Evolving laws and cosmological energy

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We couple the issue of evolution in the laws of physics with that of violations of energy conservation. Avoiding a time dependence in terms of coordinate time, we define evolution as a function of time variables canonically dual to "constants" (such as Λ, the Planck mass, or the gravitational coupling), mimicking a procedure associated with one formulation of unimodular gravity. We then introduce variability via a dependence of *other* fundamental "constants" on these clocks. Although this is not needed, sharper results are obtained if this procedure violates local Lorentz invariance, which we define in the spirit of Horava-Lifshitz theories (modifying a $3 + 1$ split action, so that a Lorentz invariant 4D reassembly is no longer possible). We find that variability in the "laws of physics" generically leads to violations of energy conservation if either a matter parameter varies as a function of a gravitational clock, or a gravity parameter depends on a matter clock, with the other combinations sterile. In general the matter components associated with the varying parameter or the clock absorb or give off the violated energy. We illustrate this with a variety of clocks (ticking unimodular time, Ricci time, etc.) and parameters (mainly the gravitational and matter speed of light, but also Λ). We can accommodate in this construction (and improve on) several early varying speed of light models, allowing for variability effects related to the spatial curvature and Λ to cause creation of radiation and a hot big bang.

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I. INTRODUCTION

John Wheeler reputedly stated that "everything comes out of higgledy-piggledy," to convey the view that the laws of Nature are subject to (possibly random) mutation, rather than being set in stone [1]. In this paper we investigate whether such a mutability could be the origin of the matter and energy in our Universe. Symmetries and conservation laws are intimately connected (as enshrined in Noether's theorem); specifically the time-translation symmetry of the laws of physics implies conservation of matter and energy. This suggests a logical connection between Wheeler's view and cosmic creation out of nothing. A major question hanging over this possibility is how to define the "time in terms of which" the laws evolve. The definition of time is closely related to the laws one accepts (to the extent that many propositions in any system are circular), so the construction of evolving laws and their associated times may indeed be higgledy-piggledy.

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In this paper we take a rather conservative view on the matter (for more radical alternatives see, for example, [2,3]). We propose interweaving two strands of thought. Along one strand, it has long been known that one can frame evolution in the physical laws in terms of variability in the fundamental constants they employ. This goes back to at least Dirac's seminal paper [4] (see [5] and references therein). Following another strand, one can obtain robust definitions of time by demoting the constants of nature to mere constants of motion (as done for the cosmological constant in unimodular gravity [6–14], specifically according to the procedure of [6]). Employing the recipe in [6], one finds that the canonical conjugates of the demoted physical constants are excellent candidates for physical, relational clocks, capable of surviving quantum gravity, among other blessings (see [15,16], for example).

As an exploratory hypothesis, in this paper we propose a "parting of constants": perhaps the fate of some constants is to provide clocks via the procedure of [6–14], whereas the fate of others is to vary in terms of such clocks. This orderly evolution might not have been to Wheeler's taste, but, as we will see, it will allow us a measure of pragmatic progress.

II. A POSSIBLE STAGE FOR EVOLUTION

We consider variability within a set of target parameters β, which could be any fundamental constant. The starting

point is an action S_0 (which can be standard general relativity or not) with all "constants" usually set to 1 restored, placed inside the space-time integral, and their different roles dissociated in the following sense. For example, c, colloquially the "speed of light," plays several roles that are usually conflated, but which have no reason to be identified once we consider variability in "c." This was stressed in [17], and applies to other constants demoted from their status: to give another example, Newton's constant G has the double role of defining the Planck scale (appearing in the gravitational commutation relations) and of gravitational coupling to matter, and the two can be dissociated [18,19]. We may thus consider a target β which includes c_P and G_P (appearing in the Planck scale, multiplying the gravity action), c_g (the c in the gravity metric), c_m (the c in the matter metric), and the coupling between matter and gravity, G_M. We may then fix some of these, identify others, or impose a constraint between them. We could also consider further parameters, such as the electron charge e, and further dissociations of roles.

A. The unimodular time prototype

As already stressed, the question then is variability as a function of what? Rather than allowing a coordinate time t to provide the brutal answer (thereby breaking time reparametrization invariance, as in [20]), we propose the use of physical, "relational" time(s). These may be defined mimicking the Henneaux and Teitelboim (HT) formulation [6] of unimodular gravity [6–12], well known for producing a physical measure of time dual to the cosmological constant Λ: the so-called 4-volume or unimodular time [6,13,14].

In the HT formulation of unimodular theory full diffeomorphism invariance is preserved (i.e., they are not restricted to volume preserving ones), but one adds to the base action S_0 an additional term:

$$S_0 \to S = S_0 + S_U = S_0 - \int d^4x\, \rho_\Lambda(\partial_\mu \mathcal{T}^\mu). \quad (1)$$

Here $\mathcal{T}^\mu$ is a density, so that the added term is diffeomorphism invariant without the need of the metric or the connection. Since the metric and connection do not appear in the new term, the Einstein equations (and other field equations) are left unchanged. In the standard theory S_0 does not depend on $\mathcal{T}^\mu$; hence, one equation of motion states the on-shell constancy of ρ_Λ. In addition, the gauge-invariant zero mode of $\mathcal{T}^0$,

$$T \equiv \int d^3x\, \mathcal{T}^0, \quad (2)$$

provides a definition of time, canonically dual to ρ_Λ. For standard general relativity we have

$$T_\Lambda = -\int d^4x\sqrt{-g} \quad (3)$$

that is, the 4-volume to the past, or unimodular time [6].

More generally [19,21], we may select a set of D constants α and perform a similar exercise:

$$S_0 \to S = S_0 + S_U = S_0 - \int d^4x\, \alpha \cdot \partial_\mu \mathcal{T}^\mu_\alpha \quad (4)$$

where the dot denotes the Euclidean inner product in D dimensional space. Just as it happens for unimodular theory, the zero modes of the zero components of the density $\mathcal{T}^\mu_\alpha$ provide definitions of time T_α, dual to the on-shell constants α. Besides including ρ_Λ in α (as in unimodular theory) we can consider a more general setting, including the "sequester" [22,23], where c_P^2/G_P (responsible for the Planck mass) is an element of α, its canonical conjugate providing a Ricci time clock.

B. Evolution potentials

We define "evolution in the laws of physics" by making the parameters β vary according to specified functions of the relational times T_α:

$$\beta = \beta(T_\alpha). \quad (5)$$

If β overlap with α this amounts to imposing a second-class constraint, but we do not consider this situation here. (The implications of such constraints vary from a technical nuisance to downright inconsistency, depending on the case; this is the reason for our "parting" the constants into two classes.) Instead, we investigate a subclass of theories where *some* constants (the α) are deconstantized so that their duals provide clocks, T_α, and where a *distinct set* of constants (the β) are allowed to vary as functions of the physical clocks provided by the former.

There are many combinations and choices for α and β (to be explored in this paper and in a sequel), but given our emphasis on local Lorentz invariance violation (LLIV) in this paper we will take as a starting point the selection (to be changed later)

$$\beta = (c_g^2, c_m^2, \ldots), \quad (6)$$

$$\alpha = \left(\rho_\Lambda, \frac{3c_P^2}{8\pi G_P}, \frac{G_M}{G_P}, \ldots\right), \quad (7)$$

where the extra factor of 6 in α_2 is included for later convenience. For standard general relativity,

$$S_0 = \int d^4x\sqrt{-g}\left[\frac{\alpha_2}{6}R + \alpha_3\mathcal{L}_M - \rho_\Lambda\right], \quad (8)$$

and the expressions for T_α are well known. Beside $T_1 \equiv T_\Lambda$, we have

$$T_2 \equiv T_R = \frac{1}{6} \int d^4x \sqrt{-g} R, \tag{9}$$

$$T_3 \equiv T_N = \int d^4x \sqrt{-g} \mathcal{L}_M, \tag{10}$$

i.e., Ricci and Newton[1] times (see [18,22–25] in addition to [6–14]). T_Λ and T_R are nothing but the "fluxes" used in the sequester model [19,21–23]. We will also toy with c_m being included in the α rather than β and other combinations.

The evolution functions $\beta(T_\alpha)$ are "givens," in the same sense that classical potentials in field theory are givens. This is supported by analogy, as we shall see in Sec. V C. If LLIV is present, the β functions can be seen as "LLIV potentials." In most cases we will assume they are step functions, modeling abrupt phase transitions, just as in [20,26,27].

