

Debye screening of non-Abelian plasmas in curved spacetimes

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(Received 4 October 2023; accepted 19 November 2023; published 18 December 2023)

Decades of analytic and computational work have demonstrated that a charge immersed in a hot plasma is screened. For both Abelian and non-Abelian interactions, the characteristic screening length $1/m_D$ is set by the so-called Debye mass $m_D \sim g_s T$, proportional to the plasma temperature T and the dimensionless gauge coupling g_s . One of the most interesting naturally occurring examples is the quark-gluon plasma (QGP) that filled the early universe prior to the QCD confinement phase transition at $t_{\text{QCD}} \sim 10^{-5}$ s. During this early epoch, regimes of strong spacetime curvature are of significant cosmological interest, such as near primordial black holes (PBHs). However, the typical description of Debye screening only applies within Minkowski spacetime and is therefore insufficient to describe the dynamics of charged plasmas near PBHs or other primordial features. We construct an effective field theory for soft modes of the gauge field A_μ^a to give a full description of Debye screening in non-Abelian plasmas within arbitrary curved spacetimes, recovering a temperature-dependent Debye mass that exhibits gravitational redshift. We then apply our results to some scenarios of cosmological interest: an expanding Friedmann-Lemaître-Robertson-Walker universe and the vicinity of a PBH immersed in a hot QGP.

DOI: [10.1103/PhysRevD.108.125010](https://doi.org/10.1103/PhysRevD.108.125010)

I. INTRODUCTION

The screening of charges in hot plasmas, for both Abelian and non-Abelian interactions, has been a topic of significant interest for decades. Upon resumming hard thermal loops, the leading-order effects yield a characteristic screening length $\lambda_D(T) = 1/m_D(T)$ set by the Debye mass $m_D(T) \sim g_s T$, where g_s is the dimensionless gauge coupling strength and T is the temperature of the plasma. (For reviews, see Refs. [1–4].)

The Debye mass $m_D(T)$ sets the characteristic scale for dynamics of collective excitations within the plasma. For example, collective oscillations of “soft” modes, with momenta $k_{\text{soft}} \sim m_D(T) \ll T$, propagate with frequency set by the plasma frequency $\omega_p \simeq m_D(T)$ [5–17]. Moreover, nontrivial spatial distributions of color charge among the soft gluons can form, with typical length scale $1/m_D(T)$ [18,19]. These effects have been studied extensively in Minkowski spacetime, using both analytic techniques and lattice simulations [1–4].

In this paper we consider Debye screening in hot plasmas within curved spacetimes, with particular interest

in cosmological applications. For example, postinflation reheating typically yields a universe filled with Standard Model particles in thermal equilibrium at a temperature as high as $T \sim \mathcal{O}(10^{14} \text{ GeV}) \sim \mathcal{O}(10^{27} \text{ K})$ by the time $t_{\text{therm}} \sim \mathcal{O}(10^{-35} \text{ s})$ after the big bang [20,21]. Such temperatures are exponentially greater than the QCD confinement scale $\Lambda_{\text{QCD}} \simeq 0.170 \text{ GeV}$. Hence at such early times, the Universe was filled with a plasma of unconfined quarks and gluons, subject to the non-Abelian dynamics of QCD [22,23].

As the Universe expanded, the temperature of the plasma fell adiabatically: $T(t)a(t) \simeq \text{constant}$, where $a(t)$ is the scale factor of the Friedmann-Lemaître-Robertson-Walker (FLRW) line element. During the radiation-dominated phase, $a(t) = (t/t_{\text{therm}})^{1/2}$. The QCD confinement transition, after which quarks and gluons remained bound in color-neutral hadronic states, occurred at $t_{\text{QCD}} \simeq 10^{-5}$ s, when $T(t_{\text{QCD}}) \simeq \Lambda_{\text{QCD}}$ [22,23].

Within the range of times $t_{\text{therm}} \ll t \ll t_{\text{QCD}}$, additional phenomena of cosmological interest could have occurred. For example, if a significant population of primordial black holes formed at early times, with masses in the range $10^{17} \text{ g} \leq M(t_c) \leq 10^{22} \text{ g}$, then they could account for the entire abundance of dark matter today [24–27]. Given the dependence of the black holes’ masses on the time of collapse t_c , the black-hole mass range of interest to account for the present dark-matter abundance corresponds to formation times $10^{-21} \text{ s} \leq t_c \leq 10^{-16} \text{ s}$. At these early times, the temperature of the plasma within which the

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primordial black holes formed would have been $10^5 \text{ GeV} \leq T(t_c) \leq 10^7 \text{ GeV}$, for which $T(t_c) \gg T(t_{\text{QCD}})$.

With such cosmological applications in mind, we study Debye screening in hot non-Abelian plasmas within arbitrary curved spacetimes, including spacetimes that need not be homogeneous and isotropic (such as in the vicinity of a primordial black hole). In such general spacetimes, the effective temperature of the plasma can develop spatial gradients, $T \rightarrow T(x)$, akin to the familiar Tolman temperature [28–30]. We identify corrections to the induced current $j_\mu^a(x)$ and to the Debye mass $m_D(T(x))$ from spacetime curvature. Much as in Minkowski spacetime, the components $A_0^a(x)$ of the gauge field acquire an effective mass in the plasma proportional to $m_D(T(x))$, whereas components $A_i^a(x)$ remain massless. (See, e.g., Refs. [31–39] on the behavior of Abelian plasmas within curved spacetimes.)

In Sec. II, we introduce an effective theory for soft modes within the plasma, generalizing the elegant formalism reviewed in Refs. [1,2] for application to curved spacetimes. In Sec. III we evaluate the induced current $j_\mu^a(x)$ for the soft modes that arises from interactions with high-energy quarks and gluons in the plasma. Section IV considers corrections to the usual expression for the Debye mass $m_D(T)$ arising from spacetime curvature. In Sec. V we apply these expressions for $j_\mu^a(x)$ and $m_D(T(x))$ to some scenarios of cosmological interest.

We restrict attention to $(3+1)$ spacetime dimensions and adopt the metric signature $(-, +, +, +)$. Greek letters $\mu, \nu = 0, 1, 2, 3$ label spacetime indices, while Latin letters $i, j = 1, 2, 3$ label spatial indices. The color-charge indices associated with the adjoint representation of the gauge group $\text{SU}(N_c)$ range over $a, b, c = 1, 2, \dots, N_c^2 - 1$ and are raised and lowered with δ_{ab} . The generators T^a of the Lie algebra for $\text{SU}(N_c)$ satisfy $[T^a, T^b] = if^{abc}T^c$, where f^{abc} are the totally antisymmetric structure constants, and the generators are normalized such that $2\text{Tr}(T^a T^b) = \delta^{ab}$. We adopt natural units in which $c = \hbar = k_B = 1$, in terms of which the reduced Planck mass is given by $M_{\text{pl}} \equiv 1/\sqrt{8\pi G} = 2.43 \times 10^{18} \text{ GeV}$.

II. EFFECTIVE THEORY FOR SOFT MODES

At early times $t \ll t_{\text{QCD}}$, when the temperature of the plasma filling the Universe satisfied $T \gg \Lambda_{\text{QCD}}$, the number density of gluon degrees of freedom likely dominated those of quarks: gluons can radiate gluons at tree level, and bosonic statistics allow gluon number densities per mode to scale as $n_k \sim 1/\alpha_s$, where $\alpha_s \equiv g_s^2/(4\pi)$ and g_s is the QCD coupling constant [40]. Given the running of α_s to lower values at higher energies, the number density of gluons could have greatly exceeded the number density of quarks within the plasma at early times [1,4]. Similar behavior has been observed in the plasmas that form immediately after relativistic heavy-ion collisions [40,41]. Hence we expect

gluons to dominate the dynamics and consider the effective action for a Yang-Mills model with gauge group $\text{SU}(N_c)$ and field strength tensor

$$F_{\mu\nu}^a = \nabla_\mu A_\nu^a - \nabla_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c, \quad (1)$$

where ∇_μ is the (spacetime) covariant derivative associated with the metric $g_{\mu\nu}(x)$.

Following the approach reviewed in Refs. [1,2], we construct an effective theory for long-wavelength excitations in the plasma within the high-temperature limit. In particular, we consider the effective action for soft modes \tilde{A}_μ^a with typical momenta $k_{\text{soft}} \sim g_s T$. Thermal and quantum corrections to the behavior of the soft modes are dominated by quanta a_μ^a with momenta $k_{\text{hard}} \sim T$. We therefore write $A_\mu^a = \tilde{A}_\mu^a + a_\mu^a$, integrate out the high-momentum modes a_μ^a , and drop the tilde on the soft modes $\tilde{A}_\mu^a \rightarrow A_\mu^a$.

As described in Refs. [1,2], the soft modes A_μ^a have large occupation numbers per mode and hence behave as effectively classical fields. The behavior of the soft modes therefore yields the mean-field dynamics for the system on length scales $\lambda \geq 1/k_{\text{soft}} \sim 1/(g_s T)$. We follow the background-field method of Ref. [1], whereby we choose to decompose the action of the gauge symmetry on a_μ^a and A_μ^a in such a way that leaves the soft modes A_μ^a unchanged. One example is to use a generalized Coulomb-type gauge-fixing term in the full theory, $\nabla_i a_i^a - g_s f^{abc} A_i^b a_c^i$ [1]. No additional gauge-fixing terms or ghosts are then required in the effective action for the soft modes.

For long length scales $\lambda \geq 1/k_{\text{soft}} \gg 1/k_{\text{hard}} \sim 1/T$, the dynamics may be described by the effective action

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[\frac{M_{\text{pl}}^2}{2} R - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a - j_\mu^a A_\mu^a + \mathcal{L}_{\text{fluid}} \right], \quad (2)$$

where $j_\mu^a(x)$ is the induced current generated by (non-Abelian) self-interactions with the high-momentum modes $a_\mu^a(x)$; the induced current j_μ^a also includes contributions from high-momentum quarks. The term $\mathcal{L}_{\text{fluid}}$ represents the contributions to the evolution of the spacetime curvature from constituents other than the soft modes. For example, the high-momentum modes in the plasma (coarse-grained over a length scale $\lambda \gg 1/k_{\text{hard}}$) behave as a neutral fluid with a radiation-dominated equation of state, and thereby contribute to the time evolution of the scale factor $a(t)$ in an FLRW background, while masses M of compact objects, such as black holes, influence the spacetime curvature in their vicinity. Each of these contributions affects $g_{\mu\nu}(x)$ and hence impacts the dynamics of the soft modes $A_\mu^a(x)$.

