Dark matter effects in modified teleparallel gravity

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This work investigates dark matter (DM) effects in compact objects in modified teleparallel gravity (MTG) in which a modification of teleparallel equivalent to general relativity is used. We applied a tetrad to the modified field equations where a set of relations is found. The conservation equation allows us to rewrite our Tolman-Oppenheimer-Volkoff equations with an effective gravitational coupling constant. As input to these new equations, we use a relativistic mean-field (RMF) model with dark matter content included, obtained from a Lagrangian density with both, hadronic and dark particle degrees of freedom, as well as the Higgs boson, used as a mediator in both sectors of the theory. Through numerical calculations, we analyze the mass-radius diagrams obtained from different parametrizations of the RMF-DM model, generated by assuming different values of the dark particle Fermi momentum and running the free parameter coming from the MTG. Our results show that it is possible for the system simultaneously support more DM content, and be compatible with recent astrophysical data provided by the LIGO and Virgo Collaborations, as well as by NASA's Neutron star Interior Composition Explorer (NICER).

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I. INTRODUCTION

The problem of galaxy rotation curves is possibly one of the most iconic indications that general relativity (GR) in the presence of ordinary matter is not the final theory of gravitation [1–3]. Indeed, in order to solve this (and other) problem(s), modifications to GR have become a trend and two main lines of research have emerged.

One of them consists in modifying the theory of gravitation. In this branch, one can keep the underlying Riemann manifold and modify the gravitational action integral, for instance, by taking an action that is proportional to a function of the scalar curvature (R) and/or the Ricci ($R_{\mu\nu}$) and Riemann ($R_{\mu\nu\rho\sigma}$) tensors: $f(R)$,$f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$, and so on [4–10]. In other proposals, the manifold is changed. Of particular interest here are the theories built on a Weitzenböck manifold—the so-called "teleparallel theories" [11–22]. In this manifold, gravitation is manifest by means of torsion instead of curvature. Many theories can be built in the Weitzenböck manifold, including one that is equivalent to GR, known as the "Teleparallel Equivalent of General Relativity" (TEGR). The gravitational action in this case is built as a specific combination of three quadratic invariants of the torsion tensor. Different combinations of these invariants lead to different theories where the equivalence with GR is no longer valid. The criticisms concerning modified theories of gravity usually claim the lack of consistency to describe simultaneously systems of stellar, galaxy, and cosmological scales [23].

The second line of research explores the modification of the matter content in the context of GR. The models propose the existence of nonordinary matter which does not interact with (ordinary) matter by means of the known interactions of the Standard Model of particles. Some candidates for dark matter may eventually interact weakly, like WIMPs [24,25]. For galaxy rotation curves, dark matter (DM) has been considered a strong candidate to solve this problem. The criticism concerning the existence of DM is the lack of evidence from the point of view of particle physics.

Usually, these two lines of research compete with each other and are considered in excluding scenarios. In other words, few proposals (as far as the authors are concerned) consider that a final solution to the problems presented by GR may be found in a "joint venture" with both modified gravity and DM. This is what shall be explored in this work, where the effects of both modified gravity and DM will be analyzed in a neutron star. This system is particularly interesting since this is an extremely dense object and the relativistic effects are very relevant.

The modified gravity model considered here is the same presented in Ref. [26], which is a teleparallel theory with arbitrary coefficients for the three quadratic invariants of the torsion tensor. For the solution of the modified Tolman-Oppenheimer-Volkoff equations generated from this theory, it is required the knowledge of suitable equations of state. For this purpose, we use here a realistic parametrization of a widely used hadronic model [27–29] with dark matter included along with protons, neutrons, and leptons. We show how the teleparallel theory favors the increasing of the dark matter amount in the system, and verify that it is possible to generate neutron stars in agreement with very recent astrophysical observational data, such as those related to gravitational waves detection, performed by LIGO and Virgo Collaboration [30–33], those furnished by the NASA's Neutron star Interior Composition Explorer (NICER) mission regarding the pulsars PSR $J0030+0451$ [34,35] and PSR $J0740+6620$ [36,37] (where we also adding the data extracted from [38] for the last one), and the PSR J0952-0607 [39], discovered by Bassa *et al.* in 2017 [40]. We present all these findings in the following way. In Sec. II we briefly revise the derivation of the teleparallel theory developed in Ref. [26]. In Sec. III we provide the main equations of state obtained from the relativistic mean-field model in which we also include interacting dark matter through the Higgs mechanism. In Sec. IV we show the mass-radius diagrams determined from the teleparallel theory and discuss the importance of this theory for the total DM content included in the system. We finalize the paper in Sec. V by presenting a summary and our concluding remarks.

