

π -axion and π -axiverse of dark QCDStephon Alexander,^{1,*} Humberto Gilmer,^{1,†} Tucker Manton,^{1,‡} and Evan McDonough^{2,§}¹*Department of Physics, Brown University, Providence, Rhode Island 02912, USA*²*Department of Physics, University of Winnipeg, Winnipeg, Manitoba R3B 2E9, Canada*

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Axions and axionlike particles are a prominent dark matter candidate, drawing motivation in part from the axiverse of string theory. Axionlike particles can also arise as composite degrees of freedom of a dark sector, for example, as dark pions in dark quantum chromodynamics. In a dark Standard Model (SM) wherein all six quark flavors are light while the photon is massive, one finds a rich low-energy spectrum of stable and ultralight particles, in the form of neutral and charged dark scalars, and complex neutral scalars analogous to the SM kaon, with mass splittings determined by the mass and charge of the dark quarks. The model finds a natural portal to the visible sector via kinetic coupling of the dark and visible photons and consequent millicharges for dark matter. The dark matter can be a mixture of all these ultralight bosonic degrees of freedom and exhibits both parity-even and parity-odd interactions, making the theory testable at a wide variety of experiments. In context of dark QCD with N_f flavors of light quarks, this scenario predicts $N_f^2 - 1$ ultralight axionlike particles—effectively an axiverse from dark QCD. This “ π -axiverse” is consistent with but makes no recourse to string theory and is complementary to the conventional string theory axiverse.

DOI: [10.1103/PhysRevD.108.123014](https://doi.org/10.1103/PhysRevD.108.123014)**I. INTRODUCTION**

Axions and axionlike particles have entered center stage as a candidate for physics beyond the Standard Model and as dark matter. Originally motivated by the strong CP problem of the Standard Model [1–3], axions are also motivated by string theory, wherein vacua of string theory are conjectured to have tens to hundreds of axionlike particles [4–6]. Many experiments are actively searching for axion dark matter, across the broad mass range $10^{-21} - 1$ eV (see Ref. [7] for a review of models and constraints).

In this work we consider an alternate path to axionlike particles, using the experimentally tested physics of the Standard Model. We consider a dark copy of the Standard Model wherein the dark quark masses are much lower than in the Standard Model, while the dark QCD scale is much higher. As proposed in Ref. [8], axionlike particles emerge as composite degrees of freedom, analogous to pions, in the confining phase of the dark QCD theory. The possibility

that ultralight dark matter could be a composite degree of freedom has been studied in only a small number of past works, see Refs. [8–10]. Analogous to the global $U(1)$ shift symmetry that protects the mass of an axion, the dark pion masses are protected by a global $SU(N_f) \times SU(N_f)$ symmetry where N_f is the number of flavors of quark in the dark QCD theory. The dark pions are cosmologically stable and can easily have masses conventionally associated with axion dark matter. This scenario differs from the “dynamical axion” models [11,12] (see also [13–16]), which use confinement of a dark $SU(N)$ to solve the strong CP problem of the Standard Model, in that it does not predict new colored particles or heavy axions, but instead utilizes ultralight dark quarks which confine into ultralight dark pions.

We will refer to dark pions that fall in the axion dark matter mass range as “ π -axions.” In the simple setup of a dark copy of the Standard Model QCD wherein all six flavors of quark are nearly massless, one finds 35π -axions. For N_f flavors one finds $N_f^2 - 1$ π -axions; for $N_f \gg 1$, this amounts to a π -axiverse, analogous to the string axiverse [4–6]. This builds on earlier work [8] in the context of $N_f = 2$, where dark pions were first proposed as an axionlike dark matter candidate.

This model finds a natural portal to the Standard Model (SM) in the form a kinetic coupling of the visible and dark sector photons, which generates a visible electric charge (“millicharge”) for the dark Standard Model fields

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including the dark quarks. This generates interactions between the charged π -axions and SM photon, and, upon integrating out the dark W and Z bosons, new interactions with photons for both the charged and complex neutral π -axions. These interactions, including both parity-odd and parity-even interactions, make the model amenable to a wide array of experimental probes.

There are many possibilities for π -axion dark matter. Depending on the quark masses, the millicharge parameter ϵ , and the dark photon mass m_γ , the dark matter can be a combination of all 35 light degrees of freedom (real neutral, complex neutral, and charged), or solely the neutral states, or a subset thereof. Remarkably, the neutral π -axions are a stable cold dark matter candidate even in the limit of unit charge $\epsilon \rightarrow 1$, due to the combination of a small π -axion mass and large decay constant. The mass splitting between the neutral π -axions is set by the masses of the quarks, and even in case of completely degenerate quark masses, one finds a mass splitting with two real neutral π -axions with mass $m_\pi = \sqrt{m_q F_\pi}$, two with mass $\sqrt{2}m_\pi$, a fourth with $\sqrt{3}m_\pi$, and a fifth with mass $\sqrt{6}m_\pi$.

The π -axion dark matter scenario is easily distinguished from conventional axion scenarios: By the dense mass spectrum that follows from the approximate $SU(N_f) \times SU(N_f)$ flavor symmetry of the underlying dark QCD theory; by the charged π -axions and neutral complex π -axions that accompany the neutral real pseudoscalar π -axions; and by the combination of parity-odd and parity-even couplings allowed by the symmetries of the model. Parity-odd couplings, accessible experimentally at experiments such as ADMX, arise from real pseudoscalar π -axions, while parity-even couplings, accessible at experiments searching for oscillations of α_e , arise from the complex neutral and charged π -axions.

The π -axiverse offers two key benefits: (1) it utilizes physics that has been observed in the laboratory, and (2) it is calculable and therefore predictive: while QCD physics calculations are challenging and one often must resort to the lattice, this challenge pales in comparison to the complications of string constructions, such as competing instanton effects. Finally, we note that the π -axiverse is compatible with string theory and indeed relatively easy to engineer, due to the similarity to the Standard Model and the decades of progress in string phenomenology.

The outline of this paper is as follows: In Sec. II we outline a dark Standard Model which provides an implementation of the π -axion scenario. In Sec. III we develop the details of the π -axions, including the mass spectrum and quark content, and in Sec. IV we develop the interactions with Standard Model photons. In Sec. V we develop the cosmology of π -axion dark matter, and in Sec. VI we focus on the observable signals through both the parity-odd and parity-even portals. We conclude in Sec. VII with a discussion of directions for future work.

II. DARK STANDARD MODEL

We consider a dark copy of the Standard Model of particle physics, including the gauge group, field content, and interactions, but with parameters of the model that are independent from those in the visible sector. We couple the dark and visible sectors via an electric millicharge of the dark fermions, arising from a kinetic mixing of the dark and visible photons.

The characteristic feature of this dark Standard Model (dSM) will be two energy scales: A UV scale to which we anchor both the Higgs vacuum expectation value (VEV) v and dark QCD scale Λ_{dQCD} ,

$$v, \Lambda_{\text{dQCD}} \gtrsim 10^{11} \text{ GeV}, \quad (1)$$

corresponding to the energy scale conventionally associated with an axion decay constant (see, e.g., [7]) and an IR scale to which we anchor the dark quark masses m_q ,

$$m_q \ll \text{eV}, \quad (2)$$

which can be realized via small quark Yukawa couplings, $y_q = m_q/v \ll 1$.

Meanwhile, we assume that the charged leptons (electron, muon, tau) of the dSM have masses at the UV scale, $m \sim v, \Lambda_{\text{dQCD}}$. This choice is made for simplicity in our analysis of the cosmology of this model. This implies a hierarchy between the lepton Yukawa couplings and the quark Yukawa couplings. The smallness of the quark Yukawa couplings is technically natural [17], on the basis of the enhanced (flavor) symmetry in the massless quark limit, which guarantees that quantum corrections to the Yukawa couplings are proportional to the couplings themselves.

Altering the Higgs VEV raises possible concern about the stability of the Higgs potential. The Higgs VEV v is related to the Higgs mass m_H and quartic self-coupling λ through the relation $v \propto m_H/\sqrt{\lambda}$. Anchoring $v \sim 10^{11} \text{ GeV}$ requires a combination of making m large or λ small. Naturalness considerations suggest that the former case is favored, as the large lepton masses would destabilize a small dark Higgs mass as in the visible hierarchy problem. In addition, the visible Higgs field exists at a point of metastable equilibrium due to its mass and the Yukawa coupling to the top quark. In contrast, the dark SM as proposed here, with $m_H \gtrsim v$ and $\lambda < 1$, sits at a point of *absolute* stability, due to the large mass and small self-coupling.

The dark W and Z bosons are anchored to the Higgs VEV, by the standard relations

$$m_W \sim \frac{1}{2} g v, \quad m_Z \sim \frac{1}{2} \sqrt{g^2 + g'^2} v, \quad (3)$$

where g and g' are the dark weak and dark hypercharge coupling constants, respectively. The freedom to set g and

g' provides some flexibility in adjusting the dark W and Z boson masses, which could in principle be much less than the Higgs VEV v . The dark neutrinos, while possibly light, are effectively sequestered by the heavy dark electroweak sector and will play no role in the model as studied here.

