Calculation of mass and width of unstable molecular state using the developed Bethe-Salpeter theory

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Applying the developed Bethe-Salpeter theory for dealing with resonance, we investigate the time evolution of molecular state composed of two vector mesons as determined by the total Hamiltonian. Then exotic meson resonance $\chi_{c0}(3915)$ is considered as a mixed state of two unstable molecular states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, and the mass and width for physical resonance $\chi_{c0}(3915)$ are calculated in the framework of relativistic quantum field theory. In this actual calculation, we minutely show how to obtain the correction for energy level of resonance and to exhibit the key features of dispersion relation in an extended Feynman diagram. The numerical results are consistent with the experimental values.

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I. INTRODUCTION

The hadronic molecule structure has been proposed to interpret the internal structure of exotic meson resonance for many years [1,2]. In previous works [1–6], molecular states were considered as meson-meson bound states and the homogeneous Bethe-Salpeter (BS) equation was frequently used to investigate molecular states. Solving homogeneous BS equations for meson-meson bound states, the authors of these works obtained the masses and BS wave functions. The mass of meson-meson bound state was regarded as the mass of exotic meson resonance. However, all decay channels of resonance should contribute to its physical mass and the correction for the energy level of the molecular state due to decay channels has seldom been considered [1–9]. Fortunately, recent fundamental research [10] noted that hadron resonance should be regarded as an unstable two-body system, and developed BS theory for dealing with the dynamics of coupled channels in the framework of relativistic quantum field theory. Though Ref. [10] illuminated the physical meaning of the developed Bethe-Salpeter theory for dealing with resonance, many details in the computational process were not presented. In this paper, we will comprehensively and systematically show the theoretical approach about unstable molecular state composed of two heavy vector mesons, and this approach is applied to investigate exotic meson resonance $\chi_{c0}(3915)$ [11], once named $X(3915)$, which is considered as a mixed state of two unstable molecular states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$.

Since resonance is an unstable state which decays spontaneously into other particles, the molecular state composed of two heavy-vector mesons should not be a stationary vector-vector bound state. To investigate this unstable two-body system, we suppose that at some given time this unstable state has been prepared to decay and then study the time evolution of this system as determined by the total Hamiltonian. This prepared state can be described by the ground-state BS wave function for the vector-vector bound state at the times $t_1 = 0$ and $t_2 = 0$. In our previous works [5,9], the most general form of BS wave functions for the bound states created by two vector fields with arbitrary spin and definite parity has been given. According to the effective theory at low-energy QCD, we have investigated the light-meson interaction with light quarks in heavy-vector mesons and obtained the interaction kernel between two light quarks in two heavy-vector mesons derived from one light-meson (σ, ω, ρ, ϕ) exchange [5,12]. Solving the BS equation with this interaction kernel, we have obtained the mass and BS wave function for the bound state composed of two vector mesons [5,13]. In this paper, we also consider the interaction kernel between two heavy quarks in two heavy mesons derived from one heavy-meson exchange. After providing the description for the prepared state, we study the time evolution of the prepared

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state and obtain the pole corresponding to resonance through the scattering-matrix element.

The crucial point of our resonance theory is that the scattering-matrix element between bound states is calculated in the framework of relativistic quantum field theory. According to the dispersion relation, the total matrix element between a final state and an initial bound state should be calculated with respect to an arbitrary value of the final-state energy [10]. It is necessary to note that the total energy of the final state extends over the real interval while the initial-state energy is specified. For the initial bound state composed of two heavy-vector mesons, we have given the generalized Bethe-Salpeter (GBS) amplitude for four-quark state describing this meson-meson structure [8,9], which should be specified. Because the value of the final-state energy is an arbitrary real number over the real interval, we may obtain several closed channels derived from the interaction Lagrangian and all open and closed channels should contribute to the mass of the physical resonance. For the exotic resonance $\chi_{c0}(3915)$, we consider three open-decay channels $J/\psi\omega$, D^+D^-, and $D^0\bar{D}^0$ and one closed channel $D^*\bar{D}^*$ from the effective interaction Lagrangian at low-energy QCD. Mandelstam's approach is applied to calculate the matrix element between bound states with respect to an arbitrary value of the final-state energy, which are exhibited by extended Feynman diagrams. Finally, we obtain the correction for the energy level of resonance $\chi_{c0}(3915)$ and the physical mass is used to calculate the decay width of the physical resonance $\chi_{c0}(3915)$.

The structure of this article is as follows. In Sec. II we give the revised general form of GBS wave functions for meson-meson bound states as four-quark states. The mass and GBS wave function for the mixed state of two bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ is obtained in instantaneous approximation. Section III gives the traditional technique to calculate the matrix element with the mass of the meson-meson bound state, which is applied to investigate the decay modes $\chi_{c0}(3915)\to J/\psi\omega$, $\chi_{c0}(3915)\to D^+D^-$ and $\chi_{c0}(3915)\to D^0\bar{D}^0$. Section IV gives the developed Bethe-Salpeter theory. In Sec. V we emphatically introduce the matrix element between bound states with respect to an arbitrary value of the final-state energy. Three open-decay channels $J/\psi\omega$, D^+D^-, $D^0\bar{D}^0$, and one closed channel $D^*\bar{D}^*$ are considered. In Sec. VI we obtain the physical mass and width for the unstable molecular state. Our numerical results are presented in Sec. VII and we make some concluding remarks in Sec. VIII.

II. GBS WAVE FUNCTION OF THE MESON-MESON BOUND STATE AS A FOUR-QUARK STATE

According to the effective theory at low-energy QCD, nonvanishing vacuum condensate causes the spontaneous breaking of chiral symmetry, which leads to the appearance of Goldstone bosons [14]. At low-energy QCD, the effective interaction Lagrangian can be regarded as Lagrangian for the interaction of light mesons with quarks. In this paper, we investigate the light-meson interaction with the light quarks in heavy mesons and the interaction Lagrangian for the coupling of light-quark fields to light-meson fields is [8]

$$\begin{aligned}\mathcal{L}_I^{\text{eff}} &= ig_0\begin{pmatrix}\bar{u} & \bar{d} & \bar{s}\end{pmatrix}\gamma_5\begin{pmatrix}\pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+\\ \sqrt{2}\pi^- & -\pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0\\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta\end{pmatrix}\begin{pmatrix}u\\ d\\ s\end{pmatrix}\\ &\quad + ig_0'\begin{pmatrix}\bar{u} & \bar{d} & \bar{s}\end{pmatrix}\gamma_\mu\begin{pmatrix}\rho^0+\omega & \sqrt{2}\rho^+ & \sqrt{2}K^{*+}\\ \sqrt{2}\rho^- & -\rho^0+\omega & \sqrt{2}K^{*0}\\ \sqrt{2}K^{*-} & \sqrt{2}\bar{K}^{*0} & \sqrt{2}\phi\end{pmatrix}_\mu\begin{pmatrix}u\\ d\\ s\end{pmatrix} + g_\sigma\begin{pmatrix}\bar{u} & \bar{d}\end{pmatrix}\begin{pmatrix}u\\ d\end{pmatrix}\sigma. \qquad (1)\end{aligned}$$

From this effective-interaction Lagrangian at low-energy QCD, we have to consider that the heavy meson is a bound state composed of a quark and an antiquark and investigate the interaction of the light meson with quarks in the heavy meson. The quark current J_μ coupling with a light-vector meson, the quark pseudoscalar density J^- coupling with light-pseudoscalar meson and the quark scalar density J coupling with σ meson can be obtained. In this section, our attention is only focused on the bound state composed of two vector mesons and some errors in previous works are revised.

A. BS wave function for bound state composed of two vector mesons

If a bound state with spin j and parity η_P is created by two Heisenberg vector fields with masses M_1 and M_2, respectively, its BS wave function is defined as

$$\begin{aligned}\chi^j_{P(\lambda\tau)}(x_1',x_2') &= \langle 0|TA_\lambda(x_1')A_\tau^\dagger(x_2')|P,j\rangle\\ &= \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(P)}}e^{iP\cdot X}\chi^j_{P(\lambda\tau)}(X'), \qquad (2)\end{aligned}$$

where P is the momentum of the bound state, $E(p) = \sqrt{\mathbf{p}^2 + m^2}$, $x_1' = (\mathbf{x}_1', it_1)$, $x_2' = (\mathbf{x}_2', it_2)$, $X = \eta_1 x_1' + \eta_2 x_2'$, $X' = x_1' - x_2'$, and $\eta_{1,2} = M_{1,2}/(M_1 + M_2)$. Making the Fourier transformation, we obtain the BS wave function in the momentum representation

$$\chi_P^j(p_1', p_2')_{\lambda\tau} = \frac{1}{(2\pi)^{3/2}} \times \frac{1}{\sqrt{2E(P)}}(2\pi)^4 \delta^{(4)}(P - p_1' + p_2')\chi_{\lambda\tau}^j(P, p), \tag{3}$$

where p is the relative momentum of two vector fields and we have $P = p_1' - p_2'$, $p = \eta_2 p_1' + \eta_1 p_2'$, where p_1' and p_2' are the momenta carried by two vector fields, respectively. The polarization tensor of the bound state $\eta_{\mu_1\mu_2\cdots\mu_j}$ can be separated,

$$\chi_{\lambda\tau}^j(P, p) = \eta_{\mu_1\mu_2\cdots\mu_j}\chi_{\mu_1\mu_2\cdots\mu_j\lambda\tau}(P, p), \tag{4}$$

where the subscripts λ and τ are derived from these two vector fields. The polarization tensor $\eta_{\mu_1\mu_2\cdots\mu_j}$ describes the spin of the bound state, which is totally symmetric, transverse, and traceless,

$$\eta_{\mu_1\mu_2\cdots} = \eta_{\mu_2\mu_1\cdots}, \qquad P_{\mu_1}\eta_{\mu_1\mu_2\cdots} = 0, \qquad \eta^{\mu_1}_{\mu_1\mu_2\cdots} = 0. \tag{5}$$

From Lorentz covariance, we have

$$\begin{aligned}
\chi_{\mu_1\cdots\mu_j\lambda\tau} = {} & p_{\mu_1}\cdots p_{\mu_j}[g_{\lambda\tau}f_1 + (P_\lambda p_\tau + P_\tau p_\lambda)f_2 + (P_\lambda p_\tau - P_\tau p_\lambda)f_3 + P_\lambda P_\tau f_4 + p_\lambda p_\tau f_5] \\
& + (p_{\{\mu_2}\cdots p_{\mu_j}g_{\mu_1\}\lambda}p_\tau + p_{\{\mu_2}\cdots p_{\mu_j}g_{\mu_1\}\tau}p_\lambda)f_6 \\
& + (p_{\{\mu_2}\cdots p_{\mu_j}g_{\mu_1\}\lambda}p_\tau - p_{\{\mu_2}\cdots p_{\mu_j}g_{\mu_1\}\tau}p_\lambda)f_7 \\
& + (p_{\{\mu_2}\cdots p_{\mu_j}g_{\mu_1\}\lambda}P_\tau + p_{\{\mu_2}\cdots p_{\mu_j}g_{\mu_1\}\tau}P_\lambda)f_8 \\
& + (p_{\{\mu_2}\cdots p_{\mu_j}g_{\mu_1\}\lambda}P_\tau - p_{\{\mu_2}\cdots p_{\mu_j}g_{\mu_1\}\tau}P_\lambda)f_9 \\
& + p_{\mu_1}\cdots p_{\mu_j}\epsilon_{\lambda\tau\xi\zeta}p_\xi P_\zeta f_{10} + p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}p_\xi f_{11} + p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi}P_\xi f_{12} \\
& + (p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta}p_\xi P_\zeta p_\tau + p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta}p_\xi P_\zeta p_\lambda)f_{13} \\
& + (p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta}p_\xi P_\zeta p_\tau - p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta}p_\xi P_\zeta p_\lambda)f_{14} \\
& + (p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta}p_\xi P_\zeta P_\tau + p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta}p_\xi P_\zeta P_\lambda)f_{15} \\
& + (p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta}p_\xi P_\zeta P_\tau - p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta}p_\xi P_\zeta P_\lambda)f_{16} \\
& + p_{\{\mu_3}\cdots p_{\mu_j}g_{\mu_1\lambda}g_{\mu_2\}\tau}f_{17} + p_{\{\mu_3}\cdots p_{\mu_j}\epsilon_{\mu_1\lambda\xi\zeta}p_\xi P_\zeta \epsilon_{\mu_2\}\tau\xi'\zeta'}p_{\xi'}P_{\zeta'}f_{18} \\
& + (p_{\{\mu_3}\cdots p_{\mu_j}g_{\mu_1\lambda}\epsilon_{\mu_2\}\tau\xi\zeta}p_\xi P_\zeta + p_{\{\mu_3}\cdots p_{\mu_j}g_{\mu_1\tau}\epsilon_{\mu_2\}\lambda\xi\zeta}p_\xi P_\zeta)f_{19} \\
& + (p_{\{\mu_3}\cdots p_{\mu_j}g_{\mu_1\lambda}\epsilon_{\mu_2\}\tau\xi\zeta}p_\xi P_\zeta - p_{\{\mu_3}\cdots p_{\mu_j}g_{\mu_1\tau}\epsilon_{\mu_2\}\lambda\xi\zeta}p_\xi P_\zeta)f_{20},
\end{aligned} \tag{6}$$

where $\{\mu_1, \ldots, \mu_j\}$ represents symmetrization of the indices $\mu_1, \ldots, \mu_j$. In fact, the relative momenta $p_{\mu_1}, \ldots, p_{\mu_j}, p_\lambda, p_\tau$ represent the orbital angular momenta. There should be 20 scalar functions $f_i(P\cdot p, p^2)(i = 1, \ldots, 20)$ in Eq. (6). In Ref. [9], three tensor structures are omitted. In this paper, these missing terms are added as the last three terms in Eq. (6). Using the transversality condition [5,12]

$$p'_{1\lambda}\chi_{\lambda\tau}^j(P, p) = p'_{2\tau}\chi_{\lambda\tau}^j(P, p) = 0 \tag{7}$$

and considering the properties of BS wave function under space reflection, we obtain the revised general form of BS wave functions for the bound states created by two massive vector fields with arbitrary spin and definite parity (see details in [5]), for $\eta_P = (-1)^j$,

$$\begin{aligned}
\chi_{\lambda\tau}^j(P, p) = {} & \frac{1}{\mathcal{N}^j}\eta_{\mu_1\cdots\mu_j}[p_{\mu_1}\cdots p_{\mu_j}(\mathcal{T}_{\lambda\tau}^1\Phi_1 + \mathcal{T}_{\lambda\tau}^2\Phi_2) \\
& + \mathcal{T}^3_{\mu_1\cdots\mu_j\lambda\tau}\Phi_3 + \mathcal{T}^4_{\mu_1\cdots\mu_j\lambda\tau}\Phi_4 \\
& + \mathcal{T}^5_{\mu_1\cdots\mu_j\lambda\tau}\Phi_5 + \mathcal{T}^6_{\mu_1\cdots\mu_j\lambda\tau}\Phi_6],
\end{aligned} \tag{8}$$

for $\eta_P = (-1)^{j+1}$,

$$\begin{aligned}
\chi_{\lambda\tau}^j(P, p) = {} & \frac{1}{\mathcal{N}^j}\eta_{\mu_1\cdots\mu_j}(p_{\mu_1}\cdots p_{\mu_j}\epsilon_{\lambda\tau\xi\zeta}p_\xi P_\zeta \Phi_1' \\
& + \mathcal{T}^7_{\mu_1\cdots\mu_j\lambda\tau}\Phi_2' + \mathcal{T}^8_{\mu_1\cdots\mu_j\lambda\tau}\Phi_3' + \mathcal{T}^9_{\mu_1\cdots\mu_j\lambda\tau}\Phi_4' \\
& + \mathcal{T}^{10}_{\mu_1\cdots\mu_j\lambda\tau}\Phi_5' + \mathcal{T}^{11}_{\mu_1\cdots\mu_j\lambda\tau}\Phi_6' + \mathcal{T}^{12}_{\mu_1\cdots\mu_j\lambda\tau}\Phi_7'),
\end{aligned} \tag{9}$$

where $\mathcal{N}^j$ is normalization, the independent tensor structures $\mathcal{T}^i_{\lambda\tau}$ are given in the Appendix, and $\Phi_i(P\cdot p, p^2)$ and $\Phi'_i(P\cdot p, p^2)$ are independent scalar functions. The scalar functions f_i in Eq. (6) are the linear combinations of Φ_i and Φ'_i.

B. Kernel between two heavy-vector mesons

In experiments [15,16], the narrow state $\chi_{c0}(3915)$, once named $Y(3940)$ and $X(3915)$, was discovered and its structure does not fit the conventional $c\bar{c}$ charmonium interpretation. Then interpretation of the $\chi_{c0}(3915)$ as a mixed state of two bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ was proposed in Refs. [3,4,7], and these theoretical works calculated the binding energy and the strong and radiative decay widths. In the following experiments [17,18], the $\chi_{c0}(3915)$ resonance, decaying to the $J/\psi\omega$ final state, was observed in two-photon collisions, and the product of the two-photon decay width and the branching fraction to $J/\psi\omega$ was measured. The value of the product of two partial decay widths $\Gamma(\chi_{c0}(3915)\to\gamma\gamma)\Gamma(\chi_{c0}(3915)\to J/\psi\omega)$ is unexpectedly large compared to other excited $c\bar{c}$ states, and this value is roughly compatible with the prediction in Ref. [7] assuming the $D^*\bar{D}^*$ bound-state model (see Refs. [17,18]). However, in experiments exotic particle $\chi_{c0}(3915)$ is resonance, so this exotic particle is an unstable state which should not be completely treated as a stationary two-body bound state and it is more reasonable to regard this exotic resonance as an unstable two-body system. In this paper, we assume that the isoscalar $\chi_{c0}(3915)$ is a mixed state of two unstable molecular states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ with spin-parity quantum numbers 0^+. There are two steps to deal with this unstable system in our theoretical frame. As the first step, we investigate the mixed state of two stable-bound states, $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$. As the second step, we study the time evolution of unstable system determined by the total Hamiltonian and obtain the correction for energy level of resonance due to decay channels.

In this section, we only investigate the mixed state of two stable-bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, and the BS wave function for this system is a linear combination of two components as

$$\chi^{D^*\bar{D}^*,j}_{\lambda\tau}(P,p)=\frac{1}{\sqrt{2}}\chi^{D^{*0}\bar{D}^{*0},j}_{\lambda\tau}(P,p)+\frac{1}{\sqrt{2}}\chi^{D^{*+}D^{*-},j}_{\lambda\tau}(P,p), \tag{10}$$

where

$$\chi^{D^{*0}\bar{D}^{*0},j}_{\lambda\tau}(P,p)=\chi^j_{\lambda\tau}(P,p)\left(-\left|\frac{1}{2},-\frac{1}{2}\right\rangle\right)^{D^{*0}}\otimes\left|\frac{1}{2},\frac{1}{2}\right\rangle^{\bar{D}^{*0}},$$
$$\chi^{D^{*+}D^{*-},j}_{\lambda\tau}(P,p)=\chi^j_{\lambda\tau}(P,p)\left(-\left|\frac{1}{2},\frac{1}{2}\right\rangle\right)^{D^{*+}}\otimes\left|\frac{1}{2},-\frac{1}{2}\right\rangle^{D^{*-}}, \tag{11}$$

and P becomes the total momentum for the mixed state of two meson-meson bound states, $\chi^j_{\lambda\tau}(P,p)$ is the component wave function in the momentum representation; $(-|\frac{1}{2},-\frac{1}{2}\rangle)\otimes|\frac{1}{2},\frac{1}{2}\rangle$ and $(-|\frac{1}{2},\frac{1}{2}\rangle)\otimes|\frac{1}{2},-\frac{1}{2}\rangle$ are the isospin wave functions of pure bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, respectively. $\chi^{D^{*0}\bar{D}^{*0},j}_{\lambda\tau}$ and $\chi^{D^{*+}D^{*-},j}_{\lambda\tau}$ represent the BS wave functions for the bound states of two-vector mesons, which are the eigenstates of the Hamiltonian without considering the coupled-channel terms. These eigenstates have the same quantum numbers. The error in Ref. [9] has been revised. As usual the momentum for the mixed state of two bound states is set as $P=(0,0,0,iM_0)$ in the rest frame.

