Heavy- and light-flavor symmetry partners of the $T_{cc}^+(3875)$, the $X(3872)$, and the $X(3960)$ from light-meson exchange saturation

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The spectrum of the charmed meson-(anti)meson system is a fundamental tool for disentangling the nature of a few exotic hadrons, including the recently discovered $T_{cc}^+(3875)$ tetraquark, the $X(3960)$, or the $X(3872)$, the nature of which is still not clear after almost two decades of its discovery. Here we consider that the charmed meson-(anti)meson short-range interaction is described by the exchange of light mesons (σ, ρ, ω). The effects of light-meson exchanges are recast into a simple contact-range theory by means of a saturation procedure, resulting in a compact description of the two-hadron interaction. From this, if the T_{cc}^+ were to be an isoscalar D^*D molecule, then there should exist an isoscalar $J = 1$ D^*D^* partner, as constrained by heavy-quark spin symmetry. Yet, within our model, the most attractive two charmed meson configurations are the isovector $J = 0$ D^*D^* molecule and its sextet $D_s^*D^*$ and $D_s^*D_s^*$ flavor partners. Finally, we find a tension between the molecular descriptions of the T_{cc}^+ and that of the $X(3872)$ and $X(3960)$, where most parameter choices suggest that if the T_{cc}^+ is purely molecular then the $X(3872)$ overbinds [or conversely, if the $X(3872)$ is a molecule the T_{cc}^+ does not bind]. This might be consequential for determining the nature of these states.

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I. INTRODUCTION

The LHCb has recently observed a state within the $D_s^+D_s^-$ invariant mass distribution of the $B^+ \to D_s^+D_s^-K^+$ decay [1]. With a mass and width of

$$M = 3956 \pm 5 \pm 10 \text{ MeV},$$
$$\Gamma = 43 \pm 13 \pm 8 \text{ MeV}, \quad (1)$$

it has been named the $X(3960)$. Its preferred quantum numbers are $J^{PC} = 0^{++}$ and it might be the same resonance as the previous $X(3930)$ state observed in the D^+D^- invariant mass distribution [2], which in turn could also happen to be the same state as the $\chi_0(3915)$ listed in the Review of Particle Physics (RPP) [3].[1] The $X(3960)$ can be interpreted as a charmed meson-antimeson state [6–10], from which it is natural to consider whether it could be related to the well-known $X(3872)$, with $J^{PC} = 1^{++}$ and whose mass and width are [3]

$$M = 3871.65 \pm 0.06 \text{ MeV},$$
$$\Gamma = 1.19 \pm 0.21 \text{ MeV}, \quad (2)$$

which has been conjectured to be a $D^*\bar{D}$ bound state [11–13], though no consensus exists yet on whether they are molecular or not [14–22].

If molecular, the previous $X(3872)$ and $X(3960)$ states would be, in principle, related to the doubly charmed tetraquark—the $T_{cc}^+(3875)$—discovered in 2021 by the LHCb Collaboration [23,24] and suspected to be a two charmed meson bound state [25–31], though predictions of the T_{cc}^+ as a compact tetraquark predate its observation by decades [32–35]. The T_{cc}^+ is extremely close to the $D^{0*}D^+$ threshold, where the mass difference $\delta m = m(T_{cc}^+) - m(D) - m(D^*)$ is

$$\delta m_{\text{BW}} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV}, \quad (3)$$

if a standard Breit-Wigner (BW) shape is used to fit the T_{cc}^+ [23], or

$$\delta m_{\text{U}} = -360 \pm 40^{+4}_{-0} \text{ keV}, \quad (4)$$

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[1]Or it might not: a recent study [4] suggests that the $X(3930)/\chi_0(3915)$ is a $J^{PC} = 2^{++}$ state and thus not identical to the $X(3960)$, while Ref. [5] proposes a possible method to differentiate whether the $X(3930)$ and $X(3960)$ are the same or not.

if it is fitted with a unitarized Breit-Wigner shape [24], where this later determination is probably more suitable for a state close to a threshold. This tetraquark is also relatively narrow, with a width of

$$\Gamma_{\rm BW} = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}, \tag{5}$$

$$\Gamma_{\rm U} = 48 \pm 2^{+0}_{-12}\ \text{keV}, \tag{6}$$

depending on the resonance profile used (where the unitarized BW shape is usually considered the most reliable of the two).

Here we will consider the $X(3872)$, $X(3960)$, and T_{cc}^{+} from the point of view of molecular spectroscopy and within the particular phenomenological model we proposed in [36]. The questions we would like to address are the following: Is it sensible to believe they are bound states? Can the T_{cc}^{+} be described with the same parameters as the $X(3872)$ and $X(3960)$? What are their partner states?

To answer these questions, we will use a simple saturation model in which the effect of the light-meson exchange forces is encapsulated in the coupling constant of a contact-range theory, modulo a proportionality constant that can be determined from a "reference state," i.e., an observed state whose nature we will assume to be a specific type of two-body bound state. The idea of a reference state can, in turn, be used to relate different molecular candidates. This is particularly relevant to the connection between the T_{cc}^{+} and the $X(3872/3960)$: if these two states can be described with the same set of parameters within a given model, this will give credence to the idea that both of them conform to the assumptions of said model.

However, as we will see, this is not the case in our model, where there is a tension between a pure molecular explanation of the T_{cc}^{+} as a two charmed meson bound state and the $X(3872/3960)$ as charmed meson-antimeson systems. That is, while the $X(3872)$ and $X(3960)$ can be easily explained as $D^{*}\bar{D}$ and $D_s\bar{D}_s$ bound or virtual states with the same parameters, this will lead to insufficient attraction in the $D^{*}D$ system as to guarantee binding (though there will be remarkable attraction nonetheless). This result would be consistent with the picture of Janc and Rosina [37], who conjectured that the binding of a tetraquark below the $D^{*}D$ threshold requires both the molecular and quark components.

II. SATURATION OF THE CONTACT-RANGE COUPLINGS

We will consider a generic two-hadron system H_1H_2, which we will describe by means of an S-wave contact-range theory containing a central and spin-spin term,

$$V_C = C = C_0 + C_1 \hat{\vec{S}}_{L1} \cdot \hat{\vec{S}}_{L2}, \tag{7}$$

where V_C is the contact-range potential in p-space, C is its strength (or coupling), which can be subdivided into C_0 and C_1, the central and spin-spin coupling constants, and $\hat{\vec{S}}_{Li} = \vec{S}_{Li}/|S_{Li}|$ is the *reduced* light-spin operator for hadron $i = 1, 2$ (where $\vec{S}_{Li}$ is the light-spin operator and S_{Li} the total light-spin of hadron $i = 1, 2$). For S-wave charmed mesons, $\hat{\vec{S}}_{Li} = \vec{\sigma}_{Li}$, with $\vec{\sigma}$ representing the Pauli matrices.

Here it is interesting to notice that the dependence on the light spin (i.e., the spin of the light $q = u, d, s$ quarks within hadrons H_1 and H_2) is a consequence of heavy-quark spin symmetry (HQSS), which implies that any dependence on the spin of heavy quarks will be suppressed by a factor of $\Lambda_{\rm QCD}/m_Q$, with $\Lambda_{\rm QCD} \approx 200$ MeV. In principle, there could be higher-spin operators (quadrupolar, octupolar, etc.), but in practice, higher order terms will be increasingly suppressed. In addition, for S-wave charmed mesons, we have $S_L = 1/2$, which precludes the appearance of the aforementioned high-spin operators.

The contact-range approximation assumes that the binding momentum of the two hadrons is not high enough to disentangle the details of the interaction binding them. That is, we do not have to consider the full two-hadron potential and all its details, but only its coarse-grained properties. We will do this by assuming that the two-hadron potential is derived from the exchange of light mesons (σ, ρ, ω), which will then saturate the coupling constants. Finally, pions are long ranged and consequently do not saturate the contact-range couplings of our model. They have also been shown to be perturbative for systems composed of two charmed mesons [38,39], a point that we have also explicitly checked with concrete calculations in our saturation model [36]. Thus, we will simply ignore their contribution, as it can be safely neglected.

