More on the spin-2 analog of the massive BF model

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The addition of mass terms in general breaks gauge symmetries which can be recovered usually via Stueckelberg fields. The massive BF model describes massive spin-1 particles while preserving the $U(1)$ symmetry without Stueckelberg fields. Replacing the spin-1 curvature (field strength) by the Riemann tensor one can define its spin-2 analog (massive"BR" model). Here we investigate the canonical structure of the free mBR model in terms of gauge invariants in arbitrary dimensions and compare with the massive BF model. We also investigate nonlinear completions of the mBR model in arbitrary dimensions. In $D = 3$ we find a nonlinear completion in the form of a bimetric model which is a subcase of a new class of bimetric models whose decoupling limit is ghost free at leading order. Their spectrum consists only of massive spin-2 particles. In arbitrary dimensions $D \geq 3$ we show that the consistency of a possible single metric completion of the mBR model is related with the consistency of a higher rank description of massless spin-1 particles in arbitrary backgrounds.

I. INTRODUCTION

The universal nature of the gravitational interaction makes the search for a possible graviton mass a fundamental subject. Severe problems in the consistency of massive gravitons like the vDVZ mass discontinuity [1,2], and the presence of ghosts [3] have been tackled by a convenient choice of the graviton potential [4]. Such work has triggered a huge amount of work in the subject of massive gravity, see [5,6]. In particular, the need of viable cosmological solutions has led to the bimetric model of [7] which besides a massive spin-2 particle contains an extra massless spin-2 particle. Regarding the phenomenology and consistency of [7], see [8,9].

The model [4] and also [7] are based on the Fierz-Pauli massive spin-2 free theory [10] which can be defined in terms of a symmetric rank-2 tensor. One might wonder how robust are the physical outputs of the present massive gravity theories against the replacement of the FP paradigm by other descriptions of free massive spin-2 particles.

The introduction of mass terms breaks the local symmetries of the massless theories in general. In the case of the metric formulation of gravity the local symmetry is reparametrization invariance which should be preserved

also in the massive theory. This is achieved in [7] via the introduction of a second metric which is a dynamical field and enlarges the spectrum of the theory as compared to [4]. One might search for a phenomenologically viable massive gravity with only massive spin-2 particles in the spectrum this is the main motivation for the present work.

In order to preserve local symmetries in a massive theory one could follow an approach already known in the spin-1 case. Instead of the usual Proca theory with Stueckelberg fields, we might follow [11] where the mass for the spin-1 field is generated via a gauge invariant coupling to a two-form field (anti-symmetric tensor). The Cremmer-Scherk or massive BF model (mBF henceforth) can be generalized to arbitrary dimensions and for the non-Abelian case [12]. Inspired by duality relations, a spin-2 analog of [11] has been suggested in [13], henceforth we call it "mBR" model since the spin-1 curvature $F_{\mu\nu}(A)$ (field strength) is replaced by a linearized Riemann curvature $R^{(L)}_{\mu\nu\alpha\beta}(h)$ and the two-form field $B_{\mu\nu}$ by a Riemann like tensor $B_{\mu\nu\alpha\beta}$.

Here we investigate in detail the particle content of the mBR model in terms of gauge invariants and compare with the mBF spin-1 model, pointing out some important differences. We go beyond the linearized truncation of [13] and investigate possible nonlinear completions of the mBR model in arbitrary D-dimensions. The linearized model in $D = 3$ has inspired us to suggest a new class of bimetric models where however the particle sepectrum only contains massive spin-2 particles as we have checked at leading order in the decoupling limit. We show that the consistency of a possible nonlinear completion of the mBR model in $D \geq 4$ is tightly connected with a consistent coupling to

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gravity of a higher rank massless spin-1 model suggested by Deser *et al.* [14].

II. LINEARIZED mBR MODEL

A. From mBF to mBR

In order to derive the linerarized massive mBR model we start from the simpler spin-1 case by constructing a master action with two vector fields (A_μ, f_μ), an antisymmetric tensor $B_{\mu\nu} = -B_{\nu\mu}$ and an arbitrary external source[1] J_μ,

$$S_M[J] = \int d^D x \left\{ -\frac{1}{4} F_{\mu\nu}^2(A) - \frac{m^2}{2} f_\mu f^\mu + \frac{m}{4} B^{\mu\nu} F_{\mu\nu}(f - A) + f_\mu J^\mu \right\}. \quad (1)$$

If we first integrate over $B_{\mu\nu}$ we arrive at the functional constraint $F_{\mu\nu}(f - A) = 0$ whose general solution can be written in terms of a scalar Stueckelberg field $f_\mu = A_\mu + \partial_\mu \varphi / m$, now integrating over f_μ we obtain the Proca theory with a Stueckelberg field which is invariant under the $U(1)$ gauge transformations $(\delta A_\mu, \delta \varphi) = (\partial_\mu \Lambda, -m\Lambda)$,

$$S_{\rm PS}[J] = \int d^D x \left\{ -\frac{1}{4} F_{\mu\nu}^2(A) - \frac{m^2}{2} \left(A_\mu + \frac{\partial_\mu \varphi}{m} \right)^2 + \left(A_\mu + \frac{\partial_\mu \varphi}{m} \right) J^\mu \right\}. \quad (2)$$

If instead, we first integrate over f_μ we arrive at the massive BF (mBF) model in D dimensions, up to quadratic terms in the external source,

$$S_{\rm mBF}[J] = \int d^D x \left\{ -\frac{1}{4} F_{\mu\nu}^2(A) + \frac{1}{8} \partial_\mu B^{\mu\nu} \partial^\lambda B_{\lambda\nu} - \frac{m}{4} B^{\mu\nu} F_{\mu\nu}(A) + B_\mu^* J^\mu + \mathcal{O}(J^2) \right\}, \quad (3)$$

where $B_\mu^* \equiv \frac{1}{m} \partial^\nu B_{\nu\mu}$. The mBF model is invariant under the independent scalar and transverse tensor ($\partial^\mu \Lambda_{\mu\nu}^t = 0$) gauge transformations:

$$\delta A_\mu = \partial_\mu \Lambda; \qquad \delta B_{\mu\nu} = \Lambda_{\mu\nu}^t. \quad (4)$$

In $D = 3$ we can write $B_{\mu\nu} = \epsilon_{\mu\nu\rho} B^\rho$ and after rotating and decoupling the vector fields B_μ and A_μ it follows that the mBF model is equivalent to a couple of topologically massive Maxwell-Chern-Simons (MCS) models [15] of opposite helicities $+1$ and -1 just like a Proca theory. In $D = 4$ we can write $B^{\mu\nu} = \epsilon^{\mu\nu\rho\gamma} B_{\rho\gamma}$ and *mBF* becomes the topologically massive *BF* model also known as the Cremmer-Scherk model [11].

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[1] We use $\eta_{\mu\nu} = (-, +, \cdots, +)$.

The master action allows us to prove the duality between $S_{\rm PS}$ and $S_{\rm mBF}$ via a map between gauge invariant vectors that we read off from the linear terms in the sources, namely: $A_\mu + \partial_\mu \varphi / m \leftrightarrow B_\mu^*$. The equations of motion $\delta S_{\rm mBF} = 0$ can be written as

$$\partial^\mu F_{\mu\nu}[A] = m^2 B_\nu^*; \qquad F_{\mu\nu}[A - B^*] = 0. \quad (5)$$

From the general solution of the second equation of (5) $A_\mu = B_\mu^* + \partial_\mu \Lambda$ back in the first equation we have a Proca-like equation $\partial^\mu F_{\mu\nu}[B^*] = m^2 B_\nu^*$ which is equivalent to the Klein-Gordon equation $(\Box - m^2) B_\mu^* = 0$ since $\partial^\mu B_\mu^* = 0$. Notice however, that the transverse condition is a dynamic equation in the Proca theory while it is a trivial identity for the dual field B_μ^*.

Now we can follow similar steps in order to derive the spin-2 version of $S_{\rm mBF}$. First, we define a master action in terms of two symmetric rank-2 tensors $(h_{\mu\nu}, f_{\mu\nu})$ and a rank-4 tensor $B_{\mu\nu\alpha\beta}$ antisymmetric by the exchange $\mu \leftrightarrow \nu$ or $\alpha \leftrightarrow \beta$ and symmetric by $[\mu\nu] \leftrightarrow [\alpha\beta]$, like the Riemann tensor. By replacing the spin-1 curvature $F_{\mu\nu}$ by the linearized Riemann curvature tensor we have

$$S_M^D[T] = \int d^D x \left\{ -\frac{1}{2} h^{\mu\nu} G_{\mu\nu}^{(L)}(h) - \frac{m^2}{4} (f_{\mu\nu} f^{\mu\nu} - f^2) + \frac{1}{2} B^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}^{(L)}(f - h) + f_{\mu\nu} T^{\mu\nu} \right\}, \quad (6)$$

where $R_{\alpha\mu\beta\nu}^{(L)}(h) = (\partial_\mu \partial_\beta h_{\alpha\nu} + \partial_\alpha \partial_\nu h_{\mu\beta} - \partial_\mu \partial_\nu h_{\alpha\beta} - \partial_\alpha \partial_\beta h_{\mu\nu})/2$.

