UV completions of Chern-Simons gravity

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Dynamical Chern-Simons (dCS) gravity is a four-dimensional parity-violating extension of general relativity. The standard mechanism to obtain this extension predicts negligible observational effects due to a large decay constant f close to the scale of grand unified theories. Here, we present two constructions of dCS that permit much smaller decay constants, ranging from sub-eV to Planck scales. That is, we show that, in the same manner as axions, dCS gravity can arise from both spontaneous and dynamical symmetry breaking. In either case, the angular part of a complex scalar field develops a pseudoscalar Yukawa interaction with a set of fermions. In the former case, the complex scalar field is a fundamental particle, and in the latter case, it is a bound state of short-wavelength fermion modes arising from strong four-Fermi self-interactions. Due to the Yukawa interaction, loop corrections with gravitons then realize a linear coupling between the angular pseudoscalar and the gravitational Chern-Simons term. The strength of this coupling is set by the Yukawa coupling constant divided by the fermion mass. Therefore, since fermions with small masses are ideal, we identify neutrinos as promising candidates. For example, if a neutrino has a mass $m_\nu \lesssim \mathrm{meV}$ and the Yukawa coupling is order unity, the dCS decay constant can be smaller than $f \sim 10^3 m_\nu \lesssim \mathrm{eV}$. We discuss other potential choices for fermions and give two examples of four-Fermi UV completions.

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I. INTRODUCTION

The Einstein-Hilbert (EH) action is the most successful description of gravity today [1,2]. However, even at energies far below the regime of quantum gravity, there is no reason to expect the EH action to be a complete descriptor of gravity from an effective field theory (EFT) perspective [3,4]. A large body of work has thus been extensively researched in order to search for its deviations [5–10]. One extension is to look for parity violation within our description of gravity, as we know parity-violating effects to already be present within the Standard Model [11,12]. The lowest-order term that encapsulates such parity violation is given by dynamical Chern-Simons (dCS) gravity, where an additional pseudoscalar field a is added that is linearly coupled to the Pontryagin density ${}^*RR$ [13,14].

The dCS gravity emerges naturally in the low-energy limit of string theory through the Green-Schwartz anomaly canceling condition with a decay constant that is typically the string scale [15,16].[1] However, such large decay constants render compact astrophysical signals and superradiance effects to be undetectable. Attempts to generate dCS through radiative corrections by explicitly including Lorentz-violating terms [17–19] have also been investigated, although it may be the case that such terms must vanish by gauge invariance [20]. The dCS gravity is also a possible solution to the gravitational analog to the strong CP problem in QCD if the pseudoscalar takes a suitable minimum [21–23]. In these cases, a gravitational axion is created either through the breaking of a global Peccei-Quinn (PQ) [24–26] or axial [27,28] symmetry.

In this work, we generalize the works of Refs. [27,28] and show that dCS gravity emerges as the low-energy limit of a wide range of global PQ or axial symmetry-breaking theories and explicitly calculate its decay constant f.

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[1]The scale of the decay constant can also depend on compactification volume factors.

In doing so, we also show that it is possible to generate the mass of any fermion involved in the symmetry-breaking process, similar to Refs. [27,29,30]. We finish by demonstrating that the dCS decay constant can be made to reach the sub-eV regime for a large range of phenomenological parameters. Specifically, we point out that, within the Standard Model, light neutrinos are great candidates for reaching this threshold.

This paper is organized as follows. First, in Sec. II, we review the definition and dynamics of dCS gravity. We then present the physics of spontaneous symmetry breaking of a global $U(1)_{\mathrm{PQ}}$ symmetry in Sec. III and confirm that it yields a Yukawa coupling between a pseudoscalar and a set of fermions. With this new coupling, we then show, in Sec. IV, that it induces a gravitational Chern-Simons interaction through a triangle diagram with fermion loops. Then, in Sec. V, we review how a single massive fermion with attractive self-interactions yields a complex scalar bound state, thus dynamically breaking the initial axial symmetry of the theory and giving rise to dCS gravity similar to Sec. IV. We quantify the parameter space of generating dCS in Sec. VI and point out possible candidates for the fermion in Sec. VII. Finally, we discuss and conclude in Sec. VIII and Sec. IX, respectively.

II. CHERN-SIMONS GRAVITY

Let κR be the EH term with $\kappa = (16\pi G)^{-1}$ and R the Ricci scalar. Then the vacuum action S of dCS gravity is given by

$$S = \int d^4x\sqrt{-g}\left[\kappa R + \frac{a}{4f}{}^*RR - \frac{1}{2}(\nabla_\mu a)(\nabla^\mu a)\right]. \quad (1)$$

