Dynamical mass generation of spin-2 fields in de Sitter space for an $O(N)$ symmetric model at large N

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We consider the strong-coupling phase in a model of $O(N)$ spin-2 field theory in de Sitter spacetime and the effective mass of spin-2 fields therein. In the strong-coupling phase, the Higuchi bound limits the mass parameter in the theory. The analysis using the large N approximation finds the critical value of the mass parameter with numerical calculation.

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I. INTRODUCTION

The massive spin-2 field theory has been studied for quite some time [1]. However, in recent years, the theory of massive graviton [2,3] as a spin-2 particle has attracted attention as a kind of modified gravitation theories. As a key to solving the cosmological constant problem, it is also considered significant to investigate the relationship between the mass of the graviton and a small cosmological constant. Besides, bigravity theories [4] and multigravity theories are also considered, and in special cases, it is also speculated that spin-2 particles (not yet discovered) other than the graviton partly plays the role of dark matter [5].

On the other hand, the existence of the Higuchi bound [6,7] is known for the mass of a spin-2 particle in maximally symmetric spacetime. It has been shown that a negative norm state appears in the spin-2 field theory below the critical mass-squared, $\frac{D-2}{D(D-1)}R$ (where R is the scalar curvature of the de Sitter space), and the theory becomes unstable (except for the massless case). This is a distinctive feature of the spin-2 field theory that is not seen in other spin fields. Therefore, in some sense, it is speculated that the study of spin-2 theory is a very important key point related to both the dark energy problem and the dark matter problem.

Now, for example, the masses of the (spin 1/2, 1) particles in the standard model is obtained by the Higgs mechanism. On the other hand, how is the mass of spin-2 particles determined? In this paper, as a very bold assumption, we consider a model in which the dynamical mass is determined from the interaction of N spin-2 fields $h^a_{\mu\nu}$ $(a = 1, \ldots, N)$. Consider N spin-2 fields $h^a_{\mu\nu}$ with $O(N)$ invariant interaction:

$$\mathcal{L}_{\rm int} = -\frac{\lambda}{8N}(h^{a\,\mu\nu}h^a_{\mu\nu} - h^{a\mu}_{\mu}h^{a\nu}_{\nu})^2, \tag{1.1}$$

where λ is the coupling constant. Then, the vacuum expectation value

$$\langle\sigma\rangle \equiv \left\langle m^2 + \frac{\lambda}{2N}(h^{a\,\mu\nu}h^a_{\mu\nu} - h^{a\mu}_{\mu}h^{a\nu}_{\nu})\right\rangle, \tag{1.2}$$

(where m is the mass parameter) is the mass squared of the spin-2 fields in the strong-coupling vacuum. For the spin-zero field model, the large N approximation as a self-consistent approach has been devised since half a century ago [8–11], which makes it possible to study the strong-coupling phase relatively easily, and we also uses this method in this paper.[1] Since the mass in the Lagrangian and the effective mass in the strong-coupling vacuum are generally different, we are most interested in how the parameter restriction by the Higuchi bound in de Sitter spacetime works.

Various no-go theories have been found for massless spin-2 field interactions [16]. However, earlier discussions have already reported that the theory of spin-2 with perfectly sound interactions requires an infinite number

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[1]The large N scalar field theory in de Sitter space has been studied in Refs. [12–14], and recently, that in three dimensional AdS space has been investigated in Ref. [15].

of massive fields anyway [17,18]. The relationship between the string theory, which involves infinite particle states, and the massive spin-2 theory has been investigated [19]. Therefore, although the model treated in the present paper does not have a number of theoretic properties such as UV completeness, we have begun our study with the expectation that the theory will have better properties in the emerging vacuum in nonperturbative regime.

In our model, defined in the next section, we proceed with the study assuming that the N spin-2 fields are independent of the graviton. However, the mixing with the graviton is conceivable as a field of the same spin-2, so in the future, we will manage to construct an effective model related to cosmology and elementary particle theory, after taking the mixing properly. Of course, the existence of large-N spin-2 fields with a degenerate mass is problematic in the literal sense. We expect that even in this simple model we can see some features of the model with a finite number of fields. We would like to leave such research as a subject for future papers.

The structure of the present paper is as follows. In Sec. II, we introduce the spin-2 model considered in this paper. In Sec. III, we consider the case of four dimensional de Sitter spacetime. The critical value of the renormalized Lagrangian mass, avoiding the Higuchi bound, is examined. Finally, we discuss the conclusion and future prospects in the last section. We have Appendix for the case of three dimensional de Sitter spacetime.

