# Constraints on monopole-dipole potential from tests of gravity

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An ultralight axion like particle (ALP) can mediate a macroscopic force with long-range monopole-dipole interactions between the Earth and the Sun, if the Earth is treated as a polarized source. Because of the geomagnetic field, there exists an estimated $10^{42}$ polarized electrons within the Earth. These electrons, in a polarized state, can interact with the unpolarized nucleons in the Sun, giving rise to a monopole-dipole potential between the Sun-Earth system. This phenomenon ultimately influences the trajectories of light and celestial bodies, resulting in observable effects such as gravitational light bending, Shapiro time delay, and perihelion precession of planets. We investigate two scenarios for constraining the monopole-dipole coupling strength. In the first scenario, we establish a constraint on the monopole-dipole strength based on a single astrophysical observation for the first time, treating the Earth as a source of polarized electrons. The perihelion precession of Earth sets an upper limit on the monopole-dipole coupling strength as $g_S g_P \lesssim 1.75 \times 10^{-16}$ for the ALP of mass $m_a \lesssim 1.35 \times 10^{-18}$ eV. This bound surpasses the limits obtained from gravitational light bending and Shapiro time delay. In the second scenario, constraints on monopole-monopole coupling strength $g_S(\lesssim 3.51 \times 10^{-25})$ arise from the perihelion precession of the planet Mars, while the limit on dipole-dipole coupling strength $g_P(\lesssim 1.6 \times 10^{-13})$ is taken from the measurement of the tip of the red giant branch in $\omega$ Centauri using Gaia DR2 data. Together, they yield a hybrid constraint on the monopole-dipole coupling strength as $g_S g_P \lesssim 5.61 \times 10^{-38}$. Our hybrid bound is 3 orders of magnitude more stringent than the Eöt-Wash experiment and 1 order of magnitude stronger than the current hybrid $(\mathrm{Lab})_{\mathrm{S}}^{\mathrm{N}} \times (\mathrm{Astro})_{\mathrm{P}}^{\mathrm{e}}$ limit.

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## I. INTRODUCTION

Ultralight pseudoscalar bosons such as axion like particles (ALPs) can mediate a long-range macroscopic force between two objects if the mass of the ALP is smaller than the inverse distance between the two bodies [1]. Unlike quantum chromodynamics (QCD) axions [2–5], the ALPs are generated by string compactifications [6]. The mass of the ALP and its symmetry breaking scale are independent of each other. These ultralight bosons couple very weakly with the Standard Model (SM) particles and hence, it is extremely challenging to search these particles in direct detection experiments. Nevertheless, many ongoing and future experiments are built to probe these particles. Several laboratories, astrophysical, and cosmological constraints on the mass and decay constant of QCD axions and ALPs are discussed in [7–30]. Ultralight ALPs can also be a promising candidate for dark matter (DM) [31–33]. The mass of the ALP can be as small as $10^{-22}$ eV and the corresponding de Broglie wavelength is of the order of the size of a dwarf galaxy (1–2 kpc) [34,35]. Therefore, the ultralight particle DM behaves as a wave. Such wave DM can solve the longstanding core-cusp problem [36–38] and evade the DM direct detection constraints [39–41]. Therefore, there is significant phenomenological importance in the search for these particles and in obtaining constraints on ALP parameters.

The mediation of ALP between two fermion currents can give rise to long-range macroscopic forces. ALP can interact with fermions through either a spin-dependent pseudoscalar coupling $(\bar{\psi}\gamma_5\psi a)$ or a spin-independent scalar coupling $(\bar{\psi}\psi a)$, where $\psi$ represents the fermion field and $a$ denotes the axion field. The spin-independent scalar current-current interaction and the spin-dependent pseudoscalar current-current interaction, mediated by the exchange of a single ALP, lead to the conventional monopole-monopole and dipole-dipole potentials, respectively. However, if ALP couples with fermion currents

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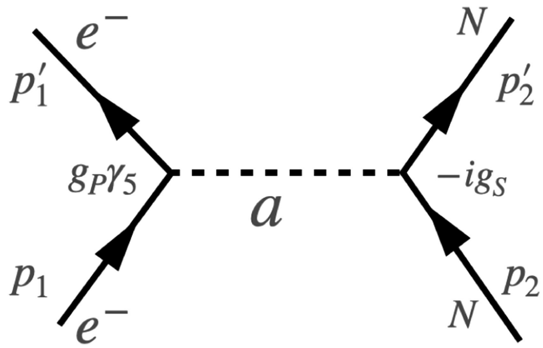


FIG. 1. Feynman diagram of $e^-N$ scattering mediated by a pseudoscalar ALP. Here, the electrons are polarized and the nucleons are unpolarized.

through a scalar coupling at one vertex and a pseudoscalar coupling at another vertex, it can mediate a long-range monopole-dipole potential. Monopole-dipole forces can also be generated by an ultralight spin 1 boson mediation which can come from a new U(1) symmetry breaking. This scenario has been extensively studied in the context of supersymmetry and grand unified theories [42,43]. The monopole-monopole, and dipole-dipole forces are parity ($P$) and time reversal ($T$) conserving. However, the search for monopole-dipole force is interesting as it can violate $P$ and $T$.

The Feynman diagram for an axion mediated monopole-dipole potential between polarized electron current and unpolarized nucleonic current is shown in Fig. 1. The expression of the monopole-dipole potential mediated by ultralight ALP between two fermion currents is given by [1,44]

$$V(r) = \frac{g_P g_S}{4\pi m_e}(\mathbf{s_1}.\hat{r})\left(\frac{m_a}{r}+\frac{1}{r^2}\right)e^{-m_a r}, \tag{1}$$

where we consider that ALP with mass $m_a$ is coupled with the polarized electron by a pseudoscalar coupling with strength $g_P$ and the ALP is also coupled with unpolarized nucleon by a scalar coupling with strength $g_S$. Here, $m_e$ is the mass of the electron, and $\mathbf{s_1}$ is the electron's spin vector. The term $\mathbf{s_1}.\hat{r}$ violates $P$ and $T$ symmetries. The derivation of Eq. (1) is given in Appendix A. We have also discussed the case where the ALP has scalar couplings with nucleons in the Sun and the Earth.

There exist several experiments dedicated to the search for parity and time reversal violating monopole-dipole potentials [45–49]. Such potential can be constrained from the torsion balance method using polarized electrons in the torsion pendulum and unpolarized nucleons in the Earth or in the Sun [50]. The bound on $g_S g_P$ obtained from this laboratory experiment is most sensitive for the axions of mass $m_a \lesssim 10^{-14}$ eV. The QUAX-$g_S g_P$ [46,51] experiment obtains a lab-lab bound on $g_S g_P$ for the mass of the axion $5\times10^{-7}$ eV $\lesssim m_a \lesssim 10^{-5}$ eV. An experiment like ARIADNE is made to search for monopole-dipole potential using a laser polarized $^3$He and a rotating tungsten source mass [52]. This lab-lab $g_S g_P$ bound is valid for axions of mass 1 μeV $\lesssim m_a \lesssim 6$ meV. In [53], polarized ultracold neutron spins and unpolarized nucleons are used to constrain such potential. This lab-lab experiment can probe axions of mass 1 meV $\lesssim m_a \lesssim 0.1$ eV. There are other laboratory experiments like SMILE ($m_a \lesssim 10^{-10}$ eV) [54], NIST ($m_a \lesssim 10^{-14}$ eV) [55], J-PARC muon $g-2$ ($m_a \lesssim 3\times10^{-14}$ eV) [56], Washington (10 μeV $\lesssim m_a \lesssim$ 10 meV) [57,58], and magnon based axion dark matter search ($m_a \lesssim 10^{-5}$ eV) [59,60] which obtain bounds on monopole-dipole interaction. The cooling of red giants and white dwarfs puts a constraint on $g_P \lesssim 1.6\times10^{-13}$ [61] and the constraint on $g_S$ obtained from the energy loss of globular cluster stars is $g_S \lesssim 1.1\times10^{-12}$ [62]. Multiplying these two numbers, one can obtain the bound on monopole-dipole coupling as $g_S g_P \lesssim 10^{-25}$ for $m_a \lesssim 10$ keV. The lab-astro bound on $g_S g_P$ is obtained from two independent experimental bounds and the bound is sensitive for $m_a \lesssim 10^{-18}$ eV. The astro-astro $g_S g_P$ bound also considers two separate observations.