The dark photon can be either massive or massless in our model and is kinetically coupled to the visible photon, generating millicharges for the dark quarks and leptons (see, e.g., [18] for a review). The dark photon mass could arise by a variety of means; e.g., through a Stückelberg mechanism, by an additional dark Higgs field, or by modifying the Higgs sector along the lines of the Georgi-Glashow model, employing a Higgs triplet. We take the dark photon mass $m_{\gamma'}$ and kinetic coupling ε to be free parameters of the model, subject to observational constraints on dark radiation (generated by annihilations of charged π -axions) and millicharged particles. For much of this work, we will focus on the case that $m_{\gamma'} \gtrsim \Lambda_{\text{dQCD}}$, in which case the primary role of the dark photon is to generate the millicharges of the dark quarks.

Finally, we come to the QCD sector of the dark Standard Model, which will be the main focus of this work. Analogous to the visible sector, we group the six dark quarks $\{u, d, s, c, b, t\}$ into three generations, characterized by mass scale $m_{\text{I}}, m_{\text{II}}, m_{\text{III}}$, and assume $m_{\text{I}} \lesssim m_{\text{II}} \lesssim m_{\text{III}}$ without loss of generality. We write the individual dark quark masses as

$$\begin{aligned} m_u &= c_1 m_{\text{I}} & m_d &= c_2 m_{\text{I}}, \\ m_s &= c_3 m_{\text{II}}, & m_c &= c_4 m_{\text{II}}, \\ m_b &= c_5 m_{\text{III}}, & m_t &= c_6 m_{\text{III}}, \end{aligned} \quad (4)$$

where the parameters $c_1, \dots, c_6$ are $\mathcal{O}(1)$ numbers. We give the dark quarks the same charge assignments as their Standard Model analogs.

Some key features of the dark Standard Model considered here in relation to the dark quark sector are:

- (i) A portal to the visible SM is provided via the dark photon kinetic mixing with the visible sector photon with coupling ε ; each dark quark gains a visible Standard Model (“milli”) electric charge $\propto \varepsilon e$.
- (ii) At low energies, the dark electroweak sector leaves traces of its full UV description in the form of otherwise-forbidden dark quark interactions, e.g., flavor-changing interactions.
- (iii) While we focus on the regime of preinflationary chiral symmetry breaking, modeling our analysis on preinflationary Peccei-Quinn symmetry breaking axion dark matter, one could relax this assumption, opening up new possibilities related to both the dark QCD phase transition and the dark electroweak phase transition, such as gravitational waves [19] and baryogenesis (see, e.g., Refs. [20,21]).

In what follows we predominantly focus on the quark sector of the theory, including the above features, returning to explicit features of the dark Standard Model only when necessary to assess self-consistency.

III. THE π -AXIVERSE OF DARK QCD

We now focus our attention on the dark quarks and dark gluons, amounting to a dark version of quantum chromodynamics (QCD). The Lagrangian, for arbitrary number of gluon colors N_c and quark flavors N_f , is given by

$$\mathcal{L}_{\text{dQCD}} = -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \sum_{n=1}^{N_f} \bar{q}_n (i \not{D} - m) q_n, \quad (5)$$

where $G_{\mu\nu}^a$ is the gluon kinetic term for $SU(N_c)$ (N_c is the number of colors), $\not{D} \equiv \gamma^\mu D_\mu$ is the gauge covariant derivative, and m is the quark mass matrix generated by the Yukawa couplings, which is chosen to be diagonal. In the massless limit, the left- and right-handed quarks can be subjected to separate chiral flavor rotations, amounting to a global symmetry group

$$U(N_f)_L \times U(N_f)_R = SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A. \quad (6)$$

At low energies, below the confinement scale Λ_{dQCD} , the theory develops a $q\bar{q}$ condensate, leading to spontaneous breaking of the chiral flavor symmetry to its diagonal subgroup,

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V. \quad (7)$$

The Goldstone bosons of chiral symmetry breaking are known as the pions, numbering in $N_f^2 - 1$ of them.¹ The quark masses in Eq. (5), generated by the Yukawa couplings of the dSM, explicitly break chiral symmetry, rendering the symmetry only approximate and in turn leading to small masses for the pions. In this work, we refer to ultralight pions, namely in the mass window of axion dark matter, as π -axions.

At leading order, the π -axions may be described by a σ -model (see Refs. [22,23]),

$$\mathcal{L}_{\text{eff}} = \frac{F_\pi^2}{4} \text{Tr}[(D_\mu U)(D^\mu U)^\dagger] + \frac{\langle q\bar{q} \rangle}{2} \text{Tr}[MU + M^\dagger U^\dagger], \quad (8)$$

¹The $q\bar{q}$ condensate also breaks $U(1)_A$, but this symmetry is anomalous and so the corresponding pseudo Nambu-Goldstone boson (corresponding to the η' of the visible SM) derives its mass separately from that of the other mesons generated by this mechanism.

where F_π is the π -axion decay constant, M is the mass matrix, $U = \exp(\frac{2\pi i}{F_\pi} \pi^a T_a)$, with T_a as the generators for $SU(N_f)$, π^a is the $N_f^2 - 1$ -dimensional vector of π -axions, and $\langle q\bar{q} \rangle$ is the condensate parameter.

While F_π , $\langle q\bar{q} \rangle$, and Λ_{dQCD} all arise from the physics of confinement, their precise relation is renormalization scheme dependent. When performing numerical estimates, we will approximate the decay constant F_π by the dQCD scale

$$F_\pi \sim \Lambda_{\text{dQCD}}, \quad (9)$$

though we note this relation is expected to hold only up to $\mathcal{O}(1)$ numerical factors, that can be precisely computed on the lattice.

The π -axion masses, on the other hand, depend on both the quark mass scale and dark QCD scale, according to the Gell-Mann–Oakes–Renner relation [24],

$$m_\pi^2 \simeq \frac{\langle q\bar{q} \rangle}{F_\pi^2} \sum_i m_{q_i}, \quad (10)$$

where m_{q_i} are the quark masses that constitute the π -axion in question and $\langle q\bar{q} \rangle \sim \Lambda_{\text{dQCD}}^3$ is the quark-antiquark condensate. This expression is less accurate when a given π -axion quark composition is the sum of more than two flavors, e.g., in the case of the SM η particle, whose flavor eigenstate involves the up, down, and strange quarks.

However, even in this case, Eq. (10) matches the observed mass up to a discrepancy of 4.6%, which is more than sufficient for our current purposes.

For the charged π -axions, the mass formula gets corrected due to loops of photons [22], both visible and dark. The charged π -axion mass can then be written as

$$m_{\pi_i^\pm}^2 \simeq \begin{cases} m_{\pi_i^0}^2 + 2\xi_i e'^2 F_\pi^2, & m_{\gamma'} < F_\pi \\ m_{\pi_i^0}^2 + 2\xi_i e'^2 \epsilon^2 F_\pi^2, & m_{\gamma'} > F_\pi, \end{cases} \quad (11)$$

where the two cases correspond to the dark photon dominating the loop correction vs the dark photon being integrated out, and the loop being dominated by a visible sector photon. Here $e = \sqrt{4\pi\alpha_e}$ is the dimensionless electric charge parameter, $e' = \sqrt{4\pi\alpha_{e'}}$ is the dimensionless dark electromagnetic (EM) coupling (where $\alpha_{e'}$ need not be $\sim \frac{1}{137}$), and ξ_i is an $\mathcal{O}(1)$ parameter unique to each π -axion, which can be positive or negative. For the majority of this work, we assume that $m_{\gamma'} > F_\pi$ in order to consider the various millicharge scenarios. We will comment on the $m_{\gamma'} < F_\pi$ case when needed and in the discussion, as it pertains to the muon $g - 2$ anomaly.

