

Dynamically assisted nonlinear Breit-Wheeler pair production in bichromatic laser fields of circular polarization

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Production of electron-positron pairs by a high-energy γ photon and a bichromatic laser wave is considered where the latter is composed of a strong low-frequency and a weak high-frequency component, both with circular polarization. An integral expression for the production rate is derived that accounts for the strong laser mode to all orders and for the weak laser mode to first order. The structure of this formula resembles the well-known expression for the nonlinear Breit-Wheeler process in a strong laser field, but includes the dynamical assistance from the weak laser mode. We analyze the dependence of the dynamical rate enhancement on the applied field parameters and show, in particular, that it is substantially higher when the two laser modes have opposite helicity.

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I. INTRODUCTION

Electron-positron pair production from vacuum by a constant electric field is a genuinely nonperturbative process that was first studied by Sauter in the early days of relativistic quantum mechanics [1]. Later on, Schwinger treated the process within the framework of quantum electrodynamics (QED) and established its famous rate $R \sim \exp(-\pi E_c/E_0)$ that has been named after him [2]. It contains the critical field of QED, $E_c = m^2 c^3 / (e\hbar) \approx 1.3 \times 10^{16}$ V/cm, and exhibits a nonanalytical, manifestly nonperturbative dependence on the applied field strength E_0 . Here, m and e denote the positron mass and charge, respectively. Intuitively, the exponential field dependence indicates a quantum mechanical tunneling from negative-energy to positive-energy states.

Pair production rates in various other strong-field configurations share the characteristic Schwinger-like form (see Refs. [3–6] for reviews). For example, pair production in homogeneous electric fields oscillating in time [7,8] and pair production in combined laser and Coulomb fields via the nonlinear Bethe-Heitler effect [9–11] show exponential dependencies on the inverse field strength as well, provided they occur in a quasistatic regime where the pair formation time is much shorter than the scale of field variations (and the applied fields are subcritical).

Because of the huge value of E_c , Schwinger pair production and its Schwinger-like variants have not been observed experimentally yet [12]. However, motivated by the enormous progress in high-power laser technology, several high-field laboratories are currently aiming at the detection of the fully nonperturbative regime of pair production. They focus, in particular, on the nonlinear Breit-Wheeler process where pairs are created by a high-energy photon colliding with a high-intensity laser wave [13–21], according to $\omega' + n\omega \rightarrow e^+e^-$, with the numbers of absorbed laser photons $n \gg 1$. The corresponding rate in the quasistatic regime ($\xi \gg 1$, $\chi \ll 1$) has the Schwinger-like form $R \sim \exp[-8/(3\chi)]$, where $\chi = 2\xi\hbar^2\omega\omega'/(m^2c^4)$ (assuming counterpropagating beams) denotes the quantum nonlinearity parameter and $\xi = eE_0/(m\omega)$ is the classical laser intensity parameter. The experimental realization of this regime—that would complement the successful observation of nonlinear Breit-Wheeler pair creation in a few-photon regime ($n \sim 5$, $\xi \lesssim 1$) at SLAC in the 1990s [22]—still represents a formidable challenge which is currently on the agenda of several high-field laboratories [23–27].

To facilitate the observation of Schwinger-like pair production, a mechanism termed *dynamical assistance* has been proposed theoretically [28]. It relies on the superposition of a very weak, but highly oscillating assisting field onto a strong (quasi)static background. Energy absorption from the assisting field can largely enhance the pair creation rate, while preserving its nonperturbative character. Dynamically assisted pair production has been studied for various field configurations, comprising the combination of static and alternating electric fields [28–32], static electric and plane-wave photonic fields [33–35] as well as two oscillating electric

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fields with largely different frequencies [36,37], including spatial field inhomogeneities [38,39]. Only few studies revealed moreover the impact of dynamical assistance in the nonlinear Bethe-Heitler [40,41] and Breit-Wheeler processes [42].

In the present paper, we study nonperturbative Breit-Wheeler pair creation with dynamical assistance. To this end, the laser field is composed of a strong low-frequency and a weak high-frequency component. By considering both field components to be circularly polarized and taking the fermion spins into account, we complement and extend the earlier study [42] where dynamically assisted nonlinear Breit-Wheeler pair creation of scalar particles has been considered in two mutually orthogonal laser field modes of linear polarization. An integral representation for the production rate will be derived within the framework of strong-field QED that includes the weak laser mode to leading order and allows to describe the absorption of one high-frequency photon from this mode during the pair production process. We will show that the latter can lead to a very strong dynamical rate enhancement and discuss its dependencies on the applied field parameters. In particular, it will be demonstrated that the circular field polarization offers an interesting additional setting option because the rate enhancement is found to be substantially larger when the two laser modes are counter- rather than corotating.

