Non-Markovian speedup dynamics of a photon induced by gravitational redshift

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(Received 10 August 2023; accepted 22 November 2023; published 13 December 2023)

Gravitational redshift can cause the distortion of photon wave packets; thus, the non-Markovian speedup evolution of a photon system must be controlled by considering gravitational redshift. We analyze the non-Markovian speedup dynamics of a photon system in the amplitude-damping channel or pure-dephasing channel under the effect of gravitational redshift, which is modelled as a beam-splitter operation. We show that the gravitational redshift weakens the non-Markovian dynamical behavior in both channels. For the amplitude-damping channel, the gravitational redshift promotes further speedup evolution of a photon system when the initial coherence of the system is nonzero. A stronger gravitational redshift and larger initial coherence make improving the evolution speed of the photon system easier. For the pure-dephasing channel, the non-Markovian dynamical behavior decreases owing to the gravitational redshift; however, the evolution speed of the photon is not influenced by the gravitational redshift.

DOI: 10.1103/PhysRevD.108.126011

I. INTRODUCTION

Recently, many theoretical models and experimental schemes have been proposed to explore the quantum nature of gravity, including the decoherence of massive particles induced by gravity [1–3], testing of the quantum nature of gravity via tabletop experiments [4], the interaction between gravity and quantum coherence in the state of light pluses propagating in curved spacetime [5], and non-Markovian speedup evolution of massive particles mediated by gravitational interaction [6,7]. The study of quantum information under relativistic effects contributes to our understanding of some key questions in quantum mechanics [8]. Extensive research has explored the influence of gravity on quantum states, particularly in quantum communications and potential applications. Some examples include photons in particular frequency bands traversing interstellar distances independent of initial-quantum-state decoherence [9,10], gravitational distortion of quantum communication [11], geometric phase acquired as a wave packet propagates along a null geodesic [12], and transformations on the photon states induced by gravitational redshift [13]. Gravitational redshift is one of the main predictions of general relativity; it has attracted research interest since its proposal [14–16], and its existence has been experimentally confirmed [17–23]. Different studies have proposed the application of gravitational redshift to astrophysics [24] and quantum information-related tasks [8,25,26].

When a photon initially prepared by the sender at a given frequency is transmitted through curved spacetime, the receiver detects it at a different frequency owing to its exposure to different local gravitational potentials. Thus, gravitational redshift affects the realistic photon with a finite bandwidth and extension. Recently, a new approach was developed to investigate the impact of gravitational redshift on the quantum states of photons. Photons were modeled as wave packets of a quantum field propagating in curved spacetime, and the relationship between the wave packets generated by sender Alice and those detected by receiver Bob was established [27–29]. The transformation of the wave packet can be interpreted as a transformation in the structure of the field mode. Moreover, the transformation of the field mode structure can be considered as a change in basis within the Hilbert space of the photon [30–33], which can be achieved through a unitary rotation in the Hilbert space or multimode mixing operation on the field operators [13,34].

In general, when a quantum system, such as a photon, travels through curved spacetime, the effect of gravitational redshift on the system should be considered. In practical research, a quantum system should be treated as an open quantum system that will inevitably be affected by the environment [35]. If the Markovian approximation is used to study the dynamics of an open quantum system, the

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information will flow continuously from the system to the environment. However, a large separation between the system and environment timescales can no longer be assumed. Information may flow back from the environment, and non-Markovian behavior may occur [36–39]. This behavior has been investigated experimentally [40–42]. Various methods have been developed to measure this backflow of information, including trace distance [43], fidelity [44], the semigroup property [45], quantum Fisher information (QFI) [46–50], entanglement [51], and quantum mutual information [52].

According to [53], non-Markovian dynamics play a significant role in accelerating the evolution of an open quantum system. The maximal speed of the quantum-state evolution can be defined by the quantum speed-limit time (QSLT), which quantifies the minimal time required for a quantum state to evolve from its initial state to its target state [54–57]. For a closed system, the unified lower bound of the QSLT is obtained through the Mandelstamm-Tamm type and Margolus-Levitin type bounds [58–60]. However, for an open system with nonunitary evolution, two quantum speed limits have been derived based on generalized Bloch vectors, which are applicable for almost all quantum states [61,62]. Compared with the other methods [63–65], the bounds obtained using this method are simpler to compute and can be experimentally obtained. Gravitational redshift plays a crucial role in the gravitational effect. However, research on the dynamics of a quantum system under the influence of gravitational redshift is still lacking. The non-Markovian speedup evolution of open quantum systems can better maintain the robustness of quantum simulators and computers against decoherence [66]. Gravitational redshift can cause the distortion of photon wavepackets and reduce fidelity [5,27–29]. Thus, the non-Markovian speedup evolution of an open quantum system must be controlled by considering gravitational redshift.

In this study, we primarily investigate the relationship between the non-Markovian speedup dynamical behavior of a photon system and gravitational redshift in the amplitude-damping channel [34–39,57] and pure-dephasing channel [54,67,68]. The gravitational redshift is introduced by a beam-splitter operation for a photon with the same optical mode at two different locations before and after propagation [27–33]. To quantify the non-Markovian speedup dynamics of the system, we employ the QSLT to quantify the quantum-evolution maximal speed of the system and define the boundary between no-speedup and speedup quantum evolution. Additionally, a method based on QFI [46,47] is used to quantify the non-Markovianity of the open photon system. In Sec. II we introduce a canonical transformation to represent the gravitational redshift. In Sec. III the definitions of the non-Markovianity and QSLT of an open system are provided, and the non-Markovian speedup evolution of a photon system under two different quantum noisy channels is investigated. The discussion and conclusions are presented in Sec. IV. Throughout the entire text, the metric signature $(-,+,+,+)$ and natural units $(\hbar = c = G = 1)$ are employed.

