

## Correlation harvesting between particle detectors in uniform motion

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(Received 29 August 2023; accepted 16 October 2023; published 27 November 2023)

We investigate the correlation harvesting protocol using two Unruh-DeWitt particle detectors moving along four classes of uniformly accelerated trajectories categorized by Letaw: linear, catenary, cusped, and circular motions. For each trajectory, two types of configurations are carried out: one possesses a stationary (time-translation invariant) Wightman function and the other is nonstationary. We find that detectors undergoing linear, catenary, and cusped motions gain fewer correlations in the nonstationary configurations compared to those in stationary configurations. Detectors in circular motion have similar behavior in both configurations. We discuss the relative suppression of correlation harvesting due to high acceleration for each case. Remarkably we find that under certain circumstances detectors in both linear and circular states of motion can harvest genuine (non-communication-assisted) entanglement even though they are in causal contact.

DOI: [10.1103/PhysRevD.108.105017](https://doi.org/10.1103/PhysRevD.108.105017)

### I. INTRODUCTION

The field of relativistic quantum information has undergone rapid development in recent years, resulting in the emergence of ground-breaking concepts such as entanglement degradation due to noninertial motion [1–3], entanglement harvesting [4–8], and the long-established Unruh effect [9–11]. The Unruh effect states that a linearly accelerating observer in Minkowski spacetime will experience a thermal bath, and this experience is indistinguishable from that of an inertial observer sitting in a thermal bath. More precisely, the temperature detected by a linearly accelerating two-level quantum system, known as the Unruh-DeWitt (UDW) particle detector [11,12], is proportional to its acceleration  $a$  and it reads

$$T_U = \frac{\hbar a}{2\pi c k_B}, \quad (1)$$

where  $\hbar$  is the reduced Planck constant,  $c$  is the speed of light, and  $k_B$  is Boltzmann's constant.

Despite its theoretical significance, the Unruh effect has yet to be verified experimentally. The main challenge hindering its experimental verification lies in the large acceleration required to produce experimentally measurable temperatures. For example, an acceleration on the

order of magnitude of  $a \approx 10^{20}$  m/s<sup>2</sup> is needed to achieve a temperature of  $T_U \sim 1$  K.

Given this situation, researchers have explored other detector trajectories that could induce a phenomenon similar to the Unruh effect. In 1981, Letaw found five classes of stationary trajectories with nonzero constant acceleration in flat spacetime [13]: linear, circular, cusped, catenary, and helix trajectories. In (3 + 1)-dimensional Minkowski spacetime, these trajectories are characterized by three parameters: two torsions and the magnitude of the proper acceleration. The effective temperatures observed by a detector undergoing these motions have been studied over subsequent years in various contexts [14–20]. Among these, the circular trajectory has attracted considerable attention for potential experimental realizations of the Unruh effect [14,15,19,21–24].

Less studied is the *entanglement harvesting* protocol for these various classes of motion, apart from detectors undergoing linearly accelerated motion, which has been extensively analyzed [8,25–28]. In this paper, we consider this problem and investigate how *two* UDW detectors can extract entanglement from the vacuum while undergoing these various types of noninertial motion.

In the entanglement harvesting protocol [4–7], two initially uncorrelated detectors interact locally with a quantum field in some state (typically the vacuum state) to extract preexisting entanglement [29,30]. More generally, detectors can harvest classical and quantum correlations in what is called the *correlation harvesting protocol*. The amount of harvested correlations is sensitive to the background spacetime [31–46] and the motion of the

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detectors [8,25–28,47–50]. Implementing this protocol is very close to implementation as recent experiments detecting correlations of the electromagnetic ground state in a ZnTe crystal have demonstrated [51–53].

In this paper we investigate the correlation harvesting protocol with detectors in four classes of uniform acceleration motion: linear, catenary, cusped, and circular trajectories. Unlike the helical case, these motions can all be realized in two spatial dimensions and so are more amenable to experimental testing [19–24]. We categorize the configurations for the detectors into two configurations: stationary, in which the Wightman function is time-translation invariant, and nonstationary, in which the Wightman function is not time-translation invariant. The Wightman functions for these two scenarios are similar; however, in nonstationary configurations, they possess an additional term that breaks time-translation invariance.

After introducing the UDW detector model in Sec. II and the four uniform acceleration trajectories in Sec. III, we then focus in Sec. IV A on a single detector following the four trajectories to examine its transition probability (or response function). We study its dependence on the magnitudes of the acceleration and torsions and numerically evaluate the effective temperature of the detector.

We then consider the correlation harvesting protocol in Sec. IV B. Specifically, we numerically evaluate the concurrence of entanglement and quantum mutual information, which measures the harvested total correlations. We find that the stationary and nonstationary configurations behave in a similar manner since their Wightman functions have terms in common. However, the amount of correlations extracted by the detectors in the nonstationary configurations differs from those of the stationary ones due to an additional term in the Wightman function. We also look into the acceleration dependence of the harvested correlations and conclude that sufficiently high accelerations prevent *any* uniformly accelerating detectors from extracting correlations. This point is consistent with previous papers that focused on linear and circular motions [8,25–28,47,48]. Finally, we show that constant acceleration makes it challenging to perform “genuine entanglement harvesting” (harvesting entanglement pre-existing in a quantum field that has no possible assistance from detector communication) in Sec. IV C. In general, genuine entanglement can be harvested from causally disconnected spacetime regions due to microcausality. For inertial detectors with Gaussian switching in Minkowski spacetime, we show that a sufficiently large energy gap allows the detectors to extract genuine entanglement from such regions [7,54]. While we find that this is generally not the case for uniformly accelerated detectors, remarkably we find small but non-negligible regions of parameter space where detectors in causal contact can harvest genuine entanglement.

Throughout this manuscript, we use the mostly plus metric convention  $(-, +, +, +)$  and the natural units  $\hbar = k_B = c = 1$ . A point in spacetime is denoted by  $\mathbf{x}$ .

## II. UNRUH-DEWITT DETECTORS

### A. Density matrix of detectors

Let us first review the correlation harvesting protocol. Consider two pointlike UDW detectors A and B with an energy gap  $\Omega_j$ ,  $j \in \{A, B\}$  between ground  $|g_j\rangle$  and excited states  $|e_j\rangle$ . These detectors interact with the quantum Klein-Gordon field  $\hat{\phi}$  along their trajectories  $\mathbf{x}_j(\tau_j) = (t(\tau_j), \mathbf{x}(\tau_j))$ , where  $\tau_j$  is the proper time of detector  $j$ .

In the interaction picture, the interaction Hamiltonian (as a generator of time translation with respect to  $\tau_j$ ) describing the coupling between detector  $j$  and  $\hat{\phi}$  is given by

$$\hat{H}_j^{\tau_j}(\tau_j) = \lambda_j \chi_j(\tau_j) \hat{\mu}_j(\tau_j) \otimes \hat{\phi}(\mathbf{x}_j(\tau_j)), \quad j \in \{A, B\}, \quad (2)$$

where  $\lambda_j$  is a coupling constant and  $\chi_j(\tau_j)$  is the switching function that governs the time dependence of the coupling. Here,  $\hat{\mu}_j(\tau_j)$  is the monopole moment given by

$$\hat{\mu}_j(\tau_j) = |e_j\rangle\langle g_j| e^{i\Omega_j\tau_j} + |g_j\rangle\langle e_j| e^{-i\Omega_j\tau_j}, \quad (3)$$

which describes each detector’s internal dynamics, and the field operator  $\hat{\phi}(\mathbf{x}_j(\tau_j))$  is pulled back along the trajectory of detector  $j$ . The superscript on  $\hat{H}_j^{\tau_j}(\tau_j)$  indicates the time translation that the Hamiltonian is generating.

The total interaction Hamiltonian  $\hat{H}_I^t(t)$  can be written as a generator of time translation with respect to the time  $t$  that is common to both detectors:

$$\hat{H}_I^t(t) = \frac{d\tau_A}{dt} \hat{H}_A^{\tau_A}(\tau_A(t)) + \frac{d\tau_B}{dt} \hat{H}_B^{\tau_B}(\tau_B(t)). \quad (4)$$

Note that the proper times  $\tau_A$  and  $\tau_B$  are each now functions of  $t$ . From this Hamiltonian, one obtains the time-evolution operator  $\hat{U}_I$  [55,56]:

$$\hat{U}_I = \mathcal{T}_t \exp\left(-i \int_{\mathbb{R}} dt \hat{H}_I^t(t)\right), \quad (5)$$

where  $\mathcal{T}_t$  is a time-ordering symbol with respect to the common time  $t$ .

