

## Observation of charmed strange meson pair production in $\Upsilon(2S)$ decays and in $e^+e^-$ annihilation at $\sqrt{s} = 10.52$ GeV

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We observe the process  $\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-$  and continuum production  $e^+e^- \rightarrow D_s^{(*)+} D_{sJ}^-$  at  $\sqrt{s} = 10.52$  GeV (and their charge conjugates) using the data samples collected by the Belle detector at KEKB, where  $D_{sJ}^-$  is  $D_{s1}(2536)^-$  or  $D_{s2}^*(2573)^-$ . Both  $D_{sJ}^-$  states are identified through their decay into  $\bar{K}\bar{D}^{(*)}$ . We measure the products of branching fractions  $\mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K}\bar{D}^{(*)})$  and the Born cross sections  $\sigma^{\text{Bom}}(e^+e^- \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K}\bar{D}^{(*)})$ , and then compare the ratios  $R_1 \equiv \mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-) / \mathcal{B}(\Upsilon(2S) \rightarrow \mu^+\mu^-)$  for  $\Upsilon(2S)$  decays and  $R_2 \equiv \sigma^{\text{Bom}}(e^+e^- \rightarrow D_s^{(*)+} D_{sJ}^-) / \sigma^{\text{Bom}}(e^+e^- \rightarrow \mu^+\mu^-)$  for continuum production. We obtain  $R_1/R_2 = 9.7 \pm 2.3 \pm 1.1$ ,  $6.8 \pm 2.1 \pm 0.8$ ,  $10.2 \pm 3.3 \pm 2.5$ , and  $3.4 \pm 2.1 \pm 0.8$  for the  $D_s^+ D_{s1}(2536)^-$ ,  $D_s^{*+} D_{s1}(2536)^-$ ,  $D_s^+ D_{s2}^*(2573)^-$ , and  $D_s^{*+} D_{s2}^*(2573)^-$  final states in the  $D_{sJ}^- \rightarrow K^-\bar{D}^{(*)0}$  modes, respectively. The measured  $R_1/R_2$  values indicate that the strong decay dominates in  $\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-$  processes. We also measure the ratios of branching fractions  $\mathcal{B}(D_{s1}(2536)^- \rightarrow K_S^0 D^*(2010)^-) / \mathcal{B}(D_{s1}(2536)^- \rightarrow K^- D^*(2007)^0) = 0.48 \pm 0.07 \pm 0.02$  and  $\mathcal{B}(D_{s2}^*(2573)^- \rightarrow K_S^0 D^-) / \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K^- D^0) = 0.49 \pm 0.10 \pm 0.02$ , which are consistent with isospin symmetry. The second ratio is the first measurement of this quantity. Here, the first uncertainties are statistical and the second are systematic.

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## I. INTRODUCTION

Much of the plethora of new quarkonium states observed in the last decades has been studied at electron-positron colliders [1]. Experiments at these colliders have collected large data samples at c.m. energies ( $\sqrt{s}$ ) corresponding to both the vector charmonium and the vector bottomonium

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states, which are produced copiously due to resonance enhancements in the cross sections. A vector quarkonium state decays electromagnetically through the annihilation process into a virtual photon or into three gluons mediated by the strong interaction. We can study the dynamics of electromagnetic charmed meson production through complementary measurements at energies above or below the quarkonium state, which are called “off resonance.” Here, only quantum electrodynamics (QED) processes contribute and thus allow measurements free from hadronic structure effects and quantum chromodynamics (QCD) related processes present for heavy quarkonia.

At  $\sqrt{s}$  significantly above the production threshold and far from quarkonium resonances, the production rates of  $e^+e^- \rightarrow q\bar{q}$  are approximately proportional to the quark charge squared, so that  $e^+e^- \rightarrow c\bar{c}$  is about 40% of the total hadronic production at  $\sqrt{s} = 10.52$  GeV [60 MeV below the  $\Upsilon(4S)$ ]. This provides an opportunity to study the charmed hadrons, including charmed mesons, charmed strange mesons, and charmed baryons. However, this kind of study has not been previously carried out. This is also the case for the Okubo-Zweig-Iizuka suppressed hadronic decays of the narrow  $\Upsilon(nS)$  states; hundreds of millions of  $\Upsilon(nS)$  events have been accumulated at Belle and BABAR, but such studies are limited. The open charm content of bottomonium hadronic decays can be used as a tool to probe the post- $b\bar{b}$ -annihilation fragmentation processes [2]. Within a QCD approach, the charm quarks are expected to be produced in  $\Upsilon(nS)$  decay only by a process in which a virtual timelike gluon of large invariant mass is produced in the initial decay process and subsequently decays into a pair of charmed hadrons [3]. Using a data sample of  $(98.6 \pm 0.9) \times 10^6$   $\Upsilon(2S)$  events, BABAR measures  $\mathcal{B}[\Upsilon(1S) \rightarrow D^{*+}X] = (2.52 \pm 0.13 \pm 0.15)\%$  [4], which is considerably larger than what is expected from  $b\bar{b}$  annihilation into a single photon. This excess is in agreement with a prediction based on splitting a virtual photon [5], but appears too small to accommodate an octet-state contribution [6]. Here and hereinafter, the first uncertainty quoted is statistical, while the second corresponds to the total systematic uncertainty. However, there are no other measurements of charm hadrons in  $\Upsilon(nS)$  decays [7]. It is argued that the suppression of charm production on the  $\Upsilon(nS)$  resonance is at least consistent with the analogous case of strangeness production on  $\psi$  and  $\psi(2S)$ , and it would be quite instructive to study the topology of events in which charm is produced [8].

Here, we present searches for  $D_s^{(*)+}D_{sJ}^-$  with the subsequent decay  $D_{sJ}^- \rightarrow \bar{K} + \bar{D}^{(*)}$  in  $\Upsilon(2S)$  decays and continuum  $e^+e^-$  annihilation, using data recorded with the Belle detector operated at the KEKB asymmetric-energy  $e^+e^-$  collider [9]. Charge-conjugated modes are implicitly included throughout the paper. For the  $\Upsilon(2S)$  data sample, we have collected data corresponding to an integrated

luminosity of  $24.7 \text{ fb}^{-1}$  at a c.m. energy corresponding to the  $\Upsilon(2S)$  resonance. We determine the number of produced  $\Upsilon(2S)$  events to be  $(158 \pm 4) \times 10^6$  using inclusive hadronic decays. The continuum production of the final states is determined using an off-resonance data sample collected using an integrated luminosity of  $89.5 \text{ fb}^{-1}$  at  $\sqrt{s} = 10.52$  GeV. We use these two data samples to separate the dynamics of electromagnetic and strong charmed hadron production at the off-resonance energy and the  $\Upsilon(2S)$  peak.

We only include the following  $D_{sJ}^-$  states, which are both established and emit a kaon in their decay:  $D_{s1}(2536)^-$  and  $D_{s2}^*(2573)^-$ ; the kaon can be charged or neutral ( $K_S^0$ ). We use the technique of partial reconstruction for the  $D_{sJ}^-$  final state: the final state is tagged through the full reconstruction of the  $D_s^{(*)+}$ , and the recoiling  $D_{sJ}^-$  is tagged by a kaon produced in the decay  $D_{sJ}^- \rightarrow \bar{K} + \bar{D}^{(*)}$ . The remaining  $\bar{D}^{(*)}$  is observed indirectly through its recoil against the  $D_s^{(*)+} - \bar{K}$  system using the known kinematics of the initial state. This circumvents the problem of low efficiencies for reconstructing  $D$  mesons associated with the large variety of possible decay processes.

## II. THE BELLE DETECTOR AND MONTE CARLO SIMULATION

The Belle detector is a large-solid-angle magnetic spectrometer [10] using a silicon vertex detector, a 50-layer central drift chamber, an array of aerogel threshold Cherenkov counters, a barrel-like arrangement of time-of-flight scintillation counters, and an electromagnetic calorimeter (ECL) composed of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux return outside the coil is instrumented to detect  $K_L^0$  mesons and identify muons. The origin of the coordinate system is defined as the position of the nominal interaction point. In the cylindrical coordinates, the  $z$  axis is aligned with the direction opposite the  $e^+$  beam and points along the magnetic field within the solenoid, and  $r$  is the radial distance.