Let D^*_l denote one of D^{*0} and D^{*+}, and $l=u$, d represents the u or d antiquark in heavy vector meson D^{*0} or D^{*+}, respectively; $\bar{D}^*_l$ denotes the antiparticle of D^*_l. From Eq. (8), we can obtain the BS wave function describing pure bound state $D^*_l\bar{D}^*_l$

$$\chi^{0^+}_{\lambda\tau}(P^{D\bar{D}},p)=\frac{1}{\mathcal{N}^{0^+}_{D\bar{D}}}[\mathcal{T}^1_{\lambda\tau}\mathcal{F}_1(P^{D\bar{D}}\cdot p,p^2)+\mathcal{T}^2_{\lambda\tau}\mathcal{F}_2(P^{D\bar{D}}\cdot p,p^2)]. \tag{12}$$

$P^{D\bar{D}}$ represents the momentum of pure bound state in the rest frame, whose fourth component is different from the one of P. This BS wave function should satisfy the equation

$$\chi^{0^+}_{\lambda\tau}(P^{D\bar{D}},p)=-\int\frac{d^4p'}{(2\pi)^4}\Delta_{F\lambda\theta}(p'_1)\mathcal{V}_{\theta\theta',\kappa'\kappa}(p,p';P^{D\bar{D}})\times\chi^{0^+}_{\theta'\kappa'}(P^{D\bar{D}},p')\Delta_{F\kappa\tau}(p'_2) \tag{13}$$

where $\mathcal{V}_{\theta\theta',\kappa'\kappa}$ is the interaction kernel, $P^{D\bar{D}}=(0,0,0,iM_{D\bar{D}})$, $p'_1=p+P^{D\bar{D}}/2$, $p'_2=p-P^{D\bar{D}}/2$, $\Delta_{F\lambda\theta}(p'_1)$ and $\Delta_{F\kappa\tau}(p'_2)$ are the propagators for the spin-1 fields, $\Delta_{F\lambda\theta}(p'_1)=(\delta_{\lambda\theta}+\frac{p'_{1\lambda}p'_{1\theta}}{M_1^2})\frac{-i}{p_1'^2+M_1^2-i\epsilon}$, $\Delta_{F\kappa\tau}(p'_2)=(\delta_{\kappa\tau}+\frac{p'_{2\kappa}p'_{2\tau}}{M_2^2})\frac{-i}{p_2'^2+M_2^2-i\epsilon}$, $M_1=M_{D^*_l}$ and $M_2=M_{\bar{D}^*_l}$. We emphasize that the kernel $\mathcal{V}$ is defined in two-body channel so $\mathcal{V}$ is not the complete interaction. The kernel in the homogeneous BS equation (13) plays a central role for making the two-body system to be a stable bound state, and the solution of the homogeneous BS equation (13) should only describe a bound state. In our approach, the BS equation for the meson-meson bound state is treated in the ladder approximation. This approximation consists of replacing the interaction kernel by its lowest-order value corresponding to the simple one-meson exchange. Though the interaction kernel $\mathcal{V}_{\theta\theta',\kappa'\kappa}$ in Eq. (13) only contains the contribution from the irreducible graph, Eq. (13) contains the contribution from irreducible and reducible graphs [19], shown as Fig. 1. Hence, the solution of the BS equation, i.e., mass and BS wave

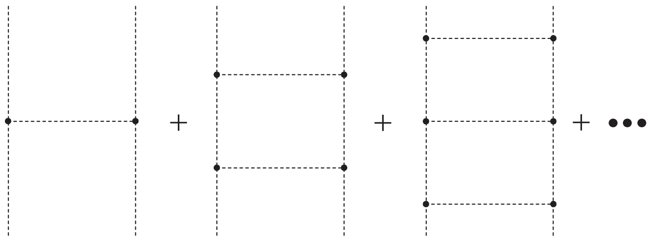

FIG. 1. Ladder approximation for the two-body propagator.

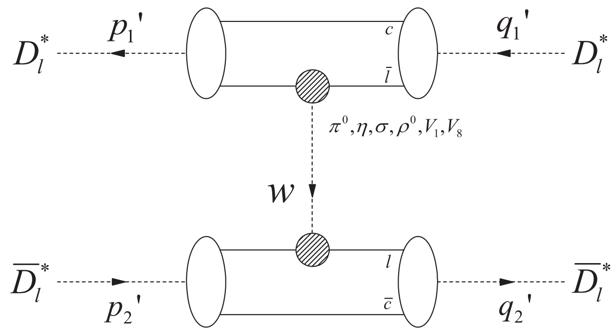

FIG. 2. Light-meson exchange between two light quarks in two heavy-vector mesons. The solid lines denote quark propagators, the filled circles represent the vertex functions for light meson and light quark, and the unfilled ellipses represent Bethe-Salpeter amplitudes.

function of the bound state, contains the contribution from irreducible and reducible graphs.

Different from the previous works about hadronic molecules, in our approach the heavy mesons in a molecular state are considered as bound states composed of a heavy quark and a light quark. From the interaction Lagrangian for the coupling of light-quark fields to light-meson fields expressed as Eq. (1), we can obtain the interaction kernel between two light quarks in two heavy mesons from one light-meson exchange. Moreover, we should consider the interaction kernel between two heavy quarks in two heavy mesons from one heavy-meson exchange. In our theoretical frame, the interaction kernel between two heavy mesons is derived from the one meson exchange between two quarks in these two heavy mesons. To construct the interaction kernel between D_l^* and $\bar{D}_l^*$, we consider the one light-meson (π^0, η, σ, ρ^0, V_1 and V_8) exchange [5,12,13] and one heavy-meson (J/ψ) exchange.

The flavor-SU(3) singlet V_1 and octet V_8 states of vector mesons mix to form the physical ω and ϕ mesons as

$$\phi = -V_8 \cos\theta + V_1 \sin\theta, \qquad \omega = V_8 \sin\theta + V_1 \cos\theta, \tag{14}$$

where the mixing angle $\theta = 38.58°$ was obtained by KLOE [20]. Then the exchanged mesons should be the octet V_8 and singlet V_1 states, and the relation of the octet-quark coupling constant g_8 and the singlet-quark coupling constant g_1 has the form

$$g_\phi = -g_8 \cos\theta + g_1 \sin\theta, \qquad g_\omega = g_8 \sin\theta + g_1 \cos\theta, \tag{15}$$

where the meson-quark coupling constants $g_\omega^2 = 2.42/2$ and $g_\phi^2 = 13.0$ were determined by QCD sum rules approach [21]. The light meson exchange between two light quarks in two heavy vector mesons is shown in Fig. 2. The filled circle of Fig. 2 represents the vertex function for light meson and light quark in heavy meson. From the Lorentz-structure, the matrix elements of quark pseudoscalar density J^-, quark scalar density J and quark current J_α can be expressed as

$$\langle VM^{\varrho}(p_1')|J^-(0)|VM^{\vartheta}(q_1')\rangle = \frac{1}{2\sqrt{E_{D_l^*}(p_1')E_{D_l^*}(q_1')}} h^{(\mathrm{p})}(w^2)\epsilon_{\varsigma\varsigma'\theta\theta'} p'_{1\varsigma} q'_{1\varsigma'} \varepsilon_\theta^{\varrho}(p_1')\varepsilon_{\theta'}^{\vartheta}(q_1'), \tag{16a}$$

$$\langle \overline{VM'}^{\varrho'}(-p_2')|J^-(0)|\overline{VM'}^{\vartheta'}(-q_2')\rangle = \frac{1}{2\sqrt{E_{\bar{D}_l^*}(-p_2')E_{\bar{D}_l^*}(-q_2')}} \bar{h}^{(\mathrm{p})}(w^2)\epsilon_{\varpi\varpi'\kappa'\kappa} p'_{2\varpi} q'_{2\varpi'} \varepsilon_\kappa^{\varrho'}(-p_2')\varepsilon_{\kappa'}^{\vartheta'}(-q_2'), \tag{16b}$$

$$\langle VM^{\varrho}(p_1')|J(0)|VM^{\vartheta}(q_1')\rangle = \frac{1}{2\sqrt{E_{D_l^*}(p_1')E_{D_l^*}(q_1')}} \times \left\{[\varepsilon^{\varrho}(p_1')\cdot\varepsilon^{\vartheta}(q_1')]h_1^{(\mathrm{s})}(w^2) - h_2^{(\mathrm{s})}(w^2)\frac{1}{M_1^2}[\varepsilon^{\varrho}(p_1')\cdot q_1'][\varepsilon^{\vartheta}(q_1')\cdot p_1']\right\}, \tag{16c}$$

$$\langle \overline{VM'}^{\varrho'}(-p_2')|J(0)|\overline{VM'}^{\vartheta'}(-q_2')\rangle = \frac{1}{2\sqrt{E_{\bar{D}_l^*}(-p_2')E_{\bar{D}_l^*}(-q_2')}} \times \left\{ [\varepsilon^{\varrho'}(-p_2')\cdot\varepsilon^{\vartheta'}(-q_2')]\bar{h}_1^{(\mathrm{s})}(w^2) - \bar{h}_2^{(\mathrm{s})}(w^2)\frac{1}{M_2^2}[\varepsilon^{\varrho'}(-p_2')\cdot(-q_2')][\varepsilon^{\vartheta'}(-q_2')\cdot(-p_2')] \right\}, \tag{16d}$$

$$\langle VM^{\varrho}(p_1')|J_\alpha(0)|VM^{\vartheta}(q_1')\rangle = \frac{1}{2\sqrt{E_{D_l^*}(p_1')E_{D_l^*}(q_1')}} \times \Big\{ [\varepsilon^{\varrho}(p_1')\cdot\varepsilon^{\vartheta}(q_1')]h_1^{(\mathrm{lv})}(w^2)(p_1'+q_1')_\alpha - h_2^{(\mathrm{lv})}(w^2)\{[\varepsilon^{\varrho}(p_1')\cdot q_1']\varepsilon_\alpha^{\vartheta}(q_1') + [\varepsilon^{\vartheta}(q_1')\cdot p_1']\varepsilon_\alpha^{\varrho}(p_1')\} - h_3^{(\mathrm{lv})}(w^2)\frac{1}{M_1^2}[\varepsilon^{\varrho}(p_1')\cdot q_1'][\varepsilon^{\vartheta}(q_1')\cdot p_1'](p_1'+q_1')_\alpha \Big\}, \tag{16e}$$

$$\langle \overline{VM'}^{\varrho'}(-p_2')|J_\beta(0)|\overline{VM'}^{\vartheta'}(-q_2')\rangle = \frac{1}{2\sqrt{E_{\bar{D}_l^*}(-p_2')E_{\bar{D}_l^*}(-q_2')}} \times \Big\{ [\varepsilon^{\varrho'}(-p_2')\cdot\varepsilon^{\vartheta'}(-q_2')]\bar{h}_1^{(\mathrm{lv})}(w^2)(-p_2'-q_2')_\beta - \bar{h}_2^{(\mathrm{lv})}(w^2)\{[\varepsilon^{\varrho'}(-p_2')\cdot(-q_2')]\varepsilon_\beta^{\vartheta'}(-q_2') + [\varepsilon^{\vartheta'}(-q_2')\cdot(-p_2')]\varepsilon_\beta^{\varrho'}(-p_2')\} - \bar{h}_3^{(\mathrm{lv})}(w^2)\frac{1}{M_2^2}[\varepsilon^{\varrho'}(-p_2')\cdot(-q_2')][\varepsilon^{\vartheta'}(-q_2')\cdot(-p_2')](-p_2'-q_2')_\beta \Big\}, \tag{16f}$$

where $p_1' = (\mathbf{p}, ip_{10}')$, $p_2' = (\mathbf{p}, ip_{20}')$, $q_1' = (\mathbf{p}', iq_{10}')$, $q_2' = (\mathbf{p}', iq_{20}')$, $w = q_1' - p_1' = q_2' - p_2'$ is the momentum of the exchanged meson and $\mathbf{w} = \mathbf{p}' - \mathbf{p}$; $h(w^2)$ and $\bar{h}(w^2)$ are scalar functions, the four-vector $\varepsilon(p)$ is the polarization vector of heavy vector meson with momentum p, and $E_{D_l^*}(p) = \sqrt{\mathbf{p}^2 + M_{D_l^*}^2}$. Taking away the external lines including normalizations and polarization vectors $\varepsilon_\theta^{\varrho}(p_1')$, $\varepsilon_{\theta'}^{\vartheta}(q_1')$, $\varepsilon_\kappa^{\varrho'}(-p_2')$, $\varepsilon_{\kappa'}^{\vartheta'}(-q_2')$, we obtain the interaction kernel from one light-meson (π^0, η, σ, ρ^0, V_1, and V_8) exchange [5,12]

$$\begin{aligned}\mathcal{V}^l_{\theta\theta',\kappa'\kappa}(p,p';P^{D\bar{D}}) ={}& h^{(\mathrm{p})}(w^2)\epsilon_{\varsigma\varsigma'\theta\theta'}p_{1\varsigma}'q_{1\varsigma'}'\left(\frac{-ig_\pi^2}{w^2+m_\pi^2}+\frac{-ig_\eta^2}{w^2+m_\eta^2}\right)\bar{h}^{(\mathrm{p})}(w^2)\epsilon_{\varpi\varpi'\kappa'\kappa}p_{2\varpi}'q_{2\varpi'}' \\ &+ h_1^{(\mathrm{s})}(w^2)\frac{-ig_\sigma^2}{w^2+M_\sigma^2}\bar{h}_1^{(\mathrm{s})}(w^2)\delta_{\theta\theta'}\delta_{\kappa'\kappa} + \left(\frac{-ig_\rho^2}{w^2+M_\rho^2}+\frac{-ig_1^2}{w^2+M_\omega^2}+\frac{-ig_8^2}{w^2+M_\phi^2}\right)\{h_1^{(\mathrm{lv})}(w^2)\bar{h}_1^{(\mathrm{lv})}(w^2) \\ &\times (p_1'+q_1')\cdot(-p_2'-q_2')\delta_{\theta\theta'}\delta_{\kappa'\kappa} - h_1^{(\mathrm{lv})}(w^2)\bar{h}_2^{(\mathrm{lv})}(w^2)\delta_{\theta\theta'}[-(p_1'+q_1')_{\kappa'}q_{2\kappa}' - p_{2\kappa'}'(p_1'+q_1')_\kappa] \\ &- h_2^{(\mathrm{lv})}(w^2)\bar{h}_1^{(\mathrm{lv})}(w^2)[q_{1\theta}'(-p_2'-q_2')_{\theta'} + (-p_2'-q_2')_\theta p_{1\theta'}']\delta_{\kappa'\kappa} + h_2^{(\mathrm{lv})}(w^2)\bar{h}_2^{(\mathrm{lv})}(w^2)[-q_{1\theta}'\delta_{\theta'\kappa'}q_{2\kappa}' \\ &+ q_{1\theta}'\delta_{\theta'\kappa}(-p_{2\kappa'}') - \delta_{\theta\kappa'}p_{1\theta'}'q_{2\kappa}' + \delta_{\theta\kappa}p_{1\theta'}'(-p_{2\kappa'}')]\},\end{aligned} \tag{17}$$

where g represents the corresponding meson-quark coupling constant, $g_\sigma = \frac{B(M_\sigma)}{f_\sigma} = \frac{299}{60}$ [22,23], $g_\rho^2 = 2.42$ [21], and these terms containing $M_{1,2}$ are neglected because the masses of heavy mesons are large. Using the method above, we can obtain the interaction kernels from one-$\rho^\pm$ exchange [13].

In this work, we consider the interaction kernel between two heavy quarks in two heavy mesons from one-J/ψ exchange and the heavy vector meson J/ψ is considered as a bound state of $c\bar{c}$. Diagrammatically, the heavy meson J/ψ exchange between two heavy quarks in two heavy vector mesons is represented by the graph of Fig. 3. In Fig. 3, the BS amplitude $\Gamma_\alpha(w,w')$ of the heavy-vector meson J/ψ is also represented by the unfilled ellipse, where w' is the relative momentum between two heavy quarks. The explicit form of BS amplitude $\Gamma_\alpha(w,w')$ for heavy vector meson J/ψ will be given in Sec. II C 1. Then, the interaction kernel from one heavy meson (J/ψ) exchange becomes

$$\mathcal{V}^h_{\theta\theta',\kappa'\kappa}(p,p';P^{D\bar{D}}) = \frac{-i}{w^2+M^2_{J/\psi}}\{h_1^{(\mathrm{hv})}(w^2)\bar{h}_1^{(\mathrm{hv})}(w^2)(p'_1+q'_1)\cdot(-p'_2-q'_2)\delta_{\theta\theta'}\delta_{\kappa'\kappa} + h_1^{(\mathrm{hv})}(w^2)\bar{h}_2^{(\mathrm{hv})}(w^2)\delta_{\theta\theta'}$$
$$\times[(p'_1+q'_1)_{\kappa'}q'_{2\kappa} + p'_{2\kappa'}(p'_1+q'_1)_\kappa] + h_2^{(\mathrm{hv})}(w^2)\bar{h}_1^{(\mathrm{hv})}(w^2)[q'_{1\theta}(p'_2+q'_2)_{\theta'} + (p'_2+q'_2)_\theta p'_{1\theta'}]\delta_{\kappa'\kappa}$$
$$- h_2^{(\mathrm{hv})}(w^2)\bar{h}_2^{(\mathrm{hv})}(w^2)[q'_{1\theta}\delta_{\theta'\kappa'}q'_{2\kappa} + q'_{1\theta}\delta_{\theta'\kappa}p'_{2\kappa'} + \delta_{\theta\kappa'}p'_{1\theta'}q'_{2\kappa} + \delta_{\theta\kappa}p'_{1\theta'}p'_{2\kappa'}]\}. \tag{18}$$

We obtain the total interaction kernel between D_l^* and $\bar{D}_l^*$ derived from one light-meson (π^0, η, σ, ρ^0, V_1, and V_8) exchange and one heavy-meson (J/ψ) exchange

$$\mathcal{V}_{\theta\theta',\kappa'\kappa} = \mathcal{V}^l_{\theta\theta',\kappa'\kappa} + \mathcal{V}^h_{\theta\theta',\kappa'\kappa}. \tag{19}$$