II. GRAVITATIONAL REDSHIFT AS A CANONICAL TRANSFORMATION

In this section, we first introduce the spacetime background. For convenience, we consider a nonrotating planet and model the spacetime background outside it using $(3+1)$-dimensional Schwarzschild spacetime [15], which is both spherically symmetric and static. The Schwarzschild vacuum metric $g_{\mu\nu}$ is

$$g_{\mu\nu} = \mathrm{diag}\left(-f(r), \frac{1}{f(r)}, r^2, r^2 \sin\theta\right), \tag{1}$$

where $f(r) = 1 - r_s/r$, $r_s = 2M$ is the Schwarzschild radius of the planet. We primarily consider the transmission of the wavepacket from the Earth to a receiver at a particular distance, and the main effect of gravity depends on the Schwarzschild radius r_s. Generally, a photon can be modeled as a wavepacket with a frequency distribution denoted as $F^{(K)}_{\Omega_{K,0}}$ [27,28]. The annihilation operator of the photon described by the different locations of observers is as follows:

$$\hat{a}_{\Omega_{K,0}}(\tau_K) = \int_0^{+\infty} d\Omega_K\, e^{-i\Omega_K\tau_K} F^{(K)}_{\Omega_{K,0}}(\Omega_K)\hat{a}_{\Omega_K}, \tag{2}$$

where the subscripts $K = A$ and B refer to the observers Alice and Bob, respectively. Ω_K is the frequency of the photon measured locally by the observer K at proper time τ_K. We also introduce the peak frequency $\Omega_{K,0}$ of the frequency distribution $F^{(K)}_{\Omega_{K,0}}$. The operator $\hat{a}_{\Omega_K}$ satisfies the canonical commutation relation

$$[\hat{a}_{\Omega_K}, \hat{a}^\dagger_{\Omega'_K}] = \delta(\Omega_K - \Omega'_K). \tag{3}$$

A wave packet $F^{(A)}_{\Omega_{A,0}}$ generated by the sender Alice at time τ_A in location r_A travels through curved spacetime, and it is detected as $F^{(B)}_{\Omega_{B,0}}$ by the receiver Bob who is at the location $r = r_B > r_A$ at time τ_B (here $\tau_B = \Delta\tau + \sqrt{f(r_B)/f(r_A)}\tau_A$). Using the relation between the annihilation operators $\hat{a}_{\Omega_A}$ and $\hat{a}_{\Omega_B}$ [27], we can obtain the relation between frequency distributions $F^{(K)}_{\Omega_{K,0}}$ in different reference frames,

$$F^{(B)}_{\Omega_{B,0}}(\Omega_B) = \sqrt[4]{\frac{f(r_B)}{f(r_A)}} F^{(A)}_{\Omega_{A,0}}\left(\sqrt{\frac{f(r_B)}{f(r_A)}}\Omega_B\right). \tag{4}$$

Owing to the curvature of spacetime and the influence of gravitational potential, the wave packet changes during its propagation. In the cases we consider, $r_B > r_A$, the wave packet frequency Ω_B measured by Bob is redshifted relative to the wave packet frequency Ω_A generated by Alice. If Alice prepares a sharp frequency mode Ω_A, Bob detects it with frequency Ω_B. The relation between Ω_B and Ω_A can be obtained by solving the eigenvalue equation for the modes considered [27,28], as follows:

$$\chi^2 = \frac{\Omega_B}{\Omega_A} = \sqrt{\frac{f(r_A)}{f(r_B)}}, \tag{5}$$

which is the well-known formula of gravitational redshift [16], and satisfies $\chi > 1$. As discussed in [27–33], photon propagation between two different locations in curved spacetime is similar to a beam-splitter operation performed on the propagating photon. Notably, the operator in Eq. (2) can be used to describe the same optical mode at two different locations before and after the propagation. Thus, the mode $\hat{a}'_{\omega_0}$ may be decomposed into the mode $\hat{a}_{\omega_0}$ and orthogonal mode $\hat{a}_\perp$:

$$\begin{pmatrix} \hat{a}'_{\omega_0} \\ \hat{a}'_\perp \end{pmatrix} = \begin{pmatrix} \cos\theta & e^{i\varphi}\sin\theta \\ -e^{-i\varphi}\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{a}_{\omega_0} \\ \hat{a}_\perp \end{pmatrix}, \tag{6}$$

where the angle θ and phase φ are obtained from the overlap of two modes. The specific forms of overlap are $\cos\theta(\chi) = |\langle 1'_{\omega_0}|1_{\omega_0}\rangle|$ and $\varphi(\chi) = \arg(\langle 1'_{\omega_0}|1_{\omega_0}\rangle)$, where $|1_{\omega_0}\rangle = \hat{a}^\dagger_{\omega_0}|0\rangle$ and $|1'_{\omega_0}\rangle = \hat{a}'^\dagger_{\omega_0}|0\rangle$. Note that the phase $\varphi(\chi)$ is set to zero for convenience and without loss of generality [33]. The quality of the channel is quantified by the fidelity $\mathcal{F} = |\Theta|^2$, with

$$\Theta = \int_0^{+\infty} d\Omega_B\, F^{(B)*}_{\Omega_{B,0}}(\Omega_B) F^{(A)}_{\Omega_{A,0}}(\Omega_B), \tag{7}$$