II. $O(N)$ SYMMETRIC SPIN-2 MODEL IN DE SITTER SPACETIME

Let us consider the following model. The action of N massive symmetric tensor field $h^a_{\mu\nu}$ on a D-dimensional maximally symmetric spacetime with the constant scalar curvature $R \equiv g^{\mu\nu}R_{\mu\nu}$, where the Ricci tensor $R_{\mu\nu}$ equals to $\frac{R}{D}g_{\mu\nu}$, is written by [20–24]

$$S_{\mathrm{FP}} = \int d^Dx\sqrt{-g}\left[-\frac{1}{2}\nabla_\mu h^a_{\nu\rho}\nabla^\mu h^{a\nu\rho} + \nabla_\nu h^a_{\mu\rho}\nabla^\mu h^{a\nu\rho} - \nabla_\mu h^a \nabla_\rho h^{a\mu\rho} + \frac{1}{2}\nabla_\mu h^a \nabla^\mu h^a + \frac{R}{D}\left(h^{a\,\mu\nu}h^a_{\mu\nu} - \frac{1}{2}h^a h^a\right) - \frac{1}{2}m_0^2(h^{a\,\mu\nu}h^a_{\mu\nu} - h^a h^a)\right], \tag{2.1}$$

where $h \equiv h^\rho_\rho = g^{\mu\nu}h_{\mu\nu}$, ∇_μ represents the covariant derivative in terms of the metric $g_{\mu\nu}$, and m_0 is the mass parameter (the bare Lagrangian mass). In addition, we assume the following $O(N)$ invariant interaction:

$$S_{\mathrm{int}} = \int d^Dx\sqrt{-g}\left[-\frac{\lambda_0}{8N}(h^{a\,\mu\nu}h^a_{\mu\nu} - h^a h^a)^2\right], \tag{2.2}$$

where λ_0 is the (bare) quartic self-interaction coupling constant. In this paper, we will not consider the possibility of $O(N)$ symmetry breaking, although it might be interesting.

To introduce the auxiliary field σ, we add the following action, which is nondynamical itself, for σ:

$$S_{\mathrm{aux}} = \int d^Dx\sqrt{-g}\left\{\frac{N}{2\lambda_0}\left[\sigma - m_0^2 - \frac{\lambda_0}{2N}(h^{a\mu\nu}h^a_{\mu\nu} - h^a h^a)\right]^2\right\}. \tag{2.3}$$

Note that the functional integral over σ is a trivial Gaussian integral.

It can be seen that the vacuum expectation value of σ becomes the square of the mass of the spin-2 fields. Performing a Gaussian integral over the spin-2 fields $h^a_{\mu\nu}$ (and the corresponding ghost fields with the gauge fixing) can then be performed in the partition function with the total action $S = S_{\mathrm{FP}} + S_{\mathrm{int}} + S_{\mathrm{aux}}$ and we obtain the one-loop effective action up to an additive constant:

$$S_{\mathrm{eff}} = \int d^Dx\sqrt{-g}\left[\frac{N}{2\lambda_0}\sigma^2 - \frac{Nm_0^2}{\lambda_0}\sigma - NL_0(\sigma)\right] \equiv \int d^Dx\sqrt{-g}[-NV(\sigma)], \tag{2.4}$$

where [25–27]

$$L_0(\sigma) \equiv \mathcal{V}_D^{-1}\left\{\frac{1}{2}\mathrm{Tr}\,\ln\left[\Delta(1,1) - 2\frac{R}{D} + \sigma\right] - \frac{1}{2}\mathrm{Tr}\,\ln\left[\Delta\left(\frac{1}{2},\frac{1}{2}\right) - 2\frac{R}{D} + \sigma\right]\right\}, \tag{2.5}$$

with $\mathcal{V}_D$ is the volume of the spacetime, $\int\sqrt{-g}d^Dx$. Here, the differential operators $\Delta(\frac{1}{2},\frac{1}{2})$ and $\Delta(1,1)$ are defined as

$$\Delta\left(\frac{1}{2},\frac{1}{2}\right)\xi_\mu \equiv -\Box\xi_\mu + R_{\mu\nu}\xi^\nu,$$
$$\Delta(1,1)\phi_{\mu\nu} \equiv -\Box\phi_{\mu\nu} + R_{\mu\tau}\phi^\tau_\nu + R_{\nu\tau}\phi^\tau_\mu - 2R_{\mu\rho\nu\tau}\phi^{\rho\tau}, \tag{2.6}$$

respectively, where $R_{\mu\rho\nu\tau}$ denotes the Riemann tensor.

The equation of motion $\frac{\delta S}{\delta\sigma} = 0$, which determines the extremum of the effective potential, $\frac{dV}{d\sigma} = 0$, leads to

$$\frac{1}{\lambda_0}\sigma - \frac{m_0^2}{\lambda_0} - \frac{dL_0(\sigma)}{d\sigma} = 0, \tag{2.7}$$

which gives the relation of the leading order in the expansion in $1/N$, thus it is independent of N. The expected value of σ can be obtained by solving this equation.