II. MODIFIED TELEPARALLEL GRAVITY

Differently from GR where gravity is represented by the curvature of spacetime, teleparallel theories describe gravity solely by the spacetime torsion. Torsion is the antisymmetric part of the connection and is represented by

$$\Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu} = T^{\rho}{}_{\mu\nu}, \tag{1}$$

and in GR, it is null. The connection components in GR are given by the Christoffel symbols, which are completely determined by the metric tensor, $g_{\mu\nu}$, and its derivatives. In this sense, $g_{\mu\nu}$ plays the role of the fundamental field in GR and others theories that are constructed on the Riemannian manifold. In teleparallel theories, the connection is called "Weitzenböck connection" and is completely determined by the tetrad field, $e^{a}{}_{\mu}$ and its derivatives; in other words, the tetrad is the fundamental field. From a geometrical point of view, the tetrad maps objects of the spacetime manifold in their corresponding counterparts of the tangent space; for instance, the spacetime metric and the tangent metric are mapped with the help of the tetrad by the contraction,

$$g_{\mu\nu} = \eta_{ab} e^{a}{}_{\mu} e^{b}{}_{\nu}. \tag{2}$$

As mentioned previously, the Weitzenböck connection is determined by the tetrad and its derivative,

$$\Gamma^{\rho}_{\mu\nu} = e^{\rho}{}_{a} \partial_{\mu} e^{a}{}_{\nu}. \tag{3}$$

While in GR the Lagrangian is essentially the Ricci scalar, in teleparallel gravity the Lagrangian is built with a linear combination of quadratic invariants of the torsion tensor, Eq. (1). There are only three quadratic invariants that can be constructed with torsion. In our model we propose a general combination of these invariants [26],

$$\mathcal{L} = -\frac{\beta_1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} - \frac{\beta_2}{2} T^{\rho\mu\nu} T_{\mu\rho\nu} + \beta_3 T^{\rho} T_{\rho}. \tag{4}$$

The difference between our Lagrangian and that from TEGR is the presence of three free parameters, namely, β_1, β_2 and β_3—in TEGR, $\beta_1 = \beta_2 = \beta_3 = 1$. The variational principle for Eq. (4) leads us to a modified set of field equations (in comparison with TEGR), although the general structure is preserved,

$$\partial_{\rho}\left(4e\Sigma_{f}{}^{\lambda\rho}\right) + 4e\Sigma_{d}{}^{\lambda\rho} T^{d}{}_{f\rho} - e e^{\lambda}_{f} \Sigma_{ijk} T^{ijk} = -2\chi e e^{\rho}_{f} T^{\lambda}{}_{\rho}. \tag{5}$$

Above, $e = \det e^{a}{}_{\mu}$ and the superpotential $\Sigma^{\mu\nu\rho}$ carries the free parameters in its definition,

$$\Sigma^{\mu\nu\rho} \equiv \frac{\beta_1}{4} T^{\mu\nu\rho} + \frac{\beta_2}{4}\left(T^{\nu\mu\rho} - T^{\rho\mu\nu}\right) + \frac{\beta_3}{2}\left(g^{\mu\rho} T^{\nu} - g^{\mu\nu} T^{\rho}\right). \tag{6}$$

In this sense, TEGR is a special case of our modified teleparallel gravity (MTG). It is worth mentioning that MTG and new general relativity (NGR) are equivalents when our free parameters relate to those ones from NGR, as can be verified in [26].

When we apply a tetrad for static spherical objects,

$$e^{a}{}_{\mu} = \begin{pmatrix} \gamma_{00} & 0 & 0 & 0 \\ 0 & \gamma_{11}\sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ 0 & \gamma_{11}\sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ 0 & \gamma_{11}\cos\theta & -r\sin\theta & 0 \end{pmatrix} \tag{7}$$

we find a set of three independent equations which are compatible with the conservation equation only under a specific condition, namely,

$$2\beta_3 - \beta_1 - \beta_2 = 0. \tag{8}$$

Equation (8) is obviously respected by TEGR. This constraint is very useful to rewrite the set of equations [cf. Eq. (9) in Ref. [26]] with a redefinition of the gravitational coupling constant as an effective one, $\frac{\chi}{\beta_3} = \bar{\chi}$. Similar approaches with the redefinition of the gravitational coupling have been investigated in the literature. In [41], for instance, the authors analyze a running gravitational constant, i.e., G has a spacetime dependence, which is relevant in the context of quantum gravity. In the present case, our redefinition is spacetime independent, but may eventually have implications concerning different applications, e.g., in a quantized version of the theory. This is a subject for future investigation. We obtain a set of equations that resemble the TOV equations of GR,

$$\begin{cases} \bar{\chi}\gamma_{11}^3\rho r^2 = 2\gamma_{11}' r + \gamma_{11}^3 - \gamma_{11} \\ -\bar{\chi}\gamma_{00}\gamma_{11}^2 p r^2 = 2\gamma_{00}' r - \gamma_{00}\gamma_{11}^2 + \gamma_{00}\,. \\ p' = -(p+\rho)\frac{1}{\gamma_{00}}\gamma_{00}' \end{cases} \tag{9}$$

The first and second equations of (9) are both dependent on the effective gravitational coupling constant. Besides, the first equation is dependent on the energy density while the second one is dependent on the pressure. The third equation depends on both the pressure and energy density and it is the covariant conservation equation of the energy-momentum tensor. As we see, the difference between MTG and TEGR lies in the free parameter β_3 dividing χ as stated above. As consequence, we do not see differences in the field equations between those of our model and those from GR in a vacuum. So, the form of the Schwarzschild solution remains as the one of GR, as well as the description of planetary motions, deflection of light rays, and other applications of this solution without matter—the difference may appear in terms of boundary conditions only. The structure of the equations allows us to use similar boundary conditions on a star to those used in GR.