The dCS extension beyond EH consists of the dynamical pseudoscalar field, a, which linearly couples to the Pontryagin density through the dCS decay constant f. The Pontryagin density is defined as

$${}^*RR = {}^*R^\rho{}_\sigma{}^{\mu\nu}R^\sigma{}_{\rho\mu\nu}, \quad (2)$$

where

$${}^*R^\rho{}_\sigma{}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}R^\rho{}_{\sigma\alpha\beta} \quad (3)$$

is Hodge dual to the Riemann tensor and $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita tensor. The Pontryagin density can also be written in terms of divergence of the Chern-Simons topological current,

$$\nabla_\mu K^\mu = \frac{1}{4}{}^*RR, \quad (4)$$

where

$$K^\mu \coloneqq \epsilon^{\mu\nu\alpha\beta}\left(\Gamma^\sigma{}_{\nu\rho}\partial_\alpha\Gamma^\rho{}_{\beta\sigma} + \frac{2}{3}\Gamma^\sigma{}_{\nu\rho}\Gamma^\rho{}_{\alpha\lambda}\Gamma^\lambda{}_{\beta\sigma}\right), \quad (5)$$

giving rise to the name "Chern-Simons gravity." Varying the action, Eq. (1), with respect to the metric yields the modified vacuum field equations,

$$G_{\mu\nu} + \frac{1}{\kappa f}C_{\mu\nu} = \frac{1}{2\kappa}T_{\mu\nu}, \quad (6)$$

which include a modification from the C tensor,

$$C^{\mu\nu} = (\nabla_\alpha a)\epsilon^{\alpha\beta\gamma(\mu}\nabla_\gamma R^{\nu)}{}_\beta + (\nabla_\alpha\nabla_\beta a){}^*R^{\beta(\mu\nu)\alpha}. \quad (7)$$

The total energy-momentum tensor $T_{\mu\nu}$ is the sum of any matter stress-energy tensor (assumed in this subsection to be zero) and the stress-energy tensor of the pseudoscalar,

$$T^{(a)}_{\mu\nu} = (\nabla_\mu a)(\nabla_\nu a) - \frac{1}{2}g_{\mu\nu}(\nabla_\lambda a)(\nabla^\lambda a). \quad (8)$$

The pseudoscalar field itself obeys the following vacuum equation of motion:

$$\Box a = -\frac{1}{4\kappa f}{}^*RR, \quad (9)$$

which can be obtained by varying the action in Eq. (1) with respect to a.

The dCS-backreacted solutions of the pseudoscalar onto a gravitational wave h lead to an amplitude birefringence between left h_L and right h_R polarizations, resulting in parity violation:

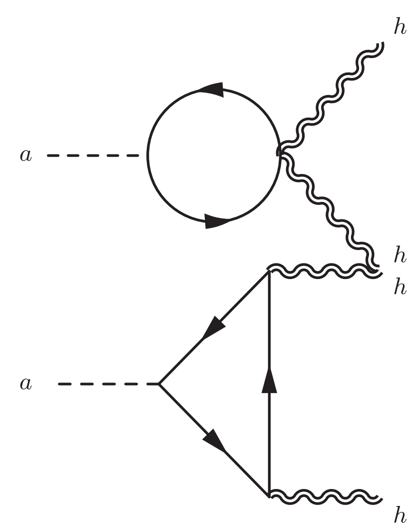


FIG. 1. One-loop diagrams for the process $a \to hh$, with a pseudoscalar particle a depicted as a dashed line and $h_{\mu\nu}$ the graviton as the solid wiggly line. The loops are generated by a fermion Ψ.

$$h_{L,R} = e^{\pm A(a,\dot{a})_{L,R}} e^{ik^\mu x_\mu}. \quad (10)$$

A key observation of dCS theory concerns the coupling f. Since f has dimensions of energy, this theory is nonrenormalizable, seen at energies $E \gg f$, and so we are therefore motivated to search for a UV completion of dCS gravity. More specifically, we expect that the dCS action be generated by loop diagrams of the UV complete theory, for example, those shown by Fig. 1. Taking these diagrams as inspiration, and imposing minimal gravitational couplings, we must thus construct a theory with a vertex interaction between the pseudoscalar and two fermions; i.e. we must obtain a pseudoscalar Yukawa interaction. In what follows, we show that fermions, and in particular neutrinos, that self-interact generate such diagrams.

III. SPONTANEOUS SYMMETRY BREAKING

We first review the mechanism to generate pseudoscalar Yukawa couplings via spontaneous symmetry breaking [31,32]. Consider the following Lagrangian of a complex scalar field and Dirac fermion with a Yukawa interaction in Minkowski spacetime, $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$,

$$\mathcal{L}_\Phi = \sum_{j=L,R} i\bar{\Psi}_j \gamma^\mu \partial_\mu \Psi_j - \tilde{m}_\Psi(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L) + \partial_\mu\Phi\partial^\mu\Phi^* - V(|\Phi|^2) - y(\Phi\bar{\Psi}_L\Psi_R + \Phi^*\bar{\Psi}_R\Psi_L), \quad (11)$$

with γ^μ the Dirac gamma matrix, $\tilde{m}_\Psi$ the Dirac fermion's bare mass, $V(x)$ a $U(1)$ symmetry-breaking potential, and y the Yukawa coupling constant (which may be attractive or repulsive). This theory has a vector symmetry, $U(1)_\mathrm{V}$, whereby fermions of vector charge q_V are rotated by angle α, $\Psi \to e^{iq_\mathrm{V}\alpha}$, and the complex scalar is unchanged, $\Phi \to \Phi$. In addition, in the massless limit $\tilde{m}_\Psi \to 0$, it also enjoys a PQ symmetry, $U(1)_\mathrm{PQ}$, whereby the fermions of PQ charge q_PQ are axially rotated by the angle β, $\Psi \to e^{iq_\mathrm{PQ}\gamma^5\beta}\Psi$, along with a compensating rotation of the complex scalar with twice the opposite PQ charge, $\Phi \to e^{-2iq_\mathrm{PQ}\beta}\Phi$. In the fermionic axial rotations, the fifth gamma matrix is defined through the relation $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