Varying S_{eff} with respect to A_μ^a yields the equations of motion

$$D^\mu F_{\mu\nu}^a = j_\nu^a, \quad (3)$$

where the covariant derivative D_μ acting on a spacetime tensor that transforms in the adjoint representation of $SU(N_c)$, $X_{\nu_1 \dots \nu_n}^a$, is defined as

$$D_\mu X_{\nu_1 \dots \nu_n}^a \equiv \nabla_\mu X_{\nu_1 \dots \nu_n}^a + g_s f^{abc} A_\mu^b X_{\nu_1 \dots \nu_n}^c. \quad (4)$$

The left-hand side of Eq. (3) can be expanded as

$$\begin{aligned} D^\mu F_{\mu\nu}^a &= \nabla^\mu F_{\mu\nu}^a + g_s f^{abc} A_b^\mu F_{\mu\nu}^c \\ &= D^\mu D_\mu A_\nu^a - g_s f^{abc} A_b^\mu \nabla_\nu A_\mu^c - R_{\mu\nu} A_a^\mu, \end{aligned} \quad (5)$$

where $R_{\mu\nu}$ is the Ricci tensor for the background spacetime. The equations of motion imply that the induced current j_μ^a must be covariantly conserved with respect to both the spacetime curvature and the $SU(N_c)$ gauge group:

$$D^\mu j_\mu^a(x) = 0, \quad (6)$$

for each color a .

The contribution to the energy-momentum tensor from the Yang-Mills field is given, as usual, by

$$T_{\mu\nu}^{(A)} = F_{\mu\lambda}^a F_{\nu}^{a\lambda} - \frac{1}{4} g_{\mu\nu} F_a^{\lambda\sigma} F_{\lambda\sigma}^a, \quad (7)$$

while the additional fluid contributes

$$T_{\mu\nu}^{\text{fluid}} = -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_{\text{fluid}}}{\delta g^{\mu\nu}}. \quad (8)$$

These terms satisfy the usual covariant conservation equations

$$\begin{aligned} \nabla_\mu T_{(A)}^{\mu\nu} &= j_\mu^a F_a^{\mu\nu}, \\ \nabla_\mu T_{\text{fluid}}^{\mu\nu} &= 0 \end{aligned} \quad (9)$$

in the presence of the induced current j_μ^a .

III. INDUCED CURRENT

The induced current $j_\mu^a(x)$ for the soft modes $A_\mu^a(x)$ arises from interactions with high-momentum quarks and gluons in the plasma. Lattice simulations have confirmed the analytic expectation that for temperatures $T \gg T_{\text{QCD}}$, high-energy quarks and gluons attain an equilibrium equation of state akin to that of a gas of noninteracting massless particles. In particular, the so-called “trace anomaly,” $(\rho - 3P)/T^4$, where ρ is the energy density and P the pressure of particles in the plasma, tends rapidly toward zero for $T \gg T_{\text{QCD}}$, and therefore $\rho \approx 3P$. (See, e.g., Refs. [42–45].)

At high temperatures, the soft quantum fields of the plasma are well approximated by classical fields, due to their large occupation numbers per mode. Furthermore, the hard (high-momentum) modes can be approximated by an ensemble of classical point particles to leading order in the coupling g_s , since they are weakly interacting. Specifically, the effects of the high-momentum particles on the soft modes within the plasma are dominated by processes involving “hard thermal loops”: one-loop diagrams with arbitrary numbers of low-momentum external legs (with $k_{\text{soft}} \sim g_s T$) and hard internal momenta (with $k_{\text{hard}} \sim T$) [10,11,46–48]. These effects can be analyzed within a mean-field approximation of the (truncated) Schwinger-Dyson equations [1,12,13,15–17], or by simply adopting classical transport equations for high-momentum particles [2,17,49–51]. In this section we adapt the latter approach to arbitrary curved spacetimes. (See also Ref. [52].)

The equations of motion for a classical point particle with color charge in a soft gauge-field background are well known [50,52–54]:

$$\begin{aligned} \frac{dx^\mu}{d\lambda} &= P^\mu, \\ \frac{dP_\mu}{d\lambda} &= -\frac{1}{2} \frac{\partial g^{\alpha\beta}}{\partial x^\mu} P_\alpha P_\beta - g_s Q^a F_{\mu\nu}^a P^\nu, \\ \frac{dQ^a}{d\lambda} &= -g_s f^{abc} A_\mu^b Q^c P^\mu. \end{aligned} \quad (10)$$

Here x^μ is the position of the particle, $P^\mu = dx^\mu/d\lambda$ its kinetic 4-momentum, λ an affine parameter, and Q^a its $SU(N_c)$ charge. We take advantage of the freedom to linearly rescale the affine parameter λ so that $dx^\mu/d\lambda$ has units of energy. (We do not explicitly rescale by the particle mass m so that our formalism can be applied to massless particles.) Notice that, due to non-Abelian self-interactions, the charges Q^a are dynamical, unlike in electromagnetism.

The induced current $j_\mu^a(x)$, summed over all hard particles, is given by [1,2]

$$j_\mu^a(x) = g_s \int dQ d\omega \frac{dx^\mu}{d\lambda} Q_a \delta f(x, p, Q), \quad (11)$$

where $\delta f(x, p, Q)$ represents the deviation from equilibrium of the distribution function for the high-momentum charge-carrying particles. The integration measure includes the momentum volume form $d\omega$ and the measure for the space of color charges dQ , subject to the physical constraints of on-shell mass condition, positivity of energy, and conservation of the $N_c - 1$ group Casimirs. The momentum measure is given below, in Eq. (36). The phase-space structure for the color-charge degrees of freedom is unaffected by the spacetime structure, so we may use the now-standard parametrization developed for previous studies within Minkowski spacetime, as reviewed, e.g., in

Ref. [2]; an explicit construction for the cases SU(2) and SU(3) is available in Sec. III of Ref. [50]. In particular, for the gauge group SU(2) the color-space measure is

$$dQ = d^3 Q c_R \delta(Q^a Q^a - q_2), \quad (12)$$

and for the gauge group SU(3) we have

$$dQ = d^8 Q c_R \delta(Q^a Q^a - q_2) \delta(d_{abc} Q^a Q^b Q^c - q_3). \quad (13)$$

For the SU(3) case, the d^{abc} are the totally symmetric constants given by $d^{abc} = 2\text{Tr}(\{T^a, T^b\}, T^c)$. The representation-dependent constant c_R is fixed by the normalization $\int dQ = 1$, while the constants q_2, q_3 are further fixed by the first and second Casimirs, respectively. Thus, the integration over color charges serves to enforce the conservation of the Casimir invariants [2,49,50].

We consider small departures from equilibrium, and therefore work perturbatively in powers of the coupling g_s , so we may write the full distribution function as

$$f(x, p, Q) = f^{(0)}(x, p) + g_s f^{(1)}(x, p, Q) + \mathcal{O}(g_s^2). \quad (14)$$

The relevant dynamics for the induced current $j_a^\mu(x)$ in Eq. (11) are captured to leading order by identifying $\delta f(x, p, Q) = g_s f^{(1)}(x, p, Q)$, as demonstrated by Refs. [49,50]. The dynamical evolution of $\delta f(x, p, Q)$ is governed by the collisionless Boltzmann equation for the full distribution function $f(x, p, Q)$, which in the framework of Hamiltonian mechanics corresponds to the conservation of $f(x, p, Q)$ along dynamical trajectories (Hamiltonian flow) in phase space.

To solve the Boltzmann equation, it will be convenient to have an explicit Hamiltonian formulation of the dynamics of the high-momentum particles. To this end, we first construct an action which yields the correct equations of motion, Eq. (10). This is nontrivial because there is no such action involving the charges Q^a directly as dynamical variables. To bypass this problem, following Refs. [54–56], we introduce new dynamical variables q^a which, like Q^a , transform under the adjoint representation of SU(N_c), but which are anticommuting (Grassmann-valued). These new variables q^a are useful as an intermediate step, in terms of which we may construct the charges Q^a as

$$Q^a = -\frac{i}{2} f^{abc} q^b q^c. \quad (15)$$

An action for a single high-momentum particle that yields the appropriate equations of motion may then be written in terms of the dynamical variables x^μ and q^a as

$$S_{1p} = \int d\lambda \left\{ \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} + \frac{i}{2} q^a \frac{dq^a}{d\lambda} - \frac{i}{2} g_s f^{abc} A_\mu^a q^b q^c \frac{dx^\mu}{d\lambda} \right\}. \quad (16)$$

The corresponding Hamiltonian is simply

$$H = \frac{1}{2} g^{\mu\nu} P_\mu P_\nu. \quad (17)$$

The kinetic momentum $P^\mu \equiv dx^\mu/d\lambda$ is related to the canonical momentum p_μ , conjugate to x^μ , by

$$P_\mu = p_\mu - g_s Q^a A_\mu^a. \quad (18)$$

Then Hamilton's equations are Eq. (10), as desired. Dynamical evolution in phase space is generated by the Liouville vector field X_H :

$$\begin{aligned} X_H &\equiv \frac{d}{d\lambda} = \frac{dx^\mu}{d\lambda} \frac{\partial}{\partial x^\mu} + \frac{dp_\mu}{d\lambda} \frac{\partial}{\partial p_\mu} + \frac{dQ^a}{d\lambda} \frac{\partial}{\partial Q^a} \\ &= P^\mu \frac{\partial}{\partial x^\mu} \Big|_P - \frac{1}{2} \frac{\partial g^{\alpha\beta}}{\partial x^\mu} \Big|_P P_\alpha P_\beta \frac{\partial}{\partial P_\mu} - g_s Q^a P^\mu F_{\mu\nu}^a \frac{\partial}{\partial P_\nu} \\ &\quad - g_s f^{abc} A_\mu^b Q^c P^\mu \frac{\partial}{\partial Q^a} \Big|_P, \end{aligned} \quad (19)$$

where, in the second line, we have written the vector field in noncanonical coordinates $\{x^\mu, P_\mu, Q^a\}$, and we have noted explicitly which derivatives are taken keeping P_μ fixed, as opposed to p_μ .