We henceforth focus on the case $N_c = 3$ and $N_f = 6$ in accordance with the dark Standard Model of the previous section. Using the $SU(6)$ generators from [25]² and defining the vector $\pi^a = (\pi_1, \dots, \pi_{35})$, we can write out the contraction $\pi^a \lambda_a$ as

$$\pi^a \lambda_a = \begin{pmatrix} \pi_3 + \frac{\pi_8}{\sqrt{3}} - \frac{\pi_{35}}{\sqrt{3}} & \pi_1 - i\pi_2 & \pi_4 - i\pi_5 & \pi_9 - i\pi_{10} & \pi_{15} - i\pi_{16} & \pi_{21} - i\pi_{22} \\ \pi_1 + i\pi_2 & -\pi_3 + \frac{\pi_8}{\sqrt{3}} - \frac{\pi_{35}}{\sqrt{3}} & \pi_6 - i\pi_7 & \pi_{11} - i\pi_{12} & \pi_{17} - i\pi_{18} & \pi_{23} - i\pi_{24} \\ \pi_4 + i\pi_5 & \pi_6 + i\pi_7 & -\frac{2\pi_8}{\sqrt{3}} - \frac{\pi_{35}}{\sqrt{3}} & \pi_{13} - i\pi_{14} & \pi_{19} - i\pi_{20} & \pi_{25} - i\pi_{26} \\ \pi_9 + i\pi_{10} & \pi_{11} + i\pi_{12} & \pi_{13} + i\pi_{14} & \pi_{29} + \frac{\pi_{34}}{\sqrt{3}} + \frac{\pi_{35}}{\sqrt{3}} & \pi_{27} - i\pi_{28} & \pi_{30} - i\pi_{31} \\ \pi_{15} + i\pi_{16} & \pi_{17} + i\pi_{18} & \pi_{19} + i\pi_{20} & \pi_{27} + i\pi_{28} & -\pi_{29} + \frac{\pi_{34}}{\sqrt{3}} + \frac{\pi_{35}}{\sqrt{3}} & \pi_{32} - i\pi_{33} \\ \pi_{21} + i\pi_{22} & \pi_{23} + i\pi_{24} & \pi_{25} + i\pi_{26} & \pi_{30} + i\pi_{31} & \pi_{32} + i\pi_{33} & \frac{\pi_{35}}{\sqrt{3}} - \frac{2\pi_{34}}{\sqrt{3}} \end{pmatrix}, \quad (12)$$

where $\lambda_a = 2T_a$.

In order to understand the quark composition of each π -axion, we imagine labeling the successive rows of Eq. (12) with the respective quark flavors u, d, s, c, b, t , while labeling the columns by the antiquarks with the same ordering. When a π -axion appears in the i th column and j th row, its composition is identified to be that of quark-antiquark labeling of that entry. For example, note that π_3 appears in the first row/column with a positive sign, and the second row/column with a negative sign, so we deduce $\pi_3 \sim u\bar{u} - d\bar{d}$. π_3 is therefore a real scalar and its own antiparticle. On the other hand, consider the π -axion $\pi_6 + i\pi_7$, which appears in the second column, third row of

Eq. (12). Its composition is thus $d\bar{s}$ and since it is complex, its antiparticle is $\pi_6 - i\pi_7$ of composition $\bar{d}s$.

The π -axion field content is given in Table I. Dark QCD where all six flavors of quark are light leads to 35π -axions. Of these, 17 are neutral and 18 are charged. Of the neutral π -axions, 5 are real pseudoscalars while the remaining 12 are grouped into 6 complex scalars. The charged π -axions are grouped into nine complex scalars. In summary, we have five real, neutral states, six complex, neutral states, and nine complex, charged states.

²Note that their form of λ_{16} has a typo, the $+i$ should be in the first column fifth row.

TABLE I. Spectrum of π -axions: Neutral real π -axions (top), complex neutral π -axions (middle), and charged π -axions (bottom). The first five π -axions are found along the diagonal of Eq. (12). The other 12 π -axions are organized into 6 complex neutral fields. Finally, 18 of the π -axions are organized into 9 charged π -axions. Here we have approximated $F_\pi \sim \Lambda_{\text{dQCD}}$ in expressing the mass formulas, and fixed $m_{\rho'} > F_\pi$ in the charged π -axion mass relation Eq. (11).

Spectrum of π -axions in dark QCD			
π -axion	Quark content	Mass squared (m_π^2)	Charge (ϵ)
Real neutral:			
π_3	$u\bar{u} - d\bar{d}$	$(c_1 + c_2)m_I F_\pi$	0
π_8	$u\bar{u} + d\bar{d} - 2s\bar{s}$	$((c_1 + c_2)m_I + c_3 m_{\text{II}})F_\pi$	0
π_{29}	$c\bar{c} - b\bar{b}$	$(c_4 m_{\text{II}} + c_5 m_{\text{III}})F_\pi$	0
π_{34}	$c\bar{c} + b\bar{b} - 2t\bar{t}$	$(c_4 m_{\text{II}} + (c_5 + c_6)m_{\text{III}})F_\pi$	0
π_{35}	$-u\bar{u} - d\bar{d} - s\bar{s} + c\bar{c} + b\bar{b} + t\bar{t}$	$((c_1 + c_2)m_I + (c_3 + c_4)m_{\text{II}} + (c_5 + c_6)m_{\text{III}})F_\pi$	0
Complex neutral:			
$\pi_6 \pm i\pi_7$	$d\bar{s}/\bar{d}s$	$(c_2 m_I + c_3 m_{\text{II}})F_\pi$	0
$\pi_9 \pm i\pi_{10}$	$u\bar{c}/\bar{u}c$	$(c_1 m_I + c_4 m_{\text{II}})F_\pi$	0
$\pi_{17} \pm i\pi_{18}$	$d\bar{b}/\bar{d}b$	$(c_2 m_I + c_5 m_{\text{III}})F_\pi$	0
$\pi_{19} \pm i\pi_{20}$	$s\bar{b}/\bar{s}b$	$(c_3 m_{\text{II}} + c_5 m_{\text{III}})F_\pi$	0
$\pi_{21} \pm i\pi_{22}$	$u\bar{t}/\bar{u}t$	$(c_1 m_I + c_6 m_{\text{III}})F_\pi$	0
$\pi_{30} \pm i\pi_{31}$	$c\bar{t}/\bar{c}t$	$(c_4 m_{\text{II}} + c_6 m_{\text{III}})F_\pi$	0
Charged:			
$\pi_1 \pm i\pi_2$	$u\bar{d}/\bar{u}d$	$(c_1 + c_2)m_I F_\pi + 2\xi_1(e\epsilon F_\pi)^2$	± 1
$\pi_4 \pm i\pi_5$	$u\bar{s}/\bar{u}s$	$(c_1 m_I + c_3 m_{\text{II}})F_\pi + 2\xi_2(e\epsilon F_\pi)^2$	± 1
$\pi_{15} \pm i\pi_{16}$	$u\bar{b}/\bar{u}b$	$(c_1 m_I + c_5 m_{\text{III}})F_\pi + 2\xi_3(e\epsilon F_\pi)^2$	± 1
$\pi_{11} \pm i\pi_{12}$	$d\bar{c}/\bar{d}c$	$(c_2 m_I + c_4 m_{\text{III}})F_\pi + 2\xi_4(e\epsilon F_\pi)^2$	∓ 1
$\pi_{23} \pm i\pi_{24}$	$d\bar{t}/\bar{d}t$	$(c_2 m_I + c_6 m_{\text{III}})F_\pi + 2\xi_5(e\epsilon F_\pi)^2$	∓ 1
$\pi_{13} \pm i\pi_{14}$	$s\bar{c}/\bar{s}c$	$(c_3 + c_4)m_{\text{II}} F_\pi + 2\xi_6(e\epsilon F_\pi)^2$	∓ 1
$\pi_{25} \pm i\pi_{26}$	$s\bar{t}/\bar{s}t$	$(c_3 m_{\text{II}} + c_6 m_{\text{III}})F_\pi + 2\xi_7(e\epsilon F_\pi)^2$	∓ 1
$\pi_{27} \pm i\pi_{28}$	$c\bar{b}/\bar{c}b$	$(c_4 m_{\text{II}} + c_5 m_{\text{III}})F_\pi + 2\xi_8(e\epsilon F_\pi)^2$	± 1
$\pi_{32} \pm i\pi_{33}$	$b\bar{t}/\bar{b}t$	$(c_5 + c_6)m_{\text{III}} F_\pi + 2\xi_9(e\epsilon F_\pi)^2$	∓ 1

IV. π -AXION-PHOTON INTERACTIONS

Experimental searches for axions are predicated on the characteristic interactions with the Standard Model photon and fermions. The π -axiverse is characterized by a mixture of parity-odd and parity-even couplings, as one might conventionally associate with an axion field and a dilaton field, respectively.

We focus on interactions with Standard Model photons. We consider four different vertices such that $\mathcal{L}_{\text{int}} = \sum_{i=1}^4 \mathcal{L}_{\text{int}}^{(i)}$, where, momentarily omitting the sum over species,

$$\mathcal{L}_{\text{int}}^{(1)} = \frac{\lambda_1}{2F_\pi} \epsilon^2(\pi^0) F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (13)$$

$$\mathcal{L}_{\text{int}}^{(2)} = \frac{\lambda_2}{2} \epsilon^2(\pi^+)(\pi^-) A_\mu A^\mu, \quad (14)$$

$$\mathcal{L}_{\text{int}}^{(3)} = \frac{\lambda_3}{2\Lambda_3^2} \epsilon^2(\pi^+)(\pi^-) F_{\mu\nu} F^{\mu\nu}, \quad (15)$$

$$\mathcal{L}_{\text{int}}^{(4)} = \frac{\lambda_4}{2\Lambda_4^2} \epsilon^2(\pi_i)(\pi_j) F_{\mu\nu} F^{\mu\nu}. \quad (16)$$

Here, $\mathcal{L}^{(1)}$ is the standard axion-photon coupling resulting from a triangle diagram, and $\mathcal{L}^{(2)}$ is the gauge covariant derivative of scalar QED, corresponding to the millicharges of the charged π -axions.