where Θ is the wave packet overlap between the distributions $F^{(B)}_{\Omega_{B,0}}(\Omega_B)$ and $F^{(A)}_{\Omega_{A,0}}(\Omega_B)$. $1 - |\Theta|^2$ indicates the probability of photon loss. If the channel is perfect, $|\Theta| = 1$. This is equivalent to the following relation: $\cos\theta = \Theta$ and $\sin^2\theta = 1 - \Theta^2$, with $\theta \in [0, \pi/2)$. Thus, we can obtain the following: an increase in θ implies an enhancement of the gravitational redshift χ, which leads to a decrease in channel quality. As discussed in [27,28], we consider the Gaussian distribution $F_{\Omega_0}(\Omega) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} e^{-\frac{(\Omega-\Omega_0)^2}{4\sigma^2}}$ (σ is the wave packet width) of the wave packet $F_{\Omega_0}(\Omega)$. From this calculation, we obtain the following:

$$\cos\theta = \sqrt{\frac{2}{\frac{1}{\chi^2} + \chi^2}} e^{-\frac{(\chi-1)^2\Omega_{B,0}^2}{4(1+\chi^4)\sigma^2}}. \tag{8}$$

For typical communication with $\Omega_{B,0} = 700$ THz and $\sigma = 1$ MHz [27,28], if Bob is far from the Earth ($\chi - 1 = 3.5 \times 10^{-10}$), the effect of gravity should be considered in the photon propagation ($\frac{(\chi-1)^2\Omega_{B,0}^2}{4(1+\chi^4)\sigma^2} \sim 7.5 \times 10^{-3}$). When Alice and Bob are in flat spacetime [$f(r_A) = f(r_B) = 1$] or they are at the same height, the channel between Alice and Bob is perfect and it will not be affected by gravitational redshift ($\chi = 1$); thus, the fidelity $\cos^2\theta = 1$. When the effect of gravitational redshift increases ($r_B > r_A$), the fidelity of the channel decreases ($\cos\theta < 1$). Thus, when $\theta = 0$, no gravitational redshift occurs (perfect overlap), and $\theta = \pi/2$ indicates the strongest gravitational-redshift effect (complete mismatch). Therefore, in the following, we use θ to quantify the strength of the gravitational redshift, and a larger θ indicates a stronger gravitational redshift.

III. NON-MARKOVIANITY AND QSLT OF A QUANTUM SYSTEM

In this section, we will analyze the effect of gravitational redshift on the non-Markovian speedup dynamics of a photon system. The gravitational redshift is introduced by Eq. (6). We first introduce the definitions of non-Markovianity and QSLT.

The non-Markovianity quantifies the backflow of information from the environment to the system. In this study, we consider a physically intuitive characterization of non-Markovianity by utilizing the average QFI flow [46,47]. Using QFI to locally characterize the non-Markovianity in different channels is effective and can be realized via quantum simulation [48]. The main objective is to quantify the non-Markovianity by measuring the non-monotonic change in QFI of the parameters encoded in quantum states during their evolution. To quantify the information content of all parameters α and ϕ encoded in the single-qubit mixed state ρ, we use the QFI matrix,

$$F(t) = \begin{pmatrix} F_\alpha(t) & F_{\alpha\phi}(t) \\ F_{\alpha\phi}(t) & F_\phi(t) \end{pmatrix}, \tag{9}$$

with $F_\alpha(t) = \mathrm{Tr}[\rho(t)L_\alpha^2]$, $F_\phi(t) = \mathrm{Tr}[\rho(t)L_\phi^2]$, and $F_{\alpha\phi}(t) = \frac{1}{2}\mathrm{Tr}[\rho(t)(L_\alpha L_\phi + L_\phi L_\alpha)]$, where L_α and L_ϕ are the symmetric logarithmic derivatives (SLD) for parameters α and ϕ, respectively. The parameters α and ϕ may be regarded as the amplitude and phase information in the qubit states, respectively. The traditional method to calculate the SLD operator is to expand it in the eigenspace of density matrix ρ [69]. The eigenvalues p_i and eigenstates $|\psi_i\rangle$ are then obtained by the spectral decomposition of the density matrix $\rho = \sum_{i=1}^N p_i|\psi_i\rangle\langle\psi_i|$, where N is the dimension of density matrix ρ. Using the spectral-decomposition form, the element of the SLD operator can be expressed by $L_{ij} = \frac{\partial_\alpha p_i}{p_i}\delta_{ij} + \frac{2(p_i - p_j)}{p_i + p_j}\langle\partial_\alpha\psi_i|\psi_j\rangle$ for $i, j \in [1, N]$, where $\langle\partial_\alpha\psi_i| = \partial_\alpha(\langle\psi_i|)$. Because the parameters are encoded in the quantum states of the system, estimating the encoded parameters in the quantum states necessitates obtaining the evolutional density matrix of the system by solving the master equation. For a single-qubit mixed state, the QFI matrix can be expressed as $F_{\alpha\phi} = \mathrm{Tr}[(\partial_\alpha\rho)(\partial_\phi\rho)] + \frac{1}{\mathrm{Det}[\rho]}\mathrm{Tr}[\rho(\partial_\alpha\rho)\rho(\partial_\phi\rho)]$, where $\mathrm{Det}[\rho]$ is the determinant of ρ [49,50]. Because $L_\alpha^\dagger = L_\alpha$ and $L_\phi^\dagger = L_\phi$, the QFI matrix $F(t)$ is Hermitian. We focus on the dynamic process with time evolution t and eliminate dependence on the parameters α and ϕ by integrating the parameters $\alpha \in [0, \pi)$ and $\phi \in [0, 2\pi)$ over a unit Bloch sphere with a uniform distribution $d\Omega = \frac{1}{4\pi}\sin\alpha d\alpha d\phi$ to obtain the averaged QFI matrix,

