We can count the lowest dimension operators of a given q by expanding the $S^2\times S$ partition function for the free fermion in characters of primary operators following [44], which we do in the Supplemental Material [29]. These operators are unique scalars when [45]

$$q = n(n+1)/2, \quad n = 1, 2, 3, \ldots, \tag{11}$$

in which case the scaling dimension is

$$\Delta^{\text{ferm}}_q = \frac{2}{3} q\sqrt{8q+1}, \tag{12}$$

which corresponds to the energy of n completely filled energy shells on $S^2\times\mathbb{R}$. We compare the monopole scaling dimensions to the dual free fermion operators in Table II. We find a match with relative error of typically 1% or less for values of q in (11) when the operator is a unique scalar. For other values we do not find such a precise match, presumably because there are other contributions to the monopole scaling dimension in this case, such as the spin-dependent degeneracy breaking terms discuss above. As q increases, the match improves for all q, especially for the unique scalar q.

IV. DISCUSSION

In this work we considered the scaling dimensions of monopoles in QED3 with N scalars and Chern-Simons level k as computed to subleading order at large N, k. When $k=0$, this computation was performed in [22], which we extended to the case of general q and $\kappa\equiv k/N$. When $k=0$ and $N=1$ the theory is dual to the $O(2)$ Wilson-Fisher theory, while when $k=N=1$ the theory is dual to a single free fermion. We found evidence of each conjectured duality by extrapolating the large-N, k results, and found matches with just a few percent relative error in each case. For the $k=0$ case, all the monopoles are unique scalars and we found good matches for all q, which extends the evidence of particle/vortex duality beyond the lowest few q considered in previous lattice studies [8,9]. For the $k=1$ case, the monopoles are only unique scalars for certain q, which is where we found matches to good precision. This match is the first quantitative evidence for 3d bosonization, and the duality we consider in fact implies a large web of other dualities as discussed in [6,7].

Looking ahead, we would like to understand better why the large-N calculation is less accurate when the dual operator is degenerate or has nonzero spin. It was observed in [25] that monopoles in scalar QED3 are degenerate at leading large N, which leads to spin-dependent degeneracy breaking contributions at subleading order. Unfortunately, we do not know how to compute this contribution for general q, κ, and even for $q=1/2$ and $\kappa=1$ where this contribution was computed in [27], it did not improve the results. In fact, this contribution was found to be negative in this case, which implies that some temperature dependent terms in the free energy must be complex, which suggests that the saddle we chose was unstable in this case. It is thus possible that a different saddle point might be required when the monopole has spin.

On the other hand, we have observed a curious coincidence that if we take the scaling dimension Δ^{ferm}_q in the free fermion theory for q in (11) when the operator is a unique scalar and analytically continue to all $q\in\mathbb{Z}/2$, then this matches to high precision to all our estimates for the monopole scaling dimension, not just q in (11) as before. For instance, for the lowest few q that are not unique scalars we get the comparison with (12)

$$\Delta_{1/2}^{\text{ferm}} = .7454, \quad \Delta_{3/2}^{\text{ferm}} = 3.606, \quad \Delta_{2}^{\text{ferm}} = 5.498,$$
$$\Delta_{1/2}^{\text{mono}} = .7211, \quad \Delta_{3/2}^{\text{mono}} = 3.535, \quad \Delta_{2}^{\text{mono}} = 5.404, \quad (13)$$

where $q = 1/2$ and $q = 3/2$ in the free fermion picture are unique operators of spin $1/2$ and $3/2$, respectively, while for $q = 2$ there are two degenerate lowest dimension operators of spins zero and 2 as shown in the Supplemental Material [29]. This analytic continuation of q could be explained in the large-q expansion [46,47], where the effective action that describes unique scalar operators must be corrected to describe more general operators [45]. Perhaps a similar correction should be added to the large-N monopole calculation for nonscalar or degenerate monopoles.

We would also like to understand better why the large-N calculation of monopole scaling dimensions works so well when computed to just subleading order [48]. For other operators in scalar QED3 that are constructed from matter fields, such as the lowest dimension singlet computed for $k = 0$ in [50], the large-N expansion seems much less accurate when compared against lattice estimates such as [8]. Perhaps the large-charge expansion [46,47] could also explain this, as this expansion was shown to work well for the scaling dimension of monopole operators [28,39]. For instance, it could be that the first couple of orders at large q only receive contributions from the first couple of orders in $1/N$.

Finally, we would like to find evidence for other 3d dualities using our method. For instance, if we extend the subleading calculation to QED3 with N fermions and nonzero k, then we could check other 3d bosonization conjectures such as the duality between the critical $O(2)$ model and QED3 with 1 fermion and $k = 1/2$ [51]. We can also consider some QED3 dualities with $N > 1$ as discussed in [52,53], or dualities with QCD3 for various gauge groups as in [57]. It would also be nice to verify some of the $\mathcal{N} = 1$ supersymmetric QED3 dualities such as [58].

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