Testing the braneworld theory with identical particles

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Various attempts to surpass the theory of general relativity start from the assumption that spacetime is not a four-dimensional but rather a higher-dimensional manifold. Among others, braneworld scenarios postulate that the spacetime we effectively observe is actually a four-dimensional brane embedded in a higher-dimensional spacetime. In general, braneworld models predict a departure from the Newton gravity law in the nonrelativistic regime. Based on this fact, we propose an experimental test that uses a pair of gravitationally interacting identical particles to determine the validity of certain braneworld models and provide numerical results that should be compared with experimental data. In particular, we consider the Randall-Sundrum braneworld model and study two cases of five-dimensional gravity theories: the Einstein-Hilbert gravity with the negative cosmological constant and the Einstein-Gauss-Bonnet (nearly-Chern-Simons) gravity.

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I. INTRODUCTION

Newton's law of gravity has been experimentally verified a long time ago [1] and since then has been repeatedly tested using various experimental setups, from tabletop experiments to cosmological observations. However, we know this law is only an approximation, as a more fundamental theory is general relativity (GR). Although its predictions have been corroborated in many different ways to a high level of precision [2,3], GR does not amount to a whole story about spacetime and gravity. The main symptom of the theory's incompleteness is the appearance of singularities, both in the center of a black hole and at the beginning of the universe. In order to solve this problem, many attempts to quantize gravity have been made [4]. The most naive procedure is based on using a linear approximation of Einstein's field equations and the standard tools of quantum field theory, which leads to a nonrenormalizable theory that cannot be trusted to arbitrary high energies. Ostensibly, some radical change in our basic assumptions has to be made when dealing with the quantum theory of gravity. Some approaches, among which string theory is perhaps the most notable one, start from a theory defined in a number of spatial dimensions greater than three. Effective, four-dimensional physics is then obtained from the Kaluza-Klein compactification [5,6] where one assumes that the additional spatial dimensions are small enough and thus experimentally inaccessible at current energies.

For phenomenological reasons, an alternative approach that does not assume the smallness of extra dimensions was proposed [7–9]. Those models are known under the name of braneworld models, and they assume that our world is a three-brane embedded in a higher-dimensional spacetime. Matter fields are usually confined on this brane, while gravity is free to propagate in all dimensions. Naturally, those models have some features that differ from the standard gravity theories. In this paper, we will propose a way to test the predictions of some braneworld models experimentally.

II. BRANEWORLD SCENARIO

For simplicity, we will focus on the so-called Randall-Sundrum (RS) II model (see Ref. [10] for a review). We start from a five-dimensional Einstein-Hilbert gravity with the negative cosmological constant, defined by the action

$$\frac{1}{16\pi G_5}\int_{\mathcal{M}_5} \mathrm{d}^5x\,\sqrt{-g}(R-2\Lambda)+\frac{1}{8\pi G_5}\int_{\partial\mathcal{M}_5}\mathrm{d}^4x\sqrt{-h}K. \tag{2.1}$$

where G_5 is the five-dimensional gravity constant related to the fundamental five-dimensional Planck scale M_P by $G_5 = 8\pi/M_P^3$. The Gibbons-Hawking-York term, with induced metric h_{ij} on the boundary, has to be included in the presence of a boundary so that the variation principle is well-posed. We then insert a brane Q of constant tension T, usually defined as a hyper-surface for which one coordinate in a preferred coordinate system is constant. The relevant term that we add to the action is

$$T\int_Q \mathrm{d}^4x\,\sqrt{-\gamma}, \tag{2.2}$$

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where γ_{ij} is the metric induced on the brane. Away from the brane, the anti-de Sitter (AdS) spacetime solves the bulk equations of motion. However, appropriate junction conditions have to be imposed that patch together solutions on both sides of the brane. Also, more complicated matter fields that are confined to the brane could be added.

