New method for measuring the ratio $\mu_p G_E/G_M$ based on the polarization transfer from the initial proton to the final electron in the $e\vec{p} \to \vec{e}p$ process

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In this paper, we propose a new method for measuring the Sachs form factors ratio ($R = \mu_p G_E/G_M$) based on the transfer of polarization from the initial proton to the final electron in the elastic $e\vec{p} \to \vec{e}p$ process, in the case when the axes of quantization of spins of the target proton at rest and of the scattered electron are parallel, i.e., when an electron is scattered in the direction of the spin quantization axis of the proton target. To do this, in the kinematics of the SANE Collaboration experiment [A. Liyanage *et al.*, Phys. Rev. C **101**, 035206 (2020).] on measuring double-spin asymmetry in the $\vec{e}\,\vec{p} \to ep$ process, using Kelly [Phys. Rev. C **70**, 068202 (2004).] and Qattan *et al.* [Phys. Rev. C **91**, 065203 (2015).] parametrizations, a numerical analysis was carried out of the dependence of the longitudinal polarization degree of the scattered electron on the square of the momentum transferred to the proton, as well as on the scattering angle of the electron. It is established that the difference in the longitudinal polarization degree of the final electron in the case of conservation and violation of scaling of the Sachs form factors can reach 70%. This fact can be used to set up polarization experiments of a new type to measure the ratio R.

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I. INTRODUCTION

Experiments on the study of electric G_E and magnetic G_M proton form factors, the so-called Sachs form factors (SFF), have been conducted since the mid-1950s [1] in ep elastic scattering of unpolarized electrons off a proton. At the same time, all experimental data on the behavior of SFF were obtained using the Rosenbluth technique (RT) based on the use of the Rosenbluth cross section (in the approximation of the one-photon exchange) for the $ep \to ep$ process in the rest frame of the initial proton [2]. With the help of RT, the dipole dependence of the SFF on the momentum transferred to the proton square Q^2 in the region $Q^2 \le 6$ GeV^2 was established [3,4]. As it turned out, G_E and G_M are related by the scaling ratio $G_M \approx \mu_p G_E$ ($\mu_p = 2.79$ is the magnetic moment of the proton), and for their ratio $R \equiv \mu_p G_E/G_M$, the approximate equality $R \approx 1$ is valid.

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Akhiezer and Rekalo [5] proposed a method for measuring the R ratio based on the phenomenon of polarization transfer from the initial electron to the final proton in the $\vec{e}p \to e\vec{p}$ process. Later this method was generalized in Ref. [6]. Precision JLab experiments [7–9], using this method, found a fairly rapid decrease in the ratio of R with an increase in Q^2, which indicates a violation of the dipole dependence (scaling) of the SFF. In the range 0.4 GeV$^2 \le Q^2 \le 5.6$ GeV^2, as it turned out, this decrease is linear. Next, more accurate measurements of the ratio R carried out in [10–14] in a wide area in Q^2 up to 8.5 GeV^2 using both the Akhiezer-Rekalo method [5] and the RT [14], only confirmed the discrepancy of the results.

In the SANE Collaboration experiment [15], the values of R were obtained by the third method [16] by extracting them from the results of measurements of double-spin asymmetry in the $\vec{e}\,\vec{p} \to ep$ process in the case, when the electron beam and the proton target are partially polarized. The degree of polarization of the proton target was $P_t = (70 \pm 5)\%$. The experiment was performed at two electron beam energies E_1, 4.725 GeV and 5.895 GeV, and two Q^2 values, 2.06 GeV^2 and 5.66 GeV^2. The extracted values of R in [15] are consistent with the results in Refs. [7–13].

In [17–22], the fourth method of measuring R is proposed based on the transfer of polarization from the initial proton to the final one in the $e\vec{p} \to e\vec{p}$ process in the case when their spins are parallel.

In this paper, the fifth method of measuring the ratio of R is proposed based on the transfer of polarization from the initial proton to the final electron in the process $e\vec{p} \to \vec{e}p$ in the case when their spins are parallel, i.e., when the electron is scattered in the direction of the spin quantization axes of the resting proton target.