C. Instantaneous approximation

1. Form factors of heavy meson

To calculate these heavy vector meson form factors $h(w^2)$ describing the heavy meson structure, we have to know the wave function of heavy vector meson D_l^* in instantaneous approximation. For heavy vector mesons, the authors of Refs. [24–27] obtained their BS amplitudes in Euclidean space:

$$\Gamma^V_\lambda(K,k) = \frac{1}{\mathcal{N}^V}\left(\gamma_\lambda + K_\lambda\frac{\gamma\cdot K}{M_V^2}\right)\varphi_V(k^2), \tag{20}$$

where K is the momentum of heavy meson, k denotes the relative momentum between quark and antiquark in heavy meson, M_V is heavy vector meson mass, $\Gamma^V_\lambda(K,k)$ is transverse ($K_\lambda\Gamma^V_\lambda(K,k)=0$), $\mathcal{N}^V$ is normalization, and $\varphi_V(k^2)$ is the scalar function fixed by providing fits to observables. The charmed meson D_l^* is composed of a c-quark and an l-antiquark. As in heavy-quark effective theory (HQET) [28], we consider that the heaviest quark carries all the heavy-meson momentum and obtain the BS wave function of the D_l^* meson

$$\chi_\lambda(K,k) = \frac{-1}{\gamma\cdot(k+K)-im_c}\frac{1}{\mathcal{N}^{D_l^*}}\left(\gamma_\lambda + K_\lambda\frac{\gamma\cdot K}{M^2_{D_l^*}}\right)\varphi_{D_l^*}(k^2)\frac{-1}{\gamma\cdot k - im_l}, \tag{21}$$

where K is set as the momentum of heavy meson in the rest frame, k becomes the relative momentum between c-quark and l-antiquark, $m_{c,l}$ are the constituent quark masses, $\varphi_{D_l^*}(k^2) = \varphi_{\bar{D}_l^*}(k^2) = \exp(-k^2/\omega^2_{D^*})$ and $\omega_{D^*} = 1.50$ GeV [27]. The components of this BS wave function are 4×4 matrices, which can be written as [29]

$$\chi_\lambda(K,k) = \Psi^S_\lambda + \Psi^V_{\lambda,\mu}\gamma_\mu + \Psi^T_{\lambda,\mu\nu}\sigma_{\mu\nu} + \Psi^{AV}_{\lambda,\mu}\gamma_\mu\gamma_5 + \Psi^{Pse}_\lambda\gamma_5, \tag{22}$$

and the coefficient corresponding to γ_μ is

$$\Psi^V_{\lambda,\mu} = \frac{1}{4}\mathrm{Tr}[\gamma_\mu\chi_\lambda(K,k)]. \tag{23}$$

Substituting Eq. (21) into (23), we can obtain the heavy vector meson wave function in instantaneous approximation

$$\Psi^{D_l^*}_{ij}(\mathbf{k}) = \int dk_4\frac{1}{\mathcal{N}^{D_l^*}}\exp\left(\frac{-\mathbf{k}^2-k_4^2}{\omega^2_{D_l^*}}\right)\frac{\mathbf{k}^2/3+k_4^2+m_cm_l}{(\mathbf{k}^2+k_4^2+m_c^2)(\mathbf{k}^2+k_4^2+m_l^2)}\delta_{ij} \quad i,j=1,2,3. \tag{24}$$

In the previous works [5,12,13], we used the method introduced in Ref. [30] and obtained the form factors for the vertices of heavy vector meson D_l^* coupling to pseudoscalar meson (π and η) [5]

$$h^{(\mathrm{p})}(w^2) = \bar{h}^{(\mathrm{p})}(w^2) = 0, \tag{25}$$

to scalar meson (σ)

$$-\frac{h_1^{(s)}(w^2)}{2E_1}=\frac{\bar{h}_1^{(s)}(w^2)}{2E_2}=F_1(\mathbf{w}^2),\qquad h_2^{(s)}(w^2)=\bar{h}_2^{(s)}(w^2)=0,$$
$$F_1(\mathbf{w}^2)=\int\frac{d^3k}{(2\pi)^3}\bar{\Psi}^{D_l^*}\left(\mathbf{k}+\frac{2E_c(k)}{E_{D_l^*}+M_{D_l^*}}\mathbf{w}\right)\sqrt{\frac{E_l(k)+m_l}{E_l(k+w)+m_l}}$$
$$\times\left\{\frac{E_l(k+w)-E_l(k)+2m_l}{2\sqrt{E_l(k+w)E_l(k)}}-\frac{\mathbf{k}\cdot\mathbf{w}}{2\sqrt{E_l(k+w)E_l(k)}[E_l(k)+m_l]}\right\}\Psi^{D_l^*}(\mathbf{k}),\tag{26}$$

and to light-vector mesons (ρ, V_1, and V_8)

$$h_1^{(\mathrm{lv})}(w^2)=h_2^{(\mathrm{lv})}(w^2)=\bar{h}_1^{(\mathrm{lv})}(w^2)=\bar{h}_2^{(\mathrm{lv})}(w^2)=F_2^{(\mathrm{lv})}(\mathbf{w}^2),\qquad h_3^{(\mathrm{lv})}(w^2)=\bar{h}_3^{(\mathrm{lv})}(w^2)=0,$$
$$F_2^{(\mathrm{lv})}(\mathbf{w}^2)=\frac{2\sqrt{E_{D_l^*}M_{D_l^*}}}{E_{D_l^*}+M_{D_l^*}}\int\frac{d^3k}{(2\pi)^3}\bar{\Psi}^{D_l^*}\left(\mathbf{k}+\frac{2E_c(k)}{E_{D_l^*}+M_{D_l^*}}\mathbf{w}\right)\sqrt{\frac{E_l(k)+m_l}{E_l(k+w)+m_l}}$$
$$\times\left\{\frac{E_l(k+w)+E_l(k)}{2\sqrt{E_l(k+w)E_l(k)}}+\frac{\mathbf{k}\cdot\mathbf{w}}{2\sqrt{E_l(k+w)E_l(k)}[E_l(k)+m_l]}\right\}\Psi^{D_l^*}(\mathbf{k}),\tag{27}$$

where $E_{c,l}(p)=\sqrt{\mathbf{p}^2+m_{c,l}^2}$ and $\Psi^{D_l^*}$ is the wave function of heavy-vector meson expressed as Eq. (24). Some errors in our previous works have been revised. From Eq. (25), we can obtain that one light pseudoscalar-meson exchange has no contribution to the interaction kernel between two heavy vector mesons for the vector-vector bound state with spin-parity quantum numbers 0^+.

In this paper, we consider that BS amplitude of heavy vector meson J/ψ has the form expressed as Eq. (20) and obtain scalar functions for the vertex of heavy vector meson D_l^* coupling to J/ψ

$$h_1^{(\mathrm{hv})}(w^2)=h_2^{(\mathrm{hv})}(w^2)=\bar{h}_1^{(\mathrm{hv})}(w^2)=\bar{h}_2^{(\mathrm{hv})}(w^2)=F_2^{(\mathrm{hv})}(\mathbf{w}^2),\qquad h_3^{(\mathrm{hv})}(w^2)=\bar{h}_3^{(\mathrm{hv})}(w^2)=0,$$
$$F_2^{(\mathrm{hv})}(\mathbf{w}^2)=\frac{2\sqrt{E_{D_l^*}M_{D_l^*}}}{E_{D_l^*}+M_{D_l^*}}\int\frac{d^3k}{(2\pi)^3}\bar{\Psi}^{D_l^*}\left(\mathbf{k}-\frac{2E_l(k)}{E_{D_l^*}+M_{D_l^*}}\mathbf{w}\right)\sqrt{\frac{E_c(k)+m_c}{E_c(k-w)+m_c}}$$
$$\times\left\{\frac{E_c(k-w)+E_c(k)}{2\sqrt{E_c(k-w)E_c(k)}}-\frac{\mathbf{k}\cdot\mathbf{w}}{2\sqrt{E_c(k-w)E_c(k)}[E_c(k)+m_c]}\right\}$$
$$\times\frac{1}{\mathcal{N}^{J/\psi}}\exp\left[\frac{-(\mathbf{k}-\mathbf{w}/2)^2-E_c^2(k)}{\omega_{J/\psi}^2}\right]\Psi^{D_l^*}(\mathbf{k}),\tag{28}$$

where $\mathcal{N}^{J/\psi}$ is normalization and $\omega_{J/\psi}=0.826$ GeV was obtained from lattice QCD (see details in Ref. [8]).

2. The extended Bethe-Salpeter equation

Substituting the BS wave function given by Eq. (12) and the kernel (19) into the BS equation (13), we find that the integral of one term on the right-hand side of (12) has a contribution to the one of itself and the other term. Ignoring the cross-terms, one can obtain two individual equations,

$$\mathcal{F}^1_{\lambda\tau}(P^{D\bar{D}}\cdot p,p^2)=-\int\frac{d^4p'}{(2\pi)^4}\Delta_{F\lambda\theta}(p_1')\mathcal{V}_{\theta\theta',\kappa'\kappa}(p,p';P^{D\bar{D}})\mathcal{F}^1_{\theta'\kappa'}(P^{D\bar{D}}\cdot p',p'^2)\Delta_{F\kappa\tau}(p_2'),\tag{29}$$

$$\mathcal{F}^2_{\lambda\tau}(P^{D\bar{D}}\cdot p,p^2)=-\int\frac{d^4p'}{(2\pi)^4}\Delta_{F\lambda\theta}(p_1')\mathcal{V}_{\theta\theta',\kappa'\kappa}(p,p';P^{D\bar{D}})\mathcal{F}^2_{\theta'\kappa'}(P^{D\bar{D}}\cdot p',p'^2)\Delta_{F\kappa\tau}(p_2'),\tag{30}$$

where $\mathcal{F}^1_{\lambda\tau}(P^{D\bar{D}}\cdot p,p^2)=\mathcal{T}^1_{\lambda\tau}\mathcal{F}_1(P^{D\bar{D}}\cdot p,p^2)$ and $\mathcal{F}^2_{\lambda\tau}(P^{D\bar{D}}\cdot p,p^2)=\mathcal{T}^2_{\lambda\tau}\mathcal{F}_2(P^{D\bar{D}}\cdot p,p^2)$. Comparing the tensor structures in both sides of Eqs. (29) and (30), respectively, we obtain

$$\mathcal{F}_1(P^{D\bar{D}}\cdot p,p^2)=\frac{1}{p_1'^2+M_1^2-i\varepsilon}\frac{1}{p_2'^2+M_2^2-i\varepsilon}\int\frac{d^4p'}{(2\pi)^4}V_1^{0^+}(p,p';P^{D\bar{D}})\mathcal{F}_1(P^{D\bar{D}}\cdot p',p'^2), \tag{31}$$

$$p_2'^2\mathcal{F}_2(P^{D\bar{D}}\cdot p,p^2)=\frac{1}{p_1'^2+M_1^2-i\varepsilon}\frac{1}{p_2'^2+M_2^2-i\varepsilon}\int\frac{d^4p'}{(2\pi)^4}V_2^{0^+}(p,p';P^{D\bar{D}})q_2'^2\mathcal{F}_2(P^{D\bar{D}}\cdot p',p'^2), \tag{32}$$

where $V_1^{0^+}(p,p';P^{D\bar{D}})$ and $V_2^{0^+}(p,p';P^{D\bar{D}})$ are derived from the interaction kernel between D_l^* and $\bar{D}_l^*$. In instantaneous approximation, we set the momentum of exchanged meson as $w=(\mathbf{w},0)$. Then Eqs. (31) and (32) become two relativistic Schrödinger-like equations (see details in Refs. [5,13])

$$\left(\frac{b_1^2(M_{D\bar{D}})}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_1^{0^+}(\mathbf{p})=\int\frac{d^3w}{(2\pi)^3}V_1^{0^+}(\mathbf{p},\mathbf{w})\Psi_1^{0^+}(\mathbf{p},\mathbf{w}), \tag{33}$$

$$\left(\frac{b_2^2(M_{D\bar{D}})}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_2^{0^+}(\mathbf{p})=\int\frac{d^3w}{(2\pi)^3}V_2^{0^+}(\mathbf{p},\mathbf{w})\Psi_2^{0^+}(\mathbf{p},\mathbf{w}), \tag{34}$$

where $\Psi_1^{0^+}(\mathbf{p})=\int dp_0\mathcal{F}_1(P^{D\bar{D}}\cdot p,p^2)$, $\Psi_2^{0^+}(\mathbf{p})=\int dp_0p_2'^2\mathcal{F}_2(P^{D\bar{D}}\cdot p,p^2)$, $\mu_R=E_1E_2/(E_1+E_2)=[M_{D\bar{D}}^4-(M_1^2-M_2^2)^2]/(4M_{D\bar{D}}^3)$, $b^2(M_{D\bar{D}})=[M_{D\bar{D}}^2-(M_1+M_2)^2][M_{D\bar{D}}^2-(M_1-M_2)^2]/(4M_{D\bar{D}}^2)$, $E_1=(M_{D\bar{D}}^2-M_2^2+M_1^2)/(2M_{D\bar{D}})$, and $E_2=(M_{D\bar{D}}^2-M_1^2+M_2^2)/(2M_{D\bar{D}})$. The potentials between D_l^* and $\bar{D}_l^*$ up to the second order of the $p/M_{D_l^*}$ expansion are

$$\begin{aligned}V_1^{0^+}(\mathbf{p},\mathbf{w})=&-F_1(\mathbf{w}^2)\frac{g_\sigma^2}{w^2+M_\sigma^2}F_1(\mathbf{w}^2)+\left[F_2^{(\mathrm{lv})}(\mathbf{w}^2)F_2^{(\mathrm{lv})}(\mathbf{w}^2)\left(\frac{g_\rho^2}{w^2+M_\rho^2}+\frac{g_1^2}{w^2+M_\omega^2}\right.\right.\\&\left.\left.+\frac{g_8^2}{w^2+M_\phi^2}\right)+F_2^{(\mathrm{hv})}(\mathbf{w}^2)F_2^{(\mathrm{hv})}(\mathbf{w}^2)\frac{1}{w^2+M_{J/\psi}^2}\right]\left(-1-\frac{4\mathbf{p}^2+5\mathbf{w}^2}{4E_1E_2}\right),\end{aligned} \tag{35}$$

$$\begin{aligned}V_2^{0^+}(\mathbf{p},\mathbf{w})=&-F_1(\mathbf{w}^2)\frac{g_\sigma^2}{w^2+M_\sigma^2}F_1(\mathbf{w}^2)\left(1-\frac{\mathbf{w}^2}{M_1^2}\right)+\left[F_2^{(\mathrm{lv})}(\mathbf{w}^2)F_2^{(\mathrm{lv})}(\mathbf{w}^2)\left(\frac{g_\rho^2}{w^2+M_\rho^2}\right.\right.\\&\left.\left.+\frac{g_1^2}{w^2+M_\omega^2}+\frac{g_8^2}{w^2+M_\phi^2}\right)+F_2^{(\mathrm{hv})}(\mathbf{w}^2)F_2^{(\mathrm{hv})}(\mathbf{w}^2)\frac{1}{w^2+M_{J/\psi}^2}\right]\\&\times\left(-1-\frac{2\mathbf{p}^2+2\mathbf{w}^2}{4M_1^2}-\frac{2\mathbf{p}^2+2\mathbf{w}^2}{4E_1E_2}\right).\end{aligned} \tag{36}$$

Solving Eqs. (33) and (34), respectively, one can obtain the eigenvalues $b_1^2(M_{D\bar{D}})$ and $b_2^2(M_{D\bar{D}})$ and the corresponding eigenfunctions $\Psi_1^{0^+}(\mathbf{p})$ and $\Psi_2^{0^+}(\mathbf{p})$. From $\Psi_1^{0^+}$ and $\Psi_2^{0^+}$, it is easy to obtain $\mathcal{F}_1$ and $\mathcal{F}_2$, respectively.

Because the cross-terms are small, we can take the ground-state BS wave function to be a linear combination of the two eigenstates $\mathcal{F}_{\lambda\tau}^{10}(P^{D\bar{D}}\cdot p,p^2)$ and $\mathcal{F}_{\lambda\tau}^{20}(P^{D\bar{D}}\cdot p,p^2)$ corresponding to the lowest energy in Eqs. (29) and (30). Then in the basis provided by $\mathcal{F}_{\lambda\tau}^{10}(P^{D\bar{D}}\cdot p,p^2)=\mathcal{T}_{\lambda\tau}^1\mathcal{F}_{10}(P^{D\bar{D}}\cdot p,p^2)$ and $\mathcal{F}_{\lambda\tau}^{20}(P^{D\bar{D}}\cdot p,p^2)=\mathcal{T}_{\lambda\tau}^2\mathcal{F}_{20}(P^{D\bar{D}}\cdot p,p^2)$, the BS wave function $\chi_{\lambda\tau}^{0^+}$ is considered as

$$\chi_{\lambda\tau}^{0^+}(P^{D\bar{D}},p)=\frac{1}{\mathcal{N}_{D\bar{D}}^{0^+}}[\mathcal{C}_1\mathcal{F}_{\lambda\tau}^{10}(P^{D\bar{D}}\cdot p,p^2)+\mathcal{C}_2\mathcal{F}_{\lambda\tau}^{20}(P^{D\bar{D}}\cdot p,p^2)]. \tag{37}$$

Substituting (37) into Eq. (13) and comparing the tensor structures in both sides, we obtain an eigenvalue equation in instantaneous approximation [5]

$$\begin{pmatrix}\frac{b_{10}^2(M_{D\bar{D}})}{2\mu_R}-\lambda & H_{12}\\ H_{21} & \frac{b_{20}^2(M_{D\bar{D}})}{2\mu_R}-\lambda\end{pmatrix}\begin{pmatrix}\mathcal{C}_1'\\ \mathcal{C}_2'\end{pmatrix}=0, \tag{38}$$

where we have the matrix elements

$$H_{12} = H_{21} = \int \frac{d^3p}{(2\pi)^3} \Psi_{10}^{0^+}(\mathbf{p})^* \int \frac{d^3w}{(2\pi)^3} \left[F_2^{(\mathrm{lv})}(\mathbf{w}^2) F_2^{(\mathrm{lv})}(\mathbf{w}^2) \left(\frac{g_\rho^2}{w^2 + M_\rho^2} + \frac{g_1^2}{w^2 + M_\omega^2} + \frac{g_8^2}{w^2 + M_\phi^2} \right) + F_2^{(\mathrm{hv})}(\mathbf{w}^2) F_2^{(\mathrm{hv})}(\mathbf{w}^2) \frac{1}{w^2 + M_{J/\psi}^2} \right] \frac{\mathbf{w}^2}{E_1 E_2} \Psi_{20}^{0^+}(\mathbf{p}, \mathbf{w}), \tag{39}$$

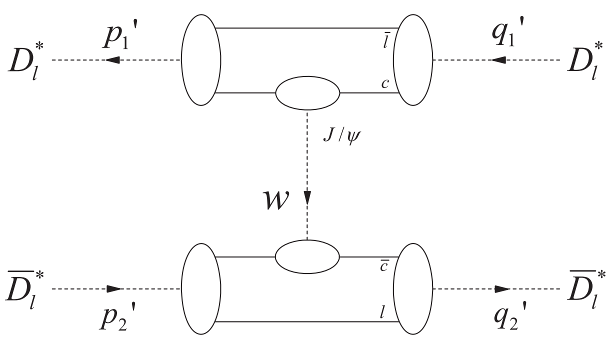

FIG. 3. Heavy-meson J/ψ exchange between two heavy quarks in two heavy-vector mesons. The unfilled ellipses represent Bethe-Salpeter amplitudes.

and $b_{10}^2(M_{D\bar{D}})/(2\mu_R)$ and $b_{20}^2(M_{D\bar{D}})/(2\mu_R)$ are the eigenvalues corresponding to the lowest energy in Eqs. (33) and (34), respectively; $\Psi_{10}^{0^+}$ and $\Psi_{20}^{0^+}$ are the corresponding eigenfunctions. From this equation, we can obtain the eigenvalues and eigenfunctions which contain the contribution from the cross-terms. Some errors in our previous works have been revised. Equations (33) and (34) can be solved numerically with these form factors, and then the eigenvalue equation (38) can be solved. The masses $M_{D\bar{D}}$ and wave functions of pure-bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ with spin-parity quantum numbers 0^+ can be obtained.