In order to derive Newton's law of gravity from the underlying relativistic theory, one has to use both the action and the associated equations of motion, and therefore, different models lead to different results. Generically, the modified gravitational potential on the four-dimensional brane has the form

$$V(r) = -\frac{Gm}{r}(1 + \Delta(r)), \tag{2.3}$$

where $G = G_5/l_{\mathrm{AdS}}$ is the effective four-dimensional gravitational constant and the five-dimensional AdS radius l_{AdS} is the characteristic bulk length scale [11]. In the case of RS II model, for large distances $r \gg \frac{1}{k}$ (k is the inverse of the bulk AdS radius, $k = 1/l_{\mathrm{AdS}}$), we have

$$\Delta(r) = \frac{2}{3k^2r^2} - \frac{4\ln kr}{k^4r^4} + \frac{(16 - 12\ln 2)}{3k^4r^4} + \cdots, \tag{2.4}$$

while for small distances, $r \ll \frac{1}{k}$, the modification is

$$\Delta(r) = \frac{4}{3\pi kr} - \frac{1}{3} - \frac{kr}{2\pi}\ln kr + 0.089237810kr + \cdots, \tag{2.5}$$

see Refs. [12,13] for more details. One can also find an approximate potential interpolating between those two extremes. It is given in [11] as

$$\Delta(r) = \frac{4}{3\pi}\left(\frac{kr\cos kr - \sin kr}{k^2r^2}\int_{+\infty}^{kr}\frac{\cos t}{t}\mathrm{d}t + \frac{\cos kr + kr\sin kr}{k^2r^2}\int_{+\infty}^{kr}\frac{\sin t}{t}\mathrm{d}t + \frac{\pi}{2k^2r^2}\right). \tag{2.6}$$

There are other braneworld models that one could also study. For example, in [14], modified Newton's potential was derived, with the following large distances behavior,

$$\Delta(r) = -\frac{e^{-2\sqrt{2}kr}}{4(\sqrt{2}-1)kr}, \tag{2.7}$$

There are also models that start from a different number of spacetime dimensions or involve more than one brane [13]. Attempts to experimentally verify those corrections have been made in the context of classical physics [15].

It is important to note that the results concerning the modification of Newton's potential follow from the tree-level computations and hence do not contain nonzero powers of $\hbar$. This is in contrast with the one-loop calculations of the gravity propagator that yields a similar result (using the ideas from AdS/CFT, those two can be seen on equal footing [16]). Namely, if we were to quantize GR (despite it being nonrenormalizable), we could obtain modifications of Newton's potential at the one-loop level, but those would be suppressed by powers of $\hbar$. The potentials that we analyse in this paper are not of this type. This means that we do not have to claim any results from quantum gravity in order to talk about the modified potentials in braneworld models; we only assume the existence of extra dimensions. Also, corrections of the form $1/r^3$ in Newton's potential are well known in GR (they are responsible for the precession of Mercury orbit). However, they are not in any way connected to the corrections we are dealing with here, which are nonrelativistic.

III. TWO IDENTICAL PARTICLES

Let us consider two identical quantum particles that mutually interact via gravitational force and are otherwise neutral—a gravitational atom. The two particles need not be elementary (like neutrinos) or even subatomic (like neutrons); we could have a pair of identical composite particles of the size of an atom or a large molecule. For sufficiently small masses (energies), classical gravity is well approximated by Newton's theory, which allows us to assume the following form of the two-particle Hamiltonian,

$$H = \frac{\boldsymbol{P}^2}{4m} + \frac{\boldsymbol{p}^2}{m} - \frac{Gm^2}{r}, \tag{3.1}$$

where $\boldsymbol{P}$ is the total momentum of the system and $\boldsymbol{p}$ is the relative momentum. For concreteness, we will from now on consider the case of two spin-$1/2$ fermions, see Fig. 1.

Eigenstates of the Hamiltonian (3.1) are of the form

$$|\psi\rangle = \frac{e^{i\vec{K}\cdot\vec{R}}}{\sqrt{V}}R_{nl}(r)Y_l^m(\theta,\varphi)\otimes|\chi\rangle. \tag{3.2}$$

We assume that the spatial volume V of the box in which we constrain our quantum system is much larger than the characteristic length scale of the bound states of the

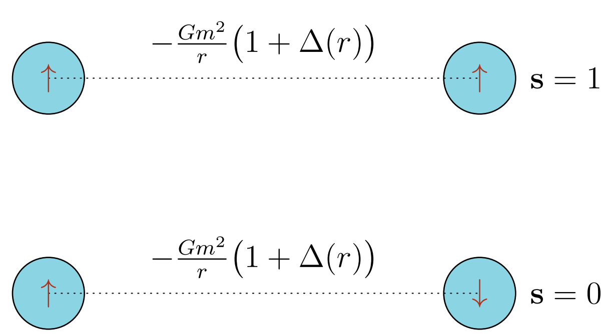

FIG. 1. Two identical spin-$\frac{1}{2}$ fermions with zero charges, interacting through a modified Newton's potential. The total energy of the system depends on the spin polarization.

