Probing invisible vector meson decay mode with the hadronic beam in the NA64 experiment at SPS CERN

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We test a novel idea of using a π^- beam in the fixed-target experiments to search for new physics in the events with missing energy. Bounds for invisible vector ρ meson decay were derived, analyzed, and compared with the current limits on searching dark matter in the accelerator based experiments. We demonstrate that the new approach can be effective tool to probe sub-GeV dark matter parameter space.

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I. INTRODUCTION

Searching for dark matter (DM) is well-motivated challenge in particle physics that stimulates experimental and theoretical efforts for decades. Study of DM phenomenology gives a unique opportunity to explain many observations in astrophysics and cosmology. In a wide range of possible DM candidates, we can mention light DM in the sub-GeV mass region which could potentially explain several observed anomalies [1,2] and could be a candidate for thermal relic dark sector. An idea of dark portals between the hidden and ordinary matter, described by the Standard Model (SM), typically implies light sub-GeV intermediate states. In particular, there are several hidden sector scenarios that have been widely discussed in literature; the Higgs portal [3,4], the tensor portal [5–7], the dark photon portal [8–10], sterile neutrino portal [11], and axion or axionlike (ALPs) portals [12,13]. In addition, we note that such models are considered typically in the framework of lepton-specific [14] or hadron-specific cases [13].

The scenarios with dark portal states predict missing energy events in reactions with leptons [9,10,15] and hadrons [16–20], including lepton-flavor violation effect [21,22]. The invisible decays play an important role in testing SM and searching for DM particles. Experimental studies of invisible hadronic decays were performed by several collaborations. In particular, the BES III Collaboration [23,24] set the constraints on the invisible branching fraction of the η, η', ω, and ϕ mesons. The *BABAR* Collaboration [25,26] studied the invisible decay modes of heavy quarkonia. The NA62 Collaboration [27] established the limits on invisible decays of π^0. Existing limits on DM from invisible decays of the vector DM mediator [28] were derived from analysis of data collected in the e^+e^- colliders and accelerator-based experiment NA64 [29,30]. Experiments which aimed for direct DM detection and probing meson decay into invisible mode may provide important signatures of sub-GeV DM [31]. Invisible meson decays can be limited by using missing energy/momentum techniques [32,33]. In the framework of missing-energy concept bounds to invisible decay in DM were obtained in [34] for such experiments as NA64 and LDMX where vector mesons are created by interaction of

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radiated photons from electron beams in the calorimeter. Many existing and future experiments for searching DM based on a use of missing energy/momenta techniques are concentrated on in setups with lepton beams colliding with fixed atomic targets. Here, the main aim is to search for missing-energy/momenta signal events which can be interpreted as potential signatures of the produced DM.

In the present paper we extend the analysis of invisible meson decays to DM by using missing-energy conception which was considered previously in Ref. [34]. Our main objective is to test the potential of missing-energy techniques for invisible meson decay for hadronic beams. The NA64 Collaboration has started to exploit this concept in experiments with hadronic beams to search for signatures of dark matter production [35,36]. For the first time the experiment will use the beam π^- mesons scattered at the active target. During last two years (runs in 2022 and 2023 with a few days of data collection), the NA64 Collaboration accumulated about 3×10^9 pions on target in order to understand potential of the NA64 detector by using the pion beam and missing-energy technique. Another aim of our paper is to estimate the sensitivity of hadronic-pion beam to search for DM implementing missing-energy techniques. In particular, we will make an estimate of observables in invisible meson decays. In our analysis we rely on the preliminary analysis of accumulated number of pions on target from the NA64 technical run (3×10^9) and make predictions for the projected statistics in the range between 5×10^{12} and 10^{14} pions on target.

The paper is organized as follows. In Sec. II we describe missing-energy conception to analyze invisible vector-meson decay mode for hadronic case beam of the NA64 experiment and estimate the yield of vector mesons in a experimental facility which can be used for analysis. In Sec. III we calculate the cross section of neutral ρ^0 vector-meson production in the π^- scattering at the nuclear target. The discussion about invisible meson decay modes to DM fermions and implementation to DM parameter space is presented in Sec. IV. Finally, in Sec. V we present our conclusions.

II. FRAMEWORK

The missing-energy conception, proposed in [37], is pretty well realized and works consistently at fixed target experiments dealing with lepton (electron and muon) beams, such as the $\mathrm{NA64}_e$ and $\mathrm{NA64}_\mu$ [38] experiments. In the future, it is planned to run several new experiments, e.g., M^3 [39,40] and LDMX [34,41–44].

For a 90% confidence level (C.L.) limit on the invisible branching ratio of the produced meson, V, in experiments where we assume zero observed signal events and background-free case, that implies

$$\mathrm{Br}(V\to\mathrm{inv})\le 2.3/N_V, \tag{1}$$

where N_V is the number of the produced vector mesons. We consider an modified experimental setup of the NA64 to estimate the ρ^0-meson production in the reaction of the π^- beam scattered at the active iron target. About 3×10^9 pions on target (πOT) in the NA64 experiment were accumulated in a short period of technical data taking at SPS/CERN. For the typical projected statistics of NA64 we will use both numbers of 5×10^{12} and 10^{14} pions on the target.

