# Quantum mismatch: A powerful measure of quantumness in neutrino oscillations

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The quantum nature of neutrino oscillations would be reflected in the mismatch between the neutrino survival probabilities with and without an intermediate observation. We propose this quantum mismatch as a measure of quantumness in neutrino oscillations. For two neutrino flavors, it inevitably performs better than the Leggett-Garg measure. For three flavors, we devise modified definitions of these two measures, which would be applicable for experiments that measure neutrino survival probabilities with negligible matter effects. The modified definitions can be used to probe deviations from expected classical behavior, even for systems with an unknown number of states. For neutrino experiments like DUNE, MINOS, and JUNO, we identify the energies where these modified measures can probe quantumness efficiently.

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## I. INTRODUCTION

Tests of quantum mechanics (QM) provide insights into the limits of local realism, which aligns with the classical world view that all properties of physical objects have values that exist independently of their measurements. For example, the Bell's inequality [1] tests for violations of the classical upper bound on correlations between measurements made on spatially separated systems. Violations of this upper bound [2,3] clearly indicate the need for QM, as they would be incompatible with the hypothesis of hidden variables [4,5].

The Leggett-Garg (LG) measure [6,7] provides another test of "quantumness" (more precisely, nonclassicality) of a system through the correlations between its measurements at different times. The Leggett-Garg inequality (LGI) tests for the interference in QM, as opposed to entanglement, which is tested by the Bell's inequality. The simplest LG measure $K_3$ employs the observation of the system at an intermediate time.

In the phenomenon of neutrino oscillations, neutrinos change their flavor ($\nu_e$, $\nu_\mu$, $\nu_\tau$) during propagation due to the interference between different mass eigenstates [8,9]. This is a unique system where QM manifests itself over hundreds and thousands of kilometers, which makes it a prime candidate for tests of QM [10–25]. Violations of LGI have been measured at neutrino oscillation experiments at MINOS [26] and Daya-Bay [27]. New physics effects on the LG measure have been discussed in [28,29].

Note that the LGI, as proposed in [6], tests for the validity of the classical assumptions of macroscopic realism and noninvasive measurements. However, in neutrino oscillation experiments, the criteria of noninvasive measurements cannot be fulfilled, as the measurement of the flavor state destroys that particular neutrino. Neutrino oscillation experiments looking for violation of LGI test for the validity of the classical assumptions of (i) macroscopic realism, (ii) time translation invariance, (iii) Markovian dynamics, and (iv) the ability to produce a given state [7]. The latter three assumptions together are often simply termed as "stationarity". In this work, for the LG measure as well as the quantum mismatch measure (to be introduced below), nonclassicality implies a violation of the combined assumption of macroscopic realism and stationarity.

A difference between observations with and without an intermediate measurement would be a natural measure of quantumness [30,31]. In this work, we introduce the "quantum mismatch" measure, $\delta P$, for ascertaining the quantum nature of neutrino oscillations. It is simply defined as the difference between neutrino survival probabilities with and without an intermediate measurement. Here, we use measurements at different energies as proxies for measurements at different times, which ensures that the "intermediate" measurement is noninvasive.

In real-world neutrino experiments, it is not possible to detect all neutrino flavors. This necessitates modification of the measures $K_3$ and $\delta P$ in the full three-flavor scenario. We identify the energies where the two modified measures $\tilde{K}_3$ and $\widetilde{\delta P}$ would be efficient in experiments.

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The flow of the paper is as follows. In Sec. II we provide the definitions of the LG measure and the quantum mismatch measure for neutrinos. In Sec. III we express these two quantum measures in terms of quantum amplitudes in the two-flavor limit. In Sec. IV we discuss how the values of the quantum measures can be determined from neutrino oscillation experiments, and compare them in the two-flavor limit. In Sec. V, we extend the formalism to more than two flavors and devise modified measures that are applicable in practical scenarios. In Sec. VI, we calculate the values of these modified measures for specific neutrino oscillation experiments and identify the energies where they can probe quantumness efficiently. We conclude with a brief discussion and overview in Sec. VII.

