Medium effects in the MIT bag model for quark matter: Self-consistent thermodynamical treatment

Suman Pal[*] and Gargi Chaudhuri[†]

Physics Group, Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700064, India
and Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400085, India

(Received 29 August 2023; accepted 16 October 2023; published 16 November 2023)

The study of strange quark matter within the framework of the density-dependent MIT bag model using the grand canonical ensemble is thermodynamically inconsistent. In this work, it is shown that if the medium effects are incorporated through a density-dependent bag pressure in the grand canonical ensemble, then the Euler relation is violated. If Euler relation is used then the minimum of energy per baryon does not occur at zero pressure. In order to overcome this inconsistency, we propose the medium effect of the strange quark matter in the form of chemical potential dependent bag pressure in the grand canonical ensemble. The density dependent bag pressure which has been used in the grand canonical ensemble so far can, however, be used in the canonical ensemble without violating the laws of thermodynamics. These prescriptions will obey the Euler relation as well as the minimum energy per baryon will coincide with the zero of pressure and hence can be considered to be self-consistent. These equations of state in the grand canonical ensemble can be further used to construct the mass-radius and other structural properties of the strange quark stars as well as hybrid stars. In our present work we have calculated the mass radius diagram of strange stars only using this formalism.

DOI: 10.1103/PhysRevD.108.103028

I. INTRODUCTION

The study of the characteristics of the strongly interacting dense matter inside neutron stars (NSs) [1–7] is a topic of contemporary interest [8–10]. It is believed that quark matter can exist inside the NS [11]. The first-principles methods however cannot be used for describing quark matter at densities relevant inside stellar cores because of the sign problem in lattice Monte Carlo simulations at nonzero chemical potentials [12] and that perturbative QCD being only effective [13] at significantly higher densities. There have been various efforts to incorporate nonperturbative effects in increasingly sophisticated models since perturbative QCD is insufficient for addressing the quark matter EoS. Various phenomenological models have been used to study quark matter recently, e.g., MIT bag model [14], quark mass density-dependent model [15–18], the Richard potential model [19], NJL model [20–28], the perturbation model [29], the field correlator method [30], the quark-cluster model [31], and many other models. These models, to some extent, have their origin in the free-particle system. Strange quark matter (SQM) plays an important role in many interesting fields for example hot and dense matter in heavy ion collision, the structure of compact stars etc. Ever since W. H. Witten suggested [32–34] that the SQM would be absolutely stable even at absolute zero temperature, there has been a lot of interest in studying it.

We consider the simple MIT bag model [14,35] for studying strange quark matter which is a bulk matter phase with u, d, and s quarks in chemical equilibrium along with a minor fraction of electrons. The bag constant (B) is added to the thermodynamical potential of the free fermion system in order to reflect the quark confinement. The bag pressure B is actually the energy density difference between the perturbative vacuum and the true vacuum [36,37] and is often considered to be constant in literature. Quark matter does not become asymptotically free immediately during or after the phase transition in contrary to the MIT bag model's *a priori* assumption that quarks are free inside the bag. In order to overcome this problem, Raha *et al.* [38] introduced the medium effects via quark mass with a density-dependent quark mass model. A similar effect can be incorporated in the bag model through a density-dependent bag pressure. It is well-known that the quarks at high densities, relevant to neutron stars or hybrid star cores, prefer asymptotic freedom [36,37]. This fact justifies that the bag pressure be density dependent rather than being a constant. There are several studies on hybrid stars with density-dependent bag models [39–41]. Therefore in the present work, we consider the similar medium dependence of the bag pressure via density. For the strange quark stars(SQS), the Bodmer-Witten conjecture

[*]sumanvecc@gmail.com
[†]gargi@vecc.gov.in

states that the absolute stability of SQM is determined in terms of matter-energy per baryon ε/ρ_B at zero pressure being smaller than that of ^{56}Fe nucleus [32–34]. From the rules of thermodynamics, pressure is given by

$$P = \rho^2 \frac{\partial}{\partial \rho}\left(\frac{\varepsilon}{\rho}\right) \tag{1}$$

According to the relation Eq. (1) minimum of $\frac{\varepsilon}{\rho}$ should occur at zero pressure. A recent study [42] of SQM using a density-dependent quark mass model solves the inconsistency problem of this model by using canonical ensemble formalism. With this formalism $(\frac{\varepsilon}{\rho})_{\rm min}$ occur exactly at zero pressure point and also the equation of state is generated via Euler relation.

$$\varepsilon = -P + \sum \mu_i \rho_i \tag{2}$$

Generally, for the calculation of the equation of state for SQM, grand canonical ensemble formalism is used, If the density-dependent bag pressure is used, then it is thermodynamically inconsistent as it violates the Euler relation Eq. (2) as described in detail in the formalism Sec. II. Therefore we propose in this work to use the more appropriate chemical potential-dependent bag pressure instead of a density-dependent bag pressure in the grand canonical ensemble. If we want to use a density-dependent bag pressure we have to use a canonical ensemble where the Euler relation is valid. Our main focus in this work is to study the quark matter EoS in different ensembles with different forms of bag pressure and thereby establish the thermodynamic consistency. Once the EoS of strange quark matter is ready, we have calculated the mass-radius diagram of the strange star using the same.

This paper is organized as follows. In Sec. II we give the detailed formalism of the equation of state in both ensembles using both density-dependent and chemical potential-dependent bag pressures. In Sec. III we show the numerical results. Finally, we summarize in Sec. IV.

II. FORMALISM

In Ref. [42], the phenomenological density-dependent quark mass model has been revisited and the thermodynamical inconsistency has been resolved within the canonical ensemble formulation. This necessitates revisiting the MIT bag Model as well where the same inconsistency is expected when one incorporates the density-dependent bag pressure in grand canonical formalism. The main motivation of this work is to remove this inconsistency and reformulate the MIT bag model in a thermodynamically consistent manner. The medium effects can be incorporated through the bag pressure by introducing its dependence on proper intensive parameter depending on the chosen ensemble. In the canonical ensemble, the proper quantity is the chemical potential whereas in the canonical ensemble density is the appropriate intensive quantity.

