Erratum: Gravitational wave detection with high frequency phonon trapping acoustic cavities [Phys. Rev. D 90, 102005 (2014)]

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We found three errors, which propogate throughout our paper with regards to the sensitivity calculation of a bulk acoustic wave (BAW) resonator to gravitational waves in the frequency range of 1 MHz to 1 GHz. The first one comes from the definition of the coordinate system used to describe the displacement from the Stevens-Tiersten theory, with the spatial amplitude distribution given by Eq. (2) in the paper. The amplitude used in the paper assumes the origin in the center of the crystal. Unfortunately, we assumed that the origin was at the bottom face of the crystal, so Fig. 1(a) in the paper should be replaced with the Fig. 1(a) in this errata. Thus, all integrals undertaken in the paper were between 0 to $2h_0$, when they should have been taken between $-h_0$ to h_0 to be consistent with the displacement used from the Stevens-Tiersten theory.

This error did not effect the calculation of effective mass given in the paper in Eq. (5), but did effect the effective sensitivity coefficient given by Eq. (12), which after taking the correct limits should be replaced with the following equation (n must be odd):

$$\tilde{\xi}_{Xn00} = \frac{\xi_{Xn00}}{h_0} = \frac{16}{n^2\pi^2}\frac{\mathrm{Erf}(\sqrt{n}\eta_x)\mathrm{Erf}(\sqrt{n}\eta_y)}{\mathrm{Erf}(\sqrt{2n}\eta_x)\mathrm{Erf}(\sqrt{2n}\eta_y)}, \quad (1)$$

which is a factor of $\frac{2}{n\pi}$ different than originally determined, this also means Fig. 2 needs to be replaced with the following figure.

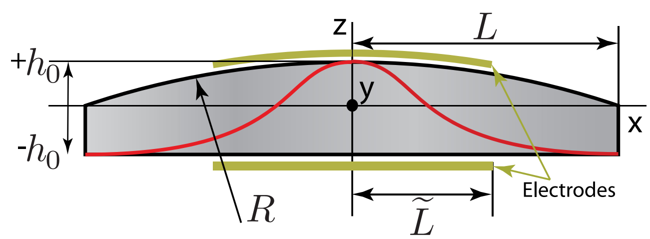

FIG. 1. (a) Side view of a curved BAW plate cavity. Red curve shows typical distribution of mode displacement along the cut in the case $m = 0$ and $p = 0$.

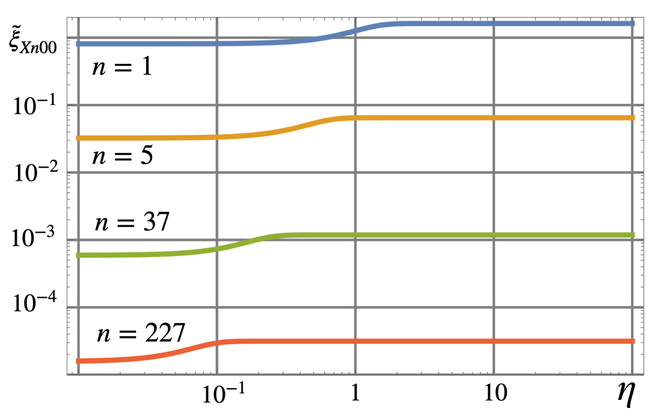

FIG. 2. Corrected gravitational wave sensitivity parameter $\tilde{\xi}_{Xn00}$ as a function of the trapping parameter η.

A second error occurred with Eq. (13), which should be replaced with

$$\sqrt{S_h^+(f)} = \frac{2}{\pi h_0 \tilde{\xi}_\lambda f}\sqrt{\frac{E_\lambda}{m_\lambda Q_\lambda \omega_\lambda}}\,[\text{strain}/\sqrt{Hz}], \tag{2}$$

to be consistent with [1–3]. Note, we include the square-root sign on the left-hand side to emphasize the units are in strain/$\sqrt{Hz}$, which is the usual convention we did not follow in the paper. The error was a factor of 2π, due to the mistake of putting frequency, f in Hz instead of rads/sec. This equation can be shown that in the limit that $\eta \to 0$ (consistent with an infinite radius of curvature and no trapping) $\tilde{\xi}_{Xn00} \to \frac{8}{\pi^2}$ and (2) becomes $\sqrt{S_h^+(f)} = \frac{\pi}{4h_0 f}\sqrt{\frac{kT}{m_\lambda Q_\lambda \omega_\lambda}}$ which is consistent with a resonant bar detector [1,3] as expected.

