Spatial dependence of Casimir friction in graphene

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The Casimir friction phenomenon involves the excitation of microscopic degrees of freedom in a medium, as a result of the relative motion of another object. When we consider a planar medium in the presence of a moving atom, excitations are expected to have a nontrivial spatial dependence, given the preferential direction set up by the atom's trajectory. Here, for a medium consisting of planar graphene, we evaluate the spatial (angular) dependence of such an effect. Using the Dirac field description, that phenomenon appears as the creation of electron-hole pairs, namely, in quantum electrodynamics terms, to pair creation. We present explicit results on the angular dependence of the fermion emission probability, as a function of the atom's velocity, and the distance of the trajectory to the sheet. This microscopic, i.e., local aspect, complements previous work on global features, like the total frictional force, which we also evaluate.

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I. INTRODUCTION

Quantum fluctuations produce macroscopic effects under the appropriate circumstances, with the Casimir effect [1] being the most celebrated example. A related kind of effect, also due to vacuum fluctuations, may arise when the bodies move, or the boundary conditions they impose become time dependent. This can lead to dissipation, via real photons excited out of the quantum vacuum, leading to what is known as the dynamical Casimir effect [2–4]. Yet another remarkable situation occurs when a purely quantum, dissipative frictional force arises between bodies moving at a constant relative velocity [5]. Here, the effect is due to the quantum degrees of freedom of the media which are excited from the vacuum, and the electromagnetic (EM) field acting as mediator. This Casimir friction effect has been studied extensively, though some calculational issues have prompted debate [6–8].

Here, we study this effect for an atom moving close to a graphene sheet, evaluating the momentum distribution of the fermion pair which is created, as a function of the parameters of the system. This study complements and extends previous work [9] in two ways: the first is that, rather than evaluating the total probability of vacuum decay, we focus on the angular aspects of the phenomenon: how the probability of detecting the fermions on the plate depends on the direction of emission, measured with respect to the trajectory of the atom. The atom moves along a trajectory which is parallel to the graphene plate, with a constant velocity. For the graphene system, we use its $2+1$ dimensional Dirac field description (see, for example [10,11]), and the atom by an electron bounded to the nucleus by a three-dimensional harmonic potential. The second way in which we introduce a novel ingredient is that the atom, in the model we use, couples to the EM field through a dipole term, plus a Röntgen term. The second term accounts for the fact that a moving electric dipole carries also a magnetic dipole moment. This term can become significant in certain scenarios, in particular in situations where there is quatum radiation, as shown in [12]. Quantum friction for two graphene sheets has been studied in [13]; note that in that situation there is no information (due to the geometry of the system) about the spatial dependence of the pair production effect. Knowledge of that dependence should, we believe, be relevant to the future design of nanodevices involving graphene.

The structure of this paper is as follows: In Sec. II, we describe the system and present the basic ingredients of our approach. Then, in Sec. III, we evaluate the probability amplitudes for the relevant elementary process contributing to friction, presenting a detailed study of its geometric (i.e., directional) properties. In Sec. IV we present our conclusions.

II. THE SYSTEM

The real-time action $\mathcal{S}$ for the system that we consider, may be conveniently written as follows:

$$\mathcal{S}[\bar{\psi},\psi,A,\mathbf{q};\mathbf{r}(t)] = \mathcal{S}_{\rm g}[\bar{\psi},\psi,A] + \mathcal{S}_{\rm a}[\mathbf{q}] + \mathcal{S}_{\rm em}[A] + \mathcal{S}_{\text{a-em}}[A,\mathbf{q};\mathbf{r}(t)], \tag{1}$$

where $\mathcal{S}_{\rm g}$ denotes a Dirac field action in $2+1$ dimensions, including its coupling to the EM field, while the terms $\mathcal{S}_{\rm a}$ and $\mathcal{S}_{\rm em}$ denote the free actions for the atom and the EM

field, respectively. $\mathcal{S}_{\text{a-em}}$ is the coupling between the atom and the EM field.

It is worth pointing out the following: graphene is, in the continuum version description which we are using here, described by a number N of flavors of four-component Dirac fields. Each one of these flavors may be thought of as composed of two spinors transforming under an irreducible representation of the $2+1$ dimensional Lorentz group, while the two flavors are mixed by parity. We recall that, in $2+1$ dimensions, a parity transformation corresponds to a reflection, rather than a spatial inversion (which is a rotation in π). For the process we study here, it will not make any difference which one of the $2N$ two-component Dirac fields is considered. Thus, we deal with just one of them and to find the result in the general case one simply multiplies the result by $2N$ (see last paragraph of Sec. III).