The first equation of Eq. (9) becomes

$$u' = \frac{1}{2}\bar{\chi}\rho r^2, \tag{10}$$

when the following change of variable is introduced:

$$\frac{1}{\gamma_{11}^2} = 1 - \frac{2u(r)}{r}. \tag{11}$$

The combination of the last two equations of (9) and (11) allows us to correlate pressure and energy density with the derivative of the pressure in presence of the effective $\bar{\chi}$,

$$p' = -\frac{p+\rho(p)}{(r^2-2ur)}\left(\frac{1}{2}\bar{\chi}pr^3 + u\right). \tag{12}$$

As expected, Eqs. (10) and (12) are analogous to those of GR (TOV equations) with the effective gravitational coupling constant. In this sense, the mass distribution of a compact object is completely determined by both expressions, and the equations of state (EOS) used as input.

III. HADRONIC MODELS WITH DARK MATTER COMPONENT

The Lagrangian density related to the hadronic part of the model used in MTG reads

$$\begin{aligned} \mathcal{L}_{\mathrm{HAD}} = {} & \bar{\psi}(i\gamma^\mu\partial_\mu - M_{\mathrm{nuc}})\psi + g_\sigma\sigma\bar{\psi}\psi - g_\omega\bar{\psi}\gamma^\mu\omega_\mu\psi \\ & - \frac{g_\rho}{2}\bar{\psi}\gamma^\mu\vec{b}_\mu\vec{\tau}\psi + \frac{1}{2}(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2) - \frac{A}{3}\sigma^3 - \frac{B}{4}\sigma^4 \\ & - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{B}^{\mu\nu}\vec{B}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{b}_\mu\vec{b}^\mu, \end{aligned} \tag{13}$$

with the nucleon and exchanged mesons (σ, ω, and ρ) fields given by ψ, σ, ω^μ, and $\vec{b}_\mu$, respectively, with masses denoted by M_{nuc}, m_σ, m_ω, and m_ρ. The free parameters of the model are the coupling constants g_σ, g_ω, g_ρ, A, and B. Finally, the tensors presented in Eq. (13) are $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ and $\vec{B}_{\mu\nu} = \partial_\mu\vec{b}_\nu - \partial_\nu\vec{b}_\mu$. We address the reader to Refs. [27–29] for more details regarding such kind of relativistic mean-field (RMF) models applied to symmetric and asymmetric nuclear matter.

Concerning DM, we remark that one possibility of coupling this component with ordinary matter comes from the Higgs sector of the theory, in which one considers, for the present case, DM particles and nucleons simultaneously exchanging Higgs bosons. This procedure has been performed recently as one can see, for instance, in Refs. [42–59]. In this case, the entire system, namely, hadrons and DM, is described by the following Lagrangian density:

$$\begin{aligned} \mathcal{L} = {} & \bar{\chi}\left(i\gamma^\mu\partial_\mu - M_\chi\right)\chi + \xi h\bar{\chi}\chi + \frac{1}{2}\left(\partial^\mu h\partial_\mu h - m_h^2h^2\right) \\ & + f\frac{M_{\mathrm{nuc}}}{v}h\bar{\psi}\psi + \mathcal{L}_{\mathrm{HAD}}, \end{aligned} \tag{14}$$

with the Dirac field χ representing the dark fermion of mass $M_\chi = 200$ GeV (lightest neutralino), and the scalar field h denoting the Higgs boson with mass $m_h = 125$ GeV. The strength of the Higgs-nucleon interaction is regulated by the quantity fM_{nuc}/v, with $v = 246$ GeV being the Higgs vacuum expectation value. The Higgs-dark particle coupling is controlled by the constant ξ. The field equations for the model are obtained by the use of the mean-field approximation [27,28], which consists in taking $\sigma \to \langle\sigma\rangle \equiv \sigma$, $\omega_\mu \to \langle\omega_\mu\rangle \equiv \omega_0$, $\vec{b}_\mu \to \langle\vec{b}_\mu\rangle \equiv b_{0(3)}$, $h \to \langle h\rangle \equiv h$. The final forms of these equations are given as follows:

$$m_\sigma^2\sigma = g_\sigma n_s - A\sigma^2 - B\sigma^3, \tag{15}$$

$$m_\omega^2\omega_0 = g_\omega n, \tag{16}$$

$$m_\rho^2 b_{0(3)} = \frac{g_\rho}{2}n_3, \tag{17}$$

$$\left[\gamma^\mu(i\partial_\mu - g_\omega\omega_0 - g_\rho b_{0(3)}\tau_3/2) - M^*\right]\psi = 0, \quad (18)$$