So far there is no single astrophysical phenomenon that can directly constrain the monopole-dipole interaction. The most stringent bound on monopole-dipole interaction is claimed by combining the best experimental bound on scalar interaction multiplied by the best astrophysical bound from stellar energy loss on the pseudoscalar interaction. It is also highlighted by the authors of [45] that in several scenarios these hybrid bounds could be overly stringent leading to a premature abandoning of the axions as an attractive theoretical prospect. There is a lack of a complete astrophysical probe of monopole-dipole potential as most of the astrophysical objects are considered to be unpolarized. Even if a polarized astrophysical object is considered, its degree of polarization is not known precisely.

In this paper, we consider the Earth as a polarized source and there are about $10^{42}$ number of polarized electrons in Earth due to the presence of Earth's geomagnetic field [63]. Here, the Earth is treated as a polarized source and the Sun is treated as an unpolarized object. The ALP has a pseudoscalar coupling with the electrons in the Earth and scalar coupling with the nucleons in the Sun. This can give rise to an axion mediated monopole-dipole potential for the Earth-Sun system. This axion mediated monopole-dipole potential can affect the geodesic of light and Earth. We obtain constraints on monopole-dipole interaction strength in this pure astrophysical scenario from the perihelion precession of Earth, gravitational light bending, and Shapiro time delay. The bounds on the monopole-dipole coupling obtained from these gravity tests are strictly valid for the range of the force greater than the Earth-Sun distance which corresponds to the mass of the axion $(m_a) \lesssim 10^{-18}$ eV.

We also consider the axion mediated monopole-monopole potential between the unpolarized nucleons in the Earth and the Sun that can similarly affect the perihelion precession of planets, gravitational light bending, and Shapiro time delay. We obtain constraints on monopole coupling from these tests of gravity. We also obtain constraints on dipole coupling from the excessive energy loss of the red giant branch. Multiplying these two couplings obtained from two different astrophysical observations, we obtain combined constraints on monopole-dipole coupling strength.

The paper is organized as follows. In Sec. II, we discuss how the Earth can be treated as a polarized source. In Sec. III we obtain the contribution of monopole dipole potential in perihelion precession of planets, gravitational light bending, and Shapiro time delay. We also obtain bounds on monopole-dipole strength from these tests of gravity with single astrophysical observation. In Sec. IV, we obtain constraints on monopole-dipole coupling strength from two different astrophysical observations. Finally, in Sec. V we conclude and discuss our results.

We use the natural system of units ($c = \hbar = 1$, where $c$ is the speed of light and $\hbar$ is the reduced Planck constant) in our paper. We also choose Newton's gravitational constant $G = 1$.

## II. EARTH AS A POLARIZED SOURCE

Recently, a long-range dipole-dipole interaction arising between two spin polarized bodies is studied where the authors have considered the Earth as a source of spin polarized electrons [63]. In the presence of the geomagnetic field, some of the electrons in paramagnetic minerals within the Earth acquire a small spin polarization. The magnitude and direction of the induced geoelectron spins depend on the Earth's material composition, geomagnetic field and temperature profile [64]. The core of the Earth is mostly made of an Fe-Ni alloy which does not contain any unpaired electron spins due to high pressure and temperature [65,66]. Hence, the Earth's core does not make any contribution to its polarization. The dominant contribution to the polarization comes from Fe, the most abundant transition metal in various oxides and silicates in the Earth's mantle and crust. Other major rock forming elements like Mg, Si, Al, and O have a negligible contribution to the Earth's polarization due to their closed electron shells. In [63], the electron spin density as a function of depth and all the mineral proportions in Earth's crust and mantle are mentioned very accurately. It is found that the unpaired electron density around $10^4$ km depth is about $10^{22}/\mathrm{cm}^3$. Hence, the total unpaired electron spins inside the Earth will be $N_e \sim 10^{22} \times 10^{27} = 10^{49}$. Most of the unpaired electrons exist in the $Fe^{2+}$ state with a total spin $s = 2$, the so-called $HS$ state. When the spin-$\frac{1}{2}$ electron in $HS$ $Fe^{2+}$ interacts with the external geomagnetic field, the spins become polarized and the polarization fraction becomes $\alpha = \frac{2\mu_B B}{kT}$, where the electron Bohr magneton is $\mu_B = \frac{e}{2m_e} = 2.94 \times 10^{-7}\ \mathrm{eV}^{-1}$, $k$ is the Boltzmann constant, $B \sim 1$ G is the Earth's magnetic field in the mantle, and $T \sim 2000$ K is the temperature. Hence, we can obtain the polarization fraction as $\alpha \sim 10^{-7}$. Therefore, the total polarized electron spins in Earth is $N_e \times \alpha \sim 10^{49} \times 10^{-7} = 10^{42}$. The value of $\alpha$ due to the Earth's magnetic field is much larger than the accidental polarization, estimated as $\alpha_{\mathrm{accidental}} \sim \frac{1}{\sqrt{N_e}} \sim 10^{-25}$. These spin polarized geoelectrons can induce a net polarization due to the Earth's magnetic field which can generate an axion mediated monopole-dipole potential for the Earth-Sun system. Such an interaction can affect the perihelion precession of the Earth, gravitational light bending, and Shapiro time delay. However, the contribution of monopole-dipole potential for these observations is limited to be no larger than the measurement uncertainty. In Fig. 2 we obtain the Feynman diagram for $e^-N$ scattering mediated by ultralight ALP for the Earth-Sun system. The ALP is coupled with the electrons in the Earth by a pseudoscalar coupling. The ALP is also coupled with unpolarized nucleons in the Sun by a scalar coupling. In the following, we obtain the contribution of monopole-dipole potential from the measurements of perihelion precession of the Earth, gravitational light bending, and Shapiro time delay.

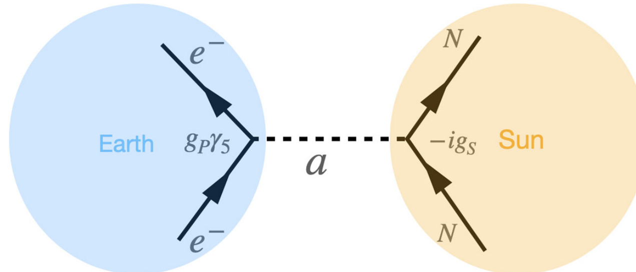


FIG. 2. Feynman diagram of $e^-N$ scattering mediated by ultralight ALP in the Earth-Sun system.