Hamiltonian (3.1) so that we can neglect the influence of this box on the energy spectrum. In case we place the system in some external (nonconstant) potential, we demand that the relevant scale of variations in the potential are large compared to any scale in (3.1) so that we can again assume that we can restrict ourselves to the case of Hamiltonian (3.1). The interchange of two particles corresponds to $\vec{r} \to -\vec{r}$, which induces the change $Y_l^m(\theta, \varphi) \to (-1)^l Y_l^m(\theta, \varphi)$. Preparing the spin state of the system in an appropriate manner (singlet or triplet), we are able to control the parity of the relative angular momentum of the eigenstates, as the total state of the two fermions has to be antisymmetric. In the case of $1/r$ potential, states with different values of the orbital number l have the same energy as long as the principal quantum number n is the same. This is related to the $SO(4)$ symmetry of the Hamiltonian (3.1). However, in the case of braneworld models, this Hamiltonian is corrected using previously introduced potentials. The new Hamiltonian has only rotational $SO(3)$ symmetry, and therefore, states with different angular momentum have different energy. We want to determine the eigenenergies of the braneworld Hamiltonians resulting from various braneworld scenarios. First, let us consider the RS II model with Einstein-Hilbert action, where the correction to Newton's potential is given by (2.6).

The energy spectrum of the unperturbed Hamiltonian (3.1) is well known, as it is the same (up to a numerical factor) as for the hydrogen atom. This means that the unperturbed energies (disregarding the center of mass energy) are given by (taking $c = 1$)

$$E_n^{(0)} = -\frac{m}{4n^2}\left(\frac{m}{m_{\mathrm{P}}}\right)^4, \tag{3.3}$$

where m_{P} is the four-dimensional Planck mass. We now consider correction to Newton's potential $-\frac{Gm}{r}\Delta(r)$ as a perturbation and use the first-order perturbation calculus to obtain corrections of the energies of the first excited ($n = 2$) level. We will see that the perturbation calculus is valid as long as $1 \ll ka_0$, where $a_0 = \frac{2\hbar^2}{Gm^3}$. As the perturbation also has $SO(3)$ symmetry, we only need to calculate diagonal matrix elements to obtain corrections to the energy levels. In Fig. 2, we present the numerical calculation for the corrections to the first excited level $n = 2$ for $l = 0$ and $l = 1$ as a function of the dimensionless parameter $1/ka_0$. To be more precise, we plot the dimensionless quantity

$$U_{n,l} = 8\frac{a_0}{k^2}\int_0^{+\infty} |\psi_{n,l}(x)|^2(-x)\Delta(x)\mathrm{d}x, \tag{3.4}$$

while the first-order corrections to the energy levels are $E'_{n,l} = \frac{Gm^2}{8a_0}U_{n,l}$. The energy difference for the first excited level $n = 2$ between $l = 1$ and $l = 0$ states in terms of $\Delta U = U_{2,1} - U_{2,0}$ is presented in Fig. 3. It is important to note that our system is nonrelativistic so we can use the Newtonian approximation. For this to be true, we must have $p \sim \frac{\hbar}{a_0} \ll mc$. In the case when the perturbation theory is not applicable, we can make the following observation. In this regime, a_0 is at least of the same order as $1/k$, or possibly larger. Wave functions peak around a_0, and this means that we can use the form of the potential for $r \ll \frac{1}{k}$. It turns out that potential $-\frac{A}{r} - \frac{B}{r^2}$ (for some constants A and B) corresponds to a Hamiltonian that we can exactly diagonalize. It can be shown that for $\frac{1}{ka_0} > \frac{3\pi}{32} \approx 0.3$, the particles would merge, and our analysis based on quantum mechanics and Newton's gravity would be inappropriate [17].