Meanwhile $\mathcal{L}^{(3)}$ and $\mathcal{L}^{(4)}$ arise as effective field operators arising from integrating out the heavy degrees of freedom in the dark Standard Model, the former interaction describing all charged π -axions and the latter describing all neutral (real and complex) π -axions. As in the visible SM, the weak bosons are electrically charged and can couple to the photon as a result. Therefore, at high energies, two neutral π -axions can interact with photons through a loop process involving weak bosons as in K - $\bar{K}$ mixing in the visible SM. The photons could be emitted by the dark $W^\pm$ or possibly by the off-shell quarks. A diagram illustrating this process is shown in Fig. 1. From this we can estimate the energy scales $\Lambda_{3,4}$ as simply $\Lambda_{3,4} \sim m_W = gv$.

The interactions (14)–(16) can describe pair creation, annihilation, or decay processes if the π -axions are two different species of the same charge. The heaviest states can proceed down a decay chain, however, the lightest complex π -axion (charged or neutral) cannot decay further and are

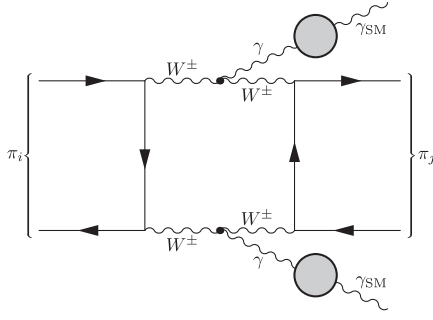


FIG. 1. A diagram illustrating a possible UV process contributing to the $\pi\pi\gamma\gamma$ effective vertices. All particles are presumed to reside in the dark SM unless explicitly noted. The gray blobs refer to kinetic mixing between the dark and visible photons.

completely stable. $\mathcal{L}_{\text{int}}^{(1)}$ is analogous to the Standard Model neutral pion interaction, which corresponds to a decay width given by

$$\Gamma_{(1)} = \varepsilon^4 \frac{\lambda_1^2 \alpha_e^2 m_i^3}{64\pi^3 F_\pi^2}. \quad (17)$$

Demanding the real, neutral π -axions be cosmologically stable can place approximate bounds on the parameters in our model, which we will discuss further in the following section.

V. COSMOLOGY OF THE π -AXIVERSE

There are many possibilities for the cosmological evolution of the π -axiverse [26–28]. The π -axions can range in mass across the whole spectrum conventionally associated with axionlike particles, including the benchmark mass for experiments such as ADMX, $m \sim 10^{-5}$ eV, “fuzzy dark matter” [29] with $m \sim 10^{-21}$ eV (see Ref. [8] for an analysis of this case), and much lighter particles which could be a subcomponent of dark matter (see Refs. [30,31]) or else contribute to dark energy.

We focus on the scenario in which the dark QCD phase transition occurs before cosmic inflation, i.e., we assume that

$$F_\pi > H_{\text{inf}}. \quad (18)$$

The relic density in π -axions can then be generated by the misalignment mechanism. Similar to the conventional cosmic history of axions [7], the π -axions are initialized after the symmetry breaking with a random value (misalignment), corresponding to the angular excitations of the system in the directions of the broken generators of the $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry, which is spontaneously broken by the dQCD condensate. The subsequent energy density is determined by the (π -)axion mass and initial misalignment. In the present context, the π -axions acquire masses due to the dark electroweak phase

transition, at which point the fields acquire a periodic potential. Since the fields have no method of selecting their position prior to the emergence of the potential, they may be misaligned with its minima and thus have differing energies with respect to each other depending on the degree of misalignment. The relative misalignment angles are a free parameter of this class of models.

Similar to a conventional axion, this scenario is constrained by cosmic microwave background isocurvature, which imposes an upper bound on the energy scale of inflation $H_{\text{inf}} \lesssim 10^{10} (F_\pi/M_{\text{pl}})$ GeV [7]. To simplify the analysis we further assume that the temperature of the Standard Model plasma is below the dark QCD energy scale at all times in the radiation dominated epoch, i.e.,

$$\Lambda_{\text{dQCD}} \sim F_\pi > T_{\text{max}}, \quad (19)$$

where T_{max} is the maximal temperature of the Standard Model plasma, which is at most $\sim \sqrt{H_{\text{inf}} M_{\text{pl}}}$. This prevents any freeze-in or freeze-out production of heavy states in the dark QCD sector, allowing us to focus solely on the light degrees of freedom.

We further demand that the π -axions are cosmologically stable and will not decay, e.g., to dark or visible sector photons. The neutral complex and charged π -axions are stable in our model, whereas the neutral can decay, into either dark photons or Standard Model photons. Decay into Standard Model photons gives the neutral scalar π -axions a lifetime,

$$\tau_\pi \approx (4 \times 10^7) \frac{F_\pi^2}{\varepsilon^4 m_\pi^3}, \quad (20)$$

while decay into dark photons is can be divided into two cases: (1) the massless dark photon, the same as above but $\varepsilon = 1$, and (2) the massive case, where decay is kinematically forbidden.

Constraints on decaying dark matter impose that the lifetime be at least $\gtrsim \mathcal{O}(10)$ times longer than the age of the Universe $\tau_U = H_0^{-1}$ (see, e.g., [32] for a detailed analysis). This is easily satisfied in our model, wherein the large hierarchy $F_\pi/m_\pi \gg 1$ is a feature of the model *a priori*. For example, a decay constant $F_\pi = 10^{11}$ GeV, mass $m_\pi = 10^{-5}$ eV, and unit ε , yields $\tau_\pi \sim 10^{29} H_0^{-1}$, well in agreement with observational constraints. Larger decay constants and smaller masses, or smaller millicharge, will only increase the neutral π -axion lifetime. It additionally follows that neutral π -axions can be a stable axionlike dark matter candidate even in the case that the dark photon is massless, since in this case the lifetime is simply Eq. (20) with $\varepsilon = 1$.

A late-time relic density of π -axions can be produced by the conventional misalignment mechanism (see Ref. [7] for a review). The total relic density in π -axions produced via misalignment is given by

$$\Omega_\pi = \frac{1}{6} (9\Omega_r)^{3/4} \frac{F_\pi^2}{M_{\text{Pl}}^2} \sum_i \left(\frac{m_{\pi_i}}{H_0} \right)^{1/2} \theta_{\pi_i}^2, \quad (21)$$

where the sum is over all light stable π -axion fields, and we assume $m_{\pi_i} > 10^{-28}$ eV for simplicity (see Ref. [7] for the relic density when $m_{\pi_i} < 10^{-28}$ eV). The sum is fixed by observation, $\Omega_{\text{DM}} h^2 = 0.12$ [33]. The relative contribution of each π -axion to the relic density depends on the mass spectrum of π -axions, which in turn encodes the dark quark masses, dark QCD scale ($\Lambda_{\text{dQCD}} \sim F_\pi$), and the millicharge ε .

Analogous to conventional axion models, enforcing that the relic density match observation leads to a relation between the dark matter mass and decay constant, however, in this case there is a *spectrum* of masses and a set of random initial misalignments. We detail the features of this spectrum in the subsections to come.

We are interested in sub-eV π -axion masses, so that at least one of the fields can evolve as a coherent scalar that survives until the present day. Demanding that the π -axions constitute an $\mathcal{O}(1)$ fraction of the observed dark matter density then enforces a lower bound on the decay constant,

$$F_\pi > 8.8 \times 10^{10} \text{ GeV}. \quad (22)$$

From this we deduce an upper bound mass range for the mass of the dark quarks for the lightest to be sub-eV,

$$m_q \lesssim 5 \times 10^{-20} \text{ eV}. \quad (23)$$

The details of the spectrum of π -axions is dictated by a combination of the quark masses and the millicharge ε . We investigate each of these in turn.

A. Parameter space for dark matter

1. Quark masses

We first consider the case wherein the millicharge makes a subdominant contribution to the π -axion masses, and we consider $\varepsilon = 0$ for simplicity.

In the case that the three generations are degenerate, $m_I = m_{\text{II}} = m_{\text{III}} = m_q$, all 35 π -axions have masses around $m_\pi \sim \sqrt{m_q F_\pi}$, with a distribution set by the c_i . In the completely degenerate case where all c_i are equal to 1, then there are solely three unique masses,

$$\begin{aligned} m_2 &\sim \sqrt{2m_q F_\pi}, \\ m_3 &\sim \sqrt{3m_q F_\pi}, \\ m_6 &\sim \sqrt{6m_q F_\pi}. \end{aligned} \quad (24)$$

All of the complex π -axions as well as, for example, π_3 and π_{29} , are composed of two unique quarks with mass m_2 . The two real π -axions π_8 and π_{29} are composed of three quarks and have mass m_3 , while π_{35} is composed of six quark flavors and is the most massive, with its mass approximately near m_6 .