C. Gauge invariance

The unimodular-like extensions do not add new local degrees of freedom. This was remarked in [6] for Λ (see also [28,29]) and is a general feature of all extensions (4). It follows from the invariance of S_U under the gauge symmetry:

$$\mathcal{T}_\Lambda^\mu \to \mathcal{T}_\Lambda^\mu + \epsilon^\mu \quad \text{with} \quad \partial_\mu \epsilon^\mu = 0. \tag{11}$$

It leads to first class constraints cancelling out the apparent new local degrees of freedom. However, the zero mode of the zeroth component

$$T_\Lambda(\Sigma) \equiv \int_\Sigma d^3x \mathcal{T}_\Lambda^0 \tag{12}$$

is gauge invariant, so globally (and in the minisuperspace approximation) we do have an extra degree of freedom. For example, in standard general relativity with a pure Λ we have no degrees of freedom under homogeneity and isotropy (de Sitter with the pregiven Λ is the only solution), whereas in unimodular theory we can specify the value of Λ.

Had we made the β (now functions of space as well) depend on the local $T^0(\mathbf{x})$ (and not just its zero mode) we would have broken the gauge symmetry. This is interesting (and was investigated in a different context in [28]), but it will not be considered here. Finally, we note that, although the gauge-invariant zero mode depends on the foliation Σ, the theory does not need to break local Lorentz invariance. However, it can do so, and the presence of a preferred Σ may be a motivation for doing so. We now explain what we mean by this, and how it relates to the choice of β.

III. BREAKING LOCAL LORENTZ INVARIANCE

We define local Lorentz invariance violation (LLIV) in the spirit of Horava-Lifshitz (HL) theory [30]. The idea is to take a local Lorentz invariant action, choose a foliation Σ, perform a $3 + 1$ split as in the Hamiltonian formulation, and then change an aspect of the split action so that a local Lorentz invariant 4D reassembly is no longer possible. The kinetic term of HL theory is an example of this. One considers a generic kinetic term of the form [30]:

$$S_K = \frac{c^3}{16\pi G} \int dt \, d^3x \sqrt{|g|} [K_{ij} K^{ij} - \lambda (K_i^i)^2], \tag{13}$$

so that when $\lambda = 1$ it reduces to the kinetic term of general relativity, and so complies with local Lorentz invariance. One then allows $\lambda \neq 1$, so that the theory no longer derives from the Einstein-Hilbert action. Although the action is still invariant under foliation-preserving (3D) diffeomorphisms and time reparametrization, it lost full 4D diffeomorphism invariance. This is merely an illustration of the process we have in mind, and we ignore other aspects of HL theory, such as detailed balance.

A. A gravitational example

In the same spirit, we take the Einstein-Cartan action subject to the operations described at the start of Sec. II:

$$S_{\text{EC}} = \int \frac{c_P^2}{32\pi G_P} \epsilon_{ABCD} e^A e^B R^{CD}[\Gamma] \tag{14}$$

(here e^A is the tetrad and $\Gamma^A{}_B$ is the spin connection). We then split it as [31]

$$S_{\text{EC}} = \int dt \, d^3x \frac{c_P^2}{16\pi G_P} [2\dot{K}_a^i E_i^a - (NH + N^a H_a + N^{ij} G_{ij})], \tag{15}$$

where K_a^i is the extrinsic curvature connection,[2] E_i^a is the densitized inverse triad, and the last three terms are the Hamiltonian, diffeomorphism, and Gauss constraints:

$$H = \frac{1}{\sqrt{h}} (K_a^j K_b^i - K_a^i K_b^j) E_i^a E_j^b - c_g^2 \sqrt{h} R_3, \tag{16}$$

$$H_a = -2D_b (K_a^i E_i^b - \delta_b^a K_c^i E_i^c), \tag{17}$$

$$G_{ij} = K_{a[j} E_{k]}^a \tag{18}$$

(where R_3 is the 3D curvature, h is the determinant of the 3-metric, and D_a the 3D covariant derivative) enforced by

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[1]Since T_3 is dual to Newton's G_N we propose this nomenclature.

[2]From which the Ashtekar connection can be built by a canonical transformation [31], should we wish to generalize the theory we are about to propose.

corresponding Lagrange multipliers N, N^a, and N^{ij}. In writing (15) we have chosen units as follows: the coordinates satisfy $[x^0] = T$ and $[x^i] = L$; the metric is dimensionless and so $[e_\mu^A] = [E^a] = [N] = [N^i] = L^0$; the extrinsic curvature has units of inverse time squared $[K_b^a] = 1/T^2$ (derivatives with respect to time), but the 3D curvature has units of inverse length squared, $[R_3] = 1/L^2$; the volume element is written in terms of time and 3D spatial volume. With these choices the prefactor of the action is proportional to c_P^2/G_P, as written in (15), and a factor of c_g^2 appears in the last term of the Hamiltonian constraint (16) (with the Hamiltonian constraint having units of $[H] = 1/T^2$).

If c_g is constant none of this matters. But if we allow its variation [even if only as a function of a global time variable, $c_g = c_g(T_\alpha)$, as in Sec. II], then we have an analogy with the kinetic term in HL for $\lambda \neq 1$. Although we started from (14), once we allow $c_g = c_g(T_\alpha)$ the split action (15) can no longer arise from (14). Any attempt to do so would produce extra terms in the derivatives of c_g. Since the connection $\Gamma^A{}_B$ has units of $1/T$, the extra terms in the action arise from

$$\int \epsilon_{ABCD} e^A e^B \Delta R^{CD} \tag{19}$$

where

$$\Delta R^A_{Bi} = \partial_\mu \frac{1}{c_g} \Gamma^A_{Bi} dx^\mu dx^i. \tag{20}$$

These terms break local Lorentz invariance. Hence the action (15) obtained by neglecting these terms when starting from the LI Eq. (14) must be LLIV.

The above choice of units is crucial to LLIV. If K_a^i and R_3 are defined with the same units, then an overall common factor of c_g appears in (16), so that the time variations in c_g can be absorbed into a redefinition of N. Also, if we define time as x^0, that is time with units of length, no extra terms appear going from (14) to (15) (but see [32]). With such choices, in effect the notation has absorbed the assumption of a fixed speed of light, with time and space rendered equivalent and the speed of light losing its meaning.

Upon quantization the gravitational commutation relations inferred from (15) are

$$[K_a^i(\boldsymbol{x}), E_j^b(\boldsymbol{y})] = i\frac{8\pi G_P \hbar}{3c_P^2} \delta_a^b \delta_j^i \delta(\boldsymbol{x} - \boldsymbol{y})$$
$$= il_P^2 c_P \delta_a^b \delta_j^i \delta(\boldsymbol{x} - \boldsymbol{y}), \tag{21}$$

where $l_P^2 = 8\pi G_P \hbar/c_P^3$ is the Planck length (note that the delta function has units of $1/L^3$, so it all matches). This justifies the statement that G_P and c_P are the relevant variables setting the Planck scale. Indeed this "speed of light," c_P, is nothing but a conversion factor between different parametrizations of the Planck scale with different dimensions, and it will be absorbed into a single parameter [α_2, as in Eq. (7)], and ignored for the rest of this paper.

B. A matter example

Theories where $\alpha_3 = G_N/G_P$ and its dual, T_N, come into play offer another example of LLIV in the same spirit [see (7) and (8) for definitions]. Integrations by parts in $\mathcal{L}_M$, which are innocuous in the standard theory, lead to different theories if α_3 is included and there is T_N dependence. This induces LLIV if the integration by parts is designed to do so.

Let us consider extension (4) applied to α_3, and a foliation Σ_t used to define the associated gauge-invariant T_N [as in (12)]. The $3 + 1$ split of this term in S_U is

$$S = S_0 - \int dt \alpha_3(t) \dot{T}_N \tag{22}$$

where we have *not* assumed homogeneity. If there is T_N dependence in S_0, then α_3 will change in time *but not in space* in this foliation, according to the corresponding Hamilton equation, $\dot{\alpha}_3 = \partial H/\partial T_N$.

Now take a massless scalar field in flat spacetime as an example. Upon a $3 + 1$ split in the same foliation its action becomes

$$S_\phi = \int d^3x \, dt \left(\dot{\phi} \Pi_\phi - \frac{N}{2} \left(\Pi_\phi^2 + (\partial_i \phi)^2 \right) \right) \tag{23}$$

and we could perform an integration by parts in time in the first term to bring this to

$$S_\phi = \int d^3x \, dt \left(\dot{\Pi}_\phi \phi - \frac{N}{2} \left(\Pi_\phi^2 + (\partial_i \phi)^2 \right) \right) \tag{24}$$

(with a $\Pi_\phi \to -\Pi_\phi$ redefinition). Both actions are (at least classically) equivalent if α_3, as defined in (8), is not deconstantized. Both lead to a massless Klein-Gordon equation. Under (22), however, it matters if in (8) we take (23) or (24) for $\mathcal{L}_M$. If S_0 depends on T_N, so that $\alpha_3 = \alpha_3(t)$ in this foliation, they are different theories. The first is still Lorentz invariant and has equation of motion $\partial_\mu(\alpha_3 \partial^\mu \phi) = 0$. The second has LLIV and satisfies

$$-\alpha_3(t)\ddot{\phi} + \nabla\phi = 0 \tag{25}$$

in this foliation. Thus, α_3 theories may be used to implement LLIV in the matter sector, even though they do not have to.