We will compute the one-loop contribution of the spin-2 fields by with the spectrum of the Laplacian on a D-sphere

S^D, as the Euclidean version of D-dimensional de Sitter spacetime. Hereafter, we set the constant curvature of the space as

$$R_{\mu\rho\nu\tau} = \frac{\Lambda}{D-1}(g_{\mu\nu}g_{\rho\tau} - g_{\mu\tau}g_{\rho\nu}), \quad R_{\mu\nu} = g^{\rho\tau}R_{\mu\rho\nu\tau} = \Lambda g_{\mu\nu}, \quad \text{and} \quad R = D\Lambda, \tag{2.8}$$

where Λ is a positive cosmological constant, while the volume is given by

$$\mathcal{V}_D = \frac{2\pi^{\frac{D+1}{2}}}{\Gamma(\frac{D+1}{2})}\left(\frac{\Lambda}{D-1}\right)^{-D/2}. \tag{2.9}$$

Then, the first derivative of L_0 is formally given by [25–32]

$$\frac{dL_0(\sigma)}{d\sigma} = \frac{\Gamma(\frac{D+1}{2})}{4\pi^{\frac{D+1}{2}}}\left(\frac{\Lambda}{D-1}\right)^{D/2} \times \left\{\sum_{\ell=0}^{\infty} d_2(\ell)\left[\frac{\Lambda}{D-1}(\ell+2)(\ell+D+1)+\sigma\right]^{-1} - \sum_{\ell=0}^{\infty} d_1(\ell)\left[\frac{\Lambda}{D-1}\ell(\ell+D+1)+\sigma\right]^{-1}\right\}, \tag{2.10}$$

where the degeneracies are

$$d_2(\ell) \equiv \frac{(D+1)(D-2)}{2}(\ell+1)(\ell+D+2)(2\ell+D+3) \times \frac{(\ell+D-1)!}{(D-1)!(\ell+3)!}, \tag{2.11}$$

and

$$d_1(\ell) \equiv (\ell+1)(\ell+D)(2\ell+D+1)\frac{(\ell+D-2)!}{(D-2)!(\ell+2)!}. \tag{2.12}$$

Note that $\frac{dL_0}{d\sigma}$ contains divergences, which must be addressed by parameter renormalization. We will show concrete methods and numerical results for four dimensional spacetime in the next section, and the results for three dimensional spacetime in the Appendix.

III. FOUR DIMENSIONS ($D = 4$)

A. Calculation of one-loop contribution

For four dimensions, the expression (2.10) becomes[2]

$$\frac{dL_0(\sigma)}{d\sigma} = \frac{\Lambda}{16\pi^2}\left\{\sum_{\ell=0}^{\infty} d_2(\ell)\left[(\ell+2)(\ell+5)+\frac{3\sigma}{\Lambda}\right]^{-1} - \sum_{\ell=0}^{\infty} d_1(\ell)\left[\ell(\ell+5)+\frac{3\sigma}{\Lambda}\right]^{-1}\right\}, \tag{3.1}$$

where

$$d_2(\ell) = \frac{5}{6}(\ell+1)(\ell+6)(2\ell+7) \quad \text{and}$$
$$d_1(\ell) = \frac{1}{2}(\ell+1)(\ell+4)(2\ell+5). \tag{3.2}$$

Using the integration formula

$$\frac{1}{z^2-\beta^2} = \int_0^{\infty} e^{-zt}\frac{\sinh\beta t}{\beta}dt, \tag{3.3}$$

$\frac{dL_0}{d\sigma}$ is expressed as

$$\frac{dL_0(\sigma)}{d\sigma} = \frac{\Lambda}{16\pi^2}\left\{\frac{5}{16}\int_{\epsilon}^{\infty}\frac{e^{-\frac{7}{2}t}(2-7e^t+7e^{2t})}{(\sinh\frac{t}{2})^4}\frac{\sinh\beta_2(\sigma)t}{\beta_2(\sigma)}dt - \frac{1}{16}\int_{\epsilon}^{\infty}\frac{e^{-\frac{5}{2}t}(1-5e^t+10e^{2t})}{(\sinh\frac{t}{2})^4}\frac{\sinh\beta_1(\sigma)t}{\beta_1(\sigma)}dt\right\}, \tag{3.4}$$

where

$$\beta_2(\sigma) \equiv \sqrt{\frac{9}{4}-\frac{3\sigma}{\Lambda}} \quad \text{and} \quad \beta_1(\sigma) \equiv \sqrt{\frac{25}{4}-\frac{3\sigma}{\Lambda}}. \tag{3.5}$$

Here, we introduced a UV regulator ϵ in order to avoid divergences at the lower limit in the integrals.