Considering the interaction kernels from one-$\rho^\pm$ exchange and using the coupled-channel approach (see details in Ref. [13]), we can calculate the mass M_0 of the mixed state of two pure bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ with 0^+. Since the mixing of component wave functions causes the change of energy, the fourth component of $P^{D\bar{D}}$ in the original BS wave function becomes the total energy of the mixed states, and $\chi_{\lambda\tau}^{0^+}(P^{D\bar{D}}, p)$ in Eq. (37) becomes

$$\chi_{\lambda\tau}^{0^+}(P,p) = \frac{1}{\mathcal{N}^{0^+}} \{ [(p_1' \cdot p_2') g_{\lambda\tau} - p_{2\lambda}' p_{1\tau}'] \mathcal{C}_1 \mathcal{F}_{10}(P \cdot p, p^2) + [p_1'^2 p_2'^2 g_{\lambda\tau} + (p_1' \cdot p_2') p_{1\lambda}' p_{2\tau}' - p_2'^2 p_{1\lambda}' p_{1\tau}' - p_1'^2 p_{2\lambda}' p_{2\tau}'] \mathcal{C}_2 \mathcal{F}_{20}(P \cdot p, p^2) \}. \tag{40}$$

We emphasize that the mass M_0 of the meson-meson bound state should not be the mass of physical resonance. Substituting Eq. (40) into (10), we obtain the BS wave function $\chi_{\lambda\tau}^{D^*\bar{D}^*,0^+}(P,p)$ for the mixed state of two bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ with 0^+.

D. GBS wave function for the four-quark state

The heavy meson is a bound state consisting of a quark and an antiquark and the meson-meson bound state is actually composed of four quarks. We have to give GBS wave function of meson-meson bound state as a four-quark state. If a bound state with spin j and parity η_P is composed of four quarks, its GBS wave function can be defined as [8]

$$\chi_P^j(x_1, x_3, x_4, x_2) = \langle 0 | T \mathcal{Q}^{\mathcal{C}}(x_1) \bar{\mathcal{Q}}^{\mathcal{A}}(x_3) \mathcal{Q}^{\mathcal{B}}(x_4) \bar{\mathcal{Q}}^{\mathcal{D}}(x_2) | P, j \rangle, \tag{41}$$

where P is the momentum of the four-quark bound state, $\mathcal{Q}$ is the quark operator and its superscript is a flavor label. From translational invariance, this GBS wave function can be written as

$$\chi_P^j(x_1, x_3, x_4, x_2) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(P)}} e^{iP\cdot X} \chi_P^j(X', x, x'), \tag{42}$$

where $X = \eta_1(\eta_1'' x_1 + \eta_3'' x_3) + \eta_2(\eta_4'' x_4 + \eta_2'' x_2)$, $X' = (\eta_1'' x_1 + \eta_3'' x_3) - (\eta_4'' x_4 + \eta_2'' x_2)$, $x = x_1 - x_3$, $x' = x_2 - x_4$, $\eta_1 + \eta_2 = 1$, $\eta_{1,3}'' = m_{\mathcal{C},\mathcal{A}}/(m_{\mathcal{C}} + m_{\mathcal{A}})$, $\eta_{2,4}'' = m_{\mathcal{D},\mathcal{B}}/(m_{\mathcal{D}} + m_{\mathcal{B}})$, and $m_{\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}}$ are the quark masses. In the momentum representation, the GBS wave function of four-quark bound state becomes

$$\chi_P^j(p_1, p_3, p_4, p_2) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(P)}} (2\pi)^4 \times \delta^{(4)}(P - p_1 + p_3 - p_4 + p_2) \times \chi^j(P, p, k, k'), \tag{43}$$

where p_1, p_3, p_4, p_2 are the momenta carried by the fields $\mathcal{Q}^{\mathcal{C}}$, $\bar{\mathcal{Q}}^{\mathcal{A}}$, $\mathcal{Q}^{\mathcal{B}}$, $\bar{\mathcal{Q}}^{\mathcal{D}}$, and p, k, k' are the conjugate variables to X', x, x', respectively, where $p = \eta_2(p_1 - p_3) + \eta_1(p_2 - p_4)$, $k = \eta_3'' p_1 + \eta_1'' p_3$, $k' = \eta_4'' p_2 + \eta_2'' p_4$. In the hadronic molecule structure, p is the relative momentum

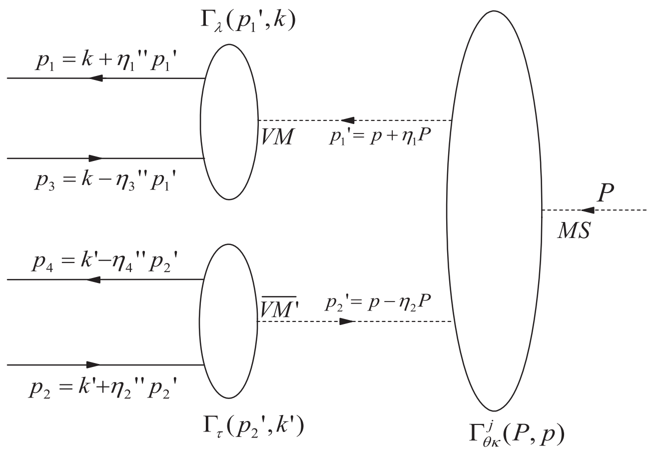

FIG. 4. Generalized Bethe-Salpeter wave function for the four-quark state in the momentum representation. The solid lines denote quark propagators, and the unfilled ellipses represent Bethe-Salpeter amplitudes.

between two mesons in molecular state, k and k' are the relative momenta between the quark and antiquark in these two mesons, respectively, shown in Fig. 4. This work is aimed at investigating the bound state composed of two vector mesons. In Fig. 4, VM represents the vector meson with mass M_1, $\overline{VM'}$ represents the antiparticle of the vector meson VM' with mass M_2, and MS represents the meson-meson bound state.

In Fig. 4, there are three two-body systems; a meson-meson bound state and two quark-antiquark bound states. We define BS wave functions of these two-body systems as $\chi^j_P(p_1', p_2')$, $\chi_{p_1'}(p_1, p_3)$, and $\chi_{p_2'}(p_4, p_2)$, respectively. The BS wave function for the bound state of two vector mesons has been given by Eq. (3) and the BS wave functions of two vector mesons are

$$\chi_{p_1'}(p_1,p_3)_\lambda = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(p_1')}} \times (2\pi)^4\delta^{(4)}(p_1'-p_1+p_3)\chi_\lambda(p_1',k), \quad (44)$$

$$\chi_{p_2'}(p_4,p_2)_\tau = \frac{1}{(2\pi)^{3/2}}\frac{1}{\sqrt{2E(p_2')}} \times (2\pi)^4\delta^{(4)}(p_2'+p_4-p_2)\chi_\tau(p_2',k'), \quad (45)$$

where p_1' and p_2' are the momenta of two vector mesons, respectively, $p_1' = p+\eta_1 P$, $p_2' = p-\eta_2 P$ and $\eta_{1,2} = M_{1,2}/(M_1+M_2)$. Applying the Feynman rules and comparing with Eq. (43), we obtain the revised GBS wave function for four-quark state describing the bound state composed of two vector mesons with arbitrary spin and definite parity [8,9]

$$\chi^j(P,p,k,k') = \chi_\lambda(p_1',k)\chi^j_{\lambda\tau}(P,p)\chi_\tau(p_2',k'). \quad (46)$$

From Eq. (21), we obtain BS wave functions of vector mesons

$$\chi_\lambda(p_1',k) = \frac{-1}{\gamma^{\mathcal{C}}\cdot p_1 - im_{\mathcal{C}}}\frac{1}{\mathcal{N}^V} \times \left(\gamma_\lambda + p'_{1\lambda}\frac{\gamma\cdot p_1'}{M_V^2}\right)\varphi_V(k^2)\frac{-1}{\gamma^{\mathcal{A}}\cdot p_3 - im_{\mathcal{A}}},$$
$$\chi_\tau(p_2',k') = \frac{-1}{\gamma^{\mathcal{B}}\cdot p_4 - im_{\mathcal{B}}}\frac{1}{\mathcal{N}^{\bar{V}'}} \times \left(\gamma_\tau + p'_{2\tau}\frac{\gamma\cdot p_2'}{M_{\bar{V}'}^2}\right)\varphi_{\bar{V}'}(k'^2)\frac{-1}{\gamma^{\mathcal{D}}\cdot p_2 - im_{\mathcal{D}}}. \quad (47)$$

In this section, we consider a mixed state of two bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ with spin-parity quantum numbers 0^+. In Fig. 4, VM and $\overline{VM'}$ become D_l^* and $\bar{D}_l^*$, respectively, and in Eq. (41) the flavor labels $\mathcal{C}=\mathcal{D}$ and $\mathcal{A}=\mathcal{B}$ represent c-quark and l-quark, respectively. From Eqs. (10), (46), and (47), we obtain the GBS wave function for meson-meson bound state as a four-quark state

$$\chi^{D^*\bar{D}^*,0^+}(P,p,k,k') = \frac{1}{\sqrt{2}}\chi_\lambda^{D^{*0}}(p_1',k)\chi_{\lambda\tau}^{D^{*0}\bar{D}^{*0},0^+}(P,p) \times\chi_\tau^{\bar{D}^{*0}}(p_2',k') + \frac{1}{\sqrt{2}}\chi_\lambda^{D^{*+}}(p_1',k) \times\chi_{\lambda\tau}^{D^{*+}D^{*-},0^+}(P,p)\chi_\tau^{D^{*-}}(p_2',k'), \quad (48)$$

where

$$\chi_\lambda^{D_l^*}(p_1',k) = \frac{-1}{\gamma\cdot p_1 - im_c}\frac{1}{\mathcal{N}^{D_l^*}}\left(\gamma_\lambda + p'_{1\lambda}\frac{\gamma\cdot p_1'}{M^2_{D_l^*}}\right)\varphi_{D_l^*}(k^2) \times\frac{-1}{\gamma\cdot p_3 - im_l},$$
$$\chi_\tau^{\bar{D}_l^*}(p_2',k') = \frac{-1}{\gamma\cdot p_4 - im_l}\frac{1}{\mathcal{N}^{\bar{D}_l^*}}\left(\gamma_\tau + p'_{2\tau}\frac{\gamma\cdot p_2'}{M^2_{\bar{D}_l^*}}\right)\varphi_{\bar{D}_l^*}(k'^2) \times\frac{-1}{\gamma\cdot p_2 - im_c}. \quad (49)$$

E. Normalization of BS wave function

1. Heavy vector meson

Here, we determine normalizations $\mathcal{N}^{D^{*0}}$ and $\mathcal{N}^{D^{*+}}$. The authors of Refs. [26,27] employed the ladder approximation to solve the BS equation for the quark-antiquark state, and the reduced normalization condition for the BS wave function of D_l^* meson given by Eq. (21) is

$$\frac{-i}{(2\pi)^4}\frac{1}{3}\int d^4k\bar{\chi}_\lambda(K,k)\frac{\partial}{\partial K_0}[S_F(k+K)^{-1}]S_F(k)^{-1}\chi_\lambda(K,k)$$
$$=(2K_0)^2, \tag{50}$$

where $S_F(p)^{-1}$ is the inverse propagator for quark field and the factor $1/3$ appears because of the sum of three transverse directions. Normalization $\mathcal{N}^{J/\psi}$ will be determined in Sec. III A.

2. Molecular state

The reduced normalization condition for $\chi^{0^+}_{\lambda\tau}(P,p)$ expressed as Eq. (40) is

$$\frac{-i}{(2\pi)^4}\int d^4p\bar{\chi}_{\tau'\lambda'}(P,p)$$
$$\times\frac{\partial}{\partial P_0}[\Delta_{F\lambda'\lambda}(p+P/2)^{-1}\Delta_{F\tau\tau'}(p-P/2)^{-1}]\chi_{\lambda\tau}(P,p)$$
$$=(2P_0)^2, \tag{51}$$

where $\Delta_{F\beta\alpha'}(p)^{-1}$ is the inverse propagator for the vector field with mass m, $\Delta_{F\beta\alpha'}(p)^{-1}=i(\delta_{\beta\alpha'}-\frac{p_\beta p_{\alpha'}}{p^2+m^2})\times(p^2+m^2)$ [8]. After determining the normalization $\mathcal{N}^{0^+}$, we automatically obtain the normalized BS wave function for the mixed state of two components $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ given by Eq. (10). Immediately, the normalized GBS wave function for meson-meson bound state as a four-quark state expressed as Eq. (48) is obtained.

III. SCATTERING MATRIX ELEMENT FROM THE FOUR-QUARK STATE TO THE FINAL STATE

In experiments two strong decay modes of $\chi_{c0}(3915)$ have been observed; $J/\psi\omega$ and D^+D^-. The narrow state $\chi_{c0}(3915)$ was discovered in 2005 [15] by the Belle Collaboration and for a long time a series of experiments [16–18,31] only observed one strong-decay mode of $\chi_{c0}(3915)$; $J/\psi\omega$ denoted as c_1'. In 2020 the LHCb Collaboration observed another decay channel D^+D^- [32] denoted as c_2'. Though the neutral channel $D^0\bar{D}^0$ still has not been observed, this neutral channel should exist for the isospin conservation, which is denoted as c_3'. In this section, we present the traditional technique to calculate decay width for these processes and revise some errors in previous works [8,9].

A. Decay channel $J/\psi\omega$ with respect to the mass of the bound state

Mandelstam's approach is a technique based on the BS wave function for evaluating the general matrix element between bound states [19]. Applying Mandelstam's approach, we have obtained the scattering matrix element from a four-quark state to a heavy meson plus a light meson [8] in the momentum representation, as shown

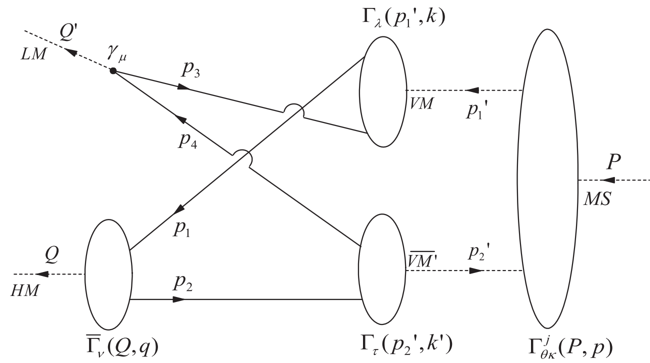

FIG. 5. The lowest-order matrix element between bound states in the momentum representation.

in Fig. 5. In this work, we retain only the lowest-order term of the two-particle irreducible Green's function. In Fig. 5, VM and $\overline{VM'}$ still represent D_l^* and $\bar{D}_l^*$, respectively; HM represents J/ψ with momentum Q ($Q^2=-M^2_{J/\psi}$) and LM represents ω with momentum Q' ($Q'^2=-M^2_\omega$). The momentum of the initial state is set as $P=(0,0,0,iM_0)$ in the rest frame, and M_0 is the mass of the mixed state of two pure-bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, which should not be the physical mass of resonance. It is necessary to emphasize that the momenta in the final state satisfy $Q+Q'=P$ in this section. Here, we consider that in the final state the light-vector meson ω is an elementary particle and the heavy-vector meson J/ψ is a bound state of $c\bar{c}$. From Eq. (47), we obtain the BS wave function of heavy-vector meson J/ψ as

$$\chi_\nu(Q,q)=\frac{-1}{\gamma\cdot(q+Q/2)-im_c}\frac{1}{\mathcal{N}^{J/\psi}}$$
$$\times\left(\gamma_\nu+Q_\nu\frac{\gamma\cdot Q}{M^2_{J/\psi}}\right)\varphi_{J/\psi}(q^2)\frac{-1}{\gamma\cdot(q-Q/2)-im_c}, \tag{52}$$

where $\varphi_{J/\psi}(q^2)=\exp(-q^2/\omega^2_{J/\psi})$ and $\omega_{J/\psi}=0.826$ GeV was determined in Ref. [8]. The reduced normalization condition for the BS wave function of J/ψ meson expressed as Eq. (52) is

$$\frac{-i}{(2\pi)^4}\frac{1}{3}\int d^4q\bar{\chi}_\nu(Q,q)$$
$$\times\frac{\partial}{\partial Q_0}[S_F(q+Q/2)^{-1}S_F(q-Q/2)^{-1}]\chi_\nu(Q,q)$$
$$=(2Q_0)^2, \tag{53}$$

where the factor $1/3$ appears for the three transverse directions are summed. The normalization $\mathcal{N}^{J/\psi}$ can be determined. These momenta in Fig. 5 become

$$p_1 = (Q+Q')/2+p+k, \qquad p_2 = (Q+Q')/2 - Q + p + k, \qquad p_3 = k, \qquad p_4 = Q'+k,$$
$$q = Q'/2 + p + k, \qquad k' = Q'+k, \qquad p'_1 = p + P/2, \qquad p'_2 = p - P/2, \qquad Q+Q' = P. \tag{54}$$

Using the Heisenberg picture, we obtain the total matrix element from the initial state $|P\,\text{in}\rangle$ to a final state $\langle Q, Q'\,\text{out}|$

$$-iR_{(c'_1;b)a}(M_0) = \langle Q, Q'\ \text{out}|P\ \text{in}\rangle = -i(2\pi)^4\delta^{(4)}(Q+Q'-P)T_{(c'_1;b)a}(M_0), \tag{55}$$

where $T_{(c'_1;b)a}(M_0)$ is the T-matrix element with mass M_0 for channel c'_1. According to Mandelstam's approach, we obtain

$$T_{(c'_1;b)a}(M_0) = \frac{ig_\omega \varepsilon^{\varrho'}_\mu(Q')\varepsilon^{\varrho}_\nu(Q)}{(2\pi)^{9/2}\sqrt{2E_{J/\psi}(Q)}\sqrt{2E_\omega(Q')}\sqrt{2E(P)}}\left(\frac{1}{\sqrt{2}}\mathcal{M}^{c'_1,D^{*0}\bar{D}^{*0}}_{\nu\mu} + \frac{1}{\sqrt{2}}\mathcal{M}^{c'_1,D^{*+}D^{*-}}_{\nu\mu}\right), \tag{56}$$

where $\varepsilon^{\varrho=1,2,3}_\nu(Q)$ and $\varepsilon^{\varrho'=1,2,3}_\mu(Q')$ are the polarization vectors of J/ψ and ω, respectively, $\varepsilon^{\varrho}(Q)\cdot Q = \varepsilon^{\varrho'}(Q')\cdot Q' = 0$, and

$$\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu} = \int \frac{d^4k d^4p}{(2\pi)^8}\frac{1}{\mathcal{N}^{J/\psi}}\frac{\varphi_{J/\psi}(q^2)}{p_2^2+m_c^2}\frac{1}{p_1^2+m_c^2}\frac{1}{\mathcal{N}^{D^*_l}}\frac{\varphi_{D^*_l}(k^2)}{p_3^2+m_l^2}\frac{1}{\mathcal{N}^{\bar{D}^*_l}}\frac{\varphi_{\bar{D}^*_l}(k'^2)}{p_4^2+m_l^2}$$
$$\times \mathrm{Tr}[(\gamma\cdot p_2 + im_c)\gamma_\nu(\gamma\cdot p_1 + im_c)\gamma_\lambda(\gamma\cdot p_3 + im_l)\gamma_\mu(\gamma\cdot p_4 + im_l)\gamma_\tau\chi^{0^+}_{\lambda\tau}(P,p)]. \tag{57}$$

Here $\chi^{0^+}_{\lambda\tau}(P,p)$ is expressed as Eq. (40). In Eq. (57) the trace of the product of 8 γ-matrices contains 105 terms and the resulting expression has been given in Appendix B of Ref. [8]. In our approach, the p integral is computed in instantaneous approximation. To calculate this tensor $\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu}$, we gave a simple method in Ref. [8]. It is obvious that the tensor $\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu}$ only depends on Q and Q', so in Minkowski space $\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu}$ can be expressed as

$$\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu} = g_{\nu\mu}U_1(Q',Q) + Q'_\nu Q_\mu U_2(Q',Q) + Q'_\nu Q'_\mu U_3(Q',Q) + Q_\nu Q'_\mu U_4(Q',Q) + Q_\nu Q_\mu U_5(Q',Q), \tag{58}$$

where $U_i(Q',Q)(i=1,\ldots,5)$ are scalar functions. The above expression is multiplied by these tensor structures $g_{\nu\mu}$, $Q'_\nu Q_\mu$, $Q'_\nu Q'_\mu$, $Q_\nu Q'_\mu$, $Q_\nu Q_\mu$, respectively; and a set of equations is obtained

$$\begin{aligned}
g_{\nu\mu}\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu} &= U'_1 = 4U_1 + (Q'\cdot Q)U_2 + Q'^2U_3 + (Q'\cdot Q)U_4 + Q^2U_5,\\
Q'_\nu Q_\mu\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu} &= U'_2 = (Q'\cdot Q)U_1 + Q'^2Q^2U_2 + Q'^2(Q'\cdot Q)U_3 + (Q'\cdot Q)^2U_4 + Q^2(Q'\cdot Q)U_5,\\
Q'_\nu Q'_\mu\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu} &= U'_3 = Q'^2U_1 + Q'^2(Q'\cdot Q)U_2 + Q'^2Q'^2U_3 + Q'^2(Q'\cdot Q)U_4 + (Q'\cdot Q)^2U_5,\\
Q_\nu Q'_\mu\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu} &= U'_4 = (Q'\cdot Q)U_1 + (Q'\cdot Q)^2U_2 + Q'^2(Q'\cdot Q)U_3 + Q^2Q'^2U_4 + Q^2(Q'\cdot Q)U_5,\\
Q_\nu Q_\mu\mathcal{M}^{c'_1,D^*_l\bar{D}^*_l}_{\nu\mu} &= U'_5 = Q^2U_1 + Q^2(Q'\cdot Q)U_2 + (Q'\cdot Q)^2U_3 + Q^2(Q'\cdot Q)U_4 + Q^2Q^2U_5,
\end{aligned} \tag{59}$$

where U'_i are numbers. Subsequently, we numerically calculate U'_i and solve this set of equations. The values of U_i can be obtained.