## II. FORMALISM AND DEFINITIONS

In Fig. 1, we schematically represent the states of a system with and without an intermediate measurement. At time $t_0 = 0$, the whole system is in the $\nu_\mu$ state (denoted by A). Over time, the neutrino flavor can either survive as $\nu_\mu$ or change to $\nu_x$. The state of the system can be measured at later times $t_1$ and $t_2$. We denote the relevant quantum amplitudes,

$$\mathcal{A}_{AX} \equiv \mathcal{A}_{\mu\mu}(0,t_1) = a_1, \qquad \mathcal{A}_{AY} \equiv \mathcal{A}_{x\mu}(0,t_1) = b_1,$$
$$\mathcal{A}_{XB} \equiv \mathcal{A}_{\mu\mu}(t_1,t_2) = a_2, \qquad \mathcal{A}_{YB} \equiv \mathcal{A}_{\mu x}(t_1,t_2) = b_2, \quad (1)$$

where $\mathcal{A}_{\beta\alpha}(t_i,t_j)$ denotes the quantum amplitude for $\nu_\alpha(t_i) \to \nu_\beta(t_j)$. The corresponding oscillation probabilities are given by $P_{\alpha\beta}(t_i,t_j) \equiv |\mathcal{A}_{\beta\alpha}(t_i,t_j)|^2$. Note that conservation of probability implies $|a_i|^2 + |b_i|^2 = 1$.

In the classical limit, the muon neutrino survival probability $P_{AB} \equiv P_{\mu\mu}(0,t_2)$ would be

$$P_{AB} = P_{AX}P_{XB} + P_{AY}P_{YB} = |a_1 a_2|^2 + |b_1 b_2|^2. \quad (2)$$

In QM, in the absence of any intermediate observation, we add the amplitudes over all possible paths, obtaining

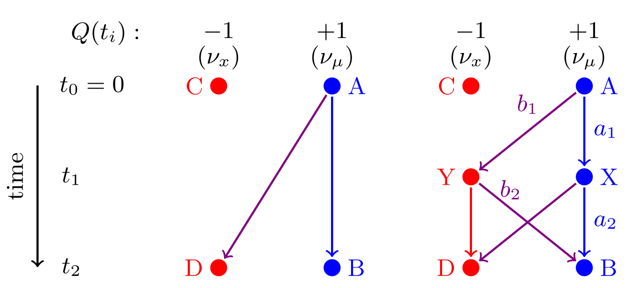


FIG. 1. Schematic representation of the states starting with $\nu_\mu$ at $t_0 = 0$ without [left] and with [right] intermediate measurements. Only two neutrino flavors, $\nu_\mu$ and $\nu_x$, are assumed.

$$P_{AB} \equiv |\mathcal{A}_{AB}|^2 = |a_1 a_2 + b_1 b_2|^2. \quad (3)$$

The simplest LG measure $K_3$ is defined through a dichotomic observable $Q(t_i)$, which can only have outcomes $\pm 1$. We define $Q(t_i) = +1$ if the detected state is $\nu_\mu$, and $Q(t_i) = -1$ for any other state $\nu_x$. The correlation function is defined as

$$\mathbb{C}_{ij} \equiv \langle Q(t_i) Q(t_j) \rangle. \quad (4)$$

The LG measure $K_3$ is then

$$K_3 \equiv \mathbb{C}_{01} + \mathbb{C}_{12} - \mathbb{C}_{02}, \quad (5)$$

where the suffixes (0, 1, 2) correspond to the times (0, $t_1$, $t_2$). In the classical scenario, $-3 \le K_3 \le 1$ [6,7], i.e., the LGI $K_3 \le 1$ is satisfied. Any observation $K_3 > 1$ would indicate the quantum nature of the system.

For the muon-neutrino survival probability, the quantum mismatch parameter $\delta P$ is

$$\delta P_{\mu\mu} = P_{AB} - \sum_{I=\mathrm{X,Y,\ldots}} P_{AI} P_{IB}, \quad (6)$$

where $I$ denotes the possible intermediate neutrino flavor states (X, Y, …) at $t_1$. In the classical scenario, the equality $\delta P_{\mu\mu} = 0$ holds. Any observation $\delta P_{\mu\mu} \neq 0$ would indicate the quantum nature of the system.