The cross section for the ρ^0 meson production in the reaction $\pi^- + (Z,A)\to\rho^0 + (Z-1,A)$ is given by the formula,

$$\begin{aligned}\sigma(\pi^- + (Z,A)\to\rho^0 + (Z-1,A))\\ = Z\sigma(\pi^- + p\to\rho^0 + n).\end{aligned} \tag{2}$$

Here we assume that the main channel of the ρ^0 production is due to positive pion captured by the nuclear target (see Fig. 1). This process is the dominant one and occurs due to the π^+ exchange in the t-channel. The second possible mechanism for the ρ^0 production could occur due annihilation of π^- from the beam with ρ^+ or two-pion pair ($\pi^+\pi^0$) radiated from the target. We will show below that the latter mechanism is strongly suppressed in comparison with the leading signature of the $\pi^+\pi^-$ annihilation.

To constrain the parameters of dark photon coupled with vector mesons [34], we need to estimate the invisible branching ratio $\mathrm{Br}(V\to\mathrm{inv})$ for vector meson production. Here we will focus on dark sector with pseudo-Dirac DM fermions, which couple to the $\mathrm{U}(1)_D$ vector mediator (dark photon). Such a coupling at low energies is described by Lagrangian

$$\mathcal{L}\supset \epsilon e A'_\mu J^\mu + g_D A'_\mu \bar{\chi}\gamma^\mu\chi, \tag{3}$$

where ϵ is the kinetic mixing parameter [45], g_D is the coupling of dark photon with dark fermions, e is the electric charge, and J_μ the electromagnetic current composed of the SM fermions. In the following we use the notation of the effective dark coupling constant $\alpha_D = g_D^2/4\pi$. The coupling of the vector ρ^0-meson with a dark photon is defined

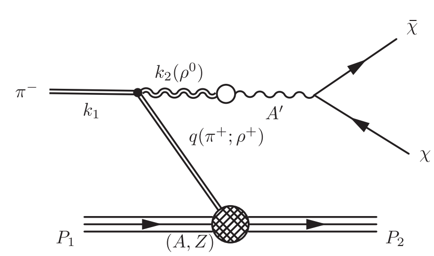

FIG. 1. Feynman diagram describing the cascade process of the ρ^0 vector meson production due to the π^- scattering at the nuclear target followed by the transition to the dark photon and DM fermions.

by an analogy with the QED photon but it has an extra factor ϵ (kinetic mixing coupling).

The width of the decay of vector meson into the dark fermion pair $V \to \bar{\chi}\chi$ is given by

$$\Gamma_{V\to\bar{\chi}\chi} = \frac{g_D^2(\epsilon e)^2}{12\pi} g_V^2 \frac{(m_V^2 + 2m_\chi^2)\sqrt{m_V^2 - 4m_\chi^2}}{(m_{A'}^2 - m_V^2)^2 + \Gamma_{A'\to\bar{\chi}\chi}^2 m_{A'}^2}, \quad (4)$$

where g_V is the vector meson coupling with the current, $m_{A'}$ and m_χ are the masses of intermediate dark photon and pseudo-Dirac DM fermion, respectively, and m_V is the mass of the vector meson. Here we use the Breit-Wigner propagator for the dark photon A' assuming that its total width is dominated by the $A' \to \bar{\chi}\chi$ mode. The decay width $\Gamma_{A'\to\bar{\chi}\chi}$ is

$$\Gamma_{A'\to\bar{\chi}\chi} = \frac{g_D^2}{12\pi} m_{A'}(1 + 2y_\chi^2)(1 - 4y_\chi^2)^{1/2}, \quad (5)$$

where $y_\chi = m_\chi/m_{A'}$.

The number of vector mesons produced by π^- beam scattering at fixed target is

$$N_\rho \simeq \pi\text{OT} \cdot \frac{\rho_T N_A}{A} L_T Z \int_0^{\theta_{\max}} d\theta \frac{d\sigma_{2\to2}}{d\theta}, \quad (6)$$

where A and Z are the atomic weight number and atomic charge number, N_A is Avogadro's number, πOT is the number of negatively charged pions accumulated on target, ρ_T is the target density, L_T is the effective thickness of the target which in the conservative scenario is assumed to be equal to the effective-pion interaction length in the target [28], and $d\sigma_{2\to2}/d\theta$ is the differential cross section of the ρ^0-meson production process on the nucleon, θ is an angle between π^- beam line and the momentum of the produced ρ^0 meson.