For the saturation of the couplings, we will consider the exchange of the scalar and vector mesons separately, as these mesons have different masses. For the scalar meson (the σ), the potential takes the form

$$V_S(\vec{q}) = -\frac{g_{S1} g_{S2}}{m_S^2 + \vec{q}^{\,2}}, \tag{8}$$

where $\vec{q}$ is the exchanged momentum, m_S is the scalar meson mass, and g_{Si} with $i = 1, 2$ is the scalar coupling for hadrons H_1 and H_2. This leads to the following contribution to the couplings:

$$C_0^S(\Lambda \sim m_S) \propto -\frac{g_{S1} g_{S2}}{m_S^2}, \tag{9}$$

$$C_1^S(\Lambda \sim m_S) \propto 0, \tag{10}$$

where we remind the reader that saturation works best for a regularization scale of the same order of magnitude as the mass of the exchanged particle (hence, $\Lambda \sim m_S$) and that we

only expect the contact-range couplings to be proportional to the potential at zero momentum, modulo an unknown proportionality constant. For the vector meson, the potential is

$$V_V(\vec{q}) = \frac{g_{V1}g_{V2}}{m_V^2+\vec{q}^2} + \frac{f_{V1}f_{V2}}{6M^2}\frac{m_V^2}{m_V^2+\vec{q}^2}\hat{\vec{S}}_{L1}\cdot\hat{\vec{S}}_{L2} + \ldots, \quad (11)$$

where we have momentarily ignored isospin, SU(3)-flavor, and G-parity factors for obtaining a more compact expression; m_V is the vector meson mass, g_{Vi} is the electric-type (E0) coupling, f_{Vi} is the magnetic-type (M1) coupling, and M is a scaling mass for the magnetic-type term, for which we will choose the nucleon mass $M = m_N = 938.9$ MeV. The dots represent S-to-D wave terms and Dirac-δ contributions that do not saturate our contact-range potential [40]. The previous potential leads to the contributions

$$C_0^V(\Lambda \sim m_V) \propto \frac{g_{V1}g_{V2}}{m_V^2}(\zeta + T_{12}), \quad (12)$$

$$C_1^V(\Lambda \sim m_V) \propto \frac{f_{V1}f_{V2}}{6M^2}(\zeta + T_{12}), \quad (13)$$

where we have now included the isospin and G-parity factors for two charmed meson systems without strangeness, with $\zeta = +1$ (-1) for the meson-meson (meson-antimeson) case and $T_{12} = \vec{\tau}_1\cdot\vec{\tau}_2$ as the isospin factor. The extension to systems with strangeness is straightforward, as it only requires one to consider the appropriate SU(3)-flavor Clebsch-Gordan coefficients.

Next, we have to combine the contributions from scalar and vector meson exchanges, where there is the problem that the respective renormalization scales at which saturation is optimal is different for the scalar and vector mesons. For this we will use a renormalization group equation (RGE) to determine how the couplings evolve when the regularization scale Λ changes. For nonrelativistic two-body theories, the RGE of a contact-range coupling $C(\Lambda)$ takes the form [41]

$$\frac{d}{d\Lambda}\langle\Psi|V_C(\Lambda)|\Psi\rangle = 0, \quad (14)$$

with $V_C(\Lambda)$ as the regularized contact-range potential, Λ as the regularization scale, and Ψ as the two-body wave function. If the wave function displays a power-law behavior of the type $\Psi(r) \propto r^{\alpha/2}$ at distances $r \sim 1/\Lambda$, the RGE will simplify to [41]

$$\frac{d}{d\Lambda}\left[\frac{C(\Lambda)}{\Lambda^\alpha}\right] = 0, \quad (15)$$

or, equivalently,

$$\frac{C(\Lambda_1)}{\Lambda_1^\alpha} = \frac{C(\Lambda_2)}{\Lambda_2^\alpha}, \quad (16)$$

with Λ_1 and Λ_2 as two values of the regularization scale. For the particular case at hand, i.e., the saturated couplings at the scalar and vector mass scales, the previous RGE leads to the following combination:

$$C(m_V) = C_V(m_V) + \left(\frac{m_V}{m_S}\right)^\alpha C_S(m_S). \quad (17)$$

For determining the exponent α we are still required to make assumptions about the power-law properties of the wave function at distances $m_V r \sim 1$, i.e., $\Psi(r) \sim r^{\alpha/2}$. From the semiclassical approximation and the Langer correction [42], we end up with $\alpha = 1$.

Putting all the pieces together, the saturated contact-range coupling is determined modulo an unknown proportionality constant and takes the following form:

$$\begin{aligned} C^{\rm sat}(\Lambda = m_V) &= C_0^{\rm sat} + C_1^{\rm sat}\hat{\vec{S}}_{L1}\cdot\hat{\vec{S}}_{L2} \\ &\propto \frac{g_{V1}g_{V2}}{m_V^2}\left[\zeta + \hat{T}_{12}\right]\left(1 + \kappa_{V1}\kappa_{V2}\frac{m_V^2}{6M^2}\hat{C}_{L12}\right) \\ &\quad - \left(\frac{m_V}{m_S}\right)^\alpha \frac{g_{S1}g_{S2}}{m_S^2}, \end{aligned} \quad (18)$$

where $\hat{T}_{12} = \hat{\vec{I}}_1\cdot\hat{\vec{I}}_2$, $\hat{C}_{L12} = \hat{\vec{S}}_{L1}\cdot\hat{\vec{S}}_{L2}$, and $\kappa_{Vi} = f_{Vi}/g_{Vi}$.

III. CALIBRATION

In this section, we calibrate the RG-saturation model. This requires us to specify the regularization of the contact-range potential and the masses and couplings of the light mesons. Yet, the most important factor to make concrete predictions is the determination of the proportionality constant for the saturated couplings. For this we have to choose a reference state—a plausible molecular candidate—from which to derive all other predictions. We will consider a few candidates—the $X(3872)$, $X(3960)$, $T_{cc}^+(3875)$, and the $D\bar{D}$ state calculated in the lattice [43]—and argue that the $X(3872)$ as a 1^{++} $D^*\bar{D}$ bound state is the most suitable choice from which to derive the molecular spectrum of the two charmed meson systems.

A. Calculating the spectrum

For the regularization, we use a regulator function $f(x)$,

$$V_C = C^{\rm sat} f\left(\frac{p'}{\Lambda}\right) f\left(\frac{p}{\Lambda}\right), \quad (19)$$

for which we will choose a Gaussian, $f(x) = e^{-x^2}$, and where Λ is the cutoff, which we take to be $\Lambda = 1$ GeV as in [36].

This potential will be plugged into the Lippmann-Schwinger equation for a bound state,

$$\phi(p) + 2\mu \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{\langle p|V_C|q\rangle}{M_{\rm p} - \frac{q^2}{2\mu} - M_{\rm th}} \phi(q) = 0, \quad (20)$$

where $\phi(p)$ refers to the vertex function, which is related to the wave function $\Psi(p)$ by the relation $(M_{\rm p} - \frac{p^2}{2\mu} - M_{\rm th})\Psi(p) = \phi(p)$, and μ is the reduced mass of the system, while $M_{\rm th}, M_{\rm p}$ are the mass of the threshold and of the pole of the two-body scattering amplitude (i.e., the mass of the bound state), respectively. In the case at hand, this equation further simplifies to

$$1 + \frac{\mu}{4\pi^2} C^{\rm sat}(\Lambda) I(\gamma_2, \Lambda) = 0, \quad (21)$$

where $\gamma_2 = \sqrt{2\mu(M_{\rm th} - M_{\rm p})}$ is the wave number of the bound state and with $I(\gamma_2, \Lambda)$ given by

$$I(\gamma_2, \Lambda) = \sqrt{2\pi}\Lambda - 2e^{2\gamma_2^2/\Lambda^2} \pi\gamma_2 \, {\rm erfc}\left(\frac{\sqrt{2}\gamma_2}{\Lambda}\right), \quad (22)$$

where ${\rm erfc}(x)$ is the complementary error function.

For calibrating the $C^{\rm sat}(\Lambda)$ coupling and its unknown proportionality constant in Eq. (18) we will use a reference state. In the doubly charmed sector, the only candidate state is the T_{cc}^+ tetraquark, while in the hidden-charm sector, in addition to the experimentally observed $X(3872)$, we may also include the recent $X(3960)$ (for which we will use the location of its pole in the scattering amplitude as determined, for instance, in [6]) or the $D\bar{D}$ and $D_s\bar{D}_s$ poles found in the lattice [43]. As the binding energy of the reference state is known, we just obtain $C_{\rm ref}^{\rm sat}$ from the bound state equation

$$1 + \frac{\mu_{\rm ref}}{4\pi^2} C_{\rm ref}^{\rm sat} I(\gamma_2, \Lambda) = 0. \quad (23)$$

After this, if we want to predict the mass of a given molecule, we calculate the ratio

$$R_{\rm mol} = \frac{\mu_{\rm mol} C_{\rm mol}^{\rm sat}}{\mu_{\rm ref} C_{\rm ref}^{\rm sat}}, \quad (24)$$

which is independent of the proportionality constant, and then from $R_{\rm mol}$ we simply solve the bound state equation

$$1 + R_{\rm mol}\left(\frac{\mu_{\rm ref}}{4\pi^2} C_{\rm ref}^{\rm sat}\right) I(\gamma_2, \Lambda) = 0, \quad (25)$$

to obtain the binding energies.