Then we can obtain the decay width with mass of meson-meson bound state for channel $J/\psi\omega$

$$\Gamma_1(M_0) = \int d^3Q d^3Q'(2\pi)^4\delta^{(4)}(Q+Q'-P)\sum_{\varrho'=1}^{3}\sum_{\varrho=1}^{3}(2\pi)^3|T_{(c'_1;b)a}(M_0)|^2, \tag{60}$$

where $P=(0,0,0,iM_0)$, $Q=(\mathbf{Q}(M_0), i\sqrt{\mathbf{Q}^2(M_0)+M^2_{J/\psi}})$, $Q'=(-\mathbf{Q}(M_0), i\sqrt{\mathbf{Q}^2(M_0)+M^2_\omega})$ and $\mathbf{Q}^2(M_0) = [M_0^2-(M_{J/\psi}+M_\omega)^2][M_0^2-(M_{J/\psi}-M_\omega)^2]/(4M_0^2)$. To calculate the decay width $\Gamma_1(M_0)$, we use the transverse condition $\varepsilon^{\varrho}(Q)\cdot Q = \varepsilon^{\varrho'}(Q')\cdot Q' = 0$ and the completeness relation. It is necessary to emphasize that the decay width $\Gamma_1(M_0)$ expressed as Eq. (60) is not the decay width of physical resonance.

B. Decay channel D^+D^- with respect to mass of bound state

Considering the lowest-order term of the two-particle irreducible Green's function, we obtain the interaction between two heavy-vector mesons derived from a light-meson exchange. Applying Mandelstam's approach, we can obtain the T-matrix element with mass M_0 for channel c_2', which can be represented graphically by Fig. 6. In Fig. 6, PM and $\overline{PM'}$ represent pseudoscalar mesons D^+ and D^-, respectively; Q_1 and Q_2 represent the momenta of final particles, $Q_1^2 = -M_{D^+}^2$, $Q_2^2 = -M_{D^-}^2$ and in this section $Q_1 + Q_2 = P$.

To simplify the computational process, we use the vertex function for the exchanged light meson, heavy pseudoscalar and vector mesons, and then Fig. 6 can be reduced to Fig. 7. From the Lorentz-structure, we obtain the matrix elements of quark scalar density J and quark current J_α between heavy pseudoscalar and vector mesons

$$\langle PM(Q_1)|J(0)|VM^\vartheta(p_1')\rangle = \frac{1}{\sqrt{2E_{D^+}(Q_1)}\sqrt{2E_{D_l^*}(p_1')}}[Q_1\cdot\varepsilon^\vartheta(p_1')]h_4^{(s)}(w^2), \tag{61a}$$

$$\langle\overline{PM'}(Q_2)|J(0)|\overline{VM'}^{\vartheta'}(-p_2')\rangle = \frac{1}{2\sqrt{E_{D^-}(Q_2)E_{\bar{D}_l^*}(-p_2')}}[Q_2\cdot\varepsilon^{\vartheta'}(-p_2')]\bar{h}_4^{(s)}(w^2), \tag{61b}$$

$$\begin{aligned}\langle PM(Q_1)|J_\alpha(0)|VM^\vartheta(p_1')\rangle = {}& \frac{1}{2\sqrt{E_{D^+}(Q_1)E_{D_l^*}(p_1')}}\{h_4^{(\mathrm{lv})}(w^2)\{[Q_1\cdot\varepsilon^\vartheta(p_1')](Q_1+p_1')_\alpha - [(Q_1+p_1')\cdot(Q_1-p_1')]\varepsilon_\alpha^\vartheta(p_1')\}\\ & - h_5^{(\mathrm{lv})}(w^2)\{[Q_1\cdot\varepsilon^\vartheta(p_1')](Q_1-p_1')_\alpha - (Q_1-p_1')^2\varepsilon_\alpha^\vartheta(p_1')\}\},\end{aligned} \tag{61c}$$

$$\begin{aligned}\langle\overline{PM'}(Q_2)|J_\beta(0)|\overline{VM'}^{\vartheta'}(-p_2')\rangle = {}& \frac{1}{2\sqrt{E_{D^-}(Q_2)E_{\bar{D}_l^*}(-p_2')}}\\ & \times\{\bar{h}_4^{(\mathrm{lv})}(w^2)\{[Q_2\cdot\varepsilon^{\vartheta'}(-p_2')](Q_2-p_2')_\beta - [(Q_2-p_2')\cdot(Q_2+p_2')]\varepsilon_\beta^{\vartheta'}(-p_2')\}\\ & - \bar{h}_5^{(\mathrm{lv})}(w^2)\{[Q_2\cdot\varepsilon^{\vartheta'}(-p_2')](Q_2+p_2')_\beta - (Q_2+p_2')^2\varepsilon_\beta^{\vartheta'}(-p_2')\}\},\end{aligned} \tag{61d}$$

where $p_1' = (\mathbf{p}, ip_{10}')$, $p_2' = (\mathbf{p}, ip_{20}')$, $Q_1 = (\mathbf{Q}_D, iQ_{10})$, $Q_2 = (-\mathbf{Q}_D, iQ_{20})$, $w = p_1' - Q_1 = p_2' + Q_2 = p - (Q_1 - Q_2)/2$ is the momentum of light meson and $\mathbf{w} = \mathbf{p} - \mathbf{Q}_D$; $E_{D^\pm}(p) = \sqrt{\mathbf{p}^2 + M_{D^\pm}^2}$, $h(w^2)$ and $\bar{h}(w^2)$ are scalar functions.

Now, we introduce the vertex function for the exchanged light meson, heavy pseudoscalar and vector mesons, shown as Fig. 8. The charmed meson D^+ is composed of c-quark and d-antiquark. For heavy pseudoscalar mesons, the authors of Refs. [24–27] also gave their BS amplitudes in Euclidean space:

$$\Gamma^P(K,k) = \frac{1}{\mathcal{N}^P}i\gamma_5\varphi_P(k^2), \tag{62}$$

where K is the momentum of heavy meson, k denotes the relative momentum between quark and antiquark in the heavy meson, $\mathcal{N}^P$ is normalization, and $\varphi_P(k^2)$ is scalar function fixed by providing fits to observables. Using the approach introduced in Sec. II C 1, we can obtain the heavy-pseudoscalar meson wave function in instantaneous approximation

$$\begin{aligned}\Psi^{D^+}(\mathbf{k}) &= \int dk_4\frac{1}{4}\mathrm{Tr}\left\{\gamma_5\frac{-1}{\gamma\cdot(k+K)-im_c}\frac{1}{\mathcal{N}^{D^+}}i\gamma_5\varphi_{D^+}(k^2)\frac{-1}{\gamma\cdot k-im_d}\right\}\\ &= \int dk_4\frac{-i}{\mathcal{N}^{D^+}}\exp\left(\frac{-\mathbf{k}^2-k_4^2}{\omega_D^2}\right)\frac{\mathbf{k}^2+k_4^2+m_cm_d}{(\mathbf{k}^2+k_4^2+m_c^2)(\mathbf{k}^2+k_4^2+m_d^2)},\end{aligned} \tag{63}$$

where $m_{c,d}$ are the constituent quark masses, $\varphi_{D^+}(k^2) = \varphi_{D^-}(k^2) = \exp(-k^2/\omega_D^2)$ and $\omega_D = 1.50$ GeV [27]. Then we can apply the method given in Refs. [5,12,13] to obtain the explicit forms of these scalar functions in vertex functions for heavy pseudoscalar meson D and vector meson D_l^* coupling to scalar meson (σ)

$$
\begin{aligned}
-\frac{h_4^{(s)}(w^2)}{2E_1} &= \frac{\bar{h}_4^{(s)}(w^2)}{2E_2} = F_4(\mathbf{w}^2),\\
F_4(\mathbf{w}^2) &= \frac{1}{\sqrt{M_{D^{*+}}^2 - M_{D^+}^2}} \int \frac{d^3k}{(2\pi)^3} \bar{\Psi}^{D^+}\left(\mathbf{k} + \frac{E_c(k)}{E_{D^+}}\mathbf{w}\right)\sqrt{\frac{E_d(k)+m_d}{E_d(k+w)+m_d}}\\
&\quad \times \left\{\frac{E_d(k+w) - E_d(k) + 2m_d}{2\sqrt{E_d(k+w)E_d(k)}} - \frac{\mathbf{k}\cdot\mathbf{w}}{2\sqrt{E_d(k+w)E_d(k)}[E_d(k)+m_d]}\right\}\Psi^{D^{*+}}(\mathbf{k}),
\end{aligned}
\tag{64}
$$

and to vector meson (ρ, V_1, and V_8)

$$
\begin{aligned}
h_4^{(\mathrm{lv})}(w^2) &= h_5^{(\mathrm{lv})}(w^2) = \bar{h}_4^{(\mathrm{lv})}(w^2) = \bar{h}_5^{(\mathrm{lv})}(w^2) = F_5(\mathbf{w}^2),\\
F_5(\mathbf{w}^2) &= \frac{1}{\sqrt{M_{D_l^*}^2 - M_{D^+}^2}} \frac{2\sqrt{E_{D_l^*}E_{D^+}}}{E_{D_l^*} + E_{D^+}} \int \frac{d^3k}{(2\pi)^3} \bar{\Psi}^{D^+}\left(\mathbf{k} + \frac{E_c(k)}{E_{D^+}}\mathbf{w}\right)\sqrt{\frac{E_l(k)+m_l}{E_d(k+w)+m_d}}\\
&\quad \times \left\{\frac{E_d(k+w) + E_l(k)}{2\sqrt{E_d(k+w)E_l(k)}} + \frac{\mathbf{k}\cdot\mathbf{w}}{2\sqrt{E_d(k+w)E_l(k)}[E_l(k)+m_l]}\right\}\Psi^{D_l^*}(\mathbf{k}),
\end{aligned}
\tag{65}
$$

where $E_d(p) = \sqrt{\mathbf{p}^2 + m_d^2}$, Ψ^{D^+} and $\Psi^{D_l^*}$ are the wave functions of heavy pseudoscalar and vector mesons expressed as Eqs. (63) and (24), respectively.

Taking away the external lines including normalizations and polarization vectors $\varepsilon_\lambda^{\vartheta}(p_1')$, $\varepsilon_\tau^{\vartheta'}(-p_2')$ in Eq. (61), we obtain the interaction from one light-meson (σ, ρ^0, V_1, and V_8) exchange,

$$
\begin{aligned}
\mathcal{V}_{\lambda\tau}(Q_1, Q_2, \mathbf{p}) &= -2E_1 F_4(\mathbf{w}^2)\frac{-ig_\sigma^2}{w^2 + M_\sigma^2} 2E_2 F_4(\mathbf{w}^2) Q_{1\lambda}Q_{2\tau}\\
&\quad - 4F_5(\mathbf{w}^2)\left(\frac{-ig_\rho^2}{w^2+M_\rho^2} + \frac{-ig_1^2}{w^2+M_\omega^2} + \frac{-ig_8^2}{w^2+M_\phi^2}\right)F_5(\mathbf{w}^2)(p_1'\cdot p_2')Q_{1\lambda}Q_{2\tau},
\end{aligned}
\tag{66}
$$

where $E_1 = E_2 = M_0/2$ and $w = (\mathbf{w}, 0)$. The interaction from one-$\rho^\pm$ exchange becomes

$$
\mathcal{V}'_{\lambda\tau}(Q_1, Q_2, \mathbf{p}) = -4F_5(\mathbf{w}^2)\frac{-i2g_\rho^2}{w^2+M_\rho^2}F_5(\mathbf{w}^2)(p_1'\cdot p_2')Q_{1\lambda}Q_{2\tau}. \tag{67}
$$

These momenta in Fig. 7 become

$$
w = (\mathbf{p} - \mathbf{Q}_D(M_0), 0), \qquad Q_1 + Q_2 = P, \qquad p_1' = p + P/2, \qquad p_2' = p - P/2, \tag{68}
$$

where $P = (0,0,0,iM_0)$, $Q_1 = (\mathbf{Q}_D(M_0), iM_0/2)$, $Q_2 = (-\mathbf{Q}_D(M_0), iM_0/2)$ and $\mathbf{Q}_D^2(M_0) = [M_0^2 - (M_{D^+} + M_{D^-})^2]/4$. For decay channel D^+D^-, we obtain the total matrix element

$$
-iR_{(c_2';b)a}(M_0) = \langle Q_1, Q_2\,\mathrm{out}|P\,\mathrm{in}\rangle = -i(2\pi)^4\delta^{(4)}(Q_1 + Q_2 - P)T_{(c_2';b)a}(M_0), \tag{69}
$$

where $T_{(c_2';b)a}(M_0)$ is the T-matrix element with mass M_0 for channel c_2'. From Fig. 7, we obtain

$$
T_{(c_2';b)a}(M_0) = \frac{-i}{(2\pi)^{9/2}\sqrt{2E_{D^+}(Q_1)}\sqrt{2E_{D^-}(Q_2)}\sqrt{2E(P)}}\left(\frac{1}{\sqrt{2}}\mathcal{M}^{c_2'} + \frac{1}{\sqrt{2}}\mathcal{M}'^{c_2'}\right), \tag{70}
$$

where

$$
\mathcal{M}^{c_2'} = \int \frac{d^4p}{(2\pi)^4}\mathcal{V}_{\lambda\tau}(Q_1, Q_2, \mathbf{p})\chi_{\lambda\tau}^{0^+}(P, p), \tag{71a}
$$

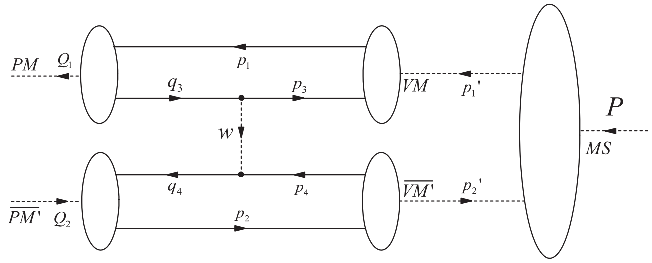

FIG. 6. Matrix element for decay channel D^+D^-. The momenta in the final state satisfy $Q_1 + Q_2 = P$. w represents the momentum of the exchanged light meson.

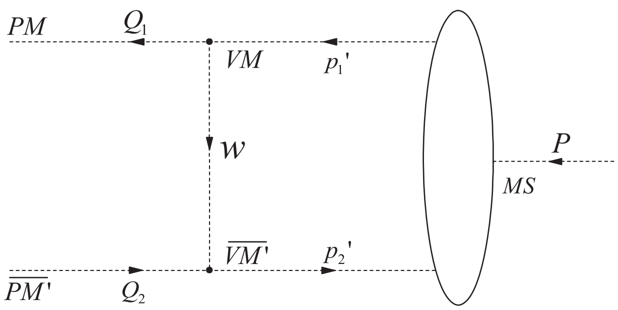

FIG. 7. Reduced matrix element for decay channel D^+D^-.

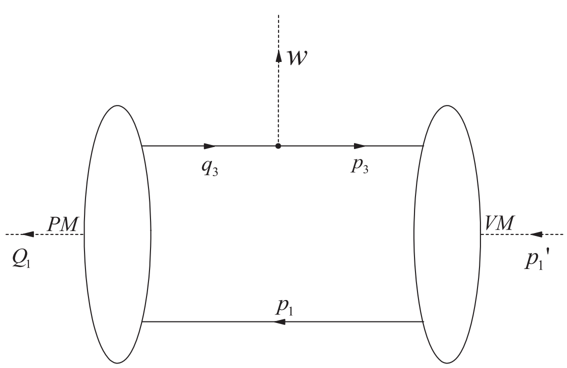

FIG. 8. Vertex function for the exchanged light meson, heavy pseudoscalar, and vector mesons.

$$\mathcal{M}'^{c_2'} = \int \frac{d^4p}{(2\pi)^4} \mathcal{V}'_{\lambda\tau}(Q_1, Q_2, \mathbf{p}) \chi^{0^+}_{\lambda\tau}(P, p). \tag{71b}$$

Here $\chi^{0^+}_{\lambda\tau}(P, p)$ is expressed as Eq. (40). The p integral is also computed in instantaneous approximation. Then, the decay width with the mass of the meson-meson bound state for channel D^+D^- becomes

$$\Gamma_2(M_0) = \int d^3Q_1 d^3Q_2 (2\pi)^4 \delta^{(4)}(Q_1 + Q_2 - P) \times (2\pi)^3 |T_{(c_2';b)a}(M_0)|^2. \tag{72}$$

The decay width $\Gamma_2(M_0)$ also is not the width of physical resonance.