Below some energy scale $\sim 4\pi F$, the shape of the potential is assumed to change so that the field falls from a minimum energy configuration at $\Phi = 0$ to one at $\Phi_\mathrm{min} = F/\sqrt{2}$, spontaneously breaking the $U(1)_\mathrm{PQ}$ symmetry. Afterwards, the dynamics of the scalar field are then best captured by performing a field redefinition around this minimum, $\Phi(x) = (1/\sqrt{2})[F + \rho(x)]\exp[ia(x)/F]$, with ρ the radial and a the angular degrees of freedom. From writing the classical PQ current with this redefinition, $\tilde{J}^\mu_\mathrm{PQ} = i\bar{\psi}\gamma^\mu\gamma^5\psi - \partial^\mu a$, one can see that the angular degree of freedom is a pseudoscalar, as the current transforms as a pseudovector. Plugging the field redefinition back into Eq. (11), and also expanding the pseudoscalar exponential, we thus obtain

$$\mathcal{L}_\Phi = \bar{\Psi}(i\gamma^\mu\partial_\mu - m_\Psi)\Psi + \frac{1}{2}\partial_\mu a\partial^\mu a - iga\bar{\Psi}\gamma^5\Psi, \quad (12)$$

with $m_\Psi = \tilde{m}_\Psi + gF$ the total mass of the Dirac field postsymmetry breaking and $g = y/\sqrt{2}$ the new pseudoscalar Yukawa coupling. In the above expression, we neglected higher-order terms of the pseudoscalar field as they are suppressed by additional powers of the energy scale F. For smaller energies than F, we also integrated out the heavy radial ρ field [which is the only degree of freedom in the potential $V(x)$]. As is standard in the Higgs mechanism, the mass of the fermion has been shifted through by the factor gF. It follows that it is possible to generate the entire mass of a massive Dirac fermion when $\tilde{m}_\Psi = 0$. Moreover, we see that, in this way, we generate a massless Nambu-Goldstone (NG) boson a with a pseudoscalar Yukawa coupling to a fermion Ψ, as desired.

IV. LOOP GENERATION OF dCS

We now place our fermion in a curved background in order to realize dCS. That is, we begin with a Dirac fermion Ψ that has both a minimal gravity and a pseudoscalar Yukawa interaction,

$$\mathcal{L}_g = \bar{\Psi}(i\gamma^b e^\mu_b D_\mu - m_\Psi)\Psi + iga\bar{\Psi}\gamma^5\Psi, \quad (13)$$

with e^μ_b an orthonormal tetrad basis, $D_\mu = \partial_\mu - (i/4)\sigma^{ab}\omega_{\mu ab}$ the covariant spinor derivative, $\sigma_{ab} = [\gamma^a, \gamma^b]$ the Lorentz group generator for Dirac fermions, and $\omega_{\mu ab} = (1/2)e^\nu_a(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu}) - (1/2)e^\nu_b(\partial_\mu e_{a\nu} - \partial_\nu e_{a\mu}) + (1/2)e^\rho_a e^\sigma_b(\partial_\sigma e_{c\rho} - \partial_\rho e_{c\sigma})e^c_\mu$ the torsion-free spin connection. Here, Greek indices indicate the global Lorentzian structure and Latin indices the local structure.

In order to generate dCS, we perform the same steps as to derive pion decay into photons and integrate out the fermion Ψ from our theory. For energies below the fermion mass m_Ψ, integrating out the field Ψ is equivalent to evaluating the Ψ-dependent Lagrangian in a fixed gravitational field. As a result, the effective Lagrangian is then

$$\mathcal{L}^\mathrm{eff}_g = gaJ_a, \quad (14)$$

$$J_a = i\langle h|\bar{\Psi}\gamma^5\Psi|h\rangle, \quad (15)$$

where J_a is the pseudoscalar composite operator, expressed as an expectation value over a gravitational field h. In order to evaluate this expectation value, we use the alternative, and equivalent, expression for the gravitational Adler-Bell-Jackiw (ABJ) anomaly,

$$\langle h|\partial_\mu \tilde{J}_5^\mu|h\rangle = -\frac{1}{384\pi^2}{}^*RR, \tag{16}$$

with $\partial_\mu \tilde{J}_5^\mu = 2m_\Psi i\bar{\Psi}\gamma^5\Psi$ the divergence of the classical axial current associated with Ψ. Note that this current is not to be confused with the total axial current $\partial_\mu J_5^\mu = \partial_\mu \tilde{J}_5^\mu + {}^*RR/(384\pi^2)$. The expression for the ABJ anomaly can be obtained in numerous different ways (e.g. Fujikawa's method) [33–35]. The equivalent perturbative triangle loop diagram is obtained by an interaction vertex between the graviton and the fermion and by evaluating the amplitude integrals. The relevant graviton-fermion interactions for the amplitude can be found in Ref. [36]. For example, a typical triangle amplitude for the decay of the pseudoscalar to gravitons is

$$\begin{aligned}\mathcal{M}(a\to hh) = \frac{ga}{16}\int d^4k e^{\mu\nu}(q_1)e^{\delta\gamma}(q_2)\\ \times[\tilde{S}(k-q_1)_\gamma\gamma_\delta\tilde{S}(k-q_1)_\nu\gamma_\mu\gamma_5]\\ +(\mu\nu\to\delta\gamma;1\to 2),\end{aligned} \tag{17}$$

where $e^{\mu\nu}$ is the graviton polarization tensor. However, since this diagram is one-loop exact, the choice of computational scheme does not alter the result. Therefore, since $J_a = [1/(2m_\Psi)]\langle h|\partial_\mu \tilde{J}_5^\mu|h\rangle$, we obtain the CS term

$$\mathcal{L}_g^{\rm eff} = -\frac{g}{384\pi^2}\frac{a}{2m_\Psi}{}^*RR. \tag{18}$$