If we relax the condition that $m_{\text{I,II,III}}$ are equal, then the spectrum splits into three distinct regions, with π -axion masses clustered around each of $\sqrt{m_I F_\pi}$, $\sqrt{m_{\text{II}} F_\pi}$, and $\sqrt{m_{\text{III}} F_\pi}$. We note that, *a priori*, the mass hierarchies are inputs of the model and thus may be set arbitrarily, but not so far apart as to spoil the flavor symmetry of the quarks.

To understand the mass spectrum, we compute the statistical average spectrum, shown in Fig. 2. We consider a scenario where $m_I < m_{\text{II}} < m_{\text{III}}$, with m_I , m_{II} , m_{III} given by 1, 3, and 6×10^{-5} eV, respectively, and with

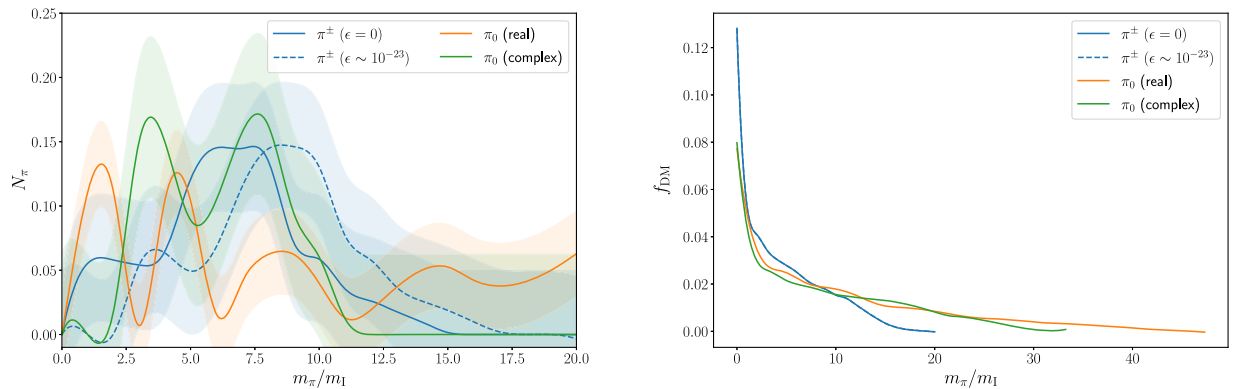


FIG. 2. Statistical mean (solid) and $\pm 1\sigma$ (shaded band) π -axion mass spectra (left) and mass distribution of fractional contribution to DM relic density (right) for m_I , m_{II} , m_{III} given by 1, 3, and 6×10^{-5} eV, respectively. Orange is the real pseudoscalar neutral π -axions, green is complex neutral π -axions, and blue is charged π -axions. The charged pions are shown with vanishing millicharge ($\varepsilon = 0$) and millicharge $\varepsilon \sim 10^{-23}$ with the solid and dashed blue lines, respectively. The dark matter relic density is nearly evenly split between the three classes of π -axions, and the distribution features a heavy tail toward larger π -axion masses. Here we assume a heavy dark photon, $m_{\gamma'} > F_\pi$.

the splitting coefficients $c_i \sim 1$ drawn from uniform distributions, $c_i = [0.7, 1.3]$. In this case, the π -axion masses are spread out across the spectrum, with larger masses possible; note that the charged and neutral complex distributions mostly mirror each other. Using Eq. (21), it is possible to compute a breakdown of the dark matter densities in terms of each species of π -axion; as expected, since the misalignment angles are *a priori* completely uncorrelated, then no one particular type dominates, as seen in Fig. 2.

2. Dark photon mass

If $m_{\gamma'} < F_\pi$, then the electromagnetic contribution to the charged pion mass is dominated by the dark photon, generating a large ($\approx F_\pi$) mass for the charged pions, see Eq. (11). On the other hand, for $m_{\gamma'} \gg F_\pi$, the charged pion mass is dominated by the visible SM photon and the charged π -axion mass is ε -suppressed as $m_\pm \sim \varepsilon F_\pi$. In the former case, the charged π -axions will decay and annihilate to dark photons before the onset of inflation, leaving no trace in the late Universe. In the latter case, the decay of the charged π -axions can be delayed, but since the decay channel is to SM photons, there is again no dark radiation constraint. As stated previously, we will proceed focusing on the case $m_{\gamma'} > F_\pi$.

3. Millicharge ε

When $\varepsilon = 0$, the charged π -axion masses are comparable in size with the neutral π -axions. In this case, the relic density is distributed evenly across all species, according to the distribution of masses and initial misalignments.

When ε is nonzero, the electromagnetic contribution to the charged masses, $\sim \varepsilon F_\pi$, can become relevant. The charged π -axions will be coherent scalars in the late Universe, i.e., have $m < \text{eV}$, only if the millicharge is below an upper bound given by $\varepsilon < \frac{1 \text{ eV}}{F_\pi}$. Given that $F_\pi \gtrsim 10^{11} \text{ GeV}$ in order for sub-eV particles to have any significant relic density, this translates to a bound $\varepsilon \lesssim 10^{-20}$ in order for the charged π -axions to enjoy an axionlike cosmological history and contribute to the dark matter (DM) relic density via their misalignment production. This bound scales with F_π as $10^{11} \text{ GeV}/F_\pi$.

Nonzero ε also allows the neutral, real π -axions to decay to two photons, via the conventional axion channel. Nonetheless, the neutral π -axions remain cosmologically stable even for $\mathcal{O}(1)$ millicharge: they have lifetime longer than the age of the Universe if

$$10^{-34} \varepsilon^4 \left(\frac{m_\pi}{10^{-5} \text{ eV}} \right)^3 \left(\frac{10^{13} \text{ GeV}}{F_\pi} \right)^2 \lesssim 1. \quad (25)$$

From this we conclude that sub-eV neutral π -axions are a stable dark matter candidate for the full range of ε and F_π we consider.

For $\varepsilon \gg 10^{-20}$, the charged π -axions will be heavy [$m_\pm \sim \varepsilon F_\pi$, see Eq. (11)] and cease to evolve as a coherent scalar field already in the very early Universe. In this case one expects a negligible relic density in charged π -axions via misalignment.

Assuming the range of F_π given by Eq. (19), this regime of ε can be further subdivided into two regimes: (1) for $\varepsilon > \sqrt{H_{\text{inf}}/M_{\text{pl}}}$, then $m_\pm > H_{\text{inf}}$ and the initial misalignment of π -axions will decay before the onset of inflation, leaving no discernible trace in the postinflationary Universe; (2) for $\varepsilon < \sqrt{H_{\text{inf}}/M_{\text{pl}}}$, the decay of the coherent charged π -axion fields will occur after inflation, and the resulting nonrelativistic charged π -axions will scatter with SM particles and annihilate to SM photons with a rate $\propto \varepsilon^2$. For a detailed discussion of constraints on ε from charged π -axions, we refer the reader to [8] and note here that the values of ε needed to engineer charged π -axions that are low mass enough to be cosmologically relevant are so small as to easily satisfy constraints.

Finally, we can consider $\varepsilon \lesssim \mathcal{O}(1)$. In this case, all of the charged π -axions will be extremely massive $m \sim F_\pi$. While there are stringent bounds on electrically charged dark matter [10,34–36], these bounds are easily satisfied for heavy masses (e.g., $m \sim F_\pi$) and particles with zero relic density. It is therefore important to check the relic density produced by means other than misalignment, such as by freeze-in or freeze-out processes [37–40]. We will consider both of these scenarios in turn in the subsequent section.

In summary, the ε dependence of the model is split into two regimes: $\varepsilon \ll 10^{-20}$ and $\varepsilon \gg 10^{-20}$, corresponding to the presence or absence of a relic density of charged π -axions produced via misalignment. For $\varepsilon \gg 10^{-20}$, the dark matter relic density is composed of real and complex neutral π -axions.

B. Freeze-out and freeze-in of charged π -axions

As one considers light charged π -axions, $m_\pm \ll F_\pi$, there is also the possibility of their freeze-in or freeze-out production. A sufficient condition to satisfy bounds on millicharged matter [36] is that the relic density generated through freeze-in or freeze-out is vanishingly small. We consider these in turn.