C. Decay channel $D^0\bar{D}^0$ with respect to mass of bound state

Since the $\chi_{c0}(3915)$ state is an isoscalar, there should exist the neutral channel $D^0\bar{D}^0$. In Figs. 6–8, PM and $\overline{PM'}$ represent pseudoscalar mesons D^0 and $\bar{D}^0$, respectively. Following the same procedure as for charged channel D^+D^-, we can obtain the T-matrix element $T_{(c_3';b)a}(M_0)$ and the decay width $\Gamma_3(M_0)$ with mass M_0 for neutral channel c_3'. The decay width $\Gamma_3(M_0)$ should not be the width of physical resonance.

IV. THE DEVELOPED BETHE-SALPETER THEORY

Sections II and III give the traditional technique to deal with molecular state in present particle physics. These masses of meson-meson bound states were regarded as masses of resonances [1–6] and used to calculate decay widths of resonances [8,9], which should not be impeccable. To deal with resonance in the framework of relativistic quantum field theory, we considered the time evolution of molecular state as determined by the total Hamiltonian and provided the developed Bethe-Salpeter theory in Ref. [10].

Because the time evolution of molecular state is determined by the total Hamiltonian, exotic meson resonance should be considered as an unstable meson-meson molecular state. According to the developed Bethe-Salpeter theory for dealing with resonance [10], this unstable state has been prepared to decay at given time, and the prepared state can be regarded as a bound state with ground-state energy. Solving the BS equation for arbitrary meson-meson bound state, one can obtain the mass M_0 and BS wave function $\chi_P(x_1', x_2')$ for this bound state with momentum $P = (\mathbf{P}, i\sqrt{\mathbf{P}^2 + M_0^2})$. Setting $t_1 = 0$ and $t_2 = 0$ in the ground-state BS wave function, we obtain a description for the prepared state (ps)

$$\mathscr{X}_a^{\text{ps}} = \chi_P(\mathbf{x}_1', t_1 = 0, \mathbf{x}_2', t_2 = 0) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2E(P)}} e^{i\mathbf{P}\cdot\mathbf{X}} \chi_P(\mathbf{X}'). \tag{73}$$

Now it is necessary to consider the total Hamiltonian

$$H = K_I + V_I, \tag{74}$$

where K_I represents the interaction responsible for the formation of stationary bound state and V_I stands for the interaction responsible for the decay of resonance. Then the time evolution of this system determined by the total Hamiltonian H has the explicit form

$$\mathscr{X}(t) = e^{-iHt}\mathscr{X}_a^{\text{ps}} = \frac{1}{2\pi i}\int_{C_2} d\epsilon e^{-i\epsilon t}\frac{1}{\epsilon - H}\mathscr{X}_a^{\text{ps}}, \quad (75)$$

where $(\epsilon - H)^{-1}$ is the Green's function and the contour C_2 runs from $ic_r + \infty$ to $ic_r - \infty$ in energy-plane. The positive constant c_r is sufficiently large that no singularity of $(\epsilon - H)^{-1}$ lies above C_2. The time-dependent wave function $\mathscr{X}(t)$ provides a complete description of the system for $t > 0$. Since $H \neq K_I$, this system should not remain in the prepared state $\mathscr{X}_a^{\text{ps}}$. Then at arbitrary time t the probability amplitude of finding the system in the state $\mathscr{X}_a^{\text{ps}}$ is

$$\mathscr{A}_a = (\mathscr{X}_a^{\text{ps}}, \mathscr{X}(t)) = \frac{1}{2\pi i}\int_{C_2} d\epsilon \frac{e^{-i\epsilon t}}{\epsilon - M_0 - (2\pi)^3 T_{aa}(\epsilon)}. \quad (76)$$

In field theory the operator $T(\epsilon)$ is just the scattering matrix with energy ϵ, and $T_{aa}(\epsilon)$ is the T-matrix element between two bound states, which is defined as

$$\langle a\,\text{out}|a\,\text{in}\rangle = \langle a\,\text{in}|a\,\text{in}\rangle - i(2\pi)^4\delta^{(4)}(P - P)T_{aa}(\epsilon). \quad (77)$$

Because of the analyticity of $T_{aa}(\epsilon)$, we define

$$T_{aa}(\epsilon) = \mathbb{D}(\epsilon) - i\mathbb{I}(\epsilon), \quad (78)$$

where ϵ approaches the real axis from above, $\mathbb{D}$ and $\mathbb{I}$ are the real and imaginary parts, respectively. In experiments, many exotic particles are narrow states and their decay widths are very small compared with their energy levels, i.e., $(2\pi)^3\mathbb{I}(M_0) \ll M_0$. This situation is ordinarily interpreted as implying that both $(2\pi)^3|\mathbb{D}(\epsilon)|$ and $(2\pi)^3\mathbb{I}(\epsilon)$ are also very small quantities, as compared to M_0. Therefore, we can expect that $[\epsilon - M_0 - (2\pi)^3 T_{aa}(\epsilon)]^{-1}$ has a pole on the second Riemann sheet

$$\epsilon_{\text{pole}} \cong M_0 + (2\pi)^3[\mathbb{D}(M_0) - i\mathbb{I}(M_0)] = M - i\frac{\Gamma(M_0)}{2}, \quad (79)$$

where $\Delta M = (2\pi)^3\mathbb{D}(M_0)$ is the correction for the energy level of resonance and $M = M_0 + (2\pi)^3\mathbb{D}(M_0)$ is the physical mass for resonance. This pole at ϵ_{pole} describes the resonance. The mass M_0 of two-body bound state is obtained by solving the homogeneous BS equation, which should not be the mass of physical resonance. $\Gamma(M_0)$ with mass M_0 also should not be the width of physical resonance, which should depend on its physical mass M. We will minutely show the computational process of T-matrix element between two bound states $T_{aa}(\epsilon)$ in the next section.

V. T-MATRIX ELEMENT $T_{aa}(\epsilon)$

When there is only one decay channel, we can use the unitarity of $T_{aa}(\epsilon)$ to obtain [33]

$$2\mathbb{I}(\epsilon) = \sum_b (2\pi)^4\delta^{(3)}(\mathbf{P}_b - \mathbf{P})\delta(E_b - \epsilon)|T_{ba}(\epsilon)|^2, \quad (80)$$

where $P_b = (\mathbf{P}_b, iE_b)$ is the total energy-momentum vector of all particles in the final state and the T-matrix element $T_{ba}(\epsilon)$ is defined as $\langle b\,\text{out}|a\,\text{in}\rangle = -i(2\pi)^4\delta^{(3)}(\mathbf{P}_b - \mathbf{P})\times \delta(E_b - \epsilon)T_{ba}(\epsilon)$. The delta function in Eq. (80) means that the energy ϵ in scattering matrix is equal to the total energy E_b of the final state, and $\sum_b$ represents summing over momenta and spins of all particles in the final state. For $E_b = \epsilon$, we also denote the total energy of the final state by ϵ and $\mathbb{I}(\epsilon)$ becomes a function of the final state energy. Using dispersion relation for the function $T_{aa}(\epsilon)$, we obtain

$$\mathbb{D}(\epsilon) = -\frac{\mathcal{P}}{\pi}\int_{\epsilon_M}^{\infty}\frac{\mathbb{I}(\epsilon')}{\epsilon' - \epsilon}d\epsilon', \quad (81)$$

where the symbol $\mathcal{P}$ means that this integral is a principal value integral and the variable of integration is the total energy ϵ' of the final state. To calculate the real part, we need calculate the function $\mathbb{I}(\epsilon')$ of value of the final state energy ϵ', which is an arbitrary real number over the real interval $\epsilon_M < \epsilon' < \infty$. As usual the momentum of initial bound state a is set as $P = (0, 0, 0, iM_0)$ in the rest frame and ϵ_M denotes the sum of all particle masses in the final state. We suppose that the final state b may contain n composite particles and n' elementary particles in decay channel c'. From Eq. (80), we have

$$\mathbb{I}(\epsilon') = \frac{1}{2}\int d^3Q'_1\cdots d^3Q'_{n'}d^3Q_1\cdots d^3Q_n(2\pi)^4 \times \delta^{(4)}(Q'_1 + \cdots + Q_n - P^{\epsilon'})\sum_{\text{spins}}|T_{(c';b)a}(\epsilon')|^2, \quad (82)$$

where $Q'_1\cdots Q'_{n'}$ and $Q_1\cdots Q_n$ are the momenta of final elementary and composite particles, respectively; $P^{\epsilon'} = (0, 0, 0, i\epsilon')$, $T_{(c';b)a}(\epsilon')$ is the T-matrix element with respect to ϵ', and $\sum_{\text{spins}}$ represents summing over spins of all particles in the final state. In Eq. (82) the energy in scattering matrix is equal to the total energy ϵ' of the final state b, which is an arbitrary real number over the real interval $\epsilon_M < \epsilon' < \infty$. The mass M_0 and BS amplitude of initial bound state a have been specified and the value of the initial state energy in the rest frame is a specified value M_0. From Eq. (82), we have $\mathbb{I}(\epsilon') > 0$ for $\epsilon' > \epsilon_M$ and $\mathbb{I}(\epsilon') = 0$ for $\epsilon' \leq \epsilon_M$, which is the reason that the integration in dispersion relation (81) ranges from ϵ_M to $+\infty$.

If there are several decay channels, we should write instead

$$\mathbb{I}(\epsilon') = \frac{1}{2}\sum_{c'}\int d^3Q'_1\cdots d^3Q'_{n'}d^3Q_1\cdots d^3Q_n(2\pi)^4 \times \delta^{(4)}(Q'_1+\cdots+Q_n-P^{\epsilon'})\sum_{\text{spins}}|T_{(c';b)a}(\epsilon')|^2, \quad (83)$$

where $\sum_{c'}$ represents summing over all open and closed channels. Because the total energy ϵ' of the final state extends from ϵ_M to $+\infty$, we may obtain several closed channels derived from the interaction Lagrangian. Assuming that resonance $\chi_{c0}(3915)$ is a mixed state of two components $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, we obtain one closed channel $D^*\bar{D}^*$ derived from the interaction Lagrangian (1), denoted as c'_4. Since bound state lies below the threshold, i.e., $M_0 < M_{D^*}+M_{\bar{D}^*}$, the closed channel c'_4 can not occur inside the physical world.

A. Channel $J/\psi\omega$ with respect to arbitrary value of the final-state energy

From Eq. (55), we obtain the total matrix element between the final-state $\langle J/\psi(Q),\omega(Q')\,\text{out}|$ and the specified initial four-quark state $|P\,\text{in}\rangle$

$$-iR_{(c'_1;b)a}(\epsilon') = \langle Q,Q'\ \text{out}|P\ \text{in}\rangle = -i(2\pi)^4\delta^{(4)}(Q+Q'-P^{\epsilon'})T_{(c'_1;b)a}(\epsilon'), \quad (84)$$

where the total energy ϵ' of the final state extends from $\epsilon_{c'_1,M}$ to $+\infty$, i.e., $\epsilon_{c'_1,M}<\epsilon'<\infty$ and $\epsilon_{c'_1,M}=M_{J/\psi}+M_\omega$. $T_{(c'_1;b)a}(\epsilon')$ is the bound state matrix element with respect to ϵ' for channel c'_1, shown as Fig. 9. It is necessary to emphasize that the energy in the two-particle irreducible Green's function is equal to the final state energy ϵ' while the mass M_0 and BS amplitude of initial bound state is specified. We have introduced extended Feynman diagram in Ref. [10] to represent arbitrary value of the final state energy. In Fig. 9, the quark momenta in left-hand side of crosses depend on the final-state energy and the momenta in right-hand side depend on the initial-state energy, i.e., $p_1-p_2-p_3+p_4=Q+Q'=P^{\epsilon'}$ and $p'_1-p'_2=P$. When $\epsilon'=M_0$, the crosses in Fig. 9 disappear and then Fig. 9 becomes Fig. 5; $T_{(c'_1;b)a}(\epsilon'=M_0)$ is the T-matrix element with mass M_0 for channel c'_1 expressed as Eq. (56). Though the T-matrix element $T_{(c'_1;b)a}(\epsilon')$ has the same form expressed as Eq. (56), these momenta should become

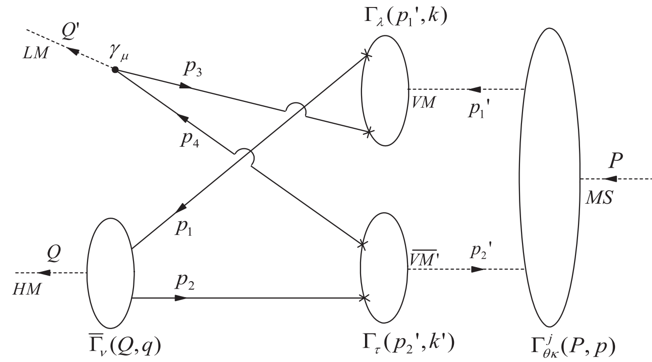

FIG. 9. Matrix element with respect to ϵ' for channel $J/\psi\omega$. The momenta in the final state satisfy $Q+Q'=P^{\epsilon'}$ and the momentum of the initial state is P. The final-state energy extends from ϵ_M to $+\infty$ while the initial-state energy is specified, and the crosses mean that the momenta of quark propagators depend on the final-state energy ϵ'.

$$p_1=(Q+Q')/2+p+k,\quad p_2=(Q+Q')/2-Q+p+k,\quad p_3=k,\quad p_4=Q'+k,$$
$$q=Q'/2+p+k,\quad k'=Q'(M_0)+k,\quad p'_1=p+P/2,\quad p'_2=p-P/2,\quad Q+Q'=P^{\epsilon'}, \quad (85)$$

where $P=(0,0,0,iM_0)$, $P^{\epsilon'}=(0,0,0,i\epsilon')$, $Q'(M_0)=(-\mathbf{Q}(M_0),i\sqrt{\mathbf{Q}^2(M_0)+M_\omega^2})$, $Q=(\mathbf{Q}(\epsilon'),i\sqrt{\mathbf{Q}^2(\epsilon')+M_{J/\psi}^2})$, $Q'=(-\mathbf{Q}(\epsilon'),i\sqrt{\mathbf{Q}^2(\epsilon')+M_\omega^2})$, and $\mathbf{Q}^2(\epsilon')=[\epsilon'^2-(M_{J/\psi}+M_\omega)^2][\epsilon'^2-(M_{J/\psi}-M_\omega)^2]/(4\epsilon'^2)$. The initial state is considered as a four-quark state, so the specified GBS amplitude of initial state should be

$$\Gamma_\lambda^{D_l^*}(p'_1,k)\chi^{0^+}_{\lambda\tau}(P,p)\Gamma_\tau^{\bar{D}_l^*}(p'_2,k'), \quad (86)$$

where k' depends on P. Then we obtain the function $\mathbb{I}_1(\epsilon')$ for channel $J/\psi\omega$

$$\mathbb{I}_1(\epsilon') = \frac{1}{2}\int d^3Qd^3Q'(2\pi)^4\delta^{(4)}(Q+Q'-P^{\epsilon'})\sum_{\varrho'=1}^{3}\sum_{\varrho=1}^{3}|T_{(c'_1;b)a}(\epsilon')|^2. \quad (87)$$

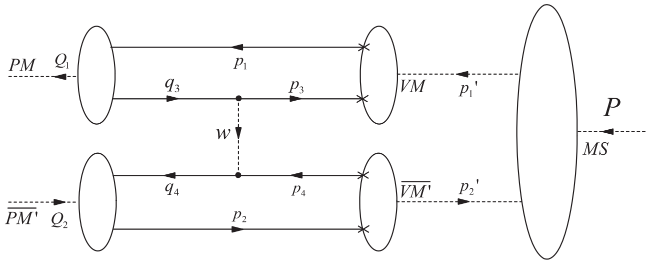

FIG. 10. Matrix element with respect to ϵ' for channel D^+D^-. The momenta in the final state satisfy $Q_1 + Q_2 = P^{\epsilon'}$ and the momentum of the initial state is P. w represents the momentum of the exchanged light meson. The crosses mean that the momenta of quark propagators and the momentum w of the exchanged light meson depend on the final-state energy ϵ'.

B. Channel D^+D^- with respect to arbitrary value of the final-state energy

The T-matrix element with respect to ϵ' for channel c_2' can be represented graphically by Fig. 10. The total energy ϵ' of the final state extends from $\epsilon_{c_2',M}$ to $+\infty$, i.e., $\epsilon_{c_2',M} < \epsilon' < \infty$ and $\epsilon_{c_2',M} = M_{D^+} + M_{D^-}$. In Fig. 10, the crosses mean that the momenta of quark propagators and the momentum w of the exchanged light meson depend on Q_1 and Q_2, i.e., $p_1 - p_2 - p_3 + p_4 = p_1 - p_2 - q_3 + q_4 = Q_1 + Q_2 = P^{\epsilon'}$ and $p_1' - p_2' = P$.

We still use the vertex function to calculate the T-matrix element with respect to ϵ' for channel c_2'. However, different from the ordinary vertex function, we should introduce the vertex function with respect to ϵ', which is shown as Fig. 11. In Fig. 11, Q_1 depends on $P^{\epsilon'}$, p_1' depends on P and the crosses mean that the momenta of quark propagators and the momentum w of the exchanged light meson depend on the final state energy ϵ'. Using the approach introduced in Sec. III B, we can obtain the explicit forms for the vertex functions with respect to ϵ', and then

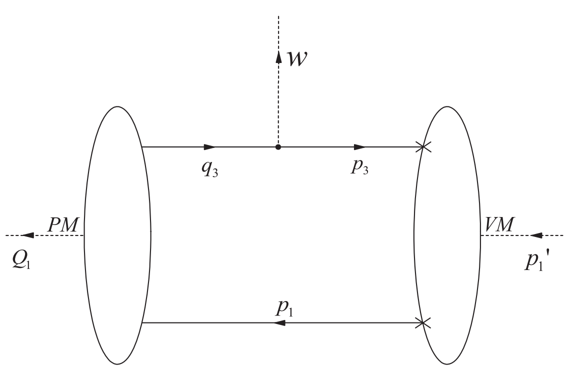

FIG. 11. Vertex function for the exchanged light meson, heavy pseudoscalar, and vector mesons with respect to ϵ'. Q_1 depends on $P^{\epsilon'}$ and p_1' depends on P. The crosses mean that the momenta of quark propagators and the momentum w of the exchanged light meson depend on the final-state energy ϵ'.

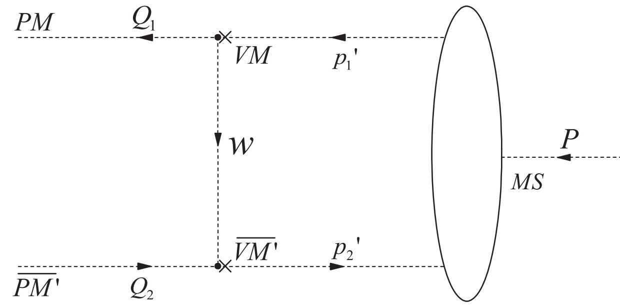

FIG. 12. Reduced matrix element with respect to ϵ' for channel D^+D^-. The crosses mean that the momentum w of the exchanged light meson depends on Q_1 and Q_2.

Fig. 10 can be reduced to Fig. 12. In Fig. 12, we have $Q_1 + Q_2 = P^{\epsilon'}$, $p_1' - p_2' = P$ and the crosses lie on the right-hand side of light meson propagator, which implies that the momentum w of the exchanged light meson depends on Q_1 and Q_2.