In this procedure, dCS is specified by two parameters: the mass of the fermion, and its Yukawa coupling to the pseudoscalar a. Moreover, connecting Eq. (18) with Eq. (1) we see that the dCS decay constant is

$$f = 192\pi^2\frac{m_\Psi}{g}. \tag{19}$$

Thus, similar to standard axions, a PQ symmetry-breaking theory also yields dCS gravity.

V. DYNAMICAL SYMMETRY BREAKING

Our goal is now to show that a theory that undergoes dynamical symmetry breaking can also give rise to dCS gravity. In order to do so, we will show that it is possible to dynamically obtain a theory of spontaneous symmetry breaking from a theory of self-interacting fermions upon condensation of said fermion. Afterwards, dCS is then obtained through the exact same manner as Sec. II. That being said, the two theories are different UV completions of dCS and thus exhibit distinct observables (e.g. the presence of self-interactions among the fermions). For simplicity, we consider the Lagrangian of a single massive fermion Ψ with attractive self-interactions in Minkowski space,

$$\mathcal{L}_\Psi = \bar{\Psi}(i\gamma^\mu\partial_\mu - \tilde{m}_\Psi)\Psi - \lambda[(\bar{\Psi}\Psi)^2 - (\bar{\Psi}\gamma^5\Psi)^2], \tag{20}$$

with γ^μ the Dirac gamma matrices, $\tilde{m}_\Psi$ the bare mass of the fermion, and λ the fermion self-interaction coupling constant. Similar to dCS, this Lagrangian is also nonrenormalizable, and thus has a cutoff Λ. Following the Nambu–Jones-Lasinio formalism presented in [37–39], an attractive four-fermion interaction will generate a new bound state Φ below this cutoff.

If we were to investigate this theory at extremely low energies $E \ll \Lambda$, where self-interactions dominate, we could perform a Hubbard-Stratonovich transformation on the fermion pair and integrate out the fermion from our theory, in accordance to the mean-field approximation. Here, we consider the intermediate regime and perform a Hubbard-Stratonovich transformation on fermion modes between the scales E and Λ, integrating them out afterwards. More specifically, we first expand the fermion modes into long ($\ell, k < E$) and short ($s, k > E$) wavelength modes, $\Psi = \Psi_\ell + \Psi_s$. Next, we multiply the path integral of our theory by a constant,

$$Z_\alpha = \int \mathcal{D}\alpha\mathcal{D}\bar{\alpha}\,\exp\left(-\int d^4x\,\tilde{m}_\Phi^2\bar{\alpha}\alpha\right), \tag{21}$$

with α an auxiliary field and $\tilde{m}_\Phi$ an as-of-yet unspecified bare mass scale. Then, we perform a field redefinition $\Phi = \alpha - \tilde{m}_\Phi^{-2}\bar{\Psi}_{R,s}\Psi_{L,s}$ in accordance with the Hubbard-Stratonovich transformation. We therefore get the new Lagrangian

$$\begin{aligned}\tilde{\mathcal{L}}_\Psi &= \mathcal{L}_\Psi - \tilde{m}_\Phi^2\bar{\alpha}\alpha\\ &= \bar{\Psi}(i\gamma^\mu\partial_\mu - \tilde{m}_\Psi)\Psi - \lambda[(\bar{\Psi}\Psi)^2 - (\bar{\Psi}\gamma^5\Psi)^2]\\ &\quad + \tilde{m}_\Phi^{-2}\bar{\Psi}_{L,s}\Psi_{R,s}\bar{\Psi}_{R,s}\Psi_{L,s} + (\Phi\bar{\Psi}_{R,s}\Psi_{L,s} + \text{H.c.})\\ &\quad + \tilde{m}_\Phi^2|\Phi|^2.\end{aligned} \tag{22}$$

Now, we integrate out the short scale modes, normalize the kinetic term, and remove the ℓ subscript from the long-wavelength modes to obtain the Lagrangian

$$\begin{aligned}\tilde{\mathcal{L}}_\Psi &= \bar{\Psi}(i\gamma^\mu\partial_\mu - \tilde{m}_\Psi)\Psi - \lambda[(\bar{\Psi}\Psi)^2 - (\bar{\Psi}\gamma^5\Psi)^2]\\ &\quad + (\partial_\mu\Phi^*)(\partial^\mu\Phi) - y(\Phi\bar{\Psi}_L\Psi_R + \text{H.c.})\\ &\quad + m_\Phi^2|\Phi|^2 - \frac{\lambda_\Phi}{4}|\Phi|^4,\end{aligned} \tag{23}$$