## III. PERIHELION PRECESSION, GRAVITATIONAL LIGHT BENDING, AND SHAPIRO TIME DELAY IN THE PRESENCE OF A MONOPOLE-DIPOLE POTENTIAL

The success of Einstein's general relativity (GR) theory has been consolidated by the observation of the perihelion precession of the Mercury planet. While orbiting around the Sun, the perihelion position of the Mercury planet shifts by a very small angle in each revolution. The dominant contribution to the perihelion shift comes from the gravitational effect of other solar bodies. There is also a subdominant contribution on perihelion shift due to the oblateness of the Sun and Lense-Thirring precession. These nonrelativistic contributions are calculated based on Newtonian mechanics which follows $\frac{1}{r^2}$ force law. However, there is about 42.9799 arcsecond/century [67,68] mismatch from the observation after including all the nonrelativistic effects in the measurement of perihelion precession of Mercury. Einstein's general relativistic calculation of perihelion precession can completely resolve this anomaly. Besides Mercury, all the other planets also experience perihelion shifts. For example, the Earth has a perihelion shift of 3.84 arcsecond/century due to GR correction. Since, the Earth is taken as a polarized source, there can be an axion mediated monopole-dipole potential for the Earth-Sun system. This axion mediated long-range potential can affect the geodesic of the Earth and contribute to its perihelion precession measurements. However, the contribution of monopole-dipole interaction should be limited to be no larger than the measurement uncertainty which is $10^{-4}$ arcsecond/century [69,70] for the Earth-Sun system. Using the perturbative method, we analytically obtain the contribution of monopole-dipole potential in perihelion shift as (see Appendix B)

$$\Delta\phi_{\text{monopole-dipole}} \simeq \frac{g_S g_P N_1 N_2}{2MD(1-\epsilon^2)M_P m_e} + \frac{g_S g_P N_1 N_2 D^2 m_a^3 (1-\epsilon^2)}{6M_P M(1+\epsilon)m_e} + \mathcal{O}((g_S g_P)^2, m_a^4). \tag{2}$$

Using the values of the solar mass $M = 1.11 \times 10^{57}$ GeV, the Sun-Earth distance $D = 0.98\ \text{AU} = 7.37 \times 10^{26}\ \text{GeV}^{-1}$, the eccentricity of the Earth-Sun orbit $\epsilon = 0.017$, the mass of the electron $m_e = 5.1 \times 10^{-4}$ GeV, the mass of the planet Earth $M_P = 3.35 \times 10^{51}$ GeV, the number of polarized electrons in Earth $N_1 = 10^{42}$, and the number of unpolarized nucleons in the Sun $N_2 = 10^{57}$, we obtain the upper bound on monopole-dipole coupling as $g_S g_P \lesssim 1.75 \times 10^{-16}$ for mass of the axion $m_a \lesssim 1.35 \times 10^{-18}$ eV. We obtain this bound by considering that the contribution of monopole-dipole potential is limited to be no larger than the perihelion precession measurement uncertainty.

Besides the perihelion precession of planets, gravitational light bending is another test of Einstein's GR theory [71,72]. When a light ray from a distant pulsar comes to Earth, then the presence of a massive object like the Sun can distort the spacetime between the light source and the Earth. The increased gravitational potential due to the presence of the Sun decreases the speed of light and the light bends. The amount of bending depends on the mass of the gravitating object (Sun) and the impact parameter. In 1915, Einstein first calculated the amount of light bending due to the presence of the Sun based on the equivalence principle. The calculated value of light bending is 1.75 arcsecond which matches well with the experiment to an uncertainty of $10^{-4}$ arcsecond [73]. The monopole dipole potential of the Earth-Sun system can affect the geodesic of light and contribute to the measurement of gravitational light bending. The contribution of the monopole-dipole potential should be limited to this uncertainty. We perturbatively calculate the contribution of monopole-dipole potential in gravitational light bending as (see Appendix C)

$$\Delta\phi_{\text{monopole-dipole}} \simeq -\frac{2m_a^3 b^3 g_S g_P N_1 N_2 \ln 2}{3M_P L^2 \times 4\pi m_e} + \frac{g_S g_P N_1 N_2}{M_P L^2 \times 4\pi m_e} \times \frac{4M}{b} - \frac{m_a^3 b^3 g_S g_P N_1 N_2}{3M_P L^2 \times 4\pi m_e} \times \frac{4M}{b} + \mathcal{O}((g_S g_P)^2, m_a^4). \tag{3}$$

We use $L^2 = MD(1-\epsilon^2)$, and the value of impact parameter $b$ as the solar radius $b \sim R_\odot = 6.96 \times 10^8\ \text{m} = 3.51 \times 10^{24}\ \text{GeV}^{-1}$. The contribution of monopole-dipole potential in the measurement of gravitational light bending should be within the measurement uncertainty and we obtain the bound on coupling as $g_S g_P \lesssim 4.25 \times 10^{-9}$ for $m_a \lesssim 1.35 \times 10^{-18}$ eV.

We also obtain constraints on monopole-dipole interaction strength from the Shapiro time delay. When a radar signal is sent from Earth to Venus and it reflects from Venus to Earth, then in this round-trip, there is a time delay in getting the signal compared to the expectation. In 1964, Irwin Shapiro calculated the amount of time delay as $2 \times 10^{-4}$ s [74,75] which agrees well with the experiment to an uncertainty of $10^{-5}$ s [76]. This time delay occurs due to the presence of strong gravitational potential near the Sun. The presence of long-range monopole-dipole potential can contribute to the Shapiro time delay. However, its contribution should be within the measurement uncertainty. We analytically calculate the contribution of monopole-dipole potential in Shapiro time delay as (see Appendix D)

$$\Delta T_{\text{monopole-dipole}} \simeq \frac{8M}{M_P r_0 E^2}\left(m_a + \frac{1}{r_0}\right)e^{-m_a r_0}\left(\frac{g_S g_P N_1 N_2}{4\pi m_e}\right) - \frac{4M}{M_P E^2 r_0^2}\left(\frac{g_S g_P N_1 N_2}{4\pi m_e}\right) + \mathcal{O}((g_S g_P)^2, m_a^2, M^2). \tag{4}$$

Using the Earth-Sun distance $r_e = D = 1.46 \times 10^{11}\ \text{m} = 7.37 \times 10^{26}\ \text{GeV}^{-1}$, the Venus-Earth distance $r_v = 1.08 \times 10^{11}\ \text{m} = 5.47 \times 10^{26}\ \text{GeV}^{-1}$, the solar radius $r_0 = R_\odot = 6.96 \times 10^8\ \text{m} = 3.51 \times 10^{24}\ \text{GeV}^{-1}$, and $E^2 \simeq \frac{L^2}{r_0^2}(1 - \frac{2M}{r_0})$, we obtain the upper bound on coupling as $g_S g_P \lesssim 1.08 \times 10^{-4}$ for $m_a \lesssim 1.35 \times 10^{-18}$ eV. There is an extra multiplicative factor of $\exp[-\frac{m_a L^2}{M}]$ in Eqs. (2)–(4) to incorporate the exponential suppression due to the large value of axion mass. In calculating the bounds on $g_S g_P$ from the above observations, we simply substitute $M \to GM$. In Fig. 3 we obtain numerically the bounds on monopole-dipole coupling from perihelion precession of the Earth (red region), gravitational light bending (blue region), and Shapiro time

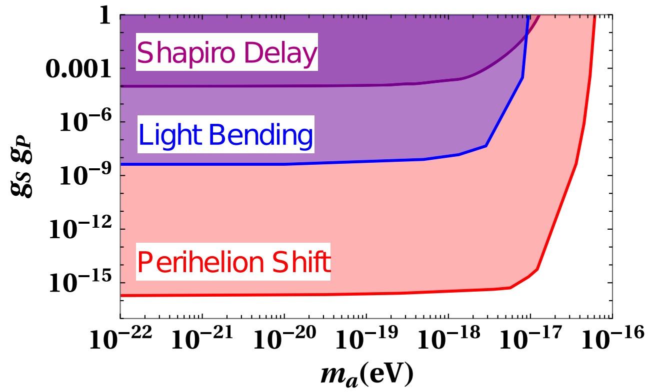


FIG. 3. Bounds on monopole-dipole interaction strength from single astrophysical observation.

delay (purple region). The shaded regions are excluded. We obtain a stronger bound on $g_S g_P$ from perihelion precession of the Earth as $g_S g_P \lesssim 1.75 \times 10^{-16}$ for the axions of mass $m_a \lesssim 1.35 \times 10^{-18}$ eV. This is the first bound on $g_S g_P$ that we obtain from a single astrophysical observation and for ALPs of mass $m_a \lesssim \mathcal{O}(10^{-18})$ eV.