Note that even when α_3 is a constant, the on-shell value of the action is different in the two theories. In the first, the on-shell Lagrangian is the pressure (should there be a potential, the Lagrangian would be $p = K - V$). In the

second, it is minus the energy density (minus the Hamiltonian, since the canonical pair terms vanish). This affects the on-shell expression for T_N.

IV. REDUCTION TO HOMOGENEITY AND ISOTROPY

Reducing the action $S = S_0 + S_U = S_G + S_M + S_U$ (where S_G and S_M are the gravity and the matter base actions) to homogeneity and isotropy we find

$$S_G = V_c \int dt\, \alpha_2 \left(\dot{b}a^2 + Na(b^2 + kc_g^2) \right), \quad (26)$$

$$S_M = V_c \int dt\, \alpha_3 \left(\dot{m}_i \phi_i - Na^3(\rho_\Lambda + \rho_M) \right), \quad (27)$$

$$S_U = V_c \int dt\, \dot{\boldsymbol{\alpha}} \cdot \boldsymbol{T} = V_c \int dt\, \dot{\rho}_\Lambda T_\Lambda + \cdots. \quad (28)$$

Here a is the expansion factor, b is the minisuperspace connection variable (an off-shell version of the Hubble parameter, since $b = \dot{a}$ on-shell, if there is no torsion), k is the spatial curvature, N is the lapse function, and $V_c = \int d^3x$ is the comoving volume of the region under study, assumed finite throughout this paper (in the quantum cosmology classical literature one usually chooses $k = 1$ and $V_c = 2\pi^2$; see [33] for a discussion of the criteria for the choice of V_c). We allow for a matter action S_M with multifluids and possibly scalar fields (see later applications), with $\rho_M = \sum_i \rho_i$ with

$$\rho_i = \frac{m_i}{a^{3(1+w_i)}} \quad (29)$$

with w_i the fluid's equation of state (here assumed constant), m_i a constant of motion in the usual theory, dual to variables ϕ_i as defined in the Lagrangian formulation of perfect fluids of [34], reduced to minisuperspace (MSS) as in [15,16,35]. We will generalize this in Sec. V B.

For this action, we therefore have total Hamiltonian

$$H = NaV_c \left[-\alpha_2(b^2 + kc_g^2) + \alpha_3(\rho_\Lambda + \rho_M)a^2 \right] \quad (30)$$

and Poisson brackets,

$$\{b, A^2\} = \frac{1}{V_c}, \quad (31)$$

$$\{\alpha_i, T_i\} = \frac{1}{V_c} \quad (32)$$

with the conjugate of b given by

$$A^2 = a^2 \alpha_2. \quad (33)$$

Upon quantization the gravitational commutation relation in MSS can be written as

$$\left[\hat{b}, \hat{a}^2 \right] = i\frac{8\pi G_P \hbar}{3V_c c_P^2} = \frac{il_P^2 c_P}{3V_c} \equiv i\mathfrak{h} c_p \quad (34)$$

which is a reduced version of (21). The Planck constant $\mathfrak{h}$ is the relevant quantity for minisuperspace quantum cosmology.

We first assume that α_2 is kept constant (this leads to a fork in the formalism, as we will investigate in Sec. VI A). Then a and A can be used interchangeably. In finding the Poisson equations, we stress that, as per the usual definition of the Poisson bracket, $\boldsymbol{\alpha}$ and $\boldsymbol{T}$ (and so also $\boldsymbol{\beta}$) are kept fixed when evaluating Poisson brackets involving b and a. As a result their equations of motion are obtained from the usual ones simply by inserting varying constants where usually they appear as fixed parameters. Hence, for all theories based on this setup (with the exclusion of those with varying α_2, as we will see), we have a Hamilton constraint (obtained by varying with respect to N) and Poisson equations for a and b:

$$b^2 + kc_g^2 = \frac{\alpha_3}{\alpha_2}\rho a^2,$$

$$\dot{a} = \{a, H\} = Nb,$$

$$\dot{b} = \{b, H\} = -\frac{\alpha_3}{2\alpha_2}(\rho + 3p)Na, \quad (35)$$

with

$$\rho = \rho_\Lambda + \rho_M = \rho_\Lambda + \sum_i \rho_i, \quad (36)$$

$$p = p_\Lambda + p_M = -\rho_\Lambda + \sum_i w_i \rho_i. \quad (37)$$

This is equivalent to directly inserting varying constants into the unmodified Friedman and Raychaudhuri equations, as in [20]:

$$\dot{a}^2 + kc_g^2 = \frac{\alpha_3}{\alpha_2}\rho a^2,$$

$$\ddot{a} = -\frac{\alpha_3}{\alpha_2}(\rho + 3p)]a$$

(in the $N = 1$ gauge for simplicity). Assuming further a fixed α_3 (not necessarily equal to 1), by dotting the first equation and comparing with the second, a consistency relation can be obtained:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = \frac{k\alpha_2}{a^2\alpha_3}\frac{dc_g^2}{dt} \quad (38)$$

(valid in any gauge). This is the central Eq. (5) in [20]:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = \frac{3kc_g^2}{4\pi G_M a^2}\frac{\dot{c}_g}{c_g} \qquad (39)$$

given that we fixed c_P (together with α_2) so the barred densities in [20] there can be dropped.

It is interesting that the most abstruse assumptions in [20] are in fact natural from the point of view of the construction in Sec. II. In [20] it was postulated that varying constants did not produce stress-energy-momentum $T_{\mu\nu}$, or change the Einstein equations, something undoubtedly odd, say, from the perspective of scalar-tensor theories. This is just what happens here for two reasons. First, the unimodular-like procedure designed in Sec. II A for producing clocks is such that the metric does not enter S_U, so no $T_{\mu\nu}$ is produced. Second the theory has a Hamiltonian structure, so that the Poisson equations for one set of variables are obtained fixing all other variables. Hence the Einstein equations in their $3 + 1$ split form are obtained fixing all other phase space variables including the time variables $\boldsymbol{T_\alpha}$ and by extension the varying constants $\boldsymbol{\beta}$.

We add that this is not a miracle and in fact it is not true in general. If we do not fix α_2, then new terms do appear in the Einstein equations, unless we go beyond minimal assumptions regarding the sympletic/Hamiltonian structure of the theory, as we will see in Sec. VI A. In order to avoid extra terms under a varying α_2 one would need to fix the α_2 multiplying first term in (26), $\dot{b}a^2$, while allowing deconstantization for the α_2 multiplying the second term and the gravitational Hamiltonian. Such a procedure goes beyond Sec. VI A and clearly requires LLIV.

V. MATTER-BASED CLOCKS

We thus justify the central equations of [20], but we can go beyond. A major weakness of [20] was an absence of information on how the energy conservation violating last term in (38) would be shared between the various matter components (including Λ). This was not an issue in scenarios where only radiation is present, as envisaged in [20]. It turns out we can be more predictive in this respect, opening up the doors to other scenarios. As a first rule of thumb, this energy is dumped into (or extracted from) whatever matter form is conjugate to the relational time variables upon which c_g depends (should the clocks be "matter clocks," as we now illustrate. Throughout this section we fix α_2 and α_3 (see next section for the associated issues).

A. Pure unimodular varying speed of light (VSL)

The simplest theory follows from $\boldsymbol{\alpha} = \rho_\Lambda$ (pure unimodular S_U) and $\boldsymbol{\beta} = c_g$, so that $c_g = c_g(T_\Lambda)$. Then, beside Eqs. (35) (which always apply), we have two more Hamilton equations:

$$\dot{T}_\Lambda = \{T_\Lambda, H\} = -N\alpha_3 a^3, \qquad (40)$$

$$\dot{\rho}_\Lambda = \{\rho_\Lambda, H\} = \frac{\partial H}{\partial T_\Lambda} = -N a\alpha_2 k\frac{dc_g^2}{dT_\Lambda}. \qquad (41)$$

The first equation just states that T_Λ is the 4-volume time if $\alpha_3 = 1$ and agrees with (3) modulo a factor of V_c. By introducing V_c in the Poisson bracket (32) we are defining *intensive* times, i.e., time per unit of spatial volume. In this case we are defining time as the 4-volume per unit of 3-volume, so that

$$T_\Lambda = -\alpha_3 \int dt N a^3, \qquad (42)$$

with units of time. The time dependence of c_g does not affect this equation, since it only cares about the Hamiltonian dependence on ρ_Λ.