II. THE HELICITY AND DIAGONAL SPIN BASES

The spin 4-vector $s = (s_0, \boldsymbol{s})$ of the fermion with 4-momentum p ($p^2 = m^2$) satisfying the conditions of orthogonality and normalization, is given by

$$s_0 = \frac{\boldsymbol{cp}}{m}, \qquad \boldsymbol{s} = \boldsymbol{c} + \frac{(\boldsymbol{cp})\boldsymbol{p}}{m(p_0 + m)}, \tag{1}$$

where $\boldsymbol{c}$ is the spin-quantization axis ($\boldsymbol{c}^2 = 1$).

Expressions (1) allow us to determine the spin 4-vector $s = (s_0, \boldsymbol{s})$ by a given 4-momentum $p = (p_0, \boldsymbol{p})$ and 3-vector $\boldsymbol{c}$. On the contrary, if the 4-vector s is known, then the spin quantization axis $\boldsymbol{c}$ is given by

$$\boldsymbol{c} = \boldsymbol{s} - \frac{s_0}{p_0 + m}\boldsymbol{p}. \tag{2}$$

At present, the most popular in high-energy physics is the helicity basis [23], in which the spin quantization axis is directed along the momentum of the particle ($\boldsymbol{c} = \boldsymbol{n} = \boldsymbol{p}/|\boldsymbol{p}|$), while the spin 4-vector reads

$$s = (s_0, \boldsymbol{s}) = (|\boldsymbol{v}|, v_0 \boldsymbol{n}), \tag{3}$$

where v_0 and $\boldsymbol{v}$ are the time and space components of the 4-velocity vector $v = p/m$ ($v^2 = 1$).

For the process under consideration

$$e(p_1) + p(q_1, s_{p_1}) \to e(p_2, s_{e_2}) + p(q_2), \tag{4}$$

where p_1, q_1 (p_2, q_2) are the 4-momenta of the initial (final) electrons and protons with masses m_0 and m, it is possible to project the spins of the initial proton and the final electron in one common direction given by [24,25]

$$\boldsymbol{a} = \boldsymbol{p}_2/p_{20} - \boldsymbol{q}_1/q_{10}. \tag{5}$$

Since the common spin quantization axis (5) defines the spin basis and is the difference of two three-dimensional vectors, the geometric image of which is the diagonal of the parallelogram, it is natural to call it the diagonal spin basis (DSB). In it, the spin 4-vectors of the initial proton s_{p_1} and the final electron s_{e_2} reads

$$s_{p_1} = \frac{m^2 p_2 - (q_1 p_2) q_1}{m\sqrt{(q_1 p_2)^2 - m^2 m_0^2}}, \tag{6}$$

$$s_{e_2} = \frac{(q_1 p_2) p_2 - m_0^2 q_1}{m_0\sqrt{(q_1 p_2)^2 - m^2 m_0^2}}. \tag{7}$$

Note that in the papers [17–22] was used the analogous DSB for the initial and final protons [26,27].

In the laboratory frame (LF), where the initial proton rests, $q_1 = (m, \boldsymbol{0})$, the spin 4-vectors (6) and (7) read

$$s_{p_1} = (0, \boldsymbol{n_2}), \qquad s_{e_2} = (|\boldsymbol{v_2}|, v_{20}\boldsymbol{n_2}), \tag{8}$$

where $\boldsymbol{n_2} = \boldsymbol{p_2}/|\boldsymbol{p_2}|$, $v_2 = (v_{20}, \boldsymbol{v_2}) = p_2/m_0$.

Using the explicit form of the spin 4-vectors (8) and formulas (2) or (5), it is easy to verify that the spin quantization axes of the initial proton and the final electron in the LF coincide with the direction of the final electron momentum

$$\boldsymbol{c}_{p_1} = \boldsymbol{c}_{e_2} = \boldsymbol{n}_2 = \boldsymbol{p_2}/|\boldsymbol{p}_2|. \tag{9}$$

In the case where $p_{10}, p_{20} \gg m_0$, the spin 4-vectors (6) and (7) read

$$s_{p_1} = \frac{m^2 p_2 - (q_1 p_2) q_1}{m(q_1 p_2)}, \qquad s_{e_2} = \frac{p_2}{m_0}. \tag{10}$$

Below, in the ultrarelativistic limit, we present the main kinematic relations used in numerical calculations of polarization effects in the $e\vec{p} \to \vec{e}p$ process in the LF.