In this work, we adopt the following conventions: both $\hbar$ and the speed of light are set equal to 1, space-time coordinates are denoted by x^μ, $\mu = 0, 1, 2, 3$, $x^0 = t$, and we use the Minkowski metric $g_{\mu\nu} \equiv \text{diag}(1, -1, -1, -1)$. Dirac's γ-matrices, on the other hand, are chosen to be in the representation $\gamma^0 \equiv \sigma_1$, $\gamma^1 \equiv i\sigma_2$, $\gamma^2 \equiv i\sigma_3$, where

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{2}$$

and

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{3}$$

where σ_i $(i = 1, 2, 3)$ denote the usual Pauli's matrices.

Let us now describe the structure of each term in the action (1), beginning with the one corresponding to the atom: the position of its center of mass, which to a very good approximation coincides with that of its nucleus, is assumed to be externally driven, and described by $\mathbf{r}(t) = (\mathbf{v}t, a)$. We have adopted a reference system fixed to the graphene plane, which occupies the $x^3 = 0$ plane, $\mathbf{v}$ denotes the (constant) velocity of the atom, which moves at a distance a from the plate. On the other hand, we assume that there is only one relevant (valence) electron in the atom, and that its position with respect to its center of mass is given by the vector: $\mathbf{q}$. In our description, therefore, the three components of this vector are the only relevant degrees of freedom in the atom. Assuming that only single transitions are relevant to the process that we are studying, the physics should be characterized by a single energy (scale). It is sufficient to take, as the classical action accounting for the free dynamics of the electron, a harmonic one:

$$\mathcal{S}_{\text{a}}[\mathbf{q}] = \int dt \left(\frac{1}{2} M \dot{\mathbf{q}}^2 - V(|\mathbf{q}|) \right) \approx \int dt \frac{M}{2} (\dot{\mathbf{q}}^2 - \Omega^2 \mathbf{q}^2), \tag{4}$$

where M is the mass of the electron and Ω characterizes the effective harmonic potential.

The free electromagnetic field has its dynamic given by the usual action, together with a gauge-fixing term

$$\mathcal{S}_{\text{em}}[A] = \int d^4x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2} (\partial_\mu A^\mu)^2 \right], \tag{5}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Graphene is a sheet of carbon atoms with a flat hexagonal crystal structure. This makes it effectively be described as a two-dimensional material. Furthermore, their electronic degrees of freedom can be described, at low energies, as Dirac fermions, and they satisfy a linear dispersion relation. That is, they behave like massless fermions that propagate with the Fermi velocity $v_F \approx 0.003$ [14]. Its action is

$$\mathcal{S}_{\text{g}}[\bar{\psi}, \psi, A] = \int d^3x_\parallel \bar{\psi}(x_\parallel) \left(i \rho^\alpha_\beta \gamma^\beta D_\alpha - m \right) \psi(x_\parallel), \tag{6}$$

where $\rho^\alpha_\beta = \text{diag}(1, v_F, v_F)$ and $D_\alpha = \partial_\alpha + ieA_\alpha(x_\parallel, 0)$. The solution to the free part takes the form

$$\psi(x) = \sum_{s=\pm} \int \frac{d^2\mathbf{p}}{2\pi} \sqrt{\frac{m}{p_0}} \Big(b(\mathbf{p}, s) u(\mathbf{p}, s) e^{-ip\cdot x} + d^\dagger(\mathbf{p}, s) \times v(\mathbf{p}, s) e^{ip\cdot x} \Big), \tag{7}$$

where $u(\mathbf{p}, s)$ and $v(\mathbf{p}, s)$ satisfy

$$\sum_{s=\pm} u(\mathbf{p}, s) \bar{u}(\mathbf{p}, s) = \frac{\rho^\alpha_\beta \gamma^\beta p_\alpha + m}{2m}, \tag{8}$$

$$\sum_{s'=\pm} v(\mathbf{q}, s') \bar{v}(\mathbf{q}, s') = \frac{\rho^\alpha_\beta \gamma^\beta q_\alpha - m}{2m}. \tag{9}$$

m is the mass gap parameter, which is almost zero for graphene. Taking the limit $m \to 0$, the dispersion relation for fermions in graphene turns out to be $p_0 \equiv E(\mathbf{p}) = v_F |\mathbf{p}|$.