If we first integrate over $B_{\mu\nu\alpha\beta}$ we obtain the linearized zero curvature condition

$$R_{\mu\nu\alpha\beta}^{(L)}(f - h) = 0, \quad (7)$$

whose general solution gives rise to vector Stueckelberg fields,

$$f_{\mu\nu}(h, \psi) = h_{\mu\nu} + \frac{\partial_\mu \psi_\nu + \partial_\nu \psi_\mu}{m}. \quad (8)$$

Back in the master action we have the Fierz-Pauli [10] theory with Stueckelberg fields which describes massive spin-2 particles with linearized reparametrization invariance $(\delta h_{\mu\nu}, \delta \psi_\mu) = (\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu, -m\epsilon_\mu)$, namely,

$$S_{\rm FPS}^D[T] = \int d^D x \left\{ -\frac{1}{2} h^{\mu\nu} G_{\mu\nu}^{(L)}(h) - \frac{m^2}{4} [f_{\mu\nu}(h, \psi) f^{\mu\nu}(h, \psi) - f^2(h, \psi)] + f_{\mu\nu}(h, \psi) T^{\mu\nu} \right\}. \quad (9)$$

On the other hand, by first integrating over $f_{\mu\nu}$ in (6) one derives the massive BR (mBR) model [13]:

$$S_{\mathrm{mBR}}^{D}[T] = \int d^D x \left\{ -\frac{1}{2} h^{\mu\nu} G_{\mu\nu}^L(h) - \frac{1}{2} B^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}^{(L)}(h) \right.$$

$$\left. + \frac{1}{m^2} \mathcal{L}_{\mathrm{DTS}}[B] + B_{\mu\nu}^* T^{\mu\nu} + \mathcal{O}(T^2) \right\} \qquad (10)$$

where the dual symmetric tensor is given by

$$B_{\mu\nu}^* = -\frac{2}{m^2} \left[\partial^\alpha \partial^\beta B_{\mu\alpha\nu\beta} - \frac{1}{(D-1)} \eta_{\mu\nu} \partial^\alpha \partial^\beta B_{\alpha\beta} \right] \qquad (11)$$

with $B^{\alpha\beta} \equiv \eta_{\mu\nu} B^{\mu\alpha\nu\beta}$ while the Lagrangian

$$\mathcal{L}_{\mathrm{DTS}}^{D}[B] = (\partial_\alpha \partial_\beta B^{\mu\alpha\nu\beta})^2 - \frac{1}{(D-1)} (\partial_\alpha \partial_\beta B^{\alpha\beta})^2 \qquad (12)$$

has been first obtained in [14] in $D = 4$ and generalized to arbitrary dimensions in [16]. It describes massless spin-1 particles like the Maxwell theory. Although of 4th order in derivatives, it is ghost free in arbitrary dimensions as shown in the Appendix, see [14] for a proof in $D = 4$. The DTS model is invariant under the following scalar and tensor transformations:

$$\delta B_{\mu\alpha\nu\beta} = (\eta_{\mu\nu} \eta_{\alpha\beta} - \eta_{\mu\beta} \eta_{\nu\alpha}) \pi + \Lambda_{\mu\alpha\nu\beta} \qquad (13)$$

where $\Lambda_{\mu\alpha\nu\beta}$ must satisfy

$$\partial^\mu \partial^\nu \Lambda_{\mu\alpha\nu\beta} = 0. \qquad (14)$$

The scalar transformation plays the role of the $U(1)$ symmetry of the Maxwell theory. Comparing with the spin-1 mBF model, we notice that the EH term (massless spin-2) is the analog of the Maxwell theory (massless spin-1) while $\mathcal{L}_{\mathrm{DTS}}$ (massless spin-1) is the analog of the antisymmetric model (massless spin-0). The spin-1 curvature $F_{\mu\nu}$ is replaced by the linearized Riemann tensor in the "BR" mixing term. Regarding the equations of motion $\delta S_{\mathrm{mBR}} = 0$, they can be written in close analogy with (5),

$$G_{\mu\nu}^{(L)}(h) = -\frac{m^2}{2} (B_{\mu\nu}^* - \eta_{\mu\nu} B^*), \qquad (15)$$

$$R_{\mu\nu\alpha\beta}^{(L)}(h - B^*) = 0. \qquad (16)$$

If we plug back in (15) the general solution of (16), i.e., $h_{\mu\nu} = B_{\mu\nu}^* + (\partial_\mu A_\nu + \partial_\nu A_\mu)/m$ we arrive exactly at Fierz-Pauli equations for the dual field:

$$G_{\mu\nu}^L(B^*) = -\frac{m^2}{2} (B_{\mu\nu}^* - \eta_{\mu\nu} B^*), \qquad (17)$$

which leads to the Klein-Gordon equation $(\Box - m^2) B_{\mu\nu}^* = 0$ and $B^* = 0 = \partial^\mu B_{\mu\nu}^*$ which are typical equations for free massive spin-2 particles. Notice that $\partial^\mu B_{\mu\nu}^* = \partial_\nu B^*$ holds

identically from the definition of the dual field, differently from the usual FP theory where it follows from the derivative of the FP equations for the fundamental field $h_{\mu\nu}$.

Since the equations of motion (17) are of fourth order in derivatives the skeptical reader may be questioning whether we have ghosts in the spectrum. Also in the spin-1 case the Proca-like equation in terms of the dual vector field B_μ^* is of 3rd order in derivatives. In order to make sure that in both $s = 1$ and $s = 2$ cases we only have the expected massive physical particles we perform in the next subsection a canonical analysis in terms of gauge invariant combinations of helicity variables in both mBF and mBR models in arbitrary D dimensions confirming our expectations.

B. Canonical structure

1. The mBF model

The canonical analysis of the mBF model via Hamiltonian methods has appeared before in [17,18] in $D = 4$ and in [19] in D dimensions. In [17,19] the Dirac method has been employed while [18] makes use of the Faddeev-Jackiw method. However, we believe that the purely Lagrangian approach presented here is simpler and clarifies the role of each degree of freedom in producing the necessary mass terms. The Lorentz invariance is not explicit but we are able to write down the action only in terms of gauge invariants, there is no need of gauge fixing. Moreover, since Hamiltonian methods become cumbersome for higher derivative theories like the mBR model, the spin-1 case works like an introduction and, more importantly, it points out that the analogy between the spin-1 and the spin-2 cases is not completely faithful.

We follow the approach used for higher order higher spin theories in $D = 3$ in [20]. The gauge invariants are built up in a constructive way from the definition of the gauge transformations. For example, in the case of the $U(1)$ symmetry we take one of the D equations $\delta A_\mu = \partial_\mu \Lambda$ for the elimination of the gauge parameter, explicitly we have[2]: $\Lambda = \partial_j \delta A_j / \nabla^2$. Plugging back in the $D - 1$ remaining equations we obtain $D - 1$ invariants $(\delta I_0(A), \delta I_j^T(A)) = (0, 0)$ where $I_0(A) = A_0 - \partial_0 \partial_j A_j / \nabla^2$ and $I_j^T(A) = \theta_{jk} A_k$, with the projection operator:

$$\theta_{jk} \equiv \delta_{jk} - \frac{\partial_j \partial_k}{\nabla^2}. \qquad (18)$$

The invariant $I_0(A)$ comes from the divergence of the electric field while $I_j^T(A)$ is given in terms of the spatially transverse components of the magnetic field which are of course the only nonvanishing ones. In general, the number

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[2]We assume vanishing fields at infinity, so $\nabla^2 A \equiv \partial_j \partial_j A = 0$ leads to $A = 0$, where A represents any field or its spacetime derivatives. Moreover, we use T for space and t for spacetime transverse quantities.

of independent gauge invariants equals the number of independent fields minus the number of independent gauge parameters, symbolically,

$$N_I = N_A - N_\Lambda. \tag{19}$$

Thus, from the $D(D-1)/2$ equations $\delta B_{\mu\nu} = \Lambda^t_{\mu\nu}$ and the $D-1$ constraints $\partial^\mu \Lambda^t_{\mu\nu} = 0$ we can find $N_I = D(D-1)/2 - [D(D-1)/2 - (D-1)] = (D-1)$ invariants[3] which turn out to be $I^t_\mu(B) \equiv \partial^\nu B_{\nu\mu}$. Now we return

to the mBF theory. Let us introduce an invertible decomposition without time derivatives in order to avoid changes in the canonical structure of the theory, namely,

$$A_0 = \gamma; \qquad A_j = v^T_j + \partial_j \theta \tag{20}$$

$$B_{0j} = b^T_j + \partial_j \psi; \qquad B_{ij} = b^T_{ij} + \partial_i c^T_j - \partial_j c^T_i \tag{21}$$

where $\partial_j v^T_j = 0 = \partial_j b^T_j = \partial_j c^T_j = \partial_j b^T_{ij} = 0$ and $b^T_{ij} = -b^T_{ji}$. So we have

$$\mathcal{L}_{\mathrm{mBF}} = \overbrace{\frac{1}{2} v^T_j \Box v^T_j - \frac{1}{2}(\gamma - \dot\theta)\nabla^2(\gamma - \dot\theta)}^{-\frac{1}{4}F^2_{\mu\nu}} \overbrace{-\frac{1}{2}\nabla^2\psi\Box\psi + \frac{1}{2}(\nabla^2 c^T_j - \dot b^T_j)^2}^{+\frac{1}{2}(\partial^\mu B_{\mu\nu})^2} + \underbrace{m(\gamma - \dot\theta)\nabla^2\psi + m(\nabla^2 c^T_j - \dot b^T_j)v^T_j}_{-\frac{m}{2}B^{\mu\nu}F_{\mu\nu}}. \tag{22}$$

The free Lagrangian is a bilinear function of the previously found invariants: $(I_0(A), I^T_j(A)) = (\gamma - \dot\theta, v^T_j)$ and $(I_0(B), I^T_j(B)) = (\nabla^2\psi, \nabla^2 c^T_j - \dot b^T_j - \partial_j\dot\psi)$. In the Maxwell theory we have $D-2$ massless propagating modes (v^T_j) and 1 nonpropagating one $(\gamma - \dot\theta)$, while in the antisymmetric tensor we have the opposite, 1 massless propagating mode (ψ) and $D-2$ nondynamic gauge invariants $(\nabla^2 c^T_j - \dot b^T_j)$. In the BF term we have a perfect match. The nondynamic modes of one theory couple to the propagating ones of the other one in order to generate the mass terms. After diagonalizing the Lagrangian, the now massive $D-1$ propagating fields (v^T_j, ψ) decouple from the nonpropagating ones giving rise to a massive spin-1 particle without extra fields or ghosts,

$$\mathcal{L}_{\mathrm{mBF}} = \frac{1}{2} v^T_j(\Box - m^2)v^T_j + \frac{1}{2}\psi(-\nabla^2)(\Box - m^2)\psi$$
$$+ \frac{C^T_j}{2} - \frac{1}{2}\Gamma\nabla^2\Gamma, \tag{23}$$

where $(C^T_j, \Gamma) \equiv (\nabla^2 c^T_j - \dot b^T_j + mv^T_j, \gamma - \dot\theta - m\psi)$ are the nondynamic invariants.