III. KINEMATICS

The energies of the final electron E_2 and proton E_{2p} are connected with $Q^2 = -(q_2 - q_1)^2$ as

$$E_2 = E_1 - 2m\tau_p, \quad E_{2p} = m(1 + 2\tau_p), \quad \tau_p = Q^2/4m^2, \tag{11}$$

where E_1 is the energy of the initial electron.

The dependencies of E_2 and Q^2 on the electron scattering angle θ_e in the LF are

$$E_2(\theta_e) = \frac{E_1}{1 + (2E_1/m)\sin^2(\theta_e/2)}, \tag{12}$$

$$Q^2(\theta_e) = \frac{4E_1^2 \sin^2(\theta_e/2)}{1 + (2E_1/m)\sin^2(\theta_e/2)}. \tag{13}$$

In ep elastic scattering an electron can be scattered by an angle of $0° \leqslant \theta_e \leqslant 180°$, while the scattering angle of the proton θ_p varies from 90° to 0°. Possible values of Q^2 lie in the range $0 \leqslant Q^2 \leqslant Q^2_{\max}$, where

$$Q^2_{\max} = \frac{4mE_1^2}{(m + 2E_1)}. \tag{14}$$

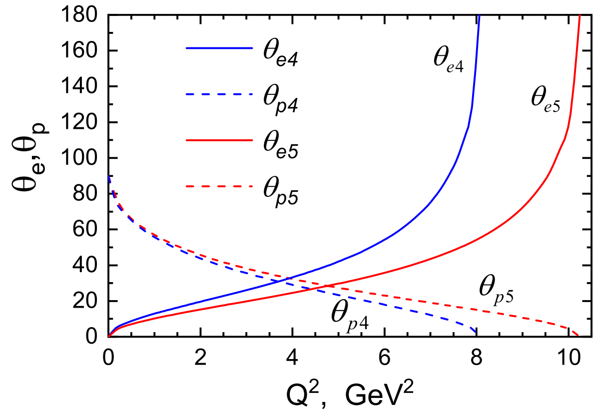

FIG. 1. Q^2 dependence of the scattering angles of the electron θ_e and the proton θ_p (in degrees) at electron-beam energies in the experiment [15]. The lines θ_{e4}, θ_{p4} (θ_{e5}, θ_{p5}) correspond to $E_1 = 4.725$ (5.895) GeV.

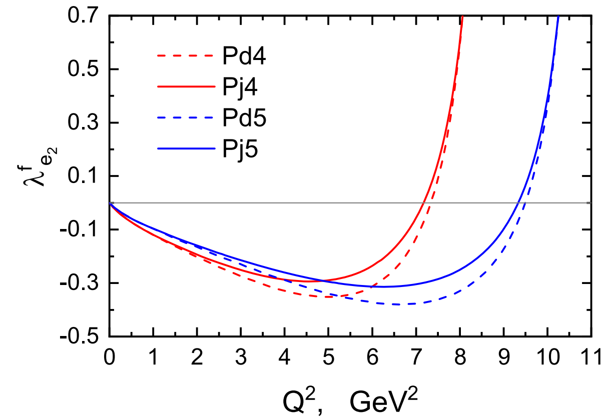

FIG. 2. Q^2 dependence of the longitudinal polarization degree of the scattered electron $\lambda^f_{e_2}$ (15) at electron beam energies in the experiment [15]. The lines $Pd4$, $Pd5$ (dashed), and $Pj4$, $Pj5$ (solid) correspond to the ratio $R = R_d$ and $R = R_j$ (20). The lines $Pd4$, $Pj4$ ($Pd5$, $Pj5$) correspond to the energies $E_1 = 4.725$ (5.895) GeV.