$$m_h^2 h = \xi n_s^{\rm DM} + f\frac{M_{\rm nuc}}{v}n_s \quad (19)$$

$$\left(\gamma^\mu i\partial_\mu - M_\chi^*\right)\chi = 0, \quad (20)$$

with $\tau_3 = 1$ for protons and -1 for neutrons, and effective dark particle and nucleon masses written as

$$M_\chi^* = M_\chi - \xi h, \quad (21)$$

and

$$M^* = M_{\rm nuc} - g_\sigma\sigma - f\frac{M_{\rm nuc}}{v}h, \quad (22)$$

respectively. The densities, or the sources of the fields, equivalently, are $n_s = n_{sp} + n_{sn}$, $n = n_p + n_n$, $n_3 = \langle\bar\psi\gamma^0\tau_3\psi\rangle = n_p - n_n = (2y_p - 1)n$,

$$n_s^{\rm DM} = \langle\bar\chi\chi\rangle = \frac{\gamma M_\chi^*}{2\pi^2}\int_0^{k_F^{\rm DM}}\frac{k^2dk}{(k^2+M_\chi^{*2})^{1/2}}, \quad (23)$$

and

$$n_{s_{p,n}} = \langle\bar\psi_{p,n}\psi_{p,n}\rangle = \frac{\gamma M^*}{2\pi^2}\int_0^{k_{F\,p,n}}\frac{k^2dk}{(k^2+M^{*2})^{1/2}}. \quad (24)$$

The symbols p, n stand for protons and neutrons, and $\gamma = 2$ is the degeneracy factor. The proton fraction is $y_p = n_p/n$, and the nucleon densities are $n_{p,n} = \gamma k_{F\,p,n}^3/(6\pi^2)$, with $k_{F\,p,n}$ and $k_F^{\rm DM}$ being, respectively, the Fermi momenta related to protons/neutrons and to the dark particle. With regard to this last quantity, we keep it as a free parameter in our analysis performed in the next section. Another possible approach for inclusion of DM in the system consists in considering DM particles interacting only through gravity in a two-fluid formalism, see for instance [60–64] for more details.

By using Eq. (14) it is possible to construct the main equations of state of the hadronic system with DM content, namely, energy density and pressure, both determined from the energy-momentum tensor associated to $\mathcal{L}$, $T^{\mu\nu}$, as $\epsilon = \langle T_{00}\rangle$ and $P = \langle T_{ii}\rangle/3$. They are explicitly written as

$$\epsilon = \frac{m_\sigma^2\sigma^2}{2} + \frac{A\sigma^3}{3} + \frac{B\sigma^4}{4} - \frac{m_\omega^2\omega_0^2}{2} - \frac{m_\rho^2 b_{0(3)}^2}{2} + g_\omega\omega_0\rho + \frac{g_\rho}{2}b_{0(3)}n_3 + \frac{m_h^2h^2}{2} + \epsilon_{\rm kin}^p + \epsilon_{\rm kin}^n + \epsilon_{\rm kin}^{\rm DM}, \quad (25)$$

and

$$P = -\frac{m_\sigma^2\sigma^2}{2} - \frac{A\sigma^3}{3} - \frac{B\sigma^4}{4} + \frac{m_\omega^2\omega_0^2}{2} + \frac{m_\rho^2 b_{0(3)}^2}{2} - \frac{m_h^2h^2}{2} + P_{\rm kin}^p + P_{\rm kin}^n + P_{\rm kin}^{\rm DM}, \quad (26)$$

with respective kinetic contributions,

$$\epsilon_{\rm kin}^{\rm DM} = \frac{\gamma}{2\pi^2}\int_0^{k_F^{\rm DM}} k^2\left(k^2+M_\chi^{*2}\right)^{1/2}dk, \quad (27)$$

$$P_{\rm kin}^{\rm DM} = \frac{\gamma}{6\pi^2}\int_0^{k_F^{\rm DM}}\frac{k^4dk}{\left(k^2+M_\chi^{*2}\right)^{1/2}}, \quad (28)$$

$$\epsilon_{\rm kin}^{p,n} = \frac{\gamma}{2\pi^2}\int_0^{k_{F\,p,n}} k^2\left(k^2+M^{*2}\right)^{1/2}dk, \quad \text{and} \quad (29)$$

$$P_{\rm kin}^{p,n} = \frac{\gamma}{6\pi^2}\int_0^{k_{Fn,p}}\frac{k^4dk}{\left(k^2+M^{*2}\right)^{1/2}}. \quad (30)$$

We use the NL3* parametrization, defined in Ref. [65], for the hadronic part of the system. This parameter set was studied recently along with more than 400 other ones and has been verified as a model compatible with neutron stars, and finite nuclei properties [66]. For the latter, it was analyzed in Ref. [66] data related to ground state binding energies, charge radii, and giant monopole resonances of a set of spherical nuclei, namely, ^{16}O, ^{34}Si, ^{40}Ca, ^{48}Ca, ^{52}Ca, ^{54}Ca, ^{48}Ni, ^{56}Ni, ^{78}Ni, ^{90}Zr, ^{100}Sn, ^{132}Sn, and ^{208}Pb.