The second equation implies that the energy source/sink term contained in (38) goes fully into the vacuum energy, since it implies [when combined with (40)]

$$\dot{\rho}_\Lambda + 3\frac{\dot{a}}{a}(\rho_\Lambda + p_\Lambda) = \dot{\rho}_\Lambda = \frac{k\alpha_2}{a^2\alpha_3}\frac{dc_g^2}{dt}. \qquad (43)$$

This is true even if there are other forms of matter ρ_i and is the simplest example of a pattern which we now uncover by generalization: the violations of energy conservation are absorbed by the matter ingredient dual to the clock.

B. The matter speed of light, c_m

Turning now to the matter speed of light, c_m, we note that it can vary (or be fixed) independently from c_g (e.g., [17,36]). For definiteness, we model matter as a mixture of isentropic fluids according to [15,16,34,35]. Imposing homogeneity and isotropy the action becomes

$$S_M = V_c \int dt \sum_i \left(a^3 n_i \dot{\Theta}_i - N a^3 \rho_i(n_i)\right). \qquad (44)$$

For each specie i, Θ_i is a Lagrange multiplier, so that n_i is the (spatial) volume density of a conserved particle number (i.e., $\Pi_i = n_i a^3$ is conserved). Thermodynamics shows that the chemical potential and pressure are given by

$$\mu_i = \frac{p_i + \rho_i}{n_i} = \frac{\partial \rho_i}{\partial n_i}, \qquad (45)$$

$$p_i = n_i \frac{\partial \rho_i}{\partial n_i} - \rho_i. \qquad (46)$$

Restoring c_m we have in general

$$\rho_i = \rho_i(n_i, c_m), \qquad (47)$$

so that

$$\mu_{ic} = \frac{\partial \rho_i}{\partial c_m^2} \qquad (48)$$

can be thought of as the speed of light "chemical" potentials, in analogy with (45). For example, for dust $\rho = n m_0(c_m) c_m^2$ (with m_0 a constant in some cases); for black body radiation $\rho = n^{4/3} \hbar c_m$. The c_m chemical potentials are a property of each matter specie just like w_i. Thus, detailed particle physics enters the cosmological argument.[3]

If we include c_m^2 in $\boldsymbol{\alpha}$, then (44) becomes

$$S_M = V_c \int dt \left[\dot{c}_m^2 T_c + \sum_i \left(\Pi_i \dot{\Theta}_i - N a^3 \rho_i \left(\frac{\Pi_i}{a^3}, c_m^2 \right) \right) \right]. \qquad (49)$$

If $w_i \neq -1$ is constant, then $\rho_i = f_i n_i^{1+w_i}$, where f_i is a dimensionful proportionality constant [34]. Hence,

$$\rho_i = n_i^{1+w_i} f_i(c_m^2) \qquad (50)$$

and we can perform a canonical transformation from $\{\Pi_i, \Theta_i\}$ to $\{m_i, T_{mi}\}$ so that

$$S_M = V_c \int dt \left[\dot{c}_m^2 T_c + \sum_i \left(\dot{m}_i T_{mi} - N \frac{m_i f_i(c_m^2)}{a^{3w_i}} \right) \right] \qquad (51)$$

leading to the time formula

$$\dot{T}_c = -N \sum_i \frac{m_i f_i'}{a^{3w_i}}. \qquad (52)$$

For a model where $c_g \in \beta$ only depends on T_c we have

$$\dot{c}_m^2 = \frac{\partial H}{\partial T_c} = -N a \alpha_2 k \frac{\partial c_g^2}{\partial T_c} \qquad (53)$$

resulting in

$$\dot{\rho}_i + 3 \frac{\dot{a}}{a} (\rho_i + p_i) = \frac{m_i \dot{f}_i}{a^{3(1+w_i)}} = \frac{m_i f_i'}{a^{3(1+w_i)}} \frac{dc_m^2}{dt}$$
$$= \frac{k \alpha_2}{a^2} - \frac{N f_i' m_i}{a^{3w_i}} \frac{dc_g^2}{dt} \qquad (54)$$

which summed over i leads to Eq. (38), as it should. But we have learnt more: in this theory the more a specie depends on c_m the more it is affected by energy conservation violations, an extension of the pattern in the previous subsection.

[3]Obviously we could complicate the model by attributing different c_{mi} to different species as in [37].

We could instead have $c_m \in \boldsymbol{\beta}$, and in particularly set $c = c_g = c_m$. We could also write $\rho_\Lambda = m_\Lambda f_\Lambda(c_m^2)$, and take m_Λ for $\boldsymbol{\alpha}$ leading to a slightly modified unimodular clock, with $\dot{T}_\Lambda = -N a^3 f_\Lambda'$. Then

$$\dot{\rho}_\Lambda = (m_\Lambda f_\Lambda)^\cdot = \dot{m}_\Lambda f_\Lambda + m_\Lambda f_\Lambda' \frac{dc_m^2}{dt}, \qquad (55)$$

whereas for the other matter components ($i \neq \Lambda$),

$$\dot{\rho}_i + 3 \frac{\dot{a}}{a} (\rho_i + p_i) = \frac{m_i \dot{f}_i}{a^{3(1+w_i)}} = \frac{m_i f_i'}{a^{3(1+w_i)}} \frac{dc_m^2}{dt}.$$

The extra terms in $\dot{c}_m$ are $\partial \boldsymbol{\beta} / \partial \boldsymbol{T}_\alpha$ terms. The first term in (55) can be expanded using

$$\dot{m}_\Lambda = \frac{\partial H}{\partial T_\Lambda} = \frac{\partial H_g}{\partial T_\Lambda} + \frac{\partial H_m}{\partial T_\Lambda}$$
$$= -N a k \alpha_2 \frac{\partial c_g^2}{\partial T_\Lambda} + N a^3 \sum_i \frac{\partial \rho_i}{\partial c_m^2} \frac{\partial c_m^2}{\partial T_\Lambda} \qquad (56)$$

so we find after some algebra

$$\dot{\rho}_\Lambda = \left(\frac{k \alpha_2}{a^2} - \sum_{i \neq \Lambda} \frac{m_i f_i'}{a^{3(1+w_i)}} \right) \frac{dc^2}{dt} \qquad (57)$$

(where we have set $c_m = c_g$). Again, summing over all the i's we get (38). Even though the net production of matter is not affected by $c_m(T_\Lambda)$, the energy equations for each species receive a source/sink term dependent on their c_m chemical potentials. If c_m is part of $\boldsymbol{\beta}$ it can be seen as an interaction potential between the different components, allowing for interchange of matter.

C. An analogy

We could use the fluids as clocks ($m_i \in \boldsymbol{\alpha}$ with conjugate T_{mi} ticking "temperature time" for radiation, proper time for dust particles, for example) and likewise with any scalar field (as was done in [15,16]). We will not explore this option in this paper because it is less fundamental than targeting proper constants; however, here we draw attention to an interesting analogy. Take a massless scalar field,

$$S_\phi = V_c \int dt \left(\dot{\phi} \Pi_\phi - N \frac{\Pi_\phi^2}{2a^3} \right), \qquad (58)$$

so the field ϕ and its conjugate Π_ϕ satisfy

$$\dot{T}_\phi \equiv \dot{\phi} = N \frac{\Pi_\phi}{a^3}, \qquad (59)$$

$$\dot{\Pi}_\phi = -\frac{\partial H}{\partial \phi}. \qquad (60)$$

It then looks as if we can use Π_ϕ as a constant $\boldsymbol{\alpha}$ and ϕ as its conjugate time, but this is at best an effective description. Nonetheless if for example we set $c_g = c_g(T_\phi)$ we find that the nonconservation equation for ϕ can be written as

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = \alpha_2 k \frac{\partial c_g^2}{\partial T_\phi}, \qquad (61)$$

since $\rho_\phi = p_\phi = \dot{\phi}^2/2$. It suggests interpreting

$$U_{\text{LLIV}} = -\frac{3k c_P^2 c_g^2(\phi)}{8\pi G_P} \qquad (62)$$

as a LLIV potential source of energy conservation violations. It makes sense to look at $\boldsymbol{\beta}(\boldsymbol{T_\alpha})$ as "potentials."

VI. GRAVITY CLOCKS

The examples in the Sec. V show a preliminary pattern. Suppose a physical clock is made of matter: it has a matter constant of motion, which is the canonical dual variable to its variable "moving hands." If the laws of nature depend on the time ticked by this clock via a gravitational parameter (c_g in our example), then there are energy conservation violations. This surplus/deficit of energy goes into the clock's constant of motion, and so into the clock system. We saw this happen if the clock is unimodular time, with Λ collecting or giving off the energy. In the case of a c_m clock, the dual constant of motion is shared by several fluids, with the total energy violation apportioned between the matter components depending on how much each contributes to the clock.