By suitable adjustments of the experimental setup, our results could be probed in the framework of the ongoing campaigns to detect the nonperturbative Breit-Wheeler process [23–27]. A corresponding observation would constitute a proof of principle of the dynamical enhancement mechanism of strong-field pair production and, as such, an important step towards future experimental studies of the dynamically assisted Schwinger effect.

It is worth mentioning that apart from rate enhancements by dynamical assistance, nonlinear Breit-Wheeler pair production in bichromatic laser fields comprises further interesting effects. In the case when both field modes have commensurate frequencies, characteristic quantum interference effects arise [43–45], whereas multiphoton threshold effects were presented for incommensurate frequencies of similar magnitude [46]. A bichromatic field configuration may, moreover, allow for additional pair production channels involving photon emission processes [47]. And, for the so-called laser-assisted Breit-Wheeler process, i.e. pair creation in the collision of high-frequency (say, γ -ray and x-ray) photons taking place in the presence of a low-frequency background laser field, pronounced redistribution effects in the created particles' phase space have been revealed [48].

Our paper is organized as follows. In Sec. II we present our analytical approach to the problem that is based on the S matrix in the Furry picture employing Dirac-Volkov states for the fermions. An expression for the rate of Breit-Wheeler pair production by absorption of an arbitrary

number of photons from the strong laser mode and a single photon from the weak laser mode is derived. The physical content of this expression is discussed in Sec. III where we illustrate the rate enhancement by dynamical assistance in the nonlinear Breit-Wheeler effect by way of numerical examples. Our conclusions are given in Sec. IV. Relativistic units with $\hbar = c = 4\pi\epsilon_0 = 1$ are used throughout, unless explicitly stated otherwise. Products of four-vectors are denoted as $(ab) = a_\mu b^\mu = a^0 b^0 - \mathbf{a} \cdot \mathbf{b}$ and Feynman slash notation is applied.

II. THEORETICAL APPROACH

In this section we present our analytical treatment of dynamically assisted Breit-Wheeler pair production in a bichromatic laser field. The latter is described by the four-potential

$$A_L^\mu(\tau) = A^\mu(\tau) + \tilde{A}^\mu(\tau) \quad (1)$$

in the radiation gauge and depends on space-time coordinates $x^\mu = (t, \mathbf{r})$ via the phase variable $\tau = (\kappa x) = t - \boldsymbol{\kappa} \cdot \mathbf{r}$, where $\kappa^\mu = (1, \boldsymbol{\kappa})$ describes the uniform wave-propagation direction along a unit vector $\boldsymbol{\kappa}$. The field is composed of the circularly polarized frequency modes

$$\begin{aligned} A^\mu(\tau) &= a[\epsilon_1^\mu \cos(\eta) + \epsilon_2^\mu \sin(\eta)], \\ \tilde{A}^\mu(\tau) &= \tilde{a}[\epsilon_1^\mu \cos(\tilde{\eta} + \tilde{\eta}_\alpha) + \sigma \epsilon_2^\mu \sin(\tilde{\eta} + \tilde{\eta}_\alpha)], \end{aligned} \quad (2)$$

that will be denoted as main mode and assisting mode, respectively, with corresponding frequencies ω and $\tilde{\omega}$ and wave vectors $k^\mu = \omega \kappa^\mu$ and $\tilde{k}^\mu = \tilde{\omega} \kappa^\mu$. The phases accordingly read $\eta = (kx) = \omega\tau$, $\tilde{\eta} = (\tilde{k}x) = \tilde{\omega}\tau$, whereas $\tilde{\eta}_\alpha$ denotes a constant phase shift between the modes. The polarization vectors satisfy $(\kappa \epsilon_i) = 0$, $(\epsilon_i \epsilon_j) = -\delta_{ij}$ for $i, j \in \{1, 2\}$. The helicity of the assisting mode is encoded in the parameter σ : the modes are corotating for $\sigma = +1$ and counter-rotating for $\sigma = -1$. The intensity parameters associated with their amplitudes are $\xi = ea/m$ and $\tilde{\xi} = e\tilde{a}/m$.