The collisionless Boltzmann equation for the distribution function $f(x, p, Q)$ can be written in terms of the Liouville vector field as $X_H[f] = 0$ and is equivalently known as the Liouville equation. Given the expansion in Eq. (14), we require that the Liouville equation be satisfied order by order in g_s :

$$P^\mu \frac{\partial f^{(0)}}{\partial x^\mu} \Big|_P - \frac{1}{2} \frac{\partial g^{\alpha\beta}}{\partial x^\mu} \Big|_P P_\alpha P_\beta \frac{\partial f^{(0)}}{\partial P_\mu} = 0 \quad (20)$$

and

$$P^\mu \frac{\partial f^{(1)}}{\partial x^\mu} \Big|_P - \frac{1}{2} \frac{\partial g^{\alpha\beta}}{\partial x^\mu} \Big|_P P_\alpha P_\beta \frac{\partial f^{(1)}}{\partial P_\mu} = Q^a P^\mu F_{\mu\nu}^a \frac{\partial f^{(0)}}{\partial P_\nu}. \quad (21)$$

We assume that all dependence of $f(x, p, Q)$ on the charges Q^a can only appear at $\mathcal{O}(g_s)$ or above, and therefore, $\partial f^{(0)}/\partial Q^a = 0$. Notice that, as emphasized in Ref. [51], the non-Abelian terms in X_H , proportional to $g_s f^{abc}$, make no contribution to the evolution of $f^{(1)}$, given our perturbative expansion in g_s .

We are free to choose any distribution function for our fluid, as long as it solves the Boltzmann equation, and its

physical meaning is compatible with small deviations from thermal equilibrium. For this reason, we aim to find a solution of the collisionless Boltzmann equation $X_H[f] = 0$ such that the zeroth order in g_s is of the usual form for a canonical ensemble, $f^{(0)} \sim \exp[-\beta E]$, for a constant β and a quantity E that may be interpreted as an energy. The equation $X_H[f] = 0$ is typically solved to $\mathcal{O}(g_s)$ by employing Green's function techniques [1,2,51]. We will instead show that one can further exploit the Hamiltonian formalism to solve efficiently for the distribution function to the same order. The two approaches are equivalent up to $\mathcal{O}(g_s)$, because quantum corrections to the classical equations of motion in Eq. (10) only arise at $\mathcal{O}(g_s^2)$ and above.

It is a classic result that Hamiltonian mechanics in the fully covariant phase space $\{x^\mu, p_\mu, Q^a\}$ with evolution parametrized by an affine λ and generated by the Hamiltonian in Eq. (17) is equivalent to Hamiltonian mechanics in the reduced phase space $\{x^i, p_i, Q^a\}$ with evolution parametrized by coordinate time $t \equiv x^0$ and generated by the reduced Hamiltonian

$$\bar{H} \equiv -p_0(t, x^i, p_i, Q^a; h). \quad (22)$$

This means that both formalisms yield the same equations of motion. This is possible thanks to the redundancy in the choice of affine parameter λ , as well as the conservation of $H(x^\mu, p_\mu, Q)$ along phase space trajectories, in the covariant formalism. In Eq. (22) we have solved for p_0 in terms of $\{t, x^i, p_i, Q^a, h\}$ using $H(x^\mu, p_\mu, Q) \equiv h$, with h a constant. In the covariant formalism, the conservation of $H(x^\mu, p_\mu, Q)$ follows from Eq. (10) and the fact that H cannot depend explicitly on λ . The reduced phase-space formalism does not yield the conservation of $H(x^\mu, p_\mu, Q^a)$, so we must impose it by hand. For details of the proof, see Sections 44 and 45 in Chapter 9 of Ref. [57].

If the reduced Hamiltonian \bar{H} from Eq. (22) does not depend explicitly on t , then it is also conserved along phase-space trajectories, $X_H[\bar{H}] = 0$, so we may exploit \bar{H} to construct a distribution function. That is, we may use the fact that any scalar function of \bar{H} will solve the collisionless Boltzmann equation to devise a valid distribution function for our fluid. We therefore introduce a quasistationary approximation: we consider only scenarios in which $\partial_t g_{\mu\nu}$ and $\partial_t A_\mu^a$ remain subdominant. This approximation is appropriate, since we are interested in the behavior of the high-momentum particles, whose dynamics are governed by the timescale $1/k_{\text{hard}} \sim 1/T$, whereas we expect the soft modes $A_\mu^a(x)$ to evolve on timescales set by $1/k_{\text{soft}} \gg 1/k_{\text{hard}}$. Likewise, our effective description can only resolve dynamics up to scales set by $1/k_{\text{soft}}$, which bounds how sharply the background spacetime can evolve within our self-consistent expansion as well. In particular, if we allowed \bar{H} to have an explicit dependence on t , then

$$X_H[\bar{H}] = \frac{1}{2} \partial_t g^{\mu\nu} P_\mu P_\nu - g_s Q^a P^\mu \partial_t A_\mu^a, \quad (23)$$

and thus a distribution function constructed from \bar{H} would yield self-consistent dynamics as long as

$$|\partial_t g^{\mu\nu}|, \frac{|\partial_t A_\mu^a|}{|A_\mu^a|} \ll k_{\text{soft}}. \quad (24)$$

Equation (24) involves coordinate-dependent quantities. However, any change in x^μ would be accompanied by changes in the conjugate momenta p_μ , such that Eq. (24) remains meaningful, given that k_{soft} is a momentum scale. This follows from the invariance of the phase-space volume under coordinate transformations. As we will see in Sec. V, this quasistationary approximation is easily satisfied in many cosmological applications of interest.

We choose a distribution function $f = \exp[-\beta_T \bar{H}]$, where β_T is a constant [58]. As explained just above, this satisfies the collisionless Boltzmann equation by construction. We will see in the next section that the zeroth-order term $f^{(0)}$ in the g_s expansion corresponds to a canonical ensemble (for fluid undergoing normal flow with respect to coordinate time t), so it is a physically reasonable choice because it represents thermal equilibrium. This term is

$$f^{(0)} = e^{\beta_T P_0}. \quad (25)$$

Given $f^{(0)}$, we may evaluate the equilibrium occupation numbers per mode in the usual way,

$$n_{\text{B,F}}^{(0)} \equiv \frac{\sum n_i e^{n_i \beta_T P_0}}{\sum e^{n_i \beta_T P_0}}, \quad (26)$$

where the sums run from $n_i = 0$ to $n_i = \infty$ for bosons and to $n_i = 1$ for fermions. Thus we obtain the Bose-Einstein and Fermi-Dirac distributions at zeroth-order:

$$n_{\text{B,F}}^{(0)} = \frac{1}{e^{-\beta_T P_0} \mp 1}. \quad (27)$$

Substituting into Eq. (21) yields

$$\begin{aligned} n_{\text{B,F}}^{(1)} &= Q^a A_0^a \frac{\partial n_{\text{B,F}}^{(0)}}{\partial P_0} \\ &= -\beta_T \frac{e^{-\beta_T P_0}}{(e^{-\beta_T P_0} \mp 1)^2} Q^a A_0^a \end{aligned} \quad (28)$$

for bosons and fermions, respectively. Summing over species and helicities, we have $\delta f = g_s(2n_{\text{B}}^{(1)} + 4N_f n_{\text{F}}^{(1)})$ in Eq. (11).

To evaluate $j_a^\mu(x)$ we first perform the Q integral with the measure in Eq. (13), which yields factors proportional to the index of the representation: $\int dQ Q_a Q_b = C_{\text{B,F}} \delta_b^a$, with

$C_B = N_c$ for bosons and $C_F = 1/2$ for fermions. Then the current becomes

$$\begin{aligned} j_a^\mu(x) &= g_s^2 \int dQ d\omega (2n_B^{(1)} + 4N_f n_F^{(1)}) Q^a P^\mu \\ &= g_s^2 A_0^a(x) \int d\omega \mathcal{N}(x, P) P^\mu, \end{aligned} \quad (29)$$

where we have defined

$$\begin{aligned} \mathcal{N}(x, P) &\equiv 2N_c \frac{\partial n_B^{(0)}}{\partial P_0} + 2N_f \frac{\partial n_F^{(0)}}{\partial P_0} \\ &= -2\beta_T \left[N_c \frac{e^{-\beta_T P_0}}{(e^{-\beta_T P_0} - 1)^2} + N_f \frac{e^{-\beta_T P_0}}{(e^{-\beta_T P_0} + 1)^2} \right]. \end{aligned} \quad (30)$$

Equation (29) includes contributions from two polarization states for each effectively massless particle, and $2N_f$ fermion species (N_f each for quarks and antiquarks). Note that, as in Minkowski spacetime [1], $j_a^\mu(x)$ is proportional to $A_0^a(x)$.

IV. DEBYE MASS

In Minkowski spacetime, the induced current reduces (in the static limit) to $j_a^\mu(x) = m_D^2 A_0^a \delta_\mu^0$, effectively giving a constant mass to the soft gluon components A_0^a , which is responsible for Debye screening [1–4]. In this section, we evaluate the induced current $j_a^\mu(x)$ of Eq. (29) and find that in spacetimes of interest, it is proportional to the square of an effective mass, $m_D^2(x)$, which has spatial dependence.

For any $(3+1)$ -dimensional globally hyperbolic spacetime, we can choose coordinates x^μ to put the metric in the Arnowitt-Deser-Misner form (see, e.g., Ref. [59])

$$\begin{aligned} ds^2 &= -N^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt) \\ &= -(N^2 - \beta_i \beta^i) dt^2 + 2\beta_i dt dx^i + \gamma_{ij} dx^i dx^j, \end{aligned} \quad (31)$$

where $x^0 = t$ is a global time function, $N(x)$ is the lapse function, and $\beta^i(x)$ the shift vector, whose indices are raised and lowered with γ_{ij} , i.e., $\beta_i = \gamma_{ij} \beta^j$. Hypersurfaces of constant time t are Cauchy surfaces by construction, with induced metric $\gamma_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, where

$$n_\mu = -(dt)_\mu = (-N, \mathbf{0}). \quad (32)$$

We normalize t such that $N^2 \rightarrow 1$ on the spatial (possibly asymptotic) boundary. The components of the inverse metric are given by

$$g^{00} = -\frac{1}{N^2}, \quad g^{0i} = \frac{\beta^i}{N^2}, \quad g^{ij} = \gamma^{ij} - \frac{\beta^i \beta^j}{N^2}. \quad (33)$$

In these coordinates, the kinetic momentum takes the form

$$P_\mu = (-Nk + \beta^i P_i, P_i), \quad P^\mu = \left(\frac{k}{N}, \gamma^{ij} P_j - \frac{k}{N} \beta^i \right), \quad (34)$$

where

$$k \equiv \sqrt{\gamma^{\mu\nu} P_\mu P_\nu} = \sqrt{\gamma^{ij} P_i P_j} \quad (35)$$

is the magnitude of the momentum projected on constant- t hypersurfaces.