Freeze-in. A detailed derivation of the relic density of heavy millicharged dark matter through freeze-in QED processes was given by one of the present authors in [8]. The main result is expressed in terms of the equation of state during reheating $w_{\text{re}} = p/\rho$ as

$$\Omega_{\pi^\pm} h^2 \simeq (5 \times 10^{-3}) \mathcal{A}(w_{\text{re}}) \varepsilon^4 \frac{\exp[10(3 - \alpha\sqrt{\beta})]}{\beta^{4/(1+w)-1/2}}, \quad (26)$$

where $\mathcal{A}(w) = \frac{(10\pi^2)^{-w/(1+w)}}{2(1+w)}$, $\alpha = \frac{m_{\pi^\pm}}{\sqrt{H_{\text{inf}} M_{\text{pl}}}}$, and $\beta = \frac{\sqrt{H_{\text{inf}} M_{\text{pl}}}}{T_{\text{reh}}}$. To guarantee a negligible relic density $\Omega_{\pi^\pm} h^2 \ll 0.12$, one

may focus on the limiting case that $\varepsilon \sim \mathcal{O}(1)$. As an example, consider $H_{\text{infl}} = 10^{10}$ GeV and reheating with $w = 0$, with a maximum temperature $T_{\text{re}} \sim 10^9$ GeV. The relic density in this case reads

$$\Omega_{\pi^\pm} h^2 \simeq 4 \times 10^{-8} \varepsilon^4 e^{-\varepsilon F_\pi / (10^{11} \text{ GeV})}, \quad (27)$$

implying that the relic density, even for $\varepsilon = 1$, is at most $\sim 10^{-7}$ for $F_\pi > 10^{11}$ GeV, and thus charged π -axions produced via freeze-in constitute an extremely small fraction of the total dark matter. From this we infer that freeze-in production of charged π -axions, for the axion dark-matter-like parameter regime $F_\pi > 10^{11}$ GeV, always yields a vastly subdominant contribution to the dark matter density.

Freeze-out. Precluding freeze-out production may be enforced by demanding that the charged π -axions are never in thermal equilibrium. The following bound on the millicharge follows:

$$\varepsilon \ll (4 \times 10^2) \left(\frac{m_{\pi^\pm}}{M_{\text{pl}}} \right)^{1/2} \left(\frac{m_{\pi^\pm}}{T_{\text{max}}} \right)^{1/4} e^{\frac{m_{\pi^\pm}}{2T_{\text{max}}}}. \quad (28)$$

In Eq. (28), T_{max} is the maximum temperature achieved during reheating [41]. We can equivalently write Eq. (28) as the transcendental relationship

$$\varepsilon \ll (6.5 \times 10^8) \left(\frac{F_\pi}{M_{\text{pl}}} \right)^2 \left(\frac{F_\pi}{T_{\text{max}}} \right) e^{2\varepsilon F_\pi / T_{\text{max}}}, \quad (29)$$

which again takes the assumption that $m_{\pi^\pm} \sim \varepsilon F_\pi$. Consider, for example, a grand unified theory (GUT) scale decay constant, $F_\pi = 10^{15}$ GeV, and a universe that reheats to a temperature well below the GUT scale, e.g., $T_{\text{max}} \sim 10^{12}$ GeV. In this case the bound on ε reads $\varepsilon \ll 10^4 e^{100\varepsilon}$. This is satisfied in the full range of ε from 0 to 1. Thus we conclude that charged π -axions are too heavy to be in thermal equilibrium, even with unit electric charge.

C. Summary of dark matter scenarios

The possibilities for dark matter can be summarized as follows:

- (i) Vanishing millicharge ($\varepsilon = 0$) and degenerate quark generation mass scale ($m_{\text{I}} = m_{\text{II}} = m_{\text{III}} = m_q$): dark matter is a combination of all 35π -axions, with masses distributed near the shared mass scale $m_\pi \sim \sqrt{m_q F_\pi}$.
- (ii) $\varepsilon = 0$ with splitting between quark generations: $m_{\text{I}} < m_{\text{II}} < m_{\text{III}}$. Dark matter is a mixture of 35π -axions, with masses clustered around the three mass scales.
- (iii) $0 < \varepsilon < 10^{-20}$: the charged π -axions get a mass correction $m_{\pi^\pm} \sim \varepsilon F_\pi$ and can become much heavier than the neutral (real and complex) π -axions, adding

additional structure to both the degenerate and nondegenerate generation cases.

- (iv) $10^{-20} \ll \varepsilon \lesssim \mathcal{O}(1)$: the charged π -axions are heavy and do not contribute to the late-time relic density. Dark matter is composed of the real and complex neutral π -axions, with masses distributed according to $m_{\text{I}}, m_{\text{II}}, m_{\text{III}}$.

VI. π -AXION DARK MATTER DETECTION PROSPECTS

The search for ultralight dark matter candidates such as axions is a multifaceted and growing field, see, e.g., [42] for a recent review. These approaches can be directly applied to π -axions, targeted at interactions of π -axions with the Standard Model that are generated both by the millicharge portal and by the heavy electroweak sector. For concreteness, here we focus on detection via photons, utilizing the interactions (13)–(16).

A. Direct detection via parity-odd portal

The long-standing experimental approach to axion direct detection has been to exploit the axion-photon coupling predicted by anomaly considerations, namely the coupling of the pseudoscalar axion to the parity-odd combination of electromagnetic field strengths $F\tilde{F} \equiv \varepsilon_{\mu\nu\sigma\rho} F^{\mu\nu} F^{\sigma\rho}$.

The effective interaction following from the triangle anomaly diagram is given by

$$\mathcal{L}_{\text{int}}^{(1)} \sim \frac{\lambda_1}{2F_\pi} \alpha_e \varepsilon^2 (\pi^0) F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (30)$$

which is of the same form as the standard axion-photon interaction $\mathcal{L}_{a\gamma\gamma} = g_{a\gamma\gamma} a F\tilde{F}/4$ [43,44], where in our case we have

$$g_{\pi\gamma\gamma} \sim \frac{\alpha_e \varepsilon^2}{F_\pi} \lambda_1. \quad (31)$$

The parameter λ_1 is analogous to the anomaly coefficient appearing in conventional axion models and, depending on the precise details of the dark Standard Model, can be $\mathcal{O}(1) - \mathcal{O}(10)$. The scaling with ε^2 reflects the millicharge portal of the dark π -axions to the Standard Model photon.

This coupling is the focus of many axion dark matter haloscope experiments both in the μeV and above mass range and the sub- μeV range (see Ref. [45] for a thorough overview of the ongoing efforts). Of note is the ADMX Collaboration [46], whose attention is to the 1 – 20 μeV axion mass range (see also [47–49] for recent proposals of plasma haloscopes). Their most recent report bounds the axion decay constant at $g_{a\gamma\gamma} \lesssim 10^{-13} \text{ GeV}^{-1}$ for the narrow mass range $m_a \sim 19.83\text{--}19.845 \mu\text{eV}$ [50], while [51] bounds $g_{a\gamma\gamma} \lesssim 10^{-15} \text{ GeV}^{-1}$ in the $m_a \sim 3\text{--}4 \mu\text{eV}$ range. The HAYSTAC Collaboration's phase 1 [52]

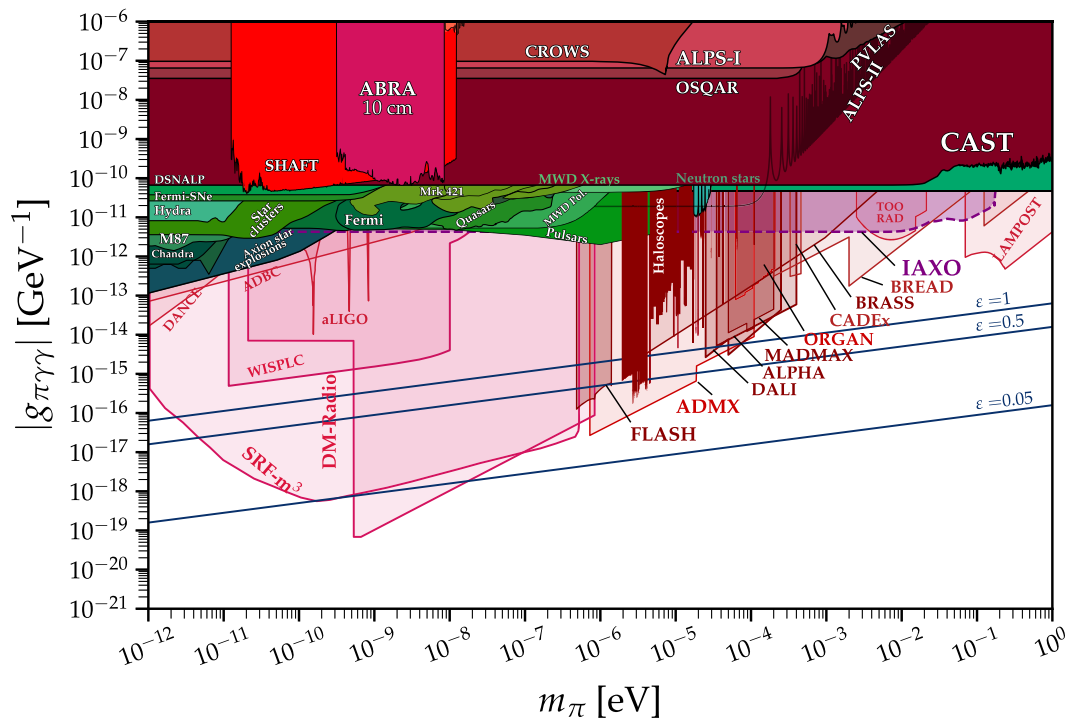


FIG. 3. An exclusion figure illustrating the parameter space limits for a simplified limit of the π -axiverse model, described in the text. For this scenario, the five neutral π -axions are assumed to have a common mass scale and equal misalignment angles $\theta_i = 1$ and to constitute all of the dark matter density. The result is that $g_{\pi\gamma\gamma} \propto \lambda \epsilon^2 \theta m_\pi^{\frac{1}{2}}$, a function of the common mass scale m_π and depends quadratically on the millicharge ϵ and linearly on the coupling λ_1 , taken to be $\lambda_1 = 1$, and misalignment angle θ_i , taken to be $\theta_i = 1$. The values of $\epsilon = 1, 0.5$, and 0.05 are shown as solid lines. Figure modified from [53].