The cross section of ρ^0-meson production plays an important role in the calculation of vector mesons flow and crucially depends on the angle $\theta_{\max}$. The maximum of the scattering angle $\theta_{\max}$ is defined by experimental cuts of the registration of the signal which can be interpreted as missing energy by transition to DM. For the NA64 experiment it is important to maintain a negligible background in a calorimeter system. The latter can be provided by adjusting the recoil energy from final neutrons or pieces of the disintegration of the atomic nucleus which implies the energy deposition inside the hadronic calorimeter. To take into account the detector response one may fix the $\theta_{\max}$ from the minimal possible background-energy emission from the recoil energy of the final neutron. The dependence of the recoil momentum of the final neutron on the scattering angle is shown in Fig. 2. We set an optimistic upper limit on the recoil momentum of the neutron to be 0.8 GeV. This limit corresponds to $\theta_{\max} \sim 0.014$ rad for the

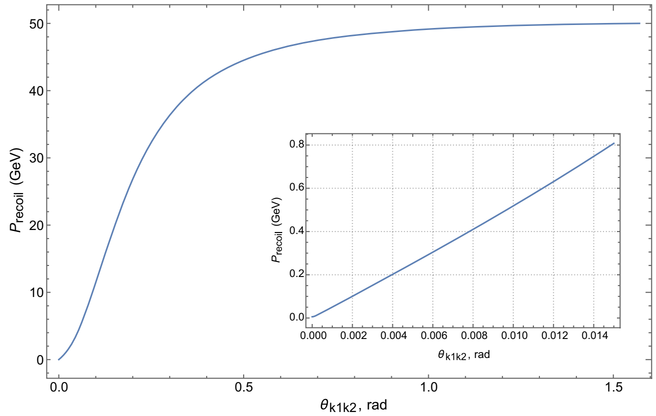

FIG. 2. Recoil momenta to nucleon at beam energy $E_{\pi^-} = 50$ GeV with creation ρ meson in final state. The small figure shows the area where recoil momenta to nucleons are less than 1 GeV and can be used for the calculation and analysis of missing energy signals.

50 GeV pion beam. For the 100 GeV pion beam, we need to use $\theta_{\max} \sim 0.008$ rad. By using those cuts on the scattering angle we obtain the optimal missing-energy cut and relatively small background which can be suppressed experimentally. This small area of angle which can be used for analysis of missing energy is connected to the kinematic of scattering of massive particles in the initial and final states. In addition, it is worth noticing a difference between our analysis and the study presented in Ref. [34] for electron beam experiments that provides the bounds on pseudo-Dirac DM from invisible vector meson decays. In Ref. [34] vector mesons are produced inside the calorimeter by interaction of a bremsstrahlung photon with the matter of the calorimeter. We note that a small typical angle of outgoing ρ^0 meson decreases the sensitivity of the hadron missing-energy experiment, but for a more accurate analysis one needs to carry out a proper Monte Carlo simulation of this process in a detector that also includes the background from recoil neutrons. For the larger angle cut one needs to take into account nuclear function and all possible transitions of the nucleus during the transfer of energy to the nuclear shell. We keep the analysis for a full possible picture of hadronic showers in the detector for future study.

We need to point out that the potential background for the invisible decays of mesons can arise from the decay of neutral mesons into neutrino-antineutrino pair. These decays are strongly suppressed from SM and the regarding decay widths are estimated to be at the level of $\Gamma(M^0 \to \nu\bar{\nu}) \lesssim 10^{-16}$ [46]. Remarkably, for the ρ^0 meson the typical bound can be set as follows: $\Gamma(\rho^0 \to \nu\bar{\nu}) \lesssim 4.2 \times 10^{-13}$ [47]. However, an experimental signatures for such decays have not been observed yet. An existence of any experimental evidence for invisible meson decay can be considered as a potential signal of new physics.

The yield of neutral-vector ρ^0 mesons at the NA64 experiment with π^- beam is shown in Table I for

TABLE I. Parameters of the fixed-target experiments NA64$_h$ for the iron target [$A = 56$, $Z = 26$, $\rho = 7.874$ (g cm^{-3}), and interaction length of pion in Fe $L_T = 20.41$ cm]: $E_{\rm beam}$ is the beam energy of pions, $\sigma_{\rm tot}$ is a total cross section, $f(\theta_{\rm max})$ is fraction of ρ^0 mesons which are produced within small angle range, $\theta \lesssim \theta_{\rm max}$, $\sigma_{\rm cut} = f(\theta_{\rm max}) \times \sigma_{\rm tot}$, πOT is a typical number of pions on accumulated on target, and N_ρ is yield of neutral ρ^0 vector mesons.