with

$$m_\Phi^2 = (\tilde{m}_\Phi^2 + \Pi_0)/\Pi_2, \tag{24}$$

$$y = \lambda\Pi_0/\Pi_2^{1/2}, \tag{25}$$

$$\lambda_\Phi = \mathbf{V}_4/\Pi_2^2, \tag{26}$$

the renormalized parameters due to the condensation at one loop. Defining $\tilde{S}(k) = (-\not{k} + m_\Psi)/(k^2 + m_\Psi^2)$ to be the Fourier transform of the full fermion propagator (including self-interaction loops),

$$\Pi = \int_E^\Lambda \frac{d^4\ell}{(2\pi)^4} \mathrm{Tr}[\tilde{S}(\ell)\tilde{S}(\ell+k) - \tilde{S}(\ell)\gamma_5\tilde{S}(\ell+k)\gamma_5] \tag{27}$$

$$\begin{aligned} &= -\frac{1}{2\pi^2}(\Lambda + E - 2k)(\Lambda - E) - \frac{2m_\Psi}{\pi^2 k}\left(k^2 - \frac{m_\Psi^4}{k^2 + 4m_\Psi^2}\right)\left[\mathrm{atan}\left(\frac{k+\Lambda}{m_\Psi}\right) - \mathrm{atan}\left(\frac{k+E}{m_\Psi}\right)\right] \\ &\quad - \frac{m_\Psi^3}{\pi^2 k}\left(1 - \frac{2m_\Psi^2}{k^2 + 4m_\Psi^2}\right)\left[\mathrm{atan}\left(\frac{\Lambda}{m_\Psi}\right) - \mathrm{atan}\left(\frac{E}{m_\Psi}\right)\right] - \frac{1}{2\pi^2}\left[(k^2 - 2m_\Psi^2) + \frac{m_\Psi^4}{k^2 + 4m_\Psi^2}\right]\log\left[\frac{m_\Psi^2 + (k+\Lambda)^2}{m_\Psi^2 + (k+E)^2}\right] \\ &\quad - \frac{1}{2\pi^2}\frac{m_\Psi^4}{k^2 + 4m_\Psi^2}\log\left[\frac{m_\Psi^2 + \Lambda^2}{m_\Psi^2 + E^2}\right] \end{aligned} \tag{28}$$

$$\equiv \Pi_0 - k^2\Pi_2 \tag{29}$$

is then the self-energy and

$$\mathbf{V}_4 = 6\int_E^\Lambda \frac{d^4\ell}{(2\pi)^4}\mathrm{Tr}[\tilde{S}(\ell)^4] \tag{30}$$

$$\begin{aligned} &= \frac{m_\Psi^2}{2\pi^2}\left[-\frac{11m_\Psi^4 + 30m_\Psi^2E^2 + 27E^4}{(m_\Psi^2 + E^2)^3}\right. \\ &\quad \left. + \frac{8m_\Psi^4}{(m_\Psi^2 + \Lambda^2)^3} - \frac{24m_\Psi^2}{(m_\Psi^2 + \Lambda^2)^2} + \frac{27}{m_\Psi^2 + \Lambda^2}\right] \\ &\quad + \frac{3}{2\pi^2}\log\left(\frac{m_\Psi^2 + \Lambda^2}{m_\Psi^2 + E^2}\right) \end{aligned} \tag{31}$$

the fermion-loop-corrected four-scalar vertex with zero external momenta. Note we have suppressed the long-wavelength subscript and ignore the renormalization of the original theory parameters for simplicity. Moreover, note that short-wavelength ($k \gg \Lambda$) modes are not dynamic as $\lim_{k/\Lambda\to\infty}\Pi_2 = 0$.

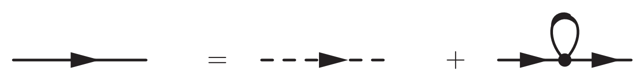
Each of the renormalized parameters has some running with energy, and so in order to precisely determine the value of these parameters at some low-energy quasifixed point, a beta function analysis must be performed. Here, for simplicity as well as to capture the general scaling of parameters, we assume the running of energy induces order one corrections to the parameters; i.e. we take $\Pi_0 \approx -\Lambda^2/(2\pi^2)$ and $\Pi_2 \approx 1/(2\pi^2)$ so that $y \approx -\lambda\Lambda^2/(\sqrt{2}\pi)$.

Restoring the ignored factors for a moment, we see that at zero external momentum, Π_0 simplifies to

$$\begin{aligned} \Pi_0 = &-\frac{1}{2\pi^2}\left[(\Lambda^2 - E^2) + m_\Psi^4\left(\frac{1}{m_\Psi^2 + E^2} - \frac{1}{m_\Psi^2 + \Lambda^2}\right)\right. \\ &\left. - 2m_\Psi^2\log\left(\frac{m_\Psi^2 + \Lambda^2}{m_\Psi^2 + E^2}\right)\right]. \end{aligned} \tag{32}$$

Solving numerically, we find that Π_0 at $E = 0$ is always negative and is also always negative at $E \sim m_\Psi$ for $m_\Psi \lesssim \Lambda$. We thus infer the mass of Φ in Eq. (24) can go from positive at $E \sim \Lambda$ to negative at $E \ll \Lambda$. Since there is a positive quartic term for Φ, Φ can then undergo spontaneous symmetry breaking.