A. Grand canonical

In the grand canonical ensemble (GE), the thermodynamic potential depends on the chemical potential(μ), volume (V), and temperature (T) and all the thermodynamical quantities can be derived from the grand canonical potential $\Omega(\mu, V, T)$. We are considering cold neutron star which implies $T = 0$.

(i) Density dependent bag pressure:
In grand canonical ensemble, pressure is defined as

$$P = -\left(\frac{\partial \Omega}{\partial V}\right)_{T,\mu} \tag{3}$$

where Ω is the grand-potential, $\Omega_{GC} = \frac{\Omega}{V}$ is grand-potential per unit volume and $N = \rho V$. We can write Eq. (3) as

$$P = -\left[\frac{\partial(\Omega_{GC} V)}{\partial(\frac{N}{\rho})}\right]_{T,\mu} = -\Omega_{GC} + \rho\left[\frac{\partial \Omega_{GC}}{\partial \rho}\right]_{T,\mu} \tag{4}$$

The density derivative term arises from the baryonic density dependence of the grand potential (as we are taking the bag pressure to be density dependent). The thermodynamical potential for the quark matter in grand canonical ensemble can be written as

$$\Omega_{GC} = -\frac{1}{\pi^2}\sum_{f=u,d,s}\int_0^{k_f}\frac{k^4}{\sqrt{k^2+m_f^2}} + B(\rho) = \sum_{f=u,d,s}\Omega_f + B(\rho) \tag{5}$$

where Ω_f is written as [35]

$$\Omega_f = -\frac{1}{4\pi^2}\left[\mu_f\sqrt{\mu_f^2 - m_f^2}\left(\mu_f^2 - \frac{5}{2}m_f^2\right) + \frac{3}{2}m_f^4 \ln\left[\frac{\mu_f + \sqrt{\mu_f^2 - m_f^2}}{m_f}\right]\right] \tag{6}$$

where Ω_f represents the free quark matter grand potential. The interactions and medium effects are taken care of by the bag pressure $B(\rho)$ [36,37]. Hence pressure in the grand canonical ensemble with density dependence bag pressure is as follows:

$$P_{GC} = -\Omega_{GC} + \rho\frac{\partial B(\rho)}{\partial \rho} \tag{7}$$

We consider two forms of the density dependence in the bag pressure.

(i) Gaussian form [36,37]:

$$B(\rho) = B_{as} + (B_0 - B_{as})e^{[-\beta_\rho(\frac{\rho}{\rho_0})^2]} \tag{8}$$

Here B_0 is the value of B at $\rho = 0$, B_{as} is the value of B for asymptotic values of ρ, β_ρ is a control parameter and ρ_0 is the saturation density.

(i) Hyperbolic form [41]:

$$B(\rho) = B_{as} + \frac{B_0}{2}\left(1 - \tan h\left(\frac{\rho - \bar{\rho}}{\Gamma_\rho}\right)\right) \tag{9}$$

B_{as}, B_0, $\bar{\rho}$, and Γ_ρ are the free parameters in hyperbolic form similarly.

The minimum of $\frac{\varepsilon}{\rho}$ occurs at zero pressure if we use the inverse Legendre transformation.

$$\varepsilon_{GC} = \Omega_{GC} + \sum_i \mu_i \rho_i \tag{10}$$

where ϵ is the energy density, μ_i is the chemical potential for the ith particle and ρ_i is the density for the ith particle but the Euler relation [Eq. (15)] is violated in this method. Ω_{GC} should be a function of μ but if bag pressure is taken to be density-dependent, then that is also not respected. So to avoid this kind of problem, we propose a proper thermodynamic treatment, where the medium effect is introduced via a chemical potential dependent bag pressure.

(i) Chemical potential dependent bag pressure:
We consider similar dependence of chemical potential in the bag pressure via Gaussian and hyperbolic as in case of density dependent B. The expression of B in the grand canonical are as follows:

(i) Gaussian form:

$$B(\mu) = B_{as} + (B_0 - B_{as})e^{[-\beta_\mu(\frac{\mu}{\mu_0})^2]} \tag{11}$$

(ii) Hyperbolic form:

$$B(\mu) = B_{as} + \frac{B_0}{2}\left[1 - \tan h\left(\frac{\mu - \bar{\mu}}{\Gamma_\mu}\right)\right]. \tag{12}$$

Here we are using a chemical potential-dependent bag pressure, so the derivative term in Eq. (4) will not arise. Hence the pressure is

$$P_{GC} = -\Omega_{GC}. \tag{13}$$

The quark number densities are given by

$$\rho_f = -\frac{\partial\Omega_{GC}}{\partial\mu_f} = \frac{k_f^3}{\pi^2} - \frac{\partial B(\mu)}{\partial\mu_f}. \tag{14}$$

The density of each flavor is modified due to the chemical potential-dependent bag pressure. This proper thermodynamical treatment where the thermodynamical potential in grand canonical ensemble depends on chemical potential and not density ensures that the Euler relation is respected unlike the previous case.

$$\varepsilon_{GC} = -P_{GC} + \sum_f \mu_f \rho_f \tag{15}$$

In the grand canonical ensemble, the energy from Eq. (15) reads as

$$\varepsilon_{GC} = \sum_f \left(\frac{1}{4\pi^2}\left[(\mu_f^2 - m_f^2)^{\frac{3}{2}}\mu_f - \frac{3}{2}m_f^2\mu_f\sqrt{m_f^2 - m_f^2} + \frac{3}{2}m_f^4 \log\left(\frac{\mu_f + \sqrt{\mu_f^2 - m_f^2}}{m_f}\right)\right]\right) + B(\mu) + \sum_f\left[\mu_f\left(\frac{k_f^3}{\pi^2} - \frac{\partial B(\mu)}{\partial\mu_f}\right)\right]. \tag{16}$$

