Causality of photon propagation under dominant energy condition in nonlinear electrodynamics

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Recently, various types of the regular black hole model are reintroduced as the solution of the Einstein equations coupled with nonlinear electrodynamics (NED). In NED, it is known that photons do not propagate along the null geodesics of the spacetime geometry, but of so-called effective geometry, which suggests the possibility of so-called "faster/slower than light" photons. We study the relation between the causality of photons and the dominant energy condition (DEC) in some static and spherically symmetric black hole spacetimes in NED. We show that if photon trajectories with a nonzero angular momentum are timelike in the spacetime geometry, DEC is always satisfied in static and spherically symmetric spacetimes in any NED that admits the Maxwell limit, and vice versa, at least, in the weak field limit. Thus, this implies that in such NED, the violation of DEC admits the existence of faster than light photons.

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I. INTRODUCTION

It is well-known that in nonlinear electrodynamics (NED), photons do not propagate along null geodesics of the spacetime geometry, but rather of another geometry which is called an effective geometry [1–3]. In such a theory, even if photons move in Minkowski spacetime, they move while feeling a "virtual" curved spacetime, whereas other massless particles move on the light cone in Minkowski spacetime. For instance, this occurs in the Euler-Heisenberg effective theory derived by the one-loop quantum correction in QED [4–10] and in the Born-Infeld electrodynamics [11,12]. Therefore, there is a possibility of faster-than-light photons and slower-than-light photons, namely, the light cone of the effective geometry lies outside the light cone of the spacetime geometry, and vice versa. Moreover, It is remarkable that in general, the velocities of photons in NED are doubled, which occurs in the former theory but does not in the latter [12].

Regular black holes (RBH), which have no singularity inside/outside an event horizon, have been studied as one of the candidates of quantum black holes since Bardeen [13] proposed the first model of asymptotically flat, static, and spherically symmetric black holes with a regular center. Subsequently, other RBHs with the same symmetry and asymptotic structure were proposed by many researchers. Remarkably, some of these are exact solutions to the Einstein equation coupled with a physical source of a magnetic monopole in NED. Recently, Fan and Wang [14] found a wide class of asymptotically flat, static, and spherically symmetric RBH solution in a certain NED, which can considered to be the generalization of the Bardeen BH [13] and the Hayward BH [15].

The propagation of photons has been studied for the Ayón-Beato-García spacetime in Ref. [2], for the Bardeen spacetime in Ref. [16] and for the Hayward spacetime in Ref. [17]. The photon orbits are also studied in rotating versions of several RBHs of NED [18]. Moreover, in Ref. [19], we discuss photons moving around regular black holes of Fan and Wang and find an unstable circular orbit of photons inside the event horizon. The purpose of this paper is to study the relation between the energy conditions and the causality of photon propagation around regular black holes in such NEDs and see whether the light cone of the effective geometry lies outside/inside the light cone of the spacetime geometry under the dominant energy condition (DEC). Furthermore, we show that in the Born-Infeld theory, where DEC is satisfied everywhere, photons with a nonzero angular momentum are always timelike for a purely magnetic/electric field. Moreover, generalizing these to arbitrary NED with the Maxwell limit, we show that under DEC, photon trajectories in static and spherically symmetric spacetimes cannot be spacelike, at least, in the weak field. In addition, we show that in any NED with the Maxwell limit if photon trajectories with a nonzero angular momentum are timelike in the spacetime geometry and null in the corresponding effective geometry, DEC is always satisfied in static and spherically symmetric spacetimes, which means that the violation of DEC leads to the existence of spacelike photons, i.e., faster than light photons.

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In the following section, we give the brief review on NED. In Sec. III, we discuss what conditions the Lagrangian density of NED should satisfy in order that four energy conditions, null energy condition (NEC), weak energy condition (WEC), DEC, and strong energy condition (SEC) are satisfied in a purely magnetic and a purely electric cases. In Sec. IV, we briefly review the known results on the effective geometry which photons in NED feel during propagating in the spacetime. In Sec. V, we first consider photons moving around the regular black hole of Fan and Wang, which can be regarded as a solution in a certain NED. We also consider Einstein-Born-Infeld theory and discuss the relation between the energy conditions and the speed of photons. In Sec. VI, we discuss the general cases with the Maxwell limit. In Sec. VII, we summarize our results and discuss possible generalization.

