# QCD-compatible supermassive inert top-down holographic mesinos at intermediate coupling

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A longstanding problem with the popular Sakai-Sugimoto holographic dual of thermal QCD is that the mesinos, the (nonsupersymmetric) fermionic partners of the mesons, are nearly isospectral with mesons and have an unsuppressed mesino-mesino-meson interaction, both being in contradiction with actual QCD. We solve this problem in a UV complete (and different) type-IIA string dual at intermediate coupling of realistic thermal QCD, in which the mesinos are shown to be much heavier than and noninteracting with mesons (the wave function/mass/interaction terms receiving no $\mathcal{M}$-theory $\mathcal{O}(R^4)$ corrections). In particular we derive a large-$N$ enhancement of the Kaluza-Klein $(KK)$ mass scale $M_{KK}$ (from $M_{KK}$ to $M_{KK}^{\rm eff} \sim N^{1+\frac{1}{\mathcal{O}(1)}} M_{KK}$) arising from the construction of the type-IIA mirror [1] of the type-IIB dual [2] of thermal QCD-like theories, as well as the generation of a one-parameter family of $M_{KK}$-independent mass scale at $\mathcal{O}(R^4)$ in the $\mathcal{M}$-theory uplift [3], wherein the parameter can be made appropriately large. We also show that the mesino-mesino-single-($\rho/\pi$)meson interactions, vanish identically in the aforementioned type-IIA holographic dual.

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## I. INTRODUCTION

One can construct gauge theories from a stack of $D$-branes and various configurations of the same. In this context, in the spirit of (nonconformal, nonsupersymmetric) gauge-gravity duality (inspired by [4]), mostly bosonic fluctuations on the world volume of $D$-branes have been considered. The type-IIA dual inclusive of the $\mathcal{O}(R^4)$ corrections—to explore the finite-$N$-limit/intermediate coupling regime of QCD—of the type-IIB dual [2] of thermal QCD-like theories, was worked out in [1,3]. As a recent example, inclusive of higher-derivative corrections to address the finite-$N$/intermediate-coupling regime (as worked out in [3]), phenomenologically-compatible low-energy coupling constants up to next-to-leading order (NLO) in the chiral expansion in $SU(3)$ chiral perturbation theory (in the chiral limit) were obtained from the Dirac-Born-Infeld (DBI) action on flavor $D6$-branes in [5]. Dirac-like action for the supersymmetric partners of mesons, the mesinos, has been obtained from a top-down approach on $Dp$-branes [6] (see [7] for the bottom-up approach). However, using the same for the Sakai-Sugimoto type-IIA dual [8] of thermal QCD, it was shown that one runs into a problem. The mesinos and mesons turn out to be approximately isospectral and their interaction is not large-$N$ suppressed [9]; both are in contradiction with real QCD. This serves as the main motivation for this paper which is to see if this issue can be resolved in the type-IIA mirror [1] at intermediate coupling [3] of the nonsupersymmetric UV-complete type-IIB dual [2] of thermal QCD-like theories. In this paper, we explicitly consider the mesino action on flavor $D6$-branes in the aforementioned type-IIA dual. We also see the effect of higher-derivative terms on the fermions relevant to holographic thermal QCD in this paper which was missing in [6]. In short, we will show that the mesinos are supermassive and do not interact with the vector/$\pi$ mesons, which is why we refer to them as weakly interacting supermassive particles (WISP), thereby not being in conflict with realistic QCD.

The following serves as a brief summary of the main results of this paper:

(1) Supermassive mesinos (Sec. III):
  (a) Dirichlet/Neumann boundary condition for the radial profile of the mesino wave function: The on shell DBI Lagrangian density $\mathcal{L}^{\rm DBI,\,D6}_{\rm on-shell}$ of the type-IIA flavor $D6$-branes [corresponding to $i: \Sigma_{(7)} \cong S^1_t \times_w \mathbb{R}^3 \times \mathbb{R}_{\geq 0} \times_w S^2_{\rm squashed}(a) \hookrightarrow M_{10}$ (the embedding of the flavor $D6$-branes in the ten-dimensional background involving a warped squashed resolved conifold) in the $\psi = 2n\pi$,

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$n = 0$, 1, 2-coordinate patches and for vanishingly small Ouyang embedding parameter in the parent type-IIB dual] obtained from the Strominger-Yau-Zaslow (SYZ) mirror of the type-IIB holographic dual of [2], in the intermediate-$N$ MQGP limit [Eq. (3)], can be shown to be vanishingly small. The mesino equation of motion (EOM),

$$\mathcal{A}\Theta + \left[\frac{\Lambda_2(\{\Gamma^\alpha\}, \mathcal{F}^{\rm IIA}, \mathcal{A})}{\mathcal{L}^{\rm DBI, D6}_{\rm on-shell}} + \frac{\Lambda_3(\{\Gamma^\alpha\}, \mathcal{F}^{\rm IIA})}{(\mathcal{L}^{\rm DBI, D6}_{\rm on-shell})^2}\right] \times \Gamma^\gamma D_\gamma \Theta = 0,$$

where $\gamma \in \{t, x^{1,2,3}, r, \theta_2, \tilde{y}\}$ indexing coordinates of the flavor $D6$-branes' world volume $\Sigma_{(7)}$, and $\mathcal{A}, \mathcal{F}^{\rm IIA}$ are defined in (21), (16) respectively; $\Lambda_{2,3}$ can be read off from (20) and can hence be approximated by a massless Dirac equation on $\Sigma_{(7)}$.

(b) Either by looking at the $SU(3)$ and the "transverse" $SU(3)$ structures on $M_6(S^1_t \times_w \mathcal{T}, \times_w$ implying a warped product, $S^1_t$ being the thermal circle and $\mathcal{T}$—deformed $T^{1,1}$—being the base of a warped non-Kähler squashed resolved conifold)/$\tilde{M}_6$ (non-Kähler warped squashed resolved conifold), or when considering the embedding of the $D6$-brane world volume $\Sigma_{(7)} \cong S^1_t \times_w (\mathbb{R}^3 \times \mathbb{R}_{\geq 0}) \times_w S^2_{\rm squashed}$ in $M_{10}$ considered either as $(S^1_t \times_w \mathbb{R}^3) \times_w \tilde{M}_6$ or $\mathbb{R}^3 \times_w (\mathbb{R}_{\geq 0} \times M_6)$, one is therefore guaranteed the existence of a pair of globally defined spinors. Using the same, and imposing antiperiodic boundary conditions along $S^1_t$, the ansatz (26) was made for the mesino spinor, and the radial profile functions therein, are solved for.

(c) For the thermal background (5) dual to thermal QCD for $T < T_c$, as well as the black hole background (4) dual to thermal QCD for $T > T_c$, we found that Dirichlet/Neumann boundary condition at $r = r_0$ (IR cutoff in the thermal background)/$r = r_h$ permitted supermassive mesinos.

(d) *Enhacement of mass scale*:

(i) Starting from the $D = 11$ supergravity Einstein's field equations in the presence of four-form $G$ fluxes of $\mathcal{M}$-theory, we explicitly show the generation of an $N$-enhanced ($N$-hanced) mass scale, thereby providing the mechanism of generation of supermassive mesinos.

(ii) Replacing the resolution parameter "$a$" of the blown-up $S^2$ by $a(r)$, substituting an ansatz: $a(r) = b + c^{\beta^0}(r - r_0) + \beta\mathcal{A}^\beta(r)$ into the Einstein's equations and estimating $r_0 \sim e^{-\kappa_{r_0} N^{1/3}}$ [10], near the $\psi = 2n\pi$, $n = 0$, 1, 2-coordinate patches, we therefore see that

$$b \sim N^{1+\frac{1}{\mathcal{O}(1)}} r_0; \qquad \mathcal{A}^\beta(r) = \mathcal{C} e^{\frac{c_{\rm linear}}{b} r},$$
$$\mathcal{C} \equiv \text{constant}. \tag{1}$$

(2) Vanishing mesino-mesino-meson interaction (Sec. V): Considering fluctuations of the vector mesons $A_{\mu \in S^1_t, \mathbb{R}^3, r} \to A^{(0)}_{\mu, r} + \delta A_{\mu, r}$ [with $A^{(0)}_{\mu = t}$ being the only nonzero background value] in the fermionic flavor $D6$-brane action and retaining terms linear in the same, performing a $KK$ expansion of the field strength fluctuation along with decomposition of the positive-chirality Majorana-Weyl mesino spinor along $M_5(t, x^{1,2,3}, r)$ and $\tilde{M}_5(\theta_{1,2}, \phi_{1,2}, \psi)$, we are able to show that no mesino-mesino-$\rho/\pi$-meson vertex is generated.

(3) Nonrenormalization of the mesino wave function and mass (Sec. III, and Appendixes B and D): With the aim of studying the effect of $\mathcal{O}(R^4)$ terms on the fermions relevant to holographic thermal QCD which was missing in [6], leads us to a nonrenormalization of the mesino wave function and mass in the sense that both turn out to be independent of the $\mathcal{O}(R^4)$ terms up to $(l_p^6/N^\alpha)$, $\alpha \geq 1$,[1] $l_p$ being the Planckian length.

The paper is organized as follows. In Sec. II, we discuss the type-IIA string dual construction of thermal QCD-like theories at intermediate coupling. In Sec. III, we show that fermionic superpartner of mesons, i.e., mesinos, are superheavy due to the generation of $N$-enhanced mass scale discussed in Sec. IV. Section V provides further evidence of superheavy mesinos because of the absence of mesino-mesino-meson interaction in type-IIA string dual. In Sec. VI, we discuss the nonrenormalization of the product of quark mass and quark condensate up to $\mathcal{O}(R)^4$. Section VII has a discussion of wave function universality in the context of glueball, meson, and graviton wave functions. The summary of the paper is provided in Sec. VIII.

There are five appendixes. Appendix A contains the discussion of baryon chemical potential. Appendix B consists of quantities appearing in the mesino EOMs of Sec. III. In Appendix C, we compute the embedding of flavor $D6$-branes in type IIA string theory inclusive of $\mathcal{O}(R^4)$ corrections. We list the constants appearing in the wave function for the black hole background in

[1] In [3], terms up to $\mathcal{O}(\frac{\beta^0}{N})$ and $\mathcal{O}(\frac{\beta}{N^\alpha})$, $0 < \alpha < 1$, $\beta \sim l_p^6$, were considered.

TABLE I. The type-IIB brane construct of [2] (NP and SP respectively denote the North Pole and South Pole of the blown-up $S^2$).

| S. No. | Branes | World volume |
|---|---|---|
| 1. | $N\ D3$ | $\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \{r = 0\}$ |
| 2. | $M\ D5$ | $\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \{r = 0\} \times S^2(\theta_1, \phi_1) \times \mathrm{NP}_{S^2_a(\theta_2,\phi_2)}$ |
| 3. | $M\ \overline{D5}$ | $\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \{r = 0\} \times S^2(\theta_1, \phi_1) \times \mathrm{SP}_{S^2_a(\theta_2,\phi_2)}$ |
| 4. | $N_f\ D7$ | $\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \mathbb{R}_+(r \in [\lvert\mu_{\mathrm{Ouyang}}\rvert^{\frac{2}{3}}, r_{\mathrm{UV}}]) \times S^3(\theta_1, \phi_1, \psi) \times \mathrm{NP}_{S^2_a(\theta_2,\phi_2)}$ |
| 5. | $N_f\ \overline{D7}$ | $\mathbb{R}^{1,3}(t, x^{1,2,3}) \times \mathbb{R}_+(r \in [\mathcal{R}_{D5/\overline{D5}} - \epsilon, r_{\mathrm{UV}}]) \times S^3(\theta_1, \phi_1, \psi) \times \mathrm{SP}_{S^2_a(\theta_2,\phi_2)}$ |

Appendix D. Finally, we summarize the top-down holographic QCD results obtained by our group in Appendix E.

## II. TYPE-IIA STRING DUAL OF THERMAL QCD-LIKE THEORIES INCLUSIVE OF $\mathcal{O}(R^4)$ CORRECTIONS

Thermal QCD-like theories refer to the equivalence class of theories that are IR confining and UV conformal with the "quarks" transforming in the fundamental representation of the symmetry groups (color and flavor). The UV-complete type-IIB string dual of such large-$N$ thermal QCD-like theories was constructed in [2]. The brane picture consists of $N$ space-time filling $D3$-branes at the tip of a warped resolved conifold, $M$ space-time filling $D5$ branes also at the tip of the conifold as mentioned above wrapping the vanishing squashed $S^2$ and at the North Pole of the resolved squashed $S^2$ of radius $a$ (resolution parameter), and space-time filling $\overline{D5}$-branes also at the tip of the conifold wrapping the abovementioned vanishing squashed $S^2(\theta_1, \phi_1)$ and at the South Pole of the resolved squashed $S^2(\theta_2, \phi_2)$. In addition, there are $N_f$ space-time filling flavor $D7$-branes wrapping the vanishing squashed $S^3(\theta_1, \phi_1, \psi)$ as well as being at the North Pole of the squashed resolved $S^2(\theta_2, \phi_2)$, dipping into the IR up to $\lvert\mu_{\mathrm{Ouyang}}\rvert^{\frac{2}{3}}$, $\lvert\mu_{\mathrm{Ouyang}}\rvert$ being the modulus of the Ouyang embedding parameter in the Ouyang embedding of the flavor $D7$-branes:

$$(9a^2r^4 + r^6)^{1/4} e^{\frac{i}{2}(\psi - \phi_1 - \phi_2)} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2} = \mu_{\mathrm{Ouyang}}. \tag{2}$$

An equal number of $\overline{D7}$ wrapping the vanishing squashed $S^3(\theta_1, \phi_1, \psi)$ and at the South Pole of the blown-up squashed $S^2(\theta_2, \phi_2)$, are also present. Equal number of $D5/D7$-branes and $\overline{D5}/\overline{D7}$-branes in the UV ensure UV conformality. The presence of $N_f$ flavor $D7$ and $\overline{D7}$-branes in the UV, implies a flavor gauge group $SU(N_f) \times SU(N_f)$ in the UV which is broken to $SU(N_f)$ due to absence of $\overline{D7}$-branes in the IR [2] (analog of chiral symmetry breaking in this brane setup). The brane construct in the type-IIB dual is summarized in Table I:

IR confinement in the gravity dual is affected by deforming the vanishing squashed $S^3$ in the conifold. Since we are interested in finite-temperature QCD, the same is effected via the black hole ($T > T_c$) and thermal ($T < T_c$) backgrounds on the gravity dual side. Due to finite temperature and finite separation of $D5$- and $\overline{D5}$-branes on the brane side, the conifold further needs also to possess an $S^2$-blow-up/resolution (with radius/resolution parameter $a$). Additionally, the 10-dimensional warp factor and fluxes include the effect of backreaction. Therefore, we conclude that string dual of thermal QCD-like theories in the large-$N$ limit involves a warped resolved deformed conifold. The additional advantage of the type-IIB dual of [2] is that in the IR, at the end of a Seiberg-like duality cascade, the number of colors $N_c$ gets identified with $M$, which in the intermediate-$N$ MQGP limit [1,11]

$$g_s \sim \frac{1}{\mathcal{O}(1)}, \qquad M, \qquad N_f \equiv \mathcal{O}(1), \qquad N > 1,$$
$$\frac{g_s M^2}{N} \ll 1, \qquad \frac{(g_s M^2)(g_s N_f)}{N} \ll 1, \tag{3}$$

can be tuned to equal 3, given that one is working in the vanishing-Ouyang-modulus limit [$\lvert\mu_{\mathrm{Ouyang}}\rvert \ll 1$ in (2)] of the embedding of the flavor $D7$-branes, $N_f$ can be set to either 2 or 3 corresponding to the lightest quark flavors [5].