B. Canonical

In the canonical ensemble, all the thermodynamical quantities can be derived from the Helmholtz free energy $F(\rho, V, T)$. At zero temperature ($T = 0$)

$$F(\rho, V) = U = \varepsilon V \tag{17}$$

where U is the total internal energy and ε is the internal energy per unit volume. The Helmholtz free energy per unit volume is given by

$$\varepsilon_C = \sum_{f=u,d,s} \frac{3}{\pi^2}\int_0^{k_f} k^2\sqrt{k^2 + m_f^2}dk + B(\rho) \tag{18}$$

where the medium effects is incorporated in bag pressure by making it density dependent since density is the appropriate intensive parameter in canonical ensemble. We can rewrite the Eq. (18) as

$$\varepsilon_C = \sum_f\left(\frac{3}{4\pi^2}\left[k_f^3\sqrt{k_f^2 + m_f^2} + \frac{1}{2}m_f^2 k_f\sqrt{k_f^2 + m_f^2} - \frac{1}{2}m_f^4 \log\left(\frac{\sqrt{k_f^2 + m_f^2} + k_f}{m_f}\right)\right]\right) + B(\rho) \tag{19}$$

Unlike the case of energy in grand canonical ensemble Eq. (16), the expression for energy in canonical does not have any derivative dependent term of bag pressure. As in the case of grand canonical ensemble, here also two forms for the density dependence is being considered. Particle number density is given by

$$\rho_f = \frac{k_f^3}{\pi^2} \tag{20}$$

Total baryon no density is

$$\rho = \frac{1}{3}(\rho_u + \rho_d + \rho_s) \tag{21}$$

The energy density at $T = 0$ is $\epsilon_c = \frac{F}{V}$

$$\varepsilon_C = \sum_f \epsilon_f + B(\rho) = \sum_f \frac{F_f}{V} + \frac{F_B}{V} \tag{22}$$

where F_f is the free energy for the contribution for the free fermions and F_B is the free energy contribution from the bag pressure.

$$P_C = -\frac{\partial F}{\partial V} = -\frac{\partial}{\partial V}\left(\sum_f F_f + F_B\right) \tag{23}$$

Pressure in this ensemble is given by

$$P_C = \left(\sum_f \rho_f^2 \frac{\partial}{\partial \rho_f}\left(\frac{\epsilon_f}{\rho_f}\right)\right) + \rho^2 \frac{\partial}{\partial \rho}\left(\frac{B(\rho)}{\rho}\right) \tag{24}$$

$$P_C = \sum_f P_f + P_{\text{bag}} \tag{25}$$

where,

$$P_f = \frac{1}{4\pi^2}\left(k_f^3\sqrt{k_f^2 + m_f^2} + \frac{1}{2}m_f^2 k_f \sqrt{k_f^2 + m_f^2} - \frac{1}{2}m_f^4 \log\left(\frac{\sqrt{k_f^2 + m_f^2} + k_f}{m_f}\right)\right) \tag{26}$$

$$P_{\text{bag}} = -B(\rho) + \rho \frac{\partial B(\rho)}{\partial \rho}. \tag{27}$$

Therefore pressure can be rewritten as

$$P_C = \frac{1}{\pi^2} \sum_{f=u,d,s} \int_0^{k_f} \frac{k^4}{\sqrt{k^2 + m_f^2}} - B(\rho) + \rho \frac{\partial B(\rho)}{\partial \rho}. \tag{28}$$

Unlike the case of grand canonical Eq. (13), pressure here has a derivative term of $B(\rho)$. The chemical potential is

$$\mu_f = \frac{\partial F}{\partial N_f} = \frac{\partial \varepsilon_f}{\partial \rho_f} + \frac{\partial B(\rho)}{\partial \rho_f} = \sqrt{k_f^2 + m_f^2} + \frac{\partial B(\rho)}{\partial \rho_f}. \tag{29}$$

In this ensemble quark chemical potential is modified due to the medium effects in MIT bag model through the density dependence. A similar effect is observed in a grand canonical ensemble where the density is modified due to the chemical potential dependent bag pressure. This modified chemical potential satisfies the Euler relation thereby establishing the validity of the method.

III. RESULTS

We focus on quark matter that might exist inside a NS. For the SQM, we include electrons also and we have to take into account the chemical equilibrium condition,

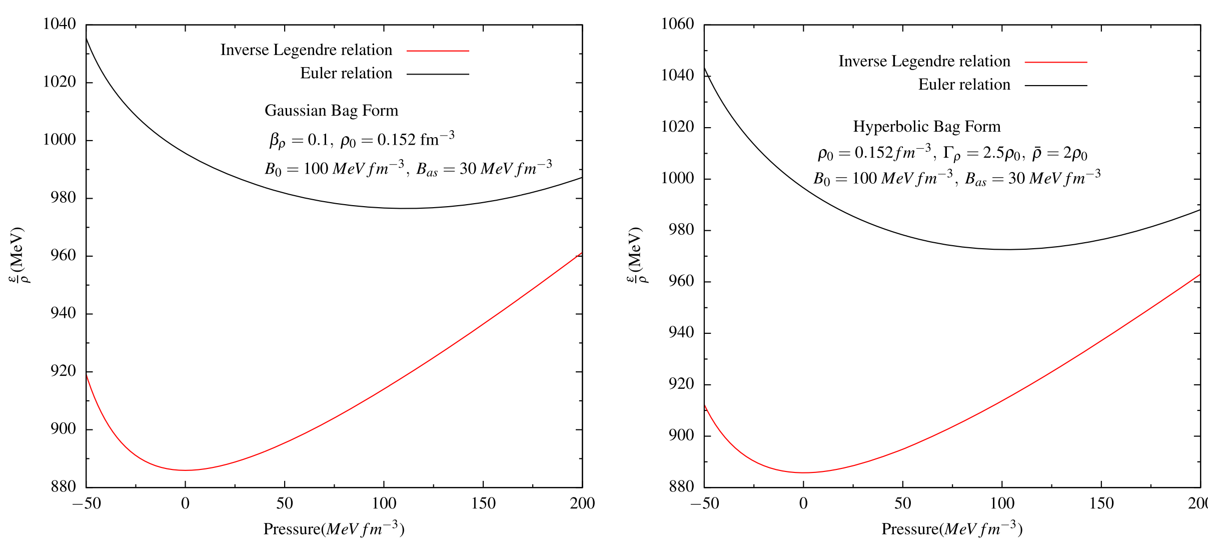

FIG. 1. Variation of energy per baryon with pressure using Euler relation and Inverse Legendre relation for Gaussian density-dependent (left) and for hyperbolic density dependent bag (right) pressure.