The spacetime metric $g_{\mu\nu}$ induces a metric in all of phase space, known as the Sasaki metric [60–62], such that the invariant momentum volume form is

$$d\omega = \frac{d^4 p}{(2\pi)^3 \sqrt{-g}}. \quad (36)$$

We restrict to the mass shell by integrating over $2\delta(P^2)\Theta(P^0)$. To lowest order in g_s , the kinetic and canonical momenta coincide, $P_\mu \rightarrow p_\mu$, so

$$\frac{d^4 p}{(2\pi)^3 \sqrt{-g}} \rightarrow \frac{d^3 p_{(i)}}{(2\pi)^3 p^0 N \sqrt{\gamma}}, \quad (37)$$

where the last step follows upon noting that $\sqrt{-g} = N\sqrt{\gamma}$. The current $j_a^\mu(x)$ in Eq. (29) may then be written

$$\begin{aligned} j_a^\mu(x) &= \frac{g_s^2 A_0^a}{(2\pi)^3 N} \\ &\times \int \frac{d^3 p_{(i)}}{\sqrt{\gamma}} \mathcal{N}(x, p) \left[\delta_0^\mu + \delta_i^\mu \left(\beta^i - \frac{N}{k} \gamma^{ij} p_j \right) \right]. \end{aligned} \quad (38)$$

We next consider a change in momentum coordinates $\tilde{k}^i \equiv (\gamma^{1/2})^{ij} p_j$, where $(\gamma^{1/2})^{ij}$ is the square matrix that satisfies $(\gamma^{1/2})_{mi} \gamma^{ij} (\gamma^{1/2})_{j\ell} = \delta_{m\ell}$. Then $d^3 p_{(i)} = \sqrt{\gamma} d^3 \tilde{k}^{(i)}$, and $k^2 = \tilde{k}^2 = \delta_{ij} \tilde{k}^i \tilde{k}^j$, which is even in the components \tilde{k}^i . One could perform an additional coordinate transformation as in Ref. [52], $p'_i = p_i - \beta_i p_0 / g_{00}$, to aid in evaluating the integral in Eq. (38). For the applications of interest to us, we will instead restrict attention to spacetimes in which the shift vector vanishes, $\beta^i = 0$. Then the integration in Eq. (38) may be performed exactly. In such cases, we find

$$j_a^0(x) = \frac{g_s^2 A_0^a}{(2\pi)^3 N} \int d^3 \tilde{k}^{(i)} \mathcal{N}(x, \tilde{k}), \quad (39)$$

$$j_a^i(x) = -\frac{g_s^2}{(2\pi)^3} A_0^a (\gamma^{1/2})^{ij} \delta_{j\ell} \int d^3 \tilde{k}^{(i)} \mathcal{N}(x, \tilde{k}) \frac{\tilde{k}^\ell}{k}. \quad (40)$$

The quantity $\mathcal{N}(x, \tilde{k})$ is even in the components \tilde{k}^i , so that $j_a^i(x)$ vanishes identically. We further note that the

momentum coordinates \tilde{k}^i are Cartesian, so $j_a^0(x)$ is equivalent to

$$\begin{aligned} j_a^0(x) &= \frac{g_s^2}{2\pi^2} \frac{A_0^a}{N} \int_0^\infty d\tilde{k} \tilde{k}^2 \mathcal{N}(x, \tilde{k}) \\ &= -\frac{m_D^2(x)}{N^2(x)} A_0^a(x) + \mathcal{O}(g_s^3), \end{aligned} \quad (41)$$

with

$$m_D^2(x) \equiv \frac{1}{6} (2N_c + N_f) \frac{g_s^2}{(\beta_T N(x))^2}. \quad (42)$$

Equation (41) corresponds to

$$j_\mu^a(x) = m_D^2(x) A_0^a \delta_\mu^0 + \mathcal{O}(g_s^3). \quad (43)$$

The induced current in Eq. (43) is equivalent to inserting an effective mass $m_D(x)$ for the A_0^a soft gluon components within the effective action of Eq. (2).

The expression for $m_D(x)$ in Eq. (42) reduces to the usual expression in Minkowski spacetime upon setting $N(x) \rightarrow 1$ and identifying $1/\beta_T = T$ with the temperature of the plasma [1–4,8,16,17,63–66]. In our case, the Boltzmann factor for the high-momentum particles is $\exp[-\beta_T \tilde{H}^{(0)}] = \exp[\beta_T N(x) k(x)]$, since we are considering spacetimes in which $\beta^i = 0$. This is exactly what one would expect for particles in a fluid that is undergoing normal flow, with four-velocity $n^\mu(x)$ and therefore local energy per particle $E \equiv -n^\mu p_\mu = k$, if we identify $T(x) \equiv 1/(\beta_T N(x))$ with the local temperature. We may therefore identify the constant $\beta_T \equiv 1/T_0$ and write

$$T(x) = \frac{T_0}{N(x)}. \quad (44)$$

Given that we have normalized t such that $N(x) \rightarrow 1$ on the spatial (possibly asymptotic) boundary, T_0 is the temperature associated with time t on the spatial boundary. We have thus recovered the familiar Tolman temperature gradient in a curved spacetime [28–30]. The Debye mass then takes the form

$$m_D^2(x) = \frac{1}{6} (2N_c + N_f) g_s^2 T^2(x), \quad (45)$$

with the local temperature $T(x)$ given by Eq. (44).

V. COSMOLOGICAL APPLICATIONS

In this section we consider a few specific examples. We begin with the familiar case of Debye screening of a non-Abelian plasma in Minkowski spacetime, to identify several regimes of interest. Next we generalize to the case of a spatially flat FLRW spacetime, which introduces a new scale (compared to the Minkowski case), set by the Hubble

radius. In the last subsection, we examine Debye screening in the vicinity of a primordial black hole.

A. Debye screening in Minkowski spacetime

We first consider the behavior of soft gluon modes $A_\mu^a(x)$ within a hot plasma in Minkowski spacetime, to clarify notation and identify several physical regimes of interest. In that case, the lapse function becomes trivial, $N(x) = 1$, and the shift vector vanishes, $\beta^i = 0$. In the static limit, $\partial_0 A_\mu^a(x) = 0$, the exact equations of motion of Eq. (5) reduce to

$$\begin{aligned} \delta_\nu^0 \{ \partial_i F_{i0}^a + g_s f^{abc} A_i^b F_{i0}^c \} \\ + \delta_\nu^i \{ \partial_k F_{ki}^a + g_s f^{abc} (A_0^b F_{i0}^c + A_k^b F_{ki}^c) \} = j_\nu^a, \end{aligned} \quad (46)$$

where repeated indices are summed and we have adopted Cartesian coordinates for the spatial sections. The induced current of Eq. (43) reduces to

$$j_\nu^a(\mathbf{x}) = m_D^2 A_0^a(\mathbf{x}) \delta_\nu^0 + \mathcal{O}(g_s^3). \quad (47)$$

A generic feature of non-Abelian field theories, which has been well studied for the case of self-interacting gauge fields in Minkowski spacetime, is the existence of monopolelike solutions among the spatial components $A_i^a(\mathbf{x})$. Such solutions can be found in simplest form for $SU(2)$ [1,67–69], and have been generalized to monopoles charged under various three-dimensional subgroups of $SU(N_c)$ [70–72]. Moreover, it is well known that self-consistent static solutions to the non-Abelian equations of motion in Minkowski spacetime generically include a Yukawa-like screened behavior for the component $A_0^a(\mathbf{x})$ combined with the Wu-Yang monopole solution for the components $A_i^a(\mathbf{x})$ [1,69]. Such solutions underscore the important physical point that only the components $A_0^a(\mathbf{x})$ acquire a nonzero mass within a medium in the state described in Sec. IV—as indicated by the form of the induced current $j_\mu^a(\mathbf{x})$ in Eq. (47)—and hence only those components undergo screening, with amplitude proportional to $\exp[-m_D r]$.

Our aim in this section is to examine how well-known field configurations in Minkowski spacetime generalize to various curved spacetimes of interest, in which additional length scales become relevant. We therefore begin by considering an exact solution to Eq. (46) that consists of a superposition of a screened component A_0^a with a Wu-Yang monopole solution for the components A_i^a . For the case of $SU(2)$, the solution takes the form [1,67–69]

$$A_0^a(\mathbf{x}) = -\frac{Q_0^a e^{-m_D r}}{r}, \quad A_i^a(\mathbf{x}) = \frac{\epsilon_{aij} \hat{x}^j}{g_s r}, \quad (48)$$

where $Q_0^a = Q_0 \hat{x}^a$, Q_0 is a constant, $\hat{x}^j \equiv x^j/r$ is a unit vector, and ϵ_{ijk} is the usual Levi-Civita symbol. Note that

the solution mixes spatial indices and color-space indices, which is straightforward for SU(2), since both $i, j = 1, 2, 3$ and $a, b = 1, 2, 3$. (To confirm that $A_\mu^a(\mathbf{x})$ in Eq. (48) satisfies Eq. (46), it is helpful to construct the projection operator $P_b^a = \delta_b^a - \hat{x}_a \hat{x}_b$, noting that $\hat{x}_a P_b^a = 0 = \hat{x}^b P_b^a$ and $\partial_j \hat{x}^i = r^{-1} P_j^i$.) For SU(3), exact monopole solutions have been found for subgroups such as $U(1) \times U(1)$ and $U(2)$, for which the components A_i^a have comparable asymptotic behavior to the solution in Eq. (48) [70–72].

The solution for $A_\mu^a(\mathbf{x})$ in Eq. (48) carries both chromoelectric charge \mathcal{Q} and chromomagnetic charge \mathcal{P} . We consider the quasilocal charges defined in terms of the chromoelectric and chromomagnetic fields, $E_i^a \equiv F_{0i}^a$ and $B_i^a \equiv -\frac{1}{2} \epsilon_{ijk} F_{jk}^a$, respectively. Specifically, we identify charges via the relations

$$E_i^a E_i^a = \frac{\mathcal{Q}^2(r)}{r^4}, \quad B_i^a B_i^a = \frac{\mathcal{P}^2(r)}{r^4}, \quad (49)$$

where the spatial and color indices are summed over. Note that the fields E_i^a and B_i^a transform covariantly in color space under gauge transformations, and therefore the bilinear combinations in Eq. (49) are gauge invariant. For the solution given in Eq. (48), the charges are

$$\mathcal{Q}(r) = \mathcal{Q}_0(1 + m_D r)e^{-m_D r}, \quad \mathcal{P}(r) = \frac{1}{g_s}, \quad (50)$$

where $\mathcal{Q}_0 = \sqrt{\mathcal{Q}_0^a \mathcal{Q}_0^a}$ is the chromoelectric charge measured at the origin. Because of screening, the value of $\mathcal{Q}(r)$ measured at a finite distance from the origin will be reduced compared to \mathcal{Q}_0 . The chromomagnetic charge, on the other hand, is not screened, and hence $\mathcal{P} = 1/g_s$ everywhere.