II. BRIEF REVIEW

Let us consider the Lagrangian density for Einstein gravity coupled with one parameter NED, which is given by

$$L = R - \mathcal{L}(\mathcal{F}), \tag{1}$$

where $\mathcal{L}$ is an arbitrary function of $\mathcal{F} \coloneqq F_{\mu\nu}F^{\mu\nu}$ with the field strength of the vector field A_μ, i.e., $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. From the action (1), the Einstein equations and the field equations for NED can be written as, respectively,

$$G^\mu{}_\nu = 2T^\mu{}_\nu, \tag{2}$$

$$\nabla_\mu(\mathcal{L}_\mathcal{F} F^{\mu\nu}) = 0, \tag{3}$$

where the energy momentum tensor for NED is given by

$$T^\mu{}_\nu = \mathcal{L}_\mathcal{F} F^{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta^\mu{}_\nu \mathcal{L}. \tag{4}$$

The static and spherically symmetric solution with a purely magnetic field is written as

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \tag{5}$$

$$f(r) = 1 - \frac{2m(r)}{r}, \tag{6}$$

$$A_\mu dx^\mu = Q_m \cos\theta d\phi, \qquad \mathcal{F} = \frac{2Q_m^2}{r^4}, \tag{7}$$

where the magnetic charge Q_m is defined by

$$Q_m \coloneqq \frac{1}{4\pi}\int_S F, \tag{8}$$

and Eq. (4) can be written as

$$(T^\mu{}_\nu) = \frac{1}{2}\mathrm{diag}\left(-\frac{1}{2}\mathcal{L}, -\frac{1}{2}\mathcal{L}, \mathcal{F}\mathcal{L}_\mathcal{F} - \frac{1}{2}\mathcal{L}, \mathcal{F}\mathcal{L}_\mathcal{F} - \frac{1}{2}\mathcal{L}\right). \tag{9}$$

From Eq. (2), the (t,t), (r,r) components and (θ,θ), (ϕ,ϕ) components are written as, respectively,

$$\mathcal{L}(\mathcal{F}(r)) = \frac{4m'}{r^2}, \tag{10}$$

$$\mathcal{L}_\mathcal{F}(\mathcal{F}(r)) = -\frac{2rm'' - 4m'}{\mathcal{F}^2 r^6} = -\frac{r^2(rm'' - 2m')}{2Q_m^4}. \tag{11}$$

Moreover, the static and spherically symmetric solution with a purely electric field is written as

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \tag{12}$$

$$f(r) = 1 - \frac{2m(r)}{r}, \tag{13}$$

$$A_\mu dx^\mu = a(r)dt, \tag{14}$$

where the function $a(r)$ cannot be written explicitly since it is not easy to solve the field equation in the electric case as in the magnetic case. The electric charge Q_e is defined by

$$Q_e \coloneqq \frac{1}{4\pi}\int_S \mathcal{L}_\mathcal{F} * F, \tag{15}$$

and Eq. (4) can be written as

$$(T^\mu{}_\nu) = \frac{1}{2}\mathrm{diag}\left(\mathcal{L}_\mathcal{F}\mathcal{F} - \frac{1}{2}\mathcal{L}, \mathcal{L}_\mathcal{F}\mathcal{F} - \frac{1}{2}\mathcal{L}, -\frac{1}{2}\mathcal{L}, -\frac{1}{2}\mathcal{L}\right). \tag{16}$$

III. ENERGY CONDITIONS IN NED

In this section, we consider the conditions which the Lagrangian $\mathcal{L}(\mathcal{F})$ should satisfy in order that four energy conditions, NEC, WEC, DEC, and SEC are satisfied in a purely magnetic case ($\mathcal{F} > 0$) and a purely electric case ($\mathcal{F} < 0$) with the same spacetime geometry.

A. Purely magnetic case

Applying, for instance, the discussion with the effective energy-momentum tensor for type I matter fields in ref. [20] to Eq. (9), we can write four energy conditions as follows:

(i) NEC, "$T_{\mu\nu}k^\mu k^\nu \geq 0$ for any null vectors k^μ", is equivalent with

$$\mathcal{F}\mathcal{L}_\mathcal{F} \geq 0. \tag{17}$$

(ii) WEC, "$T_{\mu\nu}v^\mu v^\nu \geq 0$ for any timelike vectors v^μ", is equivalent with

$$\mathcal{L} \geq 0 \quad \text{and} \quad \mathcal{F}\mathcal{L}_{\mathcal{F}} \geq 0. \tag{18}$$

(iii) DEC, "$T_{\mu\nu}v^\mu v^\nu \geq 0$ and $J_\mu J^\mu \leq 0$ for any timelike vectors v^μ and the current $J^\mu \coloneqq -T^\mu{}_\nu v^\nu$", is equivalent with

$$\mathcal{L} \geq 0, \qquad \mathcal{F}\mathcal{L}_{\mathcal{F}} \geq 0 \quad \text{and} \quad \mathcal{L} - \mathcal{F}\mathcal{L}_{\mathcal{F}} \geq 0. \tag{19}$$