IV. NEUTRON STARS PROPERTIES (MASS-RADIUS PROFILES)

In Sec. II, we demonstrated that MTG provides a set of equations totally equivalent to those from TEGR when the free parameter, β_3, is equal to unity. As stated above, different values of β_3 give us a renormalized gravitational coupling constant, as presented in the teleparallel equations (TPE), namely, Eqs. (10) and (12). In this sense, in our model, this free parameter plays a central role, and it is important to verify how it modifies the mass-radius profiles when dark matter is considered in the system. In order to obtain such profiles, it is necessary to solve the TPE and for this aim, we furnish as input the total energy density and the total pressure as input, both written as

$$\rho = \epsilon + \frac{\mu_e^4}{4\pi^2} + \frac{1}{\pi^2}\int_0^{\sqrt{\mu_\mu^2-m_\mu^2}} dk k^2\left(k^2+m_\mu^2\right)^{1/2} \quad (31)$$

and

$$p = P + \frac{\mu_e^4}{12\pi^2} + \frac{1}{3\pi^2}\int_0^{\sqrt{\mu_\mu^2-m_\mu^2}}\frac{dkk^4}{\left(k^2+m_\mu^2\right)^{1/2}}, \quad (32)$$

with the two last terms of each equation representing, respectively, massless electrons of chemical potential μ_e, and muons with mass $m_\mu = 105.7$ MeV and chemical potential μ_μ. Besides that, the system is submitted to charge neutrality and chemical equilibrium, conditions represented by

$$n_p - n_e = n_\mu \tag{33}$$

and

$$\mu_n - \mu_p = \mu_e, \tag{34}$$

where $\mu_e = (3\pi^2 n_e)^{1/3}$, $n_\mu = [(\mu_\mu^2 - m_\mu^2)^{3/2}]/(3\pi^2)$, and $\mu_\mu = \mu_e$. Electrons and muons densities are denoted by n_e and n_μ. The solution of the TPE is constrained to $p(r = 0) = p_c$ (central pressure) and $u(r = 0) = 0$. On the other hand, at the star surface, we have $p(r = R) = 0$ and $u(r = R) \equiv M$, with R defining the radius of the star. With regard to the neutron star crust, we use for this region an equation of state named BPS and provided by Baym, Pethick, and Sutherland [67], in a density range of $0.158 \times 10^{-10}\ \mathrm{fm}^{-3} \leq n \leq 0.891 \times 10^{-2}\ \mathrm{fm}^{-3}$.

In order to analyze how the neutron star properties behave with the increase of dark matter contribution to the system, an important step is to provide mass-radius diagrams with fixed values for the free parameter β_3, as depicted in Fig. 1. From this figure, it is possible to verify that the maximum neutron star mass decreases when more dark matter content is included in the system, for all β_3 values presented. This feature is observed even for the case of $\beta_3 = 1$ [Fig. 1(b), general relativity] and was reported already in recent literature, see Refs. [44–50] for instance. Consequently, the neutron stars' radii decrease with k_F^{DM} and there will be a natural limit for this quantity to enable the agreement between the mass-radius profiles and recent observational astrophysical data (band regions also displayed in the figure). Furthermore, by analyzing the MTG studied here, i.e., $\beta_3 \neq 1$, we notice that an increase of β_3 induces an increase in both, mass and radius for the neutron stars generated. This feature can be more easily identified in Fig. 2. In this figure, we plot the mass-radius profiles by keeping fixed DM content in each panel. The case of no DM included in the system, $k_F^{\mathrm{DM}} = 0$, is also shown in Fig. 2(a) for comparison. In summary, we verify that the decrease of the neutron stars' mass caused by the increase of the DM contribution is balanced by the increase of β_3. Therefore, it is possible for the system to support more DM in comparison with the general relativity case. For the sake of completeness, we provide in Table I the values obtained for β_3 that produced mass-radius diagrams fully compatible with each particular astrophysical constraint analyzed, namely, gravitational waves data related to the GW170817 [30,31], GW1908014 [32], and GW190425 [33] events, some of them provided by the LIGO and Virgo Collaboration; data from the NASA's Neutron star Interior