Now, to explore the intermediate coupling regime, the $\mathcal{O}(R^4)$ terms in 11-dimensional supergravity actions were considered in [3]. $\mathcal{M}$-theory uplift was obtained in two steps; the type-IIA Strominger-Yau-Zaslow (SYZ) mirror of type-IIB setup was first obtained, and then the former was uplifted to $\mathcal{M}$-theory. To obtain type-IIA SYZ mirror of type-IIB setup, a triple T-duality was performed along a local special Lagrangian (sLag) $T^3(x, y, z)$ where $(x, y, z)$

[2] On the gravity dual side we characterize UV($r > \mathcal{R}_{D5/\overline{D5}}$) and IR ($r < \mathcal{R}_{D5/\overline{D5}}$) in term of radial coordinate where $\mathcal{R}_{D5/\overline{D5}}$ is the boundary between UV and IR, and separation between $D5$- and $\overline{D5}$-branes.

are the toroidal analogs of $(\phi_1, \phi_2, \psi)$ which could be identified with the $T^2$-invariant sLag of [12]; in the large-complex structure limit effected by making the base $\mathcal{B}(r, \theta_1, \theta_2)$ [of a $T^3(\phi_1, \phi_2, \psi)$-fibration over $\mathcal{B}(r, \theta_1, \theta_2)$] large [1,13]. Hence, all the color and flavor $D$-branes get T-dualized to color and flavor $D6$-branes. The $\mathcal{M}$-theory uplift metric [1,3] (finite but large-$N$/intermediate coupling) of [2] (UV-complete type-IIB holographic dual of large-$N$ thermal QCD-like theories) is expressed in the following form:

$$ds_{11}^2 = e^{-\frac{2\phi^{\rm IIA}}{3}}\left[\frac{1}{\sqrt{h(r,\theta_{1,2})}}(-g(r)dt^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2) + \sqrt{h(r,\theta_{1,2})}\left(\frac{dr^2}{g(r)} + ds^2_{\rm IIA}(r,\theta_{1,2},\phi_{1,2},\psi)\right)\right] + e^{\frac{4\phi^{\rm IIA}}{3}}\left(dx^{11} + A_{\rm IIA}^{F_1^{\rm IIB}+F_3^{\rm IIB}+F_5^{\rm IIB}}\right)^2, \tag{4}$$

where the type-IIA Ramond-Ramond (RR) 1-forms, $A_{\rm IIA}^{F_{i=1,3,5}^{\rm IIB}}$ are obtained from type IIB $F_{1,3,5}^{\rm IIB}$ fluxes via the SYZ mirror of type-IIB string dual [2], $g(r) = 1 - \frac{r_h^4}{r^4}$, and $\phi^{\rm IIA}$ is the type-IIA dilaton profile. For low temperatures, i.e., $T < T_c$, the thermal gravitational dual is given by

$$ds_{11}^2 = e^{-\frac{2\phi^{\rm IIA}}{3}}\left[\frac{1}{\sqrt{h(r,\theta_{1,2})}}(-dt^2 + (dx^1)^2 + (dx^2)^2 + \tilde{g}(r)(dx^3)^2) + \sqrt{h(r,\theta_{1,2})}\left(\frac{dr^2}{\tilde{g}(r)} + ds^2_{\rm IIA}(r,\theta_{1,2},\phi_{1,2},\psi)\right)\right] + e^{\frac{4\phi^{\rm IIA}}{3}}\left(dx^{11} + A_{\rm IIA}^{F_1^{\rm IIB}+F_3^{\rm IIB}+F_5^{\rm IIB}}\right)^2, \tag{5}$$

where $\tilde{g}(r) = 1 - \frac{r_0^4}{r^4}$. One notes that $t \to x^3, x^3 \to t$ in (4) followed by a double Wick rotation in the new $x^3, t$ coordinates obtains (5); $h(r, \theta_{1,2})$ is the 10-dimensional warp factor [1,2]. This is equivalent to $-g_{tt}^{\rm BH}(r_h \to r_0) = g_{x^3x^3}{}^{\rm Thermal}(r_0)$, $g_{x^3x^3}^{\rm BH}(r_h \to r_0) = -g_{tt}{}^{\rm Thermal}(r_0)$ in the results of [3,14] (see [15] in the context of Euclidean/black $D4$-branes in type IIA). In (5), we will assume the spatial part of the solitonic $M3$ brane (which, locally, could be interpreted as solitonic $M5$-brane wrapped around a homologous sum of $S^2_{\rm squashed}$ [16]) and their world volume given by $\mathbb{R}^2(x^{1,2}) \times S^1(x^3)$ with the period of $S^1(x^3)$ given by a very large $\frac{2\pi}{M_{\rm KK}}$, where the very small $M_{\rm KK}$ is given by $\frac{2r_0}{L^2}[1 + \mathcal{O}(\frac{g_s M^2}{N})]$, $r_0$ being the very small IR cutoff in the thermal background (see also [17]) and $L = (4\pi g_s N)^{\frac{1}{4}}$. So, $\lim_{M_{\rm KK}\to 0}\mathbb{R}^2(x^{1,2}) \times S^1(x^3) = \mathbb{R}^3(x^{1,2,3})$, thereby recovering 4D physics. The working metric for the thermal background corresponding to $T < T_c$ will involve setting $\tilde{g}(r)$ to unity in (5).

The 11-dimensional supergravity action including $\mathcal{O}(R^4)$ terms used in [3] is

$$S = \frac{1}{2\kappa_{11}^2}\int_M \left[\mathcal{R} *_{11} 1 - \frac{1}{2}G_4 \wedge *_{11} G_4 - \frac{1}{6}C \wedge G \wedge G\right] + \frac{1}{\kappa_{11}^2}\int_{\partial M} d^{10}x\sqrt{h}K + \frac{1}{(2\pi)^4 3^2 2^{13}}\left(\frac{2\pi^2}{\kappa_{11}^2}\right)^{\frac{1}{3}}\int d^{11}x\sqrt{-g}\left(J_0 - \frac{1}{2}E_8\right) + \left(\frac{2\pi^2}{\kappa_{11}^2}\right)\int C_3 \wedge X_8, \tag{6}$$

where

$$J_0 = 3\cdot 2^8\left(R^{HMNK}R_{PMNQ}R_H{}^{RSP}R^Q{}_{RSK} + \frac{1}{2}R^{HKMN}R_{PQMN}R_H{}^{RSP}R^Q{}_{RSK}\right),$$
$$E_8 = \frac{1}{3!}\epsilon^{ABCM_1N_1\ldots M_4N_4}\epsilon_{ABCM_1'N_1'\ldots M_4'N_4'}R^{M_1'N_1'}{}_{M_1N_1}\ldots R^{M_4'N_4'}{}_{M_4N_4},$$
$$\kappa_{11}^2 = \frac{(2\pi)^8 l_p^9}{2}. \tag{7}$$

The equations of motion for metric and three form potential $C$ are

$$\mathrm{EOM_{MN}}:\ R_{MN}-\frac{1}{2}g_{MN}\mathcal{R}-\frac{1}{12}\left(G_{MPQR}G_N{}^{PQR}-\frac{g_{MN}}{8}G_{PQRS}G^{PQRS}\right)=-\beta\left[\frac{g_{MN}}{2}\left(J_0-\frac{1}{2}E_8\right)+\frac{\delta}{\delta g^{MN}}\left(J_0-\frac{1}{2}E_8\right)\right],$$
$$d*G=\frac{1}{2}G\wedge G+3^2 2^{13}(2\pi)^4\beta X_8, \tag{8}$$

where [18]

$$\beta\equiv\frac{(2\pi^2)^{\frac{1}{3}}(\kappa_{11}^2)^{\frac{2}{3}}}{(2\pi)^4 3^2 2^{12}}\sim l_p^6. \tag{9}$$

$R_{MNPQ}$ and $R_{MN},\mathcal{R}$ in (6) and (8) are 11-dimensional Riemann curvature tensor, Ricci tensor, and the Ricci scalar. To solve (8), the following ansatz is made:

$$g_{MN}=g^{(0)}_{MN}+\beta g^{(1)}_{MN},$$
$$C_{MNP}=C^{(0)}_{MNP}+\beta C^{(1)}_{MNP}. \tag{10}$$

The EOM for $C_{MNP}$ symbolically can be written as

$$\beta\partial(\sqrt{-g}\partial C^{(1)})+\beta\partial[(\sqrt{-g})^{(1)}\partial C^{(0)}]+\beta\epsilon_{11}\partial C^{(0)}\partial C^{(1)}$$
$$=\mathcal{O}(\beta^2)\sim 0[\text{up to }\mathcal{O}(\beta)]. \tag{11}$$

It was shown in [3], that, $C^{(1)}_{MNP}=0$ up to $\mathcal{O}(\beta)$. Therefore, the metric only receives $\mathcal{O}(R^4)$ corrections defined as

$$\delta g_{MN}=\beta g^{(1)}_{MN}=G^{\mathrm{MQGP}}_{MN}f_{MN}(r). \tag{12}$$

In general, the $\mathcal{M}$ theory metric has the following form including $\mathcal{O}(R^4)$ corrections:

$$G^{\mathcal{M}}_{MN}=G^{\mathrm{MQGP}}_{MN}(1+\beta f_{MN}(r)). \tag{13}$$

The EOMs for $f_{MN}(r)$ were solved in [3]. The type-IIA metric inclusive of $\mathcal{O}(R^4)$ corrections were obtained from the $\mathcal{M}$-theory metric by descending back to type IIA string theory, which has the following form:

$$G^{\mathrm{IIA}}_{mn}=\sqrt{G^{\mathcal{M}}_{x^{10}x^{10}}}G^{\mathrm{MQGP}}_{mn}\left(1+\frac{f_{x^{10}x^{10}}(r)}{2}+f_{mn}(r)\right). \tag{14}$$

The type-IIB dual of large-$N$ thermal QCD-like theories as constructed in [2] and its type-IIA mirror as constructed in [1,13] were successfully used to study a variety of issues in condensed matter physics, lattice/Particle Data Group (PDG)-compatible particle phenomenology, doubly holographic extension and Page curves of associated eternal black holes and $G/$(Almost)Contact(3)Metric structure classification of underlying six-, seven- and eight-folds in differential geometry (see Appendix E).

## III. SUPERMASSIVE MESINOS IN TYPE-IIA STRING THEORY

The fermionic sector of type-IIA holographic dual of QCD as constructed in [8] has the following problems. Not only are the mesinos approximately isospectral with the mesons, the single-meson-mesino-mesino interaction terms are not large-$N$ suppressed [9] (see also [19] for mesino spectroscopy degenerate with mesons in the context of [8,15]). Evidently, this is in contradiction with QCD/PDG as no mesino at the electroweak (EW) scale has thus far been observed. What we show in this section is that Dirichlet/Neumann boundary condition at the IR cutoff (for the gravity dual corresponding to $T<T_c$) or the horizon radius (for the gravity dual corresponding to $T>T_c$) is consistent with having a supermassive mesino. Further, we show an $N$-enhancement of the Kaluza-Klein mass scale via an $N$-enhancement of the resolution parameter for the thermal background ($T<T_c$), hence providing the mechanism of generation of the aforementioned supermassive mesino. Even though we have not been able to provide in (III) an analog of the $N$-enhancement of the resolution parameter (that was seen in the thermal background corresponding to $T<T_c$) for the black hole background corresponding to $T>T_c$, the following should be noted. In (III), what we were able to show for the gravity duals of both the low- and high-temperature QCD-like theories is that Dirichlet/Neumann boundary condition at the IR cutoff, horizon radius respectively in the gravity duals for $T<T_c, T>T_c$, do not fix the mesino mass. We can hence take the same to be large, and via the aforementioned $N$-enhancement of the resolution parameter in the former, we had explicitly shown the mechanism of obtaining supermassive mesinos in the thermal background. Given that we were able to show the vanishing of meson-mesino-mesino interaction in (V), even if the mesinos were of the EW scale, there still will be no contradiction with real QCD.