However, we already saw this pattern break down if a matter parameter, instead, depends on a matter clock [as with $c_m(T_\Lambda)$]. The pattern also fails to apply to gravitational parameters depending on gravitational clocks. To explore this, we need to develop formalism for T_R and T_N, bringing α_2 and α_3 into the calculation.

A. LLIV theories with Ricci and Newton clocks

The calculation in Sec. IV is different if the Planck scale α_2 is one of the $\boldsymbol{\alpha}$ and the $\boldsymbol{\beta}$ depend on its associated time [Ricci time, T_R, see (9)]. Then, it is A [as defined in (33)] and not a that should be used when computing Poisson brackets. It is useful to write the Hamiltonian as a function of A instead of a (or simply to evaluate the Euler-Lagrange equations, in this case). The equations of motion are

$$b^2 + k c_g^2 = \frac{\alpha_3}{\alpha_2}\rho a^2, \qquad (63)$$

$$\dot{a} + \frac{\dot{\alpha}_2}{2\alpha_2}a = Nb, \qquad (64)$$

$$\dot{b} = -\frac{\alpha_3}{2\alpha_2}(\rho + 3p)Na, \qquad (65)$$

and the central assumption in [20] is violated (a new term appears in the second equation). We remark that for $\alpha_3 = 1$ these equations of motion are the same as those for a nondynamical scalar field in the Jordan frame (i.e., Brans-Dicke theory in the first-order formulation with $\omega_{\text{BD}} = 0$) as studied in [38–40].[4]

A nonconservation equation can be obtained as before, dotting the Hamilton constraint (63), and using (64) and (65) to eliminate b and $\dot{b}$. It may then be convenient to define densities converted into geometrical quantitities (with units of $1/T^2$):

$$\tilde{\rho}_i = \frac{\alpha_3}{\alpha_2}\rho_i \qquad (66)$$

(and likewise for p), to find

$$\dot{\tilde{\rho}} + 3\frac{\dot{a}}{a}(\tilde{\rho} + \tilde{p}) = -\frac{\dot{\alpha}_2}{\alpha_2}\frac{\tilde{\rho} + 3\tilde{p}}{2} + \frac{k}{a^2}\frac{dc_g^2}{dt}. \qquad (67)$$

Reverting to the original variables, we have

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -\frac{\dot{\alpha}_3}{\alpha_3}\rho + \frac{\dot{\alpha}_2}{\alpha_2}\frac{\rho - 3p}{2} + \frac{k\alpha_2}{a^2\alpha_3}\frac{dc_g^2}{dt}. \qquad (68)$$

The first and last terms in the right-hand side are the same as in (38), for fixed G_P. The second term is new, and is not predicted by [20]. We also have the extra Hamilton equations:

$$\dot{T}_R = -\frac{\partial H}{\partial \alpha_2} = \frac{\alpha_3}{\alpha_2}Na^3\frac{\rho - 3p}{2}, \qquad (69)$$

$$\dot{T}_N = -\frac{\partial H}{\partial \alpha_3} = -Na^3\rho, \qquad (70)$$

$$\dot{\alpha}_2 = \frac{\partial H}{\partial T_R} = -Na\alpha_2 k \frac{\partial c_g^2}{\partial T_R}, \qquad (71)$$

$$\dot{\alpha}_3 = \frac{\partial H}{\partial T_N} = -Na\alpha_2 k \frac{\partial c_g^2}{\partial T_N}, \qquad (72)$$

where, we stress, one should keep A^2 (and not a^2) fixed evaluating the derivatives. This applies to the a appearing in the matter Hamiltonian, and it makes a crucial difference for the $\dot{T}_R$ formula. The first formula agrees with (9) reduced to MSS. The second agrees with (10), modulo the issues discussed in Sec. III B (if we do not perform a LLIV integration by parts, $\dot{T}_N = Na^3 p$, or pressure time). In the

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[4]We can also rephrase the new term in the equations as a theory with torsion, due to a nonfixed Lambda potential [as in [41,42], in this case with the even-parity torsion given by $T = \dot{\alpha}/(2N\alpha)$]. The crucial difference here is the unimodular addition, and the fact that the potentials are nonlocal functions of the conjugate of the dilaton, rather than of the dilaton itself.

last two equations we have assumed that only c_g enters $\boldsymbol{\beta}$, but these can (and will) be generalized.

Since we can replace

$$\frac{dc_g^2}{dt} = \sum_i \frac{\partial c_g^2}{\partial T_i} \dot{T}_i \qquad (73)$$

in the last term of (68), we find that the terms due to the dependence of c_g on T_R and T_N cancel out the first two terms. That is, the dependence of c_g on Ricci or Newtonian time does not lead to violations of energy conservation, with

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \qquad (74)$$

if c_g only depends on T_R and T_N (if there are other dependences, these still contribute the usual terms).

This result makes sense. A gravitational parameter depending on a clock leads to energy violations absorbed by that clock, but if the clock is gravitational there is nothing "material" to absorb the violations of energy conservation. So, there are none. In the language of natural selection, the evolution is sterile.

B. More general theories with varying Planck scale and gravitational coupling

There is a final twist in the pattern: a *matter* parameter depending on a gravitational clock. The cancellations in the previous subsection do not apply if, for example, $c_m(T_R, T_N)$. Then, Eq. (68) still holds true (since it only depends on the Einstein/gravity equations), but (71) and (72) receive a new term:

$$\dot{\alpha}_2 = \frac{\partial H}{\partial T_R} = -Na\alpha_2 k \frac{\partial c_g^2}{\partial T_R} + Na^3 \alpha_3 \frac{\partial \rho}{\partial c_m^2} \frac{\partial c_m^2}{\partial T_R},$$

$$\dot{\alpha}_3 = \frac{\partial H}{\partial T_N} = -Na\alpha_2 k \frac{\partial c_g^2}{\partial T_N} + Na^3 \alpha_3 \frac{\partial \rho}{\partial c_m^2} \frac{\partial c_m^2}{\partial T_N},$$

resulting in

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = Na^3 \frac{\partial \rho}{\partial c_m^2} \left(\frac{\alpha_3}{\alpha_2} \frac{\partial c_m^2}{\partial T_R} \frac{\rho - 3p}{2} - \frac{\partial c_m^2}{\partial T_N} \rho \right).$$

Had we considered multiple components, we would find that the general pattern is that if a matter $\boldsymbol{\beta}$ depends on a gravitational clock, then there are violations of energy conservation, apportioned between the different components by the dependence of their energy density on the relevant $\boldsymbol{\beta}$.

VII. GENERAL FORMULA

We now collect the various examples we have studied in a single general formula. We should stress that the

cancellations in Sec. VI A are not mysterious. As we will show in a sequel to this paper [43], they arise from the symplectic Hamiltonian structure of the theory alone. The same applies to the general formula we will derive in this section. It results from that structure plus the secondary constraint,

$$\dot{H} \approx 0, \qquad (75)$$

following from $H \approx 0$. This constraint usually amounts to local matter/energy density conservation and the Bianchi identities for the geometry, whereas here it will lead to a generalized nonconservation formula.

A. General formula in Einstein-Cartan theory

We first derive the general formula with reference to the Einstein-Cartan action subject to LLIV as defined in Sec. III (and the assumption of homogeneity and isotropy). The idea is to repeat the calculations in Sec. VI for a general $\boldsymbol{\beta}(T_\alpha)$. Defining

$$H = H_G + H_M = \alpha_2 H_g + \alpha_3 H_m = \alpha_2 H_g + \alpha_3 N a^3 \rho$$

so that H_g does not depend on α_2 (or α_3), and evaluating $\dot{H} = 0$ we find[5]

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = -\frac{\dot{\alpha}_3}{\alpha_3}\rho + \frac{\dot{\alpha}_2}{\alpha_2}\frac{\rho - 3p}{2}$$
$$- \frac{\alpha_2}{Na^3\alpha_3} \left(\sum_I \frac{\partial H_g}{\partial \alpha_I} \dot{\alpha}_I + \sum_K \frac{\partial H_g}{\partial \beta_K} \dot{\beta}_K \right).$$

Using the Poisson equations, the term in brackets can be written as

$$\sum_{IK} \frac{\partial H_g}{\partial \alpha_I} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \beta_K} - \frac{\partial H_g}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \alpha_I},$$

and for $I = 2, 3$ rearranged as

$$-\frac{1}{\alpha_2} \sum_{\substack{I=2,3 \\ K}} \frac{\partial}{\partial \beta_K}(H - \alpha_3 Na^3 \rho) \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \alpha_I}$$

$$= -\frac{1}{\alpha_2} \left(\dot{\alpha}_2 \dot{T}_2 + \dot{\alpha}_3 \dot{T}_3 - \alpha_3 Na^3 \sum_{\substack{I=2,3 \\ K}} \frac{\partial \rho}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \alpha_I} \right)$$

so that the first two terms generally cancel the new terms in the continuity equation. For $I \neq 2, 3$ only the matter component in H leads to a nonzero contribution. Thus

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[5]Note that we need to use Hamilton's equations to eliminate b, and this contains new terms in $\dot{\alpha}_2$; hence the need to separate them.