Whereas the magnitude of the chromomagnetic charge \mathcal{P} is fixed by $1/g_s$, the chromoelectric charge \mathcal{Q}_0 at the origin is a free parameter and can be arbitrarily larger than $1/g_s$. Such a scenario would be compatible with a collection of test charges at the origin such that the energy associated with the charges does not backreact on the spacetime itself. In that case, there will exist a self-consistent regime, $0 \leq r \leq r_{\text{abel}}$, within which $|A_0^a(\mathbf{x})| \gg |A_i^a(\mathbf{x})|$, with r_{abel} given by

$$r_{\text{abel}} \equiv \frac{1}{m_D} \ln [g_s \mathcal{Q}_0]. \quad (51)$$

For $0 \leq r \leq r_{\text{abel}}$, the system is quasi-Abelian, with $A_\mu^a(\mathbf{x}) \simeq A_0^a(\mathbf{x}) \delta_\mu^0$, and hence non-Abelian contributions of the form $f^{abc} A_\mu^a A_\nu^b$ remain subdominant. Within this regime, one may solve the exact equations of motion of Eqs. (46)–(47) with the ansatz $A_\mu^a(\mathbf{x}) = A_0^a(\mathbf{x}) \delta_\mu^0$. The solution for $A_\mu^a(\mathbf{x})$ is then of the form in Eq. (48), but with $A_i^a(\mathbf{x}) = 0$ and \mathcal{Q}_0^a an arbitrary constant vector in color space.

B. Debye screening in an FLRW spacetime

We next consider Debye screening within a hot quark-gluon plasma in an expanding FLRW spacetime at early times, prior to the QCD confinement transition ($t < t_{\text{QCD}} \sim 10^{-5}$ s). The line element for a spatially flat FLRW universe may be written

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\Omega_{(2)}^2], \quad (52)$$

where $a(t)$ is the scale factor and r is a comoving radial coordinate. The Hubble parameter is given by $H(t) \equiv \dot{a}/a$, where overdots denote derivatives with respect to cosmic time t , and the Hubble radius is $r_H = 1/H$. The presence of r_H introduces a new scale compared to Minkowski spacetime, so now we must consider H as well as k_{hard} and k_{soft} . Note also that in these coordinates the lapse function is $N(x) = 1$, so the temperature has no spatial gradients.

The term $\mathcal{L}_{\text{fluid}}$ in the effective action of Eq. (2) includes contributions from the high-momentum quarks and gluons in the quark-gluon plasma, which, as noted in Sec. III, behave to leading order as a gas of noninteracting particles with a radiation-dominated equation of state. We assume that the (coarse-grained) energy density ρ and pressure P associated with the high-momentum particles dominate $T_{\mu\nu}$, so that the Friedmann equation takes the form

$$H^2 = \frac{1}{3M_{\text{pl}}^2} \left(\frac{\pi^2}{30} g_* T^4 \right) \quad (53)$$

corresponding to a fluid of g_* effectively massless degrees of freedom in equilibrium at temperature T ; for the Standard Model at temperatures much greater than the top-quark mass ($m_t = 173$ GeV), $g_* = 106.75$ [22,23]. During the radiation-dominated phase, $a(t) \propto t^{1/2}$, so we have $H(t) = 1/(2t)$ and

$$T(t) = \left(\frac{90}{\pi^2 g_*} \right)^{1/4} \left(\frac{M_{\text{pl}}}{2t} \right)^{1/2}, \quad (54)$$

consistent with adiabatic expansion, $T(t)a(t) = \text{constant}$.

The equations of motion from Eq. (5) are

$$\begin{aligned} j_\nu^a &= \frac{1}{a^2} \delta_\nu^0 \{ \partial_i F_{i0}^a + g_s f^{abc} A_i^b F_{i0}^c \} \\ &+ \delta_\nu^i \left\{ H F_{i0}^a + \partial_0 F_{i0}^a + \frac{1}{a^2} \partial_k F_{ki}^a \right. \\ &\left. + g_s f^{abc} \left(A_0^b F_{i0}^c + \frac{1}{a^2} A_k^b F_{ki}^c \right) \right\}, \end{aligned} \quad (55)$$

where repeated indices are summed. The Hubble parameter $H(t) = \dot{a}/a$ sets the timescale over which we expect cosmological dynamics to be relevant. Notice that the terms that make Eq. (55) differ from Eq. (46) [up to factors of $a(t)$] are either proportional to $H(t)$ or include a time

derivative \dot{A}_μ^c . We consider an ansatz for $A_\mu^c(x)$ in which the only time dependence arises from the scale factor $a(t)$. By the chain rule, $dA_\mu^c/dt = \dot{a}\partial A_\mu^c/\partial a = H\partial A_\mu^c/\partial \ln a$. Therefore any term with a time derivative \dot{A}_μ^c will be proportional to H . If $H \ll k_{\text{soft}}^2/k_{\text{hard}} \sim g_s^2 T$, then all terms proportional to H , which include those with time derivatives \dot{A}_μ^c , will be subleading compared to $j_\nu^c \sim m_D^2 A_0^c \sim k_{\text{soft}}^2 k_{\text{hard}}$.

If the hierarchy $H \ll g_s^2 T$ is satisfied at early times, then it will be even more easily satisfied at later times, since $H \sim T^2/M_{\text{pl}}$ from Eq. (54) and T/M_{pl} decreases as the Universe cools down, while g_s gradually runs to larger values. Now we proceed to show that $H \ll g_s^2 T$ holds at early times of interest.

Consider, as an example, dynamics at the time $t_c \sim 10^{-21}$ s, the earliest time that a population of primordial black holes (PBHs) could have formed, if the PBHs are to constitute all of dark matter today [24–27]. At such early times, the Hubble scale $H(t_c) = 3.0 \times 10^{-4}$ GeV and the fluid filling the FLRW spacetime had a temperature $T(t_c) = 10^7$ GeV. At such high energy scales, we must take into account the running of the strong coupling $\alpha_s \equiv g_s^2/(4\pi)$. To two-loop order, the running with energy scale μ is given by [4]

$$\frac{d\alpha_s(\mu)}{d\ln\mu} = -2\alpha_s \left[b_0 \frac{\alpha_s}{4\pi} + b_1 \left(\frac{\alpha_s}{4\pi} \right)^2 \right], \quad (56)$$

with

$$\begin{aligned} b_0 &= \frac{11}{3} N_c - \frac{2}{3} N_f, \\ b_1 &= \frac{32}{3} N_c^2 - \frac{10}{3} N_c N_f - \left(\frac{N_c^2 - 1}{N_c} \right) N_f. \end{aligned} \quad (57)$$

Upon normalizing $\alpha_s(m_Z) = 0.118$ at the Z boson mass ($m_Z = 91.2$ GeV), we find $\alpha_s = 0.046$ at the energy scale $\mu = T(t_c) = 10^7$ GeV. This yields a ratio $g_s^2 T/H \sim 10^9$. Hence we may self-consistently neglect $H \ll g_s^2 T$ when considering the dynamics of the gluon modes on length scales λ_D , even at very early times in cosmic history. In other words, the FLRW spacetime is consistent with our quasistationary approximation from Eq. (24), since $|\partial_t g^{\mu\nu}| \sim H \ll g_s k_{\text{soft}} < k_{\text{soft}}$.

Having shown that we may neglect cosmological dynamics, the scale factor $a(t)$ will be approximately constant in regimes of interest. We may therefore rescale the spatial coordinates as $x^i \rightarrow x^i/a$, and thus recover identical equations of motion to those in the Minkowski background, Eq. (46). A solution is the superposition of a screened component A_0^a with a Wu-Yang monopole for the components A_i^a , which in the case of SU(2) takes a similar form to Eq. (48),

$$A_0^a(x) = -\frac{Q_0^a e^{-m_D ar}}{ar}, \quad A_i^a(x) = \frac{\epsilon_{aij} \hat{x}^j}{g_s ar}, \quad (58)$$

where we have rescaled the spatial coordinates back so that r is the comoving radial coordinate from Eq. (52). The solution for $A_\mu^a(x)$ in Eq. (58) remains consistent with the quasistationary approximation of Eq. (24) for $\lambda_D \leq r \ll r_H$.

C. Debye screening near a primordial black hole

Our final example concerns Debye screening within the hot plasma surrounding a PBH that formed early in cosmic history. PBHs form via direct gravitational collapse of primordial overdensities, and their masses are typically proportional to the Hubble mass M_H at the time of formation t_c , where $M_H(t_c) = 4\pi M_{\text{pl}}^2/H(t_c)$ is the mass contained within a Hubble radius $H^{-1}(t_c)$. Such black holes may therefore form with a huge range of masses, depending on their time of formation [24–27].

Any PBHs that formed at times $t < t_{\text{QCD}} = 10^{-5}$ s would undergo collapse amid a hot plasma of unconfined quarks and gluons. On long length scales at such early times, $\lambda \gg 1/k_{\text{soft}}$, we expect that the plasma would have attained a charge-neutral equilibrium distribution. Yet on shorter length scales, set by $\lambda_D = 1/m_D \sim 1/k_{\text{soft}} \sim 1/(g_s T)$, spatial regions of nonvanishing net color charge can form, within which the charges for most soft gluons align in color space [18,19]. In such scenarios, the PBHs would form by absorbing one or more net-color regions. Depending on the ratio of the PBH radius ($\sim GM_{\text{PBH}}$) and the Debye length ($\sim 1/m_D$) at the time of formation, PBHs therefore could form with net color charge Q_0 . Such a scenario is distinct from the examples reviewed in Refs. [73,74], which concern black hole solutions to the Einstein-Yang-Mills equations in vacuum. In our case, the PBHs are immersed in a hot active medium.