The DBI action for the fermions on flavor $D6$-branes has the following structure [6]:

$$S^f_{D_6}=\frac{T_{D_6}}{2}\int d^7\xi e^{-\Phi^{\mathrm{IIA}}}\sqrt{-\det(i^*g^{\mathrm{IIA}}+\mathcal{F}^{\mathrm{IIA}})}\bar{\Theta}(1-\Gamma_{D_6})$$
$$\times(\Gamma^\alpha D_\alpha-\Delta+L_{D_6})\Theta, \tag{15}$$

where $\Phi^{\mathrm{IIA}}$ is the type-IIA dilaton. We can define

$$\mathcal{F}^{\mathrm{IIA}}_{\alpha_1\alpha_2}=i^*B^{\mathrm{IIA}}_{\alpha_1\alpha_2}+F^{\mathrm{IIA}}_{\alpha_1\alpha_2}, \tag{16}$$

such that $B^{\rm IIA}_{\alpha_1\alpha_2}$ and $F^{\rm IIA}_{\alpha_1\alpha_2}$ are NS–NS B field (where NS is understood as Neveu-Schwarz) and gauge field restricted to the world volume of $D6$-branes. Further, $\Gamma_{D_6}$ and $L_{D_6}$ appearing in (15) are defined as[3]

$$\Gamma_{D_6} = \sum_{q+r=3} \frac{(-)^{r+1}(\Gamma_{10})^{r+1}\epsilon^{\alpha_1\ldots.\alpha_{2q}\beta_1\ldots.\beta_{2r+1}}}{q!(2r+1)!2^q\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\ldots.\mathcal{F}^{\rm IIA}_{\alpha_{2q-1}\alpha_{2q}}\Gamma_{\beta_1\ldots.\beta_{2r+1}},$$
$$\Delta = \Delta^{(1)} + \Delta^{(2)},$$
$$L_{D_6} = \sum_{q\geq 1, q+r=3} \frac{(-)^{r+1}(\Gamma_{10})^{r+1}\epsilon^{\alpha_1\ldots.\alpha_{2q}\beta_1\ldots.\beta_{2r+1}}}{q!(2r+1)!2^q\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}}\mathcal{F}_{\alpha_1\alpha_2}\ldots.\mathcal{F}_{\alpha_{2q-1}\alpha_{2q}}\Gamma_{\beta_1\ldots.\beta_{2r+1}}{}^{\gamma}D_\gamma, \tag{17}$$

where $D_m = D^{(0)}_m + W_m$, and

$$D^{(0)}_m = \nabla_m + \frac{1}{4.2!}H_{mnp}\Gamma^{np}\Gamma_{(10)},$$
$$W_m = -\frac{1}{8}e^{\Phi^{\rm IIA}}\left(\frac{1}{2}F_{np}\Gamma^{np}\Gamma_{(10)} + \frac{1}{4!}F_{npqr}\Gamma^{npqr}\right)\Gamma_m,$$
$$\Delta^{(1)} = \frac{1}{2}\left(\Gamma^m\partial_m\Phi^{\rm IIA} + \frac{1}{2.3!}H_{mnp}\Gamma^{mnp}\Gamma_{(10)}\right),$$
$$\Delta^{(2)} = \frac{1}{8}e^{\Phi^{\rm IIA}}\left(\frac{3}{2!}F_{mn}\Gamma^{mn}\Gamma_{(10)} - \frac{1}{4!}F_{mnpq}\Gamma^{mnpq}\right), \tag{18}$$

where covariant derivative is defined as $\nabla_m = \partial_m + \frac{1}{4}\Omega_m^{\underline{np}}\Gamma_{\underline{np}}$. $F_{mn}$ and $F_{mnpq}$ are field strength tensors corresponding to type-IIA $A_n$ and $A_{npq}$, and $H_{mnp} = \partial_{[m}B_{np]}$. For flavor $D6$-branes in type-IIA string theory

$$\Gamma_{\rm D6} = \frac{\epsilon^{\beta_1\ldots.\beta_7}\Gamma_{\beta_1\ldots.\beta_7}}{\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}} - \frac{\Gamma_{(10)}(\epsilon^{\alpha_1\alpha_2\beta_1\ldots\ldots\beta_5}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\Gamma_{\beta_1\ldots\ldots\beta_5})}{5!\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}}$$
$$+\frac{\epsilon^{\alpha_1\ldots\alpha_4\beta_1\beta_2\beta_3}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\mathcal{F}^{\rm IIA}_{\alpha_3\alpha_4}\Gamma_{\beta_1\beta_2\beta_3}}{48\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}} - \frac{\Gamma_{(10)}(\epsilon^{\alpha_1\ldots.\alpha_6\beta_1}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\mathcal{F}^{\rm IIA}_{\alpha_3\alpha_4}\mathcal{F}^{\rm IIA}_{\alpha_5\alpha_6}\Gamma_{\beta_1})}{48\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}},$$
$$L_{\rm D6} = -\frac{\Gamma_{(10)}(\epsilon^{\alpha_1\alpha_2\beta_1\ldots\ldots\beta_5}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\Gamma_{\beta_1\ldots\beta_5}{}^{\gamma}D_\gamma)}{240\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}} + \frac{\epsilon^{\alpha_1\ldots\alpha_4\beta_1\beta_2\beta_3}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\mathcal{F}^{\rm IIA}_{\alpha_3\alpha_4}\Gamma_{\beta_1\beta_2\beta_3}{}^{\gamma}D_\gamma}{48\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}}. \tag{19}$$

The Dirac equation for the DBI action for the fermions on flavor $D6$-branes appearing in type-IIA string dual of thermal QCD-like theories turns out to be

$$\Bigg[\mathcal{A} - \frac{\epsilon^{\alpha_1\alpha_2\beta_1\ldots.\beta_5}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\Gamma_{\beta_1\ldots.\beta_5}{}^{\gamma}D_\gamma\Gamma_{(10)}}{240\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}} - \frac{\epsilon^{\beta_1\ldots.\beta_7}\Gamma_{\beta_1\ldots.\beta_7}\mathcal{A}}{7!\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}} + \frac{4\Gamma^{\beta_1\ldots.\beta_7}\mathcal{F}^{\rm IIA}_{\beta_6\beta_7}\Gamma_{\beta_1\ldots.\beta_5}{}^{\gamma}D_\gamma\Gamma_{(10)}}{7!(-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA}))}$$
$$+\frac{\epsilon^{\alpha_1\alpha_2\beta_1\ldots.\beta_5}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\Gamma_{\beta_1\ldots.\beta_5}\Gamma_{(10)}\mathcal{A}}{5!\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}} + \frac{7!\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\mathcal{F}^{\alpha_1\alpha_2}_{\rm IIA}\Gamma^{\beta_1\ldots.\beta_5}\Gamma_{\beta_1\ldots.\beta_5}{}^{\gamma}D_\gamma\Gamma_{(10)}}{5!240(-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA}))} + \frac{\epsilon^{\alpha_1\ldots\alpha_4\beta_1\beta_2\beta_3}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\mathcal{F}^{\rm IIA}_{\alpha_3\alpha_4}\Gamma_{\beta_1\beta_2\beta_3}\mathcal{A}}{48\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}}$$
$$-\frac{\Gamma_{(10)}\mathcal{F}^2_{\rm IIA}\mathcal{F}^{\rm IIA}_{\beta_4\beta_5}\Gamma_{\beta_1\beta_2\beta_3}\Gamma^{\beta_1\ldots.\beta_5\gamma}D_\gamma}{480(-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA}))} + \frac{7!\mathcal{F}^4_{\rm IIA}\Gamma_{\beta_1\beta_2\beta_3}\Gamma^{\beta_1\beta_2\beta_3\gamma}D_\gamma}{48(-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA}))} - \frac{\Gamma_{(10)}(\epsilon^{\alpha_1\ldots.\alpha_6\beta_1}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\mathcal{F}^{\rm IIA}_{\alpha_3\alpha_4}\mathcal{F}^{\rm IIA}_{\alpha_5\alpha_6}\Gamma_{\beta_1})\mathcal{A}}{48\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}}$$
$$+\frac{\delta^{[\alpha_3}_{[\beta_2}\delta^{\alpha_4}_{\beta_3}\delta^{\alpha_5}_{\beta_4}\delta^{\alpha_6]}_{\beta_5]}\mathcal{F}^2_{\rm IIA}\mathcal{F}^{\rm IIA}_{\alpha_3\alpha_4}\mathcal{F}^{\rm IIA}_{\alpha_5\alpha_6}\Gamma_{\beta_1}\Gamma^{\beta_1\ldots\beta_5\gamma}D_\gamma}{5760(-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA}))} - \frac{\mathcal{F}^4_{\rm IIA}\mathcal{F}^{\rm IIA}_{\beta_2\beta_3}\Gamma_{\beta_1}\Gamma^{\beta_1\beta_2\beta_3\gamma}D_\gamma}{24(-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA}))} + \frac{\epsilon^{\alpha_1\ldots\alpha_4\beta_1\beta_2\beta_3}\mathcal{F}^{\rm IIA}_{\alpha_1\alpha_2}\mathcal{F}^{\rm IIA}_{\alpha_3\alpha_4}\Gamma_{\beta_1\beta_2\beta_3}{}^{\gamma}D_\gamma}{48\sqrt{-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA})}}$$
$$-\frac{\delta^{[\beta_4}_{[\alpha_1}\delta^{\beta_5}_{\alpha_2}\delta^{\beta_6}_{\alpha_3}\delta^{\beta_7]}_{\alpha_4]}\mathcal{F}^2_{\rm IIA}\mathcal{F}^{\alpha_1\alpha_2}_{\rm IIA}\mathcal{F}^{\alpha_3\alpha_4}_{\rm IIA}\Gamma^{\beta_1\beta_2\beta_3}\Gamma_{\beta_1\ldots\beta_7}{}^{\gamma}D_\gamma}{48(-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA}))} - \frac{\mathcal{F}^2_{\rm IIA}\mathcal{F}^{\beta_4\beta_5}_{\rm IIA}\Gamma_{\beta_1\ldots\beta_5}\Gamma^{\beta_1\beta_2\beta_3\gamma}D_\gamma}{120(-\det(i^*g^{\rm IIA}+\mathcal{F}^{\rm IIA}))}\Bigg]\Theta = 0, \tag{20}$$

[3]Indices, $m$, $n$, $p$ correspond to type-IIA bulk indices and $\alpha_i, \beta_i, \gamma$ etc. correspond to indices on world volume of flavor $D6$-branes.

where

$$\mathcal{A} = \Gamma^\alpha D_\alpha - \frac{1}{2}\left(\Gamma^m \partial_m \Phi^{\rm IIA} + \frac{1}{12} H_{mnp}\Gamma^{mnp}\Gamma_{(10)}\right) - \frac{1}{8} e^{\Phi^{\rm IIA}}\left(\frac{3}{2} F_{mn}\Gamma^{mn}\Gamma_{(10)} - \frac{1}{4!} F_{mnpq}\Gamma^{mnpq}\right). \tag{21}$$

In (21), $F_{mnpq}$ is the type-IIA RR four-form field strength. This in our computation is set to zero as one can show that the same can not be generated by a triple T dual of the RR $F^{IIB}_{1,3,5}$ [1]. Type-IIA NS-NS $B$ is given by [20]

$$\begin{aligned} B^{\rm IIA}\left(\theta_1 = \frac{\alpha_{\theta_1}}{N^{\frac{1}{5}}}, \theta_2 \sim \frac{\alpha_{\theta_2}}{N^{\frac{3}{10}}}\right) &= d\theta_2 \wedge d\tilde{x}\left(-\frac{2\sqrt[4]{\pi}\sqrt[4]{g_s}N^{3/4}(3\sqrt{6}\alpha_{\theta_1}^3 - 2\alpha_{\theta_1}^2\sqrt[5]{N} + 2\alpha_{\theta_2}^2)}{27\alpha_{\theta_1}^4\alpha_{\theta_2}}\right) \\ &\quad + d\theta_2 \wedge d\tilde{y}\left(\frac{2\sqrt[4]{\pi}\sqrt[4]{g_s}N^{3/4}(3\sqrt{6}\alpha_{\theta_1}^3 - 2\alpha_{\theta_1}^2\sqrt[5]{N} + 2\alpha_{\theta_2}^2)}{27\alpha_{\theta_1}^4\alpha_{\theta_2}}\right) \\ &\quad + d\theta_2 \wedge d\tilde{z}\left(-\frac{\sqrt[4]{\pi}\alpha_{\theta_2}\sqrt[4]{g_s}N^{3/20}(2(\sqrt[3]{3}-1)\alpha\sqrt[10]{N} + \sqrt[3]{3}\alpha_{\theta_2})}{3^{5/6}\alpha\sqrt{\alpha_{\theta_2}^2}}\right). \end{aligned} \tag{22}$$

When we restrict to the world volume of $D6$-branes, then only the nontrivial component that survives will be $B^{\rm IIA}_{\theta_2\tilde{y}}$. The induced metric on the world volume of $D6$-branes can be obtained from the target space metric as

$$ds^2_{\rm D6} = ds^2_5 + g^{\rm IIA}_{\theta_2\theta_2}d\theta_2^2 + g^{\rm IIA}_{\theta_2\tilde{y}}d\theta_2 d\tilde{y} + g^{\rm IIA}_{\tilde{y}\tilde{y}}d\tilde{y}^2. \tag{23}$$

Typically, the type-IIA metric is not diagonal in the basis $(x, y, z)$. Since we need the metric component along $\tilde{y}$-direction therefore, we are writing the metric in the diagonal basis in subspace $(\tilde{x}, \tilde{y}, \tilde{z})$ [20],

$$\begin{aligned} ds^2 = &\frac{2d\tilde{x}^2(9\sqrt{2}\sqrt[6]{3}\alpha_{\theta_1}N^{4/5} - 2\,3^{2/3}N)}{27\alpha_{\theta_1}^2\alpha_{\theta_2}^2} \\ &+ \frac{2d\tilde{y}^2(2\,3^{2/3}N - 9\sqrt{2}\sqrt[6]{3}\alpha_{\theta_1}N^{4/5})}{27\alpha_{\theta_1}^2\alpha_{\theta_2}^2} \\ &+ \frac{2d\tilde{z}^2(3^{2/3}\alpha_{\theta_1}^2N^{3/5} + 3^{2/3}\alpha_{\theta_2}^2N^{2/5})}{27\alpha^2\alpha_{\theta_2}^2}. \end{aligned} \tag{24}$$

$ds^2_5$ in (23) is a noncompact metric listed along $(t, x^{1,2,3}, r)$ subspace, and from (24), $g^{\rm IIA}_{\theta_2\tilde{y}} = 0$ and $g^{\rm IIA}_{\tilde{y}\tilde{y}} = \frac{2(2\,3^{2/3}N - 9\sqrt{2}\sqrt[6]{3}\alpha_{\theta_1}N^{4/5})}{27\alpha_{\theta_1}^2\alpha_{\theta_2}^2}$.

Consider the DBI action on the world volume of flavor $D6$-branes,

$$S^{\rm D6}_{\rm DBI} = -T_{D6}N_f\int_{\Sigma_{(7)}}\sqrt{-\det(i^*(g^{\rm IIA} + B^{\rm IIA}) + F^{\rm IIA})}, \tag{25}$$

$i\colon\ \Sigma_{(7)} \cong S^1_t \times_w \mathbb{R}^3 \times \mathbb{R}_{\geq 0} \times_w S^2_{\rm squashed}(a) \hookrightarrow M_{10}$ [the embedding of the flavor $D6$-branes in the 10-dimensional background involving a warped squashed resolved conifold] in the $\psi = 2n\pi, n = 0,\ 1,\ 2$-coordinate patches and vanishingly small Ouyang embedding parameter in the parent type-IIB dual. Using the induced metric on the flavor $D6$-branes as given in (23), NS-NS $B^{\rm IIA}$ as given in (22) and turning on a baryon chemical potential [by looking at the DBI action in the UV and solving for $A_t(r)$; see (A1)] corresponding to $U(1)$ subgroup of $U(N_f)$ with the associated field strength $F_{rt} = A_t'(r)$, the background $A_t(r)$ can be obtained (see Appendix A). In the IR, $\mathcal{L}^{\rm D6}_{\rm DBI,on-shell}$, for $N \sim 10^2$, can be shown to be infinitesimal.