Typically, the symmetry-breaking scale F is related to the complex scalar's quadratic and quartic terms, $F = \sqrt{4|m_\Phi|^2/\lambda_\Phi}$, once m_Φ^2 becomes negative. However, since the bare mass scale $\tilde{m}_\Phi$ in this phase is undetermined, we cannot solve for F using this expression. Therefore, in order to determine the symmetry-breaking scale, or alternatively the mass $m_\Psi = \tilde{m}_\Psi + gF$, we invoke the self-consistency condition of the fermion propagator, i.e. the gap equation, depicted below:



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The solid (dashed) arrow line is the total (bare) fermion propagator. The loop diagram is the result of the four-fermion interaction that appears in Eq. (20), which we have hidden in writing down the Lagrangian $\mathcal{L}_\Phi$.

This condition amounts to a nonperturbative loop correction, $\Delta m_\Psi = gF$, to the fermion mass. In the case of zero bare mass, $\tilde{m}_\Psi = 0$, this correction in fact generates the entire mass of the fermion at low energies, as done in Ref. [27]. More specifically, the gap equation relates the cutoff scale and self-interaction coupling constant λ to this correction and the mass m_Ψ,

$$\frac{\Delta m_\Psi}{m_\Psi}\left(\frac{2\pi^2}{\lambda\Lambda^2}\right) = 1 - \frac{m_\Psi^2}{\Lambda^2}\log\left(1 + \frac{\Lambda^2}{m_\Psi^2}\right), \tag{33}$$

with a solution possible for sufficiently strong coupling, $\lambda \geq (\Delta m_\Psi / m_\Psi)(2\pi^2/\Lambda^2)$. For large cutoffs $\Lambda \gg m_\Psi$, this bound is saturated,

$$\lambda = (2.25\ \mathrm{MeV})^{-2} \left(\frac{\Delta m_\Psi}{m_\Psi}\right) \left(\frac{10^7\ \mathrm{eV}}{\Lambda}\right)^2, \tag{34}$$

while for small cutoffs $\Lambda \ll m_\Psi$,

$$\lambda = (1.59\ \mathrm{MeV})^{-2} \left(\frac{\Delta m_\Psi}{m_\Psi}\right) \left(\frac{m_\Psi^2}{\Lambda^2}\right) \left(\frac{10^7\ \mathrm{eV}}{\Lambda}\right)^2. \tag{35}$$

Solving for F, and ignoring the lower-order terms in y, we find that

$$F \approx m_\Psi \frac{2\sqrt{2}}{\pi} \left[1 - \frac{m_\Psi^2}{\Lambda^2} \log\left(1 + \frac{\Lambda^2}{m_\Psi^2}\right)\right]. \tag{36}$$

Therefore, for large cutoffs, $F \approx 0.9 m_\Psi$, and for small cutoffs, $F \approx 0.45 \Lambda^2 / m_\Psi$. Moreover, we point out that at low cutoffs, the condition $\tilde{m}_\Psi \lesssim c\Lambda$, $c \sim 1$ translates to $(\Delta m_\Psi / m_\Psi) \gtrsim 1 - c(\Lambda / m_\Psi)$ or $\Delta m_\Psi / m_\Psi \sim 1$.

Therefore, when the cutoff is large, $\Lambda \gg m_\Psi$, we have

$$f = 1.7\ \mathrm{eV} \left(\frac{\Delta m_\Psi}{m_\Psi}\right)^{-1} \left(\frac{m_\Psi}{10^{-3}\ \mathrm{eV}}\right), \tag{37}$$

and when it is small,

$$f = 0.85\ \mathrm{eV} \left(\frac{\Delta m_\Psi}{m_\Psi}\right)^{-1} \left(\frac{\Lambda}{m_\Psi}\right) \left(\frac{\Lambda}{10^{-3}\ \mathrm{eV}}\right). \tag{38}$$

Thus, if a fermion self-interacts strongly enough $[\lambda \geq (\Delta m_\Psi / m_\Psi)(2\pi^2/\Lambda^2)]$, Chern-Simons gravity is a consequence.

VI. BOUND STATE PARAMETER SPACE

We now address the relevant parameter space for dCS gravity from dynamical symmetry breaking. That is, we identify when the fermion self-interactions are strong enough to generate a bound state and calculate the resulting fermion mass. For spontaneous symmetry breaking, all parameters are independent and calculable in a simple manner, rather than through the gap equation. We first consider the most general parametrization to generate dCS with a four-fermion interaction, and then we consider two specific examples.

A. General

From the gap equation [Eq. (33)], the main parameters in question are the mass of the fermion, m_Ψ, the cutoff scale Λ, and the self-interaction coupling strength λ. We plot the relation between these quantities to the dCS decay constant f in Figs. 2 and 3.