$$\bar{F}(t) = \begin{pmatrix} \bar{F}_\alpha(t) & \bar{F}_{\alpha\phi}(t) \\ \bar{F}_{\alpha\phi}(t) & \bar{F}_\phi(t) \end{pmatrix}, \tag{10}$$

with $\bar{F}_\alpha(t) = \int F_\alpha(t)d\Omega$, $\bar{F}_\phi(t) = \int F_\phi(t)d\Omega$, and $\bar{F}_{\alpha\phi}(t) = \int F_{\alpha\phi}(t)d\Omega$. Moreover, the averaged QFI matrix $\bar{F}(t)$ now depends only on time t and quantum dynamics Λ_t. In Markovian dynamics, information is lost to the environment in one direction; thus, the information content of the encoded parameters in the qubit state should not increase. Therefore, the averaged QFI matrix $\bar{F}(t)$ monotonically decreases with time $t \geq 0$. In others words, the derivative matrix $\frac{d}{dt}\bar{F}(t)$ is always nonpositive definite, and this can be determined by calculating the eigenvalues of the Hermitian matrix $\frac{d}{dt}\bar{F}(t)$; the eigenvalues are nonpositive, denoted as $\lambda_1(t)$ and $\lambda_2(t)$. Thus, the violation of this monotonicity is an indication of non-Markovianity [47]. A quantitative measure of non-Markovianity may be defined as follows:

$$N_{\mathrm{QFI}} = \int_{\lambda(t)>0} \lambda(t)dt, \tag{11}$$

where $\lambda(t) = \max\{\lambda_1(t), \lambda_2(t)\}$. Compared with the other methods, this method is advantageous because it does not require the selection of the initial optimal state pair. Instead, we must eliminate the dependence of the QFI matrix on parameters via averaging. Furthermore, the QFI effectively defines the accuracy of the parameter estimation using the well-known Cramér-Rao inequality [49].

QSLT is defined to quantify the bound of the minimal evolution time for an actual dynamical process from an initial state $\rho(0)$ to a target state $\rho(\tau)$; τ is set as the actual evolution time of the dynamical process. This facilitates the analysis of the maximum speed at which the dynamical process evolves [53–57]. By setting a bound on the minimum evolution time from any initial state $\rho(0)$ to a target state $\rho(\tau)$, a suitable QSLT can be effectively defined to describe the speedup evolution of the system dynamics under gravitational redshift. This helps in analyzing the maximum evolution speed of a quantum system. Campaioli *et al.* [62] reproposed a new bound from a geometric perspective using the method of states of geometric distance in a generalized Bloch sphere. The QSLT is given as follows:

$$\tau_{\mathrm{QSL}} = \frac{\|\rho(0) - \rho(\tau)\|_{\mathrm{hs}}}{\overline{\|\dot{\rho(t)}\|}}, \tag{12}$$

where $\overline{\|\dot{\rho(t)}\|} = \frac{1}{\tau}\int_0^\tau dt\|\dot{\rho(t)}\|$ and $\|X\|_{\mathrm{hs}} = \sqrt{\sum_i M_i^2}$. Here, M_i are the singular values of X. The advantage of this bound is tighter and easier to calculate for almost all quantum-evolution processes. The physical interpretation of τ_{QSL} is given as $\tau_{\mathrm{QSL}}/\tau = 1$. The evolution speed of the quantum state reaches its highest value and does not increase further. However, for $\tau_{\mathrm{QSL}}/\tau < 1$, the dynamics evolution of the quantum state may further speed up. Moreover, the smaller the τ_{QSL}/τ value, the greater the potential for the quantum speedup of system dynamics. Next, we investigate the non-Markovian speedup evolution of a photon system in two different channels under the effect of gravitational redshift.

A. Amplitude-damping channel

First, we consider a photon system (with transition frequency ω_0) interacting with a structured reservoir at zero temperature. The initial state of the reservoir is assumed to be in a vacuum state. The dynamics of the photon system can be solved exactly [34–39,57]. The Hamiltonian of the total system is $H = \omega_0\hat{a}^\dagger\hat{a} + \sum_k \omega_k\hat{b}_k^\dagger\hat{b}_k + \sum_k g_k(\hat{b}_k\hat{a}^\dagger + \hat{b}_k^\dagger\hat{a})$, where $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) operator for the photon mode, $\hat{b}_k$ ($\hat{b}_k^\dagger$) is the annihilation (creation) operator for the field mode k with frequency ω_k, and g_k is the coupling constant between the photon and reservoir with mode k. The nonunitary generator of the reduced dynamics of the system can be described as follows:

$$\mathcal{L}_t^{\mathcal{L}}(\rho_t) = \frac{\gamma_t}{2}(2\hat{a}\rho_t\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\rho_t - \rho_t\hat{a}^\dagger\hat{a}), \tag{13}$$

where γ_t is the time-dependent decay rate. With only one excitation in the entire system, the structure of the reservoir can be described by an effective Lorentzian spectral density $J(\omega) = \frac{1}{2\pi}\frac{\gamma_0\lambda}{(\omega_0-\omega)^2+\lambda^2}$, where λ is the spectral width of the reservoir, and γ_0 is the coupling strength. The initial state of the photon system is set as follows: $\cos(\alpha/2)|0\rangle + e^{i\phi}\sin(\alpha/2)|1\rangle$ with $\alpha \in [0, \pi)$ and $\phi \in [0, 2\pi)$. By considering the gravitational redshift introduced by Eq. (6) and tracing out the orthogonal mode, the state of the photon system is reduced to

$$\rho_{\mathrm{red}}^{\mathcal{L}}(0) = \begin{pmatrix} \sin^2(\frac{\alpha}{2})\cos^2\theta & \frac{1}{2}e^{i\phi}\sin\alpha\cos\theta \\ \frac{1}{2}e^{-i\phi}\sin\alpha\cos\theta & \cos^2(\frac{\alpha}{2}) + \sin^2(\frac{\alpha}{2})\sin^2\theta \end{pmatrix}. \tag{14}$$