The coefficient of the most dominant (quadratic) powers of $\frac{1}{\mathcal{L}^{\rm D6}_{\rm DBI,on-shell}}$ in (20), is proportional to $\Gamma^\gamma D_\gamma\Theta, \gamma \in \{t, x^{1,2,3}, r, \theta_2, \tilde{y}\}|_{\{\tilde{x}=0,\tilde{z}={\rm constant}\}}$ where $(\tilde{x}, \tilde{y}, \tilde{z})$ diagonalize $T^3(x, y, z)$ of (II). One can further show that in the MQGP limit, $E^\gamma_{\underline{a}}\Gamma^{\underline{a}}D_\gamma\Theta \approx 0$. The non-Kähler sixfold $M_6 = S^1_t \times_w \mathcal{T}$ ($\times_w$ implying a warped product), $S^1_t$ being the thermal circle and $\mathcal{T}$—deformed $T^{1,1}$—being the base of a warped non-Kähler squashed resolved conifold, was shown to possess an $SU(3)$ structure in [3], with another "transverse" $SU(3)$ structure induced from the (almost) contact metric structure [11] arising from the $G_2$ structure of warped product of the $\mathcal{M}$-theory circle and $M_6$. Further, the non-Kähler warped-squashed resolved conifold $\tilde{M}_6$ in the type-IIA dual also possesses an $SU(3)$ structure [3,13]. Either way, one is therefore guaranteed the existence of a pair of globally defined spinors $\Theta_{1,0}$ and $\Theta_{2,0}$ [either by looking at the $SU(3)$ and the "transverse" $SU(3)$ structures on $M_6/\tilde{M}_6$ or when considering the embedding of the $D6$-brane world volume $\Sigma_{(7)} \cong S^1_t \times_w (\mathbb{R}^3 \times \mathbb{R}_{\geq 0}) \times_w S^2_{\rm squashed}$ in $M_{10}$ considered either as $(S^1_t \times_w \mathbb{R}^3) \times_w \tilde{M}_6$ or $\mathbb{R}^3 \times_w (\mathbb{R}_{\geq 0} \times M_6)$]. Making an ansatz,

$$\Theta_i(x^\mu, y^m) = \Theta_i(t, x^1, r, \theta_2) = \sum_{n:-\infty}^{\infty} T_n(t) e^{-\sqrt{-1}px^1} R_{n,i}(r)(1+\beta f_i(\theta_2))\Theta_{i,0}, \quad i = 1, 2, \tag{26}$$

where $\beta \sim l_p^6$ ($l_p$ being the Planckian length) and assuming $T_n(t) = e^{i(2n+1)\pi T t}$ (as one imposes antiperiodic boundary conditions on the fermions along the thermal circle thereby breaking supersymmetry [21]) implying $\Theta(t + 1/T, r) = -\Theta(t, r)$, (and after a double Wick rotation along $t, x^1$, $p^2 = -m^2_{\rm Mesino}$ with $m_{\rm Mesino}$ being the nonsupersymmetric mesino mass in the holographic dual of $\rm QCD_{Mesino}$) analogous to the relation between the killing spinors $\epsilon_{1,2}$ for a supersymmetric $D6$-brane in flat space, $\epsilon_1 = \Gamma^{\underline{89\,10}}\epsilon_2$, we will impose, by hand, and for our nonsupersymmetric model, $\Theta_{1,0} = \Gamma^{\underline{68\,10}}\Theta_{2,0}$, in a curved space, where $\Theta_{1,2} \equiv \frac{1}{2}(\mathbf{1} + / - \Gamma^{(10)})\Theta$, $\Gamma^{(10)} \equiv \prod_{\underline{A}=0}^{9}\Gamma^{\underline{A}}$; $\underline{A}$, with $A = 1, \ldots, 10$ denoting the 10-dimensional tangent space indices.

The most dominant spin-connection terms in the IR are contained in $E^r_{\underline{5}}\Gamma^{\underline{5}}D_r\Theta$, in particular $\omega_r^{\underline{7\,10}}/\omega_r^{\underline{8\,10}}$ respectively for the thermal (TH), black-hole (BH) backgrounds. Consequently, substituting (26) into Θ's EOM (details given in this section and Appendix B), the same at $\mathcal{O}(\beta)$ is

$$\Bigg[i(2n+1)\pi T R_{2,n}(r) f_2(\theta_2) + \frac{E_{\underline{7}}^{\theta_2}}{E_{\underline{1}}^{t}}\Gamma^{\underline{17}}R_{2,n}(r)f_2'(\theta_2) + \frac{E_{\underline{5}}^{r}}{E_{\underline{1}}^{t}}\Gamma^{\underline{15}}R'_{2,n}(r)f_2(\theta_2) - ip\frac{E_{\underline{2}}^{x^1}}{E_{\underline{1}}^{t}}\Gamma^{\underline{12}}R_{2,n}(r)f_2(\theta_2)\Bigg]\Theta_{2,0} - \mathcal{J}(r)R_{1,n}(r)f_1(\theta_2)\Gamma^{\underline{15678}}\Theta_{1,0} = 0, \tag{27}$$

with $\mathcal{J} \equiv \omega_r^{\underline{7\,10}}\frac{E^r_{\underline{5}}}{E^t_{\underline{1}}}$ ($E^M_{\underline{a}}$ being the frames $E^M_{\underline{a}}g_{MN}E^N_{\underline{b}} = \eta_{\underline{ab}}$) for the TH background; for the BH background, $\Gamma^{\underline{15678}}$ in the second line of (27) is to be replaced by $\Gamma^{\underline{156}}$ with $\mathcal{J} \equiv \omega_r^{\underline{8\,10}}\frac{E^r_{\underline{5}}}{E^t_{\underline{1}}}$. Note, we have disregarded all $\mathcal{O}(\frac{\beta}{N^\alpha}), \alpha \geq 1$ terms (see footnote 1) and therefore there are no $\beta$ corrections in $\frac{E^{\theta_2}_{\underline{7}}}{E^t_{\underline{1}}}, \frac{E^r_{\underline{5}}}{E^t_{\underline{1}}}, \frac{E^{x^1}_{\underline{2}}}{E^t_{\underline{1}}}$. One thus sees that the only consistent solution for $f_i(\theta_2)$ is $f_i(\theta_2) = 0$ for the TH/BH backgrounds.

Defining $u \equiv \sqrt{r - r_0}$, the EOM for $R_{n,2}(r)$ for the TH-type-IIA background with $\Gamma^{\underline{15}}\Theta_{2,0} = \Theta_{2,0}, \Gamma^{\underline{12}}\Theta_{2,0} = \Theta_{2,0}$, can be recast into a Schrödinger-like equation [where, $a_1, b_1, \mathcal{A}_{\Theta_2}, \mathcal{B}_{\Theta_2}, \mathcal{A}_{\Theta_2'}, \mathcal{B}_{\Theta_2'}$ are defined in (B5)],

$$\chi''_{2,n}(u) + V(u)\chi_{2,n}(u) = 0, \tag{28}$$

where

$$V(u) = -\frac{3}{4u^2} + \frac{\mathcal{A}_{\Theta_2'}}{a_1 u} - \frac{\mathcal{A}_{\Theta_2'}}{a_1^2} + \mathcal{O}(u), \tag{29}$$

and

$$R_{2,n}(u) = \sqrt{u}(a_1 + b_1 u^2)^{-\frac{\mathcal{B}_{\Theta_2'}}{2b_1}} e^{-\frac{\mathcal{A}_{\Theta_2'}\tan^{-1}(\frac{\sqrt{b_1}u}{\sqrt{a_1}})}{\sqrt{a_1}\sqrt{b_1}}}\chi_{2,n}(u). \tag{30}$$

The solution of (28) is given by

$$\chi_{2,n}(u) = c_{1,n}M_{\frac{1}{2},1}\left(\frac{2\mathcal{A}_{\Theta_2'}u}{a_1}\right) + c_{2,n}W_{\frac{1}{2},1}\left(\frac{2\mathcal{A}_{\Theta_2'}u}{a_1}\right). \tag{31}$$

One, therefore obtains

$$R_{2,n}(r \sim r_0) = \frac{c_{2,n}a_1^{\frac{1}{2}-\frac{\mathcal{B}_{\Theta_2'}}{2b_1}}}{\sqrt{2}\sqrt{\mathcal{A}_{\Theta_2'}}} - \frac{(r-r_0)a_1^{-\frac{\mathcal{B}_{\Theta_2'}}{2b_1}-\frac{3}{2}}(a_1\mathcal{B}_{\Theta_2'}c_{2,n} + \mathcal{A}^2_{\Theta_2'}(4c_{2,n} - 8c_{1,n}))}{2\sqrt{2}\sqrt{\mathcal{A}_{\Theta_2'}}} + \mathcal{O}((r-r_0)^{3/2}). \tag{32}$$

From (B5), one sees the absence of $\mathcal{O}(R^4)$ corrections in (32). One also sees from (32) that one can impose a Dirichlet boundary condition at $r = r_0$ (thereby setting $c_2 = 0$) for all and hence superheavy mesinos ($M_{\rm Mesino}$).

For the BH background assuming $\Gamma^{\underline{15}}\Theta_{2,0} = \Theta_{2,0}$, $\Gamma^{\underline{12}}\Theta_{2,0} = \Theta_{2,0}$, implying, $\Gamma^{\underline{25}}\Theta_{2,0} = \Theta_{2,0}$, in the IR (i.e., near $r = r_h$), redefining $u \equiv \sqrt{r - r_h}$, the solution of the EOM for $R_{2,n}(u)$ is

$$R_{2,n}(u) = u^\Lambda[c_1 U(\mu_1, \mu_2, \mu_3 u) + c_2 L^{\mu_2 - 1}_{-\mu_1}(\mu_3 u)], \tag{33}$$

where $\Lambda, \mu_{1,2,3}$ are defined in (D1), and $p = M_{\rm Mesino}\frac{r_h}{\sqrt{g_s N}}$[4] is contained in the $\mathcal{O}(\frac{\beta}{N})$ term in $\mu_3$, which hence remains undetermined as $\mathcal{O}(\frac{\beta}{N})$ terms are dropped (see footnote 1). One can show that $\lim_{u\to 0}u^\Lambda c_1 U(\mu_1, \mu_2, \mu_3 u)$ is singular. One hence can not impose Dirichlet or Neumann boundary condition at $r = r_h$ if $c_2 = 0$. Now,

[4]Glueball and meson masses at high temperatures were obtained respectively in [20,22] in units of $\frac{r_h}{\sqrt{g_s N}}$.

$$L^{\mu_2-1}_{-\mu_1}(u) = \frac{\Gamma(\mu_2-\mu_1)}{\Gamma(1-\mu_1)\Gamma(\mu_2)} - \frac{\Gamma(\mu_2-\mu_1)}{\Gamma(-\mu_1)\Gamma(\mu_2+1)}u + \mathcal{O}(u^2), \tag{34}$$

implying $\lim_{u\to 0} u^{\Lambda} c_2 L^{\mu_2-1}_{-\mu_1}(\mu_3 u) = 0$, implying the Dirichlet boundary condition is identically satisfied $\forall\, M_{\rm Mesino}$ including very large $M_{\rm Mesino}$. It is extremely nontrivial that the $\mu_i$s receive no $\mathcal{O}(\beta)$ corrections up to $\mathcal{O}(\frac{\beta}{N^{\alpha_{\mu_i}}})$, $\alpha_{\mu_i} \geq 1$ [see (D1)].

The absence of $\mathcal{O}(R^4)$ corrections is essentially a reflection of the fact that the $SL(2,Z)$ completion of the effective $R^4$ interaction terms in type-IIB supergravity leads to an interesting nonrenormalization theorem that forbids perturbative corrections beyond one loop in the zero-instanton sector [23].

What we now address in Sec. IV is how an $N$-enhancement of the mass scale $M_{\rm KK} = \frac{r_0}{\sqrt{4\pi g_s N}}$ [5] is obtained which therefore explains how one could obtain supermassive $M_{\rm Mesino}$.

## IV. GENERATION OF $N$-HANCED MASS SCALE FOR $T < T_c$

In this section, starting from the $D = 11$ supergravity Einstein's field equations in the presence of four-form $G$ fluxes of $\mathcal{M}$-theory[5]—the first in (8) (also given in Appendix E)—we explicitly show the generation of an '$N$-hanced mass scale, thereby providing the mechanism of generation of supermassive mesinos.

Replacing the resolution parameter "$a$" of the blown-up $S^2$ by $a(r)$, substituting an ansatz $a(r) = b + c^{\beta^0}(r - r_0) + \beta\mathcal{A}^{\beta}(r)$ into EOM$_{MN}$ in (8) ($b$ being a "bare" resolution parameter), and estimating $r_0 \sim e^{-\kappa_{r_0} N^{1/3}}$ [10], near the $\psi = 2n\pi, n = 0,\ 1,\ 2$-coordinate patch, yields the following:

(1)

$$\begin{aligned}&{\rm EOM}_{tt,x^1x^1/x^2x^2}:\ b \sim \kappa_{tt/x^ix^i/rr} N^{10/9} e^{-\kappa_{r_0}(N)(3+0.5\kappa_{r_0}(N)N^{1/3})N^{1/3}} r_0;\\ &\mathcal{A}^{\beta}(r) \sim e^{\frac{c^{\beta}}{b}r}\mathcal{C}_1,\end{aligned} \tag{35}$$

with $\kappa_{tt/x^ix^i/rr} \gg 1, \kappa_{r_0}(N = 10^2) = \frac{1}{\mathcal{O}(1)} - \mathcal{O}(1)$, one obtains $b \gg r_0$ and in principle an $r_0$-independent true bare resolution parameter proptional to $\beta$. The EOM$_{x^3x^3}$ near $r = r_0$ does not constrain $b$.

(2)

$$ {\rm EOM}_{rr}:\ b \sim \tilde{\kappa}_{rr} N^{11/9} e^{\kappa_{r_0}N^{1/3} + 1.25\sqrt{1.57 + 0.55\log r_0 - 0.5(\log r_0)^2}} r_0; \tag{36}$$

and for an appropriate $\kappa_{r_0} \sim \frac{1}{\mathcal{O}(1)}:\ 1.57 + 0.55\log r_0 - 0.5(\log r_0)^2 > 0$, and $N \sim 10^2$, one regains the result for $b$ as obtained in the first equation in (35).