## III. TWO-FLAVOR LIMIT

In the limit of two neutrino flavors ($2\nu$ limit), conservation of probability implies $P_{x\mu} = P_{\mu x}$. The correlation function $\mathbb{C}_{ij}$ then becomes

$$\mathbb{C}_{ij}^{(2\nu)} = 2P_{\mu\mu}(t_i,t_j) - 1. \quad (7)$$

The LG measure $K_3$ can be calculated as

$$K_3^{(2\nu)} = 2(P_{AX} + P_{XB} - P_{AB}) - 1$$
$$= 1 - 4|b_1|^2|b_2|^2 - 4\mathrm{Re}[b_1^\star b_2^\star a_1 a_2]. \quad (8)$$

Clearly, the quantity responsible for a possible violation of the classical bound ($K_3 \le 1$) is the interference term

$$\mathcal{I}^{(2\nu)} \equiv \mathrm{Re}[b_1^\star b_2^\star a_1 a_2]. \quad (9)$$

The quantum mismatch measure $\delta P$ in the $2\nu$ limit is

$$\delta P_{\mu\mu}^{(2\nu)} = P_{AB} - (P_{AX}P_{XB} + P_{AY}P_{YB}) = 2\mathcal{I}^{(2\nu)}, \quad (10)$$

which is the same interference term. However, while $K_3^{(2\nu)} > 1$ is needed to indicate quantumness, $\delta P_{\mu\mu}^{(2\nu)} \neq 0$ is enough to do the same. Note that $K_3^{(2\nu)} > 1$ necessitates

$\mathcal{I}^{(2\nu)} < -|b_1|^2|b_2|^2$, whereas for all $\mathcal{I}^{(2\nu)} \neq 0$, we obtain $\delta P_{\mu\mu}^{(2\nu)} \neq 0$. Therefore, the $\delta P$ measure always succeeds in indicating quantumness whenever the LG measure does. A similar $2\nu$ condition, focusing on the conversion channel, is obtained in [32] using the no-signaling-in-time argument.

## IV. TWO-FLAVOR QUANTUM MEASURES AT NEUTRINO OSCILLATION EXPERIMENTS

The quantum measures discussed above need measurements of the system corresponding to three different time intervals $\Delta t_{10} \equiv t_1 - t_0$, $\Delta t_{21} \equiv t_2 - t_1$, and $\Delta t_{20} \equiv t_2 - t_0$. In a fixed-baseline neutrino experiment, measurements at multiple time intervals are not possible. However, this obstacle may be overcome through the following procedure.

One of the major characteristics of our procedure is that we do not use the particular form of the $\Delta t/E$ dependence of the neutrino oscillation probabilities, which arises from quantum mechanical arguments. We use only the special theory of relativity, which naturally leads to a dependence on the ratio $\Delta t/E$. The knowledge of the exact form of this dependence is not needed for our procedure to work. This will enable us to probe deviations from classicality using fixed-baseline experiments.

Consider the evolution of a particle with mass $m$ during time interval $\Delta\tau$ in its rest frame. If this particle has an energy $E$ in the lab frame, the same evolution will be observed for a time interval $\Delta t = (\Delta\tau/m)E$, by the special theory of relativity. Thus, the evolution of a neutrino in the lab frame depends only on the ratio $\Delta t/E$.