On the other hand, the interaction action for the full system is

$$\mathcal{S}_{\text{int}}[\bar{\psi}, \psi, A, \mathbf{q}; \mathbf{r}(t)] = e \int d^4x \Big[\rho^\sigma_\omega \bar{\psi}(x_\parallel) \gamma^\omega \psi(x_\parallel) A_\sigma(x) \delta(x^3) + \mathbf{q}(x^0)(\mathbf{E}(x) + \mathbf{v} \times \mathbf{B}(x)) \times \delta^{(3)}(\mathbf{x} - \mathbf{r}(x^0)) \Big]. \tag{10}$$

The first term accounts for the coupling between the graphene, that lives in the plane $z \equiv x^3 = 0$, and the electromagnetic field, present in all space but evaluated in the plane of the graphene. The second term gives the coupling of the dipolar momentum of the atom, localized at $\mathbf{r}(t)$, with the electromagnetic field, taking into account relativistic corrections up to order $|\mathbf{v}|/c$ due to the movement of the dipole [15].

III. PROBABILITY AMPLITUDES FOR QUANTUM FRICTION

The only role that the vacuum EM field plays in the processes that we study is to mediate the excitations of the microscopic degrees of freedom belonging to the two material objects involved in the phenomenon. Thus, there will not be photons in the initial and final states. Therefore, we shall consider the normalized initial ($|\mathrm{i}\rangle$) and final ($|\mathrm{f}\rangle$) quantum states given by

$$|\mathrm{i}\rangle = |0_\mathrm{a}\rangle \otimes |0_\mathrm{em}\rangle \otimes |0_\mathrm{g}\rangle, \tag{11}$$

$$|\mathrm{f}\rangle = \hat{a}_i^\dagger |0_\mathrm{a}\rangle \otimes |0_\mathrm{em}\rangle \otimes \left(\frac{2\pi}{L}\right)^2 \hat{b}^\dagger(\mathbf{p},s)\hat{d}^\dagger(\mathbf{q},s')|0_\mathrm{g}\rangle, \tag{12}$$

respectively. That is, the system is initially at rest, while in the final one the atom is in an excited state, corresponding to an electron excitation for one of the three harmonic oscillator modes: the one in the direction given by the index i. For graphene, we assume a fermion-antifermion pair; the fermion having momentum $\mathbf{p}$ and spin s, while for the antifermion those quantum numbers are $\mathbf{q}$ and s'.

For the states above, the first nontrivial contribution to the amplitude for the transition between them appears to the second order in the interaction action in (10), as it stems from the usual perturbative expansion for the evolution operator. Besides, for the states that we are using, the only nonvanishing contractions, via Wick's theorem, follow from the "crossed" contributions. Namely, contributions involving the coupling of the atom to the EM field and also the interaction between the Dirac field and the EM field.

The resulting matrix element of the S-matrix to this order then becomes

$$\begin{aligned}\mathcal{M}_i(\mathbf{p},\mathbf{q},s,s') = \frac{i}{2!}\langle \mathrm{f}|\mathbb{T}\left(\hat{S}^2_\mathrm{int}\right)|\mathrm{i}\rangle = ie^2\langle \mathrm{f}| \int d^4x \int d^4y \\ \times \mathbb{T}\Big[\rho^\sigma_\omega \bar{\psi}(x_\parallel)\gamma^\omega \psi(x_\parallel) A_\sigma(x)\delta(x^3)\mathbf{q}(y^0) \\ \cdot \Big(\mathbf{E}(y) + \mathbf{v}\times\mathbf{B}(y)\Big) \\ \times \delta^{(3)}(\mathbf{y} - \mathbf{r}(y^0))\Big]|\mathrm{i}\rangle. \end{aligned} \tag{13}$$

Wick's theorem also requires the knowledge of the contractions:

$$\underbrace{a_j q_k(t)} = \frac{1}{\sqrt{2M\Omega}} e^{i\Omega t}\delta_{jk} \tag{14}$$

$$\underbrace{b(\mathbf{p},s)\bar{\psi}(x_\parallel)} = \frac{1}{2\pi}\sqrt{\frac{m}{p_0}} e^{ip\cdot x_\parallel}\bar{u}(\mathbf{p},s) \tag{15}$$

$$\underbrace{d(\mathbf{q},s')\psi(x_\parallel)} = \frac{1}{2\pi}\sqrt{\frac{m}{q_0}} e^{iq\cdot x_\parallel} v(\mathbf{q},s'). \tag{16}$$