For $E_1 = 4.725$ (5.895) GeV the value of $Q^2_{\max}$ equal to 8.066 (10.247) GeV^2. The results of calculations of the Q^2 dependence of the scattering angles θ_e and θ_p at this electron beam energies are plotted in Fig. 1. They correspond to lines with labels θ_{e4}, θ_{p4} and θ_{e5}, θ_{p5}.

IV. POLARIZATION EFFECTS

In the one-photon exchange approximation, the longitudinal polarization degree transferred from the initial proton to the final electron calculated in the DSB (6), (7) reads

$$\lambda^f_{e_2} = \lambda_{p_1} \frac{\mu_p \tau_p((Y_3/Y_1)R + \mu_p(Y_4/Y_1))}{R^2 + \mu_p^2 \tau_p (Y_2/Y_1)}, \qquad (15)$$

where λ_{p_1} is the initial-proton polarization degree.

In the case, where $p_{10}, p_{20} \gg m_0$, expressions for Y_i ($i = 1, \ldots 4$) in LF read

$$Y_1 = 8m^2(2E_1E_2 - mE_-), \qquad (16)$$

$$Y_2 = 8m^2(E_1^2 + E_2^2 + mE_-), \qquad (17)$$

$$Y_3 = -(2m/E_2)Y_1, \qquad (18)$$

$$Y_4 = 8m^2E_+E_-(m - E_2)/E_2, \qquad (19)$$

where $E_\pm = E_1 \pm E_2$.

The formulas (16)–(19) were used to numerically calculate the Q^2 dependence of the longitudinal polarization degree of the scattered electron $\lambda^f_{e_2}$ (15) as well as the dependence on the scattering angle of the electron at electron beam energies and the polarization degree of the proton target in the experiment [15] as while conserving the scaling of the SFF in the case of a dipole dependence $R = R_d$ ($R_d = 1$), and in case of its violation. In the latter case, the parametrization $R = R_j$ from the paper [28] was used

$$R_j = (1 + 0.1430Q^2 - 0.0086Q^4 + 0.0072Q^6)^{-1}, \qquad (20)$$

and also the parametrization of Kelly ($R = R_k$) from [29], formulas for which we omit. The calculation results are presented by graphs in Figs. 2 and 3. Note that in these figures there are no lines corresponding to the parametrization of Kelly [29] since calculations using R_j and R_k give almost identical results.

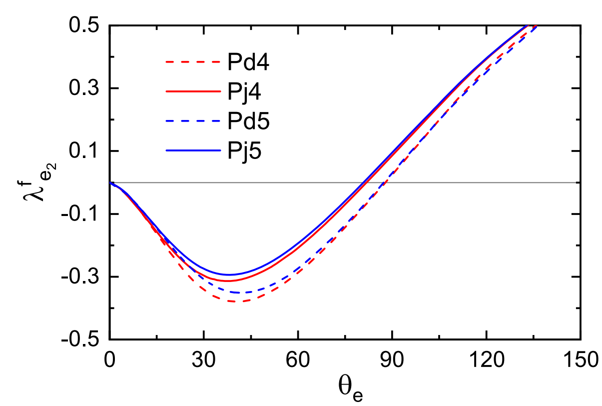

FIG. 3. Angular dependence of the transferred to the electron polarization $\lambda^f_{e_2}$ (15) at electron beam energies used in the experiment [15] on the scattering angle of the electron θ_e, expressed in degrees. The marking of lines $Pd4$, $Pd5$, $Pj4$, and $Pj5$ is the same as in Fig. 2.