We simulate the full chain  $\Upsilon(2S)/e^+e^- \rightarrow D_s^{(*)+}D_{sJ}^-$ , in which  $D_{sJ}^-$  is  $D_{s1}(2536)^-$  or  $D_{s2}^*(2573)^-$ , using the EvtGen generator [11]. We simulate the angular distributions of  $D_s^{(*)+}D_{sJ}^-$  according to the  $J^P$  quantum numbers of  $D_s^{(*)+}$  and  $D_{sJ}^-$ . Here, we take  $J^P = 1^-$  for  $D_s^{*+}$  according to the recent BESIII measurement [12]. Four decay modes of  $D_{sJ}^-$  are simulated:  $K^- + \bar{D}^0$ ,  $K_S^0 + D^-$ ,  $K^- + \bar{D}^*(2007)^0$ , and  $K_S^0 + D^*(2010)^-$ . Again, the  $D$  mesons [ $\bar{D}^0$ ,  $D^-$ ,  $\bar{D}^*(2007)^0$ , and  $D^*(2010)^-$ ] are not reconstructed but determined in the recoil of the  $D_s^{(*)+}$  and the kaon from the  $D_{sJ}^-$  decay, so that the decays of  $D$  mesons are inclusive. We simulate the response of the Belle detector using a GEANT3-based Monte Carlo (MC) technique [13].

### III. EVENT SELECTION CRITERIA AND RECONSTRUCTION

We search for the tagging  $D_s^+$  using six final states:  $\phi\pi^+$ ,  $K_S^0 K^+$ ,  $\bar{K}^*(892)^0 K^+$ ,  $\rho^+\phi$ ,  $\eta\pi^+$ , and  $\eta'\pi^+$ . The decay of  $D_s^{*+}$  only proceeds through  $D_s^{*+} \rightarrow D_s^+\gamma$ .

We reconstruct  $D_s^{(*)+}D_{sJ}^-$  final states after initially selecting well-measured charged tracks and photon candidates. A well-measured charged track has an impact parameter  $dr < 1.5$  cm in the  $r-\phi$  plane with respect to the interaction point and a displacement  $|dz| < 5$  cm in the  $r-z$  plane. We require a transverse momentum larger than  $0.1$  GeV/ $c$ . We identify each charged track by combining the information from different detector subsystems and form the likelihood  $\mathcal{L}_i$  [14] for each particle species  $i$ , denoting  $\pi$  or  $K$ . Tracks with  $\mathcal{R}_K = \frac{\mathcal{L}_K}{\mathcal{L}_K + \mathcal{L}_\pi} > 0.6$  are treated as kaons, while those with  $\mathcal{R}_K < 0.4$  are assumed to be pions. The identification efficiency is about 95% for both  $K$  and  $\pi$ , with a 5% misidentification probability. Photons are identified as ECL clusters that do not align with any charged track.

We reconstruct the  $K_S^0$ ,  $\phi$ ,  $\bar{K}^*(892)^0$ , and  $\rho^+$  candidates in their respective decay channels into  $\pi^+\pi^-$ ,  $K^+K^-$ ,  $K^-\pi^+$ , and  $\pi^+\pi^0$ . For candidate  $K_S^0$  mesons, we use pairs of oppositely charged particles originating from a common vertex and assign the pion-mass hypothesis. We use a multivariate technique to improve the purity of the  $K_S^0$  candidate sample by rejecting combinatorial background [15], which we identify with neural network-based algorithms [16]. For the invariant mass ( $M_{\pi^+\pi^-}$ ) of  $\pi^+\pi^-$  pairs we obtain a resolution of  $\sigma \approx 5$  MeV/ $c^2$ , and we define the signal region for  $K_S^0$  by  $|M_{\pi^+\pi^-} - m_{K_S^0}| < 3\sigma$ . Here,  $m_{K_S^0}$  is the nominal mass of the  $K_S^0$  [7]. Correspondingly, we choose the range of all signal mass windows to have  $\Delta m = \pm 3\sigma$  around their respective nominal masses [7], unless stated otherwise. The corresponding resolution for the invariant mass of  $K^+K^-$  pairs,  $M_{K^+K^-}$ , is  $\sigma \approx 3.3$  MeV/ $c^2$ . The  $\bar{K}^*(892)^0$  meson has a natural width of 47.3 MeV, which is much larger than the experimental resolution for  $K^-\pi^+$  pairs. We define the  $\bar{K}^*(892)^0$  signal region to be  $|M_{K^-\pi^+} - m_{\bar{K}^*(892)^0}| < 105$  MeV/ $c^2$ , where  $M_{K^-\pi^+}$  is the invariant mass of  $K^-\pi^+$  and  $m_{\bar{K}^*(892)^0}$  is the nominal mass of  $\bar{K}^*(892)^0$  [7]. Since the width of the  $\rho^+$  of about 150 MeV is dominated by the natural width, the signal region is selected by  $|M_{\pi^+\pi^0} - m_{\rho^+}| < 200$  MeV/ $c^2$ , in which  $M_{\pi^+\pi^0}$  is the invariant mass of  $\pi^+\pi^0$  and  $m_{\rho^+}$  is the nominal mass of  $\rho^+$  [7].

We combine pairs of photons to form  $\pi^0$  candidates. For this, we require the energies of photons ( $E_\gamma$ ) from  $\pi^0$  decays to exceed  $E_\gamma > 25$  MeV in the barrel ( $32.2^\circ < \theta < 128.7^\circ$ ) and  $E_\gamma > 50$  MeV in the end caps ( $12.0^\circ < \theta < 31.4^\circ$  or  $131.5^\circ < \theta < 157.1^\circ$ ) of the ECL, with the polar angles specified in the laboratory frame. The two-photon mass

resolution for  $M_{\gamma\gamma}$  is  $\sigma \approx 5$  MeV/ $c^2$ . We reconstruct  $\eta$  mesons from their decays into  $\pi^+\pi^-\pi^0$  and  $\gamma\gamma$ . In the  $\gamma\gamma$  mode, we require  $E_\gamma > 150$  MeV. The corresponding mass resolution for  $M_{\pi^+\pi^-\pi^0}$  is  $\sigma \approx 4$  MeV/ $c^2$  and for  $M_{\gamma\gamma}$  the resolution is  $\sigma \approx 13.5$  MeV/ $c^2$ . For the selection of  $\eta'$  candidates, we use a combination of  $\eta$  and  $\pi^+\pi^-$  pairs. The invariant mass resolution for  $\eta\pi^+\pi^-$  is  $\sigma_{\eta\pi^+\pi^-} \approx 5$  MeV/ $c^2$ .

In Figs. 1(a) and 1(c), we show the combined distribution  $M_{h_1h_2}$  of  $M_{\phi\pi^+}$ ,  $M_{K_S^0 K^+}$ ,  $M_{\bar{K}^*(892)^0 K^+}$ ,  $M_{\rho^+\phi}$ ,  $M_{\eta\pi^+}$ , and  $M_{\eta'\pi^+}$  from the  $\Upsilon(2S)$  data sample (upper row) and the continuum data sample (lower row). We do not apply a mass constraint for  $\pi^0$ ,  $\eta$ , or  $\eta'$ . Instead, we take the advantage of the mass difference. Taking the  $D_s^+ \rightarrow \eta\pi^+$  with  $\eta \rightarrow \gamma\gamma$  as an example, we use  $M_{\eta\pi^+} = M_{\gamma\gamma\pi^+} - M_{\gamma\gamma} + m_\eta$ , where the invariant mass  $M_{\gamma\gamma\pi^+}$  ( $M_{\gamma\gamma}$ ) is calculated from the sum of the 4-momenta of  $\gamma\gamma\pi^+$  ( $\gamma\gamma$ ). In this way, the mass resolution of the  $D_s^+$  signal in  $M_{\eta\pi^+}$  is improved from 19.7 to 13.0 MeV/ $c^2$  according to signal MC simulation. We fit the  $D_s^+$  signal in  $M_{h_1h_2}$  with a Gaussian function and describe the background through a second-order polynomial function. We obtain a mass resolution of  $\sigma_{D_s^+} = 6.7 \pm 0.1$  MeV/ $c^2$  in data, which is used to define the signal region for  $D_s^+$ , while the corresponding resolution is 6.5 MeV/ $c^2$  in signal MC simulations. Besides the  $D_s^+$  signal, we also define sideband regions through  $|M_{h_1h_2} - m_{D_s^+} \pm 9\sigma_{D_s^+}| < 3\sigma_{D_s^+}$ . Since the fraction of multicomination in  $D_s^+$  reconstruction is only about 3%, we allow multiple candidates of  $D_s^+$  in one event.