(iv) SEC, "$(T_{\mu\nu} - \frac{1}{2}T^\lambda{}_\lambda g_{\mu\nu})v^\mu v^\nu \geq 0$ for any timelike vectors v^μ" is equivalent with

$$\mathcal{L} \geq 0, \qquad \mathcal{F}\mathcal{L}_{\mathcal{F}} \geq 0 \quad \text{and} \quad 2\mathcal{F}\mathcal{L}_{\mathcal{F}} - \mathcal{L} \geq 0. \tag{20}$$

B. Purely electric case

Similarly, the energy-momentum tensor (16) leads to the following:

(i) NEC, "$T_{\mu\nu}k^\mu k^\nu \geq 0$ for any null vectors k^μ", is equivalent with

$$\mathcal{F}\mathcal{L}_{\mathcal{F}} \leq 0. \tag{21}$$

(ii) WEC, "$T_{\mu\nu}v^\mu v^\nu \geq 0$ for any timelike vectors v^μ", is equivalent with

$$\mathcal{F}\mathcal{L}_{\mathcal{F}} \leq 0 \quad \text{and} \quad 2\mathcal{L}_{\mathcal{F}}\mathcal{F} - \mathcal{L} \leq 0. \tag{22}$$

(iii) DEC, "$T_{\mu\nu}v^\mu v^\nu \geq 0$ and $J_\mu J^\mu \leq 0$ for any timelike vectors v^μ and the current $J^\mu \coloneqq -T^\mu{}_\nu v^\nu$", is equivalent with

$$\mathcal{F}\mathcal{L}_{\mathcal{F}} \leq 0, \quad \text{and} \quad \mathcal{L}_{\mathcal{F}}\mathcal{F} - \mathcal{L} \leq 0. \tag{23}$$

(iv) SEC, "$(T_{\mu\nu} - \frac{1}{2}T^\lambda{}_\lambda g_{\mu\nu})v^\mu v^\nu \geq 0$ for any timelike vectors v^μ", is equivalent with

$$\mathcal{F}\mathcal{L}_{\mathcal{F}} \leq 0 \quad \text{and} \quad \mathcal{L} \leq 0. \tag{24}$$

IV. EFFECTIVE GEOMETRY AND TIMELIKE PHOTONS

As mentioned previously, in NED given by the Lagrangian $\mathcal{L}(\mathcal{F})$, photons do not propagate along null geodesics in the spacetime geometry, but rather in the corresponding effective geometry [1]. In Ref. [1], by using the Hadamard method, the propagation of low-energy photons can be described by the evolution of the wave front, i.e., characteristic surface $S = \text{const}$, across which the electromagnetic field is continuous but the first derivative is not. In Ref. [16], the alternative method, the eikonal approximation for photons, was used. Under the short-wave approximation in NED, where the Faraday tensor can be regarded as local plane waves,

$$F_{\mu\nu} = \left(F^{(0)}_{\mu\nu} + \frac{\varepsilon}{i}F^{(1)}_{\mu\nu} + \mathcal{O}(\epsilon^2) + \cdots\right)e^{\frac{i}{\varepsilon}S} \qquad (\varepsilon \ll 1), \tag{25}$$

from Eq. (3) and the Bianchi equations, one can show that the gradient of the phase S, $k_\mu \coloneqq \nabla_\mu S$, must satisfy

$$\tilde{g}^{\mu\nu}k_\mu k_\nu = 0, \tag{26}$$

where $\tilde{g}_{\mu\nu}$ is the metric of the effective geometry, which is given by

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{4\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}F^\mu{}_\alpha F^{\alpha\nu}, \tag{27}$$

with $\mathcal{L}_{\mathcal{F}F} \coloneqq d^2\mathcal{L}/d\mathcal{F}^2$. For a static and spherically symmetric spacetime with a magnetic charge, the corresponding effective metric is denoted by

$$(\tilde{g}^{(m)}_{\mu\nu}) = \mathrm{diag}\left(-f, \frac{1}{f}, \frac{r^2}{\Phi^{(m)}}, \frac{r^2\sin^2\theta}{\Phi^{(m)}}\right), \tag{28}$$

where $\Phi^{(m)} \coloneqq 1 + 2\mathcal{L}_{\mathcal{F}\mathcal{F}}\mathcal{F}/\mathcal{L}_{\mathcal{F}}|_{\text{magnetic}}$. On the other hand, in the electric case, the effective metric is given by

$$(\tilde{g}^{(e)}_{\mu\nu}) = \mathrm{diag}\left(-\frac{f}{\Phi^{(e)}}, \frac{1}{f\Phi^{(e)}}, r^2, r^2\sin^2\theta\right), \tag{29}$$