	$E_{\rm beam}$ (GeV)	$\sigma_{\rm tot}(b)$	$f(\theta_{\rm max})$	$\sigma_{\rm cut}$ (μb)	πOT	$\theta_{\rm max}$	N_ρ
NA64$_h$:	50	0.113	3.1×10^{-6}	0.35	3×10^9	0.014	1.8×10^3
NA64$_h$:	100	0.117	4.7×10^{-7}	0.054	3×10^9	0.008	0.29×10^3
NA64$_h$:	50	0.113	3.1×10^{-6}	0.35	5×10^{12}	0.014	3.1×10^6
NA64$_h$:	100	0.117	4.7×10^{-7}	0.054	5×10^{12}	0.008	0.48×10^6
NA64$_h$:	50	0.113	7×10^{-6}	0.79	3×10^9	0.022	4.1×10^3
NA64$_h$:	50	0.113	7×10^{-6}	0.79	5×10^{12}	0.022	6.9×10^6
NA64$_h$:	20	0.104	6.7×10^{-4}	69.68	3×10^9	0.05	5.2×10^4
NA64$_h$:	20	0.104	6.7×10^{-4}	69.68	5×10^{12}	0.05	8.7×10^7

$E_{\rm beam} = 50$ GeV and 100 GeV. Besides, in Table I we also show the typical fraction $f(\theta_{\rm max})$ of ρ^0 mesons that can be produced within two benchmark angle ranges, $\theta \lesssim \theta_{\rm max} = 0.014$ rad and $\theta \lesssim \theta_{\rm max} = 0.022$ rad. It corresponds to the typical neutron recoil momenta at the level of 0.8 GeV and 1.2 GeV, respectively. The target of the experiment is a hadronic calorimeter which represents four or three modules in 48 layers (2.5 mm of iron plates and 4 mm of scintillator). The possible signature of the neutron recoil momentum of 1.2 GeV can be deposited in the hadronic calorimeter at the level of $\simeq$1 GeV; it implies $\simeq$0.2 GeV is transferred to the nucleus as a typical energy of the nucleus excitation. It is important to note, that small recoil energy of neutron can be achieved by decreasing the energy of the pion beam. This scenario for 20 GeV of pion beam is shown in Table I for the neutron recoil momentum of 1 GeV.

In our calculations we use formulas for the cross section of the ρ^0-meson production which is calculated in the next section. Estimate of the ρ^0 production is obtained for current and ultimate statistics and for two possible values of the pion energy in a beam in a narrow angle of meson production.

III. VECTOR-MESON PRODUCTION

In this section we briefly discuss formalism and obtain an expression for differential cross section of ρ^0 vector-meson production in the $\pi^- + p \to \rho^0 + n$ reaction. Our formalism is based on Lagrangians that includes nucleons $N = (p, n)$, pseudoscalar mesons $\pi^\pm$, vector ρ^μ mesons and A_μ photons,

$$\mathcal{L}_{\pi NN} = g_{\pi NN}\bar{N} i\gamma_5 \vec{\pi}\,\vec{\tau} N, \tag{7}$$

$$\mathcal{L}_{\rho\pi\pi} = i g_{\rho\pi\pi}\rho^\mu(\partial_\mu\pi^\dagger\pi - \pi^\dagger\partial_\mu\pi) \tag{8}$$

$$\mathcal{L}_{\rho NN} = g_{\rho NN}\bar{N}\gamma_\mu\rho^{\mu,a}\vec{\tau}_a N, \tag{9}$$

$$\mathcal{L}_{\pi\rho\rho} = g_{\pi\rho\rho}\pi F_\rho^{\mu\nu}\tilde{F}_\rho^{\alpha\beta}. \tag{10}$$

Here $F_\rho^{\mu\nu}$ and $\tilde{F}_\rho^{\alpha\beta}$ are the strength tensors and dual tensor of vector meson field, respectively, γ_μ and γ_5 are Dirac matrices. The couplings occurring in the above equation are

$$g_\rho = 2F_\pi^2 g_{\rho\pi\pi}, \tag{11}$$

$$g_{\rho\pi\pi}^2 = \frac{m_\rho^2}{2F_\pi^2}, \tag{12}$$

where $g_{\rho NN} = g_{\rho\pi\pi}$, $g_{\pi\rho\rho} = g_\rho^2/(4\pi F_\pi)$, $g_{\pi NN} = g_A m_N/F_\pi$, $g_A = 1.275$ is the nucleon axial charge and $F_\pi = 92.4$ MeV is the pion decay constant [28,48,49]. The Lagrangian describing the transition of the neutral vector meson to a dark photon is

$$\mathcal{L}_{\rho-A'} = e\epsilon g_\rho \rho_\mu A'^\mu, \tag{13}$$

can be derived from Lagrangian defining the $\rho - \gamma$ coupling [10,50] using well-known shift of the electromagnetic field $A^\mu \to A^\mu + \epsilon A'^\mu$.

To extend our formalism to higher energies, and specifically to the kinematical region of large s and small t with $s \gg |t|$, we should take into account that the corresponding matrix elements should have the s- and t-dependence dictated by the Regge formalism (see e.g., Refs. [51,52]). In particular, at high energies based on the Regge model the $g_{\rho\pi\pi}$ coupling should scale as the $\sqrt{s}$ and therefore, we can relate the high- and low-energy $g_{\rho\pi\pi}$ couplings by a scaling factor $\sqrt{s}$. On the other hand, it is known from AdS/QCD (see Refs. [53,53]) that the $g_{\rho\gamma}$ transition coupling scales at large s as $1/\sqrt{s}$. Therefore, in the calculation of the matrix element/cross section describing the cascade process of the ρ^0-meson production at large s followed by the transition to the dark photon and DM fermions we have a cancellation of the scaling factors $\sqrt{s}$ and $1/\sqrt{s}$ taking into account the large s-behavior of the $g_{\rho\pi\pi}$ and $g_{\rho\gamma}$ transition couplings. Hence, effectively we can use in our calculations the low-energy couplings $g_{\rho\pi\pi}$ and

$g_{\rho\gamma}$ dictated by the Lagrangian displayed in Eq. (3) keeping in mind that high s behavior of the mentioned couplings is compensated.