## IV. CONSTRAINTS ON MONOPOLE-DIPOLE COUPLING FROM TWO DIFFERENT ASTROPHYSICAL OBSERVATIONS

In this section, we obtain constraints on monopole-dipole coupling from two different astrophysical observations. In Fig. 4, we consider monopole-monopole coupling of axions with unpolarized nucleons in the planet and the Sun that can change the perihelion precession of planets, gravitational light bending, and Shapiro time delay within the measurement uncertainty. The potential due to axion mediated nucleon-nucleon scattering in the Earth-planet system is $\frac{g_S^2 N_1 N_2}{4\pi r} e^{-m_a r}$, where $N_1$ and $N_2$ are the numbers of nucleons in the Sun and the planet, respectively. Hence, the perihelion shift due to the axion mediated monopole-monopole potential between the Sun and the planet is [77]

$$\Delta\phi_{\text{monopole-monopole}} \simeq \frac{g_S^2 N_1 N_2 m_a^2 D^2 (1-\epsilon^2)}{4M_P (M + \frac{g_S^2 N_1 N_2}{4\pi M_P})(1+\epsilon)} + \mathcal{O}(g_S^3, m_a^3), \tag{5}$$

where $M_P$ is the mass of the planet, $M$ is the mass of the Sun, and $D$ is the semimajor axis of the planetary orbit with eccentricity $\epsilon$. The contribution of axion mediated monopole-monopole potential should be limited to be no larger than the perihelion precession measurement uncertainty. We obtain the stronger bound on $g_S$ for the planet Mars [77] and its value is $g_S \lesssim 3.51 \times 10^{-25}$ for the mass of the axion $m_a \lesssim 1.35 \times 10^{-18}$ eV.

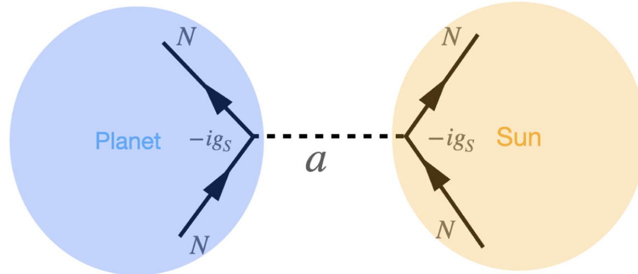


FIG. 4. Monopole coupling of axions with nucleons in the Sun-planet system.

The bending of light due to the axion mediated monopole potential is [29]

$$\Delta\phi_{\text{monopole-monopole}} \simeq \frac{g_S^2 N_1 N_2 b}{2\pi M_P L^2}(1 - 0.347 m_a^2 b^2) - \frac{g_S^2 N_1 N_2 M m_a^2 b^2}{2\pi M_P L^2} + \mathcal{O}(g_S^3, m_a^3). \tag{6}$$

We obtain the constraint on axion monopole coupling from the gravitational light bending as $g_S \lesssim 5.82 \times 10^{-23}$ for the axions of mass $m_a \lesssim 1.35 \times 10^{-18}$ eV.

Similarly, the contribution of axion mediated monopole potential in Shapiro time delay is [29]

$$\begin{aligned}\Delta T_{\text{monopole-monopole}} \simeq\; & 2b_0 c_0(-1 + c_0 M)(r_e + r_v) \\ & + \frac{b_0 c_0^2}{2}(r_e^2 + r_v^2) + 2b_0 - 4c_0 M b_0 \\ & + 2a_0(r_e + r_v) + \frac{b_0}{24}(48 \\ & + 36 c_0^2 r_0^2 [E_i(-c_0 r_e) + E_i(-c_0 r_v)]) \\ & + \mathcal{O}(g_S^3, m_a^3),\end{aligned} \tag{7}$$

where $a_0 = \frac{g_S^2 N_1 N_2 e^{-m_a r_0}}{4\pi M_P E^2 r_0}$, $b_0 = \frac{g_S^2 N_1 N_2}{4\pi M_P E^2}$, and $c_0 = m_a$.

We obtain the constraint on axion monopole coupling from the Shapiro time delay as $g_S \lesssim 3.59 \times 10^{-22}$ for the axion mass $m_a \lesssim 1.35 \times 10^{-18}$ eV. There is an extra multiplicative factor of $\exp[-\frac{m_a L^2}{M}]$ in Eqs. (5)–(7) if we solve the perihelion shift, light bending, and Shapiro time delay numerically for the axion mediated monopole-monopole potential to incorporate the exponential suppression due to large values of axion mass.

The bound on the axion electron pseudoscalar coupling can be obtained from the cooling of red giant stars and white dwarfs. The axion electron coupling allows the stellar energy loss by the bremsstrahlung $(e + Ze \to e + Ze + a)$ and Compton process $(\gamma + e \to e + a)$ [78,79]. The excessive energy loss due to these processes will delay the helium ignition in the red giant stars. Therefore the tip of the red giant branch becomes brighter. The measurement of

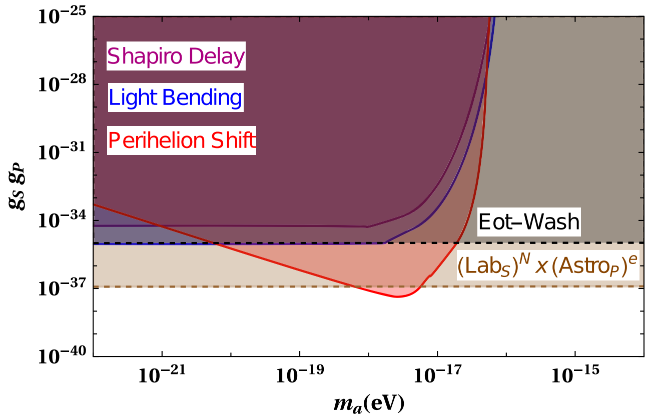


FIG. 5. Bounds on monopole-dipole interaction strength from two different astrophysical observations.

the tip of the red giant branch in the $\omega$ Centaury from Gaia DR2 data put a bound on the axion-electron coupling as $g_P \lesssim 1.6\times10^{-13}$ for the mass of the axions $m_a \lesssim$ 10 keV [61].

To obtain the bound on monopole-dipole coupling $(g_S g_P)$, we take the product of the bounds on $g_S$ obtained from the tests of gravity (perihelion precession of planet, gravitational light bending, and Shapiro time delay) and $g_P$ obtained from the energy loss from the red giant branch. In Fig. 5 we obtain the bounds on $g_S g_P$ from two different astrophysical observations. The perihelion precession of the planet Mars and red giant branch give the bound on monopole-dipole coupling as $g_S g_P \lesssim 5.61\times10^{-38}$. We also obtain the bound on $g_S g_P$ from gravitational light bending and red giant branch as $g_S g_P \lesssim 9.31\times10^{-36}$. Lastly, the bound on $g_S g_P$ obtained from Shapiro time delay and red giant branch is $g_S g_P \lesssim 5.74\times10^{-35}$. These bounds are only valid for the mass of the axion $m_a \lesssim 1.35\times10^{-18}$ eV. We obtain the stronger bound on $g_S g_P$ from the perihelion precession of the planet Mars and energy loss of the red giant branch. The shaded regions in Fig. 5 are excluded. The bound $g_S g_P \lesssim 5.61\times10^{-38}$ is 3 orders of magnitude stronger than the Eöt-Wash experiment [80] and 1 order of magnitude stronger than the $(\mathrm{Lab})^{\mathrm{N}}_{\mathrm{S}} \times (\mathrm{Astro})^{\mathrm{e}}_{\mathrm{P}}$ limit [45].