2. The mBR model

Turning off the sources and redefining $B_{\mu\alpha\nu\beta} \to mB_{\mu\alpha\nu\beta}/2$, the Lagrangian for the linearized mBR model (10) is given by:

$$\mathcal{L}_{\mathrm{mBR}} = \frac{1}{2}\left(\frac{1}{2}h_{\mu\nu}\Box h^{\mu\nu} - \frac{1}{2}h\Box h + h\partial_\mu\partial_\nu h^{\mu\nu} - h^{\mu\nu}\partial_\mu\partial^\lambda h_{\lambda\nu}\right)$$
$$+ \frac{1}{4}(\partial_\mu\partial_\nu B^{\mu\alpha\nu\beta})^2 - \frac{1}{4(D-1)}(\partial_\alpha\partial_\beta B^{\alpha\beta})^2$$
$$- \frac{m}{4}B^{\mu\alpha\nu\beta}R^{(L)}_{\mu\alpha\nu\beta}(h). \tag{24}$$

The linearized EH theory, first line of (24), is invariant under linearized reparametrizations $\delta h_{\mu\nu} = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$. We use D of those $D(D+1)/2$ equations to determine the gauge parameters ξ_μ in terms of $\delta h_{\mu\nu}$ and plugging back the result in the transformations the remaining equations furnish $N_I = D(D+1)/2 - D = D(D-1)/2$ gauge invariants. Using $(\xi_0, \xi_j) = (\omega, \xi^T_j + \partial_j\xi)$ and the decomposition:

$$h_{\mu\nu} \to \begin{cases} h_{00} = \rho \\ h_{0i} = \gamma^T_i + \partial_i\theta \\ h_{ij} = h^{TT}_{ij} + \partial_i\psi^T_j + \partial_j\psi^T_i + \theta_{ij}\nabla^2\psi + \partial_i\partial_j\varphi \end{cases} \tag{25}$$

where the superscript TT means transverse and traceless, i.e. $\partial_i h^{TT}_{ij} = 0 = \delta_{ij}h^{TT}_{ij}$, we can write down the linearized EH theory in terms of gauge invariants,[4] up to total derivatives we have:

$$\mathcal{L}_{\mathrm{LEH}} = (\sqrt{-g}R)_{hh} = \frac{1}{4}h^{TT}_{ij}\Box h^{TT}_{ij} - \frac{1}{2}\Gamma_j\nabla^2\Gamma_j$$
$$+ \frac{(D-1)(D-2)}{4}\Phi\Box\Phi + \frac{(D-2)}{2}\Phi R^{(L)} \tag{26}$$

where the $D(D-3)/2$ invariants h^{TT}_{ij} represent the propagating degrees of freedom of the graviton while the next $(D-2) + 2$ invariants:

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[3]Specifically, we have decomposed a general antisymmetric field as $(\Lambda_{0i}, \Lambda_{ij}) = (\lambda^T_i + \partial_i\phi, \Lambda^T_{ij} + \partial_i\omega^T_j - \partial_j\omega^T_i)$ and used the transversality condition $\partial^\mu\Lambda^t_{\mu\nu} = 0$ to get rid of redundant components $(\phi, \omega^T_i) = (0, \dot\lambda^T_i/\nabla^2)$.

[4]Compare with [21] for $D = 4$.

$$\Gamma_j^T = \gamma_j^T - \dot{\psi}_j^T; \qquad \Phi = \nabla^2 \varphi \qquad (27)$$

$$R^{(L)} = \partial^\mu \partial^\nu h_{\mu\nu} - \Box h = \nabla^2[\rho - 2\dot{\theta} + \ddot{\varphi} + (2 - D)\Box\psi] \qquad (28)$$

are nonpropagating. In total we have the expected $D(D-1)/2$ gauge invariants. Notice that (27) and (28)

make clear that each invariant can be indeed associated with an independent field. In particular $\delta S_{\text{LEH}}/\delta\rho = 0$ establishes that the "would be" propagating mode Φ vanishes. Consequently, from $\delta S_{\text{LEH}}/\delta\Phi = 0$ we have vanishing scalar curvature ($R^{(L)} = 0$) as expected from the Einstein equations in the vacuum.

Regarding the rest of the Lagrangian (24), we introduce the general decomposition:

$$B_{\mu\alpha\nu\beta} \rightarrow \begin{cases} B_{[0i][0j]} = B_{ij}^T + \partial_i B_j^T + \partial_j B_i^T + 2\partial_i \partial_j Z \\ B_{[0i][jk]} = B_{i[jk]}^{pT} + \partial_j C_{ik}^{pT} - \partial_k C_{ij}^{pT} \\ B_{[ij][kl]} = B_{[ij][kl]}^T + [\partial_i D_{j[kl]}^T + \partial_k D_{l[ij]}^T + \partial_i(\partial_k W_{jl}^T - \partial_l W_{jk}^T) + (i \leftrightarrow j, k \leftrightarrow l)] \end{cases} \qquad (29)$$

where $(B_{ij}^T, W_{ij}^T) = (B_{ji}^T, W_{ji}^T)$ while $C_{ij}^{pT} \neq C_{ji}^{pT}$. All spatial tensor fields are transverse in all indices except the ones in the second line of (29) which are only partially transverse (pT), namely, $\partial_j B_{i[jk]}^{pT} = 0 = \partial_j C_{ij}^{pT}$ but $\partial_i B_{i[jk]}^{pT} \neq 0$ and $\partial_i C_{ij}^{pT} \neq 0$. In the Appendix we show that $\mathcal{L}_{\text{DTS}}$ in (12) can be written in terms of $(D+1)(D-2)/2$ gauge invariants, see (A14), which split into a transverse vector V_j^T representing $D-2$ propagating massless modes, in agreement with a massless spin-1 field, plus $(D-1)(D-2)/2$ nonpropagating modes represented by a transverse symmetric spatial tensor W_{ij}^T, which we further decompose into its trace $W = \delta^{ij} W_{ij}^T$ and its traceless

and transverse piece $W_{ij}^{TT} \equiv W_{ij}^T - \theta_{ij} W/(D-2)$, see (A11) and (A13).

Regarding the BR term, there is an important difference with the spin-1 case. Namely, the BF term does not break any of the symmetries (4) of the rest of the model while the BR term breaks the scalar symmetry in (13). Since we have one less gauge parameter we end up with one more gauge invariant besides (V_j^T, W_{ij}^T). We find it by noticing that $\partial^i \partial^j \delta B_{[0i][0j]} = \partial^i \partial^j \Lambda_{[0i][0j]} = \partial^\mu \partial^\nu \Lambda_{[0\mu][0\nu]} = 0$ by virtue of (14). Back in the first line of (29) we have our last invariant $\delta Z = 0$, which only appears in the BR term. Finally, from (26) and (A14) and working out the BR term, we have for the whole mBR model:

$$\mathcal{L}_{\text{mBR}} = \frac{1}{4} h_{ij}^{TT} \Box h_{ij}^{TT} - \frac{1}{2} \Gamma_j^T \nabla^2 \Gamma_j^T + \frac{(D-1)(D-2)}{4} \Phi \Box \Phi + \frac{(D-2)}{2} \Phi R^{(L)}$$
$$+ \frac{1}{4}(W_{ij}^{TT})^2 + \frac{1}{2} V_j^T(-\nabla^2)\Box V_j^T + \frac{W^2}{4(D-1)(D-2)}$$
$$+ \underbrace{\frac{m}{2} W_{ij}^{TT} h_{ij}^{TT} + m\Gamma_j^T \nabla^2 V_j^T + \frac{m}{2} W\Phi + mZ\nabla^2 R^{(L)}}_{-\frac{m}{4} B^{\mu\alpha\nu\beta} R_{\mu\alpha\nu\beta}^L(h)}. \qquad (30)$$

Integrating over the nonpropagating fields W_{ij}^{TT}, Γ_j^T, and W we generate mass terms for the helicities $\pm 2, \pm 1, 0$ of the massive spin-2 particle respectively,

$$\mathcal{L}_{\text{mBR}} = \frac{1}{4} h_{ij}^{TT}(\Box - m^2) h_{ij}^{TT} + \frac{1}{2} V_j^T(-\nabla^2)(\Box - m^2) V_j^T + \frac{(D-1)(D-2)}{4} \Phi(\Box - m^2)\Phi, \qquad (31)$$

while the integral over Z produces the constraint $R^{(L)} = 0$ which eliminates h_{00}. We have not written down the nonpropagating modes in (31). They correspond to quadratic terms in field redefinitions of W_{ij}^{TT}, Γ_j^T, and W. Thus, we have the same number of propagating and nonpropagating gauge invariant modes in the final

Lagrangian just like in the spin-1 case. However, the analogy between the mBF (22) and the mBR model (30) is not totally faithful. Although one might think at first sight of a perfect match between propagating and nonpropagating modes $(h_{ij}^{TT}, V_j^T, \Phi) \leftrightarrow (W_{ij}^{TT}, \Gamma_j^T, W)$, we recall that Φ is a nonpropagating gauge invariant in the LEH theory, it

only becomes a dynamic field because of the last term in (30) which annihilates the effect of the term $\Phi R^{(L)}$ which sets $\Phi = 0$. So in the spin-2 case the role of the BR term is not only to couple dynamic with nondynamic fields but also to turn a nonpropagating mode into a propagating one and this is only possible because the scalar symmetry ($U(1)$ symmetry) of the DTS model is broken giving rise to the Z invariant.

Since the linearized mBR model is a consistent ghost free description of massive spin-2 particles with linearized reparametrization invariance, it is natural to try to go beyond the free theory and develop a consistent nonlinear version of the model invariant under general coordinate transformation. In order to prepare the ground for an

analysis via decoupling limit, see [5,6], in the next subsection we take a closer look at the pure massless limit starting with the simpler $D = 3$ case.