We consider a scenario in which a PBH forms with a small net charge Q_0 ; the case of larger Q_0 is treated in Ref. [75]. In particular, we consider the regime

$$Q_0^2 \ll GM_{\text{PBH}}^2. \quad (59)$$

Within this regime, backreaction on spacetime from the energy associated with the enclosed charge Q_0 remains subdominant, and we may consider the dynamics of soft gluon modes $A_\mu^a(x)$ within a fixed background geometry. (For $Q_0^2 \sim GM_{\text{PBH}}^2$, one must solve the Einstein field equations as well as the equations of motion for $A_\mu^a(x)$, which we consider in separate work [76].)

We consider a spherically symmetric spacetime with a black hole of mass M_{PBH} at the origin surrounded by hot plasma with a radiation-dominated equation of state. Our formalism holds in a quasistatic limit, which requires that M_{PBH} not change appreciably over timescales set by $1/k_{\text{soft}}$. Two competing effects could change M_{PBH} : evaporation

due to Hawking radiation (which would reduce M_{PBH} over time) and accretion from the surrounding medium (which would increase M_{PBH} over time). For the regime of interest, we find that both of these effects are negligible over the relevant dynamical timescale, and hence we may neglect \dot{M}_{PBH} .

Consider first evaporation from Hawking radiation. Because we are considering PBHs with modest charge, subject to the inequality of Eq. (59), we may approximate the Hawking temperature based on that for an (uncharged) Schwarzschild black hole of mass M_{PBH} . (Any net enclosed charge would decrease the black hole's surface gravity, and hence its Hawking temperature, compared to the zero-charge case, thus rendering evaporation even less efficient.) The Hawking temperature for a Schwarzschild black hole is given by [59]

$$T_H = \frac{M_{\text{pl}}^2}{M_{\text{PBH}}}. \quad (60)$$

The typical mass for a PBH is set by the mass enclosed within a Hubble volume $M_H(t_c)$ at the time of collapse t_c [77–80]:

$$M_{\text{PBH}}(t_c) = \gamma M_H(t_c) = 4\gamma \sqrt{\frac{90}{g_*}} \left(\frac{M_{\text{pl}}}{T_c} \right)^2 M_{\text{pl}}, \quad (61)$$

where $\gamma \simeq 0.2$ and T_c is the temperature of the plasma at the time t_c . The values of M_{PBH} relevant to account for dark matter lie within the range $10^{17} \text{ g} \leq M_{\text{PBH}} \leq 10^{22} \text{ g}$ [24–27]; from Eq. (61), these correspond to plasma temperatures $10^5 \text{ GeV} \leq T_c \leq 10^7 \text{ GeV}$. Meanwhile, for masses within the dark-matter range, Eq. (60) yields $10^{-9} \text{ GeV} \leq T_H \leq 10^{-4} \text{ GeV}$, exponentially lower than the temperature of the surrounding plasma. Such PBHs will therefore be net absorbers (rather than emitters) around the time of their formation.

Next we may consider accretion. One might suppose that if the fluid were moving with respect to the black hole, M_{PBH} would increase by absorbing some of the nearby fluid. Nonetheless, even if the relative speed between the fluid and the black hole approached the speed of light, and the black hole absorbed all the fluid in its path, then M_{PBH} would only increase by a fraction $\Delta M/M_{\text{PBH}} \sim \mathcal{O}(1)\mathcal{R}T/M_{\text{pl}}$ over an entire Hubble time, where \mathcal{R} is the ratio of the black hole radius to the Debye screening length and T the temperature of the plasma. As in the previous section, we are interested in early times at which the plasma temperature could have been as high as $T \sim 10^7 \text{ GeV}$, in which case $T/M_{\text{pl}} \sim 10^{-11}$. We consider black holes for which the ratio \mathcal{R} is not exponentially larger than one, so that the PBH forms with some small residual color charge, and therefore $\Delta M/M$ remains negligible. A more formal

calculation of the Bondi accretion rate for PBHs in this scenario yields the same conclusion [81–84].

Within the regime indicated in Eq. (59), the spacetime can be described by the McVittie line element, which reduces to an ordinary Schwarzschild spacetime near the origin and asymptotes to a spatially flat FLRW spacetime at large distances [85–87]. A convenient parametrization may be written [87]

$$ds^2 = -f(t, r)dt^2 + \left[\frac{a(t)\{dr + H(t)rdt\}}{\sqrt{f(t, r)}} - H(t)a(t)rdt \right]^2 + a^2(t)r^2 d\Omega_{(2)}^2, \quad (62)$$

with

$$f(t, r) \equiv 1 - \frac{r_s}{a(t)r} \quad (63)$$

and $r_s \equiv 2GM_{\text{PBH}}$ the usual Schwarzschild radius.

We found in Sec. VB that in cosmological scenarios of interest, $H \ll m_D$, even at very early times. For the remainder of this section, we will therefore set $H \sim 0$, for which $a(t) \sim \text{constant}$ (which we will scale to 1). In that limit, the lapse function reduces to $N(x) \rightarrow \sqrt{f(r)}$, with $f(r) = 1 - r_s/r$, and the shift vector vanishes, $\beta^i \rightarrow 0$.

Next we consider an appropriate range for the enclosed charge Q_0 , subject to the constraint in Eq. (59). The shortest scales that can be resolved within our effective field theory (EFT) are set by $\lambda_D = 1/m_D$, so we restrict $r_s \geq \lambda_D$, which in turn requires $M_{\text{PBH}} \geq 1/(2Gm_D)$. Then Eq. (59) corresponds to the upper bound

$$Q_0 \ll \frac{1}{\sqrt{2}\alpha_s} \frac{M_{\text{pl}}}{T} \sim 10^{12} \quad (64)$$

around $t_c \sim 10^{-21} \text{ s}$. Meanwhile, as discussed around Eq. (51), if $Q_0 \gg 1/g_s$, then the soft modes A_μ^a will assume a quasi-Abelian form, with $|A_0^a| \gg |A_i^a|$, for $r < r_{\text{abel}}$. The estimate of r_{abel} in Eq. (51) holds in homogeneous spacetimes, for which the temperature has no spatial gradients. In the present case, given Eq. (44), $m_D(x) \sim g_s T_0 / \sqrt{f(r)}$. Following the same steps that led to Eq. (51), we find

$$\frac{r_{\text{abel}}}{r_s} = \frac{1}{2} \left[1 + \sqrt{1 + \left(\frac{2\tilde{\lambda}_D}{r_s} \ln[g_s Q_0] \right)^2} \right], \quad (65)$$

where $\tilde{\lambda}_D$ is the Debye length associated with the temperature T_0 . For $r_s/\tilde{\lambda}_D = 3.5$ and $Q_0 \sim 10^4$, this yields $r_{\text{abel}} \sim 3r_s$, while $Q_0 \sim 10^{10}$ yields $r_{\text{abel}} \sim 7r_s$.

The last departure to consider from the previous examples concerns the effect of the local temperature $T(x)$ on the coupling α_s . Since the local temperature $T(r)$ increases as r approaches r_s , the local QCD strength runs toward zero as

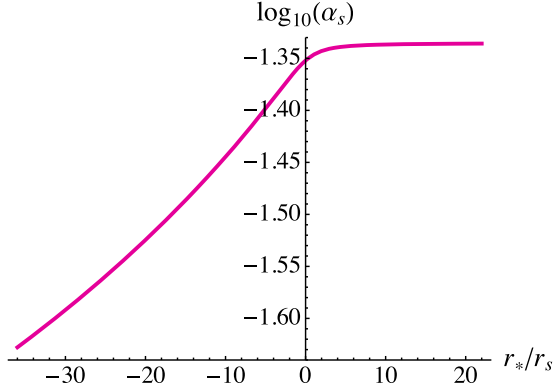


FIG. 1. The QCD coupling strength $\alpha_s \equiv g_s^2/(4\pi)$ runs to lower values near the event horizon of a black hole, as the temperature of the surrounding plasma increases. Here r_* is the tortoise radial coordinate, $r_* \equiv r + r_s \ln[(r/r_s) - 1]$; $r_*/r_s = -36$ corresponds to $r/r_s = 1 + 10^{-16}$. We have set $T_0 = 10^7$ GeV on the asymptotic boundary.

one approaches the black hole event horizon: an example of asymptotic freedom within an inhomogeneous spacetime. To quantify the effect, we use Eq. (56) for the running of α_s with energy scale $\mu \rightarrow T(r)$. To resolve the behavior of α_s near r_s , we adopt the “tortoise” radial coordinate, $r_* \equiv r + r_s \ln[(r/r_s) - 1]$, for which $r = r_s$ corresponds to $r_* \rightarrow -\infty$ [59]. See Fig. 1.

In the vicinity of a primordial black hole, the evolution of the soft modes $A_\mu^a(x)$ is therefore characterized by a hierarchy of length scales:

$$\lambda_D < r_s < r_{\text{abel}} \ll r_H. \quad (66)$$

For $r_s \leq r \leq r_{\text{abel}}$, the soft modes will evolve as a quasi-static quasi-Abelian system within a fixed background spacetime. In that regime, $A_\mu^a(x) \simeq A_0^a(r)\delta_\mu^0 + \mathcal{O}(r/r_{\text{abel}})$, and the equations of motion in Eq. (5) reduce to

$$\partial_r^2 A_0^a(r) + \frac{2}{r} \partial_r A_0^a(r) - \frac{\tilde{m}_D^2}{f^2(r)} A_0^a(r) = 0, \quad (67)$$

where $\tilde{m}_D = 1/\tilde{\lambda}_D$ is the Debye mass associated with the temperature T_0 . Including the running of g_s with r , we may solve Eq. (67) numerically, with a typical example shown in Fig. 2. The component $A_0^a(r)$ undergoes strong screening for $r \gtrsim r_s$, since more plasma gathers near the event horizon, yielding a higher density and temperature than at locations $r \gg r_s$. Far from the black hole, $A_0^a(r)$ asymptotes to similar behavior as found in Secs. VA and VB, with $A_0^a \sim \exp[-m_D r]/r$. Note that an observer at $r \gtrsim r_s$ would measure an effective charge $Q(r)$ much smaller than the charge Q_0 contained within the black hole.