In this work we consider two different channels of the $2 \to 2$ processes with ρ^0 vector meson production. The dominant channel is induced by the exchange of the π^+ mesons radiated off target in the t-channel. The second channel occurs due the t-channel exchange of the ρ^+ or the $\pi^-\pi^0$ pair. Both channels are shown in Fig. 1.

One can consider $2 \to 2$ process in the approximation of a small scattering angle. In this case the Mandelstam variables are

$$\begin{aligned} s + t + u &= 2m_N + m_\pi^2 + m_\rho^2, \\ t &= m_\pi^2 + m_\rho^2 - 2(k_1 k_2) \\ &\approx m_\rho^2 \frac{(x-1)}{x} + m_\pi^2(1-x) - \theta_{k_1 k_2}^2 x E_{k_1}^2, \\ s &= m_\pi^2 + 2m_N E_{k_1} + m_N^2, \\ u &= m_\rho^2 + m_N^2 + 2E_{k_1} m_N x, \end{aligned} \tag{14}$$

and the recoil energy for the final neutron is

$$E_{p_2} = \frac{2m_N^2 - t}{2m_N}, \tag{15}$$

where $x = E_{k_2}/E_{k_1}$ is fraction of pion-beam energy beam transferred to the outgoing vector meson, that can be associated with the typical missing energy, $x \simeq 1$ for relatively small angle, $\theta \ll 1$, of the produced ρ^0, E_{k_1} is energy of pion beam, E_{k_2} is energy of the dark photon, m_N, m_π, and m_ρ are the nucleon, pion and ρ-meson masses, respectively.

The differential cross section for the $2 \to 2$ process is

$$d\sigma_{2\to 2} = \frac{1}{2(4j)} \sum_{s(P_1)} \sum_{\lambda'} |M|^2 dF_2, \tag{16}$$

where $j = \sqrt{E_{k1}^2 m_N^2 - m_N^2 m_\pi^2}$ is invariant flow, dF_2 is the corresponding phase space factor,

$$dF_2 = \frac{1}{16\pi s} \lambda^{\frac{1}{2}}(s, m_\pi^2, m_N^2)\, d\cos\theta_{k_1 k_2}, \tag{17}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källen kinematical triangle function. The matrix element squared is provided below. We conservatively assume that the maximum scattering angle of ρ^0 is determined by typical cuts of the NA64$_h$ experiment. The angle $\theta_{k_1 k_2}$ is connected with the fraction x from the conservation laws of energy and momentum.

A. The dominant channel

For the dominant process with π^+ exchange, the matrix element squared in the laboratory frame has the following form:

$$\begin{aligned} \frac{1}{2} \sum_{s(P_1)} \sum_{\lambda'} |M|^2 = \frac{g_{\pi NN}^2 g_{\rho\pi\pi}^2}{2(t - m_\pi^2)^2} \frac{t}{2m_\rho^2} [(m_\pi^2 - t)^2 \\ - m_\rho^2(2m_\pi^2 - m_\rho^2 - 4m_N E_{k_1}(1-x))], \end{aligned} \tag{18}$$

where the sum over the polarizations of massive vector boson is given by

$$\sum_{\lambda_\rho} \epsilon_\rho^\mu(\lambda_\rho) \epsilon_\rho^\nu(\lambda_\rho) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_\rho^2}. \tag{19}$$

In this process the contribution of neutral vector mesons with $I^J(J^{PC}) = 0^-(1^{--})$ (like ω^0 meson) are strongly suppressed due to the G-parity conservation.

B. The subdominant process

The second process is a reaction with the exchange of ρ^+ meson or loop processes with a $\pi^0\pi^+$ exchange. This process is suppressed if one compares it with the first process which was considered in Sec. III A. This difference can be explained due to the typical factors arising from propagators $1/(t - m_\pi^2)^2$ and $1/(t - m_\rho^2)^2$ at small negative t. In particular, the suppression factor is proportional to the $\sim m_\pi^4/m_\rho^4 \sim 10^{-3}$ term.