We also need, as another ingredient to construct the amplitude, the contraction between the gauge field A_σ and $C_i \equiv (\mathbf{E} + \mathbf{v}\times\mathbf{B})_i$. This can be computed by using the free propagator of the gauge field, which in the Feynman gauge is

$$\underbrace{A_\mu(x)A_\nu(y)} = G_{\mu\nu}(x-y) = \int \frac{d^4k}{(2\pi)^4}\frac{-ig_{\mu\nu}}{k^2+i\epsilon}e^{-ik\cdot(x-y)}. \tag{17}$$

Putting together the previous elements, after a lengthy but otherwise straightforward calculation we find that the transition amplitude becomes

$$\mathcal{M}_i(\mathbf{p},\mathbf{q},s,s') = \delta\Big(\chi(\mathbf{p},\mathbf{q})\Big)K^\sigma I_{\sigma i}(p+q), \tag{18}$$

where we have introduced

$$K^\sigma = \left(\frac{2\pi}{L}\right)^2 \frac{ie^2 m}{(2\pi)^2\sqrt{2M\Omega p_0 q_0}}\rho^\sigma_\omega \bar{u}(\mathbf{p},s)\gamma^\omega v(\mathbf{q},s'), \tag{19}$$

$$I_{\sigma i}(p+q) = \pi e^{-a\sqrt{-(p+q)^2}}\begin{cases} \frac{1}{\sqrt{-(p+q)^2}}\Big(\Omega\eta_{\sigma i} + (p+q)_i(\eta_{\sigma 0} + v_\sigma)\Big) & \text{for } i = 1,2 \\ i(\eta_{\sigma 0} + v_\sigma) & \text{for } i = 3 \end{cases} \tag{20}$$

and

$$\chi(\mathbf{p},\mathbf{q}) \equiv \Omega + v_F(|\mathbf{p}| + |\mathbf{q}|) - (\mathbf{p}+\mathbf{q})\cdot\mathbf{v}. \tag{21}$$

Note that the last object, being the argument of a Dirac's δ function, provides important kinematic information about the process. Firstly, the *total* momentum that appears in graphene, as a consequence of the created pair, has a positive component in the direction of the atom's velocity. Another observation is that, for this process to happen, the velocity of the atom should be greater than Fermi's velocity in graphene, i.e., $|\mathbf{v}| > v_F$.

Besides, $(p+q)^2 < 0$, so that $p+q$ must be a spacelike momentum. In other words, the Coulombian part of the EM interaction is prevalent. At this point, it is worth mentioning some relevant observations regarding Lorentz invariance. It is well known that fermions in graphene have a "relativistic" dispersion relation, with v_F playing the role of the speed of light, and with spacetime reduced to $2+1$ dimensions. They behave like massless particles moving with a velocity v_F on the plane. Since v_F is less than the speed of light in the vacuum, they can have a total spacelike momenta without involving nonphysical superluminical particles. That reconciles the fact that the momentum $p+q$ is spacelike with the creation of real particles. Furthermore, $(p+q)^2 < 0$ is consistent with the fact that the final state contains no real photons.

The transition we have up to now corresponds to a final state in which the spin of the fermions, their momenta, and the orientation of the excitation of the atom have a specific value. One is usually interested in the knowledge of the probability as a function of momentum, regardless of the spin of the fermions, and of the direction of the harmonic excitation on the electron in the atom. This amounts to adding the probability densities $|\mathcal{M}_i(\mathbf{p},\mathbf{q},s,s')|^2$ for every value of s and orientation i. The resulting probability per unit time is then a function of the two momenta $\mathbf{p}$ and $\mathbf{q}$:

$$\mathcal{P}(\mathbf{p},\mathbf{q}) = \frac{1}{T}\sum_{s,s'}\sum_{i=1}^{3}|\mathcal{M}_i(\mathbf{p},\mathbf{q},s,s')|^2. \tag{22}$$