One can then use a perturbative analysis and obtain the final density matrix of a joint system  $\mathcal{H}_A \otimes \mathcal{H}_B$ , where  $\mathcal{H}_j$  is a Hilbert space for detector  $j$ . Assuming a small coupling strength,  $\lambda \ll 1$ , the Dyson series expansion of  $\hat{U}_I$  reads

$$\hat{U}_I = \mathbb{1} + \hat{U}_I^{(1)} + \hat{U}_I^{(2)} + \mathcal{O}(\lambda^3), \quad (6a)$$

$$\hat{U}_I^{(1)} = -i \int_{-\infty}^{\infty} dt \hat{H}_I^t(t), \quad (6b)$$

$$\hat{U}_1^{(2)} = - \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \hat{H}_1^t(t_1) \hat{H}_1^t(t_2). \quad (6c)$$

Assuming that the initial state  $\rho_0$  of the detector-field system is

$$\rho_0 = |g_A\rangle\langle g_A| \otimes |g_B\rangle\langle g_B| \otimes |0\rangle\langle 0|, \quad (7)$$

where  $|0\rangle$  is the vacuum state of the field, one finds the final total density matrix  $\rho_{\text{tot}}$  after the interaction to be

$$\begin{aligned} \rho_{\text{tot}} &= \hat{U}_1 \rho_0 \hat{U}_1^\dagger \\ &= \rho_0 + \rho^{(1,1)} + \rho^{(2,0)} + \rho^{(0,2)} + \mathcal{O}(\lambda^4), \end{aligned} \quad (8)$$

where  $\rho^{(i,j)} = \hat{U}^{(i)} \rho_0 \hat{U}^{(j)\dagger}$  and all the odd-power terms in  $\lambda$  vanish [7]. Then the final density matrix of the detectors,  $\rho_{AB}$ , is obtained by tracing out the field part:  $\rho_{AB} = \text{Tr}_\phi[\rho_{\text{tot}}]$ . By employing the basis  $|g_A g_B\rangle = [1, 0, 0, 0]^T$ ,  $|g_A e_B\rangle = [0, 1, 0, 0]^T$ ,  $|e_A g_B\rangle = [0, 0, 1, 0]^T$ ,  $|e_A e_B\rangle = [0, 0, 0, 1]^T$ , the density matrix  $\rho_{AB}$  reads

$$\rho_{AB} = \begin{bmatrix} 1 - \mathcal{L}_{AA} - \mathcal{L}_{BB} & 0 & 0 & \mathcal{M}^* \\ 0 & \mathcal{L}_{BB} & \mathcal{L}_{AB}^* & 0 \\ 0 & \mathcal{L}_{AB} & \mathcal{L}_{AA} & 0 \\ \mathcal{M} & 0 & 0 & 0 \end{bmatrix} + \mathcal{O}(\lambda^4), \quad (9)$$

where

$$\begin{aligned} \mathcal{L}_{ij} &= \lambda^2 \int_{\mathbb{R}} d\tau_i \int_{\mathbb{R}} d\tau'_j \chi_i(\tau_i) \chi_j(\tau'_j) e^{-i\Omega(\tau_i - \tau'_j)} \\ &\quad \times W(\mathbf{x}_i(\tau_i), \mathbf{x}_j(\tau'_j)), \end{aligned} \quad (10a)$$

$$\begin{aligned} \mathcal{M} &= -\lambda^2 \int_{\mathbb{R}} d\tau_A \int_{\mathbb{R}} d\tau_B \chi_A(\tau_A) \chi_B(\tau_B) e^{-i\Omega(\tau_A + \tau_B)} \\ &\quad \times [\Theta(t(\tau_A) - t(\tau_B)) W(\mathbf{x}_A(\tau_A), \mathbf{x}_B(\tau_B)) \\ &\quad + \Theta(t(\tau_B) - t(\tau_A)) W(\mathbf{x}_B(\tau_B), \mathbf{x}_A(\tau_A))], \end{aligned} \quad (10b)$$

where  $\Theta(t)$  is the Heaviside step function and  $W(\mathbf{x}, \mathbf{x}') := \langle 0 | \hat{\phi}(\mathbf{x}) \hat{\phi}(\mathbf{x}') | 0 \rangle$  is the vacuum Wightman function. In  $(3+1)$ -dimensional Minkowski spacetime, the Wightman function reads

$$W(\mathbf{x}, \mathbf{x}') = -\frac{1}{4\pi^2} \frac{1}{(t - t' - i\epsilon)^2 - (\mathbf{x} - \mathbf{x}')^2}, \quad (11)$$

where  $\epsilon$  is the UV cutoff. The elements  $\mathcal{L}_{jj}$ ,  $j \in \{A, B\}$  are the so-called transition probabilities (or response functions), which describe the probability of a detector transitioning from the ground to excited states,  $|g_j\rangle \rightarrow |e_j\rangle$ . The off-diagonal elements  $\mathcal{M}$  and  $\mathcal{L}_{AB}$  are responsible for

harvesting entanglement and quantum mutual information, respectively, as we shall see in the next subsection.

Throughout this paper, we use a Gaussian switching function

$$\chi_j(\tau_j) = e^{-\tau_j^2/2\sigma^2}, \quad (12)$$

where  $\sigma > 0$  is the characteristic Gaussian width, which has the units of time. We will use  $\sigma$  to make all quantities unitless (such as  $\Omega\sigma$ ).

## B. Correlation measure

Let us introduce two measures for correlation: concurrence  $\mathcal{C}_{AB}$  and quantum mutual information  $I_{AB}$ .

Concurrence is a measure of entanglement [57,58]. Let  $\rho_{AB}$  be the density matrix of a two-qubit system. We first define a matrix  $\tilde{\rho}_{AB}$  as

$$\tilde{\rho}_{AB} := (\hat{\sigma}_y \otimes \hat{\sigma}_y) \rho_{AB}^* (\hat{\sigma}_y \otimes \hat{\sigma}_y), \quad (13)$$

where  $\hat{\sigma}_y$  is the Pauli- $y$  operator and  $\rho_{AB}^*$  is the complex conjugate of  $\rho_{AB}$ . Then by denoting  $w_i \in \mathbb{R}$  ( $i = 1, 2, 3, 4$ ) as eigenvalues of a Hermitian operator  $\sqrt{\sqrt{\rho_{AB}} \tilde{\rho}_{AB} \sqrt{\rho_{AB}}}$ , the concurrence is defined as follows:

$$\begin{aligned} \mathcal{C}_{AB} &:= \max\{0, w_1 - w_2 - w_3 - w_4\}, \\ &\quad (w_1 \geq w_2 \geq w_3 \geq w_4). \end{aligned} \quad (14)$$

The concurrence is zero if and only if the state  $\rho_{AB}$  is separable. In the case of our density matrix (9), the concurrence is known to be

$$\mathcal{C}_{AB} = 2 \max\{0, |\mathcal{M}| - \sqrt{\mathcal{L}_{AA} \mathcal{L}_{BB}}\} + \mathcal{O}(\lambda^4). \quad (15)$$

Quantum mutual information [59], on the other hand, quantifies the amount of total correlation, both classical and quantum. Quantum mutual information  $I_{AB}$  between two qubits A and B up to second order in  $\lambda$  is [7]

$$\begin{aligned} I_{AB} &= \mathcal{L}_+ \ln \mathcal{L}_+ + \mathcal{L}_- \ln \mathcal{L}_- \\ &\quad - \mathcal{L}_{AA} \ln \mathcal{L}_{AA} - \mathcal{L}_{BB} \ln \mathcal{L}_{BB} + \mathcal{O}(\lambda^4), \end{aligned} \quad (16)$$

where

$$\mathcal{L}_\pm := \frac{1}{2} \left( \mathcal{L}_{AA} + \mathcal{L}_{BB} \pm \sqrt{(\mathcal{L}_{AA} - \mathcal{L}_{BB})^2 + 4|\mathcal{L}_{AB}|^2} \right). \quad (17)$$

Note that, while concurrence (15) vanishes when the ‘‘noise term’’  $\sqrt{\mathcal{L}_{AA} \mathcal{L}_{BB}}$  exceeds the nonlocal element  $|\mathcal{M}|$ , the mutual information becomes zero when  $|\mathcal{L}_{AB}| = 0$ . In addition, if  $\mathcal{C}_{AB} = 0$  but the mutual information is non-vanishing, then the extracted correlation by the detectors is either classical correlation or nondistillable entanglement.

### III. UNIFORM ACCELERATION TRAJECTORIES

#### A. Single detector trajectory classification

The most well-known trajectory for a uniformly accelerating (i.e.,  $a = \text{const}$ ) pointlike particle is linear accelerated motion. However, Letaw pointed out that there are, in fact, five classes of uniformly accelerated trajectories, excluding the case where  $a = 0$ . Along with the linear case, the other classes are circular, catenary, cusped, and helix [13]. Consider a trajectory in  $(3 + 1)$ -dimensional Minkowski spacetime. Such a trajectory can be characterized by three geometric invariants: the curvature  $a(\tau)$ , which represents the magnitude of proper acceleration, the first torsion  $b(\tau)$ , and the second torsion (also known as hyper-torsion)  $\nu(\tau)$  of the worldline. The torsions  $b(\tau)$  and  $\nu(\tau)$  correspond to the proper angular velocities in a given tetrad frame [13]. Assuming that these invariants are constants, the trajectory becomes stationary. In a nutshell, these motions are characterized by the following:

- (1) linear:  $a \neq 0, b = \nu = 0$ ;
- (2) catenary:  $a > b, \nu = 0$ ;
- (3) cusped:  $a = b, \nu = 0$ ;
- (4) circular:  $a < b, \nu = 0$ ; and
- (5) helix:  $\nu \neq 0$ .

In this subsection, we review these trajectories and consider the corresponding vacuum Wightman functions. We will suppress the UV cutoff  $\epsilon$  for readability.

#### 1. Linear motion

The linear acceleration motion of a detector is defined solely by the constant acceleration  $a$ , with all other parameters set to zero. The trajectory reads

$$\mathbf{x}(\tau) = \left( \frac{1}{a} \sinh(a\tau), \frac{1}{a} \cosh(a\tau), 0, 0 \right), \quad (18)$$

and the Wightman function along this trajectory is given by

$$W_{\text{lin}}(\Delta\tau) = -\frac{1}{4\pi^2} \frac{1}{\frac{4}{a^2} \sinh^2\left(\frac{a\Delta\tau}{2}\right)}, \quad (19)$$

where  $\Delta\tau := \tau - \tau'$ .