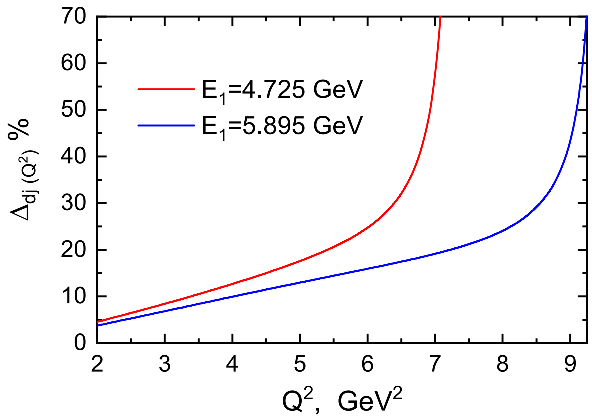

FIG. 4. Q^2 dependence of the relative difference Δ_{dj} (21) at electron beam energies $E_1 = 4.725$ GeV (red line) and $E_1 = 5.895$ GeV (blue line). For all lines, the degree of polarization of the proton target was taken to be the same $P_t = 0.70$.

Q^2 dependence of the longitudinal polarization degree of the scattered electron $\lambda_{e_2}^f$ (15) is plotted in Fig. 2, on which the lines $Pd4$, $Pd5$ (dashed) and $Pj4$, $Pj5$ (solid) are constructed for $R = R_d$ and $R = R_j$ (20). At the same time, the red lines $Pd4$, $Pj4$ and the blue lines $Pd5$, $Pj5$ correspond to the energy of the electron beam $E_1 = 4.725$ and 5.895 GeV. For all lines in Fig. 2 the degree of polarization of the proton target $P_t = 0.70$.

As can be seen from Fig. 2, the function $\lambda_{e_2}^f(Q^2)$ (15) takes negative values for most of the allowed values and has a minimum for some of them. On a smaller part of the allowed values adjacent to $Q^2_{\max}$ and amounting to approximately 9% of $Q^2_{\max}$, it takes on positive values. At the boundary of the spectrum at $Q^2 = Q^2_{\max}$, the polarization transferred to the electron is equal to the polarization of the proton target, $\lambda_{e_2}^f(Q^2_{\max}) = P_t = 0.70$.

Figure 3 shows the angular dependence of the transferred to the electron polarization $\lambda_{e_2}^f$ (15) on the scattering angle of the electron (θ_e), expressed in degrees, at electron beam energies in the experiment [15]. The degree of polarization of the proton target was taken the same for all lines $P_t = 0.70$.

The parametrizations of Qattan [28] and Kelly [29] allow us to calculate the relative difference Δ_{dj} between the polarization effects in the $e\vec{p} \to \vec{e}p$ process in the case of conservation and violation of the SFF scaling, as well as in the effects between these parametrizations Δ_{jk},

$$\Delta_{dj} = \left|\frac{\mathrm{Pd} - \mathrm{Pj}}{\mathrm{Pd}}\right|, \qquad \Delta_{jk} = \left|\frac{\mathrm{Pj} - \mathrm{Pk}}{\mathrm{Pj}}\right|, \tag{21}$$

where P_d, P_j, and P_k are the polarizations calculated by formula (15) for $\lambda_{e_2}^f$ when using the corresponding parametrizations R_d, R_j, and R_k. The results of calculations of Δ_{dj} at electron beam energies of 4.725 and 5.895 GeV are shown in Fig. 4.

It follows from the graphs in Fig. 4 that the relative difference between the polarization transferred from the initial proton to the final electron in the $e\vec{p} \to \vec{e}p$ process in the case of conserving and violation of the scaling of the SFF can reach 70%, which can be used to set up a polarization experiment by measuring the ratio R.

Numerical values of the polarization transferred to the final electron in the $e\vec{p} \to \vec{e}p$ process for the three considered parametrizations of the ratio R at E_1 and Q^2 used in the experiment [15], are presented in Table I. In it, the columns of values P_d, P_j, and P_k correspond to the dipole dependence R_d, parametrizations R_j (20) and R_k [29]; columns Δ_{dj}, Δ_{jk} correspond to the relative difference (21) (expressed in percent) at electron beam energies of 4.725 GeV and 5.895 GeV and two values of Q^2 equal to 2.06 GeV^2 and 5.66 GeV^2. It follows from Table I that the relative difference between $Pj5$ and $Pd5$ at $Q^2 = 2.06$ GeV^2 is 4.1% and between $Pj4$ and $Pd4$ it is 4.8%. At $Q^2 = 5.66$ GeV^2, the difference increases and becomes equal to 14.9% and 21.7%, respectively. Note that the relative difference Δ_{jk} between P_j and P_k for all E_1 and Q^2 in Table I is less than 1%.