The behavior of the curves in Figs. 3 and 5 can be effectively explained by considering Eqs. (2)–(7). The expression of $(\Delta\phi)_{\text{monopole-dipole}}$ in Eq. (2) depends on two terms. The first term is independent of the axion like particle's (ALP) mass, while the second term depends on the ALP mass. For ALP masses at the lower end of the spectrum, we can disregard the mass-dependent term, resulting in flat curves within this range. This same scenario applies to all the curves depicted in Figs. 3 and 5, except for the curve representing the perihelion precession of the planet in Fig. 5. In this case, at the lower mass range, the curve does not remain flat; instead, it exhibits an apparent slope. This is attributed to the fact that in Eq. (5), $(\Delta\phi)_{\text{monopole-monopole}}$ solely comprises a term dependent on the ALP mass. The monopole-monopole coupling is approximately inversely proportional to the ALP mass. Hence, in the lower mass region the curve shows a negative slope. In the high ALP mass region, the exponential suppression term $\exp[-\frac{m_a L^2}{M}]$ dominates as usual.

## V. CONCLUSIONS AND DISCUSSIONS

In this paper, we obtain constraints on monopole-dipole coupling strength from single astrophysical observations such as the perihelion precession of Earth, gravitational light bending, and Shapiro time delay. These bounds are strictly valid for the ALP mass $m_a \lesssim 1.35\times10^{-18}$ eV. Because of the presence of a geomagnetic field, $10^{42}$ number of electrons can be polarized in Earth and ALP mediated monopole-dipole force can act between the Earth and the Sun. We obtain a stronger bound on monopole-dipole coupling strength from perihelion precession of the Earth as $g_S g_P \lesssim 1.75\times10^{-16}$ from a single astrophysical observation.

The previous lab-astro bounds on $g_S g_P$ obtained in the literature are derived from two different observations. In these studies, the monopole coupling $g_S$ and the dipole coupling $g_P$ are measured independently from two different observations and they are simply multiplied to get a bound on $g_S g_P$. To get the bound in this way is overly stringent and may not be completely reliable if the axion changes its behavior in different environments. The bounds on $g_S g_P$ obtained from lab-lab experiments are only valid for the axions of mass μeV $\lesssim m_a \lesssim$ meV.

In this work, we have obtained the first bounds on $g_S g_P$ from single astrophysical observations. In all of these observations, the massless limit gives the stronger bound on monopole-dipole coupling strength. In the massless limit, the perihelion shift is inversely proportional to the Sun-planet distance. This means planets which are closer to the Sun will put the best bound on $g_S g_P$. However, to achieve an improved bound on $g_S g_P$ from perihelion precession, one needs to calculate accurately the number of polarized spins in those planets.

The bounds on the monopole-dipole couplings that we obtain from perihelion precession, gravitational light bending, and Shapiro time delay are the order of magnitude calculations. These bounds strongly depend on the number of polarized electrons in the Earth which is not a fixed quantity at all its layers. In fact, this number depends on the magnetic field and temperature at each layer of the Earth which varies with its depth. Hence, at the massless limit, the monopole-dipole coupling strength will not be a fixed quantity and it should have different values at different depths. We obtain the number of polarized electrons in Earth as $10^{42}$ by taking the average values of Earth's magnetic field, temperature, and the number density of unpaired electrons which we have taken as fixed quantities.

Therefore, our bounds on monopole-dipole couplings are constant at the massless limits. This is the first study to probe monopole-dipole coupling from single astrophysical observations for $m_a \lesssim 1.35 \times 10^{-18}$ eV. Our bounds on the monopole-dipole coupling are the order of magnitude calculation and can be significantly improved by accurate incorporation of the number of polarized spins at each layer of Earth from geochemical and geological surveys. Such analyses are important to probe these long-range spin-dependent interactions.

We also obtain constraints on monopole-dipole coupling strength from two different astrophysical observations. We consider monopole coupling of axions with unpolarized nucleons in the Earth and the Sun to obtain bounds on monopole coupling from perihelion precession of the planet Mars, gravitational light bending, and Shapiro time delay. Multiplying these monopole couplings with the dipole coupling obtained from excessive energy loss of the red giant branch, we derive the monopole-dipole coupling strength. For $m_a \lesssim 1.35 \times 10^{-18}$ eV, we obtain $g_S g_P \lesssim 5.61 \times 10^{-38}$ from perihelion precession and red giant branch. This bound is 3 orders of magnitude stronger than the Eöt-Wash experiment and 1 order of magnitude stronger than the current $(\mathrm{Lab})^{\mathrm{N}}_{\mathrm{S}} \times (\mathrm{Astro})^{\mathrm{e}}_{\mathrm{P}}$ limit. In comparison to the pseudoscalar coupling of electrons in the Earth, the scalar coupling of nucleons in the Earth does not get stronger in the massless limit for the perihelion precession measurements. In fact, in this case the scalar coupling gets stronger for the planets which are further away from the Sun.

We can also constrain the axion mediated monopole-dipole coupling between nucleonic currents. The cooling of hot neutron star HESS J1731-347 puts bound on axion nucleon pseudoscalar coupling as $g_P^N \lesssim 2.8 \times 10^{-10}$. We also obtain axion nucleon scalar coupling as $g_S^N \lesssim 3.51 \times 10^{-25}$. Combining these two couplings, we obtain the bound on the monopole-dipole coupling strength for only nucleonic currents as $g_S^N g_P^N \lesssim 9.83 \times 10^{-35}$ for the mass of the axions $m_a \lesssim 1.35 \times 10^{-18}$ eV. This bound is better than the projected ARIADNE experiment [81] and $(\mathrm{Lab})^{\mathrm{N}}_{\mathrm{S}} \times (\mathrm{Astro})^{\mathrm{N}}_{\mathrm{P}}$ by a factor of 2 [45]. Future space missions with better precision can significantly improve the bounds of monopole-dipole couplings. These ultralight axions can be promising candidates for fuzzy dark matter.

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## APPENDIX A: MONOPOLE-DIPOLE POTENTIAL DUE TO POLARIZED ELECTRON AND UNPOLARIZED NUCLEON SCATTERING

In Fig. 1 we show the Feynman diagram of $e^-N$ scattering mediated by pseudoscalar ALP ($a$). The axion is coupled to the polarized electron with coupling constant $g_P$ and to the unpolarized nucleon with coupling constant $g_S$. Hence, the amplitude of the above process becomes

$$\begin{aligned} i\mathcal{M} &= \bar{u}_{s_1'}(p_1') g_P \gamma_5 u_{s_1}(p_1) \frac{i}{q^2 - m_a^2} \bar{u}_{s_2'}(p_2')(-i g_S) u_{s_2}(p_2) \\ &= \frac{g_P g_S}{q^2 - m_a^2} \bar{u}_{s_1'}(p_1') \gamma_5 u_{s_1}(p_1) \bar{u}_{s_2'}(p_2') u_{s_2}(p_2), \end{aligned} \tag{A1}$$

where $q = p_1 - p_1' = p_2' - p_2$. In the nonrelativistic (NR) limit, all three momentum components are much smaller than the mass of the particle ($m$) and, hence, the energy of the particle is $E \approx m$. We also choose the normalization condition $u_{s'}^\dagger(p) u_s(p) = \delta_{ss'}$. We can write the positive energy spinor in the NR limit as

$$u_s(p) = \left(1 - \frac{\gamma_i p_i}{2m}\right)\chi_s + \mathcal{O}(p^2), \tag{A2}$$