Now, noting that

$$\frac{\sinh\beta t}{\beta} = t + \frac{\beta^2}{6}t^3 + O(t^5), \tag{3.6}$$

we renormalize the divergent integral by defining the renormalized parameters as follows[3]:

$$\frac{m^2}{\lambda} = \frac{m_0^2}{\lambda_0} + \frac{\Lambda}{16\pi^2}\left\{\frac{5}{16}\int_{\epsilon}^{\infty}\frac{e^{-\frac{7}{2}t}(2-7e^t+7e^{2t})}{(\sinh\frac{t}{2})^4}\left(t+\frac{3}{8}t^3\right)dt - \frac{1}{16}\int_{\epsilon}^{\infty}\frac{e^{-\frac{5}{2}t}(1-5e^t+10e^{2t})}{(\sinh\frac{t}{2})^4}\left(t+\frac{25}{24}t^3\right)dt\right\}, \tag{3.7}$$

and

[2] In the limit of $\Lambda \to 0$, we find that $\frac{dL_0(\sigma)}{d\sigma} = \frac{1}{8\pi^2}\int\frac{k^3dk}{k^2+\sigma} = \frac{g}{2}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2+\sigma}$ with $g = 2$. According to [25], the absence of the discontinuity between the massive case and massless case at the one-loop level has been noted (thus, $g = 2$ instead of $g = 5$).

[3] We adopt a kind of "minimal subtraction" of divergences in the integral in this paper.

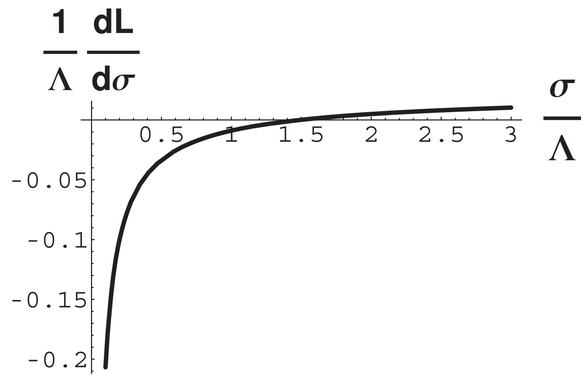

FIG. 1. $\frac{1}{\Lambda}\frac{dL}{d\sigma}$ for the four dimensional case as the function of $\frac{\sigma}{\Lambda}$.

$$\frac{1}{\lambda}=\frac{1}{\lambda_0}+\frac{1}{32\pi^2}\left\{\frac{5}{16}\int_{\epsilon}^{\infty}\frac{e^{-\frac{7}{2}t}(2-7e^t+7e^{2t})}{(\sinh\frac{t}{2})^4}t^3dt\right.$$
$$\left.-\frac{1}{16}\int_{\epsilon}^{\infty}\frac{e^{-\frac{5}{2}t}(1-5e^t+10e^{2t})}{(\sinh\frac{t}{2})^4}t^3dt\right\}. \tag{3.8}$$

Then, the equation of motion for σ (the gap equation) is rewritten as

$$\frac{1}{\lambda}\sigma-\frac{m^2}{\lambda}-\frac{dL(\sigma)}{d\sigma}=0, \tag{3.9}$$

where

$$\frac{dL(\sigma)}{d\sigma}\equiv\frac{dL_0(\sigma)}{d\sigma}-\left(\frac{m^2}{\lambda}-\frac{m_0^2}{\lambda_0}\right)+\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)\sigma. \tag{3.10}$$

After the renormalization, $\frac{dL(\sigma)}{d\sigma}$ is calculable by the integral by setting $\epsilon\to 0$.

We show $\frac{1}{\Lambda}\frac{dL}{d\sigma}$ as the function of $\frac{\sigma}{\Lambda}$ in Fig. 1, which is obtained by numerical integration in the case of four dimensions. We should notice that $\frac{dL}{d\sigma}$ is calculable without regard to the Higuchi bound for $\sigma>0$. It can be seen that L is convex downward as a function of σ, since $\frac{1}{\Lambda}\frac{dL}{d\sigma}$ is monotonously increasing as shown in Fig. 1. The minimum of L is located at $\sigma/\Lambda\approx 1.5$.

B. Negative coupling constant λ

For the $O(N)$ scalar theory, it is reported that there appears the $O(N)$ symmetric stable ground state when the renormalized coupling λ is negative [9–11]. Therefore, we first examine the case with $\lambda<0$ in our model.[4]

Since the equation of motion can be read as, for $\lambda<0$,

$$\frac{m^2}{\Lambda}=\frac{\sigma}{\Lambda}+|\lambda|\frac{1}{\Lambda}\frac{dL}{d\sigma}\qquad(\lambda<0), \tag{3.11}$$

[4]Most recently, a preprint [33] has appeared. The preprint includes a recent discussion on the negative coupling constant in scalar field theory and some important references.