similarly reported the bound $g_{a\gamma\gamma} \lesssim 10^{-15} \text{ GeV}^{-1}$ for $m_a \sim 23.15\text{--}24 \text{ } \mu\text{eV}$. There are various other probes that place an upper bound on $g_{a\gamma\gamma}$ across the larger mass range of $m_a \sim 10^{-14}\text{--}10^{-5} \text{ eV}$, see, e.g., the collection of constraint plots publicly available at [53]. Throughout that mass range, smaller values of $g_{a\gamma\gamma}$ are consistent with data. Due to the millicharge parameter ε in Eq. (31), our model can easily accommodate such small couplings.

In our model, there are five different π -axions that participate in the interaction, mainly π_3 , π_8 , π_{29} , π_{34} , and π_{35} (see Table I), each of which can have a sizable relic density produced via misalignment. ADMX therefore has the potential to see five distinct resonances corresponding to the five different masses. In the completely degenerate case where $m_{\text{I}} = m_{\text{II}} = m_{\text{III}} \equiv m$, and all the c_i are equal, then $m_{\pi_3} = m_{\pi_{29}} \sim m_2$, $m_{\pi_8} = m_{\pi_{34}} \sim m_3$, and $m_{\pi_{35}} \sim m_6$ where m_2 , m_3 , and m_6 are defined in Eq. (24). This implies that exactly three resonances would be observed at approximately $1.4\sqrt{mF_\pi}$, $1.7\sqrt{mF_\pi}$, and $2.45\sqrt{mF_\pi}$; these may be more or less degenerate depending on the resolution of the detector. In the more general case of fully nondegenerate quark masses, there will be exactly five peaks which potentially have greater than $\mathcal{O}(1)$ separation.

As an illustrative example, proposed limits arising from this scenario are shown in Fig. 3, a modified version of the

figure in [53]. Here we consider only the constraints on a single signal, without considering the integrated effect of five correlated signals of the neutral π -axions. The expected couplings as a function of ε are shown as solid lines, with the shading indicating the range of possible observations due to the spread of values. The fully degenerate scenario was implemented using Eq. (21), assuming the neutral π -axions constitute the entirety of the DM so that $\Omega_\pi h^2 = \Omega_{\text{DM}} h^2 = 0.12$ and that all misalignment angles are degenerate and equal. This allows the extraction of F_π as a function of m_π , the common π -axion mass scale in this scenario, and thus the computation of $g_{\pi\gamma\gamma}$ as a function of the mass and millicharge. In general, $g_{\pi\gamma\gamma} \propto \lambda \varepsilon^2 \theta_i m_\pi^{-\frac{1}{2}}$. For concreteness in Fig. 3, we assume $\lambda_1 = 1$, $\theta_i = 1$, and display various curves for ε .

Parts of the parameter space of this scenario are already excluded by experiments such as ADMX, especially for greater values of the millicharge such as $\epsilon \sim 1$; smaller values are mostly beyond experimental reach for many present and future experiments. DM-Radio can probe some regions of this parameter space, particularly for $m_\pi \sim 10^{-10}$ – 10^{-7} . Reducing λ_1 or ϵ , or decreasing the misalignment angle θ_i moves this scenario out of almost all present experimental exclusions. It is worth emphasizing that this benchmark scenario is simplified, and that a more thorough

treatment would require statistical averages over distributions of parameters.

B. Atomic oscillations via the parity-even portal

An alternative, more recent approach to ultralight dark matter detection is to exploit the possibility of coupling to parity-even combinations of fields (see, e.g., [54]), such as couplings to $F_{\mu\nu}F^{\mu\nu}$. We consider the parity-even interactions of the π -axions given by Eqs. (14)–(16). All of the π -axions participate in the parity-even portal. It was observed in [55] that a simple F^2 coupling induces a small oscillation in the fine structure constant α_e . Here we apply this to the π -axion scenario.

Consider homogeneous field profiles similar to the standard axion case,

$$\begin{aligned}\pi_i^r(t) &= \pi_{i,0}^r \cos(m_i t + \delta_i), \\ \pi_j^c(t) &= \pi_{j,0}^c e^{i\theta_j} \cos(m_j t + \delta_j),\end{aligned}\quad (32)$$

where the r and c superscripts denote real or complex, the $\pi_0^{r/c}$ are dimensionful constants which we take to be real, the θ_j are global phases, and the δ_j are initial, random phases.

The interactions formally include a sum over species but, for simplicity, let us discuss the diagonal terms $\sim |\pi|^2 F^2$. The correction to the fine structure constant is [56]

$$\alpha_e(t) = \alpha_e \left(1 + \frac{2\lambda e^2}{\Lambda^2} \sum_i |\pi_{i,0}|^2 \cos^2(m_i t + \delta_i) \right), \quad (33)$$

for each species that still is in abundance today [the factor of λ/Λ^2 in Eq. (33) is λ_3/Λ_3^2 for charged species and λ_4/Λ_4^2 for neutral species]. The amplitudes are related to the local dark matter density as

$$\pi_{i,0} \simeq \frac{\sqrt{2\rho_{\text{DM}}^i}}{m_{\pi_i}}, \quad (34)$$

where ρ_{DM}^i is the contribution from π_i . As is clear from Fig. 2, the lightest π -axions will contribute most to the local energy density, thus their amplitudes will be largest contribution to Eq. (33) according to Eq. (34).

The experiment discussed in [56] states that the oscillations in α_e are potentially observable for an amplitude as small as 10^{-16} , which translates to the requirement

$$\frac{\rho_{\text{DM}}^i e^2}{\Lambda^2 m_{\pi_i}^2} \gtrsim 10^{-15}. \quad (35)$$

A recent estimate for the local dark matter density is given in [57] as $\rho_{\text{DM,loc}} \sim 0.39 \pm 0.09 \text{ GeV/cm}^3$. For the example of fuzzy dark matter, with $m = 10^{-21} \text{ eV}$, composing all of the relic density, then $\Lambda < 10^8 \text{ eV}$ in

order to be observable. Assuming $\Lambda \sim m_W \sim gv$, with $v \sim F_\pi \sim 10^{16} \text{ GeV}$ for fuzzy dark matter, this requires a small gauge coupling $g < 10^{-8}$. There is a broad parameter space of π -axion mass, millicharge ϵ , and dark W boson mass m_W , where the oscillations can be in the observable range.

Our model is distinct from the standard axion case again due to the potential for multiple contributions in the sum (33), which encodes the tightly packed discreteness of π -axion masses. Any observed oscillations in α_e would not be able to be fit with a single frequency or amplitude, even in the case where the dark quarks are fully degenerate, as the initial phases δ_i as well as the masses, will be different.

C. Parametric resonance of photons in axion stars

Finally, we consider the impact of the coherent oscillations of the π -axion fields on excitations of the Standard Model photon field. As discussed in [7], the axion couplings to photon can lead to a parametric resonance of the latter, which is enhanced in dense environments. This phenomenon is well studied in the case of conventional axions, see Refs. [58–63], where it can allow for indirect detection via various astrophysical signatures. Parametric resonance has also been studied in the context of ultralight millicharged dark matter in [64]. Additionally, the gravitationally induced resonance of axion fields and gravitons has been studied in [65,66]. Here we study the possibility of parameter resonance production of photons in the π -axiverse.