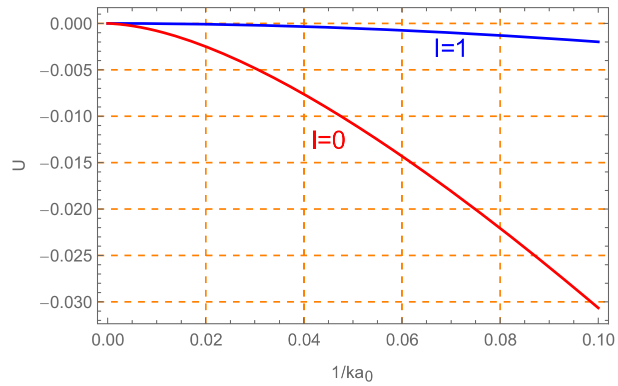

FIG. 2. Corrections to the energies of the first excited level $n = 2$ for $l = 0$ (red line) and $l = 1$ (blue line).

A. Chern-Simons gravity

So far, we assumed that the five-dimensional theory is well-described by Einstein-Hilbert gravity. In five dimensions, one can also consider Chern-Simons (CS) gravity, defined in the metric formulation as

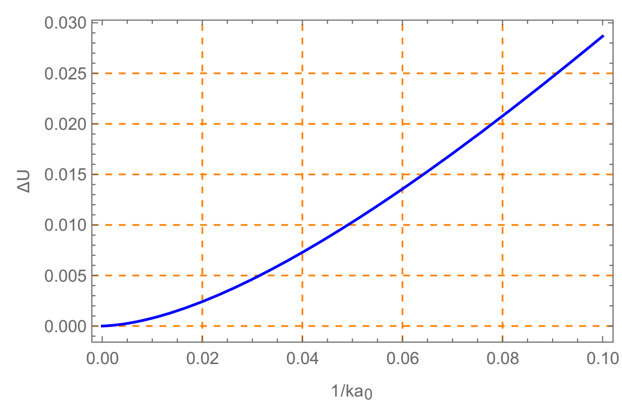

FIG. 3. Difference in energies between $l = 1$ and $l = 0$ state for the first excited level $n = 2$ in the Einstein-Hilbert case.

$$S_{\mathrm{CS}} = \frac{1}{16\pi G_5}\int \mathrm{d}^5x\sqrt{-g}\Big[R - 2\Lambda + \frac{1}{4k^2}(R^2 - 4R^{\mu\nu}R_{\nu\mu} + R^{\mu\nu\rho\sigma}R_{\rho\sigma\mu\nu})\Big]. \tag{3.5}$$

This theory has an enlarged $SO(4,2)$ symmetry group [18]. CS theory has nonvanishing torsion, but here we restrict to the case of torsion-less geometries, as they are much better understood. More generally, the CS action (3.5) can be generalized by substituting the constant parameter $\frac{1}{4k^2}$ with some general parameter $\frac{\alpha}{4k^2}$, thus obtaining the Einstein-Gauss-Bonnet theory. One can calculate the modification of Newton's potential for this generalized gravity theory [11] and obtain that the CS case exhibits no corrections. Moreover, if we take α close to the CS value, the corrections are small enough so that perturbation theory can be used for all values of $1/ka_0$.

The correction to the potential takes the form [11]

$$\Delta_\alpha(r) = \frac{4(1-\alpha)}{3\pi(1+\alpha)}\Bigg(\frac{(\beta x\cos(\beta x) - \sin(\beta x))\int_{+\infty}^{\beta x}\frac{\cos(t)}{t}\mathrm{d}t}{x^2} + \frac{(\cos(\beta x) + \beta x\sin(\beta x))\int_{+\infty}^{\beta x}\frac{\sin(t)}{t}\mathrm{d}t}{x^2} + \frac{\pi}{2x^2} - \frac{\beta}{x} + \beta^2\gamma\left(\sin(\beta\gamma x)\int_{+\infty}^{\beta\gamma x}\frac{\cos(t)}{t}\mathrm{d}t - \cos(\beta\gamma x)\int_{+\infty}^{\beta\gamma x}\frac{\sin(t)}{t}\mathrm{d}t\right)\Bigg), \tag{3.6}$$