For the couplings of the light-meson with the charmed mesons, we will resort to a series of well-known phenomenological relations. For the vector mesons (ρ, ω, K^*, and ϕ) we have simply made use of the mixing of these mesons with the electromagnetic current (vector meson dominance [44–46]) as a way to determine the g_V and κ_V (E0 and M1) couplings: we can match g_V and κ_V to the charge and magnetic moment of the particular hadron in which we are interested. This results in $g_V = 2.9$ and $\kappa_V = 2.85$, as explained in [36]. For the scalar meson, the linear σ model [47] predicts $g_{SNN} = \sqrt{2} m_N / f_\pi \simeq 10.2$ for the nucleon, where m_N is the nucleon mass and $f_\pi \simeq 132$ MeV is the pion weak decay constant. For the charmed meson, which contains one light quark instead of three, we assume the quark model relation $g_S = g_{Sqq} = g_{SNN}/3 = 3.4$, i.e., that the coupling of the σ is proportional to the number of light quarks within the hadron.

In the strange sector, the contributions coming from the exchange of the K^* and ϕ vector mesons have a shorter range than the ones from the ρ and ω mesons. In analogy to Eq. (17), their contribution will be slightly suppressed in comparison to the nonstrange vector mesons. If we take the $D_s^{(*)}D_s^{(*)}$ and $D_s^{(*)}\bar{D}_s^{(*)}$ systems as an illustrative example, the saturated coupling will be

$$C_{\rm mol}^{\rm sat} \propto \left(\frac{m_V}{m_\phi}\right)^\alpha \frac{g_V^2}{m_\phi^2}[2\zeta]\left(1 + \kappa_V^2 \frac{m_\phi^2}{6M^2}\hat{C}_{L12}\right) - \left(\frac{m_V}{m_S}\right)^\alpha \frac{g_S^2}{m_S^2}, \quad (26)$$

where we follow the conventions of Eq. (17), except for the couplings, for which we use the values mentioned in the previous paragraph. As can be appreciated, the ϕ vector meson contribution is suppressed by a $(m_V/m_\phi)^\alpha$ factor with respect to the ρ and ω contributions that we use as our baseline. For the masses of the vector mesons, we use $m_V = (m_\rho + m_\omega)/2 = 775$, $m_{K^*} = 890$, and $m_\phi = 1020$ MeV. The mass of the scalar meson is taken to be $m_S = 475$ MeV, i.e., the middle value of the (400–550) MeV range listed in the RPP [3].

We also assume that the coupling of the scalar to the s quark is approximately the same as to the u and d quarks: $g_S = g_{Suu} = g_{Sdd} = g_{Sss}$. This assumption works well when comparing the $D\bar{D}$ and $D_s\bar{D}_s$ systems predicted in the lattice [43]: a good prediction of the $D_s\bar{D}_s$ from the $D\bar{D}$ (or vice versa) requires the coupling of the σ to be similar to the strange and nonstrange quarks. Indeed, if we use the location of the $D\bar{D}$ bound state as input, we predict the $D_s\bar{D}_s$ state to be located at

$$M_p - 2m(D_s) = (-1.0)^V [0.2 - 0.5i] \text{ MeV}, \quad (27)$$

where the first (parentheses) and second (brackets) value correspond to a single ($D_s\bar{D}_s$) and coupled ($D\bar{D}$–$D_s\bar{D}_s$ – $D^*\bar{D}^*$) channel calculation. Notice that while in the single channel case we obtain a virtual state close to the $D_s\bar{D}_s$ threshold (which we indicate with the V superscript),

in the coupled channel calculation the pole is located in the (I,II) Riemann sheet of channels $D\bar{D}$ and $D_s\bar{D}_s$, respectively. This is to be compared with $M_p - 2m(D_s) = -0.2^{+0.16}_{-4.9} - 0.27^{+2.5}_{-0.15}\frac{i}{2}$ MeV in [43], which is located in sheets (II,I) for 70% of the bootstrap samples or in sheets (I,II) for the rest. We note that this choice is also necessary for reproducing the $Z_{cs}(3985)$ as a $D^*\bar{D}_s - D\bar{D}_s^*$ molecule, as we previously discussed in Ref. [48].

It is also interesting to notice that the calculation above is not only relevant for the choice of the scalar coupling with the s quark, but also for the relative importance of coupled channel effects. Indeed, the difference in the position of the $D_s\bar{D}_s$ pole is merely on the order of 1 MeV for the single and coupled channel cases, which suggest a minor role for the coupled channel dynamics. This conclusion agrees with our previous numerical results in [36]. Additionally, the $\mathcal{O}(1\text{ MeV})$ estimation for the coupled channel effects happen to be smaller than the uncertainties of our model, which we explain in the next few lines. For the previous reasons we will not further consider coupled channel effects when calculating the molecular spectrum.

B. Error estimations

For calculating the expected errors of the model, we will consider two sources of uncertainty. The first is the light-meson exchange potential itself, for which the parameters are not that well known. In principle, this would include a lot of uncertainties coming from each individual parameter, but in practice, the single largest source of uncertainty is the scalar meson. Its large width and the uncertainty in its mass are both well-known issues within the light-meson exchange picture, for which several solutions exist [49–52] (for a more detailed explanation, we refer to Appendix B of [36]). Here, in particular, we consider the finding that the exchange of a wide meson can be effectively approximated by a narrow one after a redefinition of its parameters [49,50].

For modeling the error derived from the scalar meson, we will simply vary its mass within the range listed in the RPP [3], i.e., 400–550 MeV, leading to

$$\Delta M_{\text{OBE}} = M_p(m_S \pm \Delta m_S) - M_p(m_S), \tag{28}$$

with $m_S = 475 \pm 75$ MeV, where this error is asymmetric. Numerically, this is equivalent to changing the magnitude of C^{sat} by up to 30%–40% depending on the specific molecular configuration, which is, for instance, comparable with the relative errors used for the light-meson exchange potential within the one boson exchange (OBE) model of [53].

The second source of uncertainty is the contact-range approximation itself, the accuracy of which depends on the comparison between the momentum scale of the predicted bound state and the masses of the exchanged mesons. That is, we will assume a relative error of size γ_2/m_V,

$$\Delta M_{\text{contact}} = B_{\text{mol}}\left(\frac{\gamma_2}{m_V}\right), \tag{29}$$

which happens to be symmetric (and in most cases smaller than the error coming from the uncertainty in the scalar meson mass) and where $B_{\text{mol}} = M_{\text{th}} - M_p$ refers to the binding energy. Finally, we will sum these two errors in quadrature.

C. Reference states

For the reference state, we have considered four possible choices: the $X(3872)$, the $X(3960)$, the $T_{cc}^+(3875)$, and the 0^{++} $D\bar{D}$ bound state found in the lattice [43]. The specific inputs we will use are as follows:

(a) For the $X(3872)$, we will consider it as a 1^{++} $D^*\bar{D}$ bound state with the mass as determined in the RPP [3], i.e., 3871.65 ± 0.06 MeV, which we round up to 3871.7 MeV.

(b) For the $X(3960)$, we will refer to the recent theoretical analysis of [6], which considers the $X(3960)$ to be either a virtual or bound 0^{++} $D_s\bar{D}_s$ state. The determination of the pole mass in Ref. [6] happens to be the same in the virtual and bound state cases, yielding 3928 ± 3 MeV. However, we will only consider the virtual state solution as the input for our calculations. This is because other input choices already predict a bound $X(3960)$ in the same mass range of Ref. [6].

(c) For the $T_{cc}^+(3875)$, we will use its mass as determined from the unitarized Breit-Wigner shape in [24], i.e., by the δm in Eq. (4). As we are using the isospin symmetric limit, this will give a bound state energy of $B = 1.065$ MeV.