#### 2. Circular motion

The circular trajectory is defined by  $a$  and  $b$  satisfying  $a < b$ . Let us begin with a commonly used trajectory

$$\mathbf{x}(\tau) = (\gamma\tau, R \cos(\omega\gamma\tau), R \sin(\omega\gamma\tau), 0), \quad (20)$$

where  $R$ ,  $\omega$ , and  $\gamma$  are the radius of the circular motion, angular velocity, and the Lorentz factor defined as  $\gamma := 1/\sqrt{1 - v^2}$ , respectively. Here,  $v := R\omega (\leq 1)$  is the speed of the detector. Introducing the acceleration of the detector  $a = R\omega^2\gamma^2$ , these parameters can be related by

$$\omega = \sqrt{\frac{a}{(1 + aR)R}}, \quad (21a)$$

$$\gamma = \sqrt{1 + aR}, \quad (21b)$$

$$v = \sqrt{\frac{aR}{1 + aR}}. \quad (21c)$$

In terms of the acceleration  $a$  and the torsion  $b$ , we can further express  $\omega$  and  $v$  as

$$\omega = b(1 - a^2/b^2), \quad v = a/b,$$

respectively. The Wightman function is then

$$W_{\text{cir}}(\Delta\tau) = -\frac{1}{4\pi^2} \frac{1}{\gamma^2 \Delta\tau^2 - 4R^2 \sin^2(\omega\gamma\Delta\tau/2)}. \quad (22)$$

#### 3. Cusped motion

Cusped motion is described by the acceleration and torsion with  $a = b$ . The trajectory reads

$$\mathbf{x}(\tau) = \left( \tau + \frac{1}{6} a^2 \tau^3, \frac{1}{2} a \tau^2, \frac{1}{6} a^2 \tau^3, 0 \right), \quad (23)$$

and the corresponding Wightman function is

$$W_{\text{cus}}(\Delta\tau) = -\frac{1}{4\pi^2} \frac{1}{\Delta\tau^2 + \frac{a^2}{12} \Delta\tau^4}. \quad (24)$$

#### 4. Catenary motion

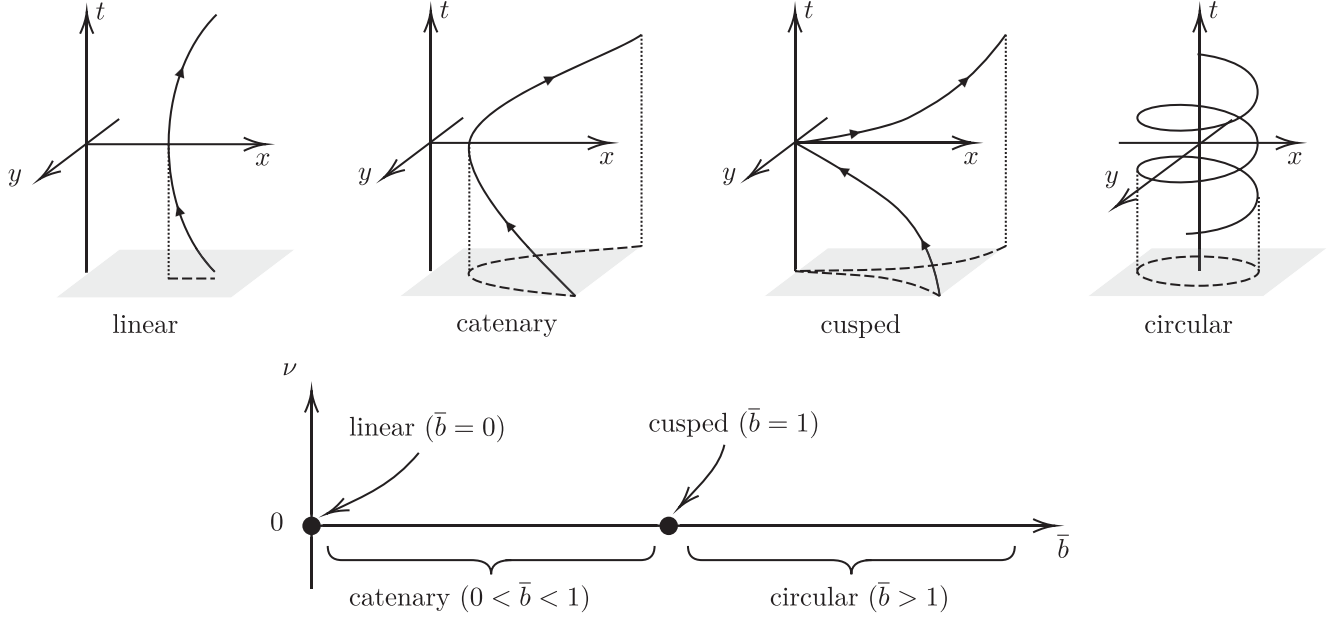
Catenary motion can be characterized by  $a$  and  $b$  with  $a > b$ . The trajectory is given by

$$\mathbf{x}(\tau) = \left( \frac{a}{a^2 - b^2} \sinh(\sqrt{a^2 - b^2}\tau), \frac{a}{a^2 - b^2} \cosh(\sqrt{a^2 - b^2}\tau), \frac{b\tau}{\sqrt{a^2 - b^2}}, 0 \right), \quad (25)$$

and the Wightman function reads

$$W_{\text{cat}}(\Delta\tau) = -\frac{1}{4\pi^2} \frac{1}{-\frac{b^2 \Delta\tau^2}{a^2 - b^2} + \frac{4a^2}{(a^2 - b^2)^2} \sinh^2\left(\frac{\sqrt{a^2 - b^2} \Delta\tau}{2}\right)}. \quad (26)$$

We immediately see that catenary motion reduces to the linear motion as  $b \rightarrow 0$ . Catenary motion also reduces to cusped motion as  $b \rightarrow a$  after a coordinate transformation consisting of a Lorentz boost and a translation [18].

FIG. 1. Four trajectories characterized by  $\bar{b} \equiv b/a$ .

### 5. Helix motion

Finally, helix motion is a combination of circular and linear acceleration motions characterized by three parameters,  $a$ ,  $b$ , and  $\nu$ :

$$\mathbf{x}(\tau) = \left( \frac{\mathcal{P}}{\Gamma_+} \sinh(\Gamma_+ \tau), \frac{\mathcal{P}}{\Gamma_+} \cosh(\Gamma_+ \tau), \frac{\mathcal{Q}}{\Gamma_-} \cos(\Gamma_- \tau), \frac{\mathcal{Q}}{\Gamma_-} \sin(\Gamma_- \tau) \right), \quad (27)$$

where  $\mathcal{P} := \Xi/\Gamma$ ,  $\mathcal{Q} := ab/\Xi\Gamma$ , and

$$\Xi^2 := \frac{1}{2}(\Gamma^2 + a^2 + b^2 + \nu^2), \quad (28a)$$

$$\Gamma^2 := \Gamma_+^2 + \Gamma_-^2, \quad \Gamma_{\pm}^2 := \sqrt{A^2 + B^2} \pm A, \quad (28b)$$

$$A := \frac{1}{2}(a^2 - b^2 - \nu^2), \quad B := av. \quad (28c)$$

The Wightman function reads

$$W_{\text{hel}}(\Delta\tau) = -\frac{1}{4\pi^2} \frac{4\mathcal{P}^2}{\Gamma_+^2} \sinh^2\left(\frac{\Gamma_+ \Delta\tau}{2}\right) - \frac{4\mathcal{Q}^2}{\Gamma_-^2} \sin^2\left(\frac{\Gamma_- \Delta\tau}{2}\right). \quad (29)$$

Note that the trajectory and the corresponding Wightman function reduce to the aforementioned trajectories when  $\nu \rightarrow 0$ . In this sense, the helix is the general motion that contains other motions.

### 6. Wightman function at $\nu=0$

We now turn our attention to the special case where  $\nu = 0$ . Although the Wightman functions for linear, circular, catenary, and cusped motions may initially appear to take different forms, they can actually be expressed in a unified manner. Let  $\bar{b} \equiv b/a$  with the condition that  $a \neq 0$ . The Wightman functions for all trajectories with  $\nu = 0$  can be written in the following compact form:

$$W_{\nu=0}(\Delta\tau) = -\frac{1}{4\pi^2} \frac{1}{-\frac{\bar{b}^2}{1-\bar{b}^2} \Delta\tau^2 + \frac{4}{(1-\bar{b}^2)^2 a^2} \sinh^2\left(\frac{\sqrt{1-\bar{b}^2} a \Delta\tau}{2}\right)}. \quad (30)$$

The parameter  $\bar{b}$  serves to specify the particular trajectory, as illustrated in Fig. 1: linear ( $\bar{b} = 0$ ), catenary ( $0 < \bar{b} < 1$ ), cusped ( $\bar{b} = 1$ ), and circular ( $\bar{b} > 1$ ). For circular motion, we employ the identity  $\sin(ix) = i \sinh(x)$ . Note that one obtains the Wightman function for the cusped motion, as given in (24), by performing a series expansion around  $\bar{b} = 1$ .

The corresponding transition probability  $\mathcal{L}_{jj}$ ,  $j \in \{A, B\}$ , in (9) reads

$$\mathcal{L}_{jj} = \lambda^2 \sigma \sqrt{\pi} \int_{\mathbb{R}} du e^{-u^2/4\sigma^2} e^{-i\Omega u} W_{\nu=0}(u). \quad (31)$$

### B. Two detectors in uniform acceleration

We now consider two UDW detectors A and B, both undergoing uniform acceleration. In particular, we categorize the detector configurations into two classes: stationary (time-translation invariant) and nonstationary scenarios.