where $\Phi^{(e)} \coloneqq 1 + 2\mathcal{L}_{\mathcal{F}\mathcal{F}}\mathcal{F}/\mathcal{L}_{\mathcal{F}}|_{\text{electric}}$. As shown in Ref. [21], from the duality in NED for the same metric,

$$\begin{aligned} \mathcal{L}^2_{\mathcal{F}}\mathcal{F}|_{\text{electric}} &= -\mathcal{F}|_{\text{magnetic}}, \\ \mathcal{L}_{\mathcal{F}}|_{\text{magnetic}} &= (\mathcal{L}_{\mathcal{F}})^{-1}|_{\text{electric}}, \end{aligned} \tag{30}$$

one can show

$$\Phi^{(m)} = \frac{1}{\Phi^{(e)}}, \tag{31}$$

which means that two effective geometries for the electric and magnetic solutions are related by the conformal transformation,

$$\tilde{g}^{(e)}_{\mu\nu} = \Phi^{(m)}\tilde{g}^{(m)}_{\mu\nu}, \tag{32}$$

and hence, have the same causal structure. Thus, although the effective metrics for both the electrically and magnetically charged spacetimes are different, the photon trajectories coincide in both effective geometries. Therefore, it is sufficient to discuss the magnetic solution only. Let us denote $\Phi^{(m)}$ with Φ simply as

$$\tilde{g}^{\mu\nu} = \mathrm{diag}\left(-\frac{1}{f}, f, \frac{\Phi}{r^2}, \frac{\Phi}{r^2\sin^2\theta}\Phi\right), \tag{33}$$

where

$$\Phi \coloneqq 1 + \frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}. \tag{34}$$

If k_μ satisfies $\tilde{g}^{\mu\nu}k_\mu k_\nu = 0$, then the norm of k_μ in the spacetime geometry is given by

$$g^{\mu\nu}k_\mu k_\nu = -\frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{r^2\mathcal{L}_{\mathcal{F}}}\left((k_\theta)^2 + \frac{(k_\phi)^2}{\sin^2\theta}\right). \tag{35}$$

Thus, under NEC and WEC, $\mathcal{F}\mathcal{L}_{\mathcal{F}} \geq 0$, and if

$$\mathcal{L}_{\mathcal{F}\mathcal{F}} < 0, \tag{36}$$

the trajectories of photons with $(k^\theta, k^\phi) \neq (0,0)$ can become timelike, namely, photons propagate on the timelike characteristic surfaces of $S = \text{const}$ because the gradient, $k^\mu = \nabla^\mu S$ is spacelike. We should note that for photons moving in the radial direction, the trajectories are also null even in the spacetime geometry.

V. EXAMPLES

We consider two examples of BHs in NED, regular Fan-Wang (FW) black holes and Eisntein-Born-Infeld BHs.

A. Regular black holes

As the first example, we consider the magnetic FW black holes [14], which are solutions to Einstein equations coupled with NED given by the Lagrangian,

$$\mathcal{L}(\mathcal{F}) = \frac{4\mu}{\alpha}\frac{(\alpha\mathcal{F})^{\frac{\nu+3}{4}}}{(1+(\alpha\mathcal{F})^{\frac{\nu}{4}})^{\frac{\mu+\nu}{\nu}}}, \tag{37}$$

where $\mu > 0$, $\nu > 0$ are dimensionless constants and $\alpha > 0$ has the parameter of the theory with the dimension of length squared. We note $\mathcal{F} > 0$ for a purely magnetic case. The Bardeen BHs [13] and Hayward BHs [15] are solutions in NED with $(\mu,\nu) = (3,2)$ and $(\mu,\nu) = (3,3)$, respectively. In terms of $x \coloneqq (\alpha\mathcal{F})^{\nu/4}$, the energy conditions are denoted by

(i) NEC $\Leftrightarrow \mathcal{F}\mathcal{L}_{\mathcal{F}} > 0 \Leftrightarrow \varepsilon_{\mathrm{NEC}} \coloneqq \nu + 3 - (\mu-3)x > 0$,
(ii) WEC $\Leftrightarrow \mathcal{F}\mathcal{L}_{\mathcal{F}} > 0,\ \mathcal{L} > 0 \Leftrightarrow \varepsilon_{\mathrm{NEC}} > 0$,
(iii) DEC $\Leftrightarrow \mathcal{F}\mathcal{L}_{\mathcal{F}} > 0, \quad \mathcal{L} > 0, \quad \mathcal{L} - \mathcal{F}\mathcal{L}_{\mathcal{F}} > 0 \Leftrightarrow \varepsilon_{\mathrm{NEC}} > 0,\ \varepsilon_{\mathrm{DEC}} \coloneqq 1 - \nu + (\mu+1)x > 0$,
(iv) SEC $\Leftrightarrow \mathcal{F}\mathcal{L}_{\mathcal{F}} > 0, \quad \mathcal{L} > 0, \quad 2\mathcal{F}\mathcal{L}_{\mathcal{F}} - \mathcal{L} > 0 \Leftrightarrow \varepsilon_{\mathrm{NEC}} > 0,\ \varepsilon_{\mathrm{SEC}} \coloneqq 1 + \nu - (\mu-1)x > 0$,