where $x = kr$, and β and γ are numerical constants defined in [11]; for $\alpha = 0.95$ they are given as $\beta \approx 3.86443$ and $\gamma \approx 0.637448$. Again, we are using approximate potential, with approximate numerical values of the parameters, as our goal is to demonstrate the possibility of detecting extra dimensions and provide some rough theoretical data that could be made more precise with more advanced numerical techniques. In Fig. 4, we present corrections to the first excited energy level $n = 2$ for $l = 0$ and $l = 1$ in the case $\alpha = 0.95$. Note that the form of the corrections is as expected. The energy is shifted more for greater values of $1/ka_0$, but the rate of this shift is decreasing. Also, for large $1/ka_0$, the energy difference between the two levels decreases. This can be deduced from the asymptotic form of the gravitational potential. In Fig. 5, we present the energy difference between the two states.

Finally, we can draw the energy difference for the first excited state $n = 2$ between $l = 1$ and $l = 0$, obtaining Fig. 5. Due to the approximate nature of our constants, previous graphs may not give the best numerical values for large $1/ka_0$, but the form of the graph should be correct.

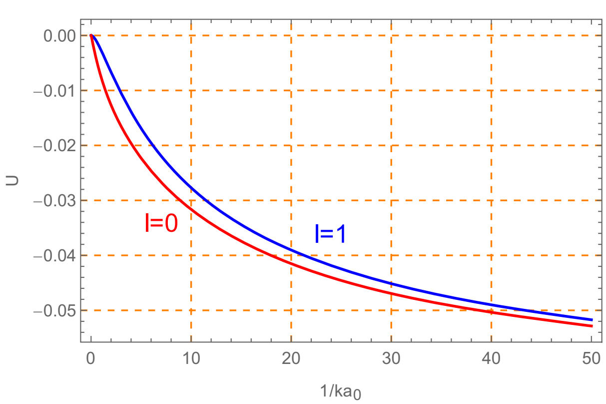

FIG. 4. Corrections to the energies of the first excited level $n = 2$ for $l = 0$ (red line) and $l = 1$ (blue line) in nearly-CS case.

IV. TESTING THE BRANEWORLD HYPOTHESIS

Let us now propose a way to empirically verify whether the consequences of the braneworld hypothesis are valid or not. The procedure is essentially based on the fact that the beyond-Newtonian gravitational potential (of the kind we considered above) lifts the orbital degeneracy of the energy levels for a pair of gravitationally interacting particles. Consider, for example, the following two states of a pair of identical spin-$\frac{1}{2}$ particles,

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}|2,0,0\rangle \otimes (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \tag{4.1}$$

$$|\Psi_2\rangle = |2,1,0\rangle \otimes |\uparrow\uparrow\rangle. \tag{4.2}$$

Both states are antisymmetric as a whole, the particles being fermions. The first state has the antisymmetric singlet state in the spin sector, while the orbital part is symmetric ($l = 0$). On the other hand, the second state has a symmetric spin part and an antisymmetric orbital part

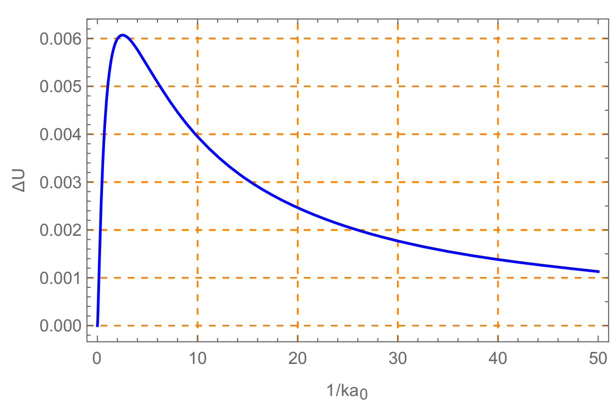

FIG. 5. Difference in energies between $l = 1$ and $l = 0$ state for the first excited level $n = 2$ in the nearly-CS case.