(d) For the 0^{++} $D\bar{D}$ bound state, we use the binding energy calculated in the lattice [43], i.e., $B = 4.0^{+5.0}_{-3.7}$ MeV.

We will not propagate the error in the input masses, as they usually generate smaller uncertainties than the scalar meson or the contact-range approximation. For choosing which one of these four inputs to use, we will compare the predictions derived from each of the inputs for the spectroscopy of a few molecular candidates and for the decays of the T_{cc}^+ tetraquark, as we will explain in the following lines.

D. Spectroscopy comparison

First, we consider the predictions for the spectroscopy of the charmed meson-(anti)meson molecular candidates depending on whether the reference state is the $X(3872)$, the $X(3960)$, the T_{cc}, or the $D\bar{D}$ state found in the lattice. We list these predictions in Table I for the molecular configurations for which there exists a clear candidate state (observed either in the experiment or the lattice).

The reason for comparing the predictions for different inputs is that we want to check the internal consistency of the molecular hypothesis for the most common candidates in our model. What we find is that the $X(3872)$ and $X(3960)$ can be explained with the same set of parameters

TABLE I. Basic set of predictions for known molecular candidates observed in experiments or calculated in the lattice and their mutual compatibility. We consider three sets of predictions depending on whether we use the $X(3872)$, the $X(3960)$, or the $T_{cc}(3875)$ as the input (or reference state) for our saturation model. "Input" refers to the input state used to determine the proportionality constant in Eq. (18), "system" indicates the particular two-meson system under consideration, $I(J^{P(C)})$ refers to its spin, parity, and C-parity (when applicable), S to its strangeness, $R_{\rm mol}$ is the central value ($m_S = 475$ MeV) of the molecular ratio defined in Eq. (24), $B_{\rm mol}$ is the central value of the binding energy, $M_{\rm mol}$ is the mass of the molecular state with uncertainties (calculated as explained in Sec. III B), "candidate" refers to a known resonance that might correspond to the two-meson system we are considering, and $M_{\rm candidate}$ is the mass of such a candidate (this includes not only experimental results, but also theoretical analyses of the results and lattice data). The superscript V above the binding energy indicates a virtual state, while a (B) attached to the upper error of the mass indicates that the state can change from virtual to bound within the uncertainties of the model. All the binding energies and masses are in units of MeV.

Input	System	$I(J^{P(C)})$	S	$R_{\rm mol}$	$B_{\rm mol}$	$M_{\rm mol}$	Candidate	$M_{\rm candidate}$
$X(3872)$	D^*D	$0\ (1^+)$	0	0.45	$(24.0)^V$	3852^{+17}_{-24}	T_{cc}	3875.7
	$D\bar{D}$	$0\ (0^{++})$	0	0.70	$(1.5)^V$	$3733.0^{+1.2}_{-1.9}$	⋯	$3730.5^{+3.7}_{-5.0}$ [43]
	$D_s\bar{D}_s$	$0\ (0^{++})$	0	0.51	$(15)^V$	3922^{+11}_{-15}	$X(3960)$	$3930^{+3.8}_{-2.0}$ [43], $(3928 \pm 3)^{V/B}$ [6]
	$D^*\bar{D}$	$0\ (1^{++})$	0	1.00	4.1	Input	$X(3872)$	3871.7
	$D^*\bar{D}^*$	$0\ (2^{++})$	0	1.04	5.5	4011.6 ± 0.7	⋯	$4014.3 \pm 4.0 \pm 1.5$ [54]
	$D^*\bar{D}$	$1\ (1^+)$	0	0.44	$(26)^V$	3850^{+19}_{-26}	$Z_c(3900)$	$(3831 - 3844)^V$ [55]
	$D^*\bar{D}_s - D\bar{D}_s^*$	$\frac{1}{2}\ (1^+)$	−1	0.45	$(24)^V$	3955^{+17}_{-25}	$Z_{cs}(3985)$	$(3971 - 3974)^V$ [56]
$X(3960)$	D^*D	$0\ (1^+)$	0	0.88	$(16.2)^V$	$3859.6^{+4.5}_{-5.4}$	T_{cc}	3875
	$D\bar{D}$	$0\ (0^{++})$	0	1.38	$(0.0)^V$	$3734.5^{+0.0(B)}_{-1.7}$	⋯	$3730.5^{+3.7}_{-5.0}$ [43]
	$D_s\bar{D}_s$	$0\ (0^{++})$	0	1.00	$(8.7)^V$	Input [6]	$X(3960)$	$3930^{+3.8}_{-2.0}$ [43], $(3928 \pm 3)^{V/B}$ [6]
	$D^*\bar{D}$	$0\ (1^{++})$	0	1.97	9.8	$3865.9^{+8.5}_{-17.3}$	$X(3872)$	3871.7
	$D^*\bar{D}^*$	$0\ (2^{++})$	0	2.04	12	4005^{+10}_{-19}	⋯	$4014.3 \pm 4.0 \pm 1.5$ [54]
	$D^*\bar{D}$	$1\ (1^+)$	0	0.88	$(17.9)^V$	$3857.9^{+5.5}_{-6.5}$	$Z_c(3900)$	$(3831 - 3844)^V$ [55]
	$D^*\bar{D}_s - D\bar{D}_s^*$	$\frac{1}{2}\ (1^+)$	−1	0.88	$(15.8)^V$	$3962.4^{+4.8}_{-5.8}$	$Z_{cs}(3985)$	$(3971 - 3974)^V$ [56]
$T_{cc}(3875)$	D^*D	$0\ (1^+)$	0	1.00	1.065	Input	T_{cc}	3874.7
	$D\bar{D}$	$0\ (0^{++})$	0	1.57	32	3703^{+20}_{-30}	⋯	$3730.5^{+3.7}_{-5.0}$ [43]
	$D_s\bar{D}_s$	$0\ (0^{++})$	0	1.14	5.0	$3931.7^{+1.8}_{-2.7}$	$X(3960)$	$3930^{+3.8}_{-2.0}$ [43], $(3928 \pm 3)^{V/B}$ [6]
	$D^*\bar{D}$	$0\ (1^{++})$	0	2.23	90	3786^{+70}_{-86}	$X(3872)$	3871.7
	$D^*\bar{D}^*$	$0\ (2^{++})$	0	2.31	95	3922^{+72}_{-90}	⋯	$4014.3 \pm 4.0 \pm 1.5$ [54]
	$D^*\bar{D}$	$1\ (1^+)$	0	0.97	0.6	$3875.2^{+0.2}_{-0.2}$	$Z_c(3900)$	$(3831 - 3844)^V$ [55]
	$D^*\bar{D}_s - D\bar{D}_s^*$	$\frac{1}{2}\ (1^+)$	−1	1.00	1.0	$3977.1^{+0.3}_{-0.2}$	$Z_{cs}(3985)$	$(3971 - 3974)^V$ [56]
$D\bar{D}$ (lattice)	D^*D	$0\ (1^+)$	0	0.64	$(4.5)^V$	$3871.3^{+4.0}_{-9.0}$	T_{cc}	3874.7
	$D\bar{D}$	$0\ (0^{++})$	0	1.0	4.0	3730.5	⋯	$3730.5^{+3.7}_{-5.0}$ [43]
	$D_s\bar{D}_s$	$0\ (0^{++})$	0	0.72	$(1.0)^V$	$3935.7^{+1.0(B)}_{-3.5}$	$X(3960)$	$3930^{+3.8}_{-2.0}$ [43], $(3928 \pm 3)^{V/B}$ [6]
	$D^*\bar{D}$	$0\ (1^{++})$	0	1.42	30	3845 ± 13	$X(3872)$	3871.7
	$D^*\bar{D}^*$	$0\ (2^{++})$	0	1.47	34	3983 ± 15	⋯	$4014.3 \pm 4.0 \pm 1.5$ [54]
	$D^*\bar{D}$	$1\ (1^+)$	0	0.63	$(5.4)^V$	$3870.4^{+4.8}_{-10.4}$	$Z_c(3900)$	$(3831 - 3844)^V$ [55]
	$D^*\bar{D}_s - D\bar{D}_s^*$	$\frac{1}{2}\ (1^+)$	−1	0.64	$(4.4)^V$	$3973.8^{+4.1}_{-9.6}$	$Z_{cs}(3985)$	$(3971 - 3974)^V$ [56]

within the uncertainties of our model, but this is not the case for the T_{cc}. This indicates that if the $X(3872)$ or the $X(3960)$ were to be explained in purely molecular terms, the T_{cc} would require additional attraction to bind, which might very well come from nonmolecular components. Alternatively, were the T_{cc} to be purely molecular, then it would be difficult to explain the mass of the $X(3872)$ and $X(3960)$ within the molecular picture in our saturation model.