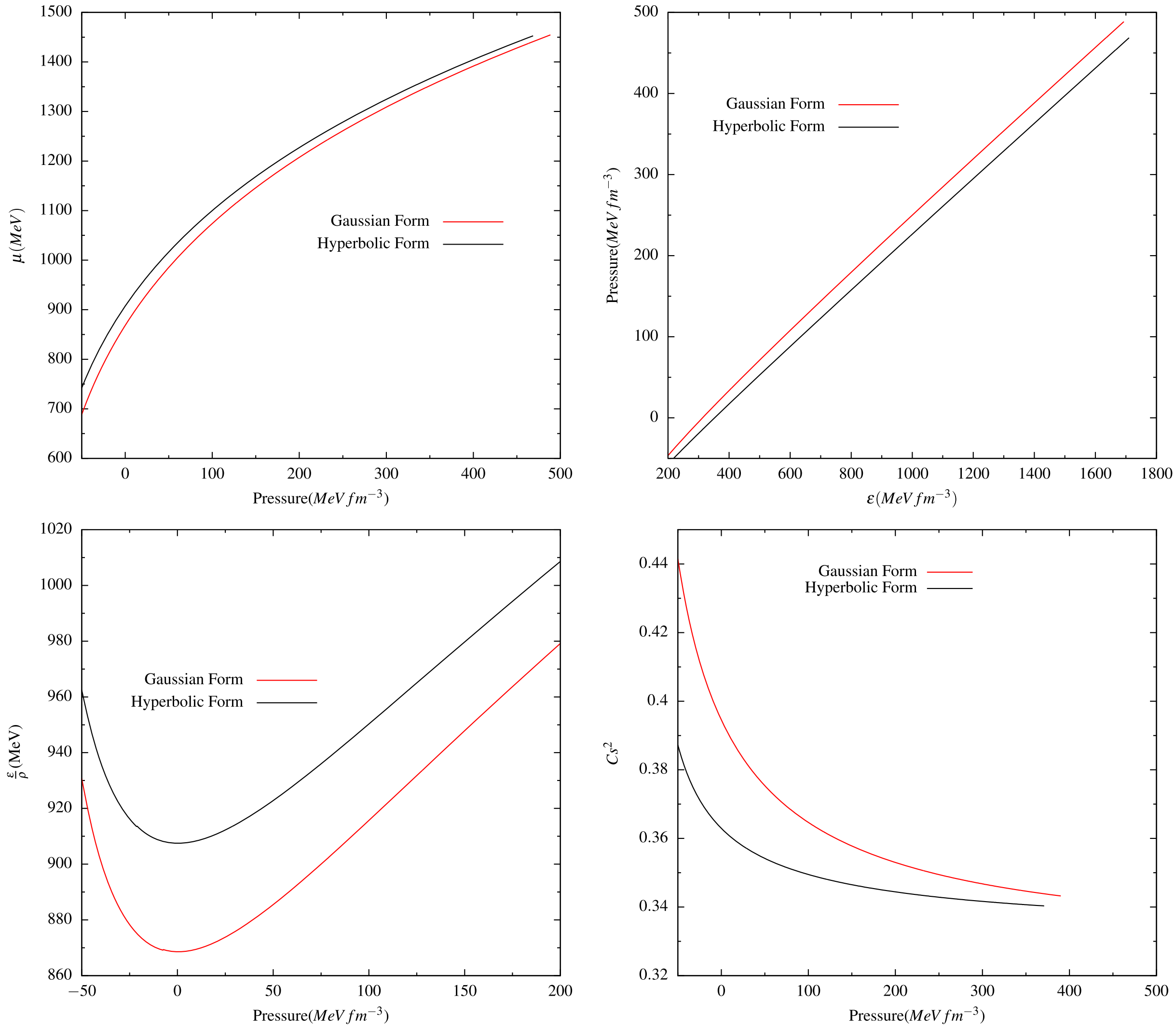

FIG. 2. Equation of state and speed of sound in the grand canonical ensemble for Gaussian and hyperbolic chemical potential dependent bag pressure, variation of chemical potential with pressure (upper left), variation of pressure with energy density (upper right), variation of $\frac{\varepsilon}{\rho}$ with pressure (lower left), and variation of speed of sound with pressure (lower right) with $B_0 = 100$ MeV fm^{-3}, $B_{as} = 30$ MeV fm^{-3}, $\beta_\mu = 1.0$, $\mu_0 = 1000$ MeV, $\Gamma_\mu = 1000$ MeV, $\bar{\mu} = 800$ MeV.

$\mu_d = \mu_u + \mu_e = \mu_s$, the charge neutrality condition, $\frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \frac{1}{3}\rho_s - \rho_e = 0$ and baryon no density conservation $\rho = \frac{1}{3}(\rho_u + \rho_d + \rho_s)$. In our calculation, we take $m_u = 5.0$ MeV, $m_d = 7.0$ MeV, and $m_s = 95.0$ MeV [43].

A. Grand canonical ensemble

First, we study the thermodynamic stability condition for the SQM in a grand canonical ensemble with density-dependent bag pressure shown in Fig. 1. From Eq. (1), the minimum of $\frac{\varepsilon}{\rho}$ should occur at zero pressure. In Fig. 1 we use both Gaussian and hyperbolic density-dependent bag pressure. In Fig. 1 we see that if we use Euler relation Eq. (15) then minimum of $\frac{\varepsilon}{\rho}$ does not occur at zero pressure but if inverse Legendre transformation Eq. (10) is used, then minimum occurs at zero pressure but it violates the rules of thermodynamics.