The matrix element squared of the process with ρ^+-meson exchange is

$$\begin{aligned} \frac{1}{2} \sum_{s(P_1)} \sum_{\lambda'} |M|^2 = \frac{g_{\rho NN}^2 g_{\rho\rho\pi}^2}{2(t - m_\rho^2)^2} [2m_N^2(m_\pi^2 - m_\rho^2)^2 + 2E_{k1} m_N (m_\pi^2 - m_\rho^2)^2 (x-1) - 4E_{k1}^2 m_N^2 (m_\pi^2 + m_\rho^2)(x-1)^2 \\ + 2m_N t(m_N(t - 2m_\pi^2 - 2m_\rho^2) + E_{k1}(m_\pi^2 - m_\rho^2)(1+x) + 2E_{k1}^2 m_N(1+x^2))]. \end{aligned} \tag{20}$$

This channel provides a negligible yield of ρ^0 meson, so that we do not take into account this term for the calculation of the bounds on dark photon parameter space.

IV. BOUNDS

In order to illustrate the results on the expected reach of $\mathrm{NA64}_h$ we introduce the dimensionless parameter $y = \alpha_D \epsilon^2 (m_\chi/m_{A'})^4$ [41] which is convenient to use for the thermal target DM parameter space. In particular, by exploiting this parameter $y = \alpha_D \epsilon^2 (m_\chi/m_{A'})^4$, we can compare the existed and projected limits of $\mathrm{NA64}_h$ with the typical relic DM parameter space.

In Fig. 3 we show the constraints at 90% C.L. on dark photon couplings from neutral vector meson for conservative number of pions on target 10^9 (few days of data taking) and projected future statistics that corresponds to the 5×10^{12} pions on target implying the $\mathrm{NA64}_h$ pion beam design. Limits are derived for two benchmark sets of the DM parameters, $\alpha_D = 0.5$ and $\alpha_D = 0.1$ for $m_{A'} = 3m_\chi$. The typical limit for the conservative statistics is comparable with bounds which are obtained from direct the production of ω and ϕ mesons at e^+e^- collider [24]. The similar results have been also obtained for the case of electron fixed target experiments ($\mathrm{NA64}_e$ and LDMX) that implies the search for DM in the missing energy signatures described in Ref. [34].

We emphasize that the thermal relic curves for Dirac, Majorana, and scalar DM in Fig. 3 are not affected by the rescaling of α_D, since the typical coupling y is determined by the thermal DM density [see e.g., Eq. (44) in Ref. [41] for details]. However, the limits for the accelerator-based experiments ($\mathrm{NA64}_e$ and *BABAR*) in terms of y are sensitive to the variation of α_D since the typical bounds in $(\epsilon, m_{A'})$ plane extracted from $\mathrm{NA64}_e$ [29] and *BABAR* [54] data should be appropriately rescaled to (y, m_χ) parameter space. In addition, the limits from $\mathrm{NA64}_h$ in Fig. 3 are sensitive to the variation of the α_D parameter only in the resonance region $m_{A'} \simeq m_V$ and are insensitive out of the resonance area for $m_{A'} \ll m_V$. The latter can be justified from Eqs. (1), (4), and (6).

Moreover, for the dark photons it is important to obtain the bound on the kinetic mixing parameter ϵ that is originated from the mixing of hidden spin-1 boson with ρ^0 meson. That type of coupling results in the invisible vector-meson decay to dark photon $\rho \to A' \to \chi\bar{\chi}$. Here we would like to note that for projected statistics $\sim 5\times 10^{12}$ πOT the direct dark-photon production results in a relatively weak bounds of the kinetic mixing parameter. The resonant production of DM is more effective due to the amplified magnitude of the cross section near the resonant dark photon mass term. In this area bounds are more strict.

We should underline important advantages for analyzing invisible vector meson decay by using pion beams and missing-energy techniques in the fixed target experiments. In particular, for ρ^0 meson production the $\pi^+\pi^-$ channel is dominant [55,56]. Besides, this invisible decay of ρ^0 meson coupled with dark photon implies the interaction coupling of spin-1 hidden boson with quarks. As a result, for the ultimate statistics of $\mathrm{NA64}_h$ at the level of $\simeq 10^{14}\pi$OT one can obtain a relatively strong bounds on the typical DM thermal target parameter $y = \alpha_D \epsilon^2 (m_\chi/m_{A'})^4$, bound have a same order of bounds as expected ultimate limit for electron beam of the $\mathrm{NA64}_e$ experiment. For statistics of $\sim 5\times 10^{12}\pi$OT the bounds will be less then those were obtained for ultimate statistics of $\mathrm{NA64}_e$ experiment. The optimistic bound of LDMX (10^{18} EOT) can rule out the expected reach of $\mathrm{NA64}_h$ for the ultimate statistics at the level of $10^{14}\pi$OT (see, e.g., Ref. [34] for detail). The comparisons of the regarding expected limits are shown in Fig. 4. In this picture we use the bounds obtained in Ref. [34] for the ultimate limit of invisible vector decay at $\mathrm{NA64}_e$ and LDMX experiments. These bounds for the