where $\chi_s$ is a normalized eigenvector satisfying $\chi_s^\dagger \gamma^0 = \chi_s^\dagger$ and $\gamma_0 \chi_s = \chi_s$. Here, $\gamma_i$ denotes the Dirac gamma matrices and $i$ runs from 1 to 3. Hence, in the NR limit, we can calculate the following bilinear terms using Eq. (A2) as

$$\bar{u}_{s_2'}(p_2') u_{s_2}(p_2) = 1, \bar{u}_{s_1'}(p_1') \gamma_5 u_{s_1}(p_1) = \frac{1}{2m_e} \chi_{s_1'}^\dagger \boldsymbol{\sigma}.\mathbf{q} \chi_{s_1}, \tag{A3}$$

where $\boldsymbol{\sigma}$ denotes the Pauli spin vector and $m_e$ denotes the mass of the polarized electron. We can write the amplitude [Eq. (A1)] of the $e^-N$ scattering process as

$$\mathcal{M} = \frac{i g_P g_S}{|\mathbf{q}|^2 + m_a^2} \bar{u}_{s_1'}(p_1') \gamma_5 u_{s_1}(p_1) \bar{u}_{s_2'}(p_2') u_{s_2}(p_2), \tag{A4}$$

where we can write $q^2 = q^{02} - |\mathbf{q}|^2$, and $|q^0| \ll |\mathbf{q}|$ in the NR limit. Using, Eq. (A3) we can write the potential for $e^-N$ scattering as

$$V(r) = -\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}.\mathbf{r}} \left(\frac{i g_P g_S}{|\mathbf{q}|^2 + m_a^2}\right) \frac{\mathbf{s_1}.\mathbf{q}}{m_e}, \tag{A5}$$

where the spin vector is $\mathbf{s_1} = \frac{\boldsymbol{\sigma}}{2}$. Therefore, the potential becomes

$$\begin{aligned} V(r) &= -\frac{g_P g_S}{m_e} (\mathbf{s_1}.\boldsymbol{\nabla}) \int \frac{d^3q}{(2\pi)^3} \frac{1}{|\mathbf{q}|^2 + m_a^2} e^{i\mathbf{q}.\mathbf{r}} \\ &= -\frac{g_P g_S}{m_e} (\mathbf{s_1}.\boldsymbol{\nabla}) \frac{1}{4\pi r} e^{-m_a r} \\ &= \frac{g_P g_S}{4\pi m_e} (\mathbf{s_1}.\hat{\boldsymbol{r}}) \left(\frac{m_a}{r} + \frac{1}{r^2}\right) e^{-m_a r}. \end{aligned} \tag{A6}$$

This is the expression for monopole-dipole potential which can act between a polarized electron and an unpolarized nucleon.

## APPENDIX B: PERIHELION PRECESSION OF EARTH IN THE PRESENCE OF A LONG-RANGE MONOPOLE-DIPOLE POTENTIAL

If Earth contains polarized electrons then long-range monopole-dipole potential can act between the Earth and the Sun. This new long-range force mediated by ultralight ALPs can contribute to the perihelion precession of Earth. However, its contribution is limited to be no larger than the uncertainty in the measurement of perihelion precession. For a timelike particle, we can write $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$, where $g_{\mu\nu}$ is the metric tensor for the Schwarzschild background spacetime. In the presence of a long-range monopole-dipole potential, we can write

$$\frac{E^2-1}{2} = \frac{\dot{r}^2}{2} + \frac{L^2}{2r^2} - \frac{ML^2}{r^3} - \frac{M}{r} - \frac{\beta E m_a}{M_P r}e^{-m_a r} - \frac{\beta E}{M_P r^2}e^{-m_a r}, \tag{B1}$$

where $\dot{r} = \frac{L}{r^2}\frac{dr}{d\phi}$, $M$ and $M_P$ are the masses of the Sun and the Earth, respectively, and $\beta = \frac{g_S g_P N_1 N_2}{4\pi m_e}$. $N_1$ and $N_2$ are the numbers of polarized electrons in the Earth and unpolarized nucleons in the Sun, respectively. We have also neglected the $\mathcal{O}(\beta^2)$ term because the coupling for the monopole-dipole potential is small. $E$ is a constant of motion which is termed as the total energy per unit rest mass for a timelike geodesic relative to an observer in rest frame at infinity. The total energy of the system per unit mass for a very small eccentric orbit in the presence of a monopole-dipole potential is

$$E \approx 1 - \frac{M}{2D} - \frac{g_S g_P N_1 N_2}{4\pi m_e}e^{-m_a D}\left(\frac{m_a}{M_P D} + \frac{m_a^2}{2M_P} + \frac{1}{M_P D^2}\right), \tag{B2}$$

and $L$ is another constant of motion which is the angular momentum per unit mass of the system. In Eq. (B1), the first term on the right-hand side denotes the kinetic energy part, the second term denotes the centrifugal potential part, the third term arises due to the contribution of GR, the fourth term denotes the Newtonian potential, and the last two terms appear due to the contribution of monopole-dipole potential. We can write Eq. (B1) in terms of reciprocal coordinate $u = \frac{1}{r}$ as

$$\left(\frac{du}{d\phi}\right)^2 + u^2 = \frac{E^2-1}{L^2} + 2Mu^3 + \frac{2Mu}{L^2} + \frac{2\beta E m_a u}{L^2 M_P}e^{-\frac{m_a}{u}} + \frac{2\beta E u^2}{L^2 M_P}e^{-\frac{m_a}{u}}, \tag{B3}$$

where $\phi$ denotes the azimuthal coordinate. Expanding the exponential term in Eq. (B3) and taking the derivative with respect to $\phi$, we obtain

$$\frac{d^2u}{d\phi^2} + u = \frac{M}{L^2} + 3Mu^2 + \frac{2\beta E u}{L^2 M_P} - \frac{\beta E m_a^3}{3L^2 M_P u^2}. \tag{B4}$$

To solve this second order differential equation we consider $u = u_0(\phi) + \Delta u(\phi)$, where $u_0(\phi)$ is the solution for Newton's theory and $\Delta u(\phi)$ is the solution due to the contribution of GR and monopole-dipole potential. Hence, we can write

$$\frac{d^2u_0}{d\phi^2} + u_0 = \frac{M}{L^2}. \tag{B5}$$

The solution of Eq. (B5) becomes

$$u_0(\phi) = \frac{M}{L^2}(1 + \epsilon\cos\phi), \tag{B6}$$

where $\epsilon$ is the eccentricity of the Earth-Sun elliptic orbit. The differential equation for $\Delta u$ is

$$\frac{d^2\Delta u}{d\phi^2} + \Delta u = \frac{3M^3}{L^4}(1 + \epsilon^2\cos^2\phi + 2\epsilon\cos\phi) + \frac{2\beta M E}{L^4 M_P}(1 + \epsilon\cos\phi) - \frac{\beta E m_a^3 L^2}{3M_P M^2(1 + \epsilon^2\cos\phi + 2\epsilon\cos\phi)}. \tag{B7}$$

The solution of Eq. (B7) becomes

$$\Delta u = \frac{3M^3}{L^4}\epsilon\phi\sin\phi + \frac{\beta M E}{L^4 M_P}\epsilon\phi\sin\phi + \frac{\beta E m_a^3 L^2}{3M_P M^2}\frac{\epsilon\sin\phi}{(1-\epsilon^2)^{\frac{3}{2}}} \times \frac{\sqrt{1-\epsilon^2}}{(1+\epsilon)}\phi, \tag{B8}$$

where we keep terms which are linear in $\phi$ and hence contribute to the perihelion precession of Earth. Hence, the total solution of Eq. (B4) becomes

$$u = u_0(\phi) + \Delta u(\phi) = \frac{M}{L^2}(1 + \epsilon\cos\phi) + \frac{3M^3}{L^4}\epsilon\phi\sin\phi + \frac{\beta M E}{L^4 M_P}\epsilon\phi\sin\phi + \frac{\beta E m_a^3 L^2}{3M_P M^2}\frac{\epsilon\sin\phi}{(1+\epsilon)(1-\epsilon^2)}. \tag{B9}$$