The sum over spins and oscillator directions allows one to produce a more explicit expression for that probability per unit time. Indeed, by using (8), (9) plus the $2+1$ dimensional Dirac matrices trace relation,

$$\mathrm{tr}\{\gamma^\sigma\gamma^\mu\gamma^\lambda\gamma^\nu\} = 2(\eta^{\sigma\mu}\eta^{\lambda\nu} - \eta^{\sigma\lambda}\eta^{\mu\nu} + \eta^{\sigma\nu}\eta^{\lambda\mu}), \tag{23}$$

the result for $\mathcal{P}$ may be put in the form

$$\mathcal{P}(\mathbf{p},\mathbf{q};\Omega,a) = \frac{e^{-2a\sqrt{-(p+q)^2}}}{\Omega v_F^2|\mathbf{p}||\mathbf{q}|}\delta\Big(\chi(\mathbf{p},\mathbf{q};\Omega)\Big)F(\mathbf{p},\mathbf{q};\Omega), \tag{24}$$

with

$$\begin{aligned}F(\mathbf{p},\mathbf{q};\Omega) = \frac{1}{-(p+q)^2}\Big\{&(|\mathbf{p}+\mathbf{q}|^2-(p+q)^2)((p_0-v_F^2\mathbf{p}\cdot\mathbf{v})(q_0-v_F^2\mathbf{q}\cdot\mathbf{v})\\ &+-(1-v_F^2v^2)(p_0q_0-v_F^2\mathbf{p}\cdot\mathbf{q})/2)+\Omega^2v_F^2p_0q_0\\ &+\Omega v_F^2(\mathbf{p}+\mathbf{q})\cdot((q_0-v_F^2\mathbf{q}\cdot\mathbf{v})\mathbf{p}+(p_0-v_F^2\mathbf{p}\cdot\mathbf{v})\mathbf{q}-(p_0q_0-v_F^2\mathbf{p}\cdot\mathbf{q})\mathbf{v})\Big\}\Big|_{\text{on shell}}.\end{aligned} \tag{25}$$

We have used the "on shell" expression to mean that the fermion satisfies the dispersion relations of real particles in graphene: $p_0 = v_F|\mathbf{p}|$ and $q_0 = v_F|\mathbf{q}|$. In order to further clarify the dependence of the result on all of the relevant parameters of the model, we have also made explicit the dependence on the dimensional parameters a and Ω.

In spite of its cumbersome appearance, it is not difficult to verify (as a consistency check) analytically that $F(\mathbf{p},\mathbf{q};\Omega) \geq 0$ when the Röntgen term coupling is ignored. One should use, in order to verify that inequality, the relation $\Omega = -v_F(|\mathbf{p}|+|\mathbf{q}|)$. With the Röntgen term included, we have verified numerically that $F(\mathbf{p},\mathbf{q};\Omega) \geq 0$.

In order to have a more explicit knowledge of the angular dependence of the effect, we begin by introducing modules and angles for the relevant vectors. First, we note that

$$F(\mathbf{p},\mathbf{q};\Omega) = v_F^2|\mathbf{p}||\mathbf{q}|f(\mathbf{p},\mathbf{q};\Omega), \tag{26}$$

so that

$$\mathcal{P}(\mathbf{p},\mathbf{q};\Omega,a) = \Omega^{-1}\delta\Big(\chi(\mathbf{p},\mathbf{q};\Omega)\Big)\underbrace{e^{-2a\sqrt{-(p+q)^2}}f(\mathbf{p},\mathbf{q};\Omega)}_{g(\mathbf{p},\mathbf{q};\Omega,a)} \tag{27}$$

and

$$\chi(\mathbf{p},\mathbf{q};\Omega) = \Omega + |\mathbf{p}|(v_F - v\cos\theta_p) + |\mathbf{q}|(v_F - v\cos\theta_q) = 0. \tag{28}$$

The δ-function may then be used to fix the value of $|\mathbf{q}|$, since

$$\delta(\chi(\mathbf{p},\mathbf{q};\Omega)) = \frac{\delta\Big(|\mathbf{q}| - q_0(\mathbf{p},\theta_q;\Omega)\Big)}{|v_F - v\cos\theta_q|}, \tag{29}$$

where we have introduced

$$q_0(\mathbf{p},\theta_q;\Omega) = \frac{\Omega + |\mathbf{p}|(v_F - v\cos\theta_p)}{v\cos\theta_q - v_F} \equiv \frac{s(\mathbf{p};\Omega)}{v\cos\theta_q - v_F}. \tag{30}$$