C. The $D = 3$ case and the massless limit

Since the mBR model describes massive spin-2 particles with reparametrization invariance, it might be related with the new massive gravity (NMG) theory of [22] for $D = 3$. Indeed, see [13], in $D = 3$ we can write the Riemann tensor in terms of the Einstein tensor $R_{\mu\nu\alpha\beta} = \epsilon_{\mu\nu\lambda}\epsilon_{\alpha\beta\sigma}G^{\lambda\sigma}$. So one can write down the mBR model (6) in $D = 3$, without sources, in terms of three symmetric rank-2 tensors:

$$S_{\text{mBR}}^{D=3} = \int d^3x\left\{-\frac{1}{2}h^{\mu\nu}G_{\mu\nu}^{(L)}(h) - \frac{m^2}{4}(f_{\mu\nu}^2 - f^2) + \tilde{B}^{\mu\nu}G_{\mu\nu}^{(L)}(f - h)\right\} \tag{32}$$

$$= \int d^3x\left\{-\frac{1}{2}h_+^{\mu\nu}G_{\mu\nu}^{(L)}(h_+) + \frac{1}{2}\tilde{B}^{\mu\nu}G_{\mu\nu}^{(L)}(\tilde{B}) - \frac{m^2}{4}(f_{\mu\nu}^2 - f^2) + f^{\mu\nu}G_{\mu\nu}^{(L)}(\tilde{B})\right\}, \tag{33}$$

where $(h_+)_{\mu\nu} \equiv h_{\mu\nu} + \tilde{B}_{\mu\nu}$ and we have introduced the following symmetric tensor which has the same number of independent components of $B_{\alpha\beta\lambda\sigma}$ in $D = 3$:

$$\tilde{B}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\alpha\beta}\epsilon^{\nu\lambda\sigma}B_{\alpha\beta\lambda\sigma} = \tilde{B}^{\nu\mu}. \tag{34}$$

Integrating over $f_{\mu\nu}$ in (33) and neglecting the first EH term which decouples and has no particle content in $D = 3$, we obtain the linearized version of the NMG theory [22]:

$$S_{\text{mBR}}^{D=3} = \int d^3x\left\{+\frac{1}{2}\tilde{B}^{\mu\nu}G_{\mu\nu}^{(L)}(\tilde{B}) + \frac{1}{m^2}\mathcal{L}_K[\tilde{B}]\right\} \tag{35}$$

with

$$\mathcal{L}_K = \frac{1}{4}\Box\tilde{B}_{\mu\nu}\Box\tilde{B}^{\mu\nu} - \frac{1}{2}\Box\tilde{B}_{\mu\nu}\partial^\mu\partial_\alpha\tilde{B}^{\alpha\nu} + \frac{1}{8}(\partial_\mu\partial_\nu\tilde{B}^{\mu\nu})^2$$
$$+ \frac{1}{4}\Box\tilde{B}\partial_\mu\partial_\nu\tilde{B}^{\mu\nu} - \frac{1}{8}(\Box\tilde{B})^2 \tag{36}$$

$$= \left[R_{\mu\nu}^2(\gamma) - \frac{3}{8}R(\gamma)^2\right]_{\tilde{B}\tilde{B}}. \tag{37}$$

We have considered $\tilde{B}_{\mu\nu}$ as the fluctuation of a metric about the flat space: $\gamma_{\mu\nu} = \eta_{\mu\nu} - \tilde{B}_{\mu\nu}$. The first term in (35) becomes the linearized version of the EH theory with the sign opposite to the usual one. As a first step to check the particle content of the NMG model beyond the linearized theory (35) one might try to look at the leading order in the decoupling limit where $m \to 0$ and $m_p \to \infty$ as we later

explain, see [23]. Notice however, that the fourth order theory (35) is singular at $m \to 0$. This is also the case of the D-dimensional mBR model in (10). If instead, we simply abandon the canonical mass dimension and redefine $B_{\mu\alpha\nu\beta} \to mB_{\mu\alpha\nu\beta}$, when $m \to 0$ we will be left only with $\mathcal{L}_k$ which is equivalent to the Maxwell theory in $D = 3$ and has $D - 2 = 1$ degree of freedom instead of two degrees of freedom as expected for a parity doublet of spin-2 in $D = 3$. If we try $m \to 0$ directly in the 2nd order theories (32) or (33) we have zero degrees of freedom. So without the extra fields, we do not have a smooth massless limit. This point has been investigated in [23]. Going back to the second order version[5] (33),

$$S_{\text{mBR}}^{(D=3)} = \int d^3x\left\{+\frac{1}{2}\tilde{B}^{\mu\nu}G_{\mu\nu}^{(L)}(\tilde{B}) - f^{\mu\nu}G_{\mu\nu}^{(L)}(\tilde{B})\right.$$
$$\left. - \frac{m^2}{4}(f_{\mu\nu}^2 - f^2)\right\}, \tag{38}$$

we notice that the mass term breaks the symmetry $(\delta f_{\mu\nu}, \delta h_{\mu\nu}) = (\partial_\mu\zeta_\nu + \partial_\nu\zeta_\mu, 0)$. In order to recover it one introduces auxiliary compensating (Stueckelberg) fields via:

$$f_{\mu\nu} \to f_{\mu\nu} + \frac{\partial_\mu A_\nu + \partial_\nu A_\mu}{m} + \frac{2\partial_\mu\partial_\nu\varphi}{m^2}. \tag{39}$$

---

[5]We have neglected the first term of (33) which has no particle content.

After taking $m \to 0$ one has:

$$S_{\mathrm{mBR}}^{(D=3)}(m \to 0) = \int d^3x \left\{ +\frac{1}{2} \tilde{B}^{\mu\nu} G_{\mu\nu}^{(L)}(\tilde{B}) - f^{\mu\nu} G_{\mu\nu}^{(L)}(\tilde{B}) \right.$$
$$\left. -\frac{1}{4} F_{\mu\nu}^2(A) - f_{\mu\nu}(\partial^\mu \partial^\nu \varphi - \eta^{\mu\nu} \Box \varphi) \right\}.$$
(40)

Integrating over $f_{\mu\nu}$ one obtains:

$$G_{\mu\nu}^{(L)}(\tilde{B}) = \eta_{\mu\nu} \Box \varphi - \partial_\mu \partial_\nu \varphi \qquad (41)$$

whose general solution is given by

$$\tilde{B}_{\mu\nu} = 2\eta_{\mu\nu} \varphi + \text{pure gauge.} \qquad (42)$$

Back in (40) one finds [23]:

$$S_{\mathrm{mBR}}^{(D=3)}(m \to 0) = \int d^3x \left\{ -\frac{1}{4} F_{\mu\nu}^2(A) + 2\varphi \Box \varphi \right\}. \qquad (43)$$

The Maxwell theory in $D = 3$ has only one degree of freedom, it is dual to a scalar field, we end up with a total of two degrees of freedom which is the expected number of degrees of a parity invariant massive spin-2 field in $D = 3$ (parity doublet). So we have a smooth massless limit. This is similar to the spin-1 master action (1) after $f_\mu \to f_\mu + \partial_\mu \varphi / m$ and $m \to 0$ which leads to Maxwell plus a scalar field again, corresponding to $D - 2 + 1 = D - 1$ degrees of freedom, in agreement with the Proca theory.

Now we go back to the mBR model in D-dimensions in its second order version (6). After inserting Stueckelberg fields we have

$$S_{\mathrm{BRStuec}}^D = \int d^D x \left\{ -\frac{1}{2} h^{\mu\nu} G_{\mu\nu}^{(L)}(h) - \frac{1}{2} B^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}^{(L)}(h) + \frac{1}{2} B^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}^{(L)}(f) - \frac{1}{4} F_{\mu\nu}^2(A) \right.$$
$$\left. -\frac{m^2}{4}(f_{\mu\nu}f^{\mu\nu} - f^2) - mf_{\mu\nu}(\partial^\mu A^\nu - \eta^{\mu\nu}\partial_\alpha A^\alpha) - f_{\mu\nu}(\partial^\mu \partial^\nu \varphi - \eta^{\mu\nu}\Box \varphi) \right\}.$$
(44)

After $m \to 0$ we obtain up to total derivatives,

$$S_{\mathrm{BR}}^D(m \to 0) = \int d^D x \left\{ -\frac{1}{2} h^{\mu\nu} G_{\mu\nu}^{(L)}(h) + h^{\mu\nu}\partial^\alpha\partial^\beta B_{\mu\alpha\nu\beta} - f^{\mu\nu}\partial^\alpha\partial^\beta B_{\mu\alpha\nu\beta} - \frac{1}{4} F_{\mu\nu}^2(A) - f_{\mu\nu}(\partial^\mu \partial^\nu \varphi - \eta^{\mu\nu}\Box \varphi) \right\}. \qquad (45)$$

Integrating over the field $B_{\mu\alpha\nu\beta}$ we have

$$R_{\mu\nu\alpha\beta}^{(L)}(f) = R_{\mu\nu\alpha\beta}^{(L)}(h) \qquad (46)$$

whose general solution is given by $f_{\mu\nu} = h_{\mu\nu} + \partial_\mu\chi_\nu + \partial_\nu\chi_\mu$, where χ_μ are arbitrary pure gauge vector fields. Back in (45) we get:

$$S_{m\to 0}^D = \int d^D x \left\{ -\frac{1}{2} h^{\mu\nu} G_{\mu\nu}^{(L)}(h) + h^{\mu\nu}(\eta_{\mu\nu}\Box\varphi - \partial_\mu\partial_\nu\varphi) \right.$$
$$\left. -\frac{1}{4} F_{\mu\nu}^2(A) \right\}. \qquad (47)$$

After a conformal redefinition $h_{\mu\nu} \to \bar{h}_{\mu\nu} + \frac{2}{(D-2)}\eta_{\mu\nu}\varphi$ we have a diagonal form which splits into spin-2, spin-1, and spin-0 sectors,

$$S_{m\to 0}^D = \int d^D x \left\{ -\frac{1}{2} \bar{h}^{\mu\nu} G_{\mu\nu}^{(L)}(\bar{h}) - \frac{1}{4} F_{\mu\nu}^2(A) + \frac{(D-1)}{(D-2)}\varphi\Box\varphi \right\}. \qquad (48)$$

In D dimensions the linearized Einstein-Hilbert term possess $D(D-3)/2$ degrees of freedom which altogether with $(D-2)+1$ degrees of freedom of the vector and scalar sectors lead to $(D+1)(D-2)/2$ which is the same number of independent modes of the massive spin-2 Fierz-Pauli [10] theory described by a symmetric rank-2 tensor $(D(D+1)/2)$ constrained by the $(D+1)$ FP conditions: $\partial^\mu h_{\mu\nu} = 0 = \eta^{\mu\nu} h_{\mu\nu}$. At $D = 4$ we end up with $5 = 2s + 1$ degrees of freedom as expected. In summary, the D-dimensional mBR model with Stueckelberg fields via (39) has a smooth massless limit like the FP theory with Stueckelberg fields.