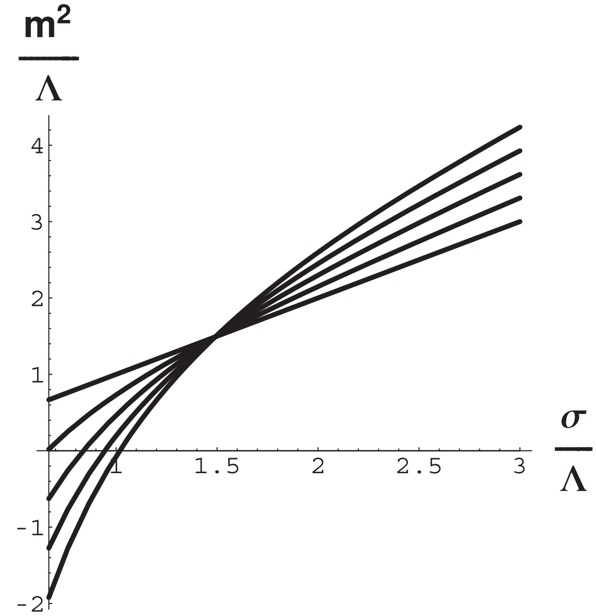

FIG. 2. The relation between $\frac{m^2}{\Lambda}$ and $\frac{\sigma}{\Lambda}$ for some specific values of the coupling $\lambda(<0)$. The curves correspond to $|\lambda|=0$, 30, 60, 90, 120, whose leftmost points line up from top to bottom.

we can exhibit the relation between $\frac{m^2}{\Lambda}$ and the vacuum expectation value of $\frac{\sigma}{\Lambda}$ for some specific values of the coupling λ in Fig. 2 (where we use the simple σ instead of $\langle\sigma\rangle$ for the vacuum expectation value). We show the region where $\sigma/\Lambda>2/3$, above the Higuchi bound [6,7]. The lower limit of m^2/Λ as a function of $|\lambda|$, which yields $\sigma/\Lambda>2/3$, is shown in Fig. 3.

C. Positive coupling constant λ

For $\lambda>0$, the equation of motion for σ reads

$$\frac{m^2}{\Lambda}=\frac{\sigma}{\Lambda}-\lambda\frac{1}{\Lambda}\frac{dL}{d\sigma}\qquad(\lambda>0). \tag{3.12}$$

We show the relation between $\frac{m^2}{\Lambda}$ and the vacuum expectation value of $\frac{\sigma}{\Lambda}$ for some specific values of the

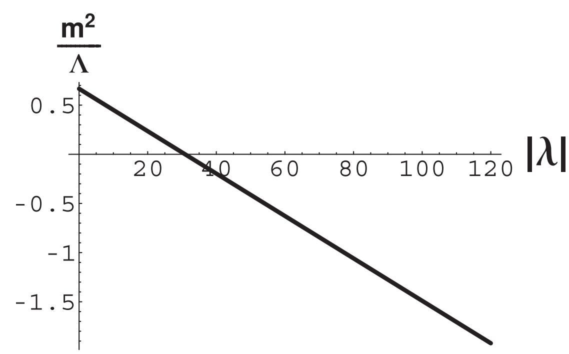

FIG. 3. The lower limit of m^2/Λ for $\sigma/\Lambda>2/3$, as a function of $|\lambda|$.

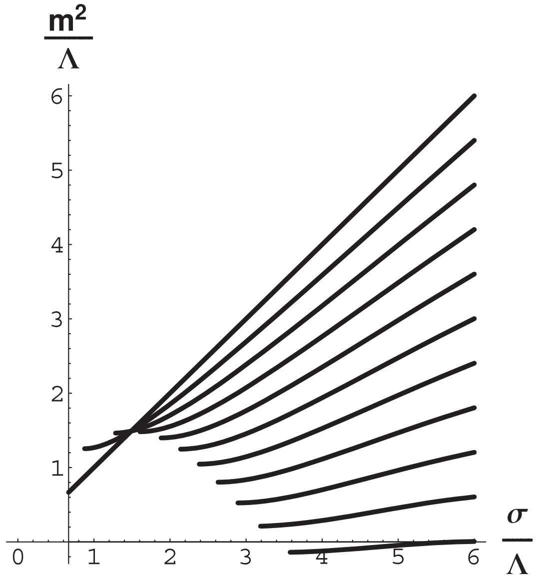

FIG. 4. The relation between $\frac{m^2}{\Lambda}$ and $\frac{\sigma}{\Lambda}$ for some specific values of the coupling $\lambda(>0)$. The curves correspond to $\lambda = 0, 30, 60, 90, 120, 150, 180, 210, 240$, 270, 300, whose left-most points line up from left to right.

coupling λ in Fig. 4. We find the region where $\sigma/\Lambda > 2/3$, above the Higuchi bound [6,7]. The increasing lines in the figure correspond to the extremum of the action, which is continuously connected to that in the limit of $\lambda \to 0$. The lower limit of m^2/Λ as a function of λ, which yields $\sigma/\Lambda > 2/3$, is shown in Fig. 5.