We can also write Eq. (B9) as

$$u = \frac{M}{L^2}[1 + \epsilon\cos\phi(1-\gamma)], \tag{B10}$$

where

$$\gamma = \frac{3M^2}{L^2} + \frac{\beta}{L^2 M_P} + \frac{\beta L^4 m_a^3}{3M_P M^3} \frac{1}{(1+\epsilon)(1-\epsilon^2)}. \tag{B11}$$

Here, we take $E \approx 1$ as other terms in Eq. (B2) are very small compared to 1. Here, $D$ denotes the semimajor axis of the orbit. As $\phi \to \phi + 2\pi$, $u$ is not the same. Therefore, Earth does not follow its previous orbit. Hence, the change in the azimuthal angle or the perihelion shift becomes

$$\Delta\phi = \frac{2\pi}{1-\gamma} - 2\pi = 2\pi\gamma = \frac{6\pi M^2}{L^2} + \frac{2\pi\beta}{L^2 M_P} + \frac{2\pi\beta L^4 m_a^3}{3M_P M^3} \frac{1}{(1+\epsilon)(1-\epsilon^2)}. \tag{B12}$$

Substituting $L^2 = MD(1-\epsilon^2)$, and $\beta = \frac{g_S g_P N_1 N_2}{4\pi m_e}$, we obtain

$$\Delta\phi = \frac{6\pi M}{D(1-\epsilon^2)} + \frac{g_S g_P N_1 N_2}{2MD(1-\epsilon^2) M_P m_e} + \frac{g_S g_P N_1 N_2 D^2 m_a^3 (1-\epsilon^2)}{6M_P M(1+\epsilon) m_e}. \tag{B13}$$

Equation (B13) is the general expression for the perihelion shift due to monopole-dipole potential between a polarized object and an unpolarized object. The first term on the right-hand side arises due to the GR contribution in perihelion shift and its value for Earth is 3.84 arcsecond/century. The last two terms arise due to the contribution of monopole-dipole potential. In the $g_S g_P \to 0$ limit, we get back the standard GR term. Hence, the contribution of monopole-dipole potential in perihelion shift is

$$\Delta\phi_{\text{monopole-dipole}} \simeq \frac{g_S g_P N_1 N_2}{2MD(1-\epsilon^2) M_P m_e} + \frac{g_S g_P N_1 N_2 D^2 m_a^3 (1-\epsilon^2)}{6M_P M(1+\epsilon) m_e} + \mathcal{O}((g_S g_P)^2, m_a^4). \tag{B14}$$

## APPENDIX C: GRAVITATIONAL LIGHT BENDING IN THE PRESENCE OF A LONG-RANGE MONOPOLE-DIPOLE POTENTIAL

Light follows the null geodesic which is given by

$$g_{\mu\nu} V^\mu V^\nu = 0, \tag{C1}$$

where $V^\mu = \frac{dx^\mu}{d\lambda}$ is the tangent vector along the path parametrized by $x^\mu(\lambda)$, where $\lambda$ is the affine parameter. For a Schwarzschild background and planar motion, the conserved quantities are $E = (1 - \frac{2M}{r})\dot{t}$ and $L = r^2\dot{\phi}$. Here, $E$ and $L$ denote the total energy and angular momentum per unit mass of the system, respectively. We can write the null geodesic in terms of these conserved quantities as

$$\frac{E^2}{2} = \frac{L^2}{2}\left(\frac{du}{d\phi}\right)^2 + \frac{L^2 u^2}{2}(1-2Mu), \tag{C2}$$

where we use $\dot{r} = \frac{dr}{d\lambda} = \frac{L}{r^2}\frac{dr}{d\phi}$ and the reciprocal coordinate $u = \frac{1}{r}$. The presence of long-range monopole-dipole potential changes the effective potential of the Sun-Earth system as

$$V_{\text{eff}} = \frac{L^2}{2}\left(\frac{du}{d\phi}\right)^2 + \frac{L^2 u^2}{2}(1-2Mu) - \frac{\beta m_a u}{M_P} e^{-\frac{m_a}{u}} - \frac{\beta u^2}{M_P} e^{-\frac{m_a}{u}}, \tag{C3}$$

where the last two terms arise due to the presence of long-range monopole-dipole potential. Hence, Eq. (C2) becomes

$$\frac{E^2}{2} = \frac{L^2}{2}\left(\frac{du}{d\phi}\right)^2 + \frac{L^2 u^2}{2}(1-2Mu) - \frac{\beta m_a u}{M_P} e^{-\frac{m_a}{u}} - \frac{\beta u^2}{M_P} e^{-\frac{m_a}{u}}. \tag{C4}$$

Differentiating Eq. (C4) and expanding the exponential term we obtain

$$\frac{d^2 u}{d\phi^2} + u = 3Mu^2 + \frac{2\beta u}{M_P L^2} - \frac{\beta m_a^3}{3u^2 M_P L^2}. \tag{C5}$$

To solve this second order differential equation, we consider $u(\phi) = u_0(\phi) + \Delta u(\phi)$, where $u_0(\phi)$ is the solution for the complementary function and $\Delta u(\phi)$ is the solution for a particular integral. Hence, we can write

$$\frac{d^2 u_0}{d\phi^2} + u_0 = 0, \tag{C6}$$

and the solution of Eq. (C6) is $u_0 = \frac{\sin\phi}{b}$, where $b$ is the impact parameter. To find the solution to a particular integral, we can write

$$\frac{d^2 \Delta u}{d\phi^2} + \Delta u = \frac{3M \sin^2\phi}{b^2} + \frac{2\beta \sin\phi}{M_P L^2 b} - \frac{\beta m_a^3 b^2}{3M_P L^2 \sin\phi}. \tag{C7}$$

The solution of Eq. (C7) is

$$\Delta u(\phi) = \frac{3M}{2b^2}\left(1 + \frac{1}{3}\cos 2\phi\right) + \frac{2\beta}{M_P L^2 b}\left(-\frac{\phi\cos\phi}{2}\right) - \frac{\beta m_a^3 b^2}{3M_P L^2}[\cos\phi \ln|\csc\phi + \cot\phi| - 1]. \tag{C8}$$

Hence, the total solution of Eq. (C5) becomes

$$u(\phi) = \frac{\sin\phi}{b} + \frac{3M}{2b^2}\left(1 + \frac{1}{3}\cos 2\phi\right) - \frac{\beta\phi\cos\phi}{M_P L^2 b} - \frac{\beta m_a^3 b^2}{3M_P L^2}[\cos\phi \ln|\csc\phi + \cot\phi| - 1]. \tag{C9}$$

At a far distance from the Sun, $u \to 0$ as $\phi \to 0$. Hence, we can write the change in the angular coordinate as

$$\delta\phi = \frac{-\frac{2M}{b^2} + \frac{\beta m_a^3 b^2}{3M_P L^2} \ln 2}{\frac{1}{b} - \frac{\beta}{M_P L^2 b} + \frac{\beta m_a^3 b^2}{3M_P L^2}}. \tag{C10}$$

From the symmetry argument, we can claim that the contribution to $\delta\phi$ before and after the turning points is the same. Therefore, the total light bending is

$$\Delta\phi = -2\delta\phi = \frac{\frac{4M}{b^2} - \frac{2\beta m_a^3 b^2}{3M_P L^2} \ln 2}{\frac{1}{b} - \frac{\beta}{M_P L^2 b} + \frac{\beta m_a^3 b^2}{3M_P L^2}}. \tag{C11}$$