For neutrino oscillations, this dependence on $\Delta t/E$ holds in vacuum, or as long as matter effects [33,34] are negligible. In this limit, the measurements of neutrinos with the same energy at different times may be replaced by measurements of neutrinos of different energies at the same time intervals. That is, for some energy $E_0$ and time interval $\Delta t_0$, if we find $E_{10}$, $E_{20}$, and $E_{21}$ such that

$$\left(\frac{\Delta t_{10}}{E_0}, \frac{\Delta t_{21}}{E_0}, \frac{\Delta t_{20}}{E_0}\right) = \left(\frac{\Delta t_0}{E_{10}}, \frac{\Delta t_0}{E_{21}}, \frac{\Delta t_0}{E_{20}}\right), \qquad (11)$$

then the measurements at energies $E_{10}$, $E_{21}$, and $E_{20}$ can act as proxies for measurements with time intervals $\Delta t_{10}$, $\Delta t_{21}$, and $\Delta t_{20}$, respectively. Here, $\Delta t_0$ should be taken as the duration of neutrino propagation from the source to the detector. Since the three time intervals are related to each other as

$$\Delta t_{10} + \Delta t_{21} = \Delta t_{20}, \qquad (12)$$

the proxy energies need to satisfy the relation

$$1/E_{10} + 1/E_{21} = 1/E_{20}. \qquad (13)$$

In principle, for every value of $E_0$, one has a different triplet $(E_{10}, E_{21}, E_{20})$, using which the quantum measures may be defined. Using measurements at different energies makes this an effectively noninvasive measurement, which does not disrupt the system in any way.

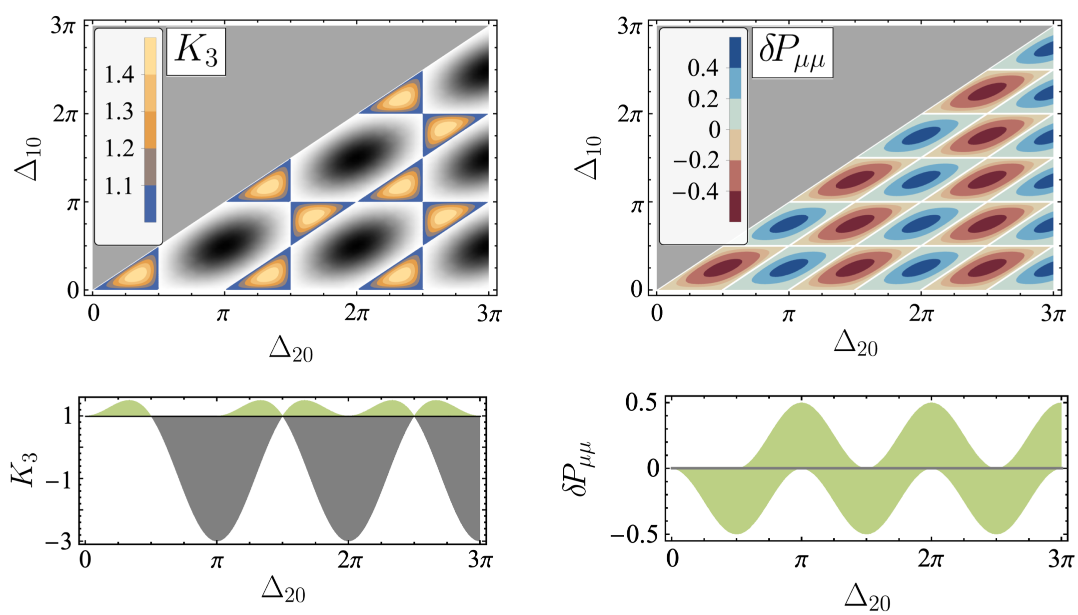


FIG. 2. Top Panel: the quantum measures $K_3$ and $\delta P_{\mu\mu}$ in the two-flavor limit, in the $(\Delta_{20}, \Delta_{10})$ plane. We have taken the mixing angle $\theta = 45°$. The upper-left solid gray triangles are unphysical regions. The black and white regions obey the classical limit. Colored regions correspond to $K_3 > 1$ and $\delta P_{\mu\mu} \neq 0$, indicating quantumness. Bottom Panel: ranges of $K_3$ and $\delta P_{\mu\mu}$ as functions of $\Delta_{20}$, for all possible $\Delta_{10}$. The dark gray (light green) regions obey (violate) classical limits.