Just positive values of q_0 are physical. Imposing $q_0(\mathbf{p},\theta_q;\Omega) > 0$ determined the allowed region $\mathcal{R}$ for

the angle between $\mathbf{q}$ and $\mathbf{v}$. Defining $\cos\alpha \equiv v_F/v$, $\mathcal{R}$ is given by

$$\theta_q \in \begin{cases} [0,\alpha) \cup (2\pi-\alpha, 2\pi) & \text{if } s(\mathbf{p};\Omega) > 0 \\ (\alpha, 2\pi-\alpha) & \text{if } s(\mathbf{p};\Omega) < 0. \end{cases} \tag{31}$$

For the probability of detecting any given particle of the pair, with momentum $\mathbf{p}$, we compute (note that the probability is symmetric under the exchange of particle by antiparticle)

$$\mathcal{P}(\mathbf{p};\Omega,a) = \int d^2\mathbf{q}\mathcal{P}(\mathbf{p},\mathbf{q};\Omega,a) = \int_{\mathcal{R}} d\theta_q \int_0^\infty d|\mathbf{q}||\mathbf{q}| \times \mathcal{P}(\mathbf{p},|\mathbf{q}|,\theta_q;\Omega,a). \tag{32}$$

Taking into account (31), the angular integral becomes

$$\int_{\mathcal{R}} d\theta_q = \Theta[s(\mathbf{p};\Omega)]\left(\int_0^\alpha d\theta_q + \int_{2\pi-\alpha}^{2\pi} d\theta_q\right) + \Theta[-s(\mathbf{p};\Omega)]\int_\alpha^{2\pi-\alpha} d\theta_q. \tag{33}$$

By using (29), we obtain

$$\begin{aligned}\mathcal{P}(\mathbf{p};\Omega,a) &= \Omega^{-1}s(\mathbf{p};\Omega)\left\{\Theta[s(\mathbf{p};\Omega)]\left(\int_0^\alpha d\theta_q + \int_{2\pi-\alpha}^{2\pi} d\theta_q\right) - \Theta[-s(\mathbf{p};\Omega)]\int_\alpha^{2\pi-\alpha} d\theta_q\right\} \\ &\times \frac{g(\mathbf{p}, q_0(\mathbf{p},\theta_q;\Omega),\theta_q;\Omega,a)}{|v\cos\theta_q - v_F|^2} \\ &= \mathcal{P}(\mathbf{p}/\Omega;1,a\Omega).\end{aligned} \tag{34}$$

The last equality above follows from the homogeneity properties of the functions involved. This will allow us to get expressions and plots, in terms of fewer parameters than one might have expected *a priori*.

An exact relation that one can see from the previous expression corresponds to finding the angle for which the probability vanishes, which sets the angular width for the production of pairs. It follows from the observation that

$$s(|\mathbf{p}|,\theta_p^0;\Omega) = 0 \;\Rightarrow\; \theta_p^0 = \arccos\left[\frac{1}{v}\left(v_F + \frac{\Omega}{|\mathbf{p}|}\right)\right] \xrightarrow{|\mathbf{p}|\gg\Omega} \alpha. \tag{35}$$

This also implies $|\mathbf{q}| = 0$, and the probability (34) vanishes.

Another probability per unit time follows by just asking for the probability density per unit angle of detecting any particle, regardless of their momentum:

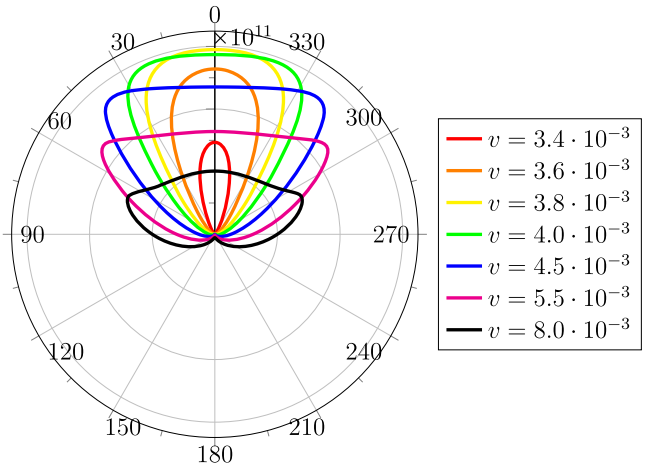

FIG. 1. Polar distribution of the probability for different velocities of the atom.

$$\mathcal{P}(\theta_p;\Omega,a) = \int_0^\infty d|\mathbf{p}||\mathbf{p}|\mathcal{P}(\mathbf{p};\Omega,a) = \Omega^2\mathcal{P}(\theta_p;1,a\Omega). \tag{36}$$

This distribution has been plotted in Fig. 1.