In the present analysis, we assume that $O(N)$ symmetry is unbroken. However, especially in the case of $m^2 < 0$, which realizes for $\lambda > \approx 289$, the unbroken $O(N)$ symmetry is problematic as in the case with the classical Higgs potential. More detailed analysis, such as Ref. [11] for scalar theory, is needed to clarify the symmetry breaking in our present model. However, since our model deals with fields with spacetime indices, similar analyses of Ref. [11] must be reconstructed, and further research is left as a future topic.

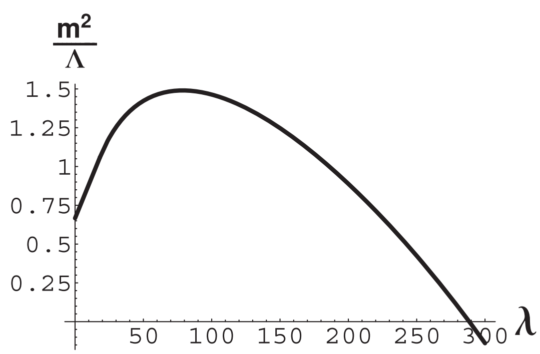

FIG. 5. The lower limit of m^2/Λ for $\sigma/\Lambda > 2/3$, as a function of λ.

IV. CONCLUSION AND OUTLOOK

In this paper, in the $O(N)$ spin-2 model, the effective mass in a strong-coupling vacuum is studied by the large N approximation, and the critical values of the mass parameter, which exceeds the Higuchi bound, are numerically estimated. If we take a large enough m^2, we can find a strong-coupling vacuum with an effective mass that exceeds the Higuchi bound.

Furthermore, for sufficiently large coupling constant, it is possible that the effective mass is created when the mass parameter is zero (if $|\lambda| > 31$ for $\lambda < 0$ and if $\lambda > 289$ for $\lambda > 0$, in the case with $D = 4$). There is also the well-known problem of discontinuity [2,3,34] at $m = 0$,[5] so this point may require additional study in the future, though the absence of the discontinuity at one-loop level is reported in Ref. [25].

The bound has been obtained in our toy model, but we hope it makes some sense in the choice of models in realistic models (including an infinite number of fields). Also, in such a case, the quantum effects of other matter fields should inevitably come into play. After incorporating them, we would like to consider the running of coupling constants and possible phase transitions. The stability of the vacuum is the most important topic that should be investigated further.

In this paper, we have also presented expressions in general D dimensions, but it may be considered that renormalization or evaluation of parameters in higher dimensions should be done more carefully than in four dimensions, so we would like to leave it as a future task. However, like the calculation of quantum effects in higher dimensional theory (e.g., [35]), it may be possible to perform numerical calculations in a similar model in odd-dimensional spacetime. We would like to consider such a issue in the future. We assumed that the $O(N)$ symmetry is not broken in this paper (for simplicity as in the initial stage of the study), but we would like to consider the case where $O(N)$ is broken in the future. We wish to examine the model when the background spacetime is also the Nariai spacetime [36], a higher-dimensional Kaluza–Klein spacetime, etc. Paying attention to whether the anisotropy can be avoided or favored, we want to study their consequences. In the future, we would like to research bold hypotheses such as the self-consistent de Sitter universe (e.g., Refs. [14,37,38]) in the strong-coupling phase.

ACKNOWLEDGMENTS

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[5]Another discontinuity is also found at $m^2 = \frac{D-2}{D-1}\Lambda$ [2,3,26].

APPENDIX: THREE DIMENSIONS ($D = 3$)

1. Calculation of one-loop contribution

As in the four dimensional case, we find the following. The formal expression of the one-loop calculation reads, for $D = 3$,

$$\frac{dL_0}{d\sigma} = \frac{\sqrt{\Lambda}}{4\sqrt{2}\pi^2}\left\{\sum_{\ell=0}^{\infty} d_2(\ell)\left[(\ell+2)(\ell+4)+\frac{2\sigma}{\Lambda}\right]^{-1} - \sum_{\ell=0}^{\infty} d_1(\ell)\left[\ell(\ell+4)+\frac{2\sigma}{\Lambda}\right]^{-1}\right\}, \tag{A1}$$

where

$$d_2(\ell) = 2(\ell+1)(\ell+5) \quad \text{and} \quad d_1(\ell) = 2(\ell+1)(\ell+3). \tag{A2}$$