The neutrino survival probability in vacuum in the $2\nu$ limit is

$$P_{\mu\mu}(t_i, t_j) = 1 - \sin^2(2\theta)\sin^2\Delta, \quad (14)$$

where $\theta$ is the mixing angle. The oscillation phase, as a function of energy, is

$$\Delta(E) = 1.27 \times (\Delta m^2 \text{ in eV}^2) \times \frac{(L \text{ in km})}{(E \text{ in GeV})}, \quad (15)$$

where $\Delta m^2$ is the mass-squared difference between the two neutrinos, and $L = c(t_j - t_i)$. Note that the dependence on the ratio $\Delta t/E$ is explicitly present above.

The top panels of Fig. 2 show the values of the quantum measures $K_3$ and $\delta P_{\mu\mu}$ in the $2\nu$ limit, in the $(\Delta_{20}, \Delta_{10})$ plane where $\Delta_{ij} \equiv \Delta(E_{ij})$. Since $\Delta_{20}$ corresponds to the largest time interval, we need $\Delta_{20} > \Delta_{10}$, making the upper-left triangles in the contour plots unphysical. The figure shows that the classical bound of $K_3 \leq 1$ would be violated in certain colored 'islands' in the parameter space, whereas the classical value of $\delta P_{\mu\mu} = 0$ would be violated for much larger regions of possible $(\Delta_{20}, \Delta_{10})$ choices.

The bottom panels of Fig. 2 further illustrate that in the $2\nu$ limit, the quantum mismatch measure $\delta P$ would be a more efficient probe of nonclassicality.

## V. DEFINING THE THREE-FLAVOR QUANTUM MEASURES

The above discussion implicitly assumes that there are only two neutrino flavors. However, as neutrinos come in three flavors $(3\nu)$, these measures will have to be modified accordingly.

Since a dichotomic observable $Q(t_i)$ is needed for the LG measure, we shall assign $Q(t_i) = -1$ for all nonmuon neutrinos, i.e., $\nu_e$ and $\nu_\tau$, as depicted in Fig. 3.

If we had the ability to detect all three neutrino flavors, i.e., if independent measurements of neutrino flavor states $X$, $Y$, and $Z$ were possible, the correlation functions $\mathbb{C}_{01}$ and $\mathbb{C}_{02}$ would take the same form as before,

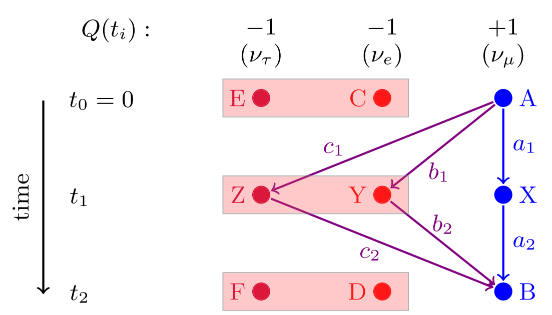


FIG. 3. Schematic representation of the states of the system with three neutrino flavors, starting with $\nu_\mu$ at $t_0 = 0$.

$$\mathbb{C}_{01}^{(3\nu)} = 2P_{AX} - 1, \qquad \mathbb{C}_{02}^{(3\nu)} = 2P_{AB} - 1. \quad (16)$$

The correlation function $\mathbb{C}_{12}$, however, would be

$$\begin{aligned}\mathbb{C}_{12}^{(3\nu)} &= P_{AX}(P_{XB} - P_{XD} - P_{XF}) \\ &\quad - \sum_{I=Y,Z} P_{AI}(P_{IB} - P_{ID} - P_{IF}) \\ &= 1 + 2(P_{AX}P_{XB} - P_{AY}P_{YB} - P_{AZ}P_{ZB}) - 2P_{AX}. \quad (17)\end{aligned}$$

The LG measure $K_3$ can then be calculated using Eq. (5).