### 1. Stationary scenario

Consider two detectors undergoing the same uniform acceleration (e.g., both linearly accelerated). The Wightman function can be made time-translation invariant, meaning it depends only on the time difference  $\Delta\tau := \tau_A - \tau_B$ , by imposing that the angle between the velocity vector of a detector and the spatial displacement vector from one detector to the other is time independent. For example, two linearly accelerating detectors along the trajectories

$$\mathbf{x}_A(\tau_A) = \left( \frac{1}{a} \sinh(a\tau_A), \frac{1}{a} \cosh(a\tau_A), 0, 0 \right), \quad (32a)$$

$$\mathbf{x}_B(\tau_B) = \left( \frac{1}{a} \sinh(a\tau_B), \frac{1}{a} \cosh(a\tau_B), L, 0 \right) \quad (32b)$$

give the following stationary Wightman function:

$$W_{\text{lin}}(\tau_A, \tau_B) = -\frac{1}{4\pi^2} \frac{1}{\frac{4}{a^2} \sinh^2\left(\frac{a\Delta\tau}{2}\right) - L^2}, \quad (33)$$

where  $L := |\mathbf{x}_A - \mathbf{x}_B|$  is the spatial separation between the two detectors. As depicted in Fig. 2, top left, each velocity vector of the detector is always perpendicular to the displacement vector  $\mathbf{x}_{AB} := \mathbf{x}_A - \mathbf{x}_B$  throughout the interaction.

We can also construct stationary Wightman functions for other motions:

#### Circular

$$\mathbf{x}_A = (\gamma\tau_A, R \cos(\omega\gamma\tau_A), R \sin(\omega\gamma\tau_A), 0), \quad (34a)$$

$$\mathbf{x}_B = (\gamma\tau_B, R \cos(\omega\gamma\tau_B), R \sin(\omega\gamma\tau_B), L). \quad (34b)$$

#### Cusped

$$\mathbf{x}_A = \left( \tau_A + \frac{1}{6} a^2 \tau_A^3, \frac{1}{2} a \tau_A^2, \frac{1}{6} a^2 \tau_A^3, 0 \right), \quad (35a)$$

$$\mathbf{x}_B = \left( \tau_B + \frac{1}{6} a^2 \tau_B^3, \frac{1}{2} a \tau_B^2, \frac{1}{6} a^2 \tau_B^3, L \right). \quad (35b)$$

#### Catenary

$$\mathbf{x}_A = \left( \frac{a}{a^2 - b^2} \sinh(\sqrt{a^2 - b^2} \tau_A), \frac{a}{a^2 - b^2} \cosh(\sqrt{a^2 - b^2} \tau_A), \frac{b\tau_A}{\sqrt{a^2 - b^2}}, 0 \right), \quad (36a)$$

$$\mathbf{x}_B = \left( \frac{a}{a^2 - b^2} \sinh(\sqrt{a^2 - b^2} \tau_B), \frac{a}{a^2 - b^2} \cosh(\sqrt{a^2 - b^2} \tau_B), \frac{b\tau_B}{\sqrt{a^2 - b^2}}, L \right). \quad (36b)$$

As for a single detector, the Wightman functions along the trajectories given above take the following compact form:

$$W_s(\tau_A, \tau_B) \equiv W_s(\Delta\tau) = -\frac{1}{4\pi^2} \frac{1}{-\frac{\bar{b}^2}{1-b^2} \Delta\tau^2 + \frac{4}{(1-b^2)^2 a^2} \sinh^2\left(\frac{\sqrt{1-b^2} a \Delta\tau}{2}\right) - L^2}, \quad (37)$$

where  $\Delta\tau := \tau_A - \tau_B$ ,  $\bar{b} \equiv b/a$  and the subscript ‘‘s’’ stands for stationary. Since the Wightman function depends only on  $\Delta\tau$ , the elements in the density matrix (9),  $\mathcal{M}$  and  $\mathcal{L}_{AB}$ , can be simplified to single integrals when the Gaussian switching function (12) is used:

$$\mathcal{M} = -2\lambda^2 \sigma \sqrt{\pi} e^{-\Omega^2 \sigma^2} \int_0^\infty du e^{-u^2/4\sigma^2} W_s(u), \quad (38a)$$

$$\mathcal{L}_{AB} = \lambda^2 \sigma \sqrt{\pi} \int_{\mathbb{R}} du e^{-u^2/4\sigma^2} e^{-i\Omega u} W_s(u). \quad (38b)$$

Here, we used the fact that the Heaviside step function in (10b) can be written as  $\Theta(t(\tau_A) - t(\tau_B)) = \Theta(\tau_A - \tau_B)$  for any of the uniform acceleration scenarios mentioned earlier.

We note that all stationary configurations can only be realized in (3 + 1) dimensions, with the exception of the linear configuration.

### 2. Nonstationary scenario

One can also consider configurations similar to those in Sec. III B 1, where the Wightman function depends not only on  $\Delta\tau$  but also on  $\Delta_+\tau := \tau_A + \tau_B$ . In this case, the Wightman function is no longer time-translation invariant (hence, nonstationary).

In particular, consider two linearly accelerating UDW detectors whose trajectories are given by

$$\mathbf{x}_A(\tau_A) = \left( \frac{1}{a} \sinh(a\tau_A), \frac{1}{a} \cosh(a\tau_A) + L, 0, 0 \right), \quad (39a)$$

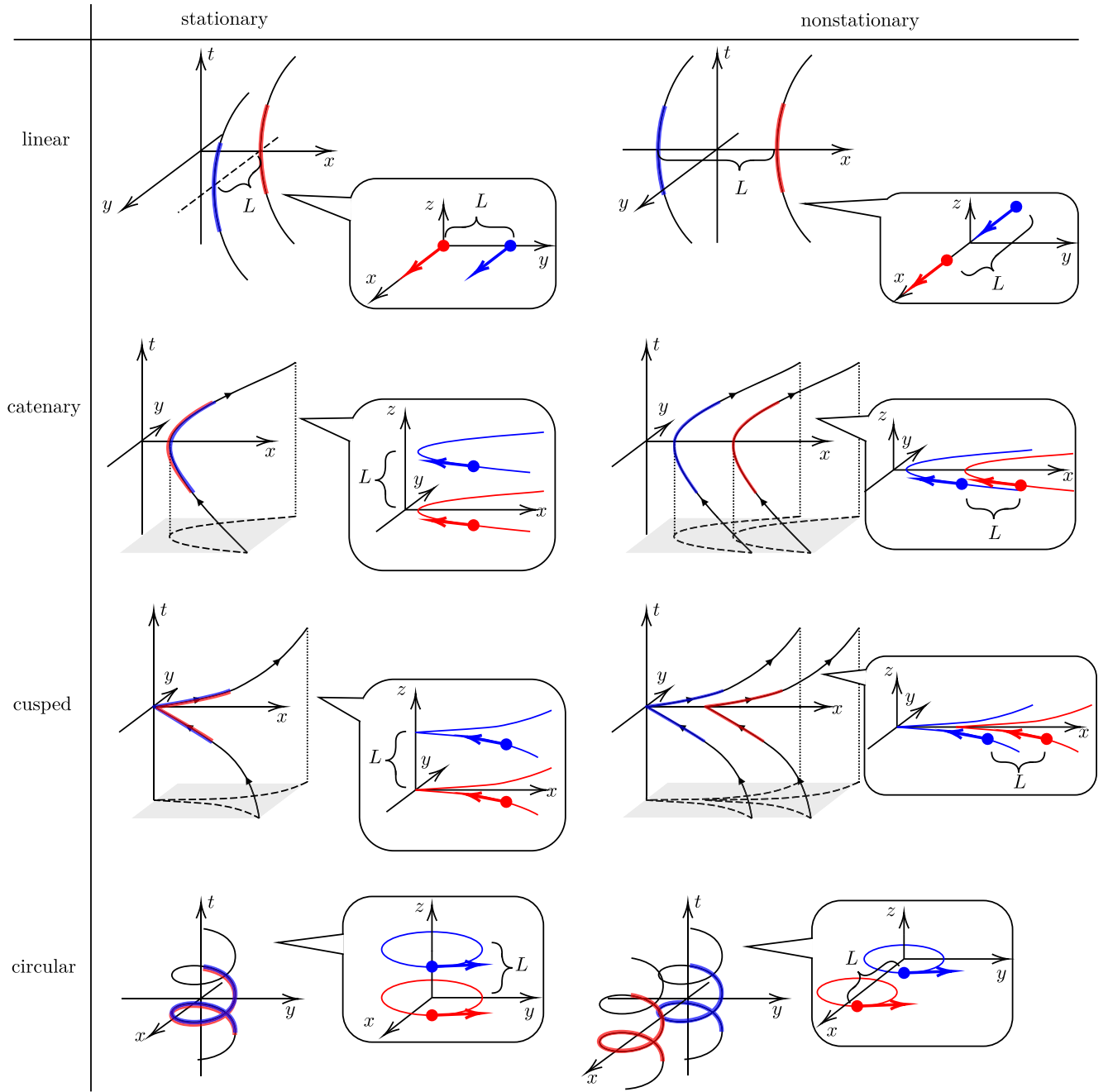


FIG. 2. Stationary and nonstationary configurations for four classes of uniformly accelerating detectors. Red and blue represent detectors A and B, respectively.

$$\mathbf{x}_B(\tau_B) = \left( \frac{1}{a} \sinh(a\tau_B), \frac{1}{a} \cosh(a\tau_B), 0, 0 \right). \quad (39b)$$

The correlation harvesting protocol along these trajectories was examined in [26,28]. The corresponding Wightman function reads

$$W_{\text{lin}}(\tau_A, \tau_B) = -\frac{1}{4\pi^2} \frac{1}{\frac{4}{a^2} \sinh^2\left(\frac{a\Delta\tau}{2}\right) - L^2 - \frac{4L}{a} \sinh\left(\frac{a\Delta\tau}{2}\right) \sinh\left(\frac{a\Delta_\pm\tau}{2}\right)}. \quad (40)$$

The term  $\Delta_+ \tau$  comes from the fact that the angle between the velocity vector and the displacement vector is time dependent ( $0^\circ$  or  $180^\circ$ ).