As the treatment in the previous section, we can find its integration form:

$$\frac{dL_0(\sigma)}{d\sigma} = \frac{\sqrt{\Lambda}}{4\sqrt{2}\pi^2}\left\{\frac{1}{4}\int_\epsilon^\infty \frac{e^{-\frac{5}{2}t}(-3+5e^t)}{(\sinh\frac{t}{2})^3}\frac{\sinh\beta_2(\sigma)t}{\beta_2(\sigma)}dt - \frac{1}{4}\int_\epsilon^\infty \frac{e^{-\frac{3}{2}t}(-1+3e^t)}{(\sinh\frac{t}{2})^3}\frac{\sinh\beta_1(\sigma)t}{\beta_1(\sigma)}dt\right\}, \tag{A3}$$

where

$$\beta_2(\sigma) \equiv \sqrt{1-\frac{2\sigma}{\Lambda}} \quad \text{and} \quad \beta_1(\sigma) \equiv \sqrt{4-\frac{2\sigma}{\Lambda}}. \tag{A4}$$

The renormalization should be done as

$$\frac{m^2}{\lambda} = \frac{m_0^2}{\lambda_0} + \frac{\sqrt{\Lambda}}{16\sqrt{2}\pi^2}\left\{\int_\epsilon^\infty \frac{e^{-\frac{5}{2}t}(-3+5e^t)}{(\sinh\frac{t}{2})^3}t\,dt - \int_\epsilon^\infty \frac{e^{-\frac{3}{2}t}(-1+3e^t)}{(\sinh\frac{t}{2})^3}t\,dt\right\}, \tag{A5}$$

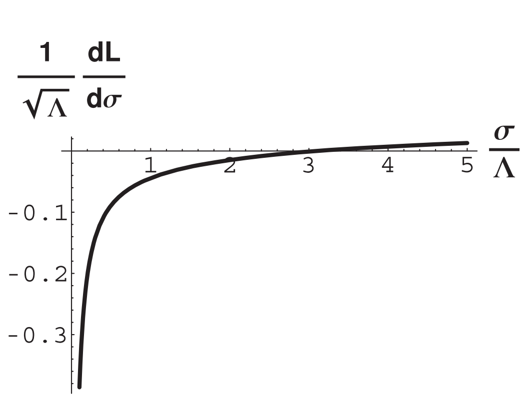

FIG. 6. $\frac{1}{\sqrt{\Lambda}}\frac{dL}{d\sigma}$ in the three dimensional case as the function of $\frac{\sigma}{\Lambda}$.

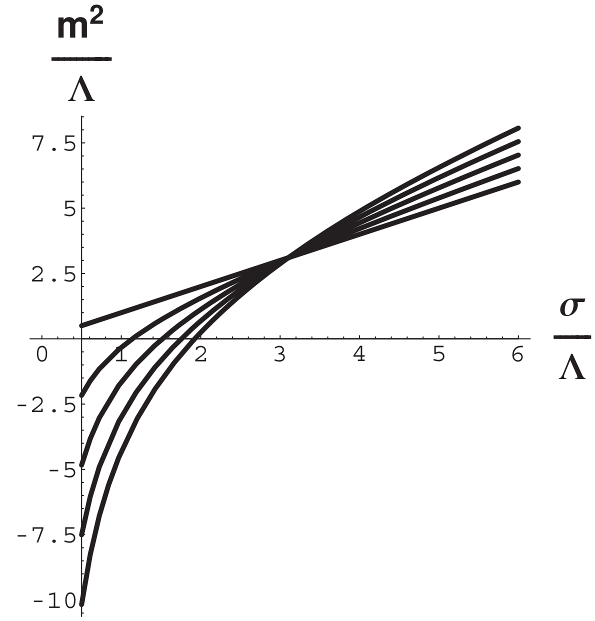

FIG. 7. The relation between $\frac{m^2}{\Lambda}$ and $\frac{\sigma}{\Lambda}$ for some specific values of the coupling $\lambda(<0)$. The lines correspond to $|\lambda|/\sqrt{\Lambda} = 0$, 30, 60, 90, 120, whose leftmost points line up from top to bottom.

and the coupling constant $\lambda_0 = \lambda$ does not undergo renormalization correction. Then, we set

$$\frac{dL(\sigma)}{d\sigma} \equiv \frac{dL_0(\sigma)}{d\sigma} - \left(\frac{m^2}{\lambda} - \frac{m_0^2}{\lambda_0}\right). \tag{A6}$$

Qualitatively, it is the same as in the case of four dimensions. Note, however, that the dimensions of the coupling constants are different. We show $\frac{1}{\sqrt{\Lambda}}\frac{dL}{d\sigma}$ as the function of $\frac{\sigma}{\Lambda}$ in Fig. 6 in the case of three dimensions. It can be seen that L is convex downward as a function of σ as in the four dimensional case. The minimum of L is located at $\sigma/\sqrt{\Lambda} \approx 3$.