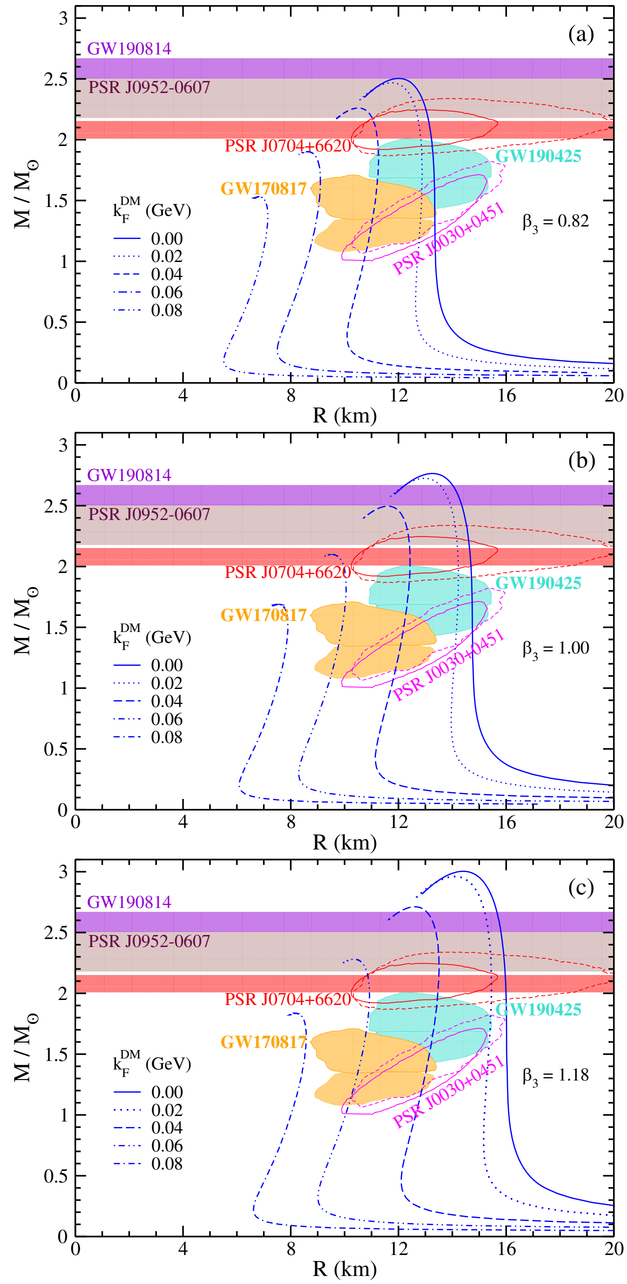

FIG. 1. Mass-radius diagrams constructed from the NL3* parametrization for different values of k_F^{DM} and β_3, namely, (a) $\beta_3 = 0.82$, (b) $\beta_3 = 1.00$, and (c) $\beta_3 = 1.18$. The contours are related to data from the NICER mission, namely, PSR J0030 + 0451 [34,35] and PSR J0740 + 6620 [36,37], the GW170817 event [30,31], and the GW190425 event [33], all of them at 90% credible level. The red (brown) horizontal lines are related to PSR J0740 + 6620 [38] (PSR J0952 + 0607 [39]). The recent observational constraint on neutron star mass, GW190814 [32], is shown as the violet horizontal lines.

Composition Explorer (NICER) mission regarding the pulsars PSR J0030 + 0451 [34,35] and PSR J0740 + 6620 [36,37], data from the latter pulsar extracted from Ref. [38], and data from PSR J0952-0607 [39].