V. EXTRACTING OF THE RATIO R

Inverting relation (15), we obtain a quadratic equation with respect to R

$$\alpha_0 R^2 - \alpha_1 R + \alpha_0\alpha_3 - \alpha_2 = 0 \tag{22}$$

with the coefficients

TABLE I. The degree of longitudinal polarization of the scattered electron $\lambda_{e_2}^f$ (15) at E_1 and Q^2 used in the experiment [15]. The values in the columns for P_d, P_j, and P_k correspond to dipole dependence, the parametrization (20) of Qattan [28] and Kelly [29]. The corresponding electron and proton scattering angles (in degrees) are given in columns for θ_e and θ_p.

E_1, GeV	Q^2, GeV^2	$\theta_e(°)$	$\theta_p(°)$	P_d	P_j	P_k	Δ_{dj}, %	Δ_{jk}, %
5.895	2.06	15.51	45.23	−0.170	−0.163	−0.163	4.1	0.0
5.895	5.66	33.57	24.48	−0.363	−0.309	−0.308	14.9	0.3
4.725	2.06	19.97	43.27	−0.207	−0.197	−0.197	4.8	0.0
4.725	5.66	49.50	19.77	−0.336	−0.263	−0.262	21.7	0.6

$$\alpha_0 = \lambda^f_{e_2}/\lambda_{p_1}, \qquad \alpha_1 = \tau_p \mu_p Y_3/Y_1,$$
$$\alpha_2 = \tau_p \mu_p^2 Y_4/Y_1, \qquad \alpha_3 = \tau_p \mu_p^2 Y_2/Y_1. \tag{23}$$

Solutions to Eq. (22) read

$$R = \frac{\alpha_1 \pm \sqrt{\alpha_1^2 - 4\alpha_0(\alpha_0\alpha_3 - \alpha_2)}}{2\alpha_0}. \tag{24}$$

They allow us to extract the ratio R from the results of an experiment to measure the polarization transferred to the electron $\lambda^f_{e_2}$ in the $e\vec{p} \to \vec{e}p$ process.

VI. CONCLUSION

In this paper, we have considered a possible method for measuring the ratio $R \equiv \mu_p G_E/G_M$ based on the transfer of polarization from the initial proton to the final electron in the $e\vec{p} \to \vec{e}p$ process, in the case when their spins are parallel, i.e., when an electron is scattered in the direction of the spin quantization axis of the resting proton target. For this purpose, in the kinematics of the SANE Collaboration experiment [15], using the parametrizations of Qattan [28] and Kelly [29], a numerical analysis was carried out of the dependence of the degree of polarization of the scattered electron on the square of the momentum transferred to the proton, as well as from the scattering angle of the electron. As it turned out, the parametrizations of Qattan [28] and Kelly [29] give almost identical results in calculations.

It is established that the difference in the degree of longitudinal polarization of the final electron in the case of conservation and violation of the SFF scaling can reach 70%, which can be used to conduct a new type of polarization experiment to measure the ratio R.

At present, an experiment to measure the longitudinal polarization degree transferred to an unpolarized electron in the $e\vec{p} \to \vec{e}p$ process seems quite real since a proton target with a high degree of polarization $P_t = (70 \pm 5)\%$ was created in principle and has already been used in the experiment [15]. For this reason, it would be most appropriate to conduct the proposed experiment at the setup used in [15] at the same $P_t = 0.70$, electron beam energies $E_1 =$ 4.725 GeV and 5.895 GeV. The difference between conducting the proposed experiment and the one in [15] consists in the fact that an incident electron beam must be unpolarized, and the detected scattered electron must move strictly along the direction of the spin quantization axis of the proton target. In the proposed experiment, it is necessary to measure only the longitudinal polarization degree of the scattered electron, what is an advantage compared to the method [5] used in JLab experiments.

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