III. NONLINEAR mBR MODELS

Before we search for nonlinear extensions of the D-dimensional mBR model we have found convenient again to first address the $D = 3$ case in the next subsection which is simpler and has allowed us to suggest a new class of bimetric models.

A. A new class of $D=3$ bimetric models

A closer look at the linearized $D=3$ mBR model (32)

$$S_{\mathrm{BR}}^{D=3}(\tilde{B},h,f) = \int d^3x \left\{ -\frac{1}{2} h^{\mu\nu} G_{\mu\nu}^{(L)}(h) - \tilde{B}^{\mu\nu} G_{\mu\nu}^{(L)}(h) \right.$$
$$\left. -\frac{m^2}{4}(f_{\mu\nu}^2 - f^2) + f^{\mu\nu} G_{\mu\nu}^{(L)}(B) \right\}, \quad (49)$$

inspires us to introduce two metrics $(g_{\mu\nu}, \gamma_{\mu\nu}) = (\eta_{\mu\nu}+h_{\mu\nu}, \eta_{\mu\nu}+B_{\mu\nu})$. While the last two terms give rise, after integrating over $f_{\mu\nu}$, to $\mathcal{L}_K$ which on its turn can be nonlinearly completed in terms of squares of curvatures, the first and second terms can be written as linearizations of linear combinations of $\sqrt{-g}R(g)$ and $\sqrt{-g}\gamma^{\mu\nu}G_{\mu\nu}(g)$ where $\gamma^{\mu\nu}=g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta}$. The field $f_{\mu\nu}$ remains an auxiliary field. In fact we can go beyond (49) and suggest a quite general Ansatz for a new class of bimetric models in $D=3$, namely, restoring the Planck mass in the action we have

$$S_{g-\gamma} = M_P \int d^3x \left\{ a\sqrt{-g}R(g) + b\sqrt{-\gamma}R(\gamma) \right.$$
$$+ c\sqrt{-g}\gamma^{\mu\nu}G_{\mu\nu}(g) + d\sqrt{-\gamma}g^{\mu\nu}G_{\mu\nu}(\gamma)$$
$$\left. + k\sqrt{-\gamma}\hat{f}^{\mu\nu}G_{\mu\nu}(\gamma) - \frac{m^2}{4}\sqrt{-\gamma}(\hat{f}_{\mu\nu}^2 - \hat{f}^2) \right\}, \quad (50)$$

where a, b, c, d, and k are arbitrary constants and $\hat{f}_{\mu\nu}$ are auxiliary fields such that $\hat{f}^{\mu\nu}=\gamma^{\mu\alpha}\gamma^{\nu\beta}\hat{f}_{\alpha\beta}$ and $\hat{f}=\gamma^{\mu\nu}\hat{f}_{\mu\nu}$. The simultaneous flat space solution: $g_{\mu\nu}=\gamma_{\mu\nu}=\eta_{\mu\nu}$; $\hat{f}_{\mu\nu}=0$ solves in general the equations of motion of (50). Introducing

$$g_{\mu\nu}=\eta_{\mu\nu}+\frac{h_{\mu\nu}}{\sqrt{M_P}}, \quad \gamma_{\mu\nu}=\eta_{\mu\nu}+\frac{b_{\mu\nu}}{\sqrt{M_P}}, \quad \hat{f}_{\mu\nu}=\frac{f_{\mu\nu}}{\sqrt{M_P}}. \quad (51)$$

At quadratic order in $D=3$ we have:

$$M_P\sqrt{-g}R(g) \to -\frac{1}{2}h^{\mu\nu}G_{\mu\nu}^{(L)}(h) \quad (52)$$

$$M_P\sqrt{-\gamma}R(\gamma) \to -\frac{1}{2}b^{\mu\nu}G_{\mu\nu}^{(L)}(b) \quad (53)$$

$$M_P\sqrt{-g}\gamma^{\mu\nu}G_{\mu\nu}(g) \to -b^{\mu\nu}G_{\mu\nu}^{(L)}(h)+\frac{5}{4}h^{\mu\nu}G_{\mu\nu}^{(L)}(h) \quad (54)$$

$$M_P\sqrt{-\gamma}g^{\mu\nu}G_{\mu\nu}(\gamma) \to -h^{\mu\nu}G_{\mu\nu}^{(L)}(b)+\frac{5}{4}b^{\mu\nu}G_{\mu\nu}^{(L)}(b) \quad (55)$$

$$M_P\sqrt{-\gamma}\hat{f}^{\mu\nu}G_{\mu\nu}(\gamma) \to f^{\mu\nu}G_{\mu\nu}^{(L)}(b) \quad (56)$$

$$M_P\sqrt{-\gamma}(\hat{f}_{\mu\nu}^2 - \hat{f}^2) \to (f_{\mu\nu}^2 - f^2). \quad (57)$$

On the right side all indices are raised with the Minkowisky metric $\eta^{\mu\nu}$. Using (52)–(57) in (50), we obtain at quadratic order:

$$S = \int d^3x \left\{ -\frac{r}{2} h^{\mu\nu}G_{\mu\nu}^{(L)}(h) - \frac{s}{2} b^{\mu\nu}G_{\mu\nu}^{(L)}(b) - tb^{\mu\nu}G_{\mu\nu}^{(L)}(h) \right.$$
$$\left. + kf^{\mu\nu}G_{\mu\nu}^{(L)}(b) - \frac{m^2}{4}(f_{\mu\nu}^2 - f^2) \right\}, \quad (58)$$

where:

$$r = a - \frac{5}{2}c, \qquad s = b - \frac{5}{2}d, \qquad t = c+d. \quad (59)$$

Since the EH theory has no content in $D=3$ we must have $k\neq0$ in order to avoid the empty content of the first three terms of (58). If $s=0=t$ the first term decouples from the $b_{\mu\nu}$ field and becomes the quadratic truncation of the EH term and the last two terms are equivalent to the "K" term of (37) which is equivalent to the Maxwell theory as shown in [24] with only one degree of freedom in $D=3$. So henceforth we assume that $k\neq0$ and s or t must be nonvanishing. If $r=0$ we must have $t=0$ otherwise the $h_{\mu\nu}$ equation of motion leads to $b_{\mu\nu}$ pure gauge and we would have no content again. Moreover if $r=0=t$ we must have $s<0$ in order that the EH term for the b-field has the "wrong sign," typical of the NMG model [22], which guarantees a physical massive spin-2 particle. If $r\neq0$ we can diagonalize the first three terms and write

$$S = \int d^3x \left\{ -\frac{r}{2}\mathcal{H}^{\mu\nu}G_{\mu\nu}^{(L)}(\mathcal{H}) + \frac{1}{2}\left(\frac{t^2}{r}-s\right)b^{\mu\nu}G_{\mu\nu}^{(L)}(b) \right.$$
$$\left. + kf^{\mu\nu}G_{\mu\nu}^{(L)}(b) - \frac{m^2}{4}(f_{\mu\nu}^2 - f^2) \right\} \quad (60)$$

where $\mathcal{H}_{\mu\nu}\equiv h_{\mu\nu}+\frac{t}{r}b_{\mu\nu}$. Since the first term (linearized Einstein-Hilbert) has no particle content, we may keep only the remaining ones which have the form of a second order formulation of the NMG model, namely,

$$S = \int d^3x \left\{ +\frac{1}{2}\left(\frac{t^2}{r}-s\right)b^{\mu\nu}G_{\mu\nu}^{(L)}(b) \right.$$
$$\left. + kf^{\mu\nu}G_{\mu\nu}^{(L)}(b) - \frac{m^2}{4}(f_{\mu\nu}^2 - f^2) \right\}. \quad (61)$$

The particle content of (61) corresponds to a physical massive spin-2 particle with two helicity states ±2 whenever the EH coefficient has the "wrong sign" and $k\neq0$,

$$\left(\frac{t^2}{r}-s\right)>0; \qquad k\neq0. \quad (62)$$

Comparing with (50) we identify the linearized mBR model (49) corresponds to $a=7/2; c=1; b=d=0;$ and $k=1$ which implies $(r,s,t)=(1,0,1)$.