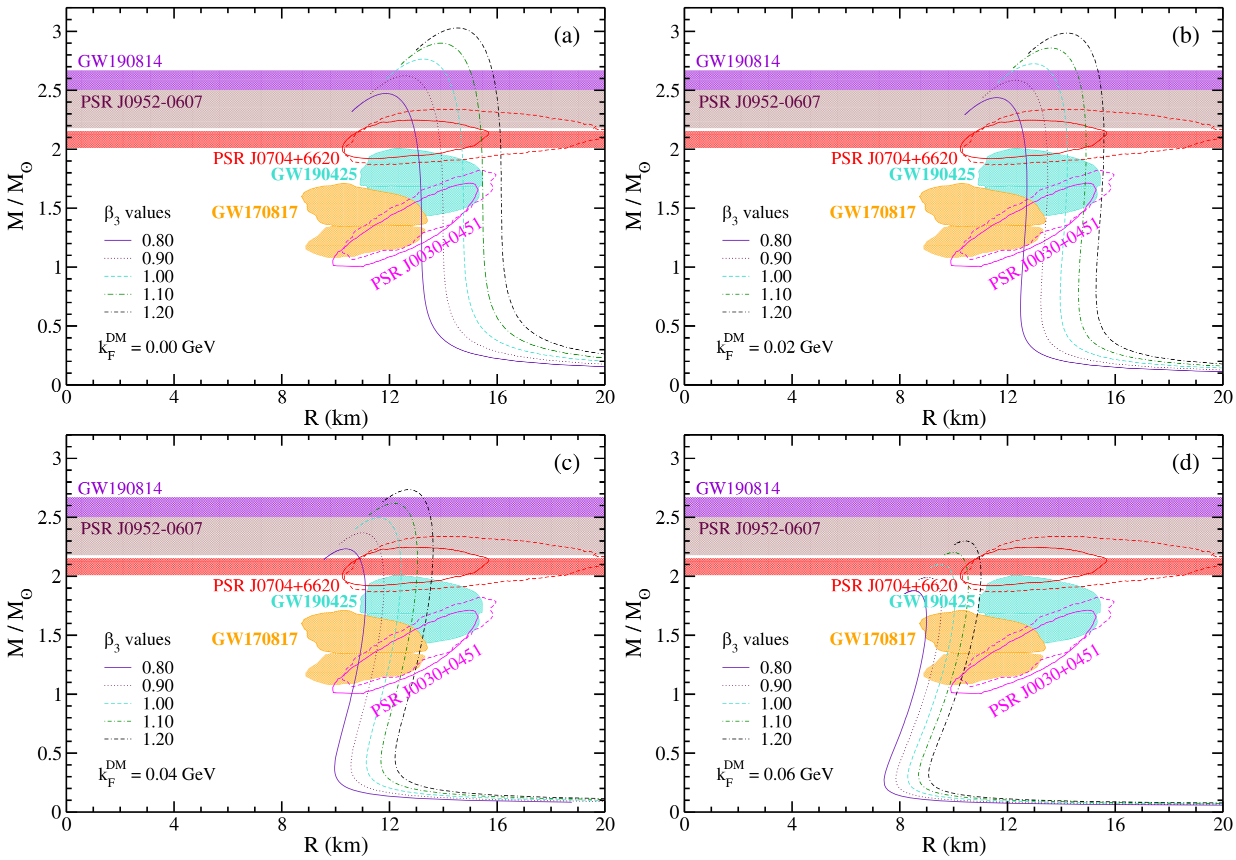

FIG. 2. Mass-radius diagrams constructed from the NL3* parametrization for different values of β_3 and k_F^{DM}, namely, (a) $k_F^{\mathrm{DM}} = 0$ (no DM included), (b) $k_F^{\mathrm{DM}} = 0.02$ GeV, and (c) $k_F^{\mathrm{DM}} = 0.04$ GeV, and (d) $k_F^{\mathrm{DM}} = 0.06$ GeV. The contours are the same exhibited in Fig 1.

V. SUMMARY AND CONCLUDING REMARKS

In this work, we have analyzed the effects of dark matter in neutron stars in the context of a modified teleparallel gravity theory. The dark particle considered here, namely, the lightest neutralino, is described by a fermion field that interacts with the Higgs boson. This mediator is also allowed to interact with ordinary matter, more specifically, with the nucleon Dirac fermion field. The equation of state of the complete fluid comprising dark matter, leptons, hadrons, and Higgs bosons is dependent on the dark matter Fermi momentum and was used to solve TOV-like equations in the context of a modified theory of gravity. The latter, by its turn, is an extension of the TEGR theory built with a linear combination with arbitrary coefficients (β_1, β_2, β_3) of the three quadratic invariants that compose the TEGR Lagrangian. In order to be compatible with the covariant conservation of the energy-momentum tensor, a constraint is imposed on these three arbitrary coefficients. This leaves us with only one parameter that can be interpreted as a modulation of the gravitational coupling constant.

With the set of teleparallel field equations and the input EOS, several mass-radius diagrams were built for different values of the parameters k_F^{DM} (Fermi momentum of the dark particle) and β_3. These curves were plotted against the contours related to data obtained from several observations/measurements (NICER, PSR J0030 + 0451, PSR J0740+6620, PSR J0920-0607, GW170817, GW190814, and GW190425), which allowed us to establish limits for β_3 for different values of k_F^{DM}. We verified that the maximum neutron star mass decreases with k_F^{DM}, while it increases with β_3. Essentially, we conclude that $\beta_3 > 1$ allows us to accommodate more dark matter content in the neutron star in comparison with the case of GR ($\beta_3 = 1$).

As a next step, we intend to use Bayesian inference in order to establish a confidence region in the parameter space k_F^{DM} vs β_3 which allows us to accommodate the available astrophysical data. Other subject that we plan to pursue in the context of our model is the analysis of the power irradiated by gravitational waves. We may expect modifications in the neutron star quadrupole moment, a nontrivial modification that can be seen, for instance, in [68]. This investigation can also be useful to impose limits for β_3. An analogous study of the multipole moments, but in the context of the usual general relativity, is detailed in [69].

TABLE I. Minimum and maximum neutron stars properties values of β_3 for different DM contributions.