The reduced density of the photon system at time t is given as follows:

$$\rho^{\mathcal{L}}_{\rm red}(t)=\begin{pmatrix}\sin^2(\frac{\alpha}{2})\cos^2\theta P_t & \frac{1}{2}\sqrt{P_t}e^{i\phi}\sin\alpha\cos\theta\\ \frac{1}{2}\sqrt{P_t}e^{-i\phi}\sin\alpha\cos\theta & 1-\sin^2(\frac{\alpha}{2})\cos^2\theta P_t\end{pmatrix}, \quad (15)$$

where $P_t = e^{-\int_0^t dt'\gamma_{t'}}$. Because the reservoir is described by an effective Lorentzian spectral $J(\omega)$, the specific form of the time-dependent decay rate is given by $\gamma_t = \frac{2\lambda\gamma_0\sinh(dt/2)}{d\cosh(dt/2)+\lambda\sinh(dt/2)}$, with $d=\sqrt{\lambda^2-2\gamma_0\lambda}$. Furthermore, the parameter P_t can be analytically obtained as $P_t = e^{-\lambda t}[\cos(dt/2)+\lambda/d\sin(dt/2)]^2$. Typically, in the weak-coupling regime ($\lambda > 2\gamma_0$), the dynamics of the photon system is Markovian and undergoes irreversible decay. In the strong-coupling regime ($\lambda < 2\gamma_0$), the behavior of the photon system can be described by non-Markovian dynamics, which involves information backflow from the environment.

Referring to the definition of non-Markovianity based on the QFI mentioned above, we first calculate the QFI of the mixed state in Eq. (9) for parameters α and ϕ, as follows: $F^{\mathcal{L}}_\alpha = P_t\cos^2\theta$, $F^{\mathcal{L}}_\phi = P_t\cos^2\theta\sin^2\alpha$, and the other values are zero. By averaging the specific values of the initial state parameters α and ϕ, we can eliminate the dependence of the QFI matrix on these parameters. Therefore, the non-Markovianity is independent of α and ϕ. The corresponding averaged QFI matrix can be calculated as follows:

$$F^{\mathcal{L}}_M=\begin{pmatrix}\bar{F}^{\mathcal{L}}_\alpha & 0\\ 0 & \bar{F}^{\mathcal{L}}_\phi\end{pmatrix}, \quad (16)$$

and we calculated the eigenvalues of the matrix $\frac{d}{dt}F^{\mathcal{L}}_M$, denoted as $\lambda_1(t)$ and $\lambda_2(t)$. Furthermore, $\lambda_t = \max\{\lambda_1(t),\lambda_2(t)\}$ was obtained as $\lambda_t = \frac{d}{dt}(P_t\cos^2\theta)$. Thus, the non-Markovianity based on the QFI can be interpreted as follows:

$$N^{\mathcal{L}}_{\rm QFI}=\int_{\dot{P}_t>0}\dot{P}_t\cos^2\theta dt. \quad (17)$$

We can clearly observe that the non-Markovianity decreases as the gravitational redshift increases. When $\alpha=\pi$, we have $\|\rho(\tau)-\rho(0)\|=\sqrt{2}\sqrt{(1-P_t)^2\cos^4\theta}$, $\|\dot{\rho}(t)\|=\sqrt{2}\sqrt{\cos^4\theta\dot{P}(t)^4}$. Thus, the ratio of the QSLT to the actual evolution time τ is as follows:

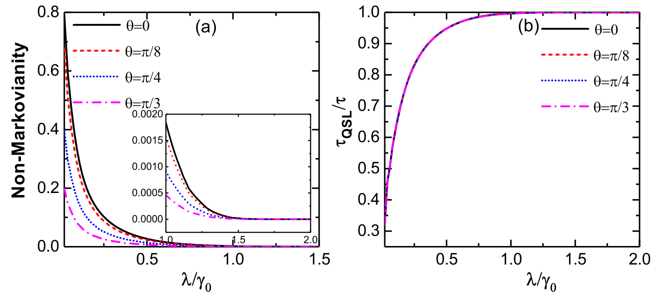

FIG. 1. By considering different θ values, (a) and (b) are the non-Markovianity and QSLT as a function of parameter λ/γ_0 in the amplitude-damping channel, respectively, where $\tau = 60$.

$$\frac{\tau_{\rm QSL}}{\tau}=\frac{\|\rho(\tau)-\rho(0)\|}{\int_0^\tau\|\dot{\rho}(t)\|dt}=\frac{1}{\frac{2N^{\mathcal{L}}_{\rm QFI}}{\cos^2\theta(1-P(\tau))}+1}. \quad (18)$$