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = \alpha_2 \sum_{\substack{I \neq 2,3 \\ K}} \frac{\partial \rho}{\partial \alpha_I} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H_g}{\partial \beta_K} - \frac{\partial \rho}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H_g}{\partial \alpha_I}$$

$$- \sum_{\substack{I=2,3 \\ K}} \frac{\partial \rho}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \alpha_I} \qquad (76)$$

with the last term being nothing but a generalization of the terms discussed in Sec. VI B. In fact, since ρ does not depend on α_2 or α_3, and $\frac{\partial H}{\partial \alpha_2} = \frac{\partial H_G}{\partial \alpha_2} = H_g$, this can be compressed into

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = \sum_{IK} \frac{\partial \rho}{\partial \alpha_I} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H_G}{\partial \beta_K} - \frac{\partial \rho}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H_G}{\partial \alpha_I}$$

$$- Na^3 \rho \sum_K \frac{\partial \rho}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_N}, \qquad (77)$$

the last term being the only one that does not fit the simple pattern.

B. Generalization

But this result is general and does not even require an action formulation: the symplectic/Hamiltonian structure is enough [43]. An abridged proof is provided here (with a more general proof presented elsewhere), by ignoring the details of gravity and deriving the same result directly from the matter degrees of freedom.

We assume a set of perfect fluids under homogeneity but that is not needed. The crucial inputs are the dependence of each species energy density ρ_i on α_I and β_K:

$$\rho_i = \rho_i(\alpha_I, \beta_K) = \rho_i\left(\frac{\Pi_i}{a^3}, \alpha_j, \beta_K\right). \qquad (78)$$

The diagonal dependence on Π_i is non-negotiable, so we separated it and gave it a different index (i) from the other α_j (within a global I index for $\boldsymbol{\alpha}$). The other dependences are specific to each theory (see Sec. V B for examples). For α_3 we assume an implementation where ρ_i does not depend on it; instead the Θ_i or T_{mi} absorb it in their definition [cf. Eqs. (49) and (51)].

Then

$$\dot{\rho}_i = \frac{\partial \rho_i}{\partial n_i} \dot{n}_i + \sum_j \frac{\partial \rho_i}{\partial \alpha_j} \dot{\alpha}_j + \sum_K \frac{\partial \rho_i}{\partial \beta_K} \dot{\beta}_K$$

with

$$\frac{\partial \rho_i}{\partial n_i} \dot{n}_i = (p_i + \rho_i)\left(\frac{\dot{\Pi}_i}{\Pi_i} - 3\frac{\dot{a}}{a}\right),$$

$$\frac{\partial \rho_i}{\partial \alpha_j} \dot{\alpha}_j = \frac{\partial \rho_i}{\partial \alpha_j} \frac{\partial H}{\partial T_j} = \sum_K \frac{\partial \rho_i}{\partial \alpha_j} \frac{\partial H}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_j},$$

$$\frac{\partial \rho_i}{\partial \beta_K} \dot{\beta}_K = -\sum_I \frac{\partial \rho_i}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \alpha_I}, \qquad (79)$$

where we have used (46) and $\Pi_i = n_i a^3$ in the first equation and Hamilton's equations for $\dot{\alpha}_j$ and $\dot{T}_I$ in the others. But the Hamilton equation for Π_i implies[6]

$$(p_i + \rho_i)\frac{\dot{\Pi}_i}{\Pi_i} = \frac{\partial \rho_i}{\partial \Pi_i} \frac{\partial H}{\partial T_{\pi i}} \qquad (80)$$

so this term fits into the same pattern as other $\boldsymbol{\alpha}$, once the metric dependence (the term in $\dot{a}/a$) has been separated. Hence

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(p_i + \rho_i) = \sum_{IK} \frac{\partial \rho_i}{\partial \alpha_I} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \beta_K} - \frac{\partial \rho_i}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \alpha_I}. \qquad (81)$$

This is the most informative equation we have. It tells us, species by species, the violations of energy conservation for arbitrary $\boldsymbol{\beta}$ depending on arbitrary times $\boldsymbol{T_\alpha}$.

Summing over i we finally get

$$\dot{\rho} + 3\frac{\dot{a}}{a}(p + \rho) = \sum_{IK} \frac{\partial \rho}{\partial \alpha_I} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \beta_K} - \frac{\partial \rho}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H}{\partial \alpha_I}$$

$$= \sum_{IK} \frac{\partial \rho}{\partial \alpha_I} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H_G}{\partial \beta_K} - \frac{\partial \rho}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_I} \frac{\partial H_G}{\partial \alpha_I}$$

$$- Na^3 \rho \sum_K \frac{\partial \rho}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_N}, \qquad (82)$$

that is, Eq. (77), derived without reference to the gravity action, and so independent of the gravitational dynamics.

The above can be written covariantly with replacements $a^3 \to \sqrt{h}$ and $3\dot{a}/a \to \theta = \dot{h}/h$. Defining $H_m = N\sqrt{h}\rho = N\sqrt{h}\sum_i \rho_i$ the general covariant expression for nonconservation is

$$n^\nu \nabla_\mu T^\mu{}_\nu = \frac{1}{N\sqrt{h}\alpha_3} \sum_{IK} \frac{\partial \beta_K}{\partial T_I} \left(\frac{\partial H_m}{\partial \alpha_I} \frac{\partial H_G}{\partial \beta_K} - \frac{\partial H_m}{\partial \beta_K} \frac{\partial H_G}{\partial \alpha_I}\right)$$

$$- N\sqrt{h}\rho \sum_K \frac{\partial \rho}{\partial \beta_K} \frac{\partial \beta_K}{\partial T_N}. \qquad (83)$$

where n^μ is the normalized normal to Σ. Modulo α_3 issues the right-hand side is just the nonlocal component of the Poisson bracket $\{H_m, H_g\}$ (as explored further in [43]).

C. General pattern

In view of this general formula we can collate and confirm the various patterns we found in the previous sections.

As its first term indicates, to get net nonconservation of energy, one possibility is for a gravitational parameter (a $\boldsymbol{\beta}$ appearing in H_G) to depend on one or more matter clocks

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[6]Or an equivalent expression for m_i, had we used it instead of Π_i; canonical transformations do not affect this formula.

(i.e., clocks dual to α appearing in at least one matter variable). This is the case in Sec. V, where c_g depends on matter clocks (T_Λ and T_c). The more a species i contributes to the dual constant of these clocks (according to a measure which is like a chemical potential for that constant), the more of a share of violations it gets. The gravitational parameter can also depend jointly on many clocks [e.g., via functions of the form $c_g(T_\Lambda, T_c)$], in which case the partial derivative with respect to the clock time controls the apportionment.

Its second and third terms open up another possibility for a net energy conservation violation: a matter parameter (a β in H_M) dependent on the time ticked by a gravity clock (dual to a α appearing in H_G, or to the prefactor α_3 of H_m, describing the strength of the gravitational coupling). This is the case in Sec. VI B, where $c_m = c_m(T_R, T_N)$. The violations are shared by the different species according to how much their energy density depends on this matter parameter.

No other combination generates net energy. A purely matter parameter dependent on a purely matter clock is similar to an interaction term, energy exchanging between different components, but with no net gain. This is the case in Sec. V B, with $c_m(T_\Lambda)$ if c_g is left fixed (or no other gravity parameter depended on T_Λ). As we will see in Sec. VIII B, this is not useless: for example Lambda could exchange energy with other forms of matter via this mechanism, leading to an interesting cosmogony and opportunities to "solve" the Λ problem.

Only a purely gravity parameter depending on a gravity clock would lead to no energy conservation violations in *any* matter component. This is the case explored in Sec. VI A, with $c_g = c_g(T_R, T_N)$ (and α_3 treated so that it does not appear in H_m). Such situation is entirely sterile from the point of view of matter production.

VIII. ILLUSTRATIVE LLIV COSMOGONIES

We finally provide some examples of scenarios for the origin of the matter in the Universe, focusing on LLIV (deferring non-LLIV scenarios to [43]). We can think of the potentials $\beta(\alpha)$ as random choices for chaotic evolution, selected in/out by their ultimate effects, specifically by whether they lead to "viable" offspring.