Similarly, other uniformly accelerating trajectories that yield a nonstationary Wightman function are as follows:

*Circular*

$$\mathbf{x}_A = (\gamma \tau_A, R \cos(\omega \gamma \tau_A) + L, R \sin(\omega \gamma \tau_A), 0), \quad (41a)$$

$$\mathbf{x}_B = (\gamma \tau_B, R \cos(\omega \gamma \tau_B), R \sin(\omega \gamma \tau_B), 0). \quad (41b)$$

*Cusped*

$$\mathbf{x}_A = \left( \tau_A + \frac{1}{6} a^2 \tau_A^3, \frac{1}{2} a \tau_A^2 + L, \frac{1}{6} a^2 \tau_A^3, 0 \right), \quad (42a)$$

$$\mathbf{x}_B = \left( \tau_B + \frac{1}{6} a^2 \tau_B^3, \frac{1}{2} a \tau_B^2, \frac{1}{6} a^2 \tau_B^3, 0 \right). \quad (42b)$$

*Catenary*

$$\mathbf{x}_A = \left( \frac{a}{a^2 - b^2} \sinh(\sqrt{a^2 - b^2} \tau_A), \frac{a}{a^2 - b^2} \cosh(\sqrt{a^2 - b^2} \tau_A) + L, \frac{b \tau_A}{\sqrt{a^2 - b^2}}, 0 \right), \quad (43a)$$

$$\mathbf{x}_B = \left( \frac{a}{a^2 - b^2} \sinh(\sqrt{a^2 - b^2} \tau_B), \frac{a}{a^2 - b^2} \cosh(\sqrt{a^2 - b^2} \tau_B), \frac{b \tau_B}{\sqrt{a^2 - b^2}}, 0 \right). \quad (43b)$$

The Wightman function for these nonstationary motions can be compactly expressed as

$$W_{\text{ns}}(\tau_A, \tau_B) = -\frac{1}{4\pi^2} \frac{1}{-\frac{\bar{b}^2}{1-\bar{b}^2} \Delta \tau^2 + \frac{4}{(1-\bar{b}^2)^2 a^2} \sinh^2\left(\frac{\sqrt{1-\bar{b}^2} a \Delta \tau}{2}\right) - L^2 - \frac{4L}{(1-\bar{b}^2)a} \sinh\left(\frac{\sqrt{1-\bar{b}^2} a \Delta \tau}{2}\right) \sinh\left(\frac{\sqrt{1-\bar{b}^2} a \Delta_+ \tau}{2}\right)} \quad (44)$$

and possesses an additional term in the denominator compared to the stationary Wightman function (37). Here, the subscript “ns” designates nonstationary. Due to this additional term, the correlations harvested by nonstationary detectors will exhibit behavior similar to those of stationary detectors.

Note that the presence of  $\Delta_+ \tau$  prevents us from reducing the double integrals in (10) into single integrals. Furthermore, all nonstationary configurations can be realized in  $(2+1)$  dimensions, except for the helix case, which we are not considering.

#### IV. NUMERICAL RESULTS

Here, we numerically compute the concurrence (15) and quantum mutual information (16) harvested by two uniformly accelerating detectors by inserting the Wightman functions (30), (37), and (44) into  $\mathcal{L}_{ij}$  and  $\mathcal{M}$  given in (10). For stationary detectors in Sec. III B 1, we utilize the expressions given by (38).

##### A. Transition probability of uniformly accelerating detectors

Let us begin by considering the transition probability  $\mathcal{L}_{jj}$  for a uniformly accelerating detector. We are particularly interested in the cases of linear ( $\bar{b} = 0$ ), catenary ( $0 < \bar{b} < 1$ ), cusped ( $\bar{b} = 1$ ), and circular ( $\bar{b} > 1$ ) motions, and their respective transition probabilities are given by (31). We consider  $\mathcal{L}_{jj}/\lambda^2$  and write the parameters in units of  $\sigma$ ,

which makes the transition probability a function of three variables:  $a\sigma$ ,  $\bar{b}$ , and  $\Omega\sigma$ . It is important to note that  $\mathcal{L}_{AA} = \mathcal{L}_{BB}$ , as we are assuming both detectors are identical.

Figure 3 depicts the transition probability  $\mathcal{L}_{jj}/\lambda^2$  as a function of the magnitude of acceleration  $a\sigma$  for fixed  $\Omega$  [Fig. 3(a)] and  $\log_{10} \bar{b}$  for different values of the acceleration [Figs. 3(b) and 3(c)]. In Fig. 3(a), the transition probabilities for a detector with  $\Omega\sigma = 2$  in linear ( $\bar{b} = 0$ ), catenary ( $\bar{b} = 0.5$ ), cusped ( $\bar{b} = 1$ ), and circular ( $\bar{b} = 2$ ) motions are shown. We find that in all these cases,  $\mathcal{L}_{jj}/\lambda^2$  increases with the acceleration  $a\sigma$ .<sup>1</sup>

However, the relationship between the transition probabilities of detectors in different uniform motions is highly nontrivial. For instance, when a detector has  $\Omega\sigma = 2$  and  $a\sigma \lesssim 5$ , as depicted in Fig. 3(a), a detector in circular motion with  $\bar{b} = 2$  shows the largest value of  $\mathcal{L}_{jj}/\lambda^2$ , whereas a detector in linear motion ( $\bar{b} = 0$ ) shows the smallest. This relation, however, flips for  $a\sigma \gtrsim 5$ . We will numerically demonstrate that such relationships depend on the interplay between  $a\sigma$  and  $\Omega\sigma$ .

In Fig. 3(b), the magnitude of the acceleration is fixed at  $a\sigma = 1$ , and  $\mathcal{L}_{jj}/\lambda^2$  is plotted as a function of  $\log_{10} \bar{b}$ . Each curve in this figure corresponds to a different value of  $\Omega\sigma$ ,

<sup>1</sup> $\mathcal{L}_{jj}/\lambda^2$  is not guaranteed to *monotonically* increase with  $a\sigma$  for a finite interaction duration. For a detector in the linearly accelerated motion, such phenomenon is known as the (weak) anti-Unruh effect [60,61].



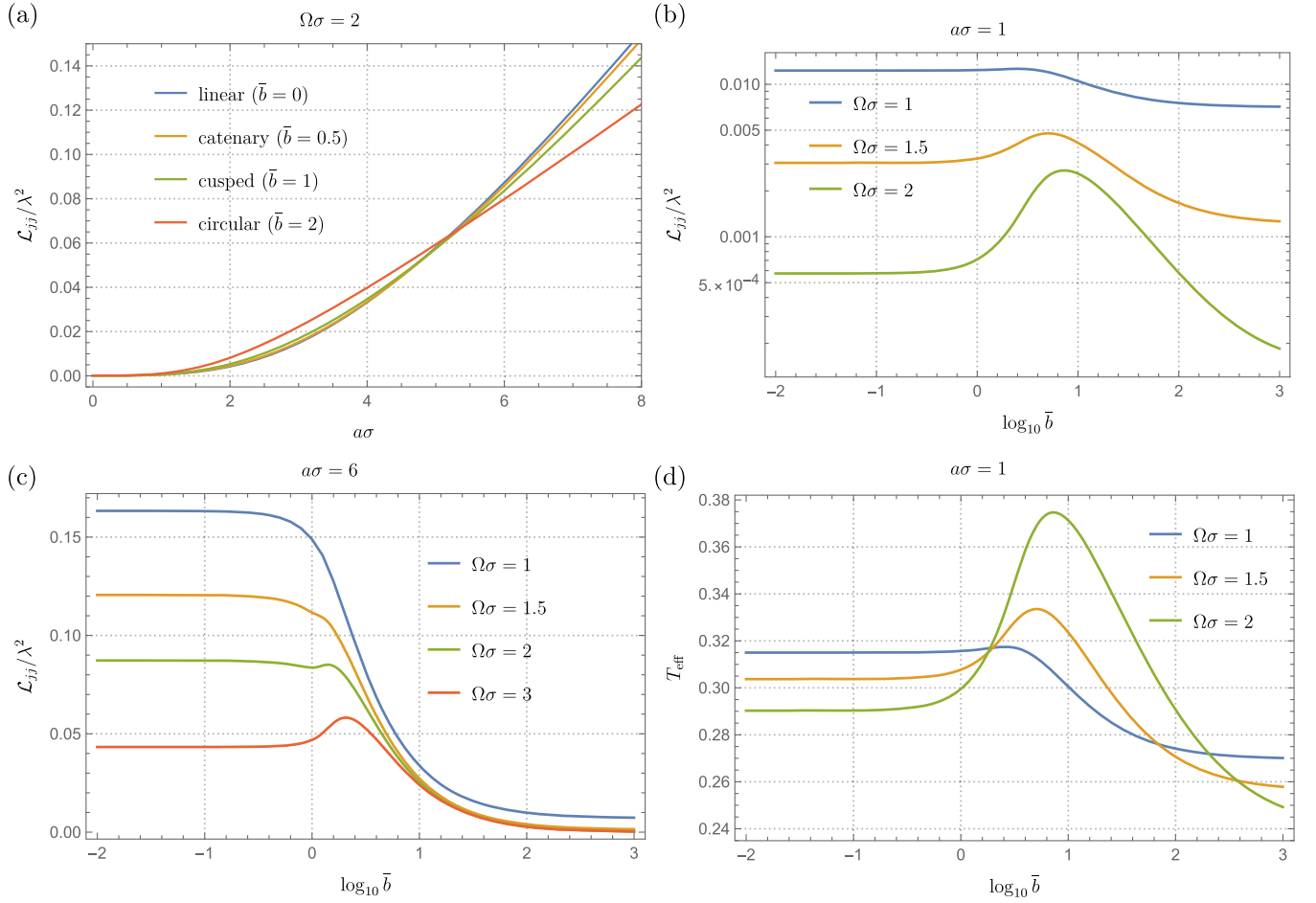


FIG. 3. (a) Transition probabilities  $\mathcal{L}_{jj}/\lambda^2$  as a function of the magnitude of acceleration  $a\sigma$  with  $\Omega\sigma = 2$ . (b)  $\mathcal{L}_{jj}/\lambda^2$  as a function of  $\log_{10} \bar{b}$  with  $a\sigma = 1$  and (c) with  $a\sigma = 6$ . (d) The effective temperature  $T_{\text{eff}}$  as a function of  $\log_{10} \bar{b}$  with  $a\sigma = 1$ .