We elaborate on this argument in more detail as follows:

(1) If we use the $X(3872)$ as the reference state, there are a few predictions in Table I on which it is worth commenting:

(a) There is a 0^{++} $D_s\bar{D}_s$ virtual state at 3922^{+11}_{-15} MeV, which might be identified with the $X(3960)$. Even though the $X(3960)$ has been found at 3956 MeV, i.e., above the $D_s\bar{D}_s$ threshold, this determination of its mass depends on the assumption that the $X(3960)$ is correctly described with a Breit-Wigner profile. As has been discussed in the literature [57,58], this is not necessarily true for bound and virtual states close to threshold. In this regard, Ref. [6] recently analyzed the $D_s^+D_s^-$ invariant mass distribution of [59] and found that the $X(3960)$ can

be described either as a virtual or bound state at about 3928 ± 3 MeV, which is compatible with our prediction in the virtual state case [but not if the $X(3960)$ turns out to be a bound state].

(b) However, if we turn our attention to the T_{cc}^{+}, there is not enough attraction to bind the $D^{*}D$ system (though it is still attractive). In this case, the T_{cc}^{+} will have to receive additional attraction from its short-range quark degrees of freedom in order to be able to bind. This scenario would be compatible with the seminal calculation by Janc and Rosina [37], which considered both the quark and meson components of the T_{cc}^{+} as necessary for generating this state. There is a series of predictions [33,34,60,61] that put the mass of the T_{cc}^{+} tetraquark above the $D^{*}D$ threshold, in which case this compact component should be able to mix with the molecular component and provide additional attraction not taken into account in our model.

(c) In addition to this, the mass of the 2^{++} partner of the $X(3872)$ is 4011.6 ± 0.8 MeV, which is compatible with the mass and quantum numbers of a 2^{++} resonance recently observed by Belle [54] ($M = 4014.3 \pm 4.0 \pm 1.5$ MeV and $J = 0$ or 2, though the signal has poor statistical significance). This 2^{++} partner, which was conjectured a decade ago [38,39], is a consequence of HQSS: if the $X(3872)$ is indeed a 1^{++} $D^{*}\bar{D}$ bound state, it should have a 2^{++} $D^{*}\bar{D}^{*}$ partner state. The location of this state is usually predicted at 4012–4013 MeV when pion interactions are neglected and 4015 MeV if the effects of the one pion exchange potential are included [39]. The width of this state has been previously estimated to be on the order of a few MeV [62] (from 0.9 to 14.0 MeV depending on the assumptions), which is in line with the width measured by Belle [54] ($\Gamma = 4 \pm 11 \pm 6$ MeV).

(d) We also obtain a $D\bar{D}$ virtual state close to threshold, which might correspond with the $D\bar{D}$ bound state found in the lattice, and we predict the Z_c and Z_{cs} states as $D^{*}\bar{D}$ and $D_s^{*}\bar{D}$ virtual states, where the masses are in line with the previous theoretical analyses of Refs. [55,56].

(2) If we use the $X(3960)$ as the reference state and assume it to be a virtual state with the mass extracted in Ref. [6], the predictions are basically compatible with the ones derived from the $X(3872)$ within errors (and thus we do not comment the results in detail). That is, provided the $X(3960)$ is a virtual state, its molecular interpretation will be compatible with that of the $X(3872)$.

(3) If we use the T_{cc}^{+} as the reference state ($D^{*}D$ molecule with $I = 0$), we find the following:

(a) The $X(3872)$ is predicted a few dozen MeV below threshold. In this scenario, the $X(3872)$ cannot be a pure molecular state within the uncertainties of our model, and there should be shorter-range components (e.g., coupling to charmonium) that push the $D^{*}\bar{D}$ pole closer to threshold.

(b) The $X(3960)$ is predicted a few MeV below threshold, in particular, at $M = 3931.6^{+1.8}_{-2.7}$ MeV, which compares well with the bound state pole determination in [6], $M = 3928 \pm 3$ MeV.

From the previous inputs, it is already evident that the molecular explanations of the $X(3872)$ and T_{cc} are not compatible within our model. The situation is more ambiguous with respect to the $X(3960)$ though, for which the virtual and bound state interpretations are compatible with a molecular $X(3872)$ and T_{cc}, respectively. The same comment applies to the $Z_c(3900)$ and $Z_{cs}(3985)$. However, if the 2^{++} state observed by Belle is eventually confirmed and happens to be the conjectured $X(4012)$, then this will favor (disfavor) the interpretation of the $X(3872)$ (T_{cc}) as molecular.

If we also include the lattice results in our discussion, we find the following:

(4) Using the $X(3872)$ as input will lead to virtual states in the $D\bar{D}$ and $D_s\bar{D}_s$ systems that are close to threshold. They are not as attractive as in the lattice calculations, but still compatible with them within errors.

(5) Using the T_{cc} as input will lead to a $D\bar{D}$ that is markedly more bound than in the lattice, though uncertainties are too large to claim that they are incompatible.

(6) If we use the lattice results as the input, it happens that the location of the $D\bar{D}$ and $D_s\bar{D}_s$ poles are compatible with each other, but predict the $X(3872)$ to be too bound by tens of MeV. The T_{cc}^{+} will not bind either in this scenario, but will still survive as a virtual state close to threshold. In this case, the importance of the quark degrees of freedom in binding will be smaller than when the $X(3872)$ is the reference state.

That is, this additional comparison does not provide clear-cut answers, though it shows a weak preference for the $X(3872)$ (instead of the T_{cc}^{+}) to be more molecular.

Regarding the conflict between the $X(3872)$ and $T_{cc}(3875)$, it is important to notice the mixing between the meson-antimeson components of the $X(3872)$ and the possible nearby χ_{c1} charmonium and the molecular $T_{cc}(3875)$ and its compact quark components. This mixing can be parametrized in terms of a potential contribution of the type

$$V = \begin{pmatrix} V_{\mathrm{mol}} & C_{\mathrm{comp}} \\ C_{\mathrm{comp}} & 0 \end{pmatrix}, \tag{30}$$

which results in a mass shift proportional to

$$\Delta M_{\rm mol} \propto -\frac{C_{\rm comp}^2}{\Delta}, \tag{31}$$

with Δ the mass difference between the (uncoupled) compact and molecular components, $\Delta = M_{\rm comp} - M_{\rm mol}$. This will make the molecular candidate lighter or heavier depending on the location of the nonmolecular component (before mixing). That is, if the original predictions of the compact component are heavier than the two-hadron threshold, the two-hadron system will increase its attraction with respect to the purely molecular scenario.

It happens that most predictions of the $\chi_{c1}(2P)$ in pure charmonium models are heavier [63–65] (usually in the 3.9–4.0 GeV ballpark) than the $X(3872)$, from which the contribution of the compact components is more likely to be attractive than repulsive. The situation is more open for a compact $cc\bar{u}\,\bar{d}$ tetraquark, as there is a comparable number of predictions below [66–69], above [33,34,60,61], or compatible with [32,70–73] the D^*D threshold. This slightly favors scenarios in which the attraction in the D^*D system is, by itself, not enough to generate a bound state below threshold (as it happens to be probable that a pure $cc\bar{u}\,\bar{d}$ state will provide the missing attraction), while it disfavors the scenarios in which the $D^*\bar{D}$ system shows overbinding [as it is improbable that the closest pure $c\bar{c}$ state will happen to be lighter than the $X(3872)$].