In QM, in the absence of any intermediate observation,

$$P_{AB} = |a_1 a_2 + b_1 b_2 + c_1 c_2|^2. \quad (18)$$

The value of $K_3$ would then be

$$K_3^{(3\nu)} = 1 - 4|b_1|^2|b_2|^2 - 4|c_1|^2|c_2|^2 - 4\mathcal{I}^{(3\nu)}, \quad (19)$$

where we have defined the $3\nu$-interference term as

$$\mathcal{I}^{(3\nu)} \equiv \mathrm{Re}[a_1^\star a_2^\star b_1 b_2] + \mathrm{Re}[b_1^\star b_2^\star c_1 c_2] + \mathrm{Re}[c_1^\star c_2^\star a_1 a_2]. \quad (20)$$

Clearly, in the absence of the interference terms, the classical bound of $K_3^{(3\nu)} \leq 1$ will be always satisfied.

For experiments where all neutrino flavors cannot be detected, $K_3^{(3\nu)}$ as defined above cannot be measured. However, a modified LG measure, observable at all experiments which can measure the muon-neutrino survival probability $P_{\mu\mu}$, can be defined as

$$\tilde{K}_3 = \tilde{\mathbb{C}}_{01} + \tilde{\mathbb{C}}_{12} - \tilde{\mathbb{C}}_{02}, \quad (21)$$

where $\tilde{\mathbb{C}}_{ij}$ is defined as

$$\tilde{\mathbb{C}}_{ij} = 2P_{\mu\mu}(t_i, t_j) - 1. \quad (22)$$

This modified definition of LG measure makes it directly applicable for those long-baseline neutrino experiments where matter effects are negligible for $P_{\mu\mu}$. We get

$$\begin{aligned}\tilde{K}_3 &= 2(P_{AX} + P_{XB} - P_{AB}) - 1 \\ &= 1 - 4|b_1|^2|b_2|^2 - 4|c_1|^2|c_2|^2 \\ &\quad - 2|b_1|^2|c_2|^2 - 2|b_2|^2|c_1|^2 - 4\mathcal{I}^{(3\nu)}. \quad (23)\end{aligned}$$

The $3\nu$-interference term allows the violation of the classical bound $\tilde{K}_3 \leq 1$, albeit for a smaller region of parameter space compared to $K_3^{(3\nu)}$. Thus, $\tilde{K}_3$ is a practical LG measure in the three-flavor system of neutrinos. It is indeed the one implicitly being used in [26,27].

Similarly, if we had the ability to detect all three neutrino flavors, the quantum mismatch measure would be