We reconstruct  $D_s^{*+}$  candidates from the above  $D_s^+$  sample using the  $\gamma D_s^+$  final state. For this, we require the photon energy to exceed  $E_\gamma > 50$  MeV in the barrel and  $E_\gamma > 100$  MeV in the end caps of the ECL. The corresponding invariant mass distributions  $M_{\gamma D_s^+}$  for  $\gamma D_s^+$  from the  $\Upsilon(2S)$  and continuum data samples are shown in Figs. 1(b) and 1(d). Here, we use  $M_{\gamma D_s^+} = M_{\gamma h_1 h_2} - M_{h_1 h_2} + m_{D_s^+}$ , where the invariant mass  $M_{\gamma h_1 h_2}$  is calculated from the sum of the 4-momenta of  $\gamma h_1 h_2$ . We fit to the  $M_{\gamma D_s^+}$  distribution between 2.07 and 2.15 GeV/ $c^2$  using two Gaussian functions for the  $D_s^{*+}$  signal and a second-order polynomial function for the background. We use  $\sigma \equiv \sqrt{f_1 \times (\sigma_1^2 + m_1^2) + f_2 \times (\sigma_2^2 + m_2^2) - m^2}$  with  $m = f_1 \times m_1 + f_2 \times m_2$  to define the mass resolution of the  $D_s^{*+}$  signals, where  $m_1$  ( $m_2$ ),  $\sigma_1$  ( $\sigma_2$ ), and  $f_1$  ( $f_2$ ) are the mean, the standard deviation, and the fraction of the first (second) Gaussian function. We obtain the mass resolution of  $\sigma_{D_s^{*+}} = 6.7 \pm 0.4$  MeV/ $c^2$  in data and 7.0 MeV/ $c^2$  in signal MC simulations, which agree well with each other. Again, in addition to the signal region for  $D_s^{*+}$ , we define sideband regions through  $|M_{\gamma D_s^+} - m_{D_s^{*+}} \pm 9\sigma_{D_s^{*+}}| < 3\sigma_{D_s^{*+}}$ . As we aim to study the  $D_s^{*+}\bar{K}$  recoil spectrum, we apply mass-constrained

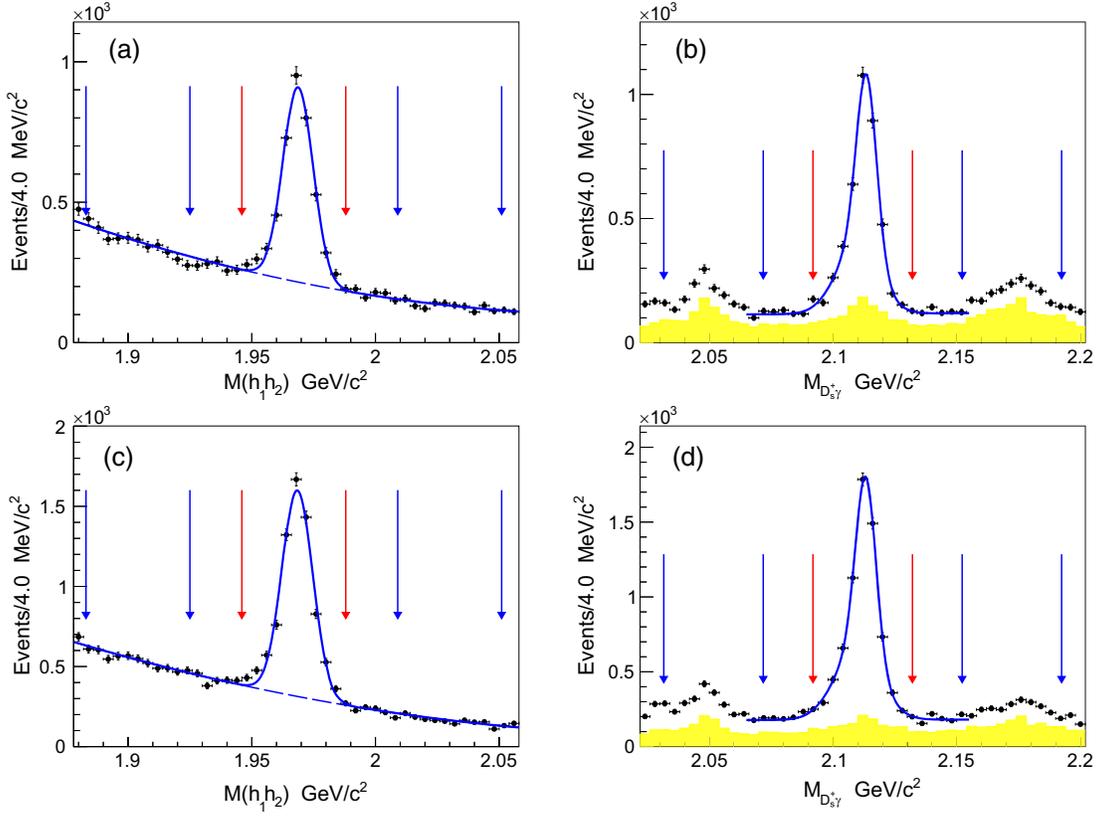


FIG. 1. Invariant mass distributions of (a),(c) the combinations of  $\phi\pi^+$ ,  $K_S^0 K^+$ ,  $\bar{K}^*(892)^0 K^+$ ,  $\rho^+\phi$ ,  $\eta\pi^+$ , and  $\eta'\pi^+$  for  $D_s^+$  candidates and (b),(d) the combinations of  $\gamma D_s^+$  for  $D_s^{(*)+}$  candidates in  $\Upsilon(2S)$  data sample (upper row) and continuum data sample (lower row). The red arrows show the signal region of  $D_s^+$  or  $D_s^{(*)+}$ , and the blue arrows show the sideband regions. The shaded histogram in (b) and (d) shows backgrounds estimated from  $D_s^+$  mass sidebands. The curves show the best fit results using Gaussian functions for the  $D_s^+$  and  $D_s^{(*)+}$  signals, respectively.

fits to the  $D_s^{*+}$  candidates in the signal region to improve their momentum resolution. We find that 10% of the events have multiple  $D_s^{*+}$  candidates. In these cases, we select the candidate with the minimum  $\chi^2$  from the mass-constraint fit. For the candidates in each  $D_s^{*+}$  mass sideband, we apply the mass constraint to the center of the sideband and select the combination with minimum  $\chi^2$  as well. To estimate the size of the peaking component in the selected  $D_s^{*+}$  sample due to the minimum  $\chi^2$  requirement, we apply the same mass constraints to events in the  $D_s^+$  sidebands. As shown in Figs. 1(b) and 1(d), the  $D_s^+$  mass sideband events can describe the peaks in the  $D_s^{*+}$  mass sidebands and therefore can be used to estimate reliably the peaking component in the  $D_s^{*+}$  mass signal region. Events with  $|M_{\gamma D_s^+} - m_{D_s^+}| < 50 \text{ MeV}/c^2$  are removed for the  $D_s^+ D_{sJ}^-$  search.

The search for  $\bar{D}^{(*)}$  requires a  $\bar{K}$  meson reconstructed in addition to  $D_s^{(*)+}$ . We determine the  $\bar{D}^{(*)}$  signal through the recoil of  $D_s^{(*)+} \bar{K}$  using the calculated mass,

$$\begin{aligned}
 M_{\bar{D}^{(*)}} &= M_{D_s^{(*)+} \bar{K}}^{\text{recoil}} \\
 &\equiv \sqrt{(E_{\text{c.m.}} - E_{D_s^{(*)+}} - E_{\bar{K}})^2 - (\vec{p}_{\text{c.m.}} - \vec{p}_{D_s^{(*)+}} - \vec{p}_{\bar{K}})^2},
 \end{aligned} \tag{1}$$

and isolate the possible production of  $D_{sJ}^-$  states in the  $\bar{K} \bar{D}^{(*)}$  final states through their recoil using the following equation:

$$\begin{aligned}
 M_{\bar{K} \bar{D}^{(*)}} &= M_{D_s^{(*)+}}^{\text{recoil}} \\
 &\equiv \sqrt{(E_{\text{c.m.}} - E_{D_s^{(*)+}})^2 - (\vec{p}_{\text{c.m.}} - \vec{p}_{D_s^{(*)+}})^2}.
 \end{aligned} \tag{2}$$

Here,  $E_{\text{c.m.}}$  and  $\vec{p}_{\text{c.m.}}$  are the energy and 3-momentum of  $e^+ e^-$  in the collision system,  $E_{D_s^{(*)+}}$  ( $E_{\bar{K}}$ ) and  $\vec{p}_{D_s^{(*)+}}$  ( $\vec{p}_{\bar{K}}$ ) are those of  $D_s^{(*)+}$  ( $\bar{K}$ ), respectively. We show the  $M_{D_s^{(*)+} \bar{K}}^{\text{recoil}}$  distributions versus  $M_{D_s^{(*)+}}^{\text{recoil}}$  from the two data samples in Figs. 2(a) and 2(b), and the signal MC simulations of  $\Upsilon(2S)$