E. Tetraquark decays

A different piece of information we might take into account is the decay of the T_{cc} into $DD\pi$ and $DD\gamma$. The decay width of the T_{cc} is expected to come mostly from the D^*D component of its wave function [74]. If we assume a wave function of the type

$$|T_{cc}\rangle = \cos\,\theta_C|D^*D\rangle + \sin\,\theta_C|cc\bar{u}\,\bar{d}\rangle, \tag{32}$$

which contains a molecular and compact component, then the actual decay width of the T_{cc} will be dominated by the D^*D components.[2] In particular, a dimensional estimation yields [74]

$$\begin{aligned}\Gamma(T_{cc}) &= \cos^2\theta_C\,\Gamma_{\rm mol} + \sin\,2\theta_C\,\Gamma_{\rm int} + \sin^2\theta_C\,\Gamma_{\rm cc} \\ &= \cos^2\theta_C\,\Gamma_{\rm mol} \times \left[1 + \mathcal{O}\left(\tan\theta_C\left(\frac{Q}{M_C}\right)^{3/2}\right)\right],\end{aligned} \tag{33}$$

where $\Gamma_{\rm mol} = \Gamma(T_{cc}(D^*D))$ and $\Gamma_{\rm cc} = \Gamma(T_{cc}(cc\bar{u}\,\bar{d}))$ are the decay widths of the molecular and compact parts of the wave function, while $\Gamma_{\rm int}$ is an interference term (not necessarily positive: despite the notation, it is not a decay width *per se*). In the second line, we have taken into account that the decay amplitude into $DD\pi$ and $DD\gamma$ scales as $1/Q^{3/2}$ and $1/M_C^{3/2}$ [74], where $Q \sim \gamma_2$ is the wave number of the T_{cc} as a D^*D two-body system (about 45 MeV in the isospin symmetric limit) and M_C is the expected natural momentum scale at which the structure of the compact tetraquark component is resolved. The naive expectation is for M_C to be considerably larger than the wave number of the molecular part of the wave function, i.e., $M_C \gg \gamma_2$. As a consequence, in a first approximation we can ignore the compact components of the wave function for calculating the decay width of the T_{cc}: from the previous scaling, we expect $\Gamma_{\rm int}/\Gamma_{\rm mol} \propto (Q/M_C)^{3/2}$ and $\Gamma_{\rm cc}/\Gamma_{\rm mol} \propto (Q/M_C)^3$. That is, if the molecular prediction for $\Gamma_{\rm mol}$ overshoots $\Gamma(T_{cc})$ we will be able to estimate the degree of *molecularness* of the T_{cc}. Provided we calculate the decays of the T_{cc} within pionless effective field theory (EFT), the second line of Eq. (33) will be valid up to next-to-leading order (NLO), as explained in [74].

Within the EFT formulation of [74] the NLO calculation of $\Gamma_{\rm mol}$ depends on the T_{cc}^+ D^*D binding energy, the D^*D effective range, and the scattering length of the final DD two-body system. The value of the scattering length and the effective range of a two-hadron system in our model is given by

$$\frac{1}{a_0} = \frac{\mu_{\rm mol}}{2\pi C_{\rm mol}^{\rm sat}} + \frac{2}{\pi}\int_0^\infty dp f^2\left(\frac{p}{\Lambda}\right), \tag{34}$$

$$\begin{aligned} r_0 = &-\frac{4}{\pi}\int_0^\infty \frac{dp}{p^2}\left(f^2\left(\frac{p}{\Lambda}\right) - f(0)\right) \\ &+ \frac{1}{a_0}\frac{d^2}{dp^2}\left[f^2\left(\frac{p}{\Lambda}\right)\right]\Big|_{p=0}, \end{aligned} \tag{35}$$

where we remind the reader that $f(x)$ refers to the regulator function. The values for $a_0(DD)$ and $r_0(D^*D, I=0)$ can, in turn, be used as input for the formulas in [74] to predict $\Gamma_{\rm mol}$. In regard to this comparison, we find the following:

(7) If the $X(3872)$ is used as input, the DD scattering length is predicted to be $a_0 = -0.26^{+0.15}_{-0.32}$ fm, which translates into a molecular decay width of

$$\Gamma_{\rm mol}^{\rm NLO} = 63.8^{+10.0}_{-7.3}\ \text{KeV}, \tag{36}$$

where the uncertainties have been calculated as in [74] (i.e., from the following sources: pion axial coupling, magnetic moments of the charmed mesons, binding energy of the T_{cc} as obtained in the unitarized Breit-Wigner parametrization δm_U, and the EFT truncation error), except for the addition in

[2]While the $J^P = 1^+$ $cc\bar{u}\,\bar{d}$ compact tetraquark is often predicted above the D^*D threshold [33,34,60,61] and thus decays into this channel, this is not the case if the tetraquark is below the D^*D threshold, which is what happens if the $T_{cc}^+(3875)$ is a mixture of compact and molecular components.

quadrature of the errors coming from the DD scattering length. This is to be compared with the decay width as extracted from the unitarized Breit-Wigner parametrization, $\Gamma_U = 48^{+2}_{-12}$ KeV [24], leading to

$$\frac{\Gamma_U}{\Gamma^{\rm NLO}_{\rm mol}} = \cos^2\theta_C = 0.75^{+0.10}_{-0.21}, \tag{37}$$

that is, this will imply that the T_{cc} is about 75% molecular. This is consistent with the fact that its location would not be reproduced from the molecular degrees of freedom alone, thus requiring contributions from other components of the wave function.

(8) In contrast, if the T^+_{cc} is used as input, its decay width happens to be too large owing to a larger DD scattering length ($a_0 = -1.15^{+0.66}_{-1.65}$ fm). By repeating the steps in the previous comparison, we find

$$\Gamma^{\rm NLO}_{\rm mol} = 85.7^{+49.5}_{-18.6}\ \text{KeV}, \tag{38}$$

which in turn leads to

$$\frac{\Gamma_U}{\Gamma^{\rm NLO}_{\rm mol}} = \cos^2\theta_C = 0.55^{+0.15}_{-0.24}, \tag{39}$$

or about 55% molecular. This implies a degree of nonmolecularness that is probably not compatible with the initial assumption that the T^+_{cc} is bound by the molecular components alone.

Thus, this comparison favors the use of the $X(3872)$ as input and a T^+_{cc} state for which binding results as a combination of the interplay between mesonic and quark degrees of freedom, as in [37]. Yet, as with the previous comparisons, caution is advised: there is a sizable level of uncertainty associated with all the comparisons we have made.

F. Choice of the reference state

All in all, from a comparison of the (partially known) spectrum of two charmed meson systems, the expected interference from nonmolecular degrees of freedom, and the decays of the T^+_{cc}, it seems that using the $X(3872)$ as input is the better choice.

This is not to say that the $X(3872)$ is purely molecular, only that the assumption that its binding can be purely explained in terms of its molecular components is more congruent with the previous information than the analogous assumption for the T^+_{cc}. Additionally, there are certain aspects of the $X(3872)$ that are better described by assuming the existence of short-range compact components [75–77]. Also, it has been argued that the effective range for the $D^*\bar{D}$ pair within the $X(3872)$ is negative [78], which might require contributions from degrees of freedom different than $D^*\bar{D}$ to the scattering amplitude. Nonetheless, we consider the $X(3872)$ to be the best choice for the reference state and compute the molecular spectrum from it. Yet, for the particular case of the $D^{(*)}D^{(*)}$ two-body system, we will consider too the predictions when the T^+_{cc} is used as the reference state. This will yield a different set of molecular states that, if eventually observed, could be used to decide between the $X(3872)$ and T^+_{cc} as the most molecular of the two.

IV. PREDICTIONS

For the predictions of the molecular spectrum of two charmed mesons, we will use the $X(3872)$ as the reference state, as previously explained. This leads to $C^{\rm sat}_{\rm ref} = -0.79$ fm^2 for the coupling of the reference state.

We will also include a modification to our model when strange-charmed mesons are included. This modification is as follows: instead of using our original formulation in Eq. (19), which implicitly assumes the same cutoff for the two hadrons within a molecular state, we will consider the possibility that each of the hadrons is better described by a different cutoff

$$V_C = C^{\rm sat} f\left(\frac{p'}{\Lambda_1}\right) f\left(\frac{p}{\Lambda_2}\right). \tag{40}$$

In particular, we will distinguish between the nonstrange and strange sectors: for the D and D^* charmed mesons we will use $\Lambda = 1.0$ GeV, just as before, but for D_s and D^*_s we will use $\Lambda = 1.2$ GeV instead. The reasons for this change are that it takes into account the more compact nature of the strange-charmed mesons and that it helps us reproduce the location of the virtual state interpretation of the $X(3960)$: with $\Lambda = 1.2$ GeV, we obtain 3927.2 MeV, which is compatible with the 3928 ± 3 MeV pole mass of [6].

With the previous choices, in the hidden-charm sector we obtain the spectrum we show in Table II. Only two systems clearly bind below threshold, the 1^{++} $D^*\bar{D}$ and 2^{++} $D^*\bar{D}^*$ configurations, yet there are a few other systems that are very close to binding: 0^{++} $D\bar{D}$, 1^{++} $D^*_s\bar{D}_s$, and 2^{++} $D^*_s\bar{D}^*_s$.