A. VSL scenarios

Foremost we find VSL scenarios relating matter production to departures from spatial flatness. For example we could take $c_m^2 \in \alpha$, $c_g^2 \in \beta$ [with $c_g(T_c)$] as in Sec. V B, and investigate a phase transition with

$$c_g^2 = c_{g-}^2 H(T_{c\star} - T_c) + c_{g+}^2 H(T_c - T_{c\star}) \qquad (84)$$

where H is the Heaviside step function and $T_\star$ is a critical time (possibly, but not necessarily of the order of the Planck

scale). From the Hamilton equations for a and b we know that for such a shock there can only be delta functions in $\ddot{a}$, so a and $\dot{a}$ must be continuous. From the Hamiltonian constraint, $a_- = a_+ = a_\star$ and $\dot{a}_- = \dot{a}_+ = \dot{a}_\star$ implies

$$\Delta\rho = \frac{k\alpha_2}{a_\star^2 \alpha_3} \Delta c_g^2, \qquad (85)$$

which could also be derived directly from (38). In such a scenario

$$
\begin{aligned}
a &= c_{g-}\sqrt{|k|}\,t \quad (t < t_\star) \\
&= c_{g-}\sqrt{|k|}\left(\frac{t}{t_\star}\right)^{1/2} \quad (t > t_\star),
\end{aligned}
\qquad (86)
$$

the only discontinuity happening in $\ddot{a}$.

This much just mimics the calculations in [20], but we can say more. The jump in c_m^2 can be computed as

$$\frac{dc_m^2}{dt} = \frac{\partial H}{\partial T_c} = -\frac{Na\alpha_2 k}{\dot{T}_c}\frac{dc_g^2}{dt} = \frac{k\alpha_2}{a^2\alpha_3\mu_c}\frac{dc_g^2}{dt}. \qquad (87)$$

This is a nonlinear equation [since $\mu_c = \mu_c(c_m^2)$] with solution

$$\int_{c_{m-}^2}^{c_{m+}^2} \mu_c\, dc_m^2 = \frac{k\alpha_2}{a_\star^2 \alpha_3}\Delta c_g^2$$

so that indeed

$$\Delta\rho = \rho(c_{m+}^2) - \rho(c_{m-}^2) = \frac{k\alpha_2}{a_\star^2 \alpha_3}\Delta c_g^2 \qquad (88)$$

i.e., Eq. (85). But we also learn that the violations are shared according to

$$\Delta\rho_i = \rho_i(m_i, c_{m+}^2) - \rho_i(m_i, c_{m-}^2) = \int_{c_{m-}^2}^{c_{m+}^2} \mu_{ic}\, dc_m^2. \qquad (89)$$

Notice that at least one m_i has to be nonzero, otherwise $\dot{T}_c = 0$ [see Eq. (52)] and c_g would never feel the step predicted in $c_g^2 = c_g^2(T_c)$. More generally $\mu_c \propto \rho$, so the less matter there is to begin with, the bigger the jump in c_m. In this theory the net violations are (85), since this only depends on the gravitational action/equations, but in addition c_m must change so that this $\Delta\rho$ is produced given the overall dependence $\rho(c_m^2)$, with the various species receiving a contribution in proportion to their individual dependence $\rho_i(c_m^2)$. We could have played this game with any other fundamental matter parameter and its clock, say e, m_e, etc.

We could instead have used the reverse situations, with $c_g^2 \in \alpha$ and $c_m^2 \in \beta$. The gravitational clock associated with c_g^2 ticks as

$$\dot{T}_{cg} = \frac{\partial H}{\partial c_g^2} = Na\alpha_2 k, \qquad (90)$$

and we could consider a step function $c_m^2 = c_m^2(T_{cg})$. As it happens there is a formal symmetry between $c_m^2 = c_m^2(T_{cg})$ and $c_g^2 = c_g^2(T_c)$, because

$$\frac{dc_m^2}{dc_g^2} = \frac{\frac{dc_m^2}{dt}}{\frac{dc_g^2}{dt}} = -\frac{\frac{\partial H}{\partial c_g^2}}{\frac{\partial H}{\partial c_m^2}} = \frac{k\alpha_2}{a^2\alpha_3\mu} \qquad (91)$$

i.e., $d\beta/d\alpha$ drops out of the calculation leading to (87), whatever choice we make for α and β. We then have the same end results (88) and (89).

The physics is very different, however. Whereas for $c_g^2(T_c)$ we need some matter for the step function to be felt ($T_c \propto \rho$), here we need $k \neq 0$, i.e., some spatial curvature. Otherwise T_{cg} would stop and c_m never feel the jump. Hence, the T_{cg} interpretation sheds light on the flatness problem. Such a curvature clock stops if $k = 0$ or if curvature becomes negligible (just as a Ricci clock stops for radiation). It also flows in opposite directions for $k = \pm 1$. Flatness therefore becomes an attractor, with $\Delta c_m^2/\Delta c_g^2$ having the sign of k/μ.

B. Lambda remittances

In the above model the Λ problem is simply the statement that $\mu_{\Lambda c}$ is much smaller than the other μ_{ic} if $\Lambda = 0$ originally (with further fine-tuning needed to address the quasi-Lambda problem [44]). While it is possible that this can be proved from first principles, we should also explore models allowing the vacuum energy to change as part of the evolution itself, dumping its energy into a form of matter. Such *vacuum remittances* can be achieved either with Λ the generator of a clock ($\rho_\Lambda \in \alpha$), with a matter β depending on T_Λ [e.g., $c_m = c_m(T_\Lambda)$]; or with $\rho_\Lambda \in \beta$ and dependent on a matter clock, such as with $\rho_\Lambda(T_c)$.

For example, with $c_m = c_m(T_\Lambda)$, we can adapt the calculations in Sec. V B to find

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + p_i) = \frac{m_i\dot{f}_i}{a^{3(1+w_i)}} = \frac{m_i f_i'}{a^{3(1+w_i)}}\frac{dc_m^2}{dt}; \quad (i \neq \Lambda)$$

$$\dot{\rho}_\Lambda = -\sum_{i\neq\Lambda}\frac{m_i f_i'}{a^{3(1+w_i)}}\frac{dc_m^2}{dt}, \qquad (92)$$

so that for a shock of the form (84)

$$\Delta\rho_\Lambda = -\sum_{i\neq\Lambda}\Delta\rho_i,$$

$$\Delta\rho_i = \rho_i(c_{m+}^2) - \rho_i(c_{m-}^2) = \int_{c_{m-}^2}^{c_{m+}^2}\mu_{ic}dc_m^2; \quad (i \neq \Lambda).$$

As expected, if we include Λ in the accounts, then no net energy is produced; however, there is energy exchange,

with a Λ remittance into the rest of the matter, its energy distributed by species according to their μ_{ic}. This could be done with any other matter parameter elevated to β.

Alternatively, we could have $\rho_\Lambda = m_\Lambda f_\Lambda(c_m^2)$ and $m_\Lambda \in \beta$ and $c_m^2 \in \alpha$ [i.e., a $m_\Lambda(T_c)$ potential]. Then

$$\frac{dc_m^2}{dt} = \frac{\partial H}{\partial T_c} = Na^3\frac{\partial\rho_\Lambda}{\partial T_c} = Na^3 f_\Lambda m_\Lambda' \qquad (93)$$

so that instead of (54) we find for $i \neq \Lambda$

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + p_i) = \frac{m_i f_i'}{a^{3(1+w_i)}}\frac{dc_m^2}{dt} = \frac{Nm_i f_i'}{a^{3w_i}}f_\Lambda m_\Lambda'$$

which, summed over i, leads to $-\dot{\rho}_\Lambda = -(\dot{m}_\Lambda f_\Lambda + m_\Lambda\dot{f}_\Lambda)$ [recall that Λ may contribute to (52)].

In both scenarios, whatever caused the hot big bang suppressed the vacuum energy. The big bang is a sign of variability involving Lambda and a matter parameter, with one of them playing the role of clock.

C. Latter-day big bangs?

Naturally within this framework one may ask why would the big bang be unique? And could this be related to why Lambda is so small compared to the Planck scale? The simplest scenario in this respect is a cascade of big bangs siphoning matter out of Lambda when Lambda resurfaces. A big bang in the past, originated by a phase transition due to vacuum domination, would be reflected in a big bang in the future when similar circumstances arise (as seems to be the case around now).

However, in such a cyclic scenario, the big bang temperature would be 10^{-32} smaller each time, which is nonsense. We should therefore renormalize the Planck mass at the start of each cycle. This could happen in two ways: either via a dependence on Ricci time, T_R, or by making α_2 a β. Imagine a $\rho_\Lambda(T_c, T_R)$. Then

$$\dot{\alpha}_2 = Na^3\frac{\partial\rho_\Lambda}{\partial T_R} \qquad (94)$$

and beside the remittance (92), we have the correction

$$\dot{\rho}_\Lambda = -\sum_{i\neq\Lambda}\frac{m_i f_i'}{a^{3(1+w_i)}}\frac{dc_m^2}{dt} + 2\frac{Na^3}{\alpha_2}\rho_\Lambda\frac{\partial\rho_\Lambda}{\partial T_R}. \qquad (95)$$

We can design $\rho_\Lambda(T_c, T_R)$ so that the Planck scale changes at each event and any new big bang event is at the Planck scale.