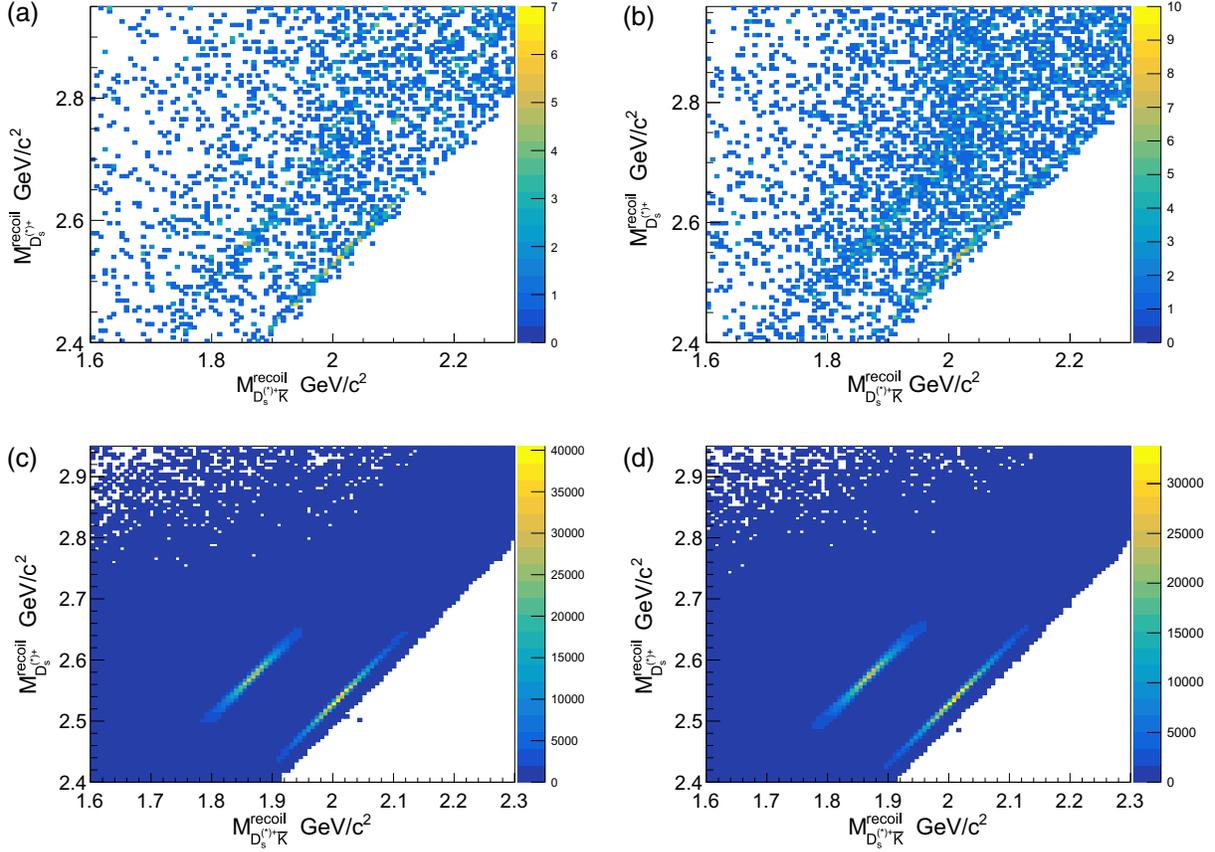


FIG. 2. The distributions of the recoil mass against  $D_s^{(*)+} \bar{K}$  versus the recoil mass against  $D_s^{(*)+}$  in (a) the  $\Upsilon(2S)$  data sample, (b) the continuum data sample at  $\sqrt{s} = 10.52$  GeV, (c) the signal MC simulation of  $\Upsilon(2S)$  decays, and (d) the signal MC simulation of continuum production at  $\sqrt{s} = 10.52$  GeV.

decays and continuum production in Figs. 2(c) and 2(d). There are clear bands in the distributions of data corresponding to the production of the  $D_{s1}(2536)^-$  signal in the  $\bar{D}^* \bar{K}$  [ $\bar{D}^*(2007)^0 K^-$  or  $D^*(2010)^- K_S^0$ ] final state and  $D_{s2}^*(2573)^-$  signal in the  $\bar{D} \bar{K}$  ( $\bar{D}^0 K^-$  or  $D^- K_S^0$ ) final state, and they agree well with the signal MC simulations. The mass resolutions of  $M_{D_s^{(*)+} \bar{K}}^{\text{recoil}}$  and  $M_{D_s^{(*)+}}^{\text{recoil}}$  are large due to the common variables  $E_{D_s^{(*)+}}$  and  $\vec{p}_{D_s^{(*)+}}$  in Eqs. (1) and (2). The mass resolution of  $\bar{D}$  from the decay of  $D_{sJ}^-$  in  $M_{D_s^{(*)+} \bar{K}}^{\text{recoil}}$  is about  $50 \text{ MeV}/c^2$ , and the signal region is defined to be  $|M_{D_s^{(*)+} \bar{K}}^{\text{recoil}} - m_{\bar{D}}| < 150 \text{ MeV}/c^2$ . We fit the  $\bar{D}^*$  mass distribution with two Gaussian functions and obtain the narrower one with a mass resolution of  $31.8 \pm 0.3 \text{ MeV}/c^2$  and a signal fraction of about 34% and the wider one with a mass resolution of  $74.2 \pm 1.0 \text{ MeV}/c^2$  and a signal fraction of about 66%. We define the signal region to be  $|M_{D_s^{(*)+} \bar{K}}^{\text{recoil}} - m_{\bar{D}^*}| < 200 \text{ MeV}/c^2$ , which has a selection efficiency of about 95%. Here,  $m_{\bar{D}}$  is the nominal mass of  $\bar{D}^0$  or  $D^-$ , and  $m_{\bar{D}^*}$  is the nominal mass of the  $\bar{D}^*(2007)^0$  or  $D^*(2010)^-$  [7]. With the events in the  $D_s^+$  or

$D_s^{*+}$  mass sidebands, no peaking background is found for the  $\bar{D}^{(*)}$  signal in the  $M_{D_s^{(*)+} \bar{K}}^{\text{recoil}}$  distributions.

To improve the mass resolution of  $M_{\bar{K} \bar{D}^{(*)}}$ , we use the following formula to replace Eq. (2):

$$M_{\bar{K} \bar{D}^{(*)}} = M_{D_s^{(*)+}}^{\text{recoil}} - M_{D_s^{(*)+} \bar{K}}^{\text{recoil}} + m_{\bar{D}^{(*)}}. \quad (3)$$

In this way, the uncertainties due to the 4-momentum of final states from  $D_s^{(*)+}$  decays are significantly reduced. From simulation, we obtain the resolutions for  $\Delta M^{\text{recoil}} \equiv M_{D_s^{(*)+}}^{\text{recoil}} - M_{D_s^{(*)+} \bar{K}}^{\text{recoil}}$  of  $\sigma_{\Delta M^{\text{recoil}}} < 5 \text{ MeV}/c^2$  for all  $D_s^{(*)+} D_{sJ}^-$  final states. In Figs. 3 and 4 we show the distributions for  $\Delta M^{\text{recoil}} + m_{\bar{D}^*}$  for  $M_{\bar{K} \bar{D}^*}$  and  $\Delta M^{\text{recoil}} + m_{\bar{D}}$  for  $M_{\bar{K} \bar{D}}$  for the two data samples. We observe clear signals for both  $D_{s1}(2536)^-$  and  $D_{s2}^*(2573)^-$ .

We determine the  $D_{sJ}^-$  signal yields,  $N_{\Upsilon(2S)}^{\text{sig}}$  of the  $\Upsilon(2S)$  decays and  $N_{\text{cont}}^{\text{sig}}$  of the continuum production at  $\sqrt{s} = 10.52$  GeV, by simultaneously fitting the  $M_{\bar{K} \bar{D}^{(*)}}$  distributions for the  $\Upsilon(2S)$  data sample and the continuum data sample and with common isospin ratios

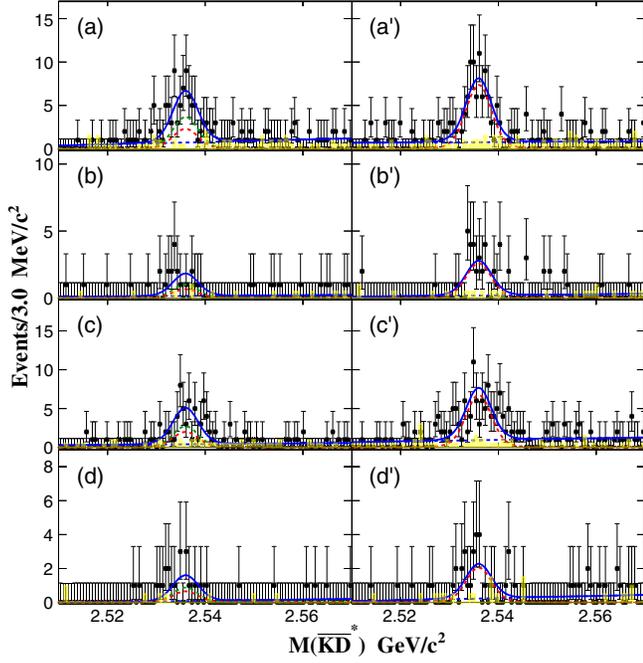


FIG. 3. The invariant mass distributions of  $\bar{K}\bar{D}^*$  calculated in the recoil mass for  $D_s^{(*)+}$  in the (a)  $D_s^+K^-\bar{D}^*(2007)^0$ , (b)  $D_s^+K_S^0D^{*(2010)-}$ , (c)  $D_s^{*+}K^-\bar{D}^*(2007)^0$ , and (d)  $D_s^{*+}K_S^0D^{*(2010)-}$  final states from the  $\Upsilon(2S)$  data sample (a)–(d) and the continuum data sample at 10.52 GeV (a')–(d'). The shaded histograms show the backgrounds estimated from the normalized  $D_s^{(*)+}$  mass sidebands. The solid curves show the best fit result; the dashed green ones are  $D_{s1}(2536)^-$  signals in  $\Upsilon(2S)$  decays, and the dashed red curves are the  $D_{s1}(2536)^-$  signals in continuum production at 10.02 GeV (a)–(d) and 10.52 GeV (a')–(d').