In order to establish thermodynamic consistency, we propose a chemical potential-dependent bag pressure, where self-consistent thermodynamic treatment is being restored. We study the equation of state in a grand canonical ensemble with both Gaussian and hyperbolic chemical potential dependent bag pressure. For this, we take $B_0 = 100$ MeV fm^{-3} and $B_{as} = 30$ MeV fm^{-3}, same for both Gaussian and hyperbolic form. For Gaussian $\beta_\mu = 1.0$, $\mu_0 = 1000$ MeV and for hyperbolic $\bar{\mu} = 800$ MeV and $\Gamma_\mu = 1000$ MeV. In Fig. 2 upper panel, variation of chemical potential with pressure and variation of pressure with energy density is being displayed. We see that the chemical potential has a similar kind of variation with energy density both for the Gaussian and hyperbolic case; the equation of state (pressure versus energy density) also has a similar pattern for both forms of the bag pressure. In the lower left panel, we show that the minimum of $\frac{\varepsilon}{\rho}$

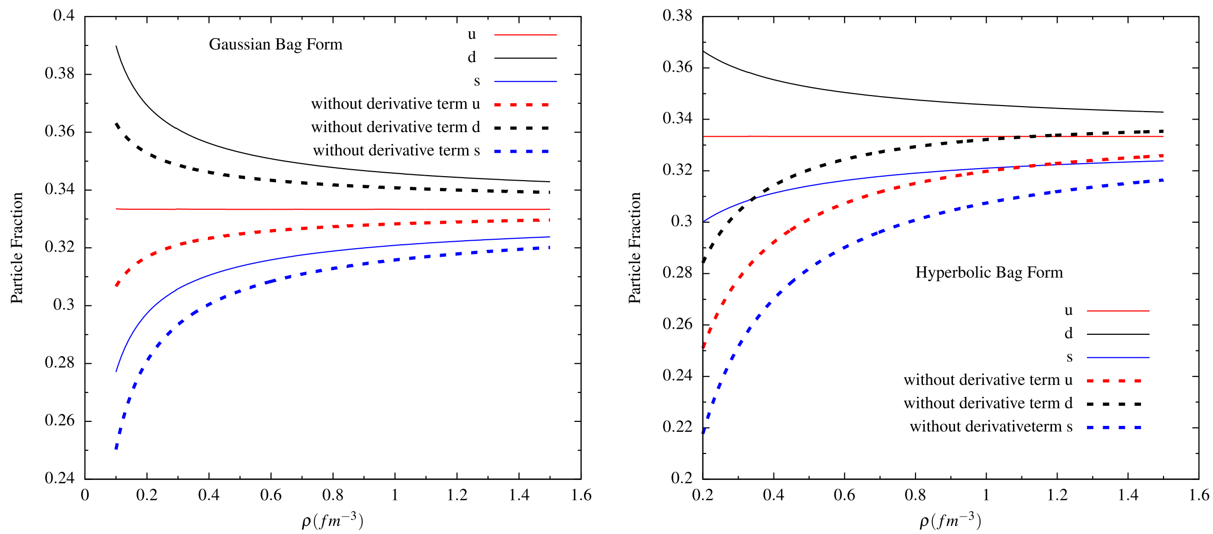

FIG. 3. Particle fraction in the grand canonical ensemble for Gaussian (left) and hyperbolic (right) chemical potential dependent bag pressure.

exactly occurs at zero pressure which will not happen in the case of density-dependent bag pressure obeying the Euler relation. In the lower right panel, variation of the speed of sound with pressure is shown; due to chemical potential dependent bag pressure, C_s^2 varies with pressure and it tends to $\frac{1}{3}$ for higher values of pressure.

In Fig. 3, we show the variation of particle fraction variation with density; the solid line represents the particle fraction according to Eq. (14) and the dotted line represents the particle fraction without taking the derivative of the bag pressure as in right hand side of Eq. (14). Particle fraction is modified due to the derivative term in the Eq. (14), and since bag pressure varies with chemical potential, it reflects in the particle fraction plot.

Bodmer-Witten conjecture [32–34] states that for SQM to be stable and be the true ground state of matter the stability is dictated by energy density per baryon, which is regulated by bag pressure. The allowed range of the different parameters (B_0, B_{as}, β_μ) in the μ dependent bag pressure (Gaussian form) have been estimated as per Bodmer-Witten conjecture as shown in Table I. The upper limit of β_μ is estimated for 2-flavor quark matter, when $\frac{\varepsilon}{\rho} \geq 930$ and lower limit of β_μ is estimated for 3-flavor quark matter, when $\frac{\varepsilon}{\rho} \leq 930$. In a similar way, we can estimate the stability range of the parameters in the hyperbolic bag pressure which is not shown here for the sake of brevity.

TABLE I. μ dependent Gaussian bag pressure in grand canonical ensemble.

B_0 (MeV fm^{-3})	B_{as} (MeV fm^{-3})	β_μ
60	25	[0.0, 0.12]
100	50	[0.48, 2.33]
100	40	[0.37, 1.45]
100	30	[0.31, 1.11]
150	50	[1.30, 3.10]
150	40	[1.08, 2.15]
150	30	[0.93, 1.71]
200	50	[1.75, 3.60]
200	40	[1.50, 2.55]
200	30	[1.35, 2.10]

B. Canonical ensemble

In the canonical ensemble, the medium effects are taken into account through a density-dependent bag pressure. This ensures thermodynamic consistency and validity of Euler's relation. For this, we take $B_0 = 100$ MeV fm^{-3} and $B_{as} = 30$ MeV fm^{-3}, same for both Gaussian and hyperbolic form. For Gaussian $\beta_\rho = 0.1$, $\rho_0 = 0.152$ fm^{-3} and for hyperbolic $\bar{\rho} = 2\rho_0$ and $\Gamma_\rho = 2.5\rho_0$. In Fig. 4 upper panel shows the variation of chemical potential with pressure and the variation of pressure with energy density. It is observed that these two variations are similar and close for the Gaussian and the hyperbolic types variation of the density-dependent bag pressure. In the lower left panel, we show that the minimum of $\frac{\varepsilon}{\rho}$ exactly occurs at zero pressure which will not happen in the case of density-dependent bag pressure in grand canonical ensemble (Fig. 1) obeying the Euler relation). Therefore if we want to use a density-dependent medium effect, we have to use a canonical approach. In the lower right panel, variation of the speed of sound with pressure due to the density-dependent medium effect is displayed. It is observed that C_s^2 tends to conformal limit at higher pressure. The variation of C_s^2 at lower values of pressure differs from that of the grand canonical since energy [See Eqs. (16) and (19)] and pressure [See Eqs. (13) and (28)] differs in both the ensembles.