(note that we could also choose some other spin state from the symmetric triplet and also some other value for the magnetic quantum number, namely ± 1). The braneworld potential (2.6) implies that states with different orbital quantum numbers l have a different energy, which is in contrast with the Newtonian case. In principle, we may also consider a superposition of the above two states,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\Psi_1\rangle + |\Psi_2\rangle), \tag{4.3}$$

and use the standard Mach-Zehnder (MZ) interferometer [19] with the first beam splitter selecting the states by the total spin projection S_z. If the energies of the states $|\Psi_1\rangle$ and $|\Psi_2\rangle$ were the same, as in the Newtonian case, the final state after passing through the MZ interferometer would remain the same. If, on the other hand, the energies of the two states were different, as predicted by the braneworld models, the final state would be

$$|\Psi_{\rm final}\rangle = \frac{1}{\sqrt{2}}(e^{i\phi}|\Psi_1\rangle + |\Psi_2\rangle), \tag{4.4}$$

where the relative phase ϕ is determined by the energy difference,

$$\phi = (E_{n=2,l=1} - E_{n=2,l=0})t/\hbar = \frac{G^2}{16\hbar^3}\Delta U m^5 t. \tag{4.5}$$

The second beam splitter in the MZ scheme should postselect the state $|\Psi\rangle$. If ϕ is nonzero, i.e. if the braneworld corrections do exist, the statistics on the detectors would be modified to $\cos^2\frac{\phi}{2}$ and $\sin^2\frac{\phi}{2}$. In this way, we can experimentally obtain the value of the energy splitting if it exists. Therefore, by preparing the particles in the superposition state $|\Psi\rangle$, simple MZ interferometry can be used to refute (or support) the braneworld scenario. If the outcome of the experiment were positive, in support of the braneworld, one would be able to estimate the initially unknown bulk parameter $k = 1/l_{\rm AdS}$. In Fig. 6, we present a scheme of the experimental proposal.

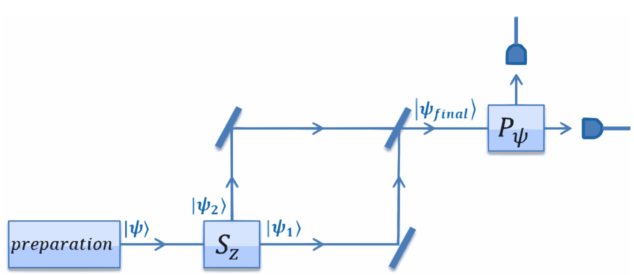

FIG. 6. The experimental setup. Interferometry is used to determine the relative phase between the two states, $|\psi_1\rangle$ and $|\psi_2\rangle$, and therefore, test the braneworld hypothesis.

Note that we could also use a pair of identical bosons instead of fermions. The main point of using identical particles was to exploit the exchange symmetry properties of their quantum state. Taking, for example, a pair of spin-1 bosons, we would have a symmetric orbital state, say $|n = 2, l = 0, m = 0\rangle$, coupled to some symmetric spin state (total spin 0 or 2), or an antisymmetric orbital state, say $|n = 2, l = 1, m = 0\rangle$, coupled to some antisymmetric (total spin 1) spin state. We could also choose fermions and bosons with other values of the spin (different from zero). What is essential is that the energy difference in orbital states with different exchange symmetry (i.e., energy splitting in the orbital quantum number l due to the modification of the gravitational potential) can be revealed by taking into account the exchange symmetry of the corresponding spin state of the gravitational atom; the overall exchange symmetry, i.e., whether the particles are fermions or bosons, is not, therefore, essential. The rest of the procedure is then virtually the same.

V. FEASIBILITY OF THE EXPERIMENT

From a purely theoretical point of view, as far as the fundamental principles of quantum mechanics and classical gravity are concerned, it seems that nothing prevents us from executing the proposed experiment. Although, at this point, we are not capable of providing precise technical details of the relevant experimental procedures, we can at least consider the orders of magnitude of the parameters involved in our analysis and thus estimate whether such an experiment is feasible.