(3)

$$\begin{aligned}&{\rm EOM}_{\theta_1\theta_1}: a(r)\ \text{determined by}:\\ &-\frac{1323\sqrt[5]{N}\alpha_{\theta_1}^2}{256\alpha_{\theta_2}^2}\\ &-\frac{729 g_s{}^3 M^2 (\frac{1}{N})^{6/5} N_f{}^2 (2187\alpha_{\theta_1}^6 + 270\sqrt{6}\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 50\alpha_{\theta_2}^4) a(r)^3 \log^3(r_0)(2rr_0\log(r_0)a'(r) + a(r)(r_0 - r)\log(r_0))}{16\pi^3 r_0^5 \alpha_{\theta_1}^2\alpha_{\theta_2}^2} = 0,\end{aligned} \tag{37}$$

whose solution is given by

$$\begin{aligned} a(r) &= \left(\frac{864 c_1 g_s{}^3 M^2 N_f{}^2 \Sigma e^{\frac{2r}{r_0}}\log^4(r_0) - 98\pi^3 N^{7/5} r r_0{}^5 \alpha_{\theta_1}^4 - 49\pi^3 N^{7/5} r_0{}^6 \alpha_{\theta_1}^4}{g_s{}^3 M^2 N_f{}^2 r^2 \Sigma \log^4(r_0)}\right)^{1/4},\\ &\sim c_1 \frac{e^{\frac{r}{2r_0}}}{\sqrt{r}} \sim \frac{c_1}{\sqrt{r_0}}\left[1 + \mathcal{O}\left(\frac{(r - r_0)^2}{r_0^2}\right)\right], \end{aligned} \tag{38}$$

[5]One can show that $E_8$-dependent terms in the same are subdominant as compared to the $J_0$-dependent terms [3].

where $\Sigma \equiv (2187\alpha_{\theta_1}^6 + 270\sqrt{6}\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 50\alpha_{\theta_2}^4)$. Recalling that $r_0 \sim e^{-\kappa_{r_0}N^{1/3}}$, we reinterpret (37) as $a(r \sim r_0) \sim c_1 e^{\frac{3\kappa_{r_0}N^{1/3}}{2}} r_0$, where for compatibity with (35) and (36), one may choose an $N$-dependent $c_1 \sim N^{(10-11)/9} e^{-\gamma N^{1/3}}$ for an appropriate $\gamma$.

(4) $\mathrm{EOM}_{\theta_1\theta_2}$

$$\lambda_3 a(r)^3(a(r) - a'(r)) + \lambda_1 a(r)^4 + \frac{\lambda_2(36a(r)^2\log(r_0) + r_0)}{{r_0}^2 - 3a(r)^2} = 0, \tag{39}$$

where

$$\begin{aligned}
\lambda_1 &\equiv -\frac{243{g_s}^3M^2(\frac{1}{N})^{11/10}N_f^2(2187\alpha_{\theta_1}^6 + 270\sqrt{6}\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 50\alpha_{\theta_2}^4)\log^4(r_0)}{8\pi^3{r_0}^4\alpha_{\theta_1}\alpha_{\theta_2}^3},\\
\lambda_2 &\equiv -\frac{1323N^{3/10}r_0\alpha_{\theta_1}^3}{256\alpha_{\theta_2}^3(\log N - 9\log(r_0))},\\
\lambda_3 &\equiv -\frac{729{g_s}^3M^2(\frac{1}{N})^{11/10}N_f^2(2187\alpha_{\theta_1}^6 + 270\sqrt{6}\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 50\alpha_{\theta_2}^4)\log^4(r_0)}{8\pi^3{r_0}^3\alpha_{\theta_1}\alpha_{\theta_2}^3}.
\end{aligned} \tag{40}$$

Defining

$$\Lambda \equiv \frac{2^{5/6}\sqrt{g_s}\sqrt[3]{M}\sqrt[3]{N_f}{r_0}^2\sqrt[6]{2187\alpha_{\theta_1}^6 + 270\sqrt{6}\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 50\alpha_{\theta_2}^4}\log^{\frac{25}{6}}(r_0)}{9\sqrt[3]{7}\sqrt{3\pi}N^{7/30}\alpha_{\theta_1}^{2/3}}, \tag{41}$$

$a(r)$ is given by

$$\begin{aligned}
&\sqrt{\Lambda + \exp\left(\frac{2(r + \lambda_3 c_1)(\Lambda(\lambda_1 + \lambda_3)(2{r_0}^2 - 9\Lambda) + 36\lambda_2\log(r_0))}{\lambda_3\Lambda({r_0}^2 - 3\Lambda)}\right)}\\
&\quad \sim \sqrt{\Lambda} \sim \frac{\sqrt[4]{g_s}\sqrt[6]{M}\sqrt[6]{N_f}\sqrt[12]{2187\alpha_{\theta_1}^6 + 270\sqrt{6}\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 50\alpha_{\theta_2}^4}\log^{\frac{25}{12}}(r_0)}{N^{7/60}\alpha_{\theta_1}^{1/3}} r_0.
\end{aligned} \tag{42}$$

(5) $\mathrm{EOM}_{\theta_1 x}$:

$$b \sim N^{23/36} e^{\frac{1}{6}\kappa_{r_0}N^{1/3}(9\kappa_{r_0}N^{1/3} + \log N)} r_0. \tag{43}$$

(6) $\mathrm{EOM}_{\theta_1 y}$:

$$b \sim N e^{\frac{3}{2}\kappa_{r_0}^2 N^{2/3}} r_0. \tag{44}$$

(7) $\mathrm{EOM}_{\theta_2 x}$:

$$b \sim \kappa_{\theta_2 y} N^{10/9} e^{-\kappa_{r_0}^2 N^{2/3} + 4\kappa_{r_0}N^{1/3}} r_0, \qquad \kappa_{\theta_2 x} \gg 1. \tag{45}$$

(8) $\mathrm{EOM}_{\theta_2 y}$:

$$b \sim N^{10/9} e^{-3\kappa_{r_0}N^{1/3} + \kappa_{r_0}^2 N^{2/3}} r_0. \tag{46}$$

(9) $\mathrm{EOM}_{\theta_2 z}$:

$$b \sim N^{10/9} e^{\kappa_{r_0}^2 N^{2/3} - 3\kappa_{r_0}N^{1/3}} r_0. \tag{47}$$

(10) $\mathrm{EOM}_{xz/yy/yz/zz}$:

$$b \sim N^{10/9} e^{\kappa_{r_0}^2 N^{2/3} - 6\kappa_{r_0}N^{1/3}} r_0. \tag{48}$$

(11) $\mathrm{EOM}_{x^{10}x^{10}}$:

$$b \sim N^{10/9} r_0. \tag{49}$$

We therefore see that the bare resolution parameter $b$ is given by

$$b \sim N^{1 + \frac{1}{\mathcal{O}(1)}} r_0; \quad a^\beta(r) = \mathcal{C} e^{\frac{c_{\rm linear}}{b} r}, \quad \mathcal{C} \equiv \text{constant}. \tag{50}$$

One hence can not obtain an $r_0$-independent $b$. One thus sees an $N$-hancement of the effective $KK$ mass scale $M_{KK}$ (from $M_{KK}$ to $M_{KK}^{\rm eff} \sim N^{1+\frac{1}{\mathcal{O}(1)}} M_{KK}$) arising from the construction of SYZ type-IIA mirror of the non-Kähler type-IIB dual [2] of thermal QCD-like theories, as well as the generation of a one-parameter ($\mathcal{C}$) family of $r_0/M_{KK}$-independent bare resolution parameter at $\mathcal{O}(R^4)$ in the $\mathcal{M}$-theory uplift involving a $G_2$-structure wherein $\mathcal{C}$ can be made appropriately large. These are the pair of reasons for generating super-massive mesinos in the fermionic sector in the string/$\mathcal{M}$ theory duals of thermal QCD at finite $N$ in [1,3].

## V. NONINTERACTING MESINOS

Given that we have seen in Sec. III that supermassive mesinos, unlike [8] (see [9]), are permissible in the type-IIA holographic dual [1] at intermediate coupling [3] of realistic thermal QCD-like theories, this already explains why mesinos have thus far not been observed near the EW scale. In this section, we will further show that mesino-mesino-single-($\rho/\pi$)meson interactions, unlike [8] (see [9]), vanish identically in the aforementioned type-IIA holographic dual. Considering fluctuations of the vector mesons $A_{\mu,r} \rightarrow A^{(0)}_{\mu,r} + \delta A_{\mu,r}$ with $A^{(0)}_{\mu=t}$ being the only nonzero background value (see Sec. III) which can be shown to be tunable so that $|F^{(0)}_{rt}| \ll 1$, implying one need only consider terms linear in $F^{(0)}_{\rm IIA}$[6] which are contained (recalling from Sec. III, $\sqrt{-\det(i^* g^{\rm IIA} + \mathcal{F}^{(0)}_{\rm IIA})} \ll 1$, $\mathcal{F}_{IIA} = i^* B_{IIA} + F$ in the large-$N$ limit) in

$$S^f_{D_6} = \frac{T_{D_6}}{2} \int d^7\xi e^{-\Phi^{\rm IIA}} \times \bar{\Theta}\left(\frac{\Gamma^{\beta_1\ldots\beta_7} \mathcal{F}^{(0)}_{{\rm IIA}\beta_6\beta_7} \Gamma_{\beta_1\ldots\beta_5}{}^{\gamma} D_\gamma \Gamma_{(10)}}{\sqrt{-\det(i^* g^{\rm IIA} + \mathcal{F}^{(0)}_{\rm IIA})}}\right)\Theta. \quad (51)$$

Considering fluctuations in the background gauge field in (51) and retaining terms linear in the same yields,

$$\delta S^f_{D_6} \sim T_{D_6} \int_{\Sigma_{(7)}} d^4x dr d\theta_2 d\tilde{y} e^{-\Phi^{\rm IIA}} \times \bar{\Theta}\left(\frac{4\Gamma^{\beta_1\ldots\beta_7} \delta\mathcal{F}^{\rm IIA}_{\beta_6\beta_7} \Gamma_{\beta_1\ldots\beta_5}{}^{\gamma} D_\gamma \Gamma_{(10)}}{\sqrt{-\det(i^* g^{\rm IIA} + \mathcal{F}^{(0)}_{\rm IIA})}}\right)\Theta. \quad (52)$$

The next step is to perform the $KK$ expansion of $\delta\mathcal{F}^{\rm IIA}_{\alpha\beta}$ and decompose spinors along $M_4$ and internal directions, and by integrating over the $\theta_2$ and $\tilde{y}$ we will get mesino-mesino-meson interaction action with couplings given in terms of radial integrals of the radial profile functions of the mesino and mesons. The usual $KK$ expansion ansatz [5] is

$$\delta A_\mu(x^\mu, r) = \sum_{n=1}^\infty \rho^{(n)}_\mu(x)\psi_n(r), \quad (53)$$

and

$$\delta A_r(x^\mu, r) = \sum_{n=0}^\infty \pi^{(n)}(x)\phi_n(r), \quad (54)$$

implies

$$\delta F_{\mu\nu} = \sum_{n=1}^\infty \tilde{F}^{(n)}_{\mu\nu}(x)\psi_n(r), \quad (55)$$

and

$$\delta F_{\mu r} = \sum_{n=0}^\infty \partial_\mu \pi^{(n)}(x^\mu)\phi_n(r) - \sum_{n=1}^\infty \rho^{(n)}_\mu(x)\dot{\psi}_n(r). \quad (56)$$

We will keep the $n = 1$ term for the vector fluctuation and $n = 0$ for the $A_r(x^\mu, r)$; hence, the degrees of freedom are the $\rho$ vector meson and the $\pi$ meson. Using the $KK$ decomposition of $\delta F_{\mu\nu}$ and $\delta F_{\mu r}$, (52) simplified as follows:

$$S^{\rm int}_{D_6} \sim T_{D_6} \int_{\Sigma_{(7)}} \left[\frac{e^{-\Phi^{\rm IIA}}}{\sqrt{-\det(i^* g^{\rm IIA} + \mathcal{F}^{(0)}_{\rm IIA})}} \bar{\Theta}(\Gamma^{\beta_1\ldots\beta_5\mu\nu} \delta\tilde{F}_{\mu\nu}\psi(r)\Gamma_{\beta_1\ldots\beta_5}{}^{\gamma} D_\gamma + \Gamma^{\beta_1\ldots\beta_5\mu r}(\partial_\mu\pi(x^\mu)\phi(r) - \rho_\mu(x^\mu)\dot{\psi}(r))\Gamma_{\beta_1\ldots\beta_5}{}^{\gamma} D_\gamma)\Theta\right]. \quad (57)$$

Using the decomposition of the 10-dimensional gamma matrices [24],

$$\Gamma^{\underline{A}=\underline{t},\underline{x^{1,2,3}},\underline{r}} = \sigma_y \otimes \mathbf{1}_4 \otimes \gamma^{\underline{A}}, \quad \Gamma^{\underline{a}=5,\ldots,9} = \sigma_x \otimes \gamma^{\underline{a}} \otimes \mathbf{1}_4, \quad (58)$$

with

$$\{\gamma^{\underline{A}}, \gamma^{\underline{B}}\} = -2\eta^{\underline{AB}}, \qquad \{\gamma^{\underline{a}}, \gamma^{\underline{b}}\} = -2\delta^{\underline{ab}}. \quad (59)$$

The 10-dimensional chirality matrix is defined as

$$\Gamma^{(10)} = \sigma_z \otimes \mathbf{1}_4 \otimes \mathbf{1}_4. \quad (60)$$

[6]Use is made of $i^* B_{\alpha_1\alpha_2} = \delta^{[\theta_1}_{\alpha_1}\delta^{\theta_2]}_{\alpha_2} B_{\theta_1\theta_2}$ and consequently, $\mathcal{F}_{rt} = F_{rt}$.