with the curve for  $\Omega\sigma = 2$  corresponding to Fig. 3(a) at  $a\sigma = 1$ . For each value of  $\Omega\sigma$  in Fig. 3(b), the transition probability has a peak for  $\log_{10} \bar{b} > 0$  (i.e.,  $\bar{b} > 1$ ) and then decreases with increasing  $\log_{10} \bar{b}$ , becoming smaller than the value for the linear case ( $\log_{10} \bar{b} \rightarrow -\infty$ ). This means that  $\mathcal{L}_{jj}/\lambda^2$  at  $a\sigma = 1$  in Fig. 3(a) increases with  $\bar{b}$  until it reaches a maximum and then decreases. We note that the presence of the peak is contingent on larger values of  $\Omega\sigma$  relative to  $a\sigma$ ; in fact, the peak does not appear for smaller energy gaps, in which case the transition probability monotonically decreases with  $\bar{b}$ , as shown in Fig. 3(b). This trend is further illustrated in Fig. 3(c), where  $a\sigma = 6$  is chosen. In this scenario, the peak is nonexistent for  $\Omega\sigma = 1$  and 1.5 (as well as for  $\Omega\sigma < 1$ ) but becomes manifest when  $\Omega\sigma \gtrsim 2$ . Thus we infer that detectors with smaller energy gaps  $\Omega\sigma$  compared to  $a\sigma$  do not have a peak in  $\mathcal{L}_{jj}(\bar{b})/\lambda^2$ .

The behavior of  $\mathcal{L}_{jj}/\lambda^2$  is related to the concept of the “effective temperature” perceived by a detector. For now, let us denote the transition probability as  $\mathcal{L}_{jj}(\Omega, \sigma)$ . The effective temperature  $T_{\text{eff}}$  is defined as

$$T_{\text{eff}}^{-1} := \frac{1}{\Omega} \ln \frac{\mathcal{L}_{jj}(-\Omega, \sigma)/\lambda^2 \sigma}{\mathcal{L}_{jj}(\Omega, \sigma)/\lambda^2 \sigma}, \quad (45)$$

where this formula is derived in the Appendix. We divide  $\mathcal{L}_{jj}(\Omega, \sigma)$  by  $\sigma$  so that it is well defined in the long interaction limit,  $\sigma \rightarrow \infty$  [61,62]. Note that if the Wightman function obeys the Kubo-Martin-Schwinger (KMS) condition [63,64], then the effective temperature converges to the KMS temperature (which is the temperature of the field formally defined in quantum field theory) in the limit  $\sigma \rightarrow \infty$ . However, in the case of finite interaction duration, the effective temperature is an estimator for the actual field temperature. For a detector in a uniform acceleration motion, the effective temperature for each scenario has been examined in, e.g., [14–20].

We plot the effective temperature  $T_{\text{eff}}$  as a function of  $\log_{10} \bar{b}$  when  $\sigma = 1$  and  $a\sigma = 1$  in Fig. 3(d), which corresponds to Fig. 3(b). We see that the locations of the peaks in  $T_{\text{eff}}$  align with those of  $\mathcal{L}_{jj}(\Omega)$  in Fig. 3(b). This suggests that, for a given acceleration and energy gap, a

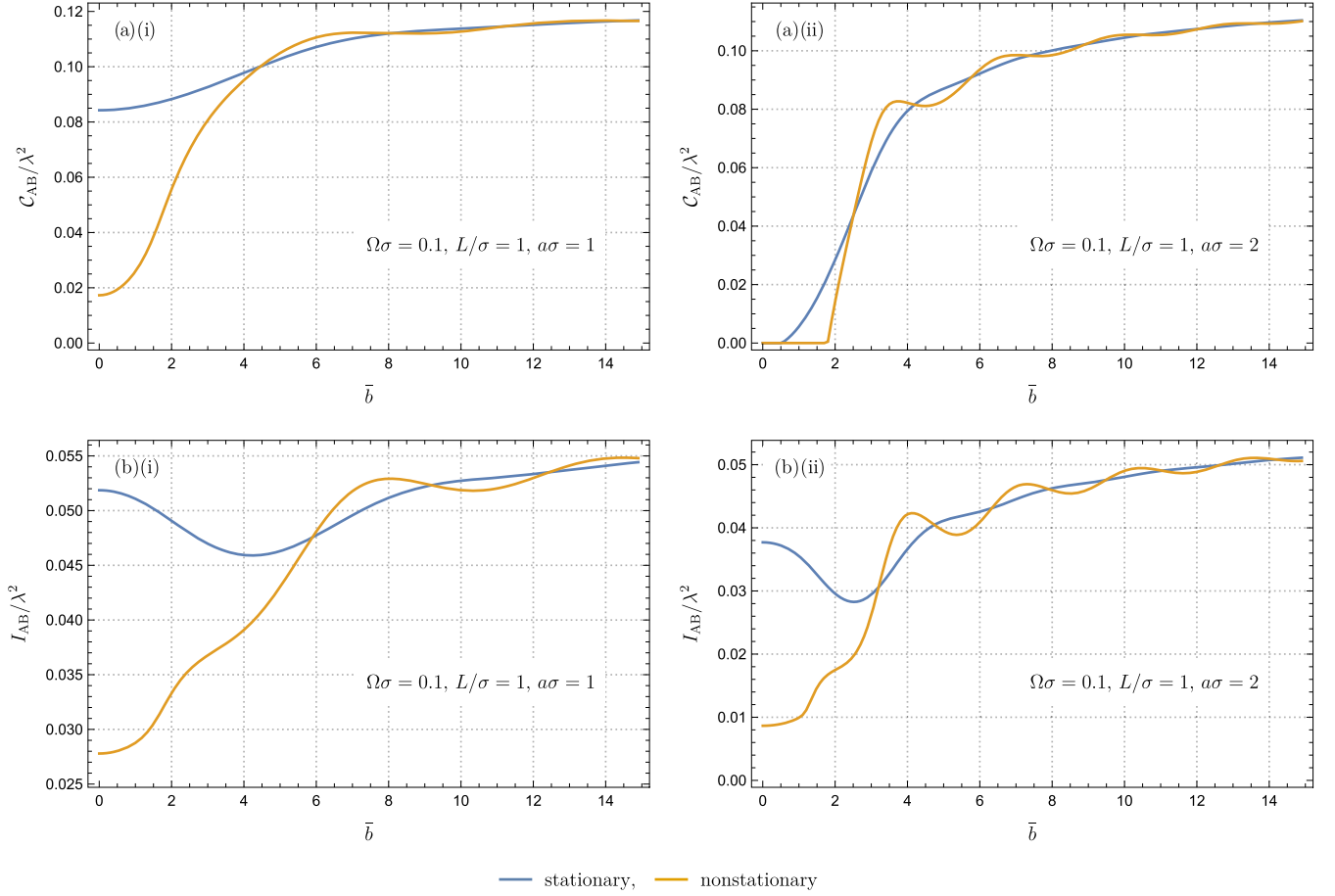


FIG. 4. Concurrence (a) and quantum mutual information (b) harvested by stationary and nonstationary detectors as a function of  $\bar{b}$ . For each case,  $\Omega\sigma = 0.1$  and  $L/\sigma = 1$ . (a)(i) and (b)(i) are, respectively,  $C_{AB}/\lambda^2$  and  $I_{AB}/\lambda^2$  when  $a\sigma = 1$ , while (a)(ii) and (b)(ii) are, respectively,  $C_{AB}/\lambda^2$  and  $I_{AB}/\lambda^2$  when  $a\sigma = 2$ .

detector in circular motion within a certain range of  $\log_{10} \bar{b}$  can register higher effective temperatures than those in other types of motion. However, as  $\bar{b} \rightarrow \infty$ , which corresponds to the speed of a detector in circular motion with  $v_{\text{circ}} (= \bar{b}^{-1}) \rightarrow 0$ , the temperature becomes colder.

### B. Concurrence and quantum mutual information between uniformly accelerating detectors

We now move on to the correlation harvesting protocol using two uniformly accelerating detectors, exploring both stationary and nonstationary configurations as described in Sec. III B.

We first examine the difference between the stationary and nonstationary configurations by plotting concurrence  $C_{AB}/\lambda^2$  and quantum mutual information  $I_{AB}/\lambda^2$  as a function of  $\bar{b}$  in Fig. 4. In these plots, we fix  $\Omega\sigma = 0.1$  and  $L/\sigma = 1$  and consider  $a\sigma = 1$  and  $a\sigma = 2$ . We notice two characteristics: (i) In the vicinity of  $\bar{b} \approx 0$ , stationary detectors consistently harvest greater correlations than nonstationary detectors, for both concurrence and mutual information. (ii) As  $\bar{b}$  becomes larger, both plots begin to

oscillate with  $\bar{b}$ , and the curve representing correlations harvested by nonstationary detectors oscillates around the curve for the stationary case. The frequency of the oscillation increases as  $a\sigma$  grows.