The major difficulty we face is how to maintain a gravitational atom, which could be built out of a pair of large composite particles, in a state of quantum superposition long enough. Some interesting recent developments concerning quantum control of large systems can be found in [20–22]. Also, to reduce the effects of decoherence, which depend on the complicated interaction with the environment, we assume that the two particles are neutral, interacting only via gravity. Moreover, our proposal has a distinct advantage in that it does not involve a direct test of the purported modification of Newtonian potential, i.e., one does not have to actually measure the strength of the gravitational interaction in order to find whether it deviates from $1/r$ (and for that reason we are not concerned with difficulties associated with high precision measurements of gravity between very small masses). The idea is to test the braneworld effect indirectly by using ordinary quantum mechanics, and for that matter, the mass scale of the gravitational atom in question is indeed important.

Our setup is designed so that an observation of the phase difference (relative phase) between states $|\psi_1\rangle$ and $|\psi_2\rangle$ signalizes a deviation from $1/r$, and the particular value of the phase difference would, if it were observed, determine the value of the AdS radius ($l_{\rm AdS} = 1/k$), which is an unknown (but fixed) parameter of the five-dimensional bulk theory of gravity. On the other hand, failing to detect the phase difference would allow us to at least set a bound on the five-dimensional AdS radius, thus narrowing the scope of possible braneworld models. It is important to note that potential experimental results essentially depend on the value of the five-dimensional parameter in question. In particular, larger values of the AdS radius (smaller k) would require a smaller mass of the particles in order to detect the same phase difference. Thus far, tabletop experiments for the Einstein-Hilbert case set the upper bound on the AdS radius at $l_{\mathrm{AdS}} \lesssim 10^{-1}$ mm, while certain cosmological observations give us $l_{\mathrm{AdS}} \lesssim 10^{-2}$ mm, see Ref. [23].

Since we do not know the value of the five-dimensional parameter l_{AdS}, we want to be able to determine it or at least significantly constrain it. The parameters that we have to consider are the mass of the particles that make up the gravitational atom, the coherence time for maintaining the quantum superposition of the gravitational atom, and the sensitivity of the MZ interferometer. By changing the values of these parameters (within the scope of current technology) and performing measurements of the phase difference, we want to find out something about the five-dimensional theory of gravity. In what follows we give our estimation of the orders of magnitude for the relevant parameters in order to assess the feasibility of the experiment (we stick to the much more familiar Einstein-Hilbert case).

The relative phase for our gravitational atom in the superposition state (4.4) is given by

$$\phi = \frac{G^2}{16\hbar^3}\Delta U m^5 t, \tag{5.1}$$

where m is the mass of each of the particles that make up the gravitational atom, t is the time it takes the gravitational atom to pass through the MZ interferometer (it has to be short enough so that the gravitational atom can pass through the MZ interferometer without decohering), and ΔU is a dimensionless parameter whose role is explained above. We set the sensitivity of the MZ interferometer at a rather modest value of $\phi = 0.01$. We thus assume that below that threshold we cannot detect the phase difference. As for the coherence time, the methodology and technology of maintaining quantum coherence for systems of relatively large mass are developing very rapidly and in various directions. Based on the recent achievements [24–26], it is reasonable to consider the case of $t = 1s$. Based on the numerical results for ΔU presented in Fig. 3, we conclude that by using particles of $m \sim 10^{-16}$ kg we can lower the upper bound for $l_{\mathrm{AdS}} = 1/k$ from the current $10^{-5}m$ down to $10^{-11}m$, which is a rather significant progress. As for the larger or smaller masses, we can say the following.

Assuming $m \sim 10^{-18}$ kg, even with the current upper bound for l_{AdS}, the resulting phase difference is notably small. This means that in our experimental proposal, we should use larger masses. However, determining the upper bound for the mass requires knowledge of the coherence time for the respective particles. This is a challenging task that is not solvable on a general basis. However, there is another aspect of the problem that also puts constraints on the mass. For that matter, we point out note that the radius of a composite particle consisting of atoms depends on its mass, and this can give us a meaningful upper bound for the mass. Namely, we need the parameter a_0 to be at least a few times greater than the size of the particles. We can see that for $m \sim 10^{-17}$ kg, we have $a_0 \sim 10^{-7}m$, suggesting that we could use particles of the type discussed, for example, in [27], see also [28]. It is unexpected that one can use composite particles with masses larger than $m \sim 10^{-16}$ kg as parameter a_0 would become too small (of the order of atomic size). Therefore, our final estimate for the mass of the (composite) particles that make up the gravitational atom is between 10^{-18} kg and 10^{-16} kg.