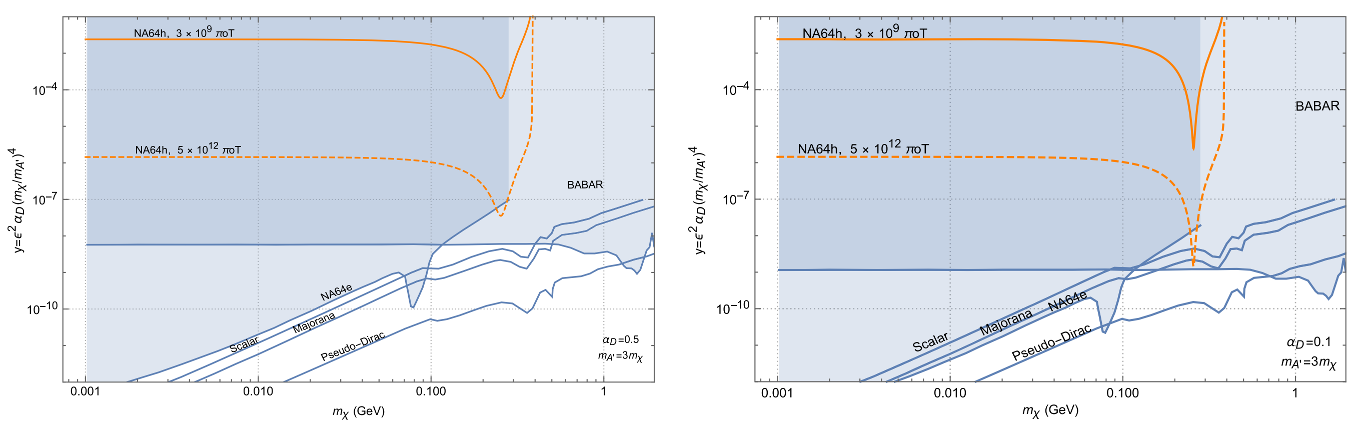

FIG. 3. The constraints on parameters space of the dark photon mediator for pseudo-Dirac DM. In both panels, we show existing limits from the last data of $\mathrm{NA64}_e$ experiment [29], our estimates for the future $\mathrm{NA64}_h$ experiment with a hadron beam [46] and constraints from the production of DM in e^+e^- collisions at *BABAR* [26]. In left panel we show the limits for ρ^0 neutral vector-meson invisible decay for current (few days of data taking) and ultimate statistics of $\mathrm{NA64}_h$ that implies $\alpha_D = 0.5$ and $m_{A'} = 3m_\chi$. In the right panel the same as in left panel but for $\alpha_D = 0.1$ and $m_{A'} = 3m_\chi$.

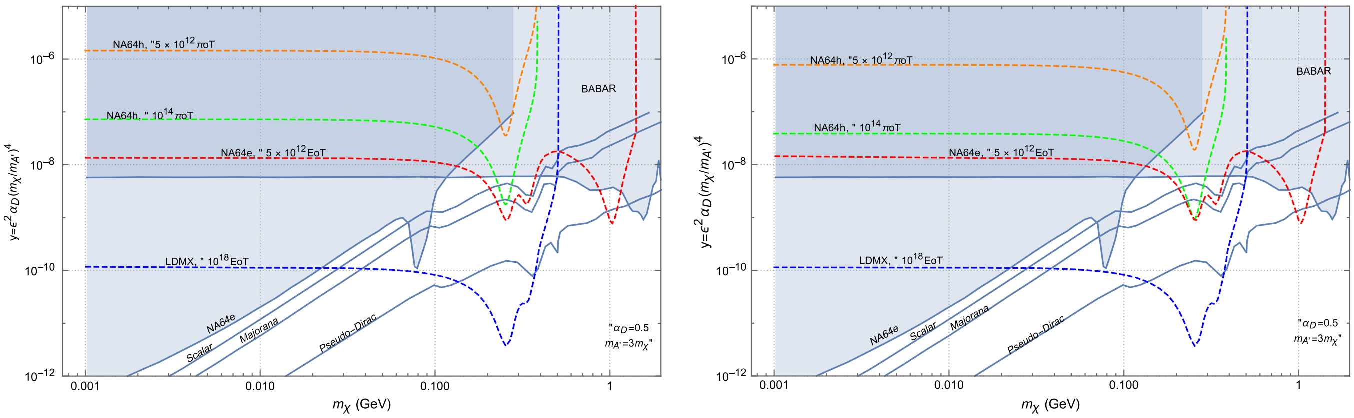

FIG. 4. The $\mathrm{NA64}_h$ projected constraints for 5×10^{12} and 10^{14} pion on target and the expected reach of $\mathrm{NA64}_e$ and LDMX experiments with an electron beam [34] for $\alpha_D = 0.5$ and $m_{A'} = 3m_\chi$. In the left panel we imply for $\mathrm{NA64}_h$ that the recoil momentum transferred from pion to the nucleon can be as small as 0.8 GeV. In right panel show the case of 1.2 GeV for the recoil momentum of nucleon.

LDMX experiment include limits from ρ and ω meson, for the NA64 experiment bound includes limits from J/ψ invisible decay too. Besides, there needs to note a difference in couplings between vector meson and dark photon with [34]. We exploit the effective field theory to fix meson couplings. We plan to expand our analysis of vector meson production for pion beam scattering at fixed target including numerical simulation and analysis of addition vector mesons. Besides, we also plan to consider real pQCD calculations in our analysis of fix target experiments with pion-beam energy at 50 GeV or 100 GeV.