where we note that the Lagrangian $\mathcal{L}$ is always positive for magnetic black holes and $\varepsilon_{\mathrm{NEC}}$ is always positive for regular FW black holes with $\mu = 3$, $\nu \geq 1$, i.e., NEC and WEC are always satisfied. The condition that timelike photons exist under WEC is written as

$$g^{\mu\nu}k_\mu k_\nu > 0 \Leftrightarrow \mathcal{L}_{\mathcal{F}\mathcal{F}} < 0$$
$$\Leftrightarrow \varepsilon_\gamma \coloneqq (\mu+1)(\mu-3)x^2 - ((3\nu+2)\mu + \nu^2 - 2\nu + 6)x + (\nu+3)(\nu-1) < 0. \tag{38}$$

In what follows, we classify four specific cases (i) $\mu = 3$, $\nu = 1$, (ii) $\mu > 3$, $\nu = 1$, (iii) $\mu = 3$, $\nu > 1$, and (iv) $\mu > 3$, $\nu > 1$.

(i) $\mu = 3$, $\nu = 1$:

$$\varepsilon_{\mathrm{NEC}} = 4, \quad \varepsilon_{\mathrm{DEC}} = 4x, \quad \varepsilon_{\mathrm{SEC}} = 2 - 2x. \tag{39}$$

Thus, we can see from these that NEC, WEC, and DEC are satisfied everywhere but SEC are satisfied only for $x \geq 1$, i.e., not satisfied in the neighborhood of the regular center of black holes. On the other hand, from

$$\varepsilon_\gamma = -20x < 0, \tag{40}$$

we find

$$g^{\mu\nu}k_\mu k_\nu > 0, \tag{41}$$

which means photons are timelike.

(ii) $\mu > 3$, $\nu = 1$:

$$\varepsilon_{\mathrm{NEC}} = 4 - (\mu-3)x, \qquad \varepsilon_{\mathrm{DEC}} = (\mu+1)x,$$
$$\varepsilon_{\mathrm{SEC}} = 2 - (\mu-1)x. \tag{42}$$

From this, we can show that NEC, WEC, DEC, and SEC are all satisfied for $x < \frac{2}{\mu-1}$, NEC, WEC, DEC only are satisfied $\frac{2}{\mu-1} < x < \frac{4}{\mu-3}$, and all energy conditions are not satisfied for $x > \frac{4}{\mu-3}$. Since in this case, ε_γ is written as

$$\varepsilon_\gamma = (\mu+1)[(\mu-3)x - 5]x, \tag{43}$$

we can summarize the energy conditions and photon causality in Table I.

(iii) $\mu = 3$, $\nu > 1$:

$$\varepsilon_{\mathrm{NEC}} = \nu + 3, \qquad \varepsilon_{\mathrm{DEC}} = 1 - \nu + 4x,$$
$$\varepsilon_{\mathrm{SEC}} = 1 + \nu - 2x. \tag{44}$$

TABLE I. The energy conditions and photon causality for $\mu > 3,\ \nu = 1$. "Yes" and "No" mean that the corresponding energy condition in each region is satisfied and is not satisfied, respectively, and "±" denote the signatures of $g^{\mu\nu}k_\mu k_\nu$.

x	$0<x<\frac{2}{\mu-1}$	$\frac{2}{\mu-1}<x<\frac{4}{\mu-3}$	$\frac{4}{\mu-3}<x<\frac{5}{\mu-3}$	$\frac{5}{\mu-3}<x<\infty$
$\varepsilon_{\rm NEC}$	+	+	−	−
$\varepsilon_{\rm DEC}$	+	+	+	+
$\varepsilon_{\rm SEC}$	+	−	−	−
ε_γ or $\mathcal{L}_{\mathcal{F}\mathcal{F}}$	−	−	−	+
NEC	Yes	Yes	No	No
WEC	Yes	Yes	No	No
DEC	Yes	Yes	No	No
SEC	Yes	No	No	No
$g^{\mu\nu}k_\mu k_\nu$	+	+	−	+