$R_{\text{iso},J} \equiv \mathcal{B}(D_{sJ}^- \rightarrow K_S^0 D^{(*)-}) / \mathcal{B}(D_{sJ}^- \rightarrow K^- \bar{D}^{(*)0})$  between the  $K_S^0 D^{(*)-}$  and  $K^- \bar{D}^{(*)0}$  final states. In the fits, we use  $N_{\Upsilon(2S)}^{\text{sig}}$  and  $N_{\text{cont}}^{\text{sig}}$  for the  $K^- \bar{D}^{(*)0}$  modes, and those of the  $K_S^0 D^{(*)-}$  modes are calculated via the isospin ratios  $R_{\text{iso},J}$  and the ratios of efficiencies and branching fractions between the  $K_S^0 D^{(*)-}$  modes and the  $K^- \bar{D}^{(*)0}$  modes. The fit function is the sum of a Breit-Wigner function (BW) convolved with a Gaussian function with a width corresponding to the mass resolution and a linear function to describe the backgrounds. The mass and width of the BW functions are fixed to the world average values for  $D_{s1}(2536)^-$  and  $D_{s2}^*(2573)^-$  [7]. The mass resolutions used in the Gaussian are obtained from MC simulations and are about 2.4 MeV/ $c^2$  (6.5 MeV/ $c^2$ ) for  $D_{s1}(2536)^-$  [ $D_{s2}^*(2573)^-$ ]. We include the branching fractions and reconstruction efficiencies corresponding to the  $D_s^{(*)+} D_{sJ}^-$  final states in the fits. The results are listed in Table I for each channel and each dataset.

We estimate the contribution of continuum production to the  $D_s^{(*)+} D_{sJ}^-$  signal in the  $\Upsilon(2S)$  data sample. For this,

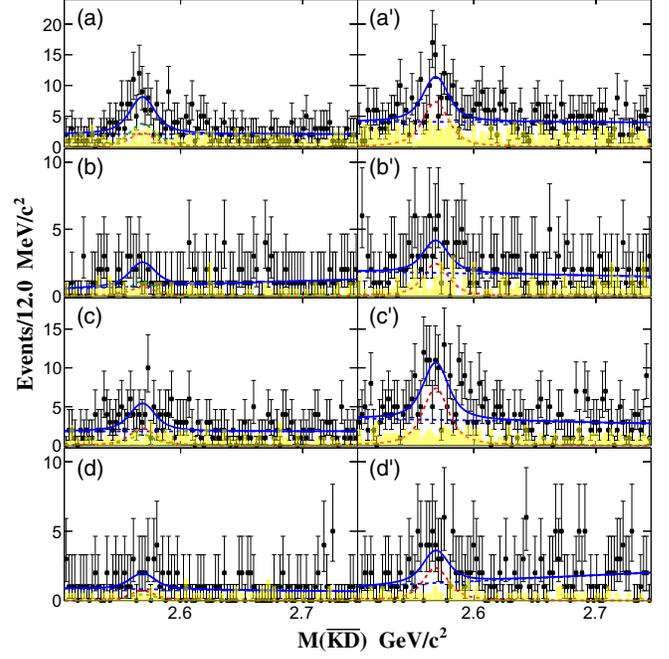


FIG. 4. The invariant mass distributions of  $\bar{K}\bar{D}$  calculated in recoil mass for  $D_s^{(*)+}$  in the (a)  $D_s^+K^-\bar{D}^0$ , (b)  $D_s^+K_S^0D^-$ , (c)  $D_s^{*+}K^-\bar{D}^0$ , and (d)  $D_s^{*+}K_S^0D^-$  final states from the  $\Upsilon(2S)$  data sample (a)–(d) and the continuum data sample at 10.52 GeV (a')–(d'). The shaded histograms show the backgrounds estimated from the normalized  $D_s^{(*)+}$  mass sidebands. The solid curves show the best fit result; the dashed green ones are  $D_{s2}^*(2573)^-$  signals in  $\Upsilon(2S)$  decays, and the dashed red curves are the  $D_{s2}^*(2573)^-$  signals in continuum production at 10.02 GeV (a)–(d) and 10.52 GeV (a')–(d').

we scale the luminosities and correct for the c.m. energy dependence of the QED cross section  $\sigma_{e^+e^-} \propto 1/s$ , resulting in a scale factor  $f_{\text{scale}} = (\mathcal{L}_{\Upsilon(2S)} \times s_{\text{cont}}) / (\mathcal{L}_{\text{cont}} \times s_{\Upsilon(2S)}) = 0.304$ . Here,  $\mathcal{L}_{\Upsilon(2S)}$  and  $\mathcal{L}_{\text{cont}}$  are the integrated luminosities of the  $\Upsilon(2S)$  data sample at  $\sqrt{s_{\Upsilon(2S)}} = 10.02$  GeV and the continuum data sample at  $\sqrt{s_{\text{cont}}} = 10.52$  GeV. Therefore, the yield of signal events produced via continuum  $e^+e^-$  annihilation in the  $\Upsilon(2S)$  data sample is  $f_{\text{scale}} \times N_{\text{cont}}^{\text{sig}}$ .

In principle, there is interference between the resonance and continuum amplitudes in the  $\Upsilon(2S)$  data sample [17]. We do not consider this effect in the simultaneous fit but consider it in the systematic uncertainty in Sec. IV.

We determine the statistical significance of  $D_{sJ}^-$  by comparing the value of  $\Delta(-2 \ln L) = -2 \ln(L_{\text{max}}/L_0)$  and the change in the number of free parameters in the fits, where  $L_{\text{max}}$  is the likelihood with  $D_{sJ}^-$  and  $L_0$  without  $D_{sJ}^-$ . The statistical significance in the  $\Upsilon(2S)$  data sample for  $D_{s1}(2536)^-$  and  $D_{s2}^*(2573)^-$  is  $6.8\sigma$  and  $4.0\sigma$ , respectively, and  $18.3\sigma$  and  $10.1\sigma$  in the continuum data sample. From these yields, we calculate the branching

TABLE I. The branching fractions of  $\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-$  decays, the Born cross sections of continuum production  $e^+e^- \rightarrow D_s^{(*)+} D_{sJ}^-$ , and the isospin ratio  $\mathcal{B}(D_{sJ}^- \rightarrow K_S^0 D^{(*)-})/\mathcal{B}(D_{sJ}^- \rightarrow K^- \bar{D}^{(*)0})$  based on the results from the simultaneous fits. Here,  $N_{\Upsilon(2S)}^{\text{sig}}$ ,  $N_{\text{cont}}^{\text{sig}}$ ,  $\mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K} \bar{D}^{(*)})$ , and  $\sigma^{\text{B}}(e^+e^- \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K} \bar{D}^{(*)})$  are described in Eqs. (4) and (5).  $\sum \varepsilon_i \mathcal{B}_i$  is the sum of the products of the reconstruction efficiencies and branching fractions in  $D_s^+$ 's six decay channels. The significance is the statistical significance of the  $D_s^{(*)+} D_{sJ}^-$  signals with  $D_{sJ}^- \rightarrow K^- D^{(*)0}$  and  $K_S^0 D^{(*)-}$  in  $\Upsilon(2S)$  decays and continuum productions. The  $K^- D^{(*)0}$  and  $K_S^0 D^{(*)-}$  modes of the  $D_{sJ}^-$  decays are connected by the isospin ratio  $\mathcal{B}(D_{sJ}^- \rightarrow K_S^0 D^{(*)-})/\mathcal{B}(D_{sJ}^- \rightarrow K^- \bar{D}^{(*)0})$  in the simultaneous fits. The systematic uncertainties of  $N^{\text{sig}}$  are of the simultaneous fits only.