In the framework of the proposal, we note that missing energy techniques require an approximate zero background for identification of missing energy signals. From Table I, one can see that decreasing beam energy to 20 GeV can increase the expected limit from invisible meson decay at the $\mathrm{NA64}_h$ experiment.

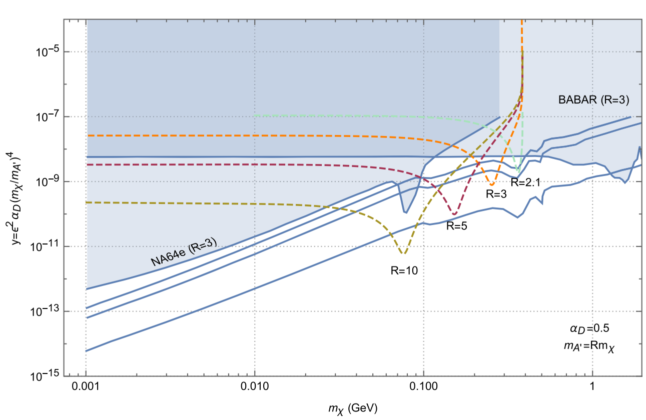

FIG. 5. Projected 90% C.L. exclusion for statistics $\sim 10^{14}$ pions on target from invisible ρ^0 neutral vector meson decay into pseudo-Dirac DM by transition via dark photon. Constraint is presented for several choices of mass ratio $R = m_{A'}/m_\chi$. Thermal targets and experimental bounds are shown for $R = 3$.

In Fig. 5 we show the sensitivity curves for various mass ratio $R = m_{A'}/m_\chi$ for the dark photons and pseudo-Dirac fermions implying the projected ultimate statistics $\sim 10^{14}$ of pions on target. Our results are in full agreement with one presented in Refs. [34,41]. In particular, the larger value of the R, the smaller typical resonant masses of DM. Moreover, one can achieve the better limits for large value of R parameter, that follows from the Breit-Wigner shape for resonance production. This behavior and moving of existing limits with change of R is connected with $y \propto 1/R^4$. The dark photon bremsstrahlung rate is proportional to $1/R^2$, whereas invisible meson decay rate is independent of $1/R$ (see Ref. [34]). If we change ratio of dark mass photons and DM fermions R in area where $m_{A'} \gg m_\chi$ we found that the bremsstrahlung rate is suppressed. Invisible meson decay does not have $1/R^n$ suppression and will give a main bound for the dark photon parameter space in a high-R scenario.

In addition to everything mentioned before, one can obtain the typical bound on the invisible branching,

$$\mathrm{Br}(\rho^0 \to \mathrm{inv}) < 1.2 \times 10^{-3} \quad \text{for } 3.3 \times 10^9\ \pi\mathrm{OT}, \qquad (21)$$

$$\mathrm{Br}(\rho^0 \to \mathrm{inv}) < 7.5 \times 10^{-7} \quad \text{for } 5 \times 10^{12}\ \pi\mathrm{OT}, \qquad (22)$$

in the framework 90% C.L. of missing-energy signature implying zero signal events and background-free case.

V. CONCLUSION

Invisible decay of the ρ^0 meson was studied by using the missing energy design of the fixed-target experiment with pion beam. We used the NA64 hadronic design with π^- beam scattered in hadronic calorimeter that serves as a target. We derived the bounds on parameter space of pseudo-Dirac DM for the proposed conservative and ultimate statistics of pions on target. We compared the regarding expected reach with typical curves of DM relic density. We analyzed two possibilities of the pion beam energy; 50 GeV and 100 GeV. The second possibility of the pion -beam energy (100 GeV) requires a more narrow angle of meson production if we want to search for missing-energy signals at small recoil energy in the background. All these cuts lead to less meson yield at high-energy pion beam. Wherein, we note that decreasing the pion-energy beam can give a chance to obtain a more strict limit to parameter DM using invisible mode of vector meson. This was tested numerically for the specific value of the pion beam energy equal to 20 GeV. That advantage of low-energy beam is connected that at the same small recoil momentum to target we will have more yield of ρ^0. It is important for using missing energy/momentum technique. Additionally, we would like to note that we propose to use a potential of missing-energy techniques in the fixed target experiment at PS/CERN with a pion-beam energy of 6 GeV. Such a configuration should be more optimal for study considering charge-exchange processes with vector-meson production.

Obtained results in the present paper led to optimistic bounds which can be made by analysis of the invisible vector-meson decay as a signal to possible DM production. We showed that the pion beam can be an effective tool for the study of DM by missing-energy/momenta technique in the fixed target experiments. In future we plan to make a more comprehensive analysis of detector in the setup of the fixed-target experiment with pion beam. Besides, we plan to study meson production at high energies in fixed-target experiments with pion beams by using both model-independent and model-dependent techniques.

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