TABLE II. The energy conditions and photon causality for $\mu = 3, \nu > 1$ "Yes" and "No" mean that the corresponding energy condition in each region is satisfied and is not satisfied, respectively, and "±" denote the signatures of $g^{\mu\nu}k_\mu k_\nu$.

x	$0<x<\frac{\nu-1}{\nu+4}$	$\frac{\nu-1}{\nu+4}<x<\frac{\nu-1}{4}$	$\frac{\nu-1}{4}<x<\frac{\nu+1}{2}$	$\frac{\nu+1}{2}<x<\infty$
$\varepsilon_{\rm NEC}$	+	+	+	+
$\varepsilon_{\rm DEC}$	−	−	+	+
$\varepsilon_{\rm SEC}$	+	+	+	−
ε_γ or $\mathcal{L}_{\mathcal{F}\mathcal{F}}$	+	−	−	−
NEC	Yes	Yes	Yes	Yes
WEC	Yes	Yes	Yes	Yes
DEC	No	No	Yes	Yes
SEC	Yes	Yes	Yes	No
$g^{\mu\nu}k_\mu k_\nu$	−	+	+	+

From this, both NEC and WEC are satisfied everywhere. DEC and SEC are satisfied for $\frac{\nu-1}{4} < x < \frac{\nu+1}{2}$, DEC is satisfied, but SEC is not for $x > \frac{\nu+1}{2}$, and SEC is satisfied, but DEC is not for $x < \frac{\nu-1}{4}$. For $\mu = 3$, ε_γ becomes

$$\varepsilon_\gamma = (\nu+3)[(\nu-1)-(\nu+4)x]. \tag{45}$$

Therefore, we can summarize the energy conditions and photon causality in Table II.

(iv) $\mu > 3,\ \nu > 1$:

NEC, WEC, DEC, and SEC are all satisfied for $\frac{\nu-1}{\mu+1} < x < \frac{\nu+1}{\mu-1}$. NEC, WEC, DEC are satisfied, but SEC is not for $\frac{\nu+1}{2} < x < \frac{\nu+3}{\mu-3}$, and NEC, WEC, SEC is satisfied, but DEC is not for $x < \frac{\nu-1}{\mu-3}$. In general, $\varepsilon_\gamma = 0$ can be solved as

$$x = x_\pm \coloneqq \frac{3\mu\nu + 2\mu + \nu^2 - 2\nu + 6 \pm \sqrt{(\mu+\nu)(5\mu\nu^2 + 4\mu\nu + 16\mu + \nu^3 - 4\nu^2 + 28\nu)}}{2(\mu+1)(\mu-3)}. \tag{46}$$

From $\varepsilon_\gamma(\frac{\nu-1}{\mu+1}) < 0$, $\varepsilon_\gamma(\frac{\nu+3}{\mu-3}) < 0$, we can see $0 < x_- < \frac{\nu-1}{\mu+1} < \frac{\nu+3}{\mu-3} < x_+$. Therefore we can summarize the energy conditions and photon causality in Table III.

In these cases, we can conclude that if we assume that DEC is satisfied, photon trajectories can be timelike in the spacetime geometry, though null in the effective geometry.

B. Einstein-Born-Infeld black holes

As the second example, we consider BHs in the Einstein-Born-Infeld theory [22], whose Lagrangian density of NED is given by

$$\mathcal{L}(\mathcal{F}) = -4\beta^2\left(1 - \sqrt{1 + \frac{\mathcal{F}}{2\beta^2}}\right), \tag{47}$$

which has the Maxwellian limit $\mathcal{L} \simeq \mathcal{F}$ in the weak field approximation $\mathcal{F} \simeq 0$. It is easy to show that the first and second derivatives have definite signatures,

$$\mathcal{L}_\mathcal{F} = \frac{1}{\sqrt{1+\frac{\mathcal{F}}{2\beta^2}}} > 0, \tag{48}$$

$$\mathcal{L}_{\mathcal{F}\mathcal{F}} = -\frac{1}{4\beta^2}\frac{1}{\sqrt{1+\frac{\mathcal{F}}{2\beta^2}}^3} < 0. \tag{49}$$

It is sufficient to consider the purely magnetic case $\mathcal{F} > 0$, in which all the energy conditions are satisfied because it is obvious that