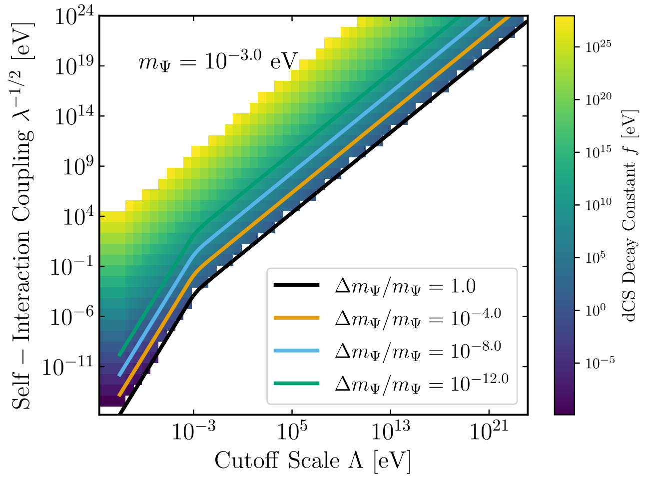

FIG. 2. The parameter space for dCS generated by a self-interacting fermion Ψ of mass $m_\Psi = 10^{-3}$ eV using Eq. (33) and Eq. (19). The black (orange) [blue] {green} solid line indicates the parameters necessary to generate a mass correction $\Delta m_\Psi / m_\Psi = 1(10^{-4})[10^{-8}]\{10^{-12}\}$. Finally, we only plot parameters that are below the Planck scale and have $\Delta m_\Psi / m_\Psi \leq 1$. Moreover, while in principle the cutoff can be made arbitrarily small, we only plot $\Lambda \gtrsim 10^{-6} m_\Psi$ for visualization purposes.

B. Scalar mediator

One scenario to create a fermion self-interaction in the form of Eq. (20) is through the interaction with a complex scalar mediator χ of mass m_χ. The interaction has a Yukawa-like form, $\mathcal{L} \supset g_\chi \chi \bar{\Psi}_L \Psi_R + \mathrm{H.c.}$ At energies $E \ll m_\chi$, a four-fermion interaction is induced with $\lambda = (g_\chi / m_\chi)^2$, indicating the cutoff of the theory is roughly $\Lambda \sim m_\chi$.

As a result, the gap equation now takes the form

$$\frac{\Delta m_\Psi}{m_\Psi} \frac{2\pi^2}{g_\chi^2} = 1 - \frac{m_\Psi^2}{m_\chi^2} \log\left(1 + \frac{m_\chi^2}{m_\Psi^2}\right). \tag{39}$$

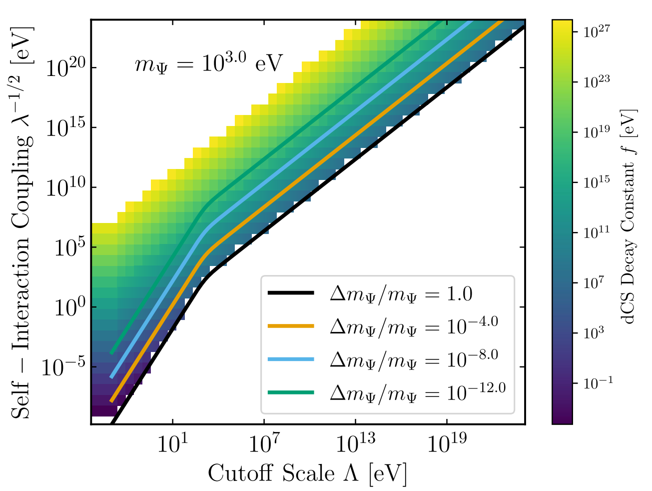

FIG. 3. Same as Fig. 2, except for $m_\Psi = 10^3$ eV.

Therefore, for $m_\chi \gg m_\Psi$,

$$g_\chi^2 = 2\pi^2 \frac{\Delta m_\Psi}{m_\Psi}, \tag{40}$$

and for $m_\chi \ll m_\Psi$,

$$g_\chi^2 = 4\pi^2 \frac{m_\Psi^2}{m_\chi^2} \frac{\Delta m_\Psi}{m_\Psi}. \tag{41}$$

Moreover, for both regimes,

$$f = 34\ \mathrm{eV} g_\chi^{-2} \left(\frac{m_\Psi}{10^{-3}\ \mathrm{eV}} \right). \tag{42}$$

C. Gravitational torsion

If the gravitational connection has nonzero torsion, a scalar four-fermion interaction is typically induced at energies below the Planck scale for all fermions [40]. In this case, the self-interaction coupling constant is $\lambda = 3\pi/\Lambda_T^2$.

VII. FERMION CANDIDATES

A. Neutrinos

Since smaller fermion masses yield larger coupling constants, we are most interested in fermions with small masses that self-interact. The ideal candidate for such a scenario is thus the neutrino. While the Standard Model induces neutrino self-interactions, these are too small to realize a condensate in nature [41]. However, beyond the Standard Model interactions both are a viable option and are well-motivated [42].

In regards to the parameter space, oscillation experiments [43], along with Planck 2018 measurements [44], imply that an active neutrino's mass must be between $0 \lesssim m_\nu \lesssim 0.05$ eV in either the normal or inverted hierarchy [45]. In addition, sterile neutrinos may have masses as small as any given active neutrino, and as large as the Planck mass [46–52]. Given the wide range of masses, connecting our results to specific neutrino self-interaction models (and therefore giving constraints on λ and Λ) is beyond the scope of our work.

B. Fermionic self-interacting dark matter

It is also possible that the fermion in question is a dark matter (DM) particle that self-interacts [53–57]. In this case, if this particle compromises all of DM, it may have masses between $0.1\ \mathrm{GeV} \leq m_\Psi \leq 10^7\ \mathrm{GeV}$ [58]. However, if instead the fermion is one of a large number $N \lesssim 10^{62}$ of particles, such as in ultralight fermionic dark matter [59], the mass of the fermion could possibly be as small as $m_\Psi \sim 10^{-14}$ eV. We note, however, that the ultralight case with self-interactions has not been studied, and so it is not definite that such a scenario is viable.