The positive-chirality 10-dimensional $\Theta$ can hence be decomposed into

$$\Theta = \uparrow \otimes \chi_{M_5(x^{0,1,2,3},r)} \otimes \psi_{\tilde{M}_5(\theta_{1,2},\phi_{1,2},\psi)}, \tag{61}$$

where $\psi_{\tilde{M}_5(\theta_{1,2},\phi_{1,2},\psi)}$ further splits into $\psi_{\tilde{M}_5} = \psi_{S^2_{\rm squashed}} \otimes \psi_{S^3_{\rm squashed}}$. Looking at the second fermionic bilinear in (57) we get

$$\bar{\Theta}\Gamma^{\beta_1\ldots\beta_5 tr}\Gamma_{\beta_1\ldots\beta_5}{}^{\gamma}D_\gamma\Theta(\beta_{i=1,\ldots,5} = x^{0,1,2,3},\theta_2,\tilde{y};\gamma = t,r)$$
$$\sim \bar{\Theta}\Gamma^t D_r\Theta + \bar{\Theta}\Gamma^r D_t\Theta. \tag{62}$$

Now, the nonvanishing $\bar{\Theta}\Gamma^{\underline{X_1}\cdots\underline{X_p}}\Theta$ involving Majorana-Weyl spinor $\Theta$ requires $p = 3, 7$ [9]. One can further show that the most dominant spin-connection component of the type $\omega_r^{\underline{ab}}$ is $\omega_r^{\underline{79}}$ and only nonvanishing spin-connection component of the type $\omega_t^{\underline{ab}}$ is $\omega_t^{\underline{x^0 r}}$. Therefore, using (58),

$$\bar{\Theta}\Gamma^{\beta_1\ldots\beta_5 tr}\Gamma_{\beta_1\ldots\beta_5}{}^{\gamma}D_\gamma\Theta \sim \bar{\Theta}\Gamma^{\underline{t}}\omega_r^{\underline{79}}\Gamma^{\underline{79}}\Theta$$
$$\propto \langle\uparrow|\sigma_y|\uparrow\rangle = 0. \tag{63}$$

Also,

$$\bar{\Theta}\Gamma^{\beta_1\ldots\ldots\beta_5\mu\nu}\delta\tilde{F}_{\mu\nu}\psi(r)\Gamma_{\beta_1\ldots\ldots\beta_5}{}^{\gamma}D_\gamma\Theta = 0, \tag{64}$$

as $\mu,\nu \in x^{1,2,3}$ and thus using (58),

$$\bar{\Theta}\Gamma^{kx^0r\theta_2\tilde{y}ij}\delta\tilde{F}_{ij}\psi(r)\Gamma_{kx^0r\theta_2\tilde{y}}{}^{i}D_i\Theta(i \neq j \neq k = x^{1,2,3})$$
$$\sim \delta\tilde{F}_{ij}\bar{\Theta}\Gamma^{ij}\Gamma^l\partial_l\Theta(l \neq k) \propto \langle\uparrow|\sigma_y|\uparrow\rangle = 0. \tag{65}$$

Hence, no mesino-mesino-$\rho/\pi$-meson vertex is generated. Together with what was argued earlier that one could have a supermassive mesino, this suggests the WISP nature of the nonsupersymmetric mesino, and consequently resolves the tension between actual QCD and top-down holographic QCD [9].

## VI. TOP-DOWN $m_{\rm quark}\langle\bar{q}q\rangle$ NONRENORMALIZATION UP TO $\mathcal{O}(R^4)$

The $\mathcal{O}(R^4)$ corrections to the $\mathcal{M}$-theory dual's metric are vanishing small in the UV [25]. The EOM of the flavor $D6$-branes' embedding, $\tilde{z} = \tilde{z}(r)$ in the IR arising from the DBI action for the flavor $D6$-branes with world volume $\Sigma_7(S^1_t \times \mathbb{R}^3 \times \mathbb{R}_{>0} \times S^2_{\rm squashed})$ embedded via $i\colon \Sigma_7 \hookrightarrow S^1_t \times \mathbb{R}^3 \times_w M_6$ [$w\equiv$ warped product] affected by $\tilde{z} = \tilde{z}(r)$ in a non-Kähler warped squashed resolved conifold $M_6$ in the type-IIA mirror of the UV-complete type-IIB dual [2] of thermal QCD-like theories, using the induced metric on flavor $D6$-branes of (23), NS-NS $B^{\rm IIA}$ of (22), can be shown to yield $\tilde{z} =$ constant, inclusive of $\mathcal{O}(\beta)$ corrections.

The DBI action in the UV is given by (disregarding overall $r$-independent factors, and hence the $\sim$)

$$\mathcal{L}^{\rm D6}_{\rm DBI} \sim \frac{r^2\sqrt{\frac{4\pi\sqrt{g_s}r^2\alpha^3_{\theta_2}(6a^2+r^2)}{9a^2+r^2} + 3\sqrt{3\pi}N^{2/5}(r^4 - {r_h}^4)\tilde{z}'(r)^2}}{{g_s}^{3/4}}, \tag{66}$$

and consequently the $\tilde{z}(r)$ EOM: $\frac{\delta\mathcal{L}^{\rm D6}_{\rm DBI}}{\delta\tilde{z}'(r)} = \mathcal{K}$ (constant) in the UV yields

$$\tilde{z}'(r) = \frac{\mathcal{C}}{r^5}, \tag{67}$$

with $\mathcal{C}$ being a constant (subsuming $g_s$- and $N$-dependent factors). One hence obtains[7]

$$\tilde{z}(r) = \mathcal{C}_1 - \frac{\tilde{\mathcal{C}}_2}{r^4} \xrightarrow{r\in{\rm UV}} \mathcal{C}_1. \tag{68}$$

As $\tilde{z}(r)$ is dimensionless, $\mathcal{C}_1$ will hence also be so, and $\mathcal{C}_2$ will have a mass dimension of four (in units of $\mathcal{R}_{D5/\overline{D5}} = D5 - \overline{D5}$-separation). By looking at fluctuations $\tilde{z} \to \tilde{z} + \delta\tilde{z}$ in the DBI action [no mass term $(\delta\tilde{z})^2$ is generated] one can show that in the UV and in the $\psi = 2n\pi, n = 0,$ 1, 2-coordinate patches and by working near, e.g., $(\theta_1,\theta_2) = (\frac{\alpha_{\theta_1}}{N^{1/5}},\frac{\alpha_{\theta_2}}{N^{3/10}})$ [10,13] (consistent with the $\mu_{\rm Ouyang}| \ll 1$-limit of the flavor $D7$-branes in the parent type-IIB dual [2]),

$$\delta\tilde{z} \xrightarrow{\rm UV} c_1 + \frac{\mathcal{C}_2}{r^4} + \mathcal{O}\left(\left(\frac{\mathcal{C}_2}{r}\right)^{12}\right). \tag{69}$$

Again, we see that the mass dimension of the coefficient $\mathcal{C}_2$ of $\frac{1}{r^4}$ is four (and $c_1$ is dimensionless). Given that one obtains an AdS$_5$ in the UV, the coefficient of $\frac{1}{r^4}$ for the massless fluctuation $\delta\tilde{z}$ is identified with a chiral condensate [26], we conjecture that $\mathcal{C}_2$ is the top-down holographic analog of the mass-dimension-four $m_q\langle\bar{q}q\rangle$. As the $\mathcal{O}(R^4)$ corrections are vanishingly small in the UV [5], $c_1$ and $\mathcal{C}_2$ receive no $\mathcal{O}(R^4)$ corrections. This is the top-down holographic analog of the RG-invariance of $m_q\langle\bar{q}q\rangle$ [27].

## VII. UNIVERSALITY IN PARTICLE WAVE FUNCTIONS IN THE IR

An intriguing universality in the wave functions of the following particle spectroscopies is noticed.

*Glueballs* [22]:

(i) $0^{-+}$ glueball: The EOM of the type-IIA RR $A$ fluctuation (to which tr$F \wedge \tilde{F}$ couples via the

[7] $\tilde{\mathcal{C}}_2 = \frac{\mathcal{C}}{4}$.

type-IIA $D4$-brane with world volume $\Sigma_{1,4}$) term, WZ term $\int_{\Sigma_{1,4}} A\wedge \mathrm{tr}F\wedge\tilde{F}$) $\partial_\nu(\sqrt{g^{\mathrm{IIA}}}g^{\mu\sigma}_{\mathrm{IIA}}g^{\nu\rho}_{\mathrm{IIA}}(\partial_{[\sigma}A_{\rho]}))=0$ (where $\mu,\nu,\ldots=a(\equiv 0,1,2,3),r,\alpha(\equiv 5,\ldots,9)$, and it was assumed $A_\mu=\delta^{\theta_2}_\mu a_{\theta_2}(r)e^{ik\cdot x}, k^2=-m^2$ as the fluctuation about the type-IIA $A_1$ that was worked out in [1]).

(ii) $0^{--}$ glueball: The EOM for the fluctuation in the type-IIB $A_{MN}=B_{MN}+iC_{MN}$ [this figures in the Weiss-Zumino term $A^{\mu\nu}d^{abc}\mathrm{Tr}(F^a_{\mu\rho}F^{b\,\rho}_{\lambda}F^{c\,\lambda}_{\nu})$], $\delta A^{MN}=\delta^M_2\delta^N_2\delta A_{23}$, $\partial_\mu(\sqrt{-g}g^{22}g^{33}g^{\mu\nu}\partial\delta A_{23})=0$.

(iii) $1^{++}$ glueball: The EOM for the radial profile function of the vector-type $\mathcal{M}$-theory metric perturbation $h_{ti}=h_{it}=g_{x^1x^1}G(r)e^{ikx^1}, i=x^2,x^3$: $R^{(1)}_{\mu\nu}\approx 0$, $R^{(1)}_{\mu\nu}$ denoting the first-order fluctuations in the Ricci tensor as a consequence of linear metric perturbations.

*Mesons* [20]: Working with the redefined radial variable $Z$: $r=r_he^Z$, after integrating out the blown-up $S^2_{\mathrm{squashed}}$ in the DBI action of the flavor type-IIA $D6$-branes and $KK$ reduction of the gauge field $A_\mu(x^\mu,Z)=\sum_{n=1}B^{(n)}_\mu(x^\mu)\alpha^{\{\mu\}}_n(Z),\mu=t,x^{i=1,2,3}$, the terms in the DBI action quadratic in the gauge field fluctuations are $\int d^4xdZ(\mathcal{V}_2(Z)F^{(n)}_{\mu\nu}F^{\mu\nu}_{(n)}\alpha^{\{\mu\}}_m(Z)\alpha^{\{\mu\}}_n(Z)+\mathcal{V}_1(Z)B^{(m)}_\mu B^{(n)}_\nu\times\dot{\alpha}^{\{\mu\}}_m\dot{\alpha}^{\{\mu\}}_n)$. The EOM for the radial profile $\alpha^{\{i\}}_m(Z)$ is $\frac{d}{dZ}(\mathcal{V}_1(Z)\dot{\alpha}^{\{i\}}_m)+2\mathcal{V}_2(Z)\mathcal{M}^2_{(m)}\alpha^{\{i\}}_m=0$, where $\mathcal{V}_1(Z)=e^{-\Phi^{IIA}}\sqrt{h}g^{ZZ}\sqrt{\det_{\theta_2,\tilde{y}}(i^*(g+B))}\sqrt{\det_{\mathbb{R}^{1,3},Z}(i^*g)}$ and $\mathcal{V}_2(Z)=e^{-\Phi^{IIA}}\frac{h}{2}\sqrt{\det_{\theta_2,\tilde{y}}(i^*(g+B))}\sqrt{\det_{\mathbb{R}^{1,3},|Z|}(i^*g)}$. The solution of $\alpha^{\{i\}}$ is given in terms of the Tricomi hypergeometric and associated Laguerre functions.

*Graviton* [28]: In the context of obtaining the Page curve of an eternal black hole from the $\mathcal{M}$-theory dual containing a black-hole in the end of the world (ETW)-brane [a hypersurface $\mathrm{AdS}^\infty_4\times_w M_6$, $\times_w$ implying a warped product, with $G_4$ fluxes threading a homologous sum of four-cycles $S^3_{\mathrm{squashed}}\times[0,1]$ and $S^2_{\mathrm{squashed}}\times S^2_{\mathrm{squashed}}$ in $M_6=M_5(\theta_{1,2},\phi_{1,2},\psi)\times S^1(x^{10})\hookrightarrow M_8^{SU(4)/\mathrm{Spin}(7)}(t,r,\theta_{1,2},\phi_{1,2},\psi,x^{10})$, with a finite tension] coupled to a nonconformal QCD bath in the doubly holographic approach, the massless graviton wavefunction with the graviton localized on the ETW-brane trapped in a volcanolike potential, is given in terms of the Tricomi hypergeometric and associated Laguerre functions.

Solutions to the EOMs for the aforementioned field fluctuations/radial profile function are given in terms of the Tricomi hypergeometric and associated Laguerre functions. The reason is that the relevant near-$r_h$ EOMs for $0^{-+},0^{--},1^{++}$-glueballs [22], and the radial profile function of the graviton wave function [28] are all of the type,

$$(r-r_h)\xi''(r)+(b+c(r-r_h))\xi'(r)+(f+(r-r_h)G)\xi(r)=0, \tag{70}$$

whose solution is given as

$$\xi(r\sim r_h)=e^{-\frac{1}{2}r(\sqrt{c^2-4G}+c)}\left[c_1U\left(\frac{b(c+\sqrt{c^2-4G})-2f}{2\sqrt{c^2-4G}},b,\sqrt{c^2-4G}(r-r_h)\right)+c_2L^{b-1}_{\frac{2f-b(c+\sqrt{c^2-4G})}{2\sqrt{c^2-4G}}}\left(\sqrt{c^2-4G}(r-r_h)\right)\right]. \tag{71}$$

In the context of the radial profile functions of vector mesons [20], and mesinos at $T>T_c$ in Eq. (27), after appropriate coordinate redefinitions, the near-horizon (IR) solutions are also given in terms of the Tricomi hypergeometric and associated Laguerre functions.

## VIII. SUMMARY

The immensely popular holographic QCD dual of [8] suffered from the longstanding problem that the Mesinos were nearly isospectral with the mesons, with nonvanishing/un(large-$N$)suppressed mesino-mesino-meson interaction [9], both in direct conflict with real QCD. What we show is that using the type-IIA Strominger-Yau-Zaslow mirror of the UV-complete [2] (unlike [8] which caters only to the IR) as constructed in [1] inclusive of $\mathcal{O}(R^4)$ corrections worked out in [3], not only is it possible to have super-massive mesinos that do not interact with the mesons, the results obtained (mesino wave function, mass, mesino-mesino-meson interaction) receive no $\mathcal{O}(R^4)$ corrections up to $\mathcal{O}(\frac{l_p^6}{N^\alpha}),\alpha\geq 1$. Thus, the WISP mesinos and nonrenormalization of their wave functions and mass up to $\mathcal{O}(R^4)$, together, apart from solving a longstanding problem, also provide a major and new insight into the fermionic sector of top-down holographic duals close to real thermal QCD.