These observations can be traced back to the form of the Wightman functions (37) and (44). Let us recall that the denominators of these expressions contain  $\sinh(x)$  when  $\bar{b} \in [0, 1)$  and transform into  $\sin(x)$  when  $\bar{b} > 1$ . Therefore, within the range  $\bar{b} \in [0, 1)$ , the correlations are characterized by an exponential pattern, while for  $\bar{b} > 1$ , an oscillatory behavior emerges. These traits explain the observation above. In particular, the suppression of correlations near  $\bar{b} \approx 0$  for nonstationary detectors can be attributed to an additional term in the denominator of (44), which is absent in the stationary Wightman function (37). This extra term diminishes the amount of harvested correlations relative to the stationary scenario and simultaneously gives rise to the oscillations noticed in the nonstationary case around the stationary one.

We next examine the acceleration dependence of concurrence  $C_{AB}$  and quantum mutual information  $I_{AB}$  as

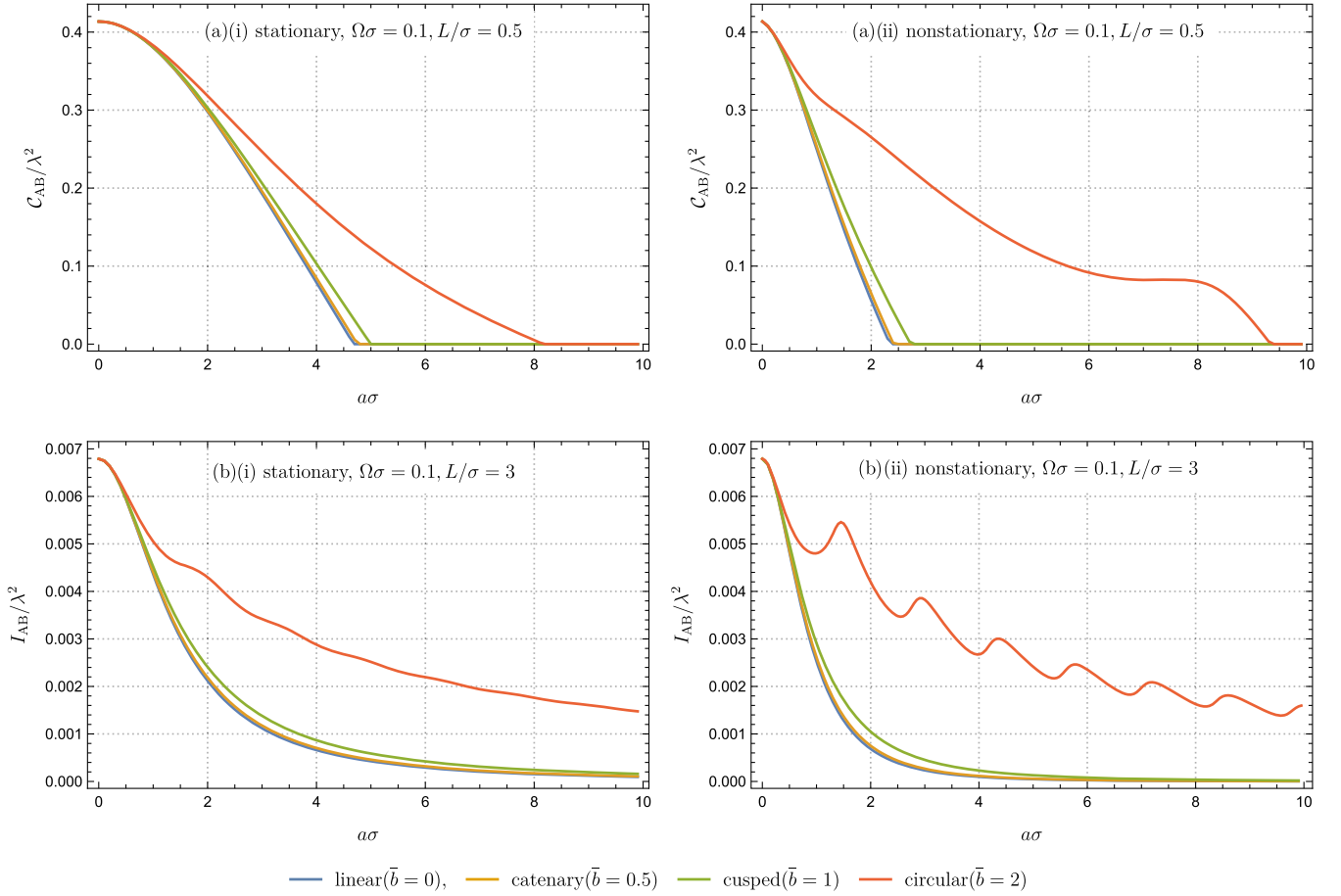


FIG. 5. Concurrence  $C_{AB}/\lambda^2$  with  $\Omega\sigma = 0.1$ ,  $L/\sigma = 0.5$  (a) and quantum mutual information  $I_{AB}/\lambda^2$  with  $\Omega\sigma = 0.1$ ,  $L/\sigma = 3$  (b) as a function of the magnitude of acceleration  $a\sigma$ . (a)(i) and (b)(i) correspond to the stationary configuration while (a)(ii) and (b)(ii) show the nonstationary one, and each figure has four curves indicating four different motions.

illustrated in Figs. 5(a) and (b), respectively. The stationary [Figs. 5(a)(i) and 5(b)(i)] and nonstationary [Figs. 5(a)(ii) and 5(b)(ii)] configurations are depicted, and all four uniformly accelerated motions, linear ( $\bar{b} = 0$ ), catenary ( $\bar{b} = 0.5$ ), cusped ( $\bar{b} = 1$ ), and circular ( $\bar{b} = 2$ ) are shown in each figure.

As we pointed out earlier, the correlations harvested by nonstationary detectors for  $\bar{b} \in [0, 1)$  [Figs. 5(a)(ii) and 5(b)(ii)] decay with increasing  $a\sigma$  faster than those extracted by the stationary detectors [Figs. 5(a)(i) and 5(b)(i)]. Meanwhile, the correlations extracted by nonstationary detectors in circular motion ( $\bar{b} > 1$ ) [Figs. 5(a)(ii) and 5(b)(ii)] exhibit oscillatory behavior around the corresponding stationary curves [Figs. 5(a)(i) and 5(b)(i)].

Another observation we make is that, for both stationary and nonstationary configurations and for any value of  $\bar{b}$ ,  $C_{AB}/\lambda^2$  becomes 0 at sufficiently high  $a\sigma$ . This can be attributed to the high transition probability at large  $a\sigma$  as shown in Fig. 3(a), leading to  $|\mathcal{M}| < \sqrt{\mathcal{L}_{AA}\mathcal{L}_{BB}}$  in (15). Furthermore, the high accelerations prevent the detectors

from extracting quantum mutual information, as  $I_{AB}/\lambda^2 \rightarrow 0$  at  $a\sigma \rightarrow \infty$  in Fig. 5(b). This indicates that any correlations cannot be harvested as  $a\sigma \rightarrow \infty$  if the detectors are uniformly accelerated. These findings are consistent with previous results [8,25–28,47,48], where linearly and circularly accelerated detectors are considered. Our paper extends these insights, providing a more general understanding that encompasses arbitrary uniformly accelerated motion.

### C. Genuine entanglement harvesting

We finally consider how much of entanglement is coming from the quantum field. It is known that the Wightman function can be decomposed into two parts: the anticommutator and the commutator of the field operator. The anticommutator part  $\langle\langle \hat{\phi}(\mathbf{x}), \hat{\phi}(\mathbf{x}') \rangle\rangle_{\rho_\phi}$  (also known as the Hadamard function), where  $\langle\cdot\rangle_{\rho_\phi}$  is the expectation value with respect to the field state  $\rho_\phi$ , depends on the state of the field  $\rho_\phi$ . Conversely, the commutator part  $\langle[\hat{\phi}(\mathbf{x}), \hat{\phi}(\mathbf{x}')]\rangle_{\rho_\phi} = [\hat{\phi}(\mathbf{x}), \hat{\phi}(\mathbf{x}')] \in \mathbb{C}$  (also known as the Pauli-Jordan function) is state independent. This means