Within our coordinate system, the Tolman temperature gradient drives $T(r) \rightarrow \infty$ as $r \rightarrow r_s$. The gradient is physical, but the divergence is an artifact of our

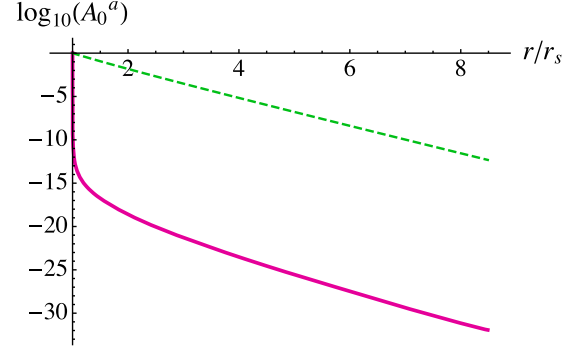


FIG. 2. The soft mode $A_0^a(r)$ (solid line) for $r_s/\tilde{\lambda}_D = 3.5$ and $T_0 = 10^7$ GeV. Given the greater density of plasma near the black hole, Debye screening is particularly effective for $r \gtrsim r_s$. For $r \gg r_s$, the behavior of $A_0^a(r)$ asymptotes to the expected slope, $\exp[-m_D r]/r$ (dashed line).

fixed-background approximation; we have not allowed the spacetime to backreact. To produce Fig. 2, we followed the example of Refs. [88–90] and evaluated the field with a boundary condition at a “stretched horizon,” $r_s + \epsilon$, rather than at r_s , with $\epsilon/r_s = 10^{-6}$. In forthcoming work [76], we address this limitation by solving the coupled Einstein field equations and equation of motion for $A_0^a(x)$ within the quasi-Abelian regime.

VI. DISCUSSION

We have generalized the description of Debye screening of charges in hot plasmas to curved spacetime backgrounds. For fluids undergoing normal flow in approximately static spacetimes, we found that the characteristic screening length $\lambda_D(x) = 1/m_D(x)$ is set by a Debye mass $m_D(x) \sim g_s T(x)$ given in Eq. (45), where g_s is the dimensionless gauge coupling. This Debye mass is the natural generalization of the Minkowski result, upon allowing the temperature to have spatial gradients due to gravitational redshift, thereby reproducing Tolman’s classic result [28–30].

To characterize Debye screening, we constructed an effective theory for long-wavelength excitations in a hot Yang-Mills plasma. We analyzed the dynamics of high-momentum particles in the plasma (with momentum $k_{\text{hard}} \sim T$) by considering classical transport equations and exploited the structure of Hamiltonian mechanics to show that non-Abelian self-interactions induce an effective local mass $m_D(x)$ for the A_0^a components of the soft modes (with momentum $k_{\text{soft}} \sim g_s T$).

We applied our results to solve for the gauge potential A_μ^a in a few cases of interest. We recovered the well-known Wu-Yang monopole solution in the Minkowski limit and generalized it to FLRW spacetimes, demonstrating the self-consistency of the quasistatic approximation in regimes of interest, for which cosmological timescales may be neglected compared to $1/k_{\text{soft}}$.

Lastly, we analyzed Debye screening in the vicinity of a primordial black hole immersed in a hot quark-gluon plasma. Such black holes can form by absorbing regions of the plasma with nonvanishing net color charge, with characteristic size set by λ_D ; hence the resulting primordial black holes can have a residual net color charge. Building on the examples involving homogeneous spacetimes, we identified a regime in which the soft modes of the gauge field outside the black hole exhibit quasi-Abelian behavior, $A_\mu^a(x) \approx A_0^a(r)\delta_\mu^0$. Given the Tolman temperature gradients, the interaction strength α_s runs to smaller values near the event horizon. Incorporating this unusual example of asymptotic freedom, we solved numerically for the soft-mode gauge potential $A_0^a(r)$, and found enhanced screening of the charge enclosed within the black hole, due to an increased density of the plasma near the event horizon.

Our examples have been restricted so far to fixed background spacetimes. Future work will focus on exploiting our EFT to study realistic cosmological scenarios involving primordial black holes. This will require solving

the coupled Einstein-Yang-Mills equations for a black hole in a hot non-Abelian plasma [76]. This formalism can also be used to consider primordial black holes with substantial QCD color charge, which could have formed at very early times in cosmic history, well before the QCD confinement transition [75].

ACKNOWLEDGMENTS

It is a pleasure to thank Chris Akers, Peter Arnold, Thomas Baumgarte, Edmund Bertschinger, Jolyon Bloomfield, Daniel Harlow, Scott Hughes, Edmond Iancu, Mikhail Ivanov, Patrick Jefferson, Jamie Kartheim, Hong Liu, Cristina Manuel, Jérôme Martin, Govert Nijs, Krishna Rajagopal, Phiala Shanahan, Vincent Vennin, and Xiaojun Yao for helpful discussions. Portions of this work were conducted in MIT's Center for Theoretical Physics and supported in part by the U.S. Department of Energy under Contract No. DE-SC0012567. E. A.-M. is also supported by a fellowship from the MIT Department of Physics.

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- [1] Jean-Paul Blaizot and Edmond Iancu, The quark gluon plasma: Collective dynamics and hard thermal loops, *Phys. Rep.* **359**, 355 (2002).
 - [2] Daniel F. Litim and Cristina Manuel, Semiclassical transport theory for nonAbelian plasmas, *Phys. Rep.* **364**, 451 (2002).
 - [3] J. I. Kapusta and Charles Gale, *Finite-Temperature Field Theory: Principles and Applications* (Cambridge University Press, Cambridge, UK, 2006).
 - [4] Alexei Bazavov and Johannes Heinrich Weber, Color screening in quantum chromodynamics, *Prog. Part. Nucl. Phys.* **116**, 103823 (2021).
 - [5] Sergei G. Matinyan, G. K. Savvidy, and N. G. Ter-Arutunian Savvidy, Classical Yang-Mills mechanics: Nonlinear color oscillations, *Sov. Phys. JETP* **53**, 421 (1981).
 - [6] V. V. Klimov, Spectrum of elementary Fermi excitations in quark gluon plasma. (In Russian). *Sov. J. Nucl. Phys.* **33**, 934 (1981).
 - [7] V. V. Klimov, Collective excitations in a hot quark gluon plasma, *Sov. Phys. JETP* **55**, 199 (1982).
 - [8] H. Arthur Weldon, Covariant calculations at finite temperature: The relativistic plasma, *Phys. Rev. D* **26**, 1394 (1982).
 - [9] Eric Braaten and Robert D. Pisarski, Resummation and gauge invariance of the gluon damping rate in hot QCD, *Phys. Rev. Lett.* **64**, 1338 (1990).
 - [10] Eric Braaten and Robert D. Pisarski, Soft amplitudes in hot gauge theories: A general analysis, *Nucl. Phys. B* **337**, 569 (1990).
 - [11] Eric Braaten and Robert D. Pisarski, Simple effective Lagrangian for hard thermal loops, *Phys. Rev. D* **45**, R1827 (1992).
 - [12] Jean Paul Blaizot and Edmond Iancu, Soft collective excitations in hot gauge theories, *Nucl. Phys. B* **417**, 608 (1994).
 - [13] Jean Paul Blaizot and Edmond Iancu, Kinetic equations for long wavelength excitations of the quark—gluon plasma, *Phys. Rev. Lett.* **70**, 3376 (1993).
 - [14] Jean-Paul Blaizot and Edmond Iancu, NonAbelian excitations of the quark—gluon plasma, *Phys. Rev. Lett.* **72**, 3317 (1994).
 - [15] Jean-Paul Blaizot and Edmond Iancu, NonAbelian plane waves in the quark—gluon plasma, *Phys. Lett. B* **326**, 138 (1994).
 - [16] Jean-Paul Blaizot, Edmond Iancu, and Rajesh R. Parwani, On the screening of static electromagnetic fields in hot QED plasmas, *Phys. Rev. D* **52**, 2543 (1995).
 - [17] Stanislaw Mrowczynski, Bjoern Schenke, and Michael Strickland, Color instabilities in the quark—gluon plasma, *Phys. Rep.* **682**, 1 (2017).
 - [18] Cristina Manuel and Stanislaw Mrowczynski, Local equilibrium of the quark gluon plasma, *Phys. Rev. D* **68**, 094010 (2003).
 - [19] Cristina Manuel and Stanislaw Mrowczynski, Whitening of the quark gluon plasma, *Phys. Rev. D* **70**, 094019 (2004).
 - [20] Mustafa A. Amin, Mark P. Hertzberg, David I. Kaiser, and Johanna Karouby, Nonperturbative dynamics of reheating after inflation: A review, *Int. J. Mod. Phys. D* **24**, 1530003 (2014).
 - [21] Rouzbeh Allahverdi *et al.*, The first three seconds: A review of possible expansion histories of the early universe, *Open J. Astrophys.* **4** (2021), 10.21105/astro.2006.16182.
 - [22] V. Mukhanov, *Physical Foundations of Cosmology* (Cambridge University Press, New York, 2005).