Now in order to go beyond the linearized approximation we investigate now the decoupling limit of (50) at leading order. Namely, following [23] we will take the following double limit while keeping the scale $\Lambda_{5/2}$ fixed,

$$m \to 0, \qquad M_P \to \infty, \qquad \Lambda_{5/2} \equiv (\sqrt{M_P} m^2)^{2/5}. \quad (63)$$

Regarding the notation, we use $\widetilde{\nabla}_\mu$ for the covariant derivative with respect to the metric $\gamma_{\mu\nu}$ while ∇_μ corresponds to the metric $g_{\mu\nu}$. Similarly to [23], we first notice

that the term $\sqrt{-\gamma} \hat{f}^{\mu\nu} G_{\mu\nu}(\gamma)$ is invariant under $\delta \hat{f}_{\mu\nu} = \widetilde{\nabla}_\mu \zeta_\nu + \widetilde{\nabla}_\nu \zeta_\mu$ which is broken by the mass term for the auxiliary fields $\hat{f}_{\mu\nu}$. In order to restore the symmetry we substitute in (50):

$$\hat{f}_{\mu\nu} \to \frac{f_{\mu\nu}}{\sqrt{M_P}} + \frac{\widetilde{\nabla}_\mu A_\nu + \widetilde{\nabla}_\nu A_\mu}{\sqrt{M_P} m} + 2 \frac{\widetilde{\nabla}_\mu \widetilde{\nabla}_\nu \varphi}{\sqrt{M_P} m^2}. \quad (64)$$

Up to total derivatives we obtain:

$$
\begin{aligned}
S_{g-\gamma} = \int d^3x \Bigg\{ & a M_P \sqrt{-g} R(g) + b M_P \sqrt{-\gamma} R(\gamma) + c M_P \sqrt{-g} \gamma^{\mu\nu} G_{\mu\nu}(g) \\
& + d M_P \sqrt{-\gamma} g^{\mu\nu} G_{\mu\nu}(\gamma) + k \sqrt{M_P} \sqrt{-\gamma} f^{\mu\nu} G_{\mu\nu}(\gamma) - \frac{m^2}{4} \sqrt{-\gamma} (\hat{f}_{\mu\nu}^2 - \hat{f}^2) \\
& - m \sqrt{-\gamma} f^{\mu\nu} (\widetilde{\nabla}_\mu A_\nu - \gamma_{\mu\nu} \widetilde{\nabla}_\alpha A^\alpha) - \sqrt{-\gamma} f^{\mu\nu} (\widetilde{\nabla}_\mu \widetilde{\nabla}_\nu \varphi - \gamma_{\mu\nu} \widetilde{\nabla}^2 \varphi) \\
& - \frac{1}{2} \sqrt{-\gamma} \Big(\widetilde{\nabla}_\mu A_\nu \widetilde{\nabla}^\mu A^\nu - \widetilde{\nabla}_\mu A^\mu \widetilde{\nabla}_\nu A^\nu \Big) + \frac{1}{2} \sqrt{-\gamma} R_{\mu\nu}(\gamma) A^\mu A^\nu \\
& + 2 \frac{\sqrt{-\gamma}}{m} R_{\mu\nu}(\gamma) A^\mu \widetilde{\nabla}^\nu \varphi + \frac{\sqrt{-\gamma}}{m^2} R_{\mu\nu}(\gamma) \widetilde{\nabla}^\mu \varphi \widetilde{\nabla}^\nu \varphi \Bigg\},
\end{aligned}
\quad (65)
$$

where we have used $[\widetilde{\nabla}_\mu, \widetilde{\nabla}_\nu] V^\mu = R_{\mu\nu}(\gamma) V^\mu$ with $V^\mu = A^\mu$ and $\widetilde{\nabla}^\mu \varphi$.

Using (51) and taking the decoupling limit we have:

$$
\begin{aligned}
S'_{g-\gamma} = \int d^3x \Bigg\{ & -\frac{r}{2} h^{\mu\nu} G_{\mu\nu}^{(L)}(h) - \frac{s}{2} b^{\mu\nu} G_{\mu\nu}^{(L)}(b) - t b^{\mu\nu} G_{\mu\nu}^{(L)}(h) - \frac{1}{4} F_{\mu\nu}^2(A) \\
& + k f^{\mu\nu} G_{\mu\nu}^{(L)}(b) - f^{\mu\nu} (\partial_\mu \partial_\nu \varphi - \eta_{\mu\nu} \Box \varphi) + \frac{1}{\Lambda_{5/2}^{5/2}} R_{\mu\nu}^{(L)}(b) \partial^\mu \varphi \partial^\nu \varphi \Bigg\}.
\end{aligned}
\quad (66)
$$

Integrating over $f_{\mu\nu}$ which appears linearly we obtain

$$k G_{\mu\nu}^{(L)}(b) = \partial_\mu \partial_\nu \varphi - \eta_{\mu\nu} \Box \varphi \quad (67)$$

whose general solution is:

$$b_{\mu\nu} = -\frac{2}{k} \eta_{\mu\nu} \varphi + \partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu. \quad (68)$$

Back in (66),

$$
\begin{aligned}
S'_{g-\gamma} = \int d^3x \Bigg\{ & -\frac{r}{2} h^{\mu\nu} G_{\mu\nu}^{(L)}(h) - \frac{2s}{k^2} \varphi \Box \varphi \\
& - \frac{t}{k} h^{\mu\nu} (\partial_\mu \partial_\nu \varphi - \eta_{\mu\nu} \Box \varphi) - \frac{1}{4} F_{\mu\nu}^2(A) \\
& + \frac{1}{k \Lambda_{5/2}^{5/2}} \Big(\Box \varphi \partial_\mu \varphi \partial^\mu \varphi + \partial_\mu \partial_\nu \varphi \partial^\mu \varphi \partial^\nu \varphi \Big) \Bigg\}.
\end{aligned}
\quad (69)
$$

After a conformal field redefinition:

$$h_{\mu\nu} \to h_{\mu\nu} + \frac{2t}{kr} \eta_{\mu\nu} \varphi. \quad (70)$$

We have a diagonal action:

$$
\begin{aligned}
S'_{g-\gamma} = \int d^3x \Bigg\{ & -\frac{r}{2} h^{\mu\nu} G_{\mu\nu}^{(L)}(h) - \frac{1}{4} F_{\mu\nu}^2(A) \\
& + \frac{2}{k^2} \left(\frac{t^2}{r} - s \right) \varphi \Box \varphi + \frac{1}{2k \Lambda_{5/2}^{5/2}} \Box \varphi \partial_\mu \varphi \partial^\mu \varphi \Bigg\}.
\end{aligned}
\quad (71)
$$

So, as in [23], we have a smooth massless limit where the scalar field has Galileon self-interaction and quadratic kinetic term with the correct sign in agreement with (62). The EH term has no content while the Maxwell theory has one degree of freedom. So the decoupling limit of the bimetric model at leading order is free of ghosts as far

as (62) holds true. Clearly, a complete nonlinear analysis including a detailed study of the Hamiltonian structure of the model (50), as the one carried out in [7] for the Hassan and Rosen 4D bimetric model, is required for a full consistency proof.

In a more general setting we can add cosmological terms to (50) like $\Lambda_g\sqrt{-g} + \Lambda_\gamma\sqrt{-\gamma}$ and look for AdS solutions. It is possible to show that there is always a region in the parameters space of the model for which AdS solutions with proportional metrics $g_{\mu\nu} \propto \gamma_{\mu\nu}$ do exist.

B. Searching for a single metric mBR model

Now we come back to the linearized $D = 3$ mBR model in (49) and consider the $\tilde{B}_{\mu\nu}$ field as an extra field appearing linearly in the action instead of a second metric fluctuation in order to prepare the ground for $D > 3$. The only metric now is $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. We arrive at the natural nonlinear generalization:

$$S_{\text{NLBR}}^{D=3} = M_P \int d^3x\sqrt{-g}\Big\{ R(g) - \tilde{B}^{\mu\nu}G_{\mu\nu}(g)$$
$$+ \tilde{B}^{\mu\nu}\mathbb{G}_{\mu\nu}(\hat{f}) - \frac{m^2}{4}(\hat{f}_{\mu\nu}^2 - \hat{f}^2)\Big\} \qquad (72)$$

where

$$\mathbb{G}_{\mu\nu}(\hat{f}) \equiv \frac{1}{2}\Big(-\nabla^2\hat{f}_{\mu\nu} + \frac{1}{2}\nabla_\mu\nabla^\alpha\hat{f}_{\alpha\nu} + \frac{1}{2}\nabla^\alpha\nabla_\mu\hat{f}_{\alpha\nu}$$
$$+ \frac{1}{2}\nabla_\nu\nabla^\alpha\hat{f}_{\alpha\mu} + \frac{1}{2}\nabla^\alpha\nabla_\nu\hat{f}_{\alpha\mu}$$
$$- \nabla_\mu\nabla_\nu\hat{f} + g_{\mu\nu}\nabla^2\hat{f} - g_{\mu\nu}\nabla_\alpha\nabla_\beta\hat{f}^{\alpha\beta}\Big). \qquad (73)$$

The symbol ∇_μ stands for the usual covariant derivative with respect to $g_{\mu\nu}$ and we use the notation $\nabla^2 = \nabla_\mu\nabla^\mu$. The Einstein-like tensor $\mathbb{G}_{\mu\nu}$ satisfies:

$$\tilde{B}^{\mu\nu}\mathbb{G}_{\mu\nu}(\hat{f}) = \hat{f}^{\mu\nu}\mathbb{G}_{\mu\nu}(\tilde{B}) + \text{total derivative}. \qquad (74)$$

The equations of motion $\delta S_{\text{NLBR}}^{D=3} = 0$ are satisfied by the flat space solution: $g_{\mu\nu} = \eta_{\mu\nu}; \hat{f}_{\mu\nu} = \tilde{B}_{\mu\nu} = 0$. Expanding about it, using

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{\sqrt{M_P}}, \quad \tilde{f}_{\mu\nu} = \frac{f_{\mu\nu}}{\sqrt{M_P}}, \quad \tilde{B}_{\mu\nu} = \frac{b_{\mu\nu}}{\sqrt{M_P}} \qquad (75)$$

we recover the linearized $D = 3$ mBR model (49) at $M_P \to \infty$. Now if we go beyond the linearized approximation and try to examine the decoupling limit as we have done in the bimetric model of the previous section there will be an important difference. First, notice the expansion

$$\mathbb{G}_{\mu\nu}(\tilde{B}) = \mathbb{G}_{\mu\nu}^{(L)}(b) + \mathbb{G}_{\mu\nu}^{(2)}(b,h) + \mathbb{G}_{\mu\nu}^{(3)}(b,h^2) + \dots \qquad (76)$$

where $\mathbb{G}_{\mu\nu}^{(n)}(b)$ is of order n in the fields. The tensor $\mathbb{G}_{\mu\nu}^{(L)}(b)$ coincides with the linearized Einstein tensor for the field $b_{\mu\nu}$, i.e., $\mathbb{G}_{\mu\nu}^{(L)}(b) = G_{\mu\nu}^{(L)}(b)$. Thus, $\partial^\mu\mathbb{G}_{\mu\nu}^{(L)}(b) = 0$, but $\partial^\mu\mathbb{G}_{\mu\nu}^{(2)}(b,h) \neq 0$. If we substitute the Stueckelberg covariant decomposition (64) in (72), after taking $m \to 0$ and $M_P \to \infty$ while keeping $\Lambda_{5/2} = (\sqrt{M_P}m^2)^{2/5}$ fixed, we obtain the following term at cubic order in the fields

$$\frac{\partial^\mu\partial^\nu\varphi\mathbb{G}_{\mu\nu}^{(2)}(b,h)}{\Lambda_{5/2}^{5/2}} \qquad (77)$$

which comes from $\sqrt{-g}\hat{f}^{\mu\nu}\mathbb{G}_{\mu\nu}(\tilde{B})$. The term (77) has more than two time derivatives and apparently introduces new degrees of freedom in the theory which will probably cause instabilities. The root of the problem is the noninvariance of the integral of $\sqrt{-g}\hat{f}^{\mu\nu}\mathbb{G}_{\mu\nu}(\tilde{B})$ under $\delta\hat{f}^{\mu\nu} = \nabla^\mu\chi^\nu + \nabla^\nu\chi^\mu$ due to $\nabla^\mu\mathbb{G}_{\mu\nu}(\tilde{B}) \neq 0$. Basically, the same problem appears in the general D-dimensional case for which we turn now.