Further, the product of the quark mass and chiral condensate may be conjectured to correspond to the coefficient of the leading nonconstant term in the flavor $D6$-branes' embedding fluctuation, with the RG-invariance of the former [27] corresponding to the nonrenormalization up to $\mathcal{O}(R^4)$ of the latter. In the end, we would also point out that there is a rather intriguing wave function universality in the form of the appearance of (appropriate) Tricomi typergeometric and associated Laguerre function in the glueball/meson/graviton (apart from mesinoic) spectroscopies.

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## APPENDIX A: FINITE BARYON CHEMICAL POTENTIAL

We explicitly show the generation of a finite baryon chemical potential. From Eq. (27) ($f(r)$ being valid $\forall\, r$), up to LO in N, $k^{\rm UV}(r) = 1–3\frac{a^2}{r^2}, f(r) = \frac{2N^{2/5}r^6}{729\pi g_s\alpha_{\theta_1}^4\alpha_{\theta_2}^2}$ and integrating $\frac{\kappa\sqrt{k^{\rm UV}(r)}}{\sqrt{\kappa^2+f^2(r)}}$, one obtains,

$$\begin{aligned} A_t(r\in{\rm UV}) &\sim \frac{1}{\sqrt{i(r^2-3a^2)}} \\ &\quad\times\Bigg\{(-1)^{2/3}\sqrt[4]{3}ar\sqrt{1-\frac{3a^2}{r^2}}\Bigg(F\left[\sin^{-1}\left(\frac{3^{3/4}\sqrt[3]{\kappa}\sqrt[6]{\frac{\pi}{2}}\sqrt{i(r^2-3a^2)}\sqrt[6]{g_s}\alpha_{\theta_1}^{2/3}\sqrt[3]{\alpha_{\theta_2}}}{ar\sqrt{15N}}\right)\Bigg|\frac{1}{2}(1-i\sqrt{3})\right] \\ &\quad-\Pi\left[\frac{\frac{3i2^{2/3}\kappa^{2/3}\sqrt[3]{\pi}\sqrt[3]{g_s}\alpha_{\theta_1}^{4/3}\alpha_{\theta_2}^{2/3}}{a^2N^{2/15}}-2(-1)^{5/6}}{2\sqrt{3}};\sin^{-1}\left(\frac{3^{3/4}\sqrt[3]{\kappa}\sqrt[6]{\frac{\pi}{2}}\sqrt{i(r^2-3a^2)}\sqrt[6]{g_s}\alpha_{\theta_1}^{2/3}\sqrt[3]{\alpha_{\theta_2}}}{ar\sqrt[15]{N}}\right)\Bigg|\frac{1}{2}(1-i\sqrt{3})\right]\Bigg)\Bigg\} \\ &\sim\frac{3\kappa\sqrt{\pi}\alpha_{\theta_1}^2\alpha_{\theta_2}(1-\frac{3a^2}{r^2})^{3/2}\sqrt{g_s}}{\sqrt{2}a^2\sqrt[5]{N}}, \end{aligned} \tag{A1}$$

$F(\phi|\mu) \equiv \int_0^\phi \frac{d\alpha}{\sqrt{1-m^2\sin^2\alpha}}$, being the incomplete elliptic integral of the first kind, and $\Pi(\nu;\phi|\mu) \equiv \int_0^\phi \frac{d\alpha}{(1-\nu^2\sin^2\alpha)\sqrt{1-\mu^2\sin^2\alpha}}$ being the incomplete integral of the fourth kind, generating a finite baryon chemical potential,

$$\mu = \frac{3\sqrt{\pi}\kappa\alpha_{\theta_1}^2\alpha_{\theta_2}\sqrt{g_s}}{\sqrt{2}a^2\sqrt[5]{N}}. \tag{A2}$$

## APPENDIX B: EOM-RELATED FOR MASSIVE MESINOS

The EOM for the radial profile $R_{2,n}(r)$ of the Mesino Θ, as defined in Eq. (26), is given by

$$\begin{aligned} &\frac{\Gamma^{\underline{15}}E^r_{\underline{5}}(r)R''_{2,n}(r)}{E^t_{\underline{1}}(r)}\Theta_{2,0}+R'_{2,n}(r)\left(\Gamma^{\underline{15}}\left(\frac{E^r_{\underline{5}}(r)}{E^t_{\underline{1}}(r)}\right)'-\frac{\Gamma^{\underline{12}}ipE^{x^1}_{\underline{2}}(r)}{E^t_{\underline{1}}(r)}-\frac{\Gamma^{\underline{12}}\mathcal{J}'(r)}{\omega_r^{\underline{8}\,\underline{10}}(r)}+2\pi i(2n+1)T\right)\Theta_{2,0} \\ &\quad+R_{2,n}(r)\Bigg(-\frac{\pi^2\Gamma^{\underline{15}}(2n+1)^2T^2E^t_{\underline{1}}(r)}{E^r_{\underline{5}}(r)}+\frac{\pi\Gamma^{\underline{25}}(2n+1)pTE^{x^1}_{\underline{2}}(r)}{E^r_{\underline{5}}(r)}+\frac{\Gamma^{\underline{12}}ipE^{x^1}_{\underline{2}}(r)\mathcal{J}'(r)}{E^t_{\underline{1}}(r)\mathcal{J}(r)} \\ &\quad+\frac{\Gamma^{\underline{15}}p^2E^{x^1}_{\underline{2}}(r)^2}{E^t_{\underline{1}}(r)^2}\omega_r^{\underline{a}\,\underline{10}}-\frac{\pi\Gamma^{\underline{25}}(2n+1)ipTE^{x^1}_{\underline{2}}(r)}{E^t_{\underline{1}}(r)}\frac{\omega_r^{\underline{a}\,\underline{10}}}{\mathcal{J}(r)}-\frac{\pi i(2n+1)T\mathcal{J}'(r)}{\mathcal{J}(r)}+\mathcal{J}(r)\omega_r^{\underline{a}\,\underline{10}}(r)\Gamma^{\underline{15}}-i\pi\left(\frac{E^{x^1}_{\underline{2}}}{E^t_{\underline{1}}}\right)'\Gamma^{\underline{12}}\Bigg)\Theta_{2,0}=0, \end{aligned} \tag{B1}$$

with $a = 7$, 8 respectively for the TH, BH backgrounds with suitable aforementioned definitions for $\mathcal{J}(r)$.

(i) $T < T_c$: Writing $M_{\rm Mesino} = \tilde{M}_{\rm Mesino}\frac{r_0}{\sqrt{g_sN}}$, the constants appearing in the Schrödinger-like EOM (30)–(32), are given as

$$a_1 \equiv \frac{r_0^2}{\sqrt{3\pi g_s N}}; \qquad a_2 \equiv \frac{23 r_0}{12\sqrt{3\pi g_s N}}; \qquad b_1 \equiv \frac{23 r_0}{12\sqrt{3\pi g_s N}}; \qquad b_6 \equiv -\frac{23\sqrt{\pi g_s N}}{4\sqrt{3} r_0^3},$$
$$\mathcal{A}_{\Theta_2'} \equiv -\frac{1511.7\sqrt{r_0}\alpha_{\theta_2}}{{g_s}^{7/2}\kappa_2 \log N M N^{2/5} {N_f}^2 \alpha_{\theta_1}^2}, \qquad \mathcal{B}_{\Theta_2'} \equiv a_2 + \frac{2^{3/2}\tilde{M}_{\rm Mesino}\pi^{1/4}}{(g_s N)^{1/4}} + 2i(2n+1)\pi T,$$
$$\mathcal{A}_{\Theta_2} \equiv \frac{-\frac{39.4\tilde{M}_{\rm Mesino}}{\sqrt[4]{g_s}\sqrt[4]{N}} + T(n(-39.5 a_6^\beta r_0 T - (125 i)) - (62.5 i) - 39.5 a_6^\beta n^2 r_0 T - 9.9 a_6^\beta r_0 T)}{r_0},$$
$$\mathcal{B}_{\Theta_2} \equiv \frac{M(\lambda_5^2 {g_s}^{15/4}\kappa_2^2 \log N^2 \tilde{M}_{\rm Mesino}(0.1n+0.1)N^{3/20}{N_f}^2 {r_0}^4 T \alpha_{\theta_2}^2 \alpha_{\theta_1}^8 - 1.1\lambda_5 {g_s}^{3/2}\kappa_2 \log N \tilde{M}_{\rm Mesino}^2 N_f {r_0}^2 \alpha_{\theta_2}\alpha_{\theta_1}^4 + 6.84 N^{3/5})}{\lambda_5^2\sqrt{g_s}\kappa_2 \log N {r_0}^{11/2}\alpha_{\theta_1}^6 \alpha_{\theta_2}^3},$$
$$\mathcal{C}_{\Theta_2} \equiv \frac{\frac{786.1\tilde{M}_{\rm Mesino}}{\sqrt[4]{g_s}\sqrt[4]{N}} + T(n(-39.5 b_6 {r_0}^2 T + (0. + 2485.6 i)) + (1242.8 i) - 39.5 b_6 n^2 {r_0}^2 T - 9.9 b_6 {r_0}^2 T)}{{r_0}^2}, \tag{B2}$$

with $\lambda_5$ being the parameter in terms of which the coframes of the relevant non-Kähler six-folds were worked out in [3], $g^{\rm IIA}_{\theta_2\theta_2}(r \sim r_0) \sim \kappa_2\sqrt{g_s N}$ and

$$a_6^\beta \equiv \frac{\sqrt{3\pi}\sqrt{g_s}\sqrt{N}}{{r_0}^2} + \frac{\beta\sqrt{g_s}M(19683\sqrt{6}\alpha_{\theta_1}^6 + 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 - 40\sqrt{6}\alpha_{\theta_2}^4)\log^3(r_0)}{4374\sqrt{\pi}\epsilon^5 \log N^4 N^{3/4} N_f {r_0}^4 \alpha_{\theta_2}^3}. \tag{B3}$$

(ii) $T > T_c$: Based on

$$E^r_{\underline{5}} = \frac{\sqrt{\frac{9a^2+r^2}{6a^2+r^2}}\sqrt{r^4 - {r_h}^4}(1 - \frac{1}{2}\beta(\mathcal{C}_{zz} - 2\mathcal{C}_{\theta_1 z} + 2\mathcal{C}_{\theta_1 x}))}{\sqrt{2}\sqrt[4]{\pi}\sqrt{g_s}\sqrt{N} r},$$
$$E^t_{\underline{1}} = \frac{\sqrt{2}\sqrt[4]{\pi}\sqrt[4]{g_s}\sqrt[4]{N} r}{\sqrt{r^4 - {r_h}^4}} + \frac{27(9b^2+1)^4\beta b^{10}\sqrt[4]{g_s}M r^2\Sigma(6a^2 + {r_h}^2)(r - 2r_h)\log^3(r_h)}{2\sqrt{2}\pi^{3/4}(3b^2-1)^5(6b^2+1)^4 \log N^4 N N_f {r_h}^4\alpha_{\theta_2}^3(9a^2 + {r_h}^2)\sqrt{r^4 - {r_h}^4}},$$
$$\omega_{\frac{8}{r}\frac{10}{}} = -\frac{7N^{3/5}}{\lambda_5 {g_s}^{3/2}\kappa_2 \log N N_f r^2 \alpha_{\theta_1}^4\alpha_{\theta_2}(r^2 - 3.3a^2)} + \frac{\kappa_{\omega_{\frac{8}{r}\frac{10}{}}} a^8\sqrt[4]{\beta}\sqrt{\mathcal{C}_{zz}}{\rm const}\lambda_5 {g_s}^{5/4} M N^{19/20} N_f\sqrt{\alpha_{\theta_1}}\sqrt{1 - \frac{{r_h}^4}{r^4}}\log(r)}{r^4\alpha_{\theta_2}^6(r^2 - 3.a^2)^2\sqrt{\frac{6a^2+r^2}{9a^2+r^2}}}, \tag{B4}$$

with $\Sigma \equiv -19683\sqrt{6}\alpha_{\theta_1}^6 - 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 40\sqrt{6}\alpha_{\theta_2}^4$, and setting consistently the $\mathcal{O}(R^4)$ corrections of $\mathcal{M}$-theory's three-form potential to zero requires $\mathcal{C}_{zz} - 2\mathcal{C}_{\theta_1 z} = 0$ and $|\mathcal{C}_{\theta_1 x}| \ll 1$ [3], we see that $E^r_{\underline{5}}$ receives no $\mathcal{O}(\beta)$ corrections. Further, the constants appearing in the EOM for massive mesinos are therefore given as

$$\mathcal{C}^{M_{\rm Mesino}}_{\frac{E^t_{\underline{1}}}{E^r_{\underline{5}}}} = \frac{\beta\sqrt{g_s}M(19683\sqrt{6}\alpha_{\theta_1}^6 + 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 - 40\sqrt{6}\alpha_{\theta_2}^4)\log^3(r_h)}{17496\sqrt{\pi}\epsilon^5(\log N)^4 N^{3/4} N_f {r_h}^3\alpha_{\theta_2}^3} + \frac{\sqrt{3\pi}\sqrt{g_s}\sqrt{N}}{4r_h},$$
$$\mathcal{C}^{M_{\rm Mesino}}_{\left(\frac{E^r_{\underline{5}}}{E^t_{\underline{1}}}\right)'} = \frac{2\beta M(-19683\sqrt{6}\alpha_{\theta_1}^6 - 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 40\sqrt{6}\alpha_{\theta_2}^4)\log^3(r_h)}{6561\pi^{3/2}\epsilon^5\sqrt{g_s}(\log N)^4 N^{7/4} N_f r_h\alpha_{\theta_2}^3} + \frac{4r_h}{\sqrt{3\pi}\sqrt{g_s}\sqrt{N}},$$
$$\mathcal{C}^{M_{\rm Mesino}}_{rt} = -\frac{14}{3\sqrt{3\pi}\sqrt{g_s}\sqrt{N}},$$
$$\mathcal{C}^{M_{\rm Mesino}}_{\frac{E^r_{\underline{5}}}{E^t_{\underline{1}}}} = \frac{4r_h}{\sqrt{3\pi}\sqrt{g_s}\sqrt{N}} + \frac{4\beta M(-19683\sqrt{6}\alpha_{\theta_1}^6 - 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 40\sqrt{6}\alpha_{\theta_2}^4)\sqrt{\frac{6a^2+{r_h}^2}{9a^2+{r_h}^2}}\log^3(r_h)}{6561\sqrt{3}\pi^{3/2}\epsilon^5\sqrt{g_s}(\log N)^4 N^{7/4} N_f r_h\alpha_{\theta_2}^3},$$