Now, substituting the values of $\tilde{\xi}_{Xn00}$ in (1) and the value of the effective mass [given by (5) in the paper] into (2) the corrected version of Eqs. (16) and (17) in the paper become

$$\sqrt{S_h^+} = n\sqrt{\frac{kT\chi}{L^2 Q}\sqrt{\frac{\rho}{\hat{c}_z^3}}}\,\frac{\sqrt{\eta_x\eta_y \mathrm{Erf}(\sqrt{2n}\eta_x)\mathrm{Erf}(\sqrt{2n}\eta_y)}}{\mathrm{Erf}(\sqrt{n}\eta_x)\mathrm{Erf}(\sqrt{n}\eta_y)}, \tag{3}$$

$$\begin{aligned}\sqrt{S_h^+} &= n\ 4.3\times 10^{-23}\,\frac{\sqrt{\eta_x\eta_y \mathrm{Erf}(\sqrt{2n}\eta_x)\mathrm{Erf}(\sqrt{2n}\eta_y)}}{\mathrm{Erf}(\sqrt{n}\eta_x)\mathrm{Erf}(\sqrt{n}\eta_y)}\\ &= n\ 4.3\times 10^{-23}\Lambda_{n,0,0}(\eta_x,\eta_y),\ \frac{[\text{strain}]}{\sqrt{\text{Hz}}}.\end{aligned} \tag{4}$$

A third error was a typo; in the proceeding paragraph we give $2h_0 = 5\times 10^{-4}$. This is wrong, we state in the paper these BAWs are about 1 mm thick, in this case $h_0 = 5\times 10^{-4}$.

These errors mean that modes of $n = 1, 3$ are more sensitive than the paper indicates, and modes of $n = 5, 7$ are of similar sensitivity as indicated in the paper. All other higher-order modes are significantly less sensitive than indicated. Thus we have to replace Fig. 5 in the paper with the following figure.

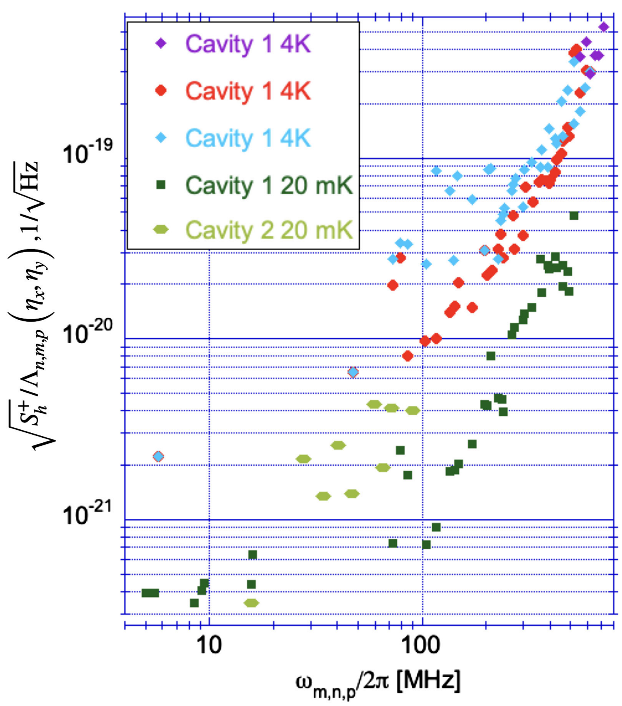

FIG. 5. Corrected normalized single-sided power spectral density of the strain sensitivity for various over tones of the longitudinal mode of two acoustical cavities at 4 K and 20 mK.

Follow up work were we built the device and saw some rare events only used lower order modes [4], so results and conclusions are not effected in any significant way in that paper.

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[1] Michael Edmund Tobar and David Gerald Blair, Sensitivity analysis of a resonant-mass gravitational wave antenna with a parametric transducer, Rev. Sci. Instrum. **66**, 2751 (1995).
[2] Note Eq. (17) in [1] was printed with a typo, a factor of $2\sqrt{2}$ error, if you follow through the calculation it is consistent with (2) and [2].
[3] Michele Maggiore, Resonant-mass detectors, in *Gravitational Waves: Volume 1: Theory and Experiments* (Oxford University Press, New York, 2007), p. 10.
[4] Maxim Goryachev, William M. Campbell, Ik Siong Heng, Serge Galliou, Eugene N. Ivanov, and Michael E. Tobar, Rare events detected with a bulk acoustic wave high frequency gravitational wave antenna, Phys. Rev. Lett. **127**, 071102 (2021).