A natural nonlinear completion of (10) is given by

$$S_{\text{BR}}^D = \frac{1}{\kappa^2}\int d^Dx\sqrt{-g}\Big\{ R(g) - \frac{1}{2}\hat{B}^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}(g)$$
$$+ \frac{1}{2}\hat{B}^{\mu\nu\alpha\beta}\mathbb{R}_{\mu\nu\alpha\beta}(\hat{f}) - \frac{m^2}{4}(\hat{f}_{\mu\nu}\hat{f}^{\mu\nu} - \hat{f}^2)\Big\} \qquad (78)$$

where

$$\mathbb{R}_{\mu\nu\alpha\beta}(\hat{f}) = \frac{1}{4}\Big(\nabla_\nu\nabla_\alpha\hat{f}_{\mu\beta} - \nabla_\nu\nabla_\beta\hat{f}_{\mu\alpha} - \nabla_\mu\nabla_\alpha\hat{f}_{\nu\beta} + \nabla_\mu\nabla_\beta\hat{f}_{\nu\alpha}\Big)$$
$$+ \frac{1}{4}\Big(\nabla_\beta\nabla_\mu\hat{f}_{\alpha\nu} - \nabla_\beta\nabla_\nu\hat{f}_{\mu\alpha}$$
$$- \nabla_\alpha\nabla_\mu\hat{f}_{\nu\beta} + \nabla_\alpha\nabla_\nu\hat{f}_{\mu\beta}\Big). \qquad (79)$$

Notice that

$$\sqrt{-g}\hat{B}^{\mu\nu\alpha\beta}\mathbb{R}_{\mu\nu\alpha\beta}(\hat{f}) = -2\sqrt{-g}\hat{f}^{\mu\nu}\mathbb{S}_{\mu\nu}(\hat{B}) + \text{total derivative} \qquad (80)$$

where we have introduced the symmetric field:

$$\mathbb{S}_{\mu\nu}(\hat{B}) \equiv \frac{1}{2}\Big(\nabla^\alpha\nabla^\beta\hat{B}_{\alpha\mu\beta\nu} + \nabla^\alpha\nabla^\beta\hat{B}_{\alpha\nu\beta\mu}\Big). \qquad (81)$$

In $D = 3$ the role of the symmetric tensor $\mathbb{S}_{\mu\nu}(\hat{B})$ is played by $\mathbb{G}_{\mu\nu}(\tilde{B})$. As in the $D = 3$ case, the symmetric tensor is not conserved in general $\nabla^\mu\mathbb{S}_{\mu\nu} \neq 0$. Consequently, new degrees of freedom show up which may destroy stability. In particular, if we apply ∇^μ on the equation of motion of $\hat{f}^{\mu\nu}$ coming from (78): $m^2(\hat{f}_{\mu\nu} - g_{\mu\nu}\hat{f}) = -2\mathbb{S}_{\mu\nu}(\hat{B})$, because of the higher time derivatives on the right side we lose the curved

space version of the flat space vector constraint $\partial^\mu f_{\mu\nu} - \partial_\nu f = 0$ which is essential for a correct counting of degrees of freedom. The same problem can be seen from a different point of view, if we Gaussian integrate over $\hat{f}^{\mu\nu}$ in (78) we obtain a curved space version of the DTS model (12), equivalent to the Maxwell one on the flat space [14], namely,

$$\mathcal{L}_{\text{DTS}}(\hat{B}) = \frac{\sqrt{-g}}{m^2} \left(\mathbb{S}_{\mu\nu}(\hat{B}) \mathbb{S}^{\mu\nu}(\hat{B}) - \frac{\mathbb{S}^2(\hat{B})}{D-1} \right) \quad (82)$$

with $\mathbb{S} = g^{\mu\nu}\mathbb{S}_{\mu\nu}$. The reader can check that part of the curved space version of the local symmetries (13) are lost. Indeed, on the flat space the restrictions (14) on the tensor gauge parameter correspond to a symmetric transverse rank-2 tensor but after replacing $\partial^\mu \to \nabla^\mu$ and symmetrizing, the restriction is just a symmetric tensor, so we have extra D restrictions on the curved space which means D less symmetries. Moreover, the curved space version of the scalar symmetry: $\delta_\pi \hat{B}_{\mu\nu\alpha\beta} = (g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}g_{\nu\beta})\pi$ is also broken since $\delta_\pi[\mathbb{S}_{\mu\nu}\mathbb{S}^{\mu\nu} - \mathbb{S}^2/(D-1)] = 2\mathbb{S}^{\mu\nu}\nabla_\mu\nabla_\nu\pi$ and the integration by parts will not vanish. So the viability of the model (78) is related with the consistency of the DTS model on curved spaces. In [14] a preliminary study of the curved space extension of gauge symmetries has been carried out without definite conclusion. Inspired by the procedure of [25] in the definition of a nonlocal gravitational model, we recall [26,27] that any symmetric tensor can be decomposed as $S_{\mu\nu} = S_{\mu\nu}^T + \nabla_\mu S_\nu + \nabla_\nu S_\mu$ where $\nabla^\mu S_{\mu\nu}^T = 0$. So we might replace $\mathbb{S}_{\mu\nu}(B) \to \mathbb{S}_{\mu\nu}^T(B) = \mathbb{S}_{\mu\nu}(B) - \nabla_\mu S_\nu - \nabla_\nu S_\mu$ in (80) where the vector field S_μ must satisfy the vector condition $\nabla^\alpha\mathbb{S}_{\alpha\mu} - \nabla^2 S_\mu - \nabla^\alpha\nabla_\mu S_\alpha = 0$ which can be implemented by a vector Lagrange multiplier. This is currently under investigation.

IV. CONCLUSIONS

The addition of mass terms usually breaks the gauge symmetry of massless theories which can be recovered by means of Stueckelberg fields. One exception to this rule is the 4D Cremmer-Scherck model [11] which describes massive spin $s = 1$ particles while preserving the $U(1)$ symmetry, which can be generalized to the non-Abelian case [12], without Stueckelberg fields. In Sec. II we have written the D-dimensional version of [11], the massive BF model (mBF) model, in terms of some Lorentz non-covariant gauge invariants. Likewise, we have written the D-dimensional generalization of the massive spin-2 model of [13], which we call the mBR model, in terms of the corresponding gauge invariants and argued that the mBR model is very much, though not exactly, a spin-2 analog of the mBF model. In both models a spin-s massless theory is coupled to a spin-(s-1) higher rank massless model by means of a gauge invariant mass term involving a spin-s curvature. We believe that there should be higher spin ($s > 2$) version of those massive models. In the $s = 1$

case the mass term (BF term) couples the massless theories without breaking any gauge symmetry while in the $s = 2$ case the BR term breaks the corresponding $U(1)$ symmetry of the higher rank spin-1 massless theory.

The mBR model is a consistent description of massive free spin-2 particles. The $D = 3$ case is rather special because the rank-4 Riemann-like tensor $B_{\mu\nu\alpha\beta}$ is equivalent to a symmetric rank-2 tensor via (34) which may be interpreted as a second metric fluctuation about flat space which has inspired us to suggest a new class of bimetric models, see (50). Following [23] we have gone beyond the linearized truncation and checked that the model is ghost free at leading order in the decoupling limit if the conditions (62) are satisfied. We can also add cosmological terms with independent cosmological constants for both metrics which leads to a rather general room to investigate the discrepancy between bulk and boundary unitarity of $D = 3$ gravity in the context of AdS$_3$/CFT$_2$ duality [24] in a purely metric formalism, differently from [28–30]. Now we are investigating a certain region in the parameters space of the model where we have found AdS solutions with proportional metrics $g_{\mu\nu} \propto \gamma_{\mu\nu}$ (in progress). In a future work we wish to study the stability of those solutions.

If, on the other hand, we stick to a single metric interpretation of a possible nonlinear completion of the $D = 3$ mBR model, see (72), it turns out that the decoupling limit reveals the appearance of extra ghostlike degrees of freedom, see (77). Basically the same problem goes thorough the $D \geq 4$ cases. We have shown that the consistency of the nonlinear model (78) is tightly connected with the consistency of the higher rank massless spin-1 model of [14] on curved backgrounds. The key point is the nonconservation of the symmetric tensor (81) on arbitrary backgrounds. We are currently investigating the replacement of $\mathbb{S}_{\mu\nu}(B)$ by a transverse version $\mathbb{S}_{\mu\nu}^T(B)$ as explained at the end of last section along the lines of [25].

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APPENDIX

1. Deser-Townsend-Siegel (DTS) model in D dimensions

The D-dimensional generalization of the $D = 4$ DTS model [14] has been suggested in [16],

$$\mathcal{L}_{\text{DTS}} = (\partial_\mu\partial_\nu B^{\mu\alpha\nu\beta})^2 - \frac{1}{(D-1)}(\partial_\alpha\partial_\beta B^{\alpha\beta})^2 \quad (A1)$$

where $B^{\alpha\beta} = \eta_{\mu\nu}B^{\mu\alpha\nu\beta}$.