	β_3		$M_{\max}$ ($M_\odot$)		$R_{\max}$ (km)		$R_{1.4}$ (km)		$R_{1.6}$ (km)	
Events	min	max	min	max	min	max	min	max	min	max
$k_F^{DM} = 0$										
GW190814	0.820	0.930	2.50359631	2.66623759	12.0172291	12.7888708	13.3525	14.2243	13.3423	14.2110
PSR J0952-0607	0.620	0.830	2.17697382	2.51881599	10.4485979	12.0859098	11.5989	13.4325	11.5595	13.4201
Lines PSR J0704 + 6620	0.530	0.610	2.01277542	2.15934634	9.65172768	10.3638020	10.7030	11.5007	10.6354	11.4581
Elipses PSR J0704 + 6620	0.540	1.940	2.03167510	3.85086465	9.74935341	18.4813080	10.808	20.6104	10.7480	20.5682
GW170817	0.360	0.830	1.65885580	2.51881599	7.95636225	12.0859098	8.717	13.4325	7.3428	13.4201
PSR J0030 + 0451	0.450	1.170	1.85465717	2.99054480	8.89420128	14.3538313	9.8325	15.9564	9.7129	15.9466
GW190425	0.560	1.100	2.06895661	2.89970446	9.92798615	13.9128246	11.0163	15.4718	10.9546	15.4677
$k_F^{DM} = 0.02$ (GeV)										
GW190814	0.840	0.960	2.49781656	2.67027831	11.8806238	12.7037411	12.9880	13.8587	13.0254	13.9021
PSR J0952-0607	0.640	0.850	2.18027306	2.51264048	10.3661957	11.9541845	11.3701	13.0552	11.3751	13.1045
Lines PSR J0704 + 6620	0.540	0.620	2.00270867	2.14593601	9.53037167	10.2087727	10.4524	11.1946	10.4218	11.1941
Elipses PSR J0704 + 6620	0.560	2.110	2.03945875	3.95879102	9.70107079	18.8505650	10.6443	20.3245	10.6204	20.3879
GW170817	0.390	0.900	1.70197511	2.58548570	8.09228802	12.2949686	8.8383	13.4310	7.2224	13.4781
PSR J0030 + 0451	0.490	1.270	1.90773892	3.07130361	9.07198238	14.6153212	9.9559	15.8664	9.8934	15.9381
GW190425	0.590	1.190	2.09337449	2.97299647	9.95231247	14.1419039	10.9261	15.3778	10.9112	15.4337
$k_F^{DM} = 0.04$ (GeV)										
GW190814	1.010	1.150	2.50870562	2.67693567	11.6610308	12.4410200	12.1719	12.9050	12.3284	13.0723
PSR J0952-0607	0.760	1.020	2.17618489	2.52109432	10.1185741	11.7144194	10.6960	12.2292	10.8054	12.3841
Lines PSR J0704 + 6620	0.650	0.750	2.01254535	2.16182041	9.35384083	10.0492163	9.9533	10.6358	10.0134	10.7365
Elipses PSR J0704 + 6620	0.700	2.940	2.08851695	4.28018618	9.71084118	19.9007397	10.3025	19.7409	10.3852	19.9734
GW170817	0.520	1.260	1.80007529	2.80203962	8.36620808	13.0260334	8.9536	13.4546	8.9291	13.6253
PSR J0030 + 0451	0.680	1.750	2.05846477	3.30223536	9.57201767	15.3552532	10.1649	15.5988	10.2395	15.8054
GW190425	0.770	1.640	2.19045520	3.19676661	10.1774836	14.8625278	10.7612	15.1503	10.8667	15.3445
$k_F^{DM} = 0.06$ (GeV)										
GW190814	1.420	1.620	2.50162792	2.67199755	11.3580246	12.1238403	11.4172	12.0707	11.6343	12.3132
PSR J0952-0607	1.080	1.450	2.18167949	2.52791548	9.90790367	11.4754133	10.1538	11.5167	10.3285	11.7464
Lines PSR J0704 + 6620	0.920	1.050	2.01359892	2.15116501	9.14256859	9.76032639	9.4690	10.0302	9.6018	10.2012
Elipses PSR J0704 + 6620	1.040	4.860	2.14089680	4.62804079	9.71743011	21.0155964	9.9891	19.3389	10.1557	19.6844
GW170817	0.760	2.090	1.83014655	3.03495455	8.31087303	13.7746029	8.6954	13.4619	8.7608	13.7331
PSR J0030 + 0451	1.120	2.890	2.22171354	3.56884742	10.0869808	16.2046146	10.3145	15.4511	10.4914	15.7632
GW190425	1.190	2.700	2.29008985	3.44953799	10.4002209	15.6645765	10.5913	15.0063	10.7776	15.3124
$k_F^{DM} = 0.08$ (GeV)										
GW190814	2.190	2.500	2.50054669	2.67167068	11.1639242	11.9200392	10.9668	11.5811	11.2282	11.8575
PSR J0952-0607	1.660	2.230	2.17704272	2.52327943	9.71061611	11.2650080	9.7836	11.0483	9.9881	11.3109
Lines PSR J0704 + 6620	1.410	1.620	2.00642300	2.15065336	8.95533466	9.60227871	9.1359	9.6852	9.2997	9.8879
Elipses PSR J0704 + 6620	1.690	8.420	2.19662666	4.90307999	9.80101395	21.8903275	9.8534	19.1368	10.0695	19.5481
GW170817	1.230	3.630	1.87398231	3.21933556	8.35858822	14.3675871	8.6139	13.4932	8.7449	13.8208
PSR J0030 + 0451	1.930	4.980	2.34742427	3.77074862	10.4772730	16.8335056	10.4136	15.3655	10.6509	15.7283
GW190425	1.960	4.680	2.36559796	3.65540791	10.5618105	16.3117752	10.4783	14.9709	10.7157	15.3339

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