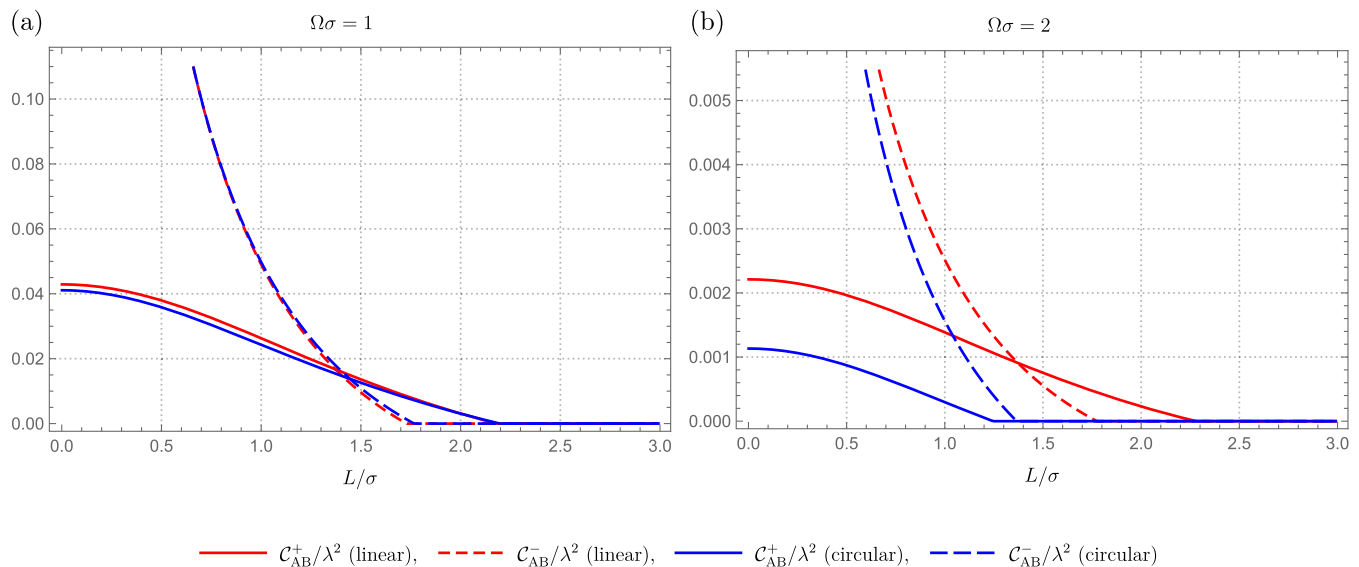


FIG. 7. Harvested and communication-assisted concurrence  $C_{AB}^{\pm}/\lambda^2$  for the stationary linear and circular configurations as a function of the detector separation  $L/\sigma$ . (a) When  $\Omega\sigma = 1$ . The linear and circular cases are very similar and the detectors can harvest genuine entanglement near  $L/\sigma = 2$ . (b) When  $\Omega\sigma = 2$ . Although the linear case does not change much compared to the  $\Omega\sigma = 1$  case, the circular case cannot harvest genuine entanglement anymore. It is the mixture of the anticommutator and commutator contributions, or in the worst case  $C_{AB}^{\pm}/\lambda^2 = 0$  around  $L/\sigma = 1.3$ .

genuine entanglement for  $L/\sigma \in (1.5, 2.2)$  since  $C_{AB}^+ > 0$  while  $C_{AB}^- = 0$  in this region. However, this is not always true as one can see from Fig. 7(b) when  $\Omega\sigma = 2$ . Here, detectors in circular motion can encounter the case where  $C_{AB}^+ = 0$  while  $C_{AB}^- > 0$ , which indicates that the generated entanglement after the interaction is purely coming from the communication and not from the field.

However genuine entanglement can still be extracted in the linear configuration.

## V. CONCLUSION

We carried out the correlation harvesting protocol using two uniformly accelerating UDW detectors in  $(3 + 1)$ -dimensional Minkowski spacetime. According to Letaw [13], trajectories with constant (nonzero) acceleration can be characterized by the magnitude of the acceleration and two torsion parameters, resulting in five classes: linear, catenary, cusped, circular, and helix motions. The first four of these classes of motion are confined to a two-dimensional spatial surface and can be regarded as specific cases of the helix motion. Since two-dimensional configurations are more amenable to experimental setups, we employed these four simpler motions for our analysis.

We first examined the transition probability of a single detector following the four trajectories in Sec. IV A. Utilizing a unified expression for the Wightman functions along these trajectories, we were able to explore the general characteristics that are common to all these motions. We found that the transition probabilities of these motions monotonically increase with the magnitude of acceleration.

Moreover, we also evaluated the effective temperature—an estimator for the temperature as observed by a detector.

We then introduced another UDW detector to consider the correlation harvesting protocol in Sec. IV B. Two configurations were explored: stationary and nonstationary configurations. In the stationary configuration, detectors are separated in the direction perpendicular to their two-dimensional spatial planes of motion. Specifically, the displacement vector pointing from one detector to the other remains orthogonal to the velocity vectors of the detectors. In such a case, the Wightman function along the stationary configuration is time-translation invariant. On the other hand, in the nonstationary configuration, the displacement vector aligns parallel to the planes of motion. This makes the Wightman function nonstationary (i.e., not time-translation invariant). Moreover, while this Wightman function shares a common term with the stationary configuration, an additional term appears that specifically characterizes the nonstationary nature of this configuration.

We found that the harvested correlations—entanglement and total correlations—behave in a distinct manner depending on the motion of the detectors. Specifically, detectors in linear, catenary, and cusped motions within the nonstationary configuration gain fewer correlations compared to those in the stationary configuration. On the other hand, in the circular motion case, both configurations exhibit similar behavior. This difference can be attributed to the Wightman functions. For linear, catenary, and cusped motions, the Wightman function contains hyperbolic functions, leading to an exponential alteration of the results. In contrast, the

Wightman function for circular motion is governed by trigonometric functions.

We also looked into the acceleration dependence of the harvested correlations and concluded (not surprisingly) that high accelerations prevent the detectors from acquiring correlations from the field. This point is consistent with previous papers [8,25–28,47,48], in which linearly and circularly accelerated detectors are considered. Our paper generalized these results to any uniformly accelerating detectors on two-dimensional spatial surfaces.

Finally, we focused on entanglement harvested by the detectors in the stationary configuration and asked how much entanglement is coming from the correlations preexisting in the field. To be precise, the entanglement coming from the commutator part of the Wightman function is state independent, which suggests that the detectors can still be correlated even if the field is not entangled [54]. Thus, it is important to examine how the anticommutator part of the Wightman function (which is state dependent) contributes to the extracted correlations. One way to eliminate the commutator contribution is to use causally disconnected detectors. However, we found that the existence of acceleration prohibits us from extracting correlations with detectors separated far away, no matter what the energy gap is. However we also found the striking result that detectors in causal contact can harvest genuine entanglement in certain parameter regimes.

Our results have important implications for experiment. Attempts to realize the Unruh effect and correlation harvesting generally rely on using laser pulses to probe what are effectively two-dimensional surfaces. To probe the effects of noninertial motion on mutual information and entanglement will therefore involve two detectors (two pulses) in nonstationary configurations, since only these can be realized in a two-dimensional setting. Experimental verification of the harvesting of genuine entanglement would be an exciting confirmation of our understanding of relativistic quantum information.

## ACKNOWLEDGMENTS

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada. K. G.-Y. is thankful to Dr. Jorma Louko for elaborating on the relationship among the uniformly accelerating trajectories.

## APPENDIX: EFFECTIVE TEMPERATURE

Here, we review the concept of effective temperature  $T_{\text{eff}}$  and clarify its relation to the KMS temperature.

Let us first review the KMS temperature. In quantum theory with separable Hilbert spaces, a trace of an operator,  $\text{Tr}[\cdot]$  is well defined. This enables us to consider the Gibbs state at the inverse temperature  $\beta$ ,  $\rho = e^{-\beta\hat{H}}/Z$ , where  $Z := \text{Tr}[e^{-\beta\hat{H}}]$  is the partition function. This is what we consider a thermal state of a system.

However in quantum field theory (QFT), a trace is generally not well defined. Instead, we identify the KMS state [63,64] as a thermal state in QFT. Specifically, if the field is in the KMS thermal state with respect to time  $\tau$  at the inverse KMS temperature  $\beta_{\text{KMS}}$ , the Wightman function satisfies

$$W(\Delta\tau - i\beta_{\text{KMS}}) = W(-\Delta\tau), \quad (\text{A1})$$

where  $\Delta\tau := \tau - \tau'$ . The Fourier transform of this equality with respect to  $\Delta\tau$  reads

$$\tilde{W}(-\omega) = e^{\beta_{\text{KMS}}\omega} \tilde{W}(\omega). \quad (\text{A2})$$

This equality in the Fourier domain is known as the detailed balance condition. Thus, the thermality of a quantum field is imprinted in these equalities.

The thermality can also be implemented in the transition probability of a UDW detector. Recall that the transition probability is written as

$$\mathcal{L} = \lambda^2 \int_{\mathbb{R}} d\tau \int_{\mathbb{R}} d\tau' \chi(\tau) \chi(\tau') e^{-i\Omega(\tau-\tau')} W(\mathbf{x}(\tau), \mathbf{x}(\tau')), \quad (\text{A3})$$

where the subscript in  $\mathcal{L}_{jj}$ ,  $j \in \{A, B\}$  is omitted for simplicity. Let us assume that the switching function  $\chi(\tau)$  has a characteristic time length  $\sigma$ . In our paper, this is the Gaussian width in  $\chi(\tau) = e^{-\tau^2/2\sigma^2}$ . It is convenient to introduce a quantity related to the transition probability known as the *response function* (divided by the characteristic time length)  $\mathcal{F}(\Omega, \sigma)$ :

$$\begin{aligned} \mathcal{L} &= \lambda^2 \sigma \mathcal{F}(\Omega, \sigma), \\ \mathcal{F}(\Omega, \sigma) &:= \frac{1}{\sigma} \int_{\mathbb{R}} d\tau \int_{\mathbb{R}} d\tau' \chi(\tau) \chi(\tau') e^{-i\Omega(\tau-\tau')} W(\mathbf{x}(\tau), \mathbf{x}(\tau')). \end{aligned} \quad (\text{A4})$$

If the field is in the KMS state and the switching function is a rapidly decreasing function, such as a Gaussian function, then the response function in the long interaction limit obeys the detailed balance relation [62]:

$$\lim_{\sigma \rightarrow \infty} \frac{\mathcal{F}(-\Omega, \sigma)}{\mathcal{F}(\Omega, \sigma)} = e^{\beta_{\text{KMS}}\Omega}. \quad (\text{A5})$$

Note that this relation holds when the long interaction limit is taken. On the other hand, if  $\sigma$  is not sufficiently long, the ratio of the response function (sometimes known as the excited-to-deexcited ratio) does not satisfy the detailed balance condition.

From this relation, one can define the *effective temperature* as

$$T_{\text{eff}}^{-1} := \frac{1}{\Omega} \ln \frac{\mathcal{F}(-\Omega, \sigma)}{\mathcal{F}(\Omega, \sigma)}. \quad (\text{A6})$$

Note that the effective temperature is not necessarily the KMS temperature. If the field is in the KMS state and the long interaction limit is taken, then the effective

temperature becomes the KMS temperature. In this sense, the effective temperature is an estimator for the field's temperature.

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