In Fig. 5, the quark chemical potential variation with baryonic chemical potential is being shown; here the solid line represents the individual chemical potential of different

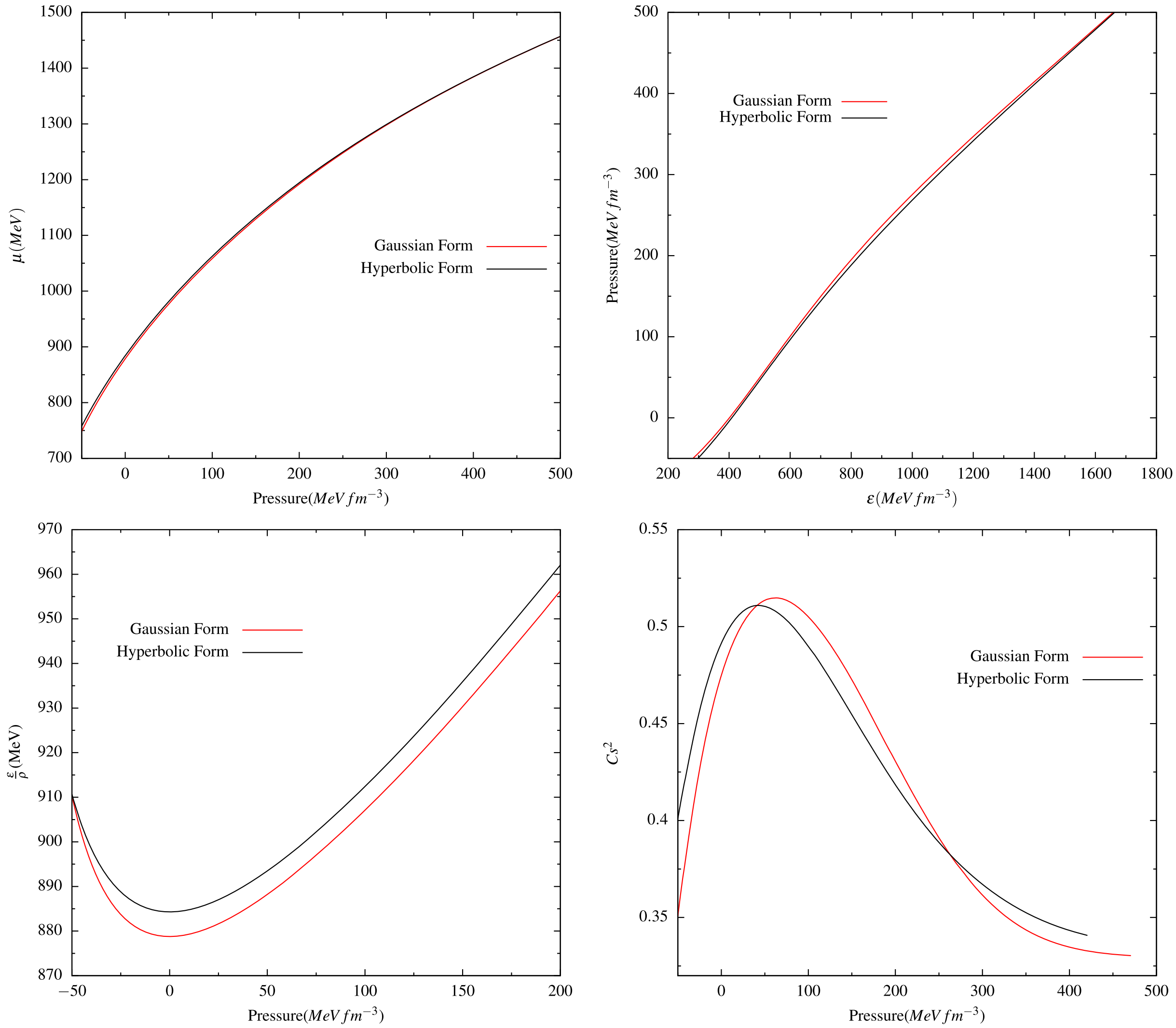

FIG. 4. Equation of state and speed of sound in the canonical ensemble for Gaussian and hyperbolic density dependent bag pressure, variation of chemical potential with pressure (upper left), variation of pressure with energy density (upper right), variation $\frac{\varepsilon}{\rho}$ with pressure (lower left), and variation speed of sound with pressure (lower right) with $B_0 = 100$ MeV fm^{-3}, $B_{as} = 30$ MeV fm^{-3}, $\beta_\rho = 0.1$, $\rho_0 = 0.152$ fm^{-3}, $\Gamma_\rho = 2.5\rho_0$, $\bar{\rho} = 2\rho_0$.