When the initial state of the system is an excited state $|1\rangle\langle 1|$ (that is, $\alpha=\pi$), we can obtain a concrete form of $\tau_{\rm QSL}/\tau$ in Eq. (18). According to [53], Deffner and Lutz concluded that the information backflow from the environment in the dynamical process [from $\rho(0)$ to $\rho(\tau)$] can lead to faster quantum evolution, and hence, to a shorter QSLT. Therefore, non-Markovian dynamics can improve the evolution speed of quantum systems. By analyzing Eq. (18) in our model, we observe that the QSLT is related to non-Markovianity, the population of the initial excited state, and gravitational redshift. Interestingly, with an increase in θ, the non-Markovianity ($N^{\mathcal{L}}_{\rm QFI}$) gradually decreases, as shown in Fig. 1(a), whereas the ratio $\frac{2N^{\mathcal{L}}_{\rm QFI}}{\cos^2\theta(1-P(\tau))}$ does not change. Thus, $\tau_{\rm QSL}/\tau$ remains unchanged. In this study, we use the parameter θ to quantify the effect of gravitational redshift. A larger θ value indicates a stronger gravitational-redshift effect. Therefore, we observe that the enhancement of gravitational redshift weakens the non-Markovian dynamics, but it does not affect the QSLT [see Fig. 1(b)]. If the system dynamic is Markovian ($N^{\mathcal{L}}_{\rm QFI}=0$), $\tau_{\rm QSL}/\tau=1$, the evolution of the quantum state of the photon system cannot be accelerated.

For $\alpha=\pi$, the coherence, as measured by l_1-norm [70,71], of the initial state of the photon system is zero, whereas for $\alpha<\pi$, the initial coherence of the photon system is nonzero. Next, we explore the effect of coherence and gravitational redshift on the speedup evolution of the system dynamics. A concrete form of $\tau_{\rm QSL}/\tau$ is given as follows:

$$\frac{\tau_{\rm QSL}}{\tau}=\frac{2\left|(1-\sqrt{P_t})\sqrt{C(\rho_0)^2+(1+\sqrt{1-C(\rho_0)^2})^2(1+\sqrt{P_t})^2\cos^2\theta}\right|}{\int_0^\tau\left|\dot{P}_t\sqrt{\frac{C(\rho_0)^2+(1+\sqrt{1-C(\rho_0)^2})^24P_t\cos^2\theta}{P_t}}\right|dt}, \quad (19)$$

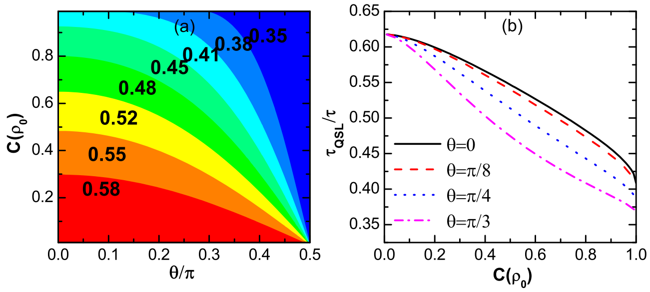

FIG. 2. The QSLT of the dynamics of the photon system in the amplitude-damping channel as a function of parameter θ and the initial coherence $C(\rho_0)$. This represents the non-Markovian-dynamics regime $\lambda = 0.1\gamma_0$ and $\tau = 60$. (a) The phase diagram of τ_{QSL}/τ as a function of θ and $C(\rho_0)$; (b) τ_{QSL}/τ as a function of $C(\rho_0)$ with different θ values.

where $C(\rho_0) = \sin\alpha$ is the initial coherence of the photon system. In Fig. 2, we present the QSLT as a function of the gravitational redshift and initial coherence in non-Markovian dynamics. When no gravitational-redshift effect occurs ($\theta = 0$), the QSLT decreases monotonically with an increase in the initial coherence. In the case of an existing gravitational-redshift effect (such as $\theta = \pi/8$), as shown in Fig. 2(b), the QSLT also decreases monotonically with an increase in the initial coherence. The value of the QSLT in the case $\theta = \pi/8$ is always smaller than that in the case $\theta = 0$. By further increasing the value of θ, the QSLT further decreases. More interestingly, the larger the value of θ, the larger the value of $C(\rho_0)$, and the smaller the value of τ_{QSL}/τ, as shown in the blue region in Fig. 2(a). Thus, we can conclude that the enhancement of the gravitational redshift and initial coherence play a positive role in improving the speedup evolution of the system in non-Markovian dynamics. Note that the gravitational effect is strongest when $\theta = \pi/2$; the QSLT reaches a minimum at this point, and the initial coherence has no effect on the QSLT at this point.

In the Markovian-dynamics regime ($\lambda = 10\gamma_0$), as shown in Fig. 3, if $\theta = 0$ and we do not consider the influence of the gravitational effect, the QSLT will decrease monotonically as the initial coherence increases. Furthermore, the gravitational redshift is weak (such as $\theta = \pi/8$); therefore, the QSLT is less than that in the case of $\theta = 0$, and it decreases with an increase in the gravitational redshift. This result is similar to that in the non-Markovian-dynamics regime. However, when the gravitational redshift increases to $\theta = \pi/4$, the QSLT first decreases monotonically to a minimum at $C(\rho_0) = 0.9$ and then increases. In the case of $\theta = \pi/3$, the QSLT reaches the same minimum at $C(\rho_0) = 0.7$ and then increases, as shown in Fig. 3(b). Figure 3(a) shows that an increase in the initial coherence increases the evolution speed of the system dynamics as $\theta < \pi/4$. However, the gravitational redshift continues to increase until $\theta > \pi/4$; thus, the stronger the initial coherence, the more difficult it becomes to improve the evolution speed of the system dynamics. Such an improvement would require an appropriate decrease in the initial coherence. This is considerably different from the non-Markovian-dynamics regime.