From Eq. (69), we obtain the total matrix element between the final state $\langle D^+(Q_1), D^-(Q_2)\,\mathrm{out}|$ and the mixed state of two pure bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$

$$-iR_{(c_2';b)a}(\epsilon') = \langle Q_1, Q_2\,\mathrm{out}|P\,\mathrm{in}\rangle = -i(2\pi)^4\delta^{(4)}(Q_1 + Q_2 - P^{\epsilon'})T_{(c_2';b)a}(\epsilon'), \tag{88}$$

where $T_{(c_2';b)a}(\epsilon')$ is the bound state matrix element with respect to ϵ' for channel c_2', shown as Fig. 12. When $\epsilon' = M_0$, the crosses in Figs. 10–12 disappear and then these three extended Feynman diagrams become Figs. 6–8, respectively; $T_{(c_2';b)a}(\epsilon' = M_0)$ is the T-matrix element with mass M_0 for channel c_2' expressed as Eq. (70). Though the T-matrix element $T_{(c_2';b)a}(\epsilon')$ has the same form expressed as Eq. (70), these momenta should become

$$w = (\mathbf{p} - \mathbf{Q}_D(\epsilon'), 0), \qquad Q_1 + Q_2 = P^{\epsilon'},$$
$$p_1' = p + P/2, \qquad p_2' = p - P/2, \tag{89}$$

where $P = (0,0,0,iM_0)$, $P^{\epsilon'} = (0,0,0,i\epsilon')$, $Q_1 = (\mathbf{Q}_D(\epsilon'), i\epsilon'/2)$, $Q_2 = (-\mathbf{Q}_D(\epsilon'), i\epsilon'/2)$ and $\mathbf{Q}_D^2(\epsilon') = [\epsilon'^2 - (M_{D^+} + M_{D^-})^2]/4$. The coefficients E_1 and E_2 in interaction $\mathcal{V}_{\lambda\tau}(Q_1, Q_2, \mathbf{p})$ given by Eq. (66) should become $E_1(\epsilon') = E_2(\epsilon') = \sqrt{\epsilon' M_0}/2$. Then we obtain the function $\mathbb{I}_2(\epsilon')$ for channel D^+D^-

$$\mathbb{I}_2(\epsilon') = \frac{1}{2}\int d^3Q_1 d^3Q_2 (2\pi)^4\delta^{(4)}(Q_1 + Q_2 - P^{\epsilon'}) \times |T_{(c_2';b)a}(\epsilon')|^2. \tag{90}$$

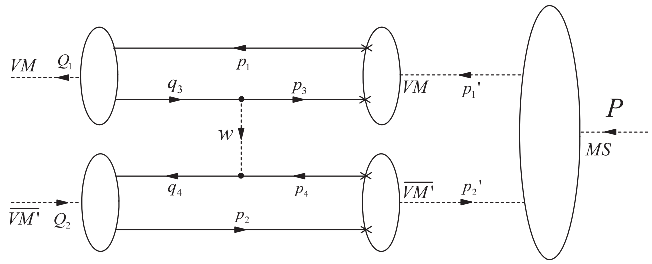

FIG. 13. Matrix element for closed channel $D_l^* \bar{D}_l^*$. The momenta in the final state satisfy $Q_1 + Q_2 = P^{\epsilon'}$ and the momentum of the initial state is P. w represents the momentum of the exchanged light meson. The crosses mean that the momenta of quark propagators and the momentum w of the exchanged light meson depend on the final-state energy ϵ'.

C. Channel $D^0\bar{D}^0$ with respect to arbitrary value of the final-state energy

In Figs. 10–12, PM and $\overline{PM'}$ represent pseudoscalar mesons D^0 and $\bar{D}^0$, respectively. Following the same procedure as for charged channel D^+D^-, we can obtain the T-matrix element $T_{(c_3';b)a}(\epsilon')$ with respect to ϵ' and the function $\mathbb{I}_3(\epsilon')$ for neutral channel c_3'. Here, the total energy ϵ' of the final state extends from $\epsilon_{c_3',M}$ to $+\infty$, i.e., $\epsilon_{c_3',M} < \epsilon' < \infty$ and $\epsilon_{c_3',M} = M_{D^0} + M_{\bar{D}^0}$.

D. Closed channel $D^*\bar{D}^*$

The final state $\langle D^*, \bar{D}^* \text{ out}|$ can be written as

$$\langle D^*, \bar{D}^* \text{ out}| = \frac{1}{\sqrt{2}} \langle D^{*0}, \bar{D}^{*0} \text{ out}| + \frac{1}{\sqrt{2}} \langle D^{*+}, D^{*-} \text{ out}|. \tag{91}$$

The total energy ϵ' of the final state extends from $\epsilon_{c_4',M}$ to $+\infty$, i.e., $\epsilon_{c_4',M} < \epsilon' < \infty$ and $\epsilon_{c_4',M} = M_{D_l^*} + M_{\bar{D}_l^*}$. Considering the lowest-order term of the two-particle irreducible Green's function, we can obtain the T-matrix element between the final state $\langle D_l^*, \bar{D}_l^* \text{ out}|$ and the initial four-quark state, which can be represented graphically by Fig. 13. In Fig. 13, VM and $\overline{VM'}$ still represent D_l^* and $\bar{D}_l^*$, respectively; Q_1 and Q_2 still represent the momenta of final particles, but $Q_1^2 = -M_{D_l^*}^2$ and $Q_2^2 = -M_{\bar{D}_l^*}^2$; the crosses mean that the momenta of quark propagators and the momentum w of the exchanged light meson depend on Q_1 and Q_2, i.e., $p_1 - p_2 - p_3 + p_4 = p_1 - p_2 - q_3 + q_4 = Q_1 + Q_2 = P^{\epsilon'}$, and $p_1' - p_2' = P$.

To calculate the T-matrix element with respect to ϵ' for channel c_4', we also introduce the form factor of heavy meson with respect to ϵ', which is shown as Fig. 14. Using the approach introduced in Sec. II C 1, we can obtain the explicit forms for the heavy meson form factors $h(w^2)$ with respect to ϵ', and then Fig. 13 can be reduced to Fig. 15. In Fig. 15, we have $Q_1 + Q_2 = P^{\epsilon'}$, $p_1' - p_2' = P$, and the crosses lie on the right-hand side of light-meson propagator, which implies that the momentum w of the exchanged light meson depends on Q_1 and Q_2.

Using the Heisenberg picture, we obtain the total matrix element between the final-state $\langle D_l^*(Q_1), \bar{D}_l^*(Q_2) \text{ out}|$ and the mixed state of two pure-bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$,

$$\begin{aligned} -iR_{(c_4';b)a}(\epsilon') &= \langle Q_1, Q_2 \text{ out}|P \text{ in}\rangle \\ &= -i(2\pi)^4 \delta^{(4)}(Q_1 + Q_2 - P^{\epsilon'}) T_{(c_4';b)a}(\epsilon'). \end{aligned} \tag{92}$$

According to Mandelstam's approach, the T-matrix element becomes

$$\begin{aligned} &T_{(c_4';b)a}(\epsilon') \\ &= \frac{1}{2} \sum_{l=u,d} \frac{-i\varepsilon_\mu^{\varrho'}(Q_2)\varepsilon_\nu^{\varrho}(Q_1)}{(2\pi)^{9/2}\sqrt{2E_{D_l^*}(Q_1)}\sqrt{2E_{\bar{D}_l^*}(Q_2)}\sqrt{2E(P)}} \\ &\quad \times (\mathcal{M}_{\nu\mu}^{c_4'} + \mathcal{M}'^{c_4'}_{\nu\mu}), \end{aligned} \tag{93}$$

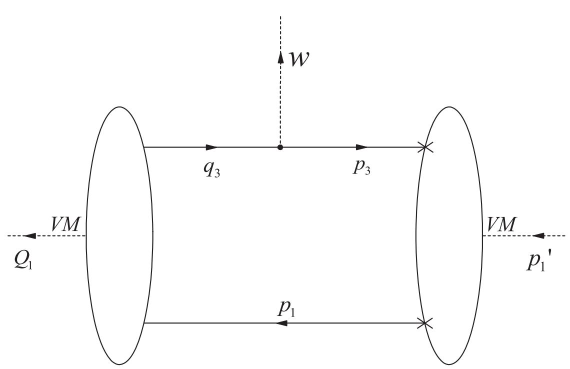

FIG. 14. The heavy-meson form factor with respect to ϵ'. Q_1 depends on $P^{\epsilon'}$ and p_1' depends on P. The crosses mean that the momenta of quark propagators and the momentum w of the exchanged light meson depend on the final state energy ϵ'.

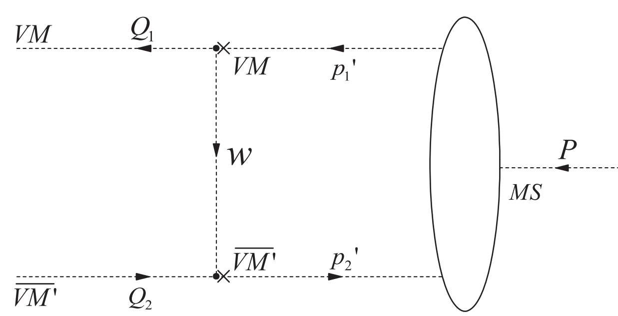

FIG. 15. Reduced matrix element for closed channel $D_l^* \bar{D}_l^*$. The crosses mean that the momentum w of the exchanged light meson depends on Q_1 and Q_2.

where $\varepsilon_\nu^{\varrho=1,2,3}(Q_1)$ and $\varepsilon_\mu^{\varrho'=1,2,3}(Q_2)$ become the polarization vectors of D_l^* and $\bar{D}_l^*$, respectively, and

$$\mathcal{M}_{\nu\mu}^{c_4'} = \int \frac{d^4p}{(2\pi)^4} \mathcal{V}_{\nu\lambda,\tau\mu}(Q_1, Q_2, \mathbf{p}) \chi_{\lambda\tau}^{0^+}(P, p), \tag{94a}$$

$$\mathcal{M}'^{c_4'}_{\nu\mu} = \int \frac{d^4p}{(2\pi)^4} \mathcal{V}'_{\nu\lambda,\tau\mu}(Q_1, Q_2, \mathbf{p}) \chi_{\lambda\tau}^{0^+}(P, p). \tag{94b}$$

Here $\chi_{\lambda\tau}^{0^+}(P, p)$ is expressed as Eq. (40), $\mathcal{V}_{\nu\lambda,\tau\mu}(Q_1, Q_2, \mathbf{p})$ and $\mathcal{V}'_{\nu\lambda,\tau\mu}(Q_1, Q_2, \mathbf{p})$ represent the interactions derived from one light meson (σ, ρ^0, V_1, and V_8) exchange and one-$\rho^\pm$ exchange, respectively.

Now, we determine the interactions $\mathcal{V}_{\nu\lambda,\tau\mu}(Q_1, Q_2, \mathbf{p})$ and $\mathcal{V}'_{\nu\lambda,\tau\mu}(Q_1, Q_2, \mathbf{p})$. Structurally similar to Eq. (16), we obtain the vertices of heavy vector mesons and light mesons derived from the light meson interaction with the light quark in heavy meson

$$\langle VM^\varrho(Q_1)|J(0)|VM^\vartheta(p_1')\rangle = \frac{1}{2\sqrt{E_{D_l^*}(Q_1)E_{D_l^*}(p_1')}} [\varepsilon^\varrho(Q_1)\cdot\varepsilon^\vartheta(p_1')] h_1^{(s)}(w^2), \tag{95a}$$

$$\langle \overline{VM'}^{\varrho'}(Q_2)|J(0)|\overline{VM'}^{\vartheta'}(-p_2')\rangle = \frac{1}{2\sqrt{E_{\bar{D}_l^*}(Q_2)E_{\bar{D}_l^*}(-p_2')}} [\varepsilon^{\varrho'}(Q_2)\cdot\varepsilon^{\vartheta'}(-p_2')] \bar{h}_1^{(s)}(w^2), \tag{95b}$$

$$\begin{aligned}\langle VM^\varrho(Q_1)|J_\alpha(0)|VM^\vartheta(p_1')\rangle = \frac{1}{2\sqrt{E_{D_l^*}(Q_1)E_{D_l^*}(p_1')}} &\{[\varepsilon^\varrho(Q_1)\cdot\varepsilon^\vartheta(p_1')] h_1^{(\mathrm{lv})}(w^2)(Q_1+p_1')_\alpha \\ &- h_2^{(\mathrm{lv})}(w^2)\{[\varepsilon^\varrho(Q_1)\cdot p_1']\varepsilon_\alpha^\vartheta(p_1') + [\varepsilon^\vartheta(p_1')\cdot Q_1]\varepsilon_\alpha^\varrho(Q_1)\}\},\end{aligned} \tag{95c}$$

$$\begin{aligned}\langle \overline{VM'}^{\varrho'}(Q_2)|J_\beta(0)|\overline{VM'}^{\vartheta'}(-p_2')\rangle = \frac{1}{2\sqrt{E_{\bar{D}_l^*}(Q_2)E_{\bar{D}_l^*}(-p_2')}} &\{[\varepsilon^{\varrho'}(Q_2)\cdot\varepsilon^{\vartheta'}(-p_2')] \bar{h}_1^{(\mathrm{lv})}(w^2)(Q_2-p_2')_\beta \\ &- \bar{h}_2^{(\mathrm{lv})}(w^2)\{[\varepsilon^{\varrho'}(Q_2)\cdot(-p_2')]\varepsilon_\beta^{\vartheta'}(-p_2') + [\varepsilon^{\vartheta'}(-p_2')\cdot Q_2]\varepsilon_\beta^{\varrho'}(Q_2)\}\},\end{aligned} \tag{95d}$$

where $w = p - (Q_1 - Q_2)/2$ is the momentum of light meson, $h(w^2)$ and $\bar{h}(w^2)$ are the heavy meson form factors with respect to ϵ'. Similarly, taking away the external lines including normalizations and polarization vectors $\varepsilon_\nu^\varrho(Q_1)$, $\varepsilon_\lambda^\vartheta(p_1')$, $\varepsilon_\mu^{\varrho'}(Q_2)$, $\varepsilon_\tau^{\vartheta'}(-p_2')$, we obtain the interaction from one light meson (σ, ρ^0, V_1, and V_8) exchange,

$$\begin{aligned}\mathcal{V}_{\nu\lambda,\tau\mu}(Q_1, Q_2, \mathbf{p}) = &-2E_1(\epsilon')F_1(\mathbf{w}^2)\frac{-ig_\sigma^2}{w^2+M_\sigma^2}2E_2(\epsilon')F_1(\mathbf{w}^2)\delta_{\nu\lambda}\delta_{\tau\mu} + \left(\frac{-ig_\rho^2}{w^2+M_\rho^2} + \frac{-ig_1^2}{w^2+M_\omega^2} + \frac{-ig_8^2}{w^2+M_\phi^2}\right) \\ &\times F_2^{(\mathrm{lv})}(\mathbf{w}^2)F_2^{(\mathrm{lv})}(\mathbf{w}^2)\{(Q_1+p_1')\cdot(Q_2-p_2')\delta_{\nu\lambda}\delta_{\tau\mu} - \delta_{\nu\lambda}[-(Q_1+p_1')_\tau p_{2\mu}' + Q_{2\tau}(Q_1+p_1')_\mu] \\ &- [p_{1\nu}'(Q_2-p_2')_\lambda + (Q_2-p_2')_\nu Q_{1\lambda}]\delta_{\tau\mu} - p_{1\nu}'\delta_{\lambda\tau}p_{2\mu}' + p_{1\nu}'\delta_{\lambda\mu}Q_{2\tau} - \delta_{\nu\tau}Q_{1\lambda}p_{2\mu}' + \delta_{\nu\mu}Q_{1\lambda}Q_{2\tau}\},\end{aligned} \tag{96}$$

where $E_1(\epsilon') = E_2(\epsilon') = \sqrt{\epsilon' M_0}/2$ and $w = (\mathbf{w}, 0)$. The interaction from one-$\rho^\pm$ exchange becomes

$$\begin{aligned}\mathcal{V}'_{\nu\lambda,\tau\mu}(Q_1, Q_2, \mathbf{p}) = &F_2^{(\mathrm{lv})}(\mathbf{w}^2)\frac{-i2g_\rho^2}{w^2+M_\rho^2}F_2^{(\mathrm{lv})}(\mathbf{w}^2) \\ &\times\{(Q_1+p_1')\cdot(Q_2-p_2')\delta_{\nu\lambda}\delta_{\tau\mu} - \delta_{\nu\lambda}[-(Q_1+p_1')_\tau p_{2\mu}' + Q_{2\tau}(Q_1+p_1')_\mu] \\ &- [p_{1\nu}'(Q_2-p_2')_\lambda + (Q_2-p_2')_\nu Q_{1\lambda}]\delta_{\tau\mu} - p_{1\nu}'\delta_{\lambda\tau}p_{2\mu}' + p_{1\nu}'\delta_{\lambda\mu}Q_{2\tau} - \delta_{\nu\tau}Q_{1\lambda}p_{2\mu}' + \delta_{\nu\mu}Q_{1\lambda}Q_{2\tau}\}.\end{aligned} \tag{97}$$

These momenta in Fig. 15 become

$$w = (\mathbf{p} - \mathbf{Q}_{D^*}(\epsilon'), 0), \qquad Q_1 + Q_2 = P^{\epsilon'},$$
$$p_1' = p + P/2, \qquad p_2' = p - P/2, \tag{98}$$

where $P = (0,0,0,iM_0)$, $P^{\epsilon'} = (0,0,0,i\epsilon')$, $Q_1 = (\mathbf{Q}_{D^*}(\epsilon'), i\epsilon'/2)$, $Q_2 = (-\mathbf{Q}_{D^*}(\epsilon'), i\epsilon'/2)$, and $\mathbf{Q}^2_{D^*}(\epsilon') = [\epsilon'^2 - (M_{D_l^*} + M_{\bar{D}_l^*})^2]/4$.

Substituting Eqs. (96) and (97) into (94), we obtain the explicit forms for tensors $\mathcal{M}^{c_4'}_{\nu\mu}$ and $\mathcal{M}'^{c_4'}_{\nu\mu}$. The p integral is also computed in instantaneous approximation. $\mathcal{M}^{c_4'}_{\nu\mu}$ and $\mathcal{M}'^{c_4'}_{\nu\mu}$ only depend on Q_1 and Q_2, which can be calculated by means of the method given in Sec. III A. Applying Eq. (82), we obtain the function $\mathbb{I}_4(\epsilon')$ for the closed channel $D^*\bar{D}^*$

$$\mathbb{I}_4(\epsilon') = \frac{1}{2}\int d^3Q_1 d^3Q_2 (2\pi)^4 \delta^{(4)}(Q_1 + Q_2 - P^{\epsilon'}) \times \sum_{\varrho'=1}^{3}\sum_{\varrho=1}^{3} |T_{(c_4';b)a}(\epsilon')|^2. \tag{99}$$

VI. PHYSICAL MASS AND WIDTH OF RESONANCE

For resonance $\chi_{c0}(3915)$, the dispersion relation (81) becomes

$$\mathbb{D}(M_0) = -\frac{\mathcal{P}}{\pi}\int_{\epsilon_{c_1',M}}^{\infty} \frac{\mathbb{I}_1(\epsilon')}{\epsilon' - M_0} d\epsilon' - \frac{\mathcal{P}}{\pi}\int_{\epsilon_{c_2',M}}^{\infty} \frac{\mathbb{I}_2(\epsilon')}{\epsilon' - M_0} d\epsilon' - \frac{\mathcal{P}}{\pi}\int_{\epsilon_{c_3',M}}^{\infty} \frac{\mathbb{I}_3(\epsilon')}{\epsilon' - M_0} d\epsilon' - \frac{1}{\pi}\int_{\epsilon_{c_4',M}}^{\infty} \frac{\mathbb{I}_4(\epsilon')}{\epsilon' - M_0} d\epsilon', \tag{100}$$

where $\epsilon_{c_1',M} = M_{J/\psi} + M_\omega$, $\epsilon_{c_2',M} = M_{D^+} + M_{D^-}$, $\epsilon_{c_3',M} = M_{D^0} + M_{\bar{D}^0}$, and $\epsilon_{c_4',M} = M_{D_l^*} + M_{\bar{D}_l^*}$. From Eq. (79), we obtain that the physical mass of resonance $\chi_{c0}(3915)$ is $M = M_0 + (2\pi)^3\mathbb{D}(M_0)$. Replacing M_0 by M in Eqs. (56) and (60), we recalculate the matrix element $T_{(c_1';b)a}(M)$ and obtain the width Γ_1 for physical decay model $\chi_{c0}(3915) \to J/\psi\omega$. Replacing M_0 by M in Eqs. (70) and (72), we recalculate the matrix element $T_{(c_2';b)a}(M)$ and obtain the width Γ_2 for physical decay model $\chi_{c0}(3915) \to D^+D^-$. For the isospin conservation, it is easy to obtain the width Γ_3 for physical decay model $\chi_{c0}(3915) \to D^0\bar{D}^0$.