(i) NEC $\Leftrightarrow \mathcal{F}\mathcal{L}_\mathcal{F} \geq 0$,
(ii) WEC $\Leftrightarrow \mathcal{F}\mathcal{L}_\mathcal{F} \geq 0,\ \mathcal{L} \geq 0$,
(iii) DEC $\Leftrightarrow \mathcal{F}\mathcal{L}_\mathcal{F} \geq 0,\ \mathcal{L} \geq 0,\ \mathcal{L} - \mathcal{F}\mathcal{L}_\mathcal{F} \geq 0$,
(iv) SEC $\Leftrightarrow \mathcal{F}\mathcal{L}_\mathcal{F} \geq 0,\ \mathcal{L} \geq 0,\ 2\mathcal{F}\mathcal{L}_\mathcal{F} - \mathcal{L} \geq 0$,

are satisfied everywhere. Therefore, from

$$g^{\mu\nu}k_\mu k_\nu = -\frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{r^2\mathcal{L}_\mathcal{F}}\left((k_\theta)^2 + \frac{(k_\phi)^2}{\sin^2\theta}\right) \geq 0, \tag{50}$$

in general, the photons propagating along null geodesics in the effective geometry move along timelike curves in the

TABLE III. The energy conditions and photon causality for $\mu > 3$, $\nu > 1$. “Yes” and ”No” mean that the corresponding energy condition in each region is satisfied and is not satisfied, respectively, and “±” denote the signatures of $g^{\mu\nu}k_\mu k_\nu$.

x	$0 < x < x_-$	$x_- < x < \frac{\nu-1}{\mu+1}$	$\frac{\nu-1}{\mu+1} < x < \frac{\nu+1}{\mu-1}$	$\frac{\nu+1}{\mu-1} < x < \frac{\nu+3}{\mu-3}$	$\frac{\nu+3}{\mu-3} < x < x_+$	$x_+ < x < \infty$
$\varepsilon_{\mathrm{NEC}}$	+	+	+	+	−	−
$\varepsilon_{\mathrm{DEC}}$	−	−	+	+	+	+
$\varepsilon_{\mathrm{SEC}}$	+	+	+	−	−	−
ε_γ or $\mathcal{L}_{\mathcal{F}\mathcal{F}}$	+	−	−	−	−	+
NEC	Yes	Yes	Yes	Yes	No	No
WEC	Yes	Yes	Yes	Yes	No	No
DEC	No	No	Yes	Yes	No	No
SEC	Yes	Yes	Yes	No	No	No
$g^{\mu\nu}k_\mu k_\nu$	−	+	+	+	−	+

spacetime geometry except for photons moving in the radial direction with $(k^\theta, k^\phi) = (0, 0)$, which propagate along null curves in both geometries.

VI. MORE GENERAL DISCUSSION

In more general cases, if we assume that the Lagrangian density with the Maxwell limit in the weak field limit has smoothness of $\mathcal{L}$ at $\mathcal{F} \to 0$, since

$$\mathcal{L} \simeq \mathcal{F} + \frac{1}{2}\mathcal{L}_{\mathcal{F}\mathcal{F}}(0)\mathcal{F}^2 + \cdots, \tag{51}$$

$$\mathcal{L}_{\mathcal{F}} \simeq 1 + \mathcal{L}_{\mathcal{F}\mathcal{F}}(0)\mathcal{F} + \cdots, \tag{52}$$

we can see

$$\mathcal{L} - \mathcal{F}\mathcal{L}_{\mathcal{F}} \simeq -\frac{1}{2}\mathcal{L}_{\mathcal{F}\mathcal{F}}(0)\mathcal{F}^2. \tag{53}$$

If we assume that DEC, $\mathcal{L} \geq 0, \mathcal{F}\mathcal{L} \geq 0, \mathcal{L} - \mathcal{F}\mathcal{L}_{\mathcal{F}} \geq 0$, are satisfied in the weak field, the second-order derivative $\mathcal{L}_{\mathcal{F}\mathcal{F}}(0)$ must be nonpositive, which means that photons can be timelike or null in the weak field such as at infinity.

On the contrary, let us assume that photon propagation can be timelike, i.e.,

$$g^{\mu\nu}k_\mu k_\nu > 0,$$

which can be classified in two cases, $\mathcal{L}_{\mathcal{F}\mathcal{F}} < 0, \mathcal{L}_{\mathcal{F}} > 0$ and $\mathcal{L}_{\mathcal{F}\mathcal{F}} > 0, \mathcal{L}_{\mathcal{F}} < 0$ from Eq. (35). The latter case does not admit the Maxwell limit, and hence, we consider only the former case. Moreover, from

$$\partial_{\mathcal{F}}(\mathcal{L} - \mathcal{F}\mathcal{L}_{\mathcal{F}}) = -\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}} > 0, \tag{54}$$

we find that

$$\mathcal{L} - \mathcal{F}\mathcal{L}_{\mathcal{F}} > (\mathcal{L} - \mathcal{F}\mathcal{L}_{\mathcal{F}})|_{\mathcal{F}=0} = 0, \quad \mathcal{L} > \mathcal{L}|_{\mathcal{F}=0} = 0, \tag{55}$$

which means that DEC holds in such a region. In other words, the violation of DEC implies the existence of “faster than light” photons.