For the doubly charmed sector, we have made two sets of predictions, depending on whether we use the $X(3872)$ or the T^+_{cc} as input, Tables III and IV, respectively. The doubly charmed molecules are constrained by (extended) Bose-Einstein symmetry, i.e., not all combinations of spin and isospin are allowed. For DD and D^*D^* this translates into the condition that $(I + J)$ must be an odd number, while for D_sD_s and $D^*_sD^*_s$ the condition is that J must be even. For the $D^{(*)}_sD^{(*)}$ there are two configurations, antitriplet and sextet, given by

$$|\bar{3}\rangle = \frac{1}{\sqrt{2}}\left[|D^{(*)}_s D^{(*)}\rangle - |D^{(*)} D^{(*)}_s\rangle\right], \tag{41}$$

TABLE II. Predictions for the $D^{(*)}\bar{D}^{(*)}$ systems when the $X(3872)$ is used as the reference state. The meaning of system, $I(J^{P(C)})$, $R_{\rm mol}$, $B_{\rm mol}$, $M_{\rm mol}$, candidate, and $M_{\rm candidate}$ is the same as in Table I. The cutoff for the D_s and D_s^* charmed-strange mesons is set to $\Lambda_s = 1.2$ GeV (instead of $\Lambda = 1.0$ GeV as in Table I). All binding energies and masses are in units of MeV.

System	$I(J^{P(C)})$	$R_{\rm mol}$	$B_{\rm mol}$	$M_{\rm mol}$	Candidate	$M_{\rm candidate}$
$D\bar{D}$	$0\ (0^{++})$	0.70	$(1.5)^V$	$3733.0^{+1.2}_{-1.9}$	⋯	$3730.5^{+3.7}_{-5.0}$ [43]
$D^*\bar{D}$	$0\ (1^{++})$	1.00	4.1	Input	$X(3872)$	3871.7
$D^*\bar{D}$	$0\ (1^{+-})$	0.46	$(22.3)^V$	3854^{+16}_{-22}	⋯	⋯
$D^*\bar{D}^*$	$0\ (0^{++})$	0.20	⋯	⋯	⋯	⋯
$D^*\bar{D}^*$	$0\ (1^{+-})$	0.48	$(18.9)^V$	3998^{+14}_{-20}	⋯	⋯
$D^*\bar{D}^*$	$0\ (2^{++})$	1.04	5.5	4011.6 ± 0.7	⋯	$4014.3 \pm 4.0 \pm 1.5$ [54]
$D\bar{D}$	$1\ (0^{++})$	0.42	$(30.2)^V$	3704^{+21}_{-28}	⋯	⋯
$D^*\bar{D}$	$1\ (1^{+\pm})$	0.44	$(26.2)^V$	3850^{+19}_{-26}	⋯	$(3831-3844)^V$ [55]
$D^*\bar{D}^*$	$1\ (0^{++},1^{-+},2^{++})$	0.46	$(22.5)^V$	3995^{+16}_{-24}	⋯	⋯
$D\bar{D}_s$	$\frac{1}{2}\ (0^+)$	0.43	$(24.7)^V$	3811^{+19}_{-28}	⋯	⋯
$D^*\bar{D}_s$	$\frac{1}{2}\ (1^+)$	0.45	$(21.0)^V$	3957^{+17}_{-26}	⋯	$(3971-3974)^V$ [56]
$D^*\bar{D}_s^*$	$\frac{1}{2}\ (0^+,1^+,2^+)$	0.47	$(17.5)^V$	4103^{+14}_{-24}	⋯	⋯
$D_s\bar{D}_s$	$0\ (0^{++})$	0.51	$(8.2)^V$	$3928.5^{+7.7}_{-15.3}$	⋯	$3930^{+3.8}_{-2.0}$ [43], $(3928 \pm 3)^{V/B}$ [6]
$D_s^*\bar{D}_s$	$0\ (1^{++})$	0.64	$(0.3)^V$	$4080.2^{+0.3(B)}_{-3.2}$	⋯	⋯
$D_s^*\bar{D}_s$	$0\ (1^{+-})$	0.42	$(21.9)^V$	4059^{+20}_{-36}	⋯	⋯
$D_s^*\bar{D}_s^*$	$0\ (0^{++})$	0.32	$(48.1)^V$	4176^{+40}_{-127}	⋯	⋯
$D_s^*\bar{D}_s^*$	$0\ (1^{+-})$	0.44	$(18.2)^V$	4206^{+17}_{-34}	⋯	⋯
$D_s^*\bar{D}_s^*$	$0\ (2^{++})$	0.66	$(0.0)^V$	$4224.4^{+0.0(B)}_{-2.2}$	⋯	⋯

TABLE III. Predictions for the $D^{(*)}D^{(*)}$ systems when the $X(3872)$ is used as the reference state. The meaning of system, $I(R)(J^P)$, $R_{\rm mol}$, $B_{\rm mol}$, and $M_{\rm mol}$ is the same as in Table I, where the only difference is that now, after isospin, we include (R) in parentheses to indicate the SU(3)-flavor representation to which a $D_s^{(*)}D^{(*)}$ state belongs (the reason being that they are not distinguishable by isospin alone in this case). We do not include candidate states and their masses as there is only one: the $T_{cc}^+(3875)$ with a mass of 3875.7 MeV. The cutoff for the D_s and D_s^* charmed-strange mesons is set to $\Lambda_s = 1.2$ GeV (instead of $\Lambda = 1.0$ GeV as in Table I). All binding energies and masses are in units of MeV.

System	$I(R)\ (J^P)$	$R_{\rm mol}$	$B_{\rm mol}$	$M_{\rm mol}$
DD	$1\ (0^+)$	0.28	⋯	⋯
D^*D	$0\ (1^+)$	0.45	$(24.2)^V$	3852^{+17}_{-24}
D^*D	$1\ (1^+)$	0.16	⋯	⋯
D^*D^*	$1\ (0^+)$	0.58	$(7.7)^V$	$4009.4^{+5.8}_{-8.6}$
D^*D^*	$0\ (1^+)$	0.47	$(20.7)^V$	3997^{+15}_{-22}
D^*D^*	$1\ (2^+)$	0.16	⋯	⋯
D_sD	$\frac{1}{2}(6)\ (0^+)$	0.34	$(49.8)^V$	3786^{+40}_{-63}
$D_s^*D - D_sD^*$	$\frac{1}{2}(\bar{3})\ (1^+)$	0.43	$(25.0)^V$	3953^{+20}_{-31}
$D_s^*D - D_sD^*$	$\frac{1}{2}(6)\ (1^+)$	0.23	⋯	⋯
$D_s^*D^*$	$\frac{1}{2}(6)\ (0^+)$	0.59	$(4.4)^V$	$4116.3^{+4.0}_{-7.1}$
$D_s^*D^*$	$\frac{1}{2}(\bar{3})\ (1^+)$	0.43	$(24.1)^V$	4097^{+19}_{-30}
$D_s^*D^*$	$\frac{1}{2}(6)\ (2^+)$	0.23	⋯	⋯
D_sD_s	$0\ (0^+)$	0.38	$(32.7)^V$	3804^{+28}_{-50}
$D_s^*D_s$	$0\ (1^+)$	0.28	⋯	⋯
$D_s^*D_s^*$	$0\ (0^+)$	0.63	$(0.4)^V$	$4224.0^{+0.4(B)}_{-4.0}$
$D_s^*D_s^*$	$0\ (2^+)$	0.29	⋯	⋯

$$|6\rangle = \frac{1}{\sqrt{2}}\left[|D_s^{(*)}D^{(*)}\rangle + |D^{(*)}D_s^{(*)}\rangle\right]. \tag{42}$$

The antitriplet and sextet configurations correspond to the SU(3)-flavor generalizations of the $I = 0$ and $I = 1$

TABLE IV. Predictions for the $D^{(*)}D^{(*)}$ systems when the T_{cc}^+ is used as the reference state. The conventions used are identical to those of Table III.