The DTS model is invariant under the transformations (13) with the restrictions (14). The number of gauge invariants built up from the field $B_{\mu\alpha\nu\beta}$ and its derivatives

via (13) is the number of independent components of $B_{\mu\alpha\nu\beta}$ minus the number of independent gauge parameters $(\Lambda_{\mu\alpha\nu\beta}, \pi)$. Since $\Lambda_{\mu\alpha\nu\beta}$ has the same index properties of $B_{\mu\alpha\nu\beta}$, the number of gauge invariants (N_I) is the number of restrictions (14) minus 1 (due to π). The restrictions (14) correspond to a symmetric transverse tensor, thus we have $N_I = D(D-1)/2 - 1 = (D+1)(D-2)/2$. In order to derive those invariants we start with the general

decomposition (29) and decompose the gauge parameter in a similar way. After requiring (14) we obtain

$$\Lambda_{[0i][0j]} = \Phi_{ij}^T + \partial_i \lambda_j^T + \partial_j \lambda_i^T \tag{A2}$$

$$\Lambda_{[0i][jk]} = \Psi_{i[jk]}^{pT} + \partial_j \Theta_{ik}^{pT} - \partial_k \Theta_{ij}^{pT} \tag{A3}$$

$$\Lambda_{[ij][kl]} = \Lambda_{[ij][kl]}^T + \left\{ \partial_i \Omega_{j[kl]}^T + \partial_k \Omega_{l[ij]}^T - \frac{\partial_i}{\nabla^4} \left[\partial_k \ddot{\Phi}_{jl}^T - \partial_l \ddot{\Phi}_{jk}^T \right] + \frac{\partial_i}{\nabla^2} \left[\partial_k \left(\dot{\Theta}_{jl}^{pT} + \dot{\Theta}_{lj}^{pT} \right) - \partial_l \left(\dot{\Theta}_{jk}^{pT} + \dot{\Theta}_{kj}^{pT} \right) \right] \right\} + (i \leftrightarrow j, k \leftrightarrow l) \tag{A4}$$

with the constraint

$$\dot{\lambda}_i^T = \partial_j \Theta_{ji}^{pT}. \tag{A5}$$

From (13) we have $\delta B_{[0i][0j]} = \Lambda_{[0i][0j]} - \delta_{ij}\pi$ which leads to

$$\pi = -\delta[2\nabla^2 \rho] \tag{A6}$$

$$\lambda_i^T = \delta b_i^T \tag{A7}$$

$$\Phi_{ij}^T = \delta[B_{ij}^T - 2(\partial_i \partial_j - \delta_{ij}\nabla^2)\rho] \tag{A8}$$

while from $\delta B_{[0i][jk]} = \Lambda_{[0i][jk]}$ we have

$$\Theta_{ij}^{pT} = \delta C_{ij}^{pT} \tag{A9}$$

$$\Psi_{i[jk]}^{pT} = \delta B_{i[jk]}^{pT}. \tag{A10}$$

Applying ∂_i in (A9) and using the constraint (A5) we find: $\dot{\lambda}_j^T = \delta[\partial_i C_{ij}^{pT}]$. Combining this result with (A7), we obtain $\delta V_j^T = 0$ where

$$V_j^T = \dot{b}_k^T - \partial_j C_{jk}^{pT}. \tag{A11}$$

On the other hand, applying $\partial_j \partial_l$ on $\delta B_{[ij][kl]} = \Lambda_{[ij][kl]} + (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})\pi$ we have

$$\ddot{\Phi}_{ik}^T + \theta_{ij}\nabla^2 \dot{\Theta}_{jk}^{pT} + \theta_{kj}\dot{\Theta}_{ji}^{pT} + \theta_{ik}\nabla^2)\pi = -\delta[\nabla^4 W_{ik}^T]. \tag{A12}$$

Substituting (A6), (A8), and (A10) in (A12) we obtain $\delta \mathbb{W}_{ij}^T = 0$ where

$$\mathbb{W}_{ij}^T = \nabla^4 W_{ij}^T + \ddot{B}_{ij}^T + \theta_{ik}\nabla^2 \dot{C}_{kj}^{pT} + \theta_{jk}\nabla^2 \dot{C}_{ki}^{pT} - 2\theta_{ij}\nabla^2 \Box\rho \tag{A13}$$.

The invariant (A11) has $D - 2$ components while in (A13) we have $(D-1)(D-2)/2$, they add up to a total of $(D+1)(D-2)/2$ gauge invariants as expected. After introducing the traceless and transverse tensor invariant $\mathbb{W}_{ij}^{TT} = \mathbb{W}_{ij}^T - \theta_{ij}\mathbb{W}/(D-2)$ where $\mathbb{W} = \delta^{ij}\mathbb{W}_{ij}^T$, the DTS Lagrangian density in D dimensions can be written in a canonically simple way:

$$\mathcal{L}_{\text{DTS}} = \frac{1}{4}(\mathbb{W}_{ij}^{TT})^2 + \frac{\mathbb{W}^2}{4(D-1)(D-2)} + \frac{1}{2}V_j^T(-\nabla^2)\Box V_j^T \tag{A14}.$$

So we have $D - 2$ propagating massless degrees of freedom (V_j^T) as expected for a massless spin-1 particle and $(D-1)(D-2)/2$ nonpropagating modes represented by $\mathbb{W}_{ij}^T$.

[1] H. van Dam and M. J. G. Veltman, Massive and massless Yang-Mills and gravitational fields, Nucl. Phys. **B22**, 397 (1970).

[2] V. I. Zakharov, Linearized gravitation theory and the graviton mass, JETP Lett. **12**, 312 (1970).

[3] D. G. Boulware and S. Deser, Inconsistency of finite range gravitation, Phys. Lett. **40B**, 227 (1972).

[4] C. de Rham, G. Gabadadze, and A. J. Tolley, Resummation of massive gravity, Phys. Rev. Lett. **106**, 231101 (2011).

[5] K. Hinterbichler, Theoretical aspects of massive gravity, Rev. Mod. Phys. **84**, 671 (2012).

[6] C. de Rham, Massive gravity, Living Rev. Relativity **17**, 7 (2014).

[7] S. Hassan and R. A. Rosen, Bimetric gravity from ghost-free massive gravity, J. High Energy Phys. 02 (2012) 126.

[8] M. Hogas and E. Mortsell, Constraints on bimetric gravity. Part II. Observational constraints, J. Cosmol. Astropart. Phys. 05 (2021) 002.

[9] M. Hogas and E. Mortsell, Constraints on bimetric gravity. Part I. Analytical constraints, J. Cosmol. Astropart. Phys. 05 (2021) 001.

[10] M. Fierz, Force-free particles with any spin, Helv. Phys. Acta **12**, 3 (1939); M. Fierz and W. Pauli, On relativistic wave equations for particles of arbitrary spin in an electromagnetic field, Proc. R. Soc. A **173**, 211 (1939).

[11] E. Cremmer and J. Scherk, Spontaneous dynamical breaking of gauge symmetry in dual models, Nucl. Phys. **B72**, 117 (1974).

[12] D. Z. Freedman and P. K. Townsend, Antisymmetric tensor gauge theories and non-linear sigma-models, Nucl. Phys. **B177**, 282 (1981).

[13] A. Khoudeir and D. Sierra, Duality invariance in massive theories, Phys. Rev. D **91**, 064015 (2015).

[14] S. Deser, P. K. Townsend, and W. Siegel, Higher rank representations of lower spin, Nucl. Phys. **B184**, 333 (1981).

[15] S. Deser, R. Jackiw, and S. Templeton, Topologically massive gauge theories, Ann. Phys. (N.Y.) **140**, 372 (1982).

[16] D. Dalmazi and R. C. Santos, Spin-1 duality in D-dimensions, Phys. Rev. D **84**, 045027 (2011).

[17] A. Lahiri, Constrained dynamics of the coupled Abelian two-form, Mod. Phys. Lett. A **08**, 2403 (1993).

[18] T. Kim, S. Kouwn, and P. Oh, Hamiltonian formalism of topologically massive electrodynamics, Mod. Phys. Lett. A **34**, 1950067 (2019).

[19] A. L. R dos Santos, D. Dalmazi, and W. de Paula, Hamiltonian analysis and positivity of a new massive spin-2 model, Classical Quantum Gravity **39**, 035013 (2022).

[20] D. Dalmazi and A. L. R. dos Santos, Higher spin analogs of linearized topologically massive gravity and linearized new massive gravity, Phys. Rev. D **104**, 085023 (2021).

[21] M. Jaccard, M. Maggiore, and E. Mitsou, Bardeen variables and hidden gauge symmetries in linearized massive gravity, Phys. Rev. D **87**, 044017 (2013).

[22] E. Bergshoeff, O. Hohm, and P. K. Townsend, Massive gravity in three dimensions, Phys. Rev. Lett. **102**, 201301 (2009).

[23] C. de Rham, G. Gabadadze, D. Pirtskhalava, A. Tolley, and I. Yavin, Nonlinear dynamics of 3D massive gravity, J. High Energy Phys. 06 (2011) 028.

[24] E. A. Bergshoeff, O. Hohm, and P. K. Townsend, More on massive 3D gravity, Phys. Rev. D **79**, 124042 (2009).

[25] M. Maggiore, Phantom dark energy from non-local infrared modifications of general relativity, Phys. Rev. D **89**, 043008 (2014).

[26] S. Deser, Covariant decomposition and the gravitational Cauchy problem, Ann. Inst. H. Poincare Phys. Theor. **7**, 149 (1967).

[27] J. J. York, Covariant decompositions of symmetric tensors in the theory of gravitation, Ann. Inst. H. Poincare Phys. Theor. **21**, 319 (1974).

[28] E. Bergshoeff, O. Hohm, W. Merbis, A. J. Routh, and P. K. Townsend, Minimal massive 3D gravity, Classical Quantum Gravity **31**, 145008 (2014).

[29] M. R. Setare, On the generalized minimal massive gravity, Nucl. Phys. **B898**, 259 (2015).

[30] M. Ozkan, Y. Pang, and P. K. Townsend, Exotic massive 3D gravity, J. High Energy Phys. 08 (2018) 035.