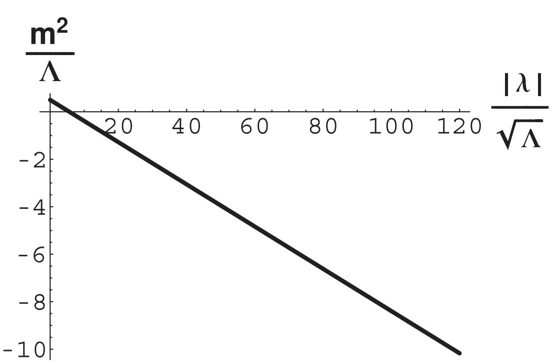

FIG. 8. The lower limit of m^2/Λ for $\sigma/\Lambda > 1/2$, as a function of $|\lambda|/\sqrt{\Lambda}$.

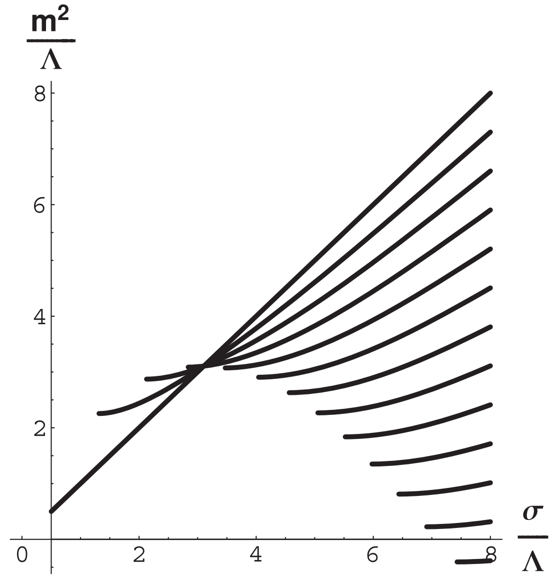

FIG. 9. the relation between $\frac{m^2}{\Lambda}$ and $\frac{\sigma}{\Lambda}$ for some specific values of the coupling $\lambda(>0)$. The lines correspond to $\lambda/\sqrt{\Lambda} = 0, 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330, 360$, whose leftmost points line up from left to right.

2. Negative coupling constant λ

The equation of motion can be read as, for $\lambda < 0$,

$$\frac{m^2}{\Lambda} = \frac{\sigma}{\Lambda} + \frac{|\lambda|}{\sqrt{\Lambda}} \frac{1}{\sqrt{\Lambda}} \frac{dL}{d\sigma}, \tag{A7}$$

where $\frac{|\lambda|}{\sqrt{\Lambda}}$ is dimensionless. We show the relation between $\frac{m^2}{\Lambda}$ and $\frac{\sigma}{\Lambda}$ for some specific values of $\frac{|\lambda|}{\sqrt{\Lambda}}$ in Fig. 7. We show

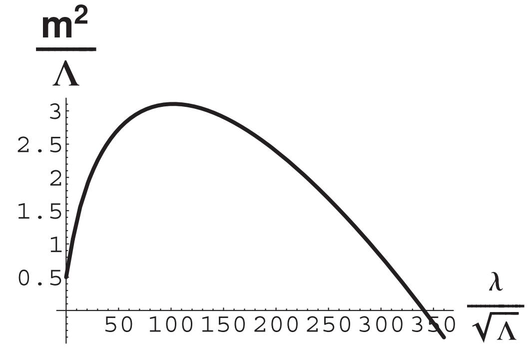

FIG. 10. The lower limit of m^2/Λ for $\sigma/\Lambda > 1/2$, as a function of $\lambda/\sqrt{\Lambda}$.

the region where $\sigma/\Lambda > 1/2$, which is above the Higuchi bound in three dimensions.

The lower limit of the mass parameter, which yields $\sigma/\Lambda > 1/2$, is shown in Fig. 8.

3. Positive coupling constant λ

The equation of motion can be read as, for $\lambda > 0$,

$$\frac{m^2}{\Lambda} = \frac{\sigma}{\Lambda} - \frac{\lambda}{\sqrt{\Lambda}} \frac{1}{\sqrt{\Lambda}} \frac{dL}{d\sigma}. \tag{A8}$$

We show the relation between $\frac{m^2}{\Lambda}$ and $\frac{\sigma}{\Lambda}$ for some specific values of $\frac{\lambda}{\sqrt{\Lambda}}$ in Fig. 9. We show the region where $\sigma/\Lambda > 1/2$, which is above the Higuchi bound in three dimensions.

The lower limit of the mass parameter, which yields $\sigma/\Lambda > 1/2$, is shown in Fig. 10.

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