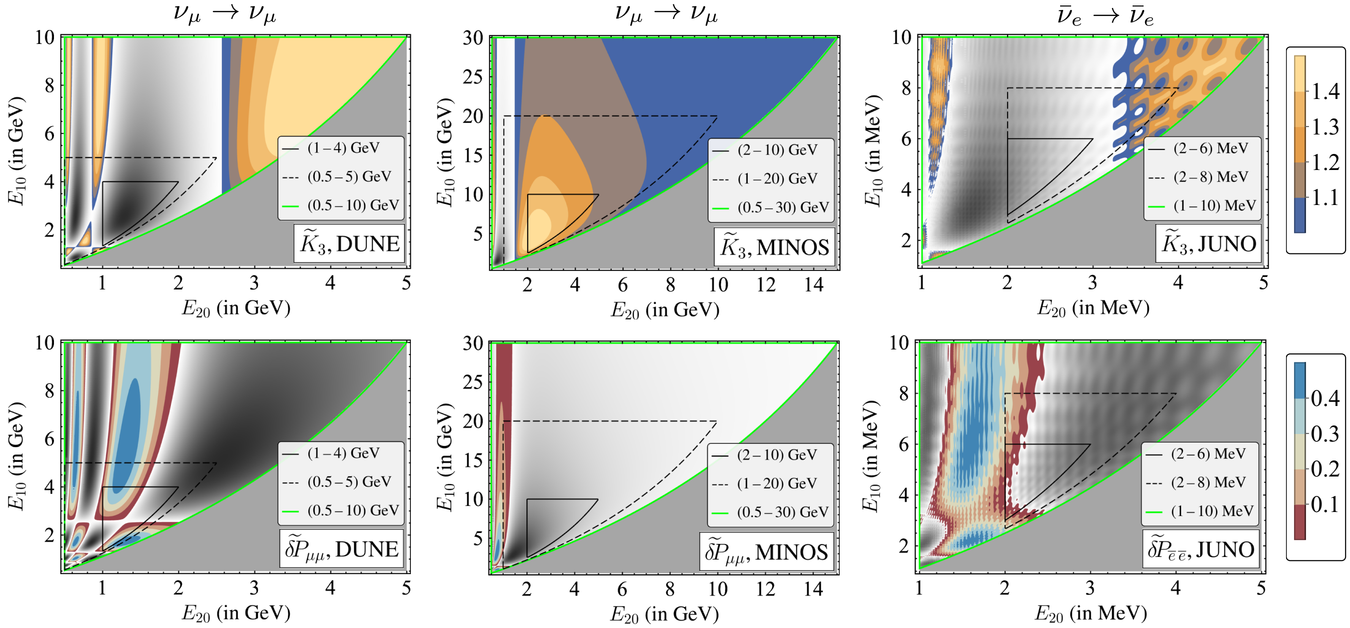


FIG. 4. The modified quantum measures $\tilde{K}_3$ [top] and $\widetilde{\delta P}$ [bottom] at DUNE [left], MINOS [middle], and JUNO [right] in the $(E_{20}, E_{10})$ plane. The quasi-triangular regions enclosed by the green boundaries indicate the parameter regions where the experiment has nonzero flux at $E_{10}$, $E_{20}$, and $E_{21}$ [see Eq. (13)]. The solid gray regions outside these green boundaries do not give an allowed $E_{21}$ value. The quasitriangular regions enclosed by dashed/solid black boundaries denote energies where the flux is higher. The black and white regions obey the classical limit. Colored regions correspond to $\tilde{K}_3 > 1$ and $\widetilde{\delta P} > 0$, indicating quantumness. Neutrino parameters are given in Eq. (26). The spotted features for JUNO are due to the coexistence of atmospheric and solar neutrino oscillations.

$$\delta P^{(3\nu)}_{\mu\mu} \equiv P_{AB} - (P_{AX}P_{XB} + P_{AY}P_{YB} + P_{AZ}P_{ZB}) = 2\mathcal{I}^{(3\nu)}. \tag{24}$$

Though this measure precisely extracts the interference term that causes violations of classical bounds, it cannot be calculated for real-world experiments. For experiments which only observe $P_{\mu\mu}$, we define the modified quantum mismatch measure

$$\widetilde{\delta P}_{\mu\mu} \equiv P_{AB} - (P_{AX}P_{XB} + (1 - P_{AX})(1 - P_{XB})) = 2\mathcal{I}^{(3\nu)} - (|b_1|^2|c_2|^2 + |b_2|^2|c_1|^2), \tag{25}$$

where we have used $P_{XB} + P_{YB} + P_{ZB} = 1$, which is true due to probability conservation. In the classical limit, i.e., in absence of the $3\nu$-interference term, $\widetilde{\delta P}_{\mu\mu} \leq 0$. Therefore, $\widetilde{\delta P}_{\mu\mu} > 0$ is a clear indicator of quantumness.

Note that the modified measure $\widetilde{\delta P}_{\mu\mu}$ has a classical upper bound as opposed to the $\delta P^{(3\nu)}_{\mu\mu}$ which would have had a fixed value of zero in the classical limit. This may make $\widetilde{\delta P}_{\mu\mu}$ appear to be a less-efficient counterpart of the two-flavor quantum mismatch measure. However, $\widetilde{\delta P}_{\mu\mu}$ is a practical measure that can be determined at real-world experiments. Moreover, it can be used to probe quantumness for any $n$-state system, even when the number of states $n$ is not known.

## VI. QUANTUM MEASURES AT NEUTRINO OSCILLATION EXPERIMENTS

Although the phenomenon of neutrino oscillation is inherently quantum, the observability of quantumness depends on the quantum measure employed as well as the parameters of the experiment. Here, we identify the energies at which the quantum nature would be observable through $\tilde{K}_3$ and $\widetilde{\delta P}$ at neutrino oscillation experiments.