Final state ( $f$ )	$N_{\Upsilon(2S)}^{\text{sig}}$		Significance ( $\sigma$ )	$\mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K} \bar{D}^{(*)}) (\times 10^{-5})$	
	$K^-$ mode	$K_S^0$ mode		$K^-$ mode	$K_S^0$ mode
$D_s^+ D_{s1}(2536)^-$	$43 \pm 9 \pm 2$	$14 \pm 3 \pm 2$	5.3	$1.6 \pm 0.3 \pm 0.2$	$0.84 \pm 0.18 \pm 0.15$
$D_s^{*+} D_{s1}(2536)^-$	$31 \pm 8 \pm 2$	$10 \pm 3 \pm 2$	4.3	$1.4 \pm 0.4 \pm 0.2$	$0.82 \pm 0.25 \pm 0.19$
$D_s^+ D_{s2}^*(2573)^-$	$51 \pm 15 \pm 5$	$17 \pm 5 \pm 5$	3.8	$1.4 \pm 0.4 \pm 0.2$	$0.69 \pm 0.20 \pm 0.22$
$D_s^{*+} D_{s2}^*(2573)^-$	$20 \pm 12 \pm 2$	$7 \pm 4 \pm 4$	1.6	$0.9 \pm 0.5 \pm 0.2$	$0.54 \pm 0.31 \pm 0.47$

...	$N_{\text{cont}}^{\text{sig}}$		...	$\sigma^{\text{B}}(e^+e^- \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K} \bar{D}^{(*)}) (\text{fb})$	
	$K^-$ mode	$K_S^0$ mode		$K^-$ mode	$K_S^0$ mode
$D_s^+ D_{s1}(2536)^-$	$86 \pm 10 \pm 2$	$28 \pm 4 \pm 2$	13.9	$67 \pm 8 \pm 6$	$34 \pm 5 \pm 4$
$D_s^{*+} D_{s1}(2536)^-$	$79 \pm 10 \pm 2$	$25 \pm 4 \pm 2$	11.8	$84 \pm 11 \pm 11$	$41 \pm 6 \pm 6$
$D_s^+ D_{s2}^*(2573)^-$	$102 \pm 17 \pm 21$	$33 \pm 8 \pm 5$	7.1	$56 \pm 9 \pm 13$	$27 \pm 6 \pm 5$
$D_s^{*+} D_{s2}^*(2573)^-$	$102 \pm 16 \pm 6$	$33 \pm 7 \pm 4$	7.6	$106 \pm 17 \pm 12$	$51 \pm 11 \pm 9$

$\sum \varepsilon_i \mathcal{B}_i$ (%)					
Final state ( $f$ )	$D_s^+ D_{s1}(2536)^-$	$D_s^{*+} D_{s1}(2536)^-$	...	$D_s^+ D_{s2}^*(2573)^-$	$D_s^{*+} D_{s2}^*(2573)^-$
$K^-$ mode	$1.63 \pm 0.07$	$1.06 \pm 0.05$	...	$1.19 \pm 0.05$	$0.77 \pm 0.03$
$K_S^0$ mode	$2.32 \pm 0.10$	$1.56 \pm 0.07$	...	$1.22 \pm 0.05$	$0.82 \pm 0.03$

Isospin ratio $\mathcal{B}(D_{sJ}^- \rightarrow K_S^0 D^{(*)-})/\mathcal{B}(D_{sJ}^- \rightarrow K^- \bar{D}^{(*)0})$	
$D_{s1}(2536)^-$ decays	$0.48 \pm 0.07 \pm 0.02$
$D_{s2}^*(2573)^-$ decays	$0.49 \pm 0.10 \pm 0.02$

fraction of  $\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-$  and the Born cross section for  $e^+e^- \rightarrow D_s^{(*)+} D_{sJ}^-$  by

$$\begin{aligned} & \mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K} \bar{D}^{(*)}) \\ &= \frac{N_{\Upsilon(2S)}^{\text{sig}}}{N_{\Upsilon(2S)} \times \sum \varepsilon_i \mathcal{B}_i}, \end{aligned} \quad (4)$$

and

$$\begin{aligned} & \sigma^{\text{B}}(e^+e^- \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K} \bar{D}^{(*)}) \\ &= \frac{N_{\text{cont}}^{\text{sig}} \times |1 - \Pi|^2}{\mathcal{L}_{\text{cont}} \times \sum \varepsilon_i \mathcal{B}_i \times (1 + \delta_{\text{ISR}})}. \end{aligned} \quad (5)$$

Here,  $i$  identifies the mode of  $D_s^+ \rightarrow h_1 h_2$  decay, while  $\varepsilon_i$  and  $\mathcal{B}_i$  are their reconstruction efficiencies and branching fractions. We calculate  $\sum \varepsilon_i \mathcal{B}_i$  according to signal MC simulations for  $\varepsilon_i$  and the world average values of  $\mathcal{B}_i$  [7]

and list the values in Table I. Their errors are mainly due to the uncertainties of  $\mathcal{B}_i$  from world averages [7], since all of the relative statistical uncertainties due to the statistics of MC simulations are less than 0.5%. Here, we take into account the branching fraction of  $K_S^0 \rightarrow \pi^+ \pi^-$  decay [7]. From the Born cross sections we can calculate the full ‘‘dressed’’ cross section through  $\sigma^{\text{dressed}} = \sigma^{\text{Born}}/|1 - \Pi|^2$ . The factor  $|1 - \Pi|^2 = 0.931$  is the vacuum polarization factor [18,19]. In addition, we have to correct for radiative effects. The radiative correction factor  $1 + \delta_{\text{ISR}}$  is determined by  $\int \sigma^{\text{dressed}}(s(1-x))F(x,s)dx/\sigma^{\text{dressed}}(s)$  and has the value 0.82, where  $F(x,s)$  is the radiative function obtained from a QED calculation with an accuracy of 0.2% [20–22].

We summarize the branching fractions of  $\Upsilon(2S)$  decays and the Born cross sections of continuum production in Table I. The number of corrected signal events in the  $\Upsilon(2S)$  data sample is  $20 \pm 12 \pm 2$  for the  $D_s^{*+} D_{s2}^*(2573)^-$  decay, from which we derive a statistical significance of only  $1.6\sigma$ .

We integrate the likelihood versus the number of  $D_s^{*+} D_{s2}^*(2573)^-$  signal events and determine its upper limit at 90% confidence level (C.L.) to be  $N^{\text{UL}}(\Upsilon(2S) \rightarrow D_s^{*+} D_{s2}^*(2573)^-) < 44$  in the  $D_{s2}^*(2573)^- \rightarrow K^- \bar{D}^0$  mode, which has been degraded by a factor of  $1/(1 - \delta_{\text{sys}})$  to account for the systematic uncertainties detailed below. We obtain an upper limit for the production in  $\Upsilon(2S)$  decay of  $\mathcal{B}^{\text{UL}}(\Upsilon(2S) \rightarrow D_s^{*+} D_{s2}^*(2573)^-) \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K^- \bar{D}^0) < 2.9 \times 10^{-5}$ .

In addition, we determine the isospin ratios  $R_{\text{iso},J}$  from the simultaneous fits to be  $R_{\text{iso},1} = 0.48 \pm 0.07 \pm 0.02$  and  $R_{\text{iso},2} = 0.49 \pm 0.10 \pm 0.02$  for the  $D_{s1}(2536)^-$  and  $D_{s2}^*(2573)^-$ , respectively. These ratios are in good agreement with the expectation from isospin symmetry, which are 0.498 and 0.497 from a calculation taking into account the phase space. Replacing the  $N_{\Upsilon(2S)}^{\text{sig}}$  and  $N_{\text{cont}}^{\text{sig}}$  of  $K^- \bar{D}^{(*)0}$  modes with those of the  $K_S^0 D^{(*)-}$  modes in the simultaneous fits, and calculating those of the  $K^- \bar{D}^{(*)0}$  modes with  $R_{\text{iso},J}$  and the ratios of efficiencies and branching fractions between the  $K^- \bar{D}^{(*)0}$  modes and the  $K_S^0 D^{(*)-}$  modes, we obtain the new fit results and calculate the  $\mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K} \bar{D}^{(*)})$  and  $\sigma^{\text{B}}(e^+ e^- \rightarrow D_s^{(*)+} D_{sJ}^-) \mathcal{B}(D_{sJ}^- \rightarrow \bar{K} \bar{D}^{(*)})$  of the  $D_{sJ}^- \rightarrow K_S^0 D^{(*)-}$  decay modes, as listed in Table I. We get the same fit results of the isospin ratios  $R_{\text{iso},J}$  of the  $D_{s1}(2536)^-$  and

$D_{s2}^*(2573)^-$  decays. We also determine the  $N^{\text{UL}}(\Upsilon(2S) \rightarrow D_s^{*+} D_{s2}^*(2573)^-) < 15$  in the  $K_S^0 D^-$  mode and  $\mathcal{B}^{\text{UL}}(\Upsilon(2S) \rightarrow D_s^{*+} D_{s2}^*(2573)^-) \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K_S^0 D^-) < 3.0 \times 10^{-5}$  at 90% C.L.