Note however that we have focused on the first excited state ($n = 2$) of our system simply because it was the simplest option to discuss. By increasing the quantum number n, we can make the separation of particles larger, thus mitigating the issue concerning the size of the particles. By extension, this would also allow us to consider masses larger than 10^{-16} kg, which were excluded, as we saw, based on the relation between the size of the particles and their separation.

VI. DISCUSSION AND CONCLUSION

The main purpose of this article was to promote the idea that a particular hypothesis regarding the structure of spacetime (the braneworld scenario), coming from high-energy physics, could be tested in a tabletop experiment in the near future by combining quantum mechanics with classical gravity.

Our proposal is based on the fact that the braneworld RS II model predicts a modification of the energy spectrum of gravitationally bound states by lifting the characteristic orbital degeneracy associated with Newtonian gravity. To test the existence of an extra dimension and probe the unknown five-dimensional parameter l_{AdS} one uses a pair of gravitationally bound neutral identical particles (fermions or bosons) whose energy spectrum depends on whether their spin state is symmetric or antisymmetric. From a practical point of view, the crucial step would be to prepare the particles in the superposition state (4.3), which might be achievable by solely manipulating the spin of the particles. A Mach-Zehnder interferometer is then used to measure the relative phase between the two branches of superposition. Detection of the relative phase signalizes a modification of the Newtonian potential in support of the braneworld scenario, yet failing to detect the relative phase would still allow us to constrain the possible braneworld models. Setting the sensitivity of the MZ interferometer at a rather modest value of $\phi = 0.01$ and assuming the coherence time of order of $1s$, the relevant mass range would be from 10^{-18} kg to 10^{-16} kg. In particular, by using particles of $m \sim 10^{-16}$ kg we can significantly lower the upper bound for $l_{\mathrm{AdS}} = 1/k$ from the current $10^{-5}m$ down to $10^{-11}m$.

We have studied two cases of five-dimensional theories of gravity, Einstein-Hilbert and nearly-Chern-Simons gravity, and presented the numerical results concerning the orbital energy splitting, which appears solely due to the braneworld hypothesis (the presence of a large extra spatial dimension). By controlling the masses of the particles that make up the gravitational atom, one can obtain a graph of the relevant energy difference and compare it to the theoretical predictions such as Figs. 3 and 5 presented above. In this way, one could discriminate between different models of five-dimensional gravity. Another proposal for testing the RS model can be found in [15]. However, we believe that gravitational atoms made out of identical particles in combination with quantum mechanics could be the most straightforward way to test the braneworld models.

A possible extension of our work would be to make a comparison to a similar experimental setup [29,30], discussing the entanglement between quantum particles induced by gravity [31–35], see also [36,37]. In particular, one could use potentials analyzed in this paper to see the differences induced by the corrections coming from the RS II model. In the case of Einstein-Hilbert gravity, and using the limit of small or large distance, this was done recently in [38,39]. It would be interesting to analyse other experimental proposals and to see the effects coming from different gravity models in the five-dimensional bulk. Let us also note an interesting case of gravitationally bound states of dark matter considered in [40]. One might suspect that the effect analyzed in this paper could also be relevant in that case. However, what we have discussed is a tabletop experiment, and we are not suggesting that dark matter, or anything similar, could be a viable resource for the particular setup we have in mind. Nevertheless, in perspective, the influence of gravity on quantum mechanical systems of the type analyzed in this paper could also be relevant for some cosmological observations.

As a final remark, we point out that our discussion here delves into the general possibility of achieving a superposition of states while simultaneously extracting information about gravitational dynamics. We have refrained from delving into the specific superposition of states proposed in our paper, as that would pose a considerable challenge that would necessitate a separate investigation, which falls outside the scope of this article. Nevertheless, our analysis indicates that there are no inherent obstacles preventing the execution of the proposed experiment, and with the positive trend of recent experimental results, it is reasonable to anticipate that the experiment we have envisioned will soon become feasible with the currently available technology. Its realization holds the promise of systematically scrutinizing the theory of large extra dimensions.

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