However, we should note that, without the Maxwell limit, the violation of DEC does not necessarily imply the existence of the spacelike propagation of photons as seen in examples shown in Tables II and III.

VII. SUMMARY AND DISCUSSION

In this paper, we have studied the causality of photon propagation in NED when the energy conditions are satisfied. As instances, we have considered the causality of photons around static and spherically symmetric Einstein-Born-Infeld BHs and well-known regular BHs such as Bardeen BHs, Hayward BHs, and Fan-Wang BHs, which can be regarded as static and spherically symmetric solutions to the Einstein equations coupled with NED. For such example, we have seen that as long as DEC is satisfied, the photon trajectories can be timelike in the spacetime geometry, though they are null in the effective geometry; i.e., the light cone of the effective geometry does not lie outside the light cone of the spacetime geometry.

In general, DEC can be interpreted as that the speed of energy flow of matter is always less than the speed of light. Hence, the existence of “faster than light photons” in NED contradicts with DEC. Indeed, we have shown that the violation of DEC always leads to the existence of such photons, at least, in NED with the Maxwell limit, where we should note that this cannot necessarily be true in NED with no Maxwell limit; we have seen this in the examples of RBHs. The timelike photons do not necessarily mean that they can become massive since as can be seen from Eq. (35), the causality depends on the directions of the propagation; i.e., its propagation in the angular directions becomes timelike but null in the radial direction.

In this paper, for simplicity, we have dealt with static and spherically symmetric spacetimes with a purely magnetic field or a purely electric field but we are not sure whether our results are also true for spacetimes with both fields, where an easy construction is shown in Ref. [23], or rotating regular black holes [24]. This deserves our future work.

Finally, we wish to comment on the consistency with the results by Gibbons and Herdeiro in [12]. As discussed by them, the eigenvalues of the effective metric are proportional to

$$\mu + \mathcal{F}, \qquad \mu + \mathcal{F}, \qquad \mu - \mathcal{F}, \qquad \mu - \mathcal{F}, \tag{56}$$

where μ is a root of the quadratic equation,

$$w\mu^2 + \mu + \omega - w(\mathcal{F} + \mathcal{G}) = 0, \tag{57}$$

where $\mathcal{G} \coloneqq F_{\mu\nu} * F^{\mu\nu}$. Here, for the one-parameter Lagrangian density $\mathcal{L}(F)$, the functions w and ω are written as

$$w \coloneqq \frac{\mathcal{L}_{\mathcal{F}\mathcal{F}}\mathcal{L}_{\mathcal{G}\mathcal{G}} - \mathcal{L}_{\mathcal{F}\mathcal{G}}^2}{\mathcal{L}_{\mathcal{F}}(\mathcal{L}_{\mathcal{F}\mathcal{F}} + \mathcal{L}_{\mathcal{G}\mathcal{G}})} = 0, \tag{58}$$

$$\omega \coloneqq \frac{\mathcal{L}_{\mathcal{F}} + \mathcal{F}(\mathcal{L}_{\mathcal{F}\mathcal{F}} - \mathcal{L}_{\mathcal{G}\mathcal{G}}) + 2\mathcal{G}\mathcal{L}_{\mathcal{F}\mathcal{G}}}{\mathcal{L}_{\mathcal{F}\mathcal{F}} + \mathcal{L}_{\mathcal{G}\mathcal{G}}} = \mathcal{F} + \frac{\mathcal{L}_{\mathcal{F}}}{\mathcal{L}_{\mathcal{F}\mathcal{F}}}; \tag{59}$$

therefore, the function μ turns out to be

$$\mu = -\mathcal{F} - \frac{\mathcal{L}_{\mathcal{F}}}{\mathcal{L}_{\mathcal{F}\mathcal{F}}}. \tag{60}$$

The velocities of photons, the ratio of spacelike to timelike eigenvalues, are given by

$$\left(1, \frac{\mu - x}{\mu + x}, \frac{\mu - x}{\mu + x}\right) = \left(1, 1 + \frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}, 1 + \frac{2\mathcal{F}\mathcal{L}_{\mathcal{F}\mathcal{F}}}{\mathcal{L}_{\mathcal{F}}}\right), \tag{61}$$

where the fact that the first component in the above equation is one means that there are two directions in which the light cone in the effective geometry touches the usual light cone in the spacetime geometry. Under DEC, the condition of $\mathcal{L}_{\mathcal{F}\mathcal{F}} < 0$ is equivalent with that the light cone in the effective geometry does not lie outside the light-cone in the spacetime geometry; i.e., photons are timelike or null in the spacetime geometry.

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