#### IV. SYSTEMATIC UNCERTAINTIES

The determination of the branching fractions in  $\Upsilon(2S)$  decays and the Born cross sections of continuum productions have various systematic uncertainties, which are listed in Table II. The particle identification uncertainty for  $K^\pm$  is 1.1% and 0.9% per  $\pi^\pm$  [14]; the uncertainty of the tracking efficiency per track is 0.35% and is added linearly; the photon reconstruction uncertainty is 2% for each photon. The uncertainties of the efficiency of mass window requirements due to data and MC differences in mass resolutions for  $\pi^0$ ,  $K_S^0$ ,  $\bar{K}^*(892)^0$ ,  $\rho^+$ ,  $\phi$ ,  $\eta$ , and  $\eta'$  are measured to be 0.2%, 0.2%, 1.0%, 1.4%, 0.1%, 1.7%, and 0.3%, respectively. We take the decay branching fractions and their uncertainties of the intermediate states  $\bar{K}^*(892)^0$ ,  $\eta$ ,  $\rho^+$ ,  $\eta'$ , and  $D_s^{(*)+}$  from Ref. [7]. We determine the efficiency of the  $D_s^+$  ( $D_s^{*+}$ ) mass window to be  $(99.9 \pm 0.1)\%$  [ $(99.8 \pm 0.1)\%$ ] in data and 97.4% (99.5%) in the simulation, and we attribute a systematic uncertainty of 2.5% (0.3%); the differences in these numbers for data and simulation reflect the different mass resolutions obtained

TABLE II. The summary of systematic uncertainties (%) of  $D_s^{(*)+} K$  reconstruction. Additional uncertainties due to the angular distributions are 6.9%, 8.5%, 8.5%, and 9.2% for the  $D_s^+ D_{s1}(2536)^-$ ,  $D_s^+ D_{s2}^*(2573)^-$ ,  $D_s^{*+} D_{s1}(2536)^-$ , and  $D_s^{*+} D_{s2}^*(2573)^-$ , respectively.

Source	$D_s^+$ reconstruction						$K^-$	$K_S^0$
	$D_s^+$ decay mode							
	$\phi\pi^+$	$K_S^0 K^+$	$\bar{K}^*(892)^0 K^+$	$\rho^+ \phi$	$\eta\pi^+(\gamma\gamma/\pi^+\pi^-\pi^0)$	$\eta'\pi^+(\gamma\gamma/\pi^+\pi^-\pi^0)$		
$K$ ID	2.20	1.10	2.20	2.20	...	...	1.10	...
$\pi$ ID	0.90	...	0.90	0.90	0.90/2.70	2.70/4.50	...	...
Tracking	1.05	1.05	1.05	1.05	0.35/1.05	1.05/1.75	0.35	...
$K_S^0$ reconstruction	...	2.23	...	...	...	...	...	2.23
$\pi^0$ reconstruction	...	...	...	2.25	2.25/...	2.25/...	...	...
Photon reconstruction	...	...	...	...	4.0/...	4.0/...	...	...
Mass windows of intermediate states	0.07	0.20	0.97	1.44	0.23/1.68	0.26/1.69	...	0.20
$\mathcal{B}$ s of intermediate-state decays	0.08	0.08	0.08	1.12	0.04/0.03	0.04/0.03	...	...
$D_s^+$ mass window	0.43	0.67	0.19	0.79	1.07	1.20	...	...
$D_s^{*+}$ mass window	0.38	1.01	0.10	0.34	0.94	0.61	...	...

Source	Reconstruction mode			
	$D_s^+ K^-$	$D_s^+ K_S^0$	$D_s^{*+} K^-$	$D_s^{*+} K_S^0$
$D_s^{(*)+} K$ reconstruction	3.2	3.2	3.3	3.4
$\mathcal{B}(D_s^{*+} \rightarrow \gamma D_s^+)$	...	...	0.7	0.7
Trigger	1.0	1.0	1.0	1.0
MC statistics	0.2	0.2	0.2	0.2
$N_{\Upsilon(2S)}$ (luminosity)	2.2(1.4)	2.2(1.4)	2.2(1.4)	2.2(1.4)
Sum in quadrature	4.0(3.6)	4.0(3.6)	4.2(3.8)	4.2(3.9)

TABLE III. The parameters  $A = \sqrt{\sigma^B(e^+e^- \rightarrow D_s^{(*)+}D_{sJ}^-)/\mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+}D_{sJ}^-)}$  from the results of the simultaneous fits and the maximum effect of interference term  $F_{\text{int}}^{\text{max}}$  in the  $\Upsilon(2S)$  decays. The values of  $A(K^-)$  and  $F_{\text{int}}^{\text{max}}(K^-)$  are for the  $K^-$  mode, and the values of  $A(K_S^0)$  and  $F_{\text{int}}^{\text{max}}(K_S^0)$  are for the  $K_S^0$  mode.

	$D_s^+D_{s1}(2536)^-$	$D_s^{*+}D_{s1}(2536)^-$	$D_s^+D_{s2}^*(2573)^-$	$D_s^{*+}D_{s2}^*(2573)^-$
$A(K^-)(\text{nb}^{1/2})$	$2.05 \pm 0.23$	$2.45 \pm 0.38$	$2.00 \pm 0.32$	$3.43 \pm 0.99$
$A(K_S^0)(\text{nb}^{1/2})$	$2.00 \pm 0.25$	$2.23 \pm 0.38$	$1.98 \pm 0.36$	$3.07 \pm 0.94$
$F_{\text{int}}^{\text{max}}(K^-)(\%)$	8.0	9.7	8.0	14.4
$F_{\text{int}}^{\text{max}}(K_S^0)(\%)$	7.9	9.0	8.2	13.2

for both. We determine these total uncertainties of  $\sum \varepsilon_i \times \mathcal{B}_i$  to be 3.2%, 3.2%, 3.3%, and 3.4% in the final states of  $D_s^+K^-\bar{D}^{(*)0}$ ,  $D_s^+K_S^0D^{(*)-}$ ,  $D_s^{*+}K^-\bar{D}^{(*)0}$ , and  $D_s^{*+}K_S^0D^{(*)-}$ , respectively. To estimate the systematic uncertainty in the angular distribution of  $D_s^{(*)+}D_{sJ}^-$ , we generate new MC samples uniformly in phase space, and half of the efficiency differences are taken to be the systematic uncertainties. We obtain the systematic uncertainties of 6.9%, 8.5%, 8.5%, and 9.2% for the  $D_s^+D_{s1}(2536)^-$ ,  $D_s^+D_{s2}^*(2573)^-$ ,  $D_s^{*+}D_{s1}(2536)^-$ , and  $D_s^{*+}D_{s2}^*(2573)^-$ , respectively. The uncertainty of the total number of  $\Upsilon(2S)$  events is 2.2%. The uncertainties in the integrated luminosities for the two data samples are 1.4% and highly correlated, but they cancel in the scale factor. We estimate the systematic uncertainty in determining the  $\mathcal{B}^{\text{UL}}(\Upsilon(2S) \rightarrow D_s^{*+}D_{s2}^*(2573)^-)\mathcal{B}(D_{s2}^*(2573)^- \rightarrow K^-\bar{D}^0)$  to be  $\delta_{\text{sys}} = 10.1\%$ . Besides those listed in Table II, there are additional systematic uncertainties of the scale factor  $f_{\text{scale}}$  and the radiative correction factor  $1 + \delta_{\text{ISR}}$ , where ISR is the initial state radiation. By changing  $s_{\text{cont}}/s_{\Upsilon(2S)}$  to  $[s_{\text{cont}}/s_{\Upsilon(2S)}]^{1.5}$ , the value of  $f_{\text{scale}}$  changes from 0.304 to 0.319, and we take 4.9% to be its systematic uncertainty. By varying the photon energy cutoff 50 MeV in the simulation of ISR, we determine the change of  $1 + \delta_{\text{ISR}}$  to be 0.01 and take 1.0% to be the conservative systematic uncertainty.

Various systematic uncertainties are considered in the simultaneous fit. We change the fit range from [2.51, 2.57] to [2.51, 2.62] GeV/ $c^2$  for the  $D_{s1}(2536)^-$  signals, and from [2.50, 2.74] to [2.50, 2.79] GeV/ $c^2$  for the

$D_{s2}^*(2573)^-$  signals. We vary the mass and width of  $D_{s1}(2536)^-$  or  $D_{s2}^*(2573)^-$  by  $1\sigma$  according to the world average values [7]. We also change the mass resolutions from the signal MC simulations by  $1\sigma$ , and the systematic uncertainties are found to be negligible.