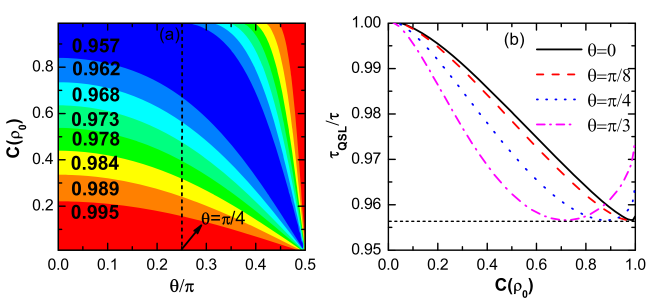

FIG. 3. The QSLT of the dynamics of the photon system in the amplitude-damping channel as a function of parameter θ and initial coherence $C(\rho_0)$. This represents the Markovian-dynamics regime $\lambda = 10\gamma_0$ and $\tau = 10$. (a) The phase diagram of τ_{QSL}/τ as a function of θ and $C(\rho_0)$; (b) τ_{QSL}/τ as a function of $C(\rho_0)$ with different θ values.

B. Ohmic-like dephasing channel

In the following, we consider an interaction between a photon system (with transition frequency ω_0) and bosonic environment with the Hamiltonian $H = \omega_0 \hat{a}^\dagger \hat{a} + \sum_k [\omega_k \hat{b}_k^\dagger \hat{b}_k + \hat{a}^\dagger \hat{a}(g_k \hat{b}_k + g_k^* \hat{b}_k^\dagger)]$, where $\hat{a}$ ($\hat{a}^\dagger$) is the annihilation (creation) operator for the photon mode, ω_k is the kth field frequency of the environmental mode, and g_k is the coupling constant between the photon and reservoir mode k. We assume that no correlation exists between the system and environment at the beginning, and the environment is initially in a zero-temperature vacuum state. The dynamics of the system can be modeled as an exactly solvable Ohmic-like dephasing model. The nonunitary generator of the reduced dynamics of the system can be described as follows [72]:

$$\mathcal{L}_t^{\mathcal{O}}(\rho_t) = \frac{\gamma_t}{2}[2\hat{a}^\dagger \hat{a} \rho_t \hat{a}^\dagger \hat{a} - (\hat{a}^\dagger \hat{a})^2 \rho_t - \rho_t (\hat{a}^\dagger \hat{a})^2]. \quad (20)$$

The bosonic environment operator is a linearly coupled sum of the coordinates of the harmonic-oscillator continuum described by the spectral function $J(\omega)$. The time-dependent decay rate is written as $\gamma_t = \int_0^\infty J(\omega) \coth(\hbar\omega/2K_B T) \frac{1-\cos(\omega t)}{\omega^2} \mathrm{d}\omega$, where T is the temperature and K_B is the Boltzmann constant. Here, we assume that the spectral density of the environment modes is described as Ohmic-like $J(\omega) = \eta \frac{\omega^s}{\omega_c^{s-1}} e^{(-\omega/\omega_c)}$, where ω_c is the cutoff frequency and η is a dimensionless coupling constant. By changing the values of the s-parameter, the types of environments are divided into sub-Ohmic ($0 < s < 1$), Ohmic ($s = 1$), and super-Ohmic ($s > 1$). Typically, when the reservoir spectrum is super-Ohmic, the memory effects that lead to information backflow and recoherence occur [67,68]. When the temperature T of the environment is zero, for $t > 0$ and $s > 0$, the dephasing rate can be written as $\gamma_t = \eta[1 - \frac{\cos[(s-1)\arctan(\omega_c t)]}{(1+\omega_c^2 t^2)^{\frac{s-1}{2}}}]\Gamma(s-1)$, where $\Gamma(s-1)$ is the Euler gamma function. As parameter s tends to 1, the dephasing rate can be written as $\gamma_t(s=1) = \eta\ln(1+\omega_c^2 t^2)$. Similarly, the initial state of the quantum system is set to $\cos(\alpha/2)|0\rangle + e^{i\phi}\sin(\alpha/2)|1\rangle$ with $\alpha \in [0,\pi)$ and $\phi \in [0, 2\pi)$. The gravitational redshift is introduced by the operation in Eq. (6), and the initial state of the system is reduced as follows:

$$\rho_{\mathrm{red}}^{\mathcal{O}}(0) = \begin{pmatrix} \sin^2(\frac{\alpha}{2})\cos^2\theta & \frac{1}{2}e^{i\phi}\sin\alpha\cos\theta \\ \frac{1}{2}e^{-i\phi}\sin\alpha\cos\theta & \cos^2(\frac{\alpha}{2}) + \sin^2(\frac{\alpha}{2})\sin^2\theta \end{pmatrix}. \tag{21}$$

Under the action of the nonunitary generator, the reduced density of the system at time t is given as follows:

$$\rho_{\mathrm{red}}^{\mathcal{O}}(t) = \begin{pmatrix} \sin^2(\frac{\alpha}{2})\cos^2\theta & \frac{1}{2}e^{i\phi}\sin\alpha\cos\theta q_t \\ \frac{1}{2}e^{-i\phi}\sin\alpha\cos\theta q_t & \cos^2(\frac{\alpha}{2}) + \sin^2(\frac{\alpha}{2})\sin^2\theta \end{pmatrix}, \tag{22}$$

where $q(t) = e^{-\gamma_t/2}$. Next, referring to the definition of non-Markovianity in Eq. (11), we first evaluate the QFI for parameters α and ϕ, as follows:

$$F_\alpha^{\mathcal{O}} = \frac{\cos^2\theta((-2+q_t^2)(1+\cos\alpha) + 2q_t^2\cos\alpha - q_t^2(-1+\cos\alpha)\cos(2\theta))}{2(-1+q_t^2)(1+\cos\alpha) - 4\sin^2(\frac{\alpha}{2})\sin^2\theta}. \tag{23}$$