System	$I(R)\ (J^P)$	$R_{\rm mol}$	$B_{\rm mol}$	$M_{\rm mol}$
DD	$1\ (0^+)$	0.63	$(9.9)^V$	$3724.6^{+7.9}_{-24.8}$
D^*D	$0\ (1^+)$	1.0	1.065	3874.7
D^*D	$1\ (1^+)$	0.35	⋯	⋯
D^*D^*	$1\ (0^+)$	1.30	12.2	$4004.9^{+5.6}_{-8.5}$
D^*D^*	$0\ (1^+)$	1.04	1.8	4015.3 ± 0.1
D^*D^*	$1\ (2^+)$	0.36	⋯	⋯
D_sD	$\frac{1}{2}(6)\ (0^+)$	0.75	$(0.8)^V$	$3834.8^{+0.8(B)}_{-6.1}$
$D_s^*D - D_sD^*$	$\frac{1}{2}(\bar{3})\ (1^+)$	0.95	2.1	$3976.1^{+1.1}_{-1.0}$
$D_s^*D - D_sD^*$	$\frac{1}{2}(6)\ (1^+)$	0.51	$(19.2)^V$	3959^{+18}_{-90}
$D_s^*D^*$	$\frac{1}{2}(6)\ (0^+)$	1.32	23.1	4098^{+11}_{-15}
$D_s^*D^*$	$\frac{1}{2}(\bar{3})\ (1^+)$	0.95	2.0	4188.7 ± 1.0
$D_s^*D^*$	$\frac{1}{2}(6)\ (2^+)$	0.51	$(18.5)^V$	4102^{+17}_{-87}
D_sD_s	$0\ (0^+)$	0.85	2.0	$3934.7^{+1.9}_{-3.2}$
$D_s^*D_s$	$0\ (1^+)$	0.63	$(2.8)^V$	$4077.8^{+2.8(B)}_{-31.0}$
$D_s^*D_s^*$	$0\ (0^+)$	1.41	52.8	4172^{+28}_{-32}
$D_s^*D_s^*$	$0\ (2^+)$	0.66	$(1.6)^V$	$4222.8^{+1.6(B)}_{-27.4}$

configurations in $D^{(*)}D^{(*)}$. Indeed, it can be readily noticed in Tables III and IV that the binding or virtual state energies of the $I=0$ ($I=1$) $D^{(*)}D^{(*)}$ and $\bar{3}$ (6) $D_s^{(*)}D^{(*)}$ molecules are predicted to be similar. This consequence of SU(3)-flavor symmetry was previously exploited in [79] to predict the $D_sD^*-D_s^*D$ molecular partners of the T_{cc}.

When we use the $X(3872)$ as input, Table III, it is apparent that the D^*D^* HQSS partner of the T_{cc}, i.e., the T_{cc}^*, does not bind and it is a virtual state not close to threshold instead. This is in contrast with the prediction of a bound T_{cc}^* when the T_{cc}^+ is used as input, Table IV. This difference in the predictions could be used to better understand the nature of the T_{cc}^+: the eventual discovery of the T_{cc}^* at about (1–2) MeV below the D^*D^* threshold would be a strong indication that the T_{cc}^+ is mostly molecular. If it were not and the binding of the T_{cc}^+ required the interplay between the mesonic and quark degrees of freedom, we would not expect the T_{cc}^* to bind: if the pole of the compact component once we remove the coupling with the D^*D channel were to be between the D^*D and D^*D^* threshold, this component will favor binding for D^*D and disfavor the formation of a D^*D^* molecule. However, this is not necessarily the only possibility: were this isolated compact component to be somewhat above the D^*D^* threshold, the HQSS expectation that the $J=1$ D^*D and D^*D^* states have the same binding energy would be strongly violated. Yet, this later possibility seems less plausible as it would require a really strong coupling between the compact and molecular components for providing strong enough attraction in the D^*D channel.

Be it as it may, this is not the only difference between the two spectra. The doubly charmed spectrum derived from the $X(3872)$ contains very few configurations close to binding, basically the sextet $J^P=0^+$ configurations containing two excited charmed mesons, i.e., the $I=1$ $J^P=0^+$ D^*D^*, the sextet 0^+ $D^*D_s^*$, and the 0^+ $D_s^*D_s^*$ molecules. In contrast, if the T_{cc}^+ were to be mostly molecular, the spectrum of possible molecular states would be much richer. For instance, all the antitriplet $J^P=1^+$ configurations will bind, including the T_{cc} and T_{cc}^* as well as their strange and hidden-strange counterparts. Again, the eventual detection of these molecules will imply a molecular T_{cc}^+. Alternatively, lattice calculations of the $D_s^{(*)}D^{(*)}$ and $D_s^{(*)}D_s^{(*)}$ systems will shed light on this issue, though for the moment only calculations in the D^*D case exist (indicating either a virtual [43,80] or bound state [81] solution for the T_{cc}^+). Yet, independent of the input state, the most attractive configuration turns out to be the $I=0$ or sextet $J^P=0^+$ D^*D^* and 0^+ $D^*D_s^*$ molecules, a conclusion that is in agreement with [82] for the nonstrange sector.

V. CONCLUSIONS

To summarize, we have considered the molecular spectrum of systems containing two S-wave charmed mesons within a contact-range theory in which the couplings are saturated by light-meson exchanges (σ, ρ, ω). The question we wanted to address was whether the $X(3872)$, $X(3960)$, and $T_{cc}^+(3875)$ can all be described with the same set of parameters. It turns out that this is not the case and that there is a tension between the molecular description of the $X(3872)$ and $T_{cc}^+(3875)$ within the saturation model we use. Basically, if the $X(3872)$ is molecular it would be difficult to explain the $T_{cc}^+(3875)$ in purely molecular terms and vice versa.

Intuitively this can be understood in terms of vector meson exchange alone, as the attraction provided by this effect is twice as big in the isoscalar $D^*\bar{D}$ system than in the D^*D one,

$$V_V(D^*\bar{D},I=0)=-4\left[\frac{g_V^2}{m_V^2+\vec{q}^{\,2}}+\frac{f_V^2}{6M^2}\frac{m_V^2}{m_V^2+\vec{q}^{\,2}}\vec{\sigma}_{L1}\cdot\vec{\sigma}_{L2}\right], \tag{43}$$

$$V_V(D^*D,I=0)=-2\left[\frac{g_V^2}{m_V^2+\vec{q}^{\,2}}+\frac{f_V^2}{6M^2}\frac{m_V^2}{m_V^2+\vec{q}^{\,2}}\vec{\sigma}_{L1}\cdot\vec{\sigma}_{L2}\right], \tag{44}$$

as derived from Eq. (11) once we include isospin factors or as in Eqs. (12) and (13) once we use the contact-range approximation. Thus, in the absence of other attractive effects, we expect a molecular $X(3872)$ to be considerably more bound than a molecular $T_{cc}^+(3875)$. The inclusion of the scalar meson, which provides the same degree of attraction in both systems, somewhat softens the previous conclusion but not necessarily as much as to avoid the tension (unless scalar meson exchange is much stronger than expected here).

From a comparison of the hidden- and open-charm two-meson molecules with known molecular candidates and the decays of the $T_{cc}^+(3875)$, we consider that it is more probable for the $X(3872)$ to be mostly molecular than for the $T_{cc}^+(3875)$. Hence, to make predictions of the two charmed meson molecular spectrum we use the $X(3872)$ as a reference or input state within our RG-saturation model. For the hidden-charm sector, this choice generates only a second molecular state that is clearly below threshold: the $J^{PC}=2^{++}$ partner of the $X(3872)$. Yet, there are a few virtual states that are extremely close to threshold and can bind within the uncertainties of our model. These include the 0^{++} $D\bar{D}$ and $D_s\bar{D}_s$ molecules, the first one corresponding with the state found in the lattice [43] and the second one with the $X(3960)$.

For the open-charm sector, the most attractive configurations are the $I=1$, $J=0^+$ D^*D^*, sextet $J=0^+$ $D_s^*D^*$, and $J=0^+$ $D_s^*D_s^*$ molecules, which are predicted as virtual states really close to threshold. However, the interesting

feature of the open-charm molecules is that their spectrum will allow us to distinguish whether the $T_{cc}^{+}(3875)$ is only partly or predominantly molecular. In the second case—a $T_{cc}^{+}(3875)$ whose binding can be explained purely in molecular terms—there will be a $I = 0$, $J = 1^{+}$ $D^{*}D^{*}$ partner state at 4015 MeV, a $T_{cc}^{*+}(4015)$. The existence of this state has been consistently predicted in models assuming that the $T_{cc}^{+}(3875)$ is predominantly molecular in the first place [83–85]. Finding the $T_{cc}^{*+}(4015)$ will thus represent a very strong hint that the $T_{cc}^{+}(3875)$ binding comes almost exclusively from its molecular components.

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