VII. NUMERICAL RESULT

Considering the isospin conservation, we employ the constituent quark masses $m_u = m_d = 0.33$ GeV, the heavy quark mass $m_c = 1.55$ GeV [30] and the meson masses $M_\sigma = 0.45$ GeV, $M_\omega = 0.782$ GeV, $M_{\rho^0} = M_{\rho^\pm} = 0.775$ GeV, $M_\phi = 1.019$ GeV, $M_{D^{*0}} = M_{D^{*+}} = 2.007$ GeV, $M_{D^0} = M_{D^+} = 1.865$ GeV, and $M_{J/\psi} = 3.097$ GeV [34]. Without an adjustable parameter, we numerically solve the eigenvalue equation (38) and obtain the masses and wave functions of pure bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ with spin-parity quantum numbers 0^+. Considering the cross terms between these two pure bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$ and using the coupled-channel approach, we obtain the mass M_0 of the mixed state with 0^+. Then M_0 and GBS wave function $\chi^{D^*\bar{D}^*,0^+}(P,p,k,k')$ given in Eq. (48) are used to evaluate the matrix elements $T_{(c_1';b)a}(M_0)$ and $T_{(c_2';b)a}(M_0)$ with the mass of the meson-meson bound state, and the decay widths $\Gamma_1(M_0)$ and $\Gamma_2(M_0)$ with the mass of the meson-meson bound state should not be the width of physical resonance. From Eqs. (56), (57), and (85), we calculate the T-matrix element $T_{(c_1';b)a}(\epsilon')$ with respect to ϵ' for channel $J/\psi\omega$. From Eqs. (66), (67), (70), (71), and (89), we calculate the T-matrix element $T_{(c_2';b)a}(\epsilon')$ with respect to ϵ' for channel D^+D^-. From Eqs. (93), (94), (96), (97) and (98), we calculate the T-matrix element $T_{(c_4';b)a}(\epsilon')$ with respect to ϵ' for closed channel $D^*\bar{D}^*$. From Eqs. (87), (90), and (99), we calculate the functions $\mathbb{I}_1(\epsilon')$ over $\epsilon_{c_1',M} < \epsilon' < \infty$, $\mathbb{I}_2(\epsilon')$ over $\epsilon_{c_2',M} < \epsilon' < \infty$, $\mathbb{I}_3(\epsilon')$ over $\epsilon_{c_3',M} < \epsilon' < \infty$, and $\mathbb{I}_4(\epsilon')$ over $\epsilon_{c_4',M} < \epsilon' < \infty$, respectively. By doing the numerical calculation, we obtain the mass correction $\Delta M = (2\pi)^3\mathbb{D}(M_0)$ and the physical mass M for resonance $\chi_{c0}(3915)$. Finally, the physical mass is used to recalculate these strong decay widths $\Gamma_1(\chi_{c0}(3915) \to J/\psi\omega)$, $\Gamma_2(\chi_{c0}(3915) \to D^+D^-)$, and $\Gamma_3(\chi_{c0}(3915) \to D^0\bar{D}^0)$. Some errors in Ref. [10] have been revised. M and Γ should be the observed mass and full width in experiments. Our numerical results for resonance $\chi_{c0}(3915)$ are in good agreement with the experimental data, which are presented in Table I.

It is necessary to emphasize that there is not an adjustable parameter in our approach. We require the meson-quark coupling constants g and the parameters ω_H in BS amplitudes of heavy mesons to calculate the mass and decay width of physical resonance. The meson-quark coupling constants can be determined by QCD sum rules approach [21], and these parameters in BS amplitudes of

TABLE I. Mass M and full width Γ for physical resonance $\chi_{c0}(3915)$. M_0 is the mass of mixed state of two bound states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, ΔM is the calculated correction due to all open and closed channels, and Γ_i is the calculated width of ith decay channel. (Dimensioned quantities in MeV).

Quantity	M_0	ΔM	M	Γ_1	Γ_2	Γ_3	Γ
This work	3952.7	−30.7	3922.0	21.8	1.5	1.5	24.8
PDG [34]			3921.7 ± 1.8				18.8 ± 3.5

heavy mesons are fixed by providing fits to observables [26,27,35]. Our approach also involves the constituent quark masses m_u, m_d, and the heavy quark mass m_c. According to the spontaneous breaking of chiral symmetry, the light quarks (u, d, s) obtain their constituent masses because the vacuum condensate is not equal to zero, and the heavy quark mass m_c is irrelevant to vacuum condensate. Normally, the value slightly greater than a third of nucleon mass is employed as the constituent mass of light quark. The value of heavy-quark mass m_c can be determined by the experimental mass of charmonium system J/ψ. Of course, the values of these parameters, including g, ω_H, m_u, m_d, and m_c, are values in respective ranges. Simultaneously varying these parameters in respective ranges, we find that the uncertainties of numerical results are at most 5%. Despite the large uncertainty of meson mass M_σ, it has been found that the uncertainties of numerical results from meson mass M_σ are also very small in our previous works [5,8,9,13] and Refs. [6,36]. Therefore, in our approach the calculated mass and decay width are uniquely determined.

Up to now, a theoretical approach from QCD to investigate resonance which is regarded as an unstable two-body system has been established. In this paper, we only explore exotic meson resonance which is considered as an unstable molecular state composed of two heavy vector mesons. The extension of our approach to more general resonances is straightforward, while the interaction Lagrangian may be modified. More importantly, it is most reasonable and fascinating to investigate resonance as far as possible from QCD. In the framework of quantum field theory, the nonperturbative contribution from the vacuum condensates can be introduced into the BS wave function [13] and the two-particle irreducible Green's function, and then the calculated mass and decay width of resonance will contain more inspiration of QCD.

VIII. CONCLUSION

Exotic resonance $\chi_{c0}(3915)$ is considered as a mixed state of two unstable molecular states $D^{*0}\bar{D}^{*0}$ and $D^{*+}D^{*-}$, and we investigate the time evolution of the meson-meson molecular state as determined by the total Hamiltonian. According to the developed Bethe-Salpeter theory, the total matrix elements for all decay channels should be calculated with respect to arbitrary value of the final state energy. Because the total energy of the final state extends from ϵ_M to $+\infty$, we consider three open decay channels $J/\psi\omega$, D^+D^-, $D^0\bar{D}^0$ and one closed channel $D^*\bar{D}^*$ from the effective interaction Lagrangian at low-energy QCD, which are exhibited by extended Feynman diagrams. Using the developed Bethe-Salpeter theory, we calculate the mass M and full width Γ of physical resonance $\chi_{c0}(3915)$, which are in good agreement with the experimental data. Obviously, our work can be extended to more general resonances.

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APPENDIX: TENSOR STRUCTURES IN THE GENERAL FORM OF BS WAVE FUNCTIONS

The tensor structures in Eqs. (8) and (9) are given below [5,9]

$$
\begin{aligned}
\mathcal{T}^1_{\lambda\tau} &= (p^2+\eta_1 P\cdot p-\eta_2 P\cdot p-\eta_1\eta_2 P^2)g_{\lambda\tau}-(p_\lambda p_\tau+\eta_1 P_\tau p_\lambda-\eta_2 P_\lambda p_\tau-\eta_1\eta_2 P_\lambda P_\tau),\\
\mathcal{T}^2_{\lambda\tau} &= (p^2+2\eta_1 P\cdot p+\eta_1^2 P^2)(p^2-2\eta_2 P\cdot p+\eta_2^2 P^2)g_{\lambda\tau}\\
&\quad+(p^2+\eta_1 P\cdot p-\eta_2 P\cdot p-\eta_1\eta_2 P^2)(p_\lambda p_\tau+\eta_1 P_\lambda p_\tau-\eta_2 P_\tau p_\lambda-\eta_1\eta_2 P_\lambda P_\tau)\\
&\quad-(p^2-2\eta_2 P\cdot p+\eta_2^2 P^2)(p_\lambda p_\tau+\eta_1 P_\lambda p_\tau+\eta_1 P_\tau p_\lambda+\eta_1^2 P_\lambda P_\tau)\\
&\quad-(p^2+2\eta_1 P\cdot p+\eta_1^2 P^2)(p_\lambda p_\tau-\eta_2 P_\lambda p_\tau-\eta_2 P_\tau p_\lambda+\eta_2^2 P_\lambda P_\tau),\\
\mathcal{T}^3_{\mu_1\cdots\mu_j\lambda\tau} &= \frac{1}{j!}p_{\{\mu_2}\cdots p_{\mu_j}g_{\mu_1\}\lambda}(p^2+2\eta_1 P\cdot p+\eta_1^2 P^2)[(p^2-2\eta_2 P\cdot p+\eta_2^2 P^2)(p+\eta_1 P)_\tau\\
&\quad-(p^2+\eta_1 P\cdot p-\eta_2 P\cdot p-\eta_1\eta_2 P^2)(p-\eta_2 P)_\tau]\\
&\quad-p_{\mu_1}\cdots p_{\mu_j}[(p^2-2\eta_2 P\cdot p+\eta_2^2 P^2)(p_\lambda p_\tau+\eta_1 P_\lambda p_\tau+\eta_1 P_\tau p_\lambda+\eta_1^2 P_\lambda P_\tau)\\
&\quad-(p^2+\eta_1 P\cdot p-\eta_2 P\cdot p-\eta_1\eta_2 P^2)(p_\lambda p_\tau+\eta_1 P_\lambda p_\tau-\eta_2 P_\tau p_\lambda-\eta_1\eta_2 P_\lambda P_\tau)],
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}^4_{\mu_1\cdots\mu_j\lambda\tau} ={}& \frac{1}{j!} p_{\{\mu_2}\cdots p_{\mu_j} g_{\mu_1\}\tau}(p^2 - 2\eta_2 P\cdot p + \eta_2^2 P^2)[(p^2 + \eta_1 P\cdot p \\
&- \eta_2 P\cdot p - \eta_1\eta_2 P^2)(p+\eta_1 P)_\lambda - (p^2 + 2\eta_1 P\cdot p + \eta_1^2 P^2)(p-\eta_2 P)_\lambda] \\
&+ p_{\mu_1}\cdots p_{\mu_j}[(p^2 + 2\eta_1 P\cdot p + \eta_1^2 P^2)(p_\lambda p_\tau - \eta_2 P_\lambda p_\tau - \eta_2 P_\tau p_\lambda + \eta_2^2 P_\lambda P_\tau) \\
&- (p^2 + \eta_1 P\cdot p - \eta_2 P\cdot p - \eta_1\eta_2 P^2)(p_\lambda p_\tau + \eta_1 P_\lambda p_\tau - \eta_2 P_\tau p_\lambda - \eta_1\eta_2 P_\lambda P_\tau)], \\
\mathcal{T}^5_{\mu_1\cdots\mu_j\lambda\tau} ={}& \frac{1}{j!}(p^2 + 2\eta_1 P\cdot p + \eta_1^2 P^2)(p^2 - 2\eta_2 P\cdot p + \eta_2^2 P^2) p_{\{\mu_3}\cdots p_{\mu_j} g_{\mu_1\lambda} g_{\mu_2\}\tau} \\
&- \frac{1}{j!} p_{\{\mu_2}\cdots p_{\mu_j} g_{\mu_1\}\tau}(p^2 - 2\eta_2 P\cdot p + \eta_2^2 P^2)(p+\eta_1 P)_\lambda \\
&- \frac{1}{j!} p_{\{\mu_2}\cdots p_{\mu_j} g_{\mu_1\}\lambda}(p^2 + 2\eta_1 P\cdot p + \eta_1^2 P^2)(p-\eta_2 P)_\tau \\
&+ p_{\mu_1}\cdots p_{\mu_j}(p_\lambda p_\tau + \eta_1 P_\lambda p_\tau - \eta_2 P_\tau p_\lambda - \eta_1\eta_2 P_\lambda P_\tau), \\
\mathcal{T}^6_{\mu_1\cdots\mu_j\lambda\tau} ={}& p_{\{\mu_3}\cdots p_{\mu_j}\epsilon_{\mu_1\lambda\xi\zeta} p_\xi P_\zeta \epsilon_{\mu_2\}\tau\xi'\zeta'} p_{\xi'} P_{\zeta'}, \\
\mathcal{T}^7_{\mu_1\cdots\mu_j\lambda\tau} ={}& -(2p^2 + \eta_1 P\cdot p - \eta_2 P\cdot p) p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi} p_\xi \\
&+ (2\eta_1\eta_2 P\cdot p + \eta_2 p^2 - \eta_1 p^2) p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi} P_\xi \\
&+ p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta} p_\xi P_\zeta p_\tau + p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta} p_\xi P_\zeta p_\lambda, \\
\mathcal{T}^8_{\mu_1\cdots\mu_j\lambda\tau} ={}& -(P\cdot p) p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi} p_\xi + p^2 p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi} P_\xi \\
&- p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta} p_\xi P_\zeta p_\tau + p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta} p_\xi P_\zeta p_\lambda, \\
\mathcal{T}^9_{\mu_1\cdots\mu_j\lambda\tau} ={}& -(2P\cdot p + \eta_1 P^2 - \eta_2 P^2) p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi} p_\xi \\
&+ P\cdot(\eta_2 p - \eta_1 P + 2\eta_1\eta_2 P) p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi} P_\xi \\
&+ p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta} p_\xi P_\zeta P_\tau + p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta} p_\xi P_\zeta P_\lambda, \\
\mathcal{T}^{10}_{\mu_1\cdots\mu_j\lambda\tau} ={}& -P^2 p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi} p_\xi + (P\cdot p) p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\tau\xi} P_\xi \\
&- p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\lambda\xi\zeta} p_\xi P_\zeta P_\tau + p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta} p_\xi P_\zeta P_\lambda, \\
\mathcal{T}^{11}_{\mu_1\cdots\mu_j\lambda\tau} ={}& (p^2 + \eta_1 P\cdot p - \eta_2 P\cdot p - \eta_1\eta_2 P^2) p_{\{\mu_3}\cdots p_{\mu_j} g_{\mu_1\lambda}\epsilon_{\mu_2\}\tau\xi\zeta} p_\xi P_\zeta \\
&- p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}\tau\xi\zeta} p_\xi P_\zeta (p-\eta_2 P)_\lambda, \\
\mathcal{T}^{12}_{\mu_1\cdots\mu_j\lambda\tau} ={}& (p^2 + \eta_1 P\cdot p - \eta_2 P\cdot p - \eta_1\eta_2 P^2) p_{\{\mu_3}\cdots p_{\mu_j} g_{\mu_1\tau}\epsilon_{\mu_2\}\lambda\xi\zeta} p_\xi P_\zeta \\
&- p_{\{\mu_2}\cdots p_{\mu_j}\epsilon_{\mu_1\}_\lambda\xi\zeta} p_\xi P_\zeta (p+\eta_1 P)_\tau.
\end{aligned}
$$

[1] E. S. Swanson, Phys. Lett. B **588**, 189 (2004).
[2] N. A. Törnqvist, Phys. Lett. B **590**, 209 (2004).
[3] X. Liu and S. L. Zhu, Phys. Rev. D **80**, 017502 (2009).
[4] C. Hidalgo-Duque, J. Nieves, and M. P. Valderrama, Phys. Rev. D **87**, 076006 (2013).
[5] X. Chen and X. Lü, Eur. Phys. J. C **75**, 98 (2015).
[6] M.-J. Zhao, Z.-Y. Wang, C. Wang, and X.-H. Guo, Phys. Rev. D **105**, 096016 (2022).
[7] T. Branz, T. Gutsche, and V. E. Lyubovitskij, Phys. Rev. D **80**, 054019 (2009).
[8] X. Chen and X. Lü, Phys. Rev. D **97**, 114005 (2018).
[9] X. Chen, X. Lü, R. Shi, X. Guo, and Q. Wang, Phys. Rev. D **101**, 014009 (2020).
[10] X. Chen and X. Lü, Eur. Phys. J. C **83**, 499 (2023).
[11] A. Esposito, A. Pilloni, and A. D. Polosa, Phys. Rep. **668**, 1 (2017).
[12] X. Chen, R. Liu, R. Shi, and X. Lü, Phys. Rev. D **87**, 065013 (2013).
[13] X. Chen, X. Lü, R. Shi, and X. Guo, Nucl. Phys. **B909**, 243 (2016).

[14] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, England, 1996), Vol. 2.
[15] S.-K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **94**, 182002 (2005).
[16] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **101**, 082001 (2008).
[17] S. Uehara *et al.* (Belle Collaboration), Phys. Rev. Lett. **104**, 092001 (2010).
[18] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. D **86**, 072002 (2012).
[19] D. Lurié, *Particles and Fields* (Interscience Publishers, New York, 1968).
[20] F. Ambrosino *et al.* (KLOE Collaboration), J. High Energy Phys. 07 (2009) 105.
[21] L. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).
[22] X. Lü, Y. Liu, and E. Zhao, Chin. Phys. Lett. **13**, 652 (1996).
[23] X. Lü, Y. Liu, and E. Zhao, Sci. China (Series A) **27**, 361 (1997).
[24] C. J. Burden, L. Qian, C. D. Roberts, P. C. Tandy, and M. J. Thomson, Phys. Rev. C **55**, 2649 (1997).
[25] P. Maris, C. D. Roberts, and P. C. Tandy, Phys. Lett. B **420**, 267 (1998).
[26] M. A. Ivanov, Y. L. Kalinovsky, and C. D. Roberts, Phys. Rev. D **60**, 034018 (1999).
[27] M. A. Ivanov, J. G. Körner, S. G. Kovalenko, and C. D. Roberts, Phys. Rev. D **76**, 034018 (2007).
[28] M. Neubert, Phys. Rep. **245**, 259 (1994).
[29] X. Chen, B. Wang, X. Li, X. Zeng, S. Yu, and X. Lü, Phys. Rev. D **79**, 114006 (2009).
[30] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Lett. B **634**, 214 (2006).
[31] A. Vinokurova *et al.* (Belle Collaboration), J. High Energy Phys. 06 (2015) 132.
[32] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. D **102**, 112003 (2020).
[33] M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).
[34] R. L. Workman *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[35] T. Kawanai and S. Sasaki, Phys. Rev. Lett. **107**, 091601 (2011).
[36] G.-J. Ding, Phys. Rev. D **79**, 014001 (2009).