- [23] D. Boyanovsky, H. J. de Vega, and D. J. Schwarz, Phase transitions in the early and the present universe, *Annu. Rev. Nucl. Part. Sci.* **56**, 441 (2006).
- [24] Bernard Carr and Florian Kühnel, Primordial black holes as dark matter: Recent developments, *Annu. Rev. Nucl. Part. Sci.* **70**, 355 (2020).
- [25] Anne M. Green and Bradley J. Kavanagh, Primordial black holes as a dark matter candidate, *J. Phys. G* **48**, 043001 (2021).
- [26] Pablo Villanueva-Domingo, Olga Mena, and Sergio Palomares-Ruiz, A brief review on primordial black holes as dark matter, *Front. Astron. Space Sci.* **8**, 87 (2021).
- [27] Albert Escrivà, Florian Kühnel, and Yuichiro Tada, Primordial black holes, [arXiv:2211.05767](https://arxiv.org/abs/2211.05767).
- [28] Richard C. Tolman, On the weight of heat and thermal equilibrium in general relativity, *Phys. Rev.* **35**, 904 (1930).
- [29] Richard Tolman and Paul Ehrenfest, Temperature equilibrium in a static gravitational field, *Phys. Rev.* **36**, 1791 (1930).
- [30] Jessica Santiago and Matt Visser, Tolman-like temperature gradients in stationary spacetimes, *Phys. Rev. D* **98**, 064001 (2018).
- [31] K. A. Holcomb and T. Tajima, General relativistic plasma physics in the early universe, *Phys. Rev. D* **40**, 3809 (1989).
- [32] C. P. Dettmann, N. E. Frankel, and V. Kowalenko, Plasma electrodynamics in the expanding Universe, *Phys. Rev. D* **48**, 5655 (1993).
- [33] Tomislav Prokopec and Richard P. Woodard, Vacuum polarization and photon mass in inflation, *Am. J. Phys.* **72**, 60 (2004).
- [34] Tomislav Prokopec and Ewald Puchwein, Photon mass generation during inflation: De Sitter invariant case, *J. Cosmol. Astropart. Phys.* **04** (2004) 007.
- [35] I. Y. Dodin and N. J. Fisch, On the evolution of linear waves in cosmological plasmas, *Phys. Rev. D* **82**, 044044 (2010).
- [36] Fedor K. Popov, Debye mass in de Sitter space, *J. High Energy Phys.* **06** (2018) 033.
- [37] Enrico Cannizzaro, Andrea Caputo, Laura Sberna, and Paolo Pani, Plasma-photon interaction in curved spacetime I: Formalism and quasibound states around nonspinning black holes, *Phys. Rev. D* **103**, 124018 (2021).
- [38] Enrico Cannizzaro, Andrea Caputo, Laura Sberna, and Paolo Pani, Plasma-photon interaction in curved spacetime. II. Collisions, thermal corrections, and superradiant instabilities, *Phys. Rev. D* **104**, 104048 (2021).
- [39] Justin C. Feng, Sumanta Chakraborty, and Vitor Cardoso, Shielding a charged black hole, *Phys. Rev. D* **107**, 044050 (2023).
- [40] Miklos Gyulassy and Larry McLerran, New forms of QCD matter discovered at RHIC, *Nucl. Phys. A* **750**, 30 (2005).
- [41] F. Scardina, M. Colonna, S. Plumari, and V. Greco, Quark-to-gluon composition of the quark-gluon plasma in relativistic heavy-ion collisions, *Phys. Lett. B* **724**, 296 (2013).
- [42] Sz. Borsanyi, G. Endrodi, Z. Fodor, S. D. Katz, and K. K. Szabo, Precision SU(3) lattice thermodynamics for a large temperature range, *J. High Energy Phys.* **07** (2012) 056.
- [43] Szabolcs Borsanyi, Zoltan Fodor, Christian Hoelbling, Sandor D. Katz, Stefan Krieg, and Kalman K. Szabo, Full result for the QCD equation of state with $2 + 1$ flavors, *Phys. Lett. B* **730**, 99 (2014).
- [44] A. Bazavov *et al.* (HotQCD Collaboration), Equation of state in $(2 + 1)$ -flavor QCD, *Phys. Rev. D* **90**, 094503 (2014).
- [45] Roman Pasechnik and Michal Šumbera, Phenomenological review on quark–gluon plasma: Concepts vs. observations, *Universe* **3**, 7 (2017).
- [46] Robert D. Pisarski, Scattering amplitudes in hot gauge theories, *Phys. Rev. Lett.* **63**, 1129 (1989).
- [47] Eric Braaten and Robert D. Pisarski, Deducing hard thermal loops from Ward identities, *Nucl. Phys. B* **339**, 310 (1990).
- [48] Eric Braaten and Robert D. Pisarski, Calculation of the gluon damping rate in hot QCD, *Phys. Rev. D* **42**, 2156 (1990).
- [49] P. F. Kelly, Q. Liu, C. Lucchesi, and C. Manuel, Deriving the hard thermal loops of QCD from classical transport theory, *Phys. Rev. Lett.* **72**, 3461 (1994).
- [50] P. F. Kelly, Q. Liu, C. Lucchesi, and C. Manuel, Classical transport theory and hard thermal loops in the quark–gluon plasma, *Phys. Rev. D* **50**, 4209 (1994).
- [51] Robert D. Pisarski, NonAbelian Debye screening, tsunami waves, and worldline fermions, *NATO Sci. Ser., C* **511**, 195 (1998).
- [52] Fernando T. Brandt, J. Frenkel, and J. C. Taylor, High temperature QCD and the classical Boltzmann equation in curved space-time, *Nucl. Phys. B* **437**, 433 (1995).
- [53] S. K. Wong, Field and particle equations for the classical Yang-Mills field and particles with isotopic spin, *Nuovo Cimento A* **65**, 689 (1970).
- [54] N. Linden, A. J. Macfarlane, and J. W. van Holten, Particle motion in a Yang-Mills field: Wong’s equations and spin $1/2$ analogs, *Czech. J. Phys.* **46**, 209 (1996).
- [55] A. P. Balachandran, Per Salomonson, Bo-Sture Skagerstam, and Jan-Olof Winnberg, Classical description of particle interacting with non-Abelian gauge field, *Phys. Rev. D* **15**, 2308 (1977).
- [56] A. Barducci, R. Casalbuoni, and L. Lusanna, Classical scalar and spinning particles interacting with external Yang-Mills fields, *Nucl. Phys. B* **124**, 93 (1977).
- [57] V. I. Arnold, *Mathematical Methods of Classical Mechanics* (Springer, New York, 1978).
- [58] Naoki Sato, The effect of spacetime curvature on statistical distributions, *Classical Quantum Gravity* **38**, 165003 (2021).
- [59] Robert M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984).
- [60] Shigeo Sasaki, On the differential geometry of tangent bundles of Riemannian manifolds, *Tohoku Math. J.* **10**, 338 (1958).
- [61] Werner Israel, Covariant fluid mechanics and thermodynamics: An introduction, in *Relativistic Fluid Dynamics*, edited by A. M. Anile and Y. Choquet-Bruhat (Springer, Berlin, 1989), pp. 152–210.
- [62] Rubén O. Acuña Cárdenas, Carlos Gabarrete, and Olivier Sarbach, An introduction to the relativistic kinetic theory on curved spacetimes, *Gen. Relativ. Gravit.* **54**, 23 (2022).
- [63] A. K. Rebhan, The Non-Abelian Debye mass at next-to-leading order, *Phys. Rev. D* **48**, R3967 (1993).
- [64] Anton K. Rebhan, Non-Abelian Debye screening in one loop resummed perturbation theory, *Nucl. Phys. B* **430**, 319 (1994).

- [65] Eric Braaten and Agustin Nieto, Free energy of QCD at high temperature, *Phys. Rev. D* **53**, 3421 (1996).
- [66] D. Boyanovsky, H. J. de Vega, and M. Simionato, Non-equilibrium quantum plasmas in scalar QED: Photon production, magnetic and Debye masses and conductivity, *Phys. Rev. D* **61**, 085007 (2000).
- [67] Tai Tsun Wu and Chen Ning Yang, Dirac's monopole without strings: Classical Lagrangian theory, *Phys. Rev. D* **14**, 437 (1976).
- [68] Tai Tsun Wu and Chen Ning Yang, Dirac monopole without strings: Monopole harmonics, *Nucl. Phys. B* **107**, 365 (1976).
- [69] R. Jackiw, Q. Liu, and C. Lucchesi, Hard thermal loops, static response and the composite effective action, *Phys. Rev. D* **49**, 6787 (1994).
- [70] A. Sinha, SU(3) magnetic monopoles, *Phys. Rev. D* **14**, 2016 (1976).
- [71] John Preskill, Magnetic monopoles, *Annu. Rev. Nucl. Part. Sci.* **34**, 461 (1984).
- [72] J. Kunz and D. Masak, Finite energy SU(3) monopoles, *Phys. Lett. B* **196**, 513 (1987).
- [73] Mikhail S. Volkov and Dmitri V. Gal'tsov, Gravitating nonAbelian solitons and black holes with Yang-Mills fields, *Phys. Rep.* **319**, 1 (1999).
- [74] Mikhail S. Volkov, Hairy black holes in the XX-th and XXI-st centuries, in *The Fourteenth Marcel Grossmann Meeting* (World Scientific, Singapore, 2017), Vol. 2, pp. 1779.
- [75] Elba Alonso-Monsalve and David I. Kaiser, Primordial black holes with QCD color charge, [arXiv:2310.16877](https://arxiv.org/abs/2310.16877).
- [76] Elba Alonso Monsalve and David I. Kaiser (to be published).
- [77] Bernard J. Carr, The primordial black hole mass spectrum, *Astrophys. J.* **201**, 1 (1975).
- [78] Jens C. Niemeyer and K. Jedamzik, Near-critical gravitational collapse and the initial mass function of primordial black holes, *Phys. Rev. Lett.* **80**, 5481 (1998).
- [79] Anne M. Green and Andrew R. Liddle, Critical collapse and the primordial black hole initial mass function, *Phys. Rev. D* **60**, 063509 (1999).
- [80] Florian Kühnel, Cornelius Rampf, and Marit Sandstad, Effects of critical collapse on primordial black-hole mass spectra, *Eur. Phys. J. C* **76**, 93 (2016).
- [81] Stuart L. Shapiro and Saul A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars: The Physics of Compact Objects* (Wiley, New York, 1983).
- [82] Jared R. Rice and Bing Zhang, Cosmological evolution of primordial black holes, *J. High Energy Astrophys.* **13–14**, 22 (2017).
- [83] V. De Luca, G. Franciolini, P. Pani, and A. Riotto, Constraints on primordial black holes: The importance of accretion, *Phys. Rev. D* **102**, 043505 (2020).
- [84] Chloe B. Richards, Thomas W. Baumgarte, and Stuart L. Shapiro, Relativistic Bondi accretion for stiff equations of state, *Mon. Not. R. Astron. Soc.* **502**, 3003 (2021); *Mon. Not. R. Astron. Soc.* **506**, 3935(E) (2021).
- [85] Nemanja Kaloper, Matthew Kleban, and Damien Martin, McVittie's Legacy: Black holes in an expanding universe, *Phys. Rev. D* **81**, 104044 (2010).
- [86] Valerio Faraoni, Andres F. Zambrano Moreno, and Angus Prain, The charged McVittie spacetime, *Phys. Rev. D* **89**, 103514 (2014).
- [87] Rudeep Gaur and Matt Visser, Cosmology in Painlevé-Gullstrand coordinates, *J. Cosmol. Astropart. Phys.* **09** (2022) 030.
- [88] K. S. Thorne and D. MacDonald, Electrodynamics in curved spacetime—3 + 1 formulation, *Mon. Not. R. Astron. Soc.* **198**, 339 (1982).
- [89] Kip S. Thorne, Richard H. Price, and Douglas A. MacDonald, *Black Holes: The Membrane Paradigm* (Yale University Press, New Haven, CT, 1986).
- [90] Douglas A. Macdonald and Wai-Mo Suen, Membrane viewpoint on black holes: Dynamical electromagnetic fields near the horizon, *Phys. Rev. D* **32**, 848 (1985).