Both parity-odd and parity-even portals will contribute to parametric resonance of photons. Including the sum over π -axion species, the Lagrangian including all interactions for the SM photon is

$$\begin{aligned}\mathcal{L}_{U(1)_{\text{EM}}}^{\text{int}} &= -e^2 \left(\frac{\lambda_3}{2\Lambda_3^2} \sum_{i,j}^{\text{charged}} \pi_i \pi_j^* + \frac{\lambda_4}{2\Lambda_4^2} \sum_{i,j}^{\text{neutral}} \pi_i \pi_j^* \right) F_{\mu\nu} F^{\mu\nu} \\ &\quad - \frac{\lambda_1}{2F_\pi} e^2 \sum_i^{\text{neutral, R}} \pi_i F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{\lambda_2 e^2 e^2}{2} \\ &\quad \times \sum_{i,j}^{\text{charged}} \pi_i \pi_j^* A_\mu A^\mu + \text{H.c.}\end{aligned}\quad (36)$$

We then take the Fourier representation of $\vec{A}$ as

$$\vec{A}(t, \vec{x}) = \sum_{\pm} \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \hat{\epsilon}_{\vec{k}, \pm} A_{\vec{k}, \pm}(t) + \text{c.c.}, \quad (37)$$

and for the π -axions, we assume the field profiles Eq. (32). The equation for the mode functions is then of the form

$$\begin{aligned}0 &= (1 + P(t)) (A_{\pm}'' + k^2 A_{\pm}) + B(t) A_{\pm}' \\ &\quad + (C_{\pm}(t) k + D(t)) A_{\pm},\end{aligned}\quad (38)$$

where explicitly,

$$P(t) = \frac{4\lambda_3}{\Lambda_3^2} \epsilon^2 \sum_{i,j}^9 \pi_{i,0}^c \pi_{j,0}^c \cos(\theta_i - \theta_j) \cos \varphi_i(t) \cos \varphi_j(t) \\ + \frac{2\lambda_4}{\Lambda_4^2} \epsilon^2 \left[2 \sum_{i,j}^6 \pi_{i,0}^c \pi_{j,0}^c \cos(\theta_i - \theta_j) + \sum_{i,j}^5 \pi_{i,0}^r \pi_{j,0}^r \right. \\ \left. + 4 \sum_{i=1}^5 \sum_{j=1}^6 \pi_{i,0}^r \pi_{j,0}^c \cos \theta_j \right] \cos \varphi_i(t) \cos \varphi_j(t), \quad (39)$$

$$B(t) = -\frac{4\lambda_3}{\Lambda_3^2} \epsilon^2 \sum_{i,j}^9 \pi_{i,0}^c \pi_{j,0}^c \cos(\theta_i - \theta_j) \\ \times (m_i \sin \varphi_i(t) \cos \varphi_j(t) + m_j \cos \varphi_i(t) \sin \varphi_j(t)) \\ - \frac{2\lambda_4}{\Lambda_4^2} \epsilon^2 \left[2 \sum_{i,j}^6 \pi_{i,0}^c \pi_{j,0}^c \cos(\theta_i - \theta_j) + \sum_{i,j}^5 \pi_{i,0}^r \pi_{j,0}^r \right. \\ \left. + 4 \sum_{i=1}^5 \sum_{j=1}^6 \pi_{i,0}^r \pi_{j,0}^c \cos \theta_j \right] (m_i \sin \varphi_i(t) \cos \varphi_j(t) \\ + m_j \cos \varphi_i(t) \sin \varphi_j(t)), \quad (40)$$

along with

$$C_{\pm}(t) = \pm \frac{2\lambda_1}{F_{\pi}} \epsilon^2 \sum_i^5 \pi_{i,0}^r m_i \sin \varphi_i(t), \quad (41)$$

defining the shorthand $\varphi_i(t) = m_i t + \delta_i$. In both (39) and (40), the sum over the first line is the nine charged species, the other three sums are over the six complex and five real neutral species, while the sum in (41) is over the five real, neutral species. The $D(t)$ function is the sum over charged species,

$$D(t) = 2\lambda_2 \epsilon^2 \sum_{i,j}^9 \pi_{i,0}^c \pi_{j,0}^c \cos(\theta_i - \theta_j) \cos \varphi_i(t) \cos \varphi_j(t). \quad (42)$$

Inserting these functions into Eq. (38) results in a special form of Hill's equation, more complicated than the Mathieu equation generally encountered in axion cosmology.

In the most general case where all of the π -axion masses differ, there will clearly be an exceedingly rich resonant structure due to the different in driving frequencies and amplitudes. However, even in the case where the dark quark masses are fully degenerate, the parametric resonance will differ from the standard axion scenario [67–72]. The sums in Eqs. (40) and (41) slightly simplify due to Eq. (34); following the discussion of mass hierarchies above, there are just three mass scales for the neutral π -axions. Since the interactions turn off when $\epsilon = 0$, we generically have nine

unique masses for the charged π -axions. In principle, the numerator in Eq. (34) can be different for all stable π -axions, even if they have the same masses, and we also have no reason to expect the initial phases δ_i to be related. In summary, even in the simplest case where the quark masses are fully degenerate, the model predicts a very rich resonant structure.

In scenarios where the frequency of the produced photons is observable, as in [59], the π -axiverse predicts a rich signal due to this resonance structure. The combined effect of these resonances would also accelerate the instability of axion stars. We leave these possibilities to future work.

VII. CONCLUSION

In this work we have developed the theory, cosmology, and observables of the π -axiverse. The π -axiverse joins the string axiverse as a possible origin for ultralight axionlike particles, which can be probed by a variety of cosmological and direct approaches, see Ref. [7] for a review. In its simplest form, the π -axiverse emerges from a dark Standard Model wherein all six flavors of quark are ultralight. For N_f flavors of light quark, this leads to $N_f^2 - 1$ ultralight π -axions. This scenario leads to multiple species of axion dark matter characterized by spectrum masses and a mixture of parity-odd and parity-even couplings, leading to complementary signals in a range of observable windows. The parameter space for this class of models is large and the signatures may be diverse. The authors are currently working on a follow-up examination of that parameter space, the production of further figures, and discussion to guide study of the model.

This scenario offers many interesting directions for future work:

Fuzzy π -axion dark matter. An ultralight boson with mass $m \sim 10^{-21}$, known as fuzzy dark matter [29], is a compelling resolution of the core-cusp problem [73,74]. However, this scenario faces tight constraints, in particular from stellar heating due to wave interference [75,76]. The latter can be ameliorated by introducing a large number of species of fuzzy dark matter fields [77,78]. The π -axiverse can naturally realize this latter scenario, via degenerate quark masses and vanishing millicharge ϵ . For an analysis of a scenario with three π -axions, we refer the reader to [8].

π -axion self-interactions. A distinguishing feature of the π -axiverse are the interactions not only with the Standard Model, but also self-interactions of π -axions, e.g., quartic interactions π^4 and $|\pi_i|^2 |\pi_j|^2$. Self-interactions of ultralight dark matter can play an important role in structure formation [79], boson stars [80], and Galactic dynamics [81,82]. The π -axion self-interactions from dark QCD physics are given by the higher-order terms of chiral perturbation theory [22,23]. The self-interactions receive additional contributions from the dark Standard Model,

e.g., after integrating out the dark W bosons. It follows that there is considerable freedom in setting the π -axion self-interactions, and these could be much larger than the conventional axion ($\sim m^2/f_a^2$). In addition, the axion quality problem that is thought to plague the QCD axion, wherein the axion mass receives large corrections from higher-order operators, could also be in play here. A future work could consider the approach of [83,84] where suitable charge assignments can cancel the effects of dangerous operators.

Direct detection of charged π -axions and charged leptons. For ϵ in an intermediate range within $10^{-20} \ll \epsilon \ll 1$, the charged π -axions may have a small enough mass yet strong enough coupling to be accessible to a variety of experimental probes of millicharged particles, see Table I of [8]. Similarly, for smaller charged lepton masses, $m \ll F_\pi$, the charged lepton masses may be accessible to experiment directly.

Direct detection of dark photons. Throughout this work, we have restricted to the case of $m_{\gamma'} > F_\pi$. However, there is an active experimental effort searching for dark photons at much lower mass scales, see Ref. [85] for a review. The detection or constraints on ϵ impact the model prediction for the charged π -axion mass and thus provide a complementary view into the model.

Indirect detection of (superheavy) baryons: As emphasized in [8], the baryons of the dark QCD in this model are a candidate for a superheavy dark matter component: the lightest baryon has mass $m_b \sim \Lambda_{\text{dQCD}} \sim F_\pi \gtrsim 10^{11}$ GeV. Superheavy dark matter is characterized by its own array of observables [86]. By independently probing the decay

constant and millicharge, this provides a complementary window into the π -axiverse.

Embedding within string theory. Finally, while the π -axiverse makes no recourse to string theory, we note that it can be straightforwardly realized in variants of the intersecting brane constructions of the Standard Model in type IIB string theory [87]. The dark quarks in this context correspond to open strings connecting two stacks of branes, with identically massless quarks occurring at the intersection. Nearly massless quarks, as considered here, therefore arise for nearly intersecting branes.

This serves a useful purpose of realizing ultralight axionlike particles in the face of the Stückelberg mechanism, realized by D3-branes at singularities [88–91] (for a recent review see Ref. [92]), wherein an anomalous $U(1)$ “eats” the axion to become massive through the Stückelberg mechanism.

We leave these and other interesting directions to future work.

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