In the $\beta \to 0$ limit, we obtain $\Delta\phi = \frac{4M}{b}$, which is the standard GR result for gravitational light bending. Hence, we can write the contribution of monopole-dipole potential in gravitational light bending as

$$\Delta\phi_{\text{monopole-dipole}} = \frac{\frac{4M}{b^2} - \frac{2m_a^3 b^2 \ln 2}{3M_P L^2} \frac{g_S g_P N_1 N_2}{4\pi m_e}}{\frac{1}{b} - \frac{1}{M_P L^2 b} \frac{g_S g_P N_1 N_2}{4\pi m_e} + \frac{m_a^3 b^2}{3M_P L^2} \frac{g_S g_P N_1 N_2}{4\pi m_e}} - \frac{4M}{b} + \mathcal{O}((g_S g_P)^2, m_a^4). \tag{C12}$$

We can also write Eq. (C12) as

$$\Delta\phi_{\text{monopole-dipole}} \simeq -\frac{2m_a^3 b^3 g_S g_P N_1 N_2 \ln 2}{3M_P L^2 \times 4\pi m_e} + \frac{g_S g_P N_1 N_2}{M_P L^2 \times 4\pi m_e} \times \frac{4M}{b} - \frac{m_a^3 b^3 g_S g_P N_1 N_2}{3M_P L^2 \times 4\pi m_e} \times \frac{4M}{b} + \mathcal{O}((g_S g_P)^2, m_a^4). \tag{C13}$$

## APPENDIX D: SHAPIRO TIME DELAY IN THE PRESENCE OF A LONG-RANGE MONOPOLE-DIPOLE POTENTIAL

When a radar signal is sent from the Earth to the Venus and the signal reflects from the Venus to the Earth then due to the presence of the Sun between the Earth and the Venus there is a time delay in the round-trip compared to the case if there is no Sun between the Earth and the Venus. We can write Eq. (C4) as

$$\frac{E^2}{2} = \frac{\dot{r}^2}{2} + \frac{L^2}{2r^2}\left(1 - \frac{2M}{r}\right) - \frac{\beta m_a}{M_P r} e^{-m_a r} - \frac{\beta}{r^2 M_P} e^{-m_a r}, \tag{D1}$$

where $\dot{r} = \frac{dr}{d\lambda} = \frac{E}{(1-\frac{2M}{r})} \frac{dr}{dt}$. Therefore, we can write Eq. (D1) as

$$\frac{E^2}{2} = \frac{E^2}{2(1-\frac{2M}{r})^2}\left(\frac{dr}{dt}\right)^2 + \frac{L^2}{2r^2}\left(1 - \frac{2M}{r}\right) - \frac{\beta m_a}{M_P r} e^{-m_a r} - \frac{\beta}{r^2 M_P} e^{-m_a r}. \tag{D2}$$

Let $r = r_0$ be the closest approach of light where $\frac{dr}{dt} = 0$. Putting $r = r_0$ and $\frac{dr}{dt} = 0$ in Eq. (D2) we obtain

$$\frac{L^2}{E^2} = \frac{r_0^2}{(1-\frac{2M}{r_0})}\left[1 + \frac{2\beta}{M_P r_0 E^2}\left(m_a + \frac{1}{r_0}\right) e^{-m_a r_0}\right]. \tag{D3}$$

In the absence of a monopole-dipole potential, Eq. (D3) becomes $\frac{L^2}{E^2} = \frac{r_0^2}{(1-\frac{2M}{r_0})}$. Using Eqs. (D2) and (D3), we can write the time taken by the light to reach from $r_0$ to $r$ as

$$t = \int_{r_0}^{r} \frac{dt}{dr} dr = \int_{r_0}^{r} dr \frac{1}{(1-\frac{2M}{r})}\left[1 - \frac{r_0^2}{r^2}\frac{(1-\frac{2M}{r})}{(1-\frac{2M}{r_0})}(1+\eta) + \frac{2\beta}{M_P r E^2}\left(m_a + \frac{1}{r}\right) e^{-m_a r}\right]^{-\frac{1}{2}}, \tag{D4}$$

where $\eta = \frac{2\beta}{M_P r_0 E^2}(m_a + \frac{1}{r_0}) e^{-m_a r_0}$. The solution of Eq. (D4) in the $r \gg r_0$ limit is

$$t_1 = \sqrt{r^2 - r_0^2} + 2M \ln\left(\frac{2r}{r_0}\right) + M - \frac{\beta}{M_P E^2}\left(\frac{M}{r_0^2} + \frac{1}{r_0}\right) + \frac{\eta r_0}{2}\left(1 + \frac{2M}{r_0}\right). \tag{D5}$$

If $r_e$ denotes the distance between the Sun and the Earth and $r_v$ denotes the distance between the Sun and the Venus, then the total time required for the signal to go from the Earth to the Venus and return to the Earth is

$$T_1 = 2t_1 = 2\left[\sqrt{r_e^2 - r_0^2} + \sqrt{r_v^2 - r_0^2} + 2M \ln\left(\frac{2r_e}{r_0}\right) + 2M \ln\left(\frac{2r_v}{r_0}\right) + 2M - \frac{2\beta M}{M_P E^2 r_0^2} - \frac{2\beta}{M_P E^2 r_0} + \eta r_0\left(1 + \frac{2M}{r_0}\right)\right]. \tag{D6}$$

If there is no massive gravitating object between Earth and Venus, then the total time required for the pulse to go from Earth to Venus and return to Earth is

$$T_2 = 2\left[\sqrt{r_e^2 - r_0^2} + \sqrt{r_v^2 - r_0^2} - \frac{2\beta}{M_P E^2 r_0} + \eta r_0\right]. \tag{D7}$$

Hence, the excess time due to GR correction and monopole-dipole potential is $\Delta T = T_1 - T_2$ and we can write

$$\Delta T = 4M\left[1+\ln\left(\frac{4r_e r_v}{r_0^2}\right)\right] - \frac{4M}{M_P E^2 r_0^2}\left(\frac{g_S g_P N_1 N_2}{4\pi m_e}\right) + \frac{8M}{M_P r_0 E^2}\left(m_a + \frac{1}{r_0}\right)e^{-m_a r_0}\left(\frac{g_S g_P N_1 N_2}{4\pi m_e}\right), \qquad \text{(D8)}$$

where we put the expressions of $\beta$ and $\eta$. If there is no monopole-dipole potential then $g_S g_P \to 0$ and we get back the standard GR contribution in Shapiro time delay as

$$\Delta T_{\mathrm{GR}} = 4M\left[1+\ln\left(\frac{4r_e r_v}{r_0^2}\right)\right]. \qquad \text{(D9)}$$

Using the Earth-Sun distance $r_e = D = 1.46\times 10^{11}$ m $= 7.37\times 10^{26}$ GeV$^{-1}$, the Venus-Earth distance $r_v = 1.08\times 10^{11}$ m $= 5.47\times 10^{26}$ GeV$^{-1}$, and the solar radius $r_0 = R_\odot = 6.96\times 10^8$ m $= 3.51\times 10^{24}$ GeV$^{-1}$, we obtain the GR contribution in Shapiro time delay as $2\times 10^{-4}$ s. Thus the contribution of monopole-dipole potential in Shapiro time delay is

$$\Delta T_{\text{monopole-dipole}} = \frac{8M}{M_P r_0 E^2}\left(m_a + \frac{1}{r_0}\right)e^{-m_a r_0}\left(\frac{g_S g_P N_1 N_2}{4\pi m_e}\right) - \frac{4M}{M_P E^2 r_0^2}\left(\frac{g_S g_P N_1 N_2}{4\pi m_e}\right) + \mathcal{O}((g_S g_P)^2, m_a^2, M^2). \qquad \text{(D10)}$$


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