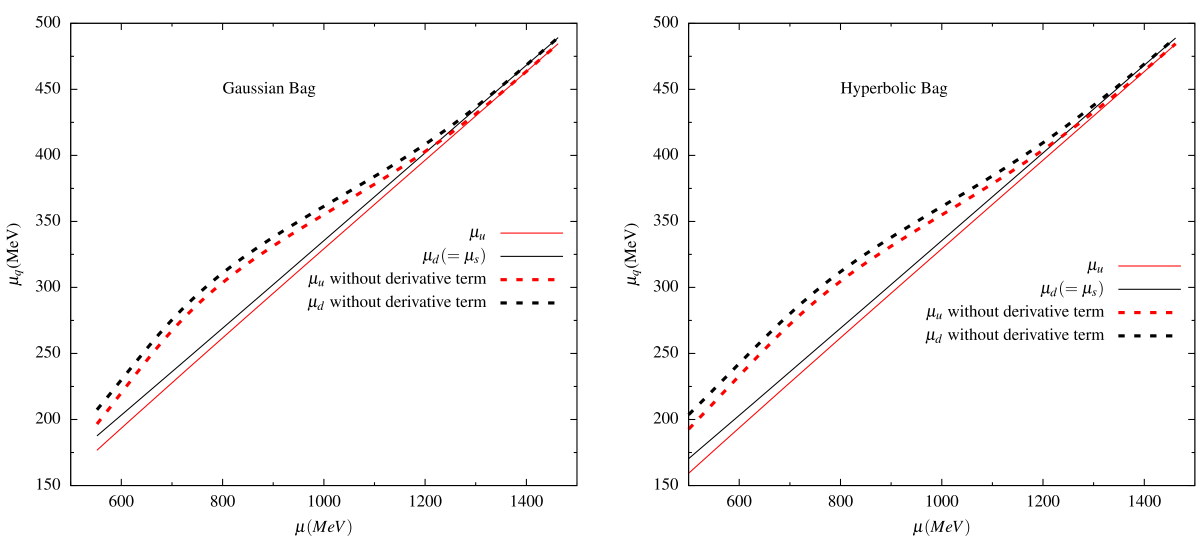

FIG. 5. Quark chemical potential in the canonical ensemble for Gaussian (left) and hyperbolic (right) density dependent bag pressure.

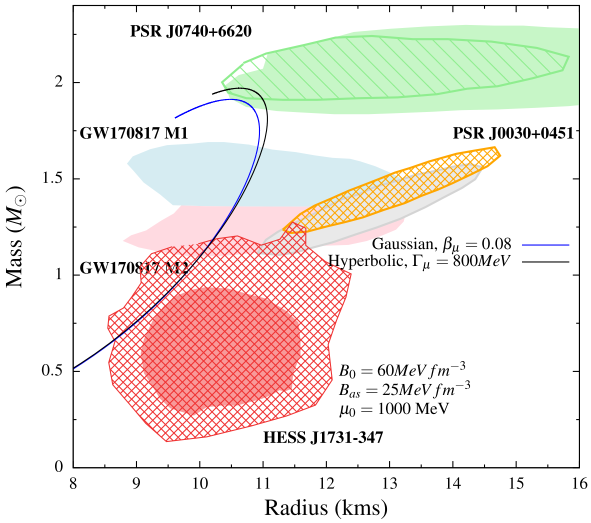

FIG. 6. Mass-radius relationship of quark star with chemical potential dependent values of bag pressure for Gaussian form of $B(\mu)$ and hyperbolic form $B(\mu)$ (left) and density dependent values of bag pressure for Gaussian form of $B(\rho)$ and hyperbolic form $B(\rho)$. Observational limits imposed from HESS J1731-347 [44] and the constraints on M-R plane prescribed from GW170817 [45] and PSR J0740 + 6620 [2] are also compared.

quarks (u,d,s), and the dotted line represents the same chemical potential without taking the derivative of the bag pressure term as shown in Eq. (29). At higher values of the chemical potential, the bag pressure is almost constant, therefore the derivative term Eq. (29) does not contribute and hence the lines merge as seen in Fig. 5.

Following the same procedure as previously stated in case of the grand canonical ensemble, the allowed free parameters in the ρ dependent bag pressure (Gaussian form) constrained by Bodmer-Witten conjecture $(B_0, B_{as}, \beta_\rho)$ is estimated as shown in Table II. In a similar way, we can estimate the stability limit of the parameters following the hyperbolic bag pressure.

C. Mass-radius diagram

Exploring the inner structures of strange quark stars will be simple once the EoS of strange quark matter is calculated. Therefore we present mass-radius configuration

TABLE II. ρ dependent Gaussian bag pressure in canonical ensemble.

B_0 (MeV fm^{-3})	B_{as} (MeV fm^{-3})	β_ρ
57	40	[0.0, 0.013]
100	50	[0.052, 0.59]
100	40	[0.041, 0.33]
100	30	[0.035, 0.23]
150	50	[0.113, 0.75]
150	40	[0.091, 0.44]
150	30	[0.077, 0.31]
200	50	[0.141, 0.85]
200	40	[0.115, 0.50]
200	30	[0.098, 0.36]

in Fig. 6. We use μ dependent bag pressure in the grand canonical ensemble in both Gaussian and hyperbolic forms and the chosen parameters are given in the left side of Fig. 6. We have also used ρ dependent bag pressure in the Canonical ensemble in both Gaussian and hyperbolic forms and the chosen parameters are given on the right side of Fig. 6. Different parameters of the MIT bag model used in the mass-radius diagram are chosen such that they satisfy the Bodmer-Witten conjecture [32] for stability of stars as well as the recent astrophysical data that are shown in Fig. 6. The calculated quark star configurations satisfy the recently obtained constraint from the low-mass compact object HESS J1731-347 and constraints on the mass and radius of compact stars from GW170817. The maximum mass constraint from PSR J0740 + 662 is satisfied for the μ dependent hyperbolic case only, though it is dependent on the parameter space. Our main focus in this work is the proper thermodynamic treatment and stability; in future, one can explore more on quark star and hybrid star properties with chemical potential dependent $B(\mu)$ (varying the parameters) and compare those with different astrophysical constraints.