$$
\begin{aligned}
\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_2^{x^1}}{E_1^t}} &= \frac{2\sqrt{2}\sqrt[4]{\pi}\sqrt[4]{g_s}\sqrt[4]{N}}{\sqrt{r_h{}^3}} - \frac{4\sqrt{\frac{2}{3}}\beta\sqrt[4]{g_s}M(-19683\sqrt{6}\alpha_{\theta_1}^6 - 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 40\sqrt{6}\alpha_{\theta_2}^4)(6a^2+r_h{}^2)\log^3(r_h)}{6561\pi^{3/4}\epsilon^5(\log N)^4 N N_f r_h{}^{7/2}\alpha_{\theta_2}^3(9a^2+r_h{}^2)},\\
\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_2^{x^1}}{E_5^r}} &= \frac{\sqrt{\frac{3}{2}}\pi^{3/4}g_s{}^{3/4}N^{3/4}}{r_h{}^{3/2}} + \frac{\beta g_s{}^{3/4}M\sqrt{\frac{1}{N}}(19683\sqrt{6}\alpha_{\theta_1}^6 + 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 - 40\sqrt{6}\alpha_{\theta_2}^4)\log^3(r_h)}{2187\sqrt{2}\sqrt[4]{\pi}\epsilon^5(\log N)^4 N_f r_h{}^{7/2}\alpha_{\theta_2}^3},\\
a_{2\beta}^{M_{\rm Mesino}} &\equiv -\frac{21.9}{\sqrt{g_s N}} + \mathcal{C}_{rt}^{M_{\rm Mesino}}\\
\mathcal{A}_1^{M_{\rm Mesino}} &= \pi^2(-(2n+1)^2)T^2\left(\frac{\beta\sqrt{g_s}M(19683\sqrt{6}\alpha_{\theta_1}^6 + 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 - 40\sqrt{6}\alpha_{\theta_2}^4)\log^3(r_h)}{17496\sqrt{\pi}\epsilon^5(\log N)^4 N^{3/4}N_f r_h{}^3\alpha_{\theta_2}^3}\right.\\
&\quad \left. + \frac{\sqrt{3\pi}\sqrt{g_s}\sqrt{N}}{4r_h}\right) - (0.+3.3i)(2n+1)T,\\
\mathcal{A}_2^{M_{\rm Mesino}} &= \frac{2\beta M(-19683\sqrt{6}\alpha_{\theta_1}^6 - 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + 40\sqrt{6}\alpha_{\theta_2}^4)\log^3(r_h)}{6561\pi^{3/2}\epsilon^5\sqrt{g_s}(\log N)^4 N^{7/4}N_f r_h\alpha_{\theta_2}^3} - \frac{1.3r_h}{\sqrt{g_s}\sqrt{N}} + \frac{4r_h}{\sqrt{3\pi}\sqrt{g_s}\sqrt{N}} + 2i\pi(2n+1)T,\\
\mathcal{B}_1^{M_{\rm Mesino}} &= 1.1i\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_2^{x^1}}{E_1^t}} + (2n+1)T\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_2^{x^1}}{E_5^r}}. \qquad \text{(B5)}
\end{aligned}
$$

## APPENDIX C: $\tilde{z}=$CONSTANT EMBEDDING OF FLAVOR $D6$-BRANES INCLUSIVE OF $\mathcal{O}(\beta)$ CORRECTIONS

The EOM for the embedding of the flavor $D6$-branes in the warped squashed resolved conifold $\tilde{z}=\tilde{z}(r)=\tilde{z}_{(0)}+\beta\tilde{z}_{(1)}$, up to $\mathcal{O}(\beta)$, is given by

$$
\begin{aligned}
&\frac{N^{3/5}N_f r^2(r^4-r_h{}^4)(\log N - 3\log(r_h))(\tilde{z}'_{(0)}+\beta\tilde{z}'_{(1)})}{4\sqrt{6}\pi^{7/4}g_s{}^{3/4}\alpha_{\theta_1}^2\alpha_{\theta_2}^{5/2}\sqrt{\frac{4\sqrt{\pi}\sqrt{g_s}r^2\alpha_{\theta_2}^3(6a^2+r^2)}{9a^2+r^2}} + 3\sqrt{3}N^{2/5}(r^4-r_h{}^4)(\tilde{z}'_{(0)}+\beta\tilde{z}'_{(1)})^2}\\
&-\frac{0.0005\beta M N^{87/20}r_h{}^5(-492.1\alpha_{\theta_1}^6 - 67.8\alpha_{\theta_2}^2\alpha_{\theta_1}^3 + \alpha_{\theta_2}^4)(r-r_h)^2\log^3(r_h)(\log N - 3\log(r_h))\tilde{z}'_{(0)}(r)}{\epsilon^5 g_s{}^{5/2}\log N^4\alpha_{\theta_1}^6\alpha_{\theta_2}^6(9a^2+r_h{}^2)} = K^{(0)}+\beta K^{(1)}. \qquad \text{(C1)}
\end{aligned}
$$

At $\mathcal{O}(\beta^0)$, (C1) yields

$$
\tilde{z}'_{(0)} = \pm\frac{8\sqrt{6}\pi^2 g_s{}^{3/4}K^{(0)}\alpha_{\theta_1}^2\alpha_{\theta_2}^{5/2}\sqrt{\frac{\sqrt{g_s}r^2\alpha_{\theta_2}^3(6a^2+r^2)}{9a^2+r^2}}}{\sqrt{N^{2/5}(r^4-r_h{}^4)(N^{4/5}N_f{}^2r^4(r^4-r_h{}^4)(\log N - 3\log(r_h))^2 - 288\sqrt{3}\pi^{7/2}g_s{}^{3/2}K^{(0)2}\alpha_{\theta_1}^4\alpha_{\theta_2}^5)}}. \qquad \text{(C2)}
$$

From (C2), one obtains $\tilde{z}(r)\in\mathbb{R}$ if $K^{(0)}=0$ (irrespective of whether one performs first a large-$N$ followed by a small-$r$ expansion or vice versa). In a similar manner, at $\mathcal{O}(\beta)$,

$$\tilde{z}_{(1)} = c_1 - \frac{4\pi^2 {g_s}^{5/4} K^{(1)} r \alpha_{\theta_1}^2 \alpha_{\theta_2}^{11/2} \sqrt{6a^2+r^2}}{N^{3/5} N_f {r_h}^4 \sqrt{9a^2+r^2}\sqrt{9a^2-{r_h}^2}\sqrt{9a^2+{r_h}^2}(\log N - 3\log(r_h))\sqrt{\frac{\sqrt{g_s} r^2 \alpha_{\theta_2}^3 (6a^2+r^2)}{9a^2+r^2}}}$$
$$\times \left(\sqrt{6}\left(\sqrt{{r_h}^2-6a^2}\sqrt{9a^2+{r_h}^2}\tan^{-1}\left(\frac{\sqrt{6a^2+r^2}\sqrt{9a^2-{r_h}^2}}{\sqrt{9a^2+r^2}\sqrt{{r_h}^2-6a^2}}\right)\right.\right.$$
$$\left.\left. + \sqrt{-6a^2-{r_h}^2}\sqrt{9a^2-{r_h}^2}\tan^{-1}\left(\frac{\sqrt{6a^2+r^2}\sqrt{9a^2+{r_h}^2}}{\sqrt{9a^2+r^2}\sqrt{-6a^2-{r_h}^2}}\right)\right) - 4\sqrt{9a^2-{r_h}^2}\sqrt{9a^2+{r_h}^2}\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}\sqrt{6a^2+r^2}}{\sqrt{9a^2+r^2}}\right)\right). \tag{C3}$$

Again, a finite real $\tilde{z}_{(1)}$ is obtained only for $K^{(1)} = 0$. This hence implies a constant $\tilde{z}$-embedding up to $\mathcal{O}(\beta)$.

## APPENDIX D: CONSTANTS APPEARING IN THE SOLUTION TO THE MESINO WAVE FUNCTION FOR $T > T_c$

The parameters $\mu_{1,2,3}, \Lambda$ in Eq. (33), are defined as follows [in the following, terms of $\mathcal{O}(\frac{\beta}{N^\alpha})$, $\alpha \geq 1$ have been dropped as the same were subdominant as compared to the order considered in the [3] ]:

$$\begin{aligned}
\mu_1 &\equiv -\frac{\mathcal{A}_2^{M_{\rm Mesino}}}{\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_5^r}{E_1^t}}} + \frac{2i\mathcal{B}_1^{M_{\rm Mesino}}}{\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_2^{x^1}}{E_1^t}}} + \frac{\sqrt{\mathcal{A}_2^{M_{\rm Mesino}\ 2} - 2\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_5^r}{E_1^t}}\mathcal{A}_2^{M_{\rm Mesino}} + \mathcal{C}^{M_{\rm Mesino}\ 2}_{\frac{E_5^r}{E_1^t}} + 4\mathcal{A}_1^{M_{\rm Mesino}}\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_5^r}{E_1^t}}}}{\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_5^r}{E_1^t}}} + 1\\
&= 2.5\sqrt{-(0.3\ + 0.4i)n + (0.1\ - 0.2i) - 0.3n^2} + (0.9\ - 0.9i)n - (0.7\ + 0.4i) - \frac{0.6(n+0.5)r_h}{\sqrt{g_s}\sqrt{N}}\\
&+\mathcal{O}\left(\frac{\beta}{N^{5/4}}\right)^{\nearrow 0},\\
\mu_2 &\equiv \frac{2\sqrt{\mathcal{A}_2^{M_{\rm Mesino}\ 2} - 2\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_5^r}{E_1^t}}\mathcal{A}_2^{M_{\rm Mesino}} + \mathcal{C}^{M_{\rm Mesino}\ 2}_{\frac{E_5^r}{E_1^t}} + 4\mathcal{A}_1^{M_{\rm Mesino}}\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_5^r}{E_1^t}}}}{\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_5^r}{E_1^t}}} + 1\\
&= 1 + 4.9\sqrt{-(0.25\ + 0.4i)n + (0.1\ - 0.2i) - 0.3n^2} + \mathcal{O}\left(\frac{\beta}{N^{5/4}}\right)^{\nearrow 0},\\
\mu_3 &\equiv \frac{i\sqrt{6}\pi^{3/4}{g_s}^{3/4}N^{3/4}pu}{{r_h}^{5/2}} + \frac{\frac{2i\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_2^{x^1}}{E_1^t}}p}{\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_5^r}{E_1^t}}}}{}\\
&= \frac{i\sqrt{6}\pi^{3/4}{g_s}^{3/4}N^{3/4}pu}{{r_h}^{5/2}} + \frac{i\sqrt{2}\beta {g_s}^{3/4}Mpu\left(19683\sqrt{6}\alpha_{\theta_1}^6 + 6642\alpha_{\theta_2}^2\alpha_{\theta_1}^3 - 40\sqrt{6}\alpha_{\theta_2}^4\right)\log^3(r_h)}{2187\sqrt[4]{\pi}\epsilon^5\log N^4\sqrt{N}N_f{r_h}^{9/2}\alpha_{\theta_2}^3}\\
&= \frac{i\sqrt{6}\pi^{3/4}{g_s}^{3/4}N^{3/4}pu}{{r_h}^{5/2}} + \mathcal{O}\left(\frac{\beta}{N}\right)^{\nearrow 0};\\
\Lambda &\equiv \mu_1 - \frac{2i\mathcal{B}_1^{M_{\rm Mesino}}}{\mathcal{C}^{M_{\rm Mesino}}_{\frac{E_2^{x^1}}{E_1^t}}} = 2.5\sqrt{-(0.3\ + 0.4i)n + (0.1\ - 0.2i) - 0.3n^2} + (-0.9i)n + (1\ - 0.4i) + \mathcal{O}\left(\frac{\beta}{N^{5/4}}\right)^{\nearrow 0}.
\end{aligned} \tag{D1}$$

## APPENDIX E: SUMMARY OF APPLICATIONS OF TOP-DOWN HOLOGRAPHIC QCD [1,3]

One of the authors (A. M.) has been working on the top-down holographic QCD for the past few years. The holographic dual of finite $N$ QCD was first constructed in [1] and then $\mathcal{O}(R^4)$ corrections to [1] were obtained in [3]. Following is the summary of results obtained in this direction:

(i) Summary of applications of [1]: In [16], transport coefficients such as shear viscosity, diffusion constant, electrical conductivity, charge susceptibility, etc., of black $M3$-banes (black $M5$-branes wrapping a homologous sum of two cycles) in the MQGP limit were obtained, and it was found that the ratio of shear viscosity-to-entropy density is $1/4\pi$. In [13], deconfinement temperature and mass scale of the first-generation quarks were obtained without the inclusion of $\mathcal{O}(R^4)$ corrections relevant to thermal QCD. Further, thermodynamic stability and $G_2$ structure of [1] and temperature dependence of electrical conductivity and charge susceptibility were also discussed in [13,29]. In this process, Einstein's law was verified by computing the ratio of electrical conductivity to charge susceptibility. For the discussion on Wiedemann-Franz law by calculating the thermal and electrical conductivities up to LO in $N$ and NLO in $N$ correction to the aforementioned transport coefficients and speed of sound from the gauge-invariant metric perturbations, see [30]. The glueball and meson spectra of finite $N$ QCD have been obtained in [20,22], respectively. Decay of glueballs into mesons ($\pi$ and $\rho$ mesons) has been discussed in [14] and for the QCD trace anomaly from $\mathcal{M}$-theory perspective, see [31].

(ii) Summary of applications of [3]: The low energy coupling constants at the NLO in chiral expansion of $SU(3)$ chiral perturbation theory (for simplicity in the chiral limit) were obtained from the aforementioned type IIA dual, in [5] where we observed a connection between higher-derivative terms and large-$N$ expansion. In the process of computing the deconfinement temperature ($T_c$) in [25,32], a novel "UV-IR" mixing, nonrenormalization $T_c$ beyond one loop in the zero instanton sector and flavor-memory effect were obtained. Further, we constructed a doubly holographic setup with a nonconformal bath in [28] to get the Page curve of the related eternal black hole from a top-down approach. One of the exciting results that we obtained in [28] is the Page curve of the relevant eternal black hole for massless gravity on the Karch-Randall brane. Massless graviton was responsible for the exponential-in-$N$ suppressed entanglement entropy from higher-derivative terms in 11-dimensional supergravity action. This provided us the connection between the mass of graviton and higher derivative terms. On the math side with the aim of classifying nonsupersymmetric thermal geometries relevant to realistic top-down holographic duals of thermal QCD-like theories, $SU(3)/G_2/SU(4)/\mathrm{Spin}(7)$-structures and (almost) contact (3) (metric) Structures on the underlying six-, seven- and eight-folds were studied in [3,11].


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