VIII. DISCUSSION

We clarify three assumptions and give four comments. First, given that we are dealing with an EFT with a very low-energy cutoff, one may worry that astrophysical or cosmological systems, such as compact binary coalescences, are characterized by larger energies. As a rough argument, the scale of the EFT breaking down at ringdown occurs at energies smaller than the Schwarzschild radius, which for ∼eV energies is far above the EFT limit. A much more detailed analysis of such scales has been done in Refs. [60,61], although they note that it is difficult to analyze the parity-violating sector.

Second, for simplicity, we only considered the self-interaction of a single neutrino. Our method of generating a neutrino bound state can be extended to multiple generations of neutrinos in a straightforward fashion, whether they are active or sterile, through promoting the self-interacting coupling constant to a self-interacting coupling matrix.

Third, we assume that renormalization will lead to order one changes in the induced condensation parameters. We base this assumption on both dimensional grounds and the results presented in other dynamical symmetry-breaking papers [38,62]. However, we note that as energies fall, the Yukawa coupling y increases. Therefore, the dCS decay constant will decrease, our results are then in fact upper bounds on the actual decay constant, and the likelihood of detecting dCS via this mechanism increases.

It may be the case that the complex scalar and fermion in Sec. III are actually fundamental new particles, rather than related to some bound state. In this case, the dCS phenomenology is completely specified by the Yukawa coupling and symmetry-breaking scale.

If the fermion in question is a neutrino, then the pseudoscalar interaction in Eq. (13) can also be generated through the breaking of a global lepton symmetry, so that $a = J$, with J the majoron [63–66]. Moreover, this association is also possible if the fermion is a dark-matter candidate [67,68]; however, the masses of the fermion typically are large $m_\Psi \gg$ GeV.

In order to create fermion self-interactions we considered the case where a real scalar mediator generates the fermion self-interactions; however, it can equally be a complex scalar, vector, or tensor. For these mediators, we expect the resulting low-energy self-interacting coupling constant to be an $\mathcal{O}(1)$ factor difference from the real scalar mediator.

Finally, we point out that sensitivity forecasts of 2G detectors to the dCS decay constant in the inspiral of black hole mergers yield $f \gtrsim 10^{-50}$ eV (or $\xi^{1/4} \lesssim 10$ km) [69–71]. Hence, it will be difficult, but not implausible, that inspiral signals will give rise to a detectable signal from our mechanism. In particular, very light neutrinos of mass $m_\nu \sim 10^{-53}$ eV yield dCS decay constants $f \sim 10^{-50}$ eV and would be detectable in systems that obey the EFT constraints. Moreover, it may be the case that backreaction of the pseudoscalar onto the evolution of the binary could imprint a more distinct signature on gravitational wave observations (alternatively, through superradiance [72]). In addition, it is unclear if merger and ringdown signals will produce larger parity-violating signals, given that the scale of the system is much smaller and the gravitational strength much larger, especially in supermassive black hole systems. Outside of individual gravitational wave events, dCS could also lead to novel interactions with stochastic gravitational wave backgrounds, such as those shown by Ref. [73]. We leave the investigation of all these possibilities for future work.

IX. CONCLUSION

In this paper, we presented two categories of UV completions to dCS gravity with decay constants that reach the sub-eV regime. The first example was a theory of a fundamental complex scalar interacting with a Dirac fermion through a Yukawa interaction, while the second example took the complex scalar to be composite, arising from the attractive chiral four-Fermi interaction of a Dirac fermion. In the latter case, we further presented two UV completions of the four-Fermi theory.

We thus see that dCS gravity can arise in the same manner as axions, as the spontaneous breaking of a chiral symmetry yields a pseudoscalar Yukawa coupling between the angular part of the complex scalar field and a Dirac fermion. The Yukawa interaction facilitates a triangle diagram between the pseudoscalar and a pair of gravitons, with the fermions going around the loop. By integrating out these fermions, the dCS interaction term is created with a decay constant that is proportional to the mass of the fermion divided by the Yukawa coupling (i.e. the vacuum expectation value of the complex scalar).

In order to explicitly show that the fermion with attractive self-interactions will form a bound state, we performed a renormalization-group analysis, integrating out the short-wavelength fermion modes. This analysis demonstrated that the bound state is in fact made up of such modes. Moreover, in doing the integration, we showed that these modes also generate quadratic and quartic potential terms for the bound state, allowing for spontaneous symmetry breaking to occur. In order to solve for the symmetry-breaking scale, we invoked the gap equation, a self-consistency condition for the fermion propagator that also can generate the entire mass of the fermion. As a result, we found that the symmetry-breaking scale at large cutoffs is roughly the scale of the fermion mass, and at small cutoffs it is roughly the square of the cutoff scale divided by the fermion mass.

With these results, we identified neutrinos as particularly optimistic candidates to generate dCS gravity within the Standard Model, due to their small masses and ability to self-interact. In particular, we found that for neutrinos of mass $m_\nu \lesssim$ meV the dCS decay constant could be less than an eV, $f \sim 10^3 m_\nu \lesssim$ eV. Other possibilities, such as an ultralight self-interacting fermion dark matter candidate, or gravity with nonzero torsion, are also candidates for dCS gravity, demonstrating a range of possible UV completions.

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