IV. SUMMARY AND CONCLUSION

In this work, we have studied the medium effects of quark matter through the bag pressure in the framework of MIT bag model. We demonstrate that if a density-dependent bag pressure is used in the grand canonical ensemble, then in the equation of state, either the Euler relation [Eq. (2)] becomes invalid or the lowest energy per baryon does not coincide with that of zero pressure. In order to overcome this inconsistency in thermodynamics, we suggest that the medium influence of SQM to be incorporated via density or chemical potential dependent bag pressure depending on the ensemble chosen. In other words, the intensive parameter which addresses the medium effects should be ensemble dependent. This work is one of the first to propose chemical potential dependent bag pressure in grand canonical ensemble. The self-consistency in thermodynamics is restored if the chemical potential dependent bag parameter is used in the grand canonical ensemble and density dependent in the canonical ensemble. In the grand canonical ensemble, density is modified due to chemical potential-dependent bag pressure whereas in the canonical ensemble, chemical potential is modified due to the density-dependent bag pressure. The main ingredient of our study is the reformulation of medium-dependent bag pressure according to the choice of ensemble which solves the inconsistency problem from thermodynamics point of view. Generally, for the infinite matter system, grand canonical ensemble is used. If one chooses to take the density-dependent medium effect, then canonical is the appropriate ensemble. We have calculated the mass radius (M-R) of strange stars using this thermodynamically consistent formalism. This can be further explored for the study of the equation of state and different structural properties of the strange stars and the hybrid stars in future.

[1] J. Antoniadis *et al.*, Science **340**, 6131 (2013).
[2] E. Fonseca *et al.*, Astrophys. J. Lett. **915**, L12 (2021).
[3] T. E. Riley *et al.*, Astrophys. J. Lett. **918**, L27 (2021).
[4] M. C. Miller *et al.*, Astrophys. J. Lett. **918**, L28 (2021).
[5] T. E. Riley *et al.*, Astrophys. J. Lett. **887**, L21 (2019).
[6] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **119**, 161101 (2017).
[7] D. Radice, A. Perego, F. Zappa, and S. Bernuzzi, Astrophys. J. Lett. **852**, L29 (2018).
[8] G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song, and T. Takatsuka, Rep. Prog. Phys. **81**, 056902 (2018).
[9] M. Oertel, M. Hempel, T. Klähn, and S. Typel, Rev. Mod. Phys. **89**, 015007 (2017).
[10] A. Lovato *et al.*, arXiv:2211.02224.
[11] E. Annala, T. Gorda, A. Kurkela, J. Nättilä, and A. Vuorinen, Nat. Phys. **16**, 907 (2020).
[12] P. de Forcrand, Proc. Sci. LAT2009 (**2009**) 010.
[13] A. Kurkela, P. Romatschke, and A. Vuorinen, Phys. Rev. D **81**, 105021 (2010).
[14] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9**, 3471 (1974).
[15] G. X. Peng, H. C. Chiang, J. J. Yang, L. Li, and B. Liu, Phys. Rev. C **61**, 015201 (1999).
[16] X. J. Wen, X. H. Zhong, G. X. Peng, P. N. Shen, and P. Z. Ning, Phys. Rev. C **72**, 015204 (2005).
[17] P.-C. Chu and L.-W. Chen, Astrophys. J. **780**, 135 (2013).
[18] O. G. Benvenuto and G. Lugones, Phys. Rev. D **51**, 1989 (1995).
[19] M. Sinha, X.-G. Huang, and A. Sedrakian, Phys. Rev. D **88**, 025008 (2013).
[20] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[21] S. P. Klevansky, Rev. Mod. Phys. **64**, 649 (1992).
[22] T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994).
[23] M. Buballa, Phys. Rep. **407**, 205 (2005).
[24] M. Buballa and M. Oertel, Phys. Lett. B **457**, 261 (1999).
[25] C. H. Lenzi, A. S. Schneider, C. Providencia, and R. M. Marinho, Phys. Rev. C **82**, 015809 (2010).
[26] M. Hanauske, L. M. Satarov, I. N. Mishustin, H. Stoecker, and W. Greiner, Phys. Rev. D **64**, 043005 (2001).
[27] Q. Wang, T. Zhao, and H. Zong, Mod. Phys. Lett. A **35**, 2050321 (2020).
[28] A. Pfaff, H. Hansen, and F. Gulminelli, Phys. Rev. C **105**, 035802 (2022).
[29] E. S. Fraga, R. D. Pisarski, and J. Schaffner-Bielich, Phys. Rev. D **63**, 121702 (2001).
[30] S. Plumari, G. F. Burgio, V. Greco, and D. Zappalà, Phys. Rev. D **88**, 083005 (2013).
[31] R.-X. Xu, Astrophys. J. Lett. **596**, L59 (2003).
[32] E. Farhi and R. L. Jaffe, Phys. Rev. D **30**, 2379 (1984).
[33] J. Torres and D. Menezes, Europhys. Lett. **101**, 42003 (2013).
[34] E. J. Ferrer, V. de la Incera, and L. Paulucci, Phys. Rev. D **92**, 043010 (2015).
[35] N. Glendenning, *Compact Stars: Nuclear Physics, Particle Physics and General Relativity*, Astronomy and Astrophysics Library (Springer, New York, 2012).
[36] G. F. Burgio, M. Baldo, P. K. Sahu, A. B. Santra, and H. J. Schulze, Phys. Lett. B **526**, 19 (2002).
[37] G. F. Burgio, M. Baldo, P. K. Sahu, and H.-J. Schulze, Phys. Rev. C **66**, 025802 (2002).
[38] S. Chakrabarty, S. Raha, and B. Sinha, Phys. Lett. B **229**, 112 (1989).
[39] D. Sen, N. Alam, and G. Chaudhuri, J. Phys. G **48**, 105201 (2021).
[40] D. Sen, N. Alam, and G. Chaudhuri, Phys. Rev. D **106**, 083008 (2022).
[41] S. Pal, S. Podder, D. Sen, and G. Chaudhuri, Phys. Rev. D **107**, 063019 (2023).
[42] G. Lugones and A. G. Grunfeld, Phys. Rev. D **107**, 043025 (2023).
[43] P. D. Group, Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[44] V. Doroshenko, V. Suleimanov, G. Pühlhofer, and A. Santangelo, Nat. Astron. **6**, 1444 (2022).
[45] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **121**, 161101 (2018).