We neglect the interference between the resonance and continuum amplitudes in  $\Upsilon(2S)$  decays in the simultaneous fit since it cannot be determined from the available data samples with an unknown relative phase between the two amplitudes. Instead, we estimate the maximum effect  $F_{\text{int}}^{\text{max}} \equiv \sigma_{\text{int}}/(\sigma_{\text{int}} + \sigma_{\Upsilon(2S)})$  according to the factor

$$A = \sqrt{\sigma^B(e^+e^- \rightarrow D_s^{(*)+}D_{sJ}^-)/\mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+}D_{sJ}^-)} \quad [17],$$

and take it as the systematic uncertainty. Here  $\sigma_{\Upsilon(2S)}$  and  $\sigma_{\text{int}}$  are the cross sections of the resonance process and the interference term in the  $\Upsilon(2S)$  decays, respectively. We obtain the values  $A \pm \Delta A$  from the fit results for  $\sigma^B(e^+e^- \rightarrow D_s^{(*)+}D_{sJ}^-)$  and  $\mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+}D_{sJ}^-)$ , which are listed in Table I, and then obtain  $F_{\text{int}}^{\text{max}}$  with the input of  $A + \Delta A$ , as shown in Table III.

## V. SUMMARY

In summary, we observe the charmed strange meson pair  $D_s^{(*)+}D_{sJ}^-$  production in  $\Upsilon(2S)$  decays and in  $e^+e^-$  annihilation at  $\sqrt{s} = 10.52$  GeV for the first time, where  $D_{sJ}^-$  is  $D_{s1}(2536)^-$  or  $D_{s2}^*(2573)^-$ . We determine the products of branching fractions for the  $D_{sJ}^-$  production in  $\Upsilon(2S)$  decays to be

$$\begin{aligned} \mathcal{B}(\Upsilon(2S) \rightarrow D_s^+D_{s1}(2536)^-)\mathcal{B}(D_{s1}(2536)^- \rightarrow K^-D^*(2007)^0) &= (1.6 \pm 0.3 \pm 0.2) \times 10^{-5}, \\ \mathcal{B}(\Upsilon(2S) \rightarrow D_s^{*+}D_{s1}(2536)^-)\mathcal{B}(D_{s1}(2536)^- \rightarrow K^-D^*(2007)^0) &= (1.4 \pm 0.4 \pm 0.2) \times 10^{-5}, \\ \mathcal{B}(\Upsilon(2S) \rightarrow D_s^+D_{s2}^*(2573)^-)\mathcal{B}(D_{s2}^*(2573)^- \rightarrow K^-D^0) &= (1.4 \pm 0.4 \pm 0.2) \times 10^{-5}, \\ \mathcal{B}(\Upsilon(2S) \rightarrow D_s^{*+}D_{s2}^*(2573)^-)\mathcal{B}(D_{s2}^*(2573)^- \rightarrow K^-D^0) &= (0.9 \pm 0.5 \pm 0.2) \times 10^{-5}, \end{aligned}$$

and

$$\begin{aligned} \mathcal{B}(\Upsilon(2S) \rightarrow D_s^+D_{s1}(2536)^-)\mathcal{B}(D_{s1}(2536)^- \rightarrow K_S^0D^*(2010)^-) &= (0.84 \pm 0.18 \pm 0.15) \times 10^{-5}, \\ \mathcal{B}(\Upsilon(2S) \rightarrow D_s^{*+}D_{s1}(2536)^-)\mathcal{B}(D_{s1}(2536)^- \rightarrow K_S^0D^*(2010)^-) &= (0.82 \pm 0.25 \pm 0.19) \times 10^{-5}, \\ \mathcal{B}(\Upsilon(2S) \rightarrow D_s^+D_{s2}^*(2573)^-)\mathcal{B}(D_{s2}^*(2573)^- \rightarrow K_S^0D^-) &= (0.69 \pm 0.20 \pm 0.22) \times 10^{-5}, \\ \mathcal{B}(\Upsilon(2S) \rightarrow D_s^{*+}D_{s2}^*(2573)^-)\mathcal{B}(D_{s2}^*(2573)^- \rightarrow K_S^0D^-) &= (0.54 \pm 0.31 \pm 0.47) \times 10^{-5}. \end{aligned}$$

We also determine the upper limit  $\mathcal{B}^{\text{UL}}(\Upsilon(2S) \rightarrow D_s^{*+} D_{s2}^*(2573)^-) \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K^- \bar{D}^0) < 2.9 \times 10^{-5}$  and  $\mathcal{B}^{\text{UL}}(\Upsilon(2S) \rightarrow D_s^{*+} D_{s2}^*(2573)^-) \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K_S^0 D^-) < 3.0 \times 10^{-5}$  at 90% C.L. We determine Born cross sections for continuum productions of the  $D_{sJ}^-$  at  $\sqrt{s} = 10.52$  GeV to be

$$\begin{aligned}\sigma^{\text{Born}}(e^+e^- \rightarrow D_s^+ D_{s1}(2536)^-) \mathcal{B}(D_{s1}(2536)^- \rightarrow K^- D^*(2007)^0) &= (67 \pm 8 \pm 6) \text{ fb}, \\ \sigma^{\text{Born}}(e^+e^- \rightarrow D_s^{*+} D_{s1}(2536)^-) \mathcal{B}(D_{s1}(2536)^- \rightarrow K^- D^*(2007)^0) &= (84 \pm 11 \pm 11) \text{ fb}, \\ \sigma^{\text{Born}}(e^+e^- \rightarrow D_s^+ D_{s2}^*(2573)^-) \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K^- D^0) &= (56 \pm 9 \pm 13) \text{ fb}, \\ \sigma^{\text{Born}}(e^+e^- \rightarrow D_s^{*+} D_{s2}^*(2573)^-) \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K^- D^0) &= (106 \pm 17 \pm 12) \text{ fb},\end{aligned}$$

and

$$\begin{aligned}\sigma^{\text{Born}}(e^+e^- \rightarrow D_s^+ D_{s1}(2536)^-) \mathcal{B}(D_{s1}(2536)^- \rightarrow K_S^0 D^*(2010)^-) &= (34 \pm 5 \pm 4) \text{ fb}, \\ \sigma^{\text{Born}}(e^+e^- \rightarrow D_s^{*+} D_{s1}(2536)^-) \mathcal{B}(D_{s1}(2536)^- \rightarrow K_S^0 D^*(2010)^-) &= (41 \pm 6 \pm 6) \text{ fb}, \\ \sigma^{\text{Born}}(e^+e^- \rightarrow D_s^+ D_{s2}^*(2573)^-) \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K_S^0 D^-) &= (27 \pm 6 \pm 5) \text{ fb}, \\ \sigma^{\text{Born}}(e^+e^- \rightarrow D_s^{*+} D_{s2}^*(2573)^-) \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K_S^0 D^-) &= (51 \pm 11 \pm 9) \text{ fb}.\end{aligned}$$

For comparison,  $\sigma^{\text{Born}}(e^+e^- \rightarrow \mu^+\mu^-) = 0.784$  nb at  $\sqrt{s} = 10.52$  GeV and  $\mathcal{B}(\Upsilon(2S) \rightarrow \mu^+\mu^-) = (1.93 \pm 0.17)\%$  [7]. We define the ratios  $R_1 \equiv \mathcal{B}(\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-) / \mathcal{B}(\Upsilon(2S) \rightarrow \mu^+\mu^-)$  for the  $\Upsilon(2S)$  decays and  $R_2 \equiv \sigma^{\text{Born}}(e^+e^- \rightarrow D_s^{(*)+} D_{sJ}^-) / \sigma^{\text{Born}}(e^+e^- \rightarrow \mu^+\mu^-)$  for the continuum productions. We obtain  $R_1/R_2 = 9.7 \pm 2.3 \pm 1.1$ ,  $6.8 \pm 2.1 \pm 0.8$ ,  $10.2 \pm 3.3 \pm 2.5$ , and  $3.4 \pm 2.1 \pm 0.8$  for

the  $D_s^+ D_{s1}(2536)^-$ ,  $D_s^{*+} D_{s1}(2536)^-$ ,  $D_s^+ D_{s2}^*(2573)^-$ , and  $D_s^{*+} D_{s2}^*(2573)^-$  final states in the  $D_{sJ}^- \rightarrow K^- \bar{D}^{(*)0}$  modes, respectively, where the uncertainty of  $\mathcal{B}(\Upsilon(2S) \rightarrow \mu^+\mu^-)$  is taken into account of the systematic uncertainties. The measured  $R_1/R_2$  values indicate that the strong decay dominates in  $\Upsilon(2S) \rightarrow D_s^{(*)+} D_{sJ}^-$  processes.

Here, we also determine the ratios of branching fractions to be

$$\begin{aligned}\mathcal{B}(D_{s1}(2536)^- \rightarrow K_S^0 D^*(2010)^-) / \mathcal{B}(D_{s1}(2536)^- \rightarrow K^- D^*(2007)^0) &= 0.48 \pm 0.07 \pm 0.02, \\ \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K_S^0 D^-) / \mathcal{B}(D_{s2}^*(2573)^- \rightarrow K^- D^0) &= 0.49 \pm 0.10 \pm 0.02,\end{aligned}$$

which are in good agreement with the expected values of 0.498 and 0.497 from isospin symmetry, considering the phase space, since with  $K_S^0$  only half of the neutral kaons can be reconstructed. The first ratio has the same precision as the world average value [7], while the second ratio is the first measurement of this quantity.

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