IX. CONCLUSIONS AND OUTLOOK

Where does all the matter in the Universe come from? In this paper we proposed that the Universe acquired its energy by virtue of "changing the rules of the game," viz.

evolution in the laws of physics. Rather than implementing evolution in terms of a coordinate time, we defined it in terms of time variables, T_α, dual to constants, α, using as a blueprint one formulation of unimodular gravity [6] (in which Λ plays the role of α). Variability was then introduced by making *other* constants, β, functions of these times, $\beta(T_\alpha)$. The main practical advantage over coordinate time dependence is that this approach specifies where the energy is credited. In general there is energy production if either a matter parameter varies in terms of a gravitational clock, or a gravity parameter evolves in terms of a matter clock, with that matter component acquiring the energy violation balance. Other combinations are sterile.

As in Wheeler's view, we can think of the evolution potentials $\beta(T_\alpha)$ as random choices for chaotic evolution, selected in or out by their ultimate effects. These could be whether "viable" offspring is produced: only evolution that implies creation of matter is beneficial, with sterile evolution edited out. In this paper we "geocentrically" qualified this statement as creation of *semiclassical* matter, but this is not strictly necessary. Perhaps the quantum cosmology of this process is far more interesting, with classical matter creation replaced by an *S*-matrix process. In Appendix A we sketch the first steps in this direction, to show that progress is possible, but defer full treatment to future work. Another offshoot is a classical realization of the Hartle-Hawking initial conditions, as sketched in Appendix B. This leads to signature change, a possibly essential feature for more detailed model building, since it affects the horizon structure of the Universe (see Appendix B).

We stress that the proposal in this paper is distinct from multidimensional time. On-shell, the various T_α are all a function of each other, so time is classically one dimensional. But the T_α start off as independent variables (something which has dramatic effects in the quantum theory [15,16,19,21]), and index evolution differently should there be a $\beta(T_\alpha)$. If there are violations of energy conservation due to evolution, it is crucial to specify in terms of which of these clocks evolution takes place (even though on-shell they are all related). This has physical effects, namely it determines into which matter component the energy goes/is taken off.

We close with a few comments on future developments. In this paper we focused on theories exhibiting local Lorentz invariance violations, defined in the spirit of Horava-Lifshitz theory: a $4 = 3 + 1$ split that leads to a $3 + 1 \neq 4$ after one fiddles with some aspect of the split theory. This author started off this article with the hunch that Lorentz invariance violation would be favored by cosmic procreation of matter, but was disabused by hard calculations. The shattered remains of this dogma lead to the sequel [43]: a repeat of this paper without recourse to local Lorentz invariance violation. We should stress that in our theories, even under LLIV and with time dependence, in

we have a sympletic/Hamiltonian structure. This is very powerful, and is responsible for most of the results in this paper.

Finally, there is the question of falsifiability. The various scenarios with matter progeny considered in Sec. VIII are all equally possible, so one may raise questions about testability. But producing copious amounts on matter on near homogeneous patches is just the starting point: one must add fluctuations. We gave special attention to scenarios with bimetric VSL ($c_m \neq c_g$) because these are known to produce very distinctive predictions for the cosmic fluctuations [45,46] (see also [47–50]). More directly, we speculate on whether violations of energy conservation might be accessible on not-so-large scales, for example during black hole formation. This is particularly relevant if there is evolution in the laws of physics in our future, as predicted in Sec. VIII C. In the presence of foliation-dependent evolution, the region near a black hole horizon acts as a crystal ball for events in our future. Any future or latter-day big bangs that might be implied will indeed happen near the black hole horizon, with possible observational consequences. This matter is currently under investigation.

APPENDIX A: QUANTUM COSMOLOGY

We briefly sketch how a quantum treatment might start. Standard theories with relational times T_α are known to convert the Wheeler-DeWitt equation into a Schrödinger equation (e.g., [15,16,19,21]). An independent claim to the same effect was made for VSL scenarios with hard break of diffeomorphism invariance [51]. For the VSL scenarios we have proposed in this paper we obtain a time-dependent Schrödinger equation, similar to what one gets in the interaction picture. Hence the classical matter creation we have reported can also be seen as quantum matter creation, and an *S*-matrix approach could be developed, as we now show.

To fix ideas, we take unimodular VSL (see Sec. VA). One can put the Hamiltonian constraint in the form [21]

$$0 = \frac{1}{b^2 + kc_g^2} a^2 - \frac{3}{\Lambda} \equiv H_0 - \phi$$

with Λ replaced by $\phi = 3/\Lambda$ and the dynamical Hamiltonian H_0 defined by this expression. Choosing ϕ (instead of Λ) for α and allowing c_g^2 to become a function of T_ϕ, upon quantization with suitable ordering we obtain the time-dependent Schrödinger equation:

$$\left[\hat{H}_0(b, T_\phi) - i\mathfrak{h}\frac{\partial}{\partial T_\phi}\right]\psi(b, T_\phi) = 0,$$

with

$$\hat{H}_0(b, T_\phi) = \frac{-i\mathfrak{h}}{b^2 + kc_g^2(T_\phi)}\frac{\partial}{\partial b}$$

following from (34) (where $\mathfrak{h}$ is defined). As is well known, if c_g does not change, one possible solution is

$$\psi(b, T_\phi; \phi) = \mathcal{N}e^{-i\frac{\phi}{\mathfrak{h}}T_\phi}\psi_s(b, \phi)$$

where

$$\psi_s(b, \phi) = e^{i\frac{\phi}{\mathfrak{h}}X_{\mathrm{CS}}(b)} = e^{i\frac{\phi}{\mathfrak{h}}\left(\frac{b^3}{3} + kc_g^2 b\right)}$$

is a minisuperspace version of the Chern-Simons-Kodama state [52–54]. This is the monochromatic solution and superpositions with different Λ are possible (see [21,54] for a discussion of normalizability and inner product). If c_g^2 changes, the solution is instead a time-dependent version of the Chern-Simons-Kodama state:

$$\psi(b, T_\phi) = \mathcal{N}\mathcal{T}\exp\left[-\frac{i}{\mathfrak{h}}\int_{T_{\phi 0}}^{T_\phi} H_0(\tilde{T}_\phi)d\tilde{T}_\phi\right]\psi(b, T_{\phi 0})$$

where $\mathcal{T}$ denotes time-ordering in T_ϕ, thus opening up a parallel with the S matrix framework.

APPENDIX B: SIGNATURE CHANGE AND EVOLUTION

The fact that the natural variable in the dynamics is often c^2 suggests that c could appear in α as a square. We could then entertain scenarios in which the sign of c^2, and so the

signature, changes and this has a curious relation to the Hartle-Hawking [55] no-boundary proposal, in a process reminiscent of [56]. This could be done with

$$c_g^2(T_\Lambda) = c_0^2 H(T_\Lambda) - c_0^2 H(-T_\Lambda) \tag{B1}$$

for $k = 1$ and $c_0^2 > 0$. Then, Eq. (43) implies

$$\Delta\rho_\Lambda = \frac{k\alpha_2}{a_\star^2\alpha_3}\Delta c_g^2 = 2c_0^2\frac{k\alpha_2}{a_\star^2\alpha_3} \tag{B2}$$

which combined with the Hamiltonian constraint implies a swap in the sign of $\rho_\Lambda = \pm\rho_{\Lambda 0}$, equivalent to a redefinition of N and

$$\alpha_2(-b^2 + kc_0^2) + \alpha_3\rho_{\Lambda 0}a^2 \approx 0.$$

For $T_\Lambda < 0$ we therefore obtain a solution to the (real) Euclidean theory: the 4-sphere. In this scenario, rather than changing signature as a function of b (as in [56]), the change happens as a function of T_Λ, that is, with proper time evolution. It is curious that Hartle and Hawking's "creation out of nothing," usually seen as an instanton and so a purely quantum process, could in fact be a classical phenomenon, within a theory with varying laws and classical signature change. This would dramatically change the usual discussion of the horizon problem, which would thus become disconnected from the flatness problem. Options in which the matter speed of light increases (rather than decreases, as in the usual solution to the horizon problem) could then be considered, as long as the sign of c^2 changed. We could also have early Universe situations where $c_m^2 > 0$ but $c_g^2 < 0$, that is Lorentzian spacetime for matter (or some of matter) and Euclidean gravity, or vice versa.

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