We see that, for velocities close to v_F, the probability becomes highly concentrated around the direction along which the atom moves. On the other hand, it widens up as the velocity increases. The area inside each curve is proportional to the total probability of this process to happen, and we see that it reaches its maximum around $v = 4.5\times10^{-3} \sim 1.5v_F$.

Another interesting quantity is the power dissipated $\mathcal{W}$, which is related to the friction force by $\mathcal{W} = vF_{\text{fr}}$. The dissipated power is the energy per unit time transmitted from the mechanical system that moves the atom to the graphene through the electromagnetic field. The energy that the graphene is receiving when a fermionic pair of momenta $\mathbf{p}$ and $\mathbf{q}$ is created is $E = v_F(|\mathbf{p}| + |\mathbf{q}|)$, so the power transmitted to graphene is proportional to

$$\mathcal{W}(\Omega,a) = \int d^2\mathbf{p}\int d^2\mathbf{q}(|\mathbf{p}| + |\mathbf{q}|)\mathcal{P}(\mathbf{p},\mathbf{q};\Omega,a) \tag{37}$$

$$\begin{aligned}&= \int_0^{2\pi} d\theta_p \int_0^\infty d|\mathbf{p}||\mathbf{p}|s(\mathbf{p};\Omega)\left\{\Theta[s(\mathbf{p};\Omega)]\right. \\ &\times \left(\int_0^\alpha d\theta_q + \int_{2\pi-\alpha}^{2\pi} d\theta_q\right) - \Theta[-s(\mathbf{p};\Omega)] \\ &\left.\times \int_\alpha^{2\pi-\alpha} d\theta_q\right\}\left(|\mathbf{p}| + \frac{s(\mathbf{p})}{v\cos\theta_q - v_F}\right) \\ &\times \frac{g(\mathbf{p}, q_0(\mathbf{p},\theta_q;\Omega),\theta_q;\Omega,a)}{|v\cos\theta_q - v_F|^2} = \Omega^3\mathcal{W}(1,a\Omega).\end{aligned} \tag{38}$$

We have not written it explicitly but this is a function of the velocity of the atom [appearing also in $\alpha, s(\cdots)$ and

$g(\cdots)]$, so we could see the dependence of the friction force on the velocity.

All our calculations have been made for a two-component Dirac field. Graphene, however, does correspond to N flavors of four-component Dirac fields, with each one of those N flavors in a reducible representation of the Lorentz group in $2+1$ dimensions. The coupling between each flavor and the EM field is the same; therefore, in this kind of process, all the results for the probabilities should be multiplied by a factor $2N$ (we assume one is interested in the probability of creating fermions of *any* flavor).

IV. CONCLUSIONS

We have presented a detailed calculation of the process that drives Casimir friction when an atom moves close to a graphene plate, presenting the angular dependence of the probability of detecting fermions, as a function of the parameters of the system. All our calculations have been presented for a single two-component Dirac flavor. Results corresponding to N four-component Dirac flavors can be obtained by multiplying our results for the probability by a global factor $4N$.

This probability of detection is a *local* aspect, as opposed to a global one, like for example the total probability of detection.

Besides the known fact that there is a velocity threshold for the effect to occur, we have found a relation which restricts, for particles with a given momentum, the angular region where they could be detected.

We have also obtained the angular probability distribution, namely, the probability density (per unit time) of detecting a given particle regardless of its momentum.

A related but different observable is the power dissipation on the graphene plate, for which we could obtain expressions depending on the velocity v and the other parameters a, Ω, v_F.

We think it is worth mentioning the following observation: since the fermion and antifermion have opposite electric charges, and the probability of detecting a fermion is identical to the one of detecting an antifermion (for the same momenta), the processes described here do not amount to the production of a net electric current. However, we suggest that a possible way to allow for the production of a net current on the sample would be to study friction in the presence of a constant and uniform magnetic field, normal to the graphene plane. Under this external condition, the particle and antiparticle will experience opposite forces when they are produced along a given direction, with the same velocity.

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