Note that the procedure outlined in Secs. IV and V, which uses only special relativity and not the explicit quantum mechanical form neutrino oscillation probabilities, is strictly valid only in vacuum. While implementing this procedure for long-baseline experiments, we shall use the survival channel probability $P_{\mu\mu}$ which is not sensitive to matter effects on neutrino oscillations.[1]

Note however, that the matter effects themselves are also quantum in nature and would not appear in the classical calculations. Therefore, deviations from the classical predictions due to matter effects would also be a sign of a nonclassical phenomenon taking place.

In Fig. 4, we show the values of the modified quantum measures $\tilde{K}_3$ and $\widetilde{\delta P}$ in terms of the energies $(E_{20}, E_{10})$. The value of $E_{21}$ can be obtained from Eq. (13). Since

[1]For example, the difference between the survival probabilities $(P_{\mu\mu})$ at DUNE with and without matter effects is less than 0.006 at any energy.

$E_{20} < E_{10}$, and since all three energies must lie within the energy range of the experiment, the solid gray regions in lower-right corners of all panels are not relevant.

For the purpose of illustration, we choose the experiments DUNE and MINOS which measure $P_{\mu\mu} \equiv P(\nu_\mu \to \nu_\mu)$. This is valid because $P_{\mu\mu}$ does not have any leading-order matter contributions [35]. We further analyze the modified quantum measures through $P_{\bar{e}\bar{e}} \equiv P(\bar{\nu}_e \to \bar{\nu}_e)$ for the reactor antineutrino experiment JUNO, where matter effects are negligible. We take neutrino mixing parameter values consistent with the global fits [36–39],

$$\Delta m_{21}^2 = 7.5\times 10^{-5}\ \mathrm{eV}^2,\quad \Delta m_{31}^2 = 2.5\times 10^{-3}\ \mathrm{eV}^2,$$
$$\theta_{13} = 8.5°,\quad \theta_{23} = 45°,\quad \theta_{12} = 33.5°,\quad \delta_{\mathrm{CP}} = -90°. \qquad (26)$$

We observe that for DUNE [40], especially in the region where we expect the maximum neutrino flux, the measure $\widetilde{\delta P}$ is much better-suited for probing quantumness than $\tilde{K}_3$. The results obtained for T2K/T2HK [41,42] are also quite similar to this.

For MINOS [43], the modified LG measure $\tilde{K}_3$ is more efficient. Note that the probe of quantum nature of neutrino oscillations at MINOS [26] implicitly uses this modified measure. We also perform the exercise for NOvA [44], and find that neither of the measures would be efficient in probing the quantum nature of neutrino oscillations at this experiment.

For JUNO [45], $\widetilde{\delta P}$ is a better measure in the energy range with higher flux. The energy resolution of the detector will play a crucial role in determining the observability of quantumness. Note that the fine pattern seen in the JUNO plots is a result of the interference between oscillations due to $\Delta m_{21}^2$ and $\Delta m_{31}^2$.

## VII. CONCLUDING REMARKS

Tests of violations of classicality—codified by the combined assumptions of (i) macroscopic realism, (ii) time translation invariance, (iii) Markovian dynamics, and (iv) the ability to produce a given state—are instrumental in probing the fundamental nature of physical systems. In this work, we introduce the quantum mismatch measure $\delta P$ for detecting quantumness in neutrino oscillations. In the two-flavor limit, this measure precisely extracts the quantum interference term.

We extend the definitions of the quantum mismatch measure $\delta P$ and the Leggett-Garg measure $K_3$ to the full three-flavor scenario. In the absence of experiments which can detect all three neutrino flavors separately, we provide modified practical definitions of both the measures, $\widetilde{\delta P}$ and $\tilde{K}_3$, that employ only the neutrino survival probabilities. In fact, the modified definitions enable us to also probe systems with an unknown number of states. We further identify the energies for which the quantum measures can efficiently probe deviations from classicality, at neutrino experiments like DUNE, MINOS, and JUNO.

The new quantum mismatch measure $\widetilde{\delta P}$ is thus a robust, practical and efficient measure, which would further advance the quest of probing quantumness at macroscopic length scales.

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