$F_\phi^{\mathcal{O}} = q_t^2\cos^2\theta\sin^2\alpha$, and the other values are zero. By averaging the specific values of the state for parameters α and ϕ, the corresponding averaged QFI matrix can be calculated as follows:

$$F_M^{\mathcal{O}} = \begin{pmatrix} \bar{F}_\alpha^{\mathcal{O}} & 0 \\ 0 & \bar{F}_\phi^{\mathcal{O}} \end{pmatrix}. \tag{24}$$

The eigenvalues of the matrix $\frac{d}{dt}F_M^{\mathcal{O}}$ can be obtained and are denoted as $\lambda_1(t)$ and $\lambda_2(t)$. Moreover, $\lambda_t = \max\{\lambda_1(t), \lambda_2(t)\}$, and we acquire $\lambda_t = \frac{2}{3}\frac{d}{dt}(q_t^2\cos^2\theta)$. Therefore, the non-Markovianity based the QFI can be interpreted as follows:

$$N_{\mathrm{QFI}}^{\mathcal{O}} = \int_{\dot{q}_t > 0} \frac{4}{3} q_t \dot{q}_t \cos^2\theta dt. \tag{25}$$

Simultaneously, we calculate the singular values σ_i of $\mathcal{L}_t^{\mathcal{O}}(\rho_t)$, denoted as $\sigma_1 = \sigma_2 = |(\dot{q}_t\cos\theta\sin\alpha)/2|$. Additionally, $\|\rho(\tau) - \rho(0)\| = |q(\tau) - q(0)|\sqrt{\sin^2\alpha(1+\cos(2\theta))/2}$. Thus, the QSLT can be reduced to

$$\tau_{\mathrm{QSL}}/\tau = \frac{|q(\tau) - q(0)|}{\int_0^\tau |\dot{q}_t| dt}. \tag{26}$$

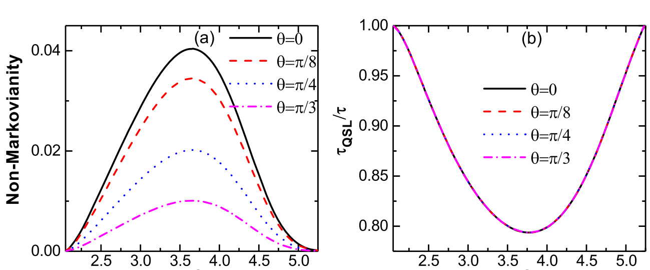

FIG. 4. Considering different parameter values of θ, (a) and (b) show the non-Markovianity and QSLT of a photon in an Ohmic-like dephasing channel as a function of parameter s, respectively, with $\tau = 60$.

Similarly, we plotted the non-Markovianity and QSLT, as shown in Fig. 4. Figure 4(a) shows that if the effect of gravitational redshift is considered, the non-Markovianity decreases as the gravitational redshift increases in the super-Ohmic case ($s > 2$). However, the QSLT in Fig. 4(b) is not affected by the gravitational redshift, as shown in Eq. (26), and it is independent of the initial coherence of the system. Notably, for the pure-dephasing channel, the non-Markovian dynamical behavior is reduced by the gravitational redshift, but the evolution speed of the photon is not influenced by the gravitational redshift.

IV. CONCLUSION

Gravitational effects are inevitable as photons propagate in curved spacetime [5,12]; thus, the process of photon propagation may be viewed as a lossy channel. The use of a beam-splitter operation introduces the gravitational redshift through a canonical transformation [27,28,31,33]. Moreover, gravitational effects have an impact on quantum communications in space. In this study, we investigated the non-Markovian speedup evolution of a photon system undergoing amplitude-damping and pure-dephasing channels under the effect of gravitational redshift. For the amplitude-damping channel, the quantum-evolution speed of the photon is related to non-Markovianity, gravitational redshift, and the initial coherence of the photon system. When the system is initially excited, the enhancement of gravitational redshift weakens the non-Markovian dynamical behavior, but does not affect the quantum evolution speed of the photon system. However, if the initial coherence of the system is nonzero, improving the non-Markovian speedup dynamics of the photon system is easier with a stronger gravitational redshift and larger initial coherence. Moreover, in the Markovian dynamics, a stronger gravitational redshift and larger initial coherence may increase the QSLT.

For the pure-dephasing channel, we also observed that the introduction of the gravitational redshift weakens the non-Markovian dynamical behavior, but it does not affect the quantum-evolution speed of the photon system. Unlike the amplitude-damping channel, the QSLT under a gravitational redshift is independent of the initial coherence of the photon system in this channel.

To explain the above results of this study, we consider that the gravitational effect can lead to the distortion of photon wave packets, and the canonical transformation in Eq. (6) describes a lossy channel reflecting the probability of photon loss. The loss of the quantum coherence of the system in such a lossy channel may be aggravated [73]. Therefore, a photon propagating in curved spacetime is affected by the gravitational effect, and the decoherence of a photon may be aggravated. This explains why the non-Markovian dynamics of a photon system may be weakened by the introduction of gravitational redshift in both channels. In addition, a canonical transformation [29,30,32] may still quantitatively introduce the gravitational redshift for a rotating planet, and the resulting conclusions are similar to those discussed above.

ACKNOWLEDGMENTS

We thank the referees for their useful suggestions that improved our paper. This work is supported by the National Natural Science Foundation of China (11974209, 12274257), the Provincial Natural Science Foundation of Shandong (ZR2020 MA086), Taishan Scholar Project of Shandong Province (China) (TSQN201812059).

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