A. Pair production amplitude

The S matrix element for nonlinear Breit-Wheeler pair production by a high-energy photon of wave vector $k'^\mu = (\omega', \mathbf{k}')$ and polarization ϵ'^μ in the bichromatic laser field (1) reads

$$S_{\text{fi}} = -ie \sqrt{\frac{2\pi}{V\omega'}} \int d^4x e^{-i(k'x)} \bar{\Psi}_{p',s'}^{(-)} \not{\epsilon}' \Psi_{p,s}^{(+)}, \quad (3)$$

with a normalization volume V . Here, $\Psi_{p',s'}^{(-)}$ and $\Psi_{p,s}^{(+)}$ denote the Volkov states for the created electron, with asymptotic four-momentum p'^μ and spin projection s' , and

the created positron, with asymptotic four-momentum p^μ and spin projection s , respectively. They are given by [15]

$$\Psi_{p,s}^{(\pm)}(x) = \sqrt{\frac{m}{Vq_L^0}} \left(1 \pm \frac{e\cancel{k}A_L}{2(\kappa p)} \right) \begin{Bmatrix} v_{p,s} \\ u_{p,s} \end{Bmatrix} \exp(iS^{(\pm)}), \quad (4)$$

with

$$S^{(\pm)} = \pm(px) + \frac{e}{(\kappa p)} \int^\tau \left[(pA_L(\tau')) \mp \frac{e}{2} A_L^2(\tau') \right] d\tau'.$$

Observe that the normalization constant of the Volkov states is chosen with respect to the effective momentum

$$q_L^\mu = p^\mu + \frac{m^2 \xi_L^2}{2(\kappa p)} \kappa^\mu, \quad (5)$$

involving the total intensity parameter $\xi_L = \sqrt{\xi^2 + \tilde{\xi}^2}$.

With these details in mind, the S matrix becomes

$$S_{\tilde{\text{fi}}} = -ieN_L \int d^4x e^{i(q_L^\mu + q_L^\mu - k^\mu)x_\mu} \bar{u}_{p',s'} M_L v_{p,s} e^{i\Phi_L}, \quad (6)$$

with the normalization factor $N_L = (\frac{m}{Vq_L^0} \frac{m}{Vq_L^0} \frac{2\pi}{V\omega})^{1/2}$, the matrix

$$M_L = \left(1 - \frac{e\cancel{k}A_L}{2(\kappa p')} \right) \not{\epsilon}' \left(1 + \frac{e\cancel{k}A_L}{2(\kappa p)} \right), \quad (7)$$

and the oscillating phase

$$\begin{aligned} \Phi_L &= z \sin(\eta - \eta_0) + \tilde{z} \sin(\tilde{\eta} + \tilde{\eta}_\alpha - \sigma\eta_0) \\ &\quad + \tilde{z}_\alpha \sin(\tilde{\eta} + \tilde{\eta}_\alpha - \sigma\eta). \end{aligned} \quad (8)$$

In the latter, we used $z = ea\sqrt{-Q_L^2}$, $\tilde{z} = e\tilde{a}\sqrt{-\tilde{Q}_L^2}$, and

$$\tilde{z}_\alpha = e^2 a \tilde{a} \left[\frac{1}{(\tilde{k}p) - \sigma(kp)} + \frac{1}{(\tilde{k}p') - \sigma(kp')} \right],$$

with $Q_L^\mu = q_L^\mu/(kq_L) - q_L^\mu/(kq_L)$, $\tilde{Q}_L^\mu = q_L^\mu/(\tilde{k}q_L) - q_L^\mu/(\tilde{k}q_L')$, and the angle η_0 being determined by

$$\cos(\eta_0) = ea \frac{(\epsilon_1 Q_L)}{z}, \quad \sin(\eta_0) = ea \frac{(\epsilon_2 Q_L)}{z}. \quad (9)$$

By virtue of the Volkov states (4), the S matrix (3) contains both modes of the bichromatic laser field to all orders. For our purposes, however, it is possible to simplify

this general expression. Being interested in the Breit-Wheeler process with dynamical assistance, we shall assume from now on that $\tilde{\xi} \ll 1 \lesssim \xi$ and $\omega \ll \tilde{\omega}$ [49]. We may therefore expand the S matrix in powers of the assisting mode amplitude according to [50]

$$S_{\tilde{\text{fi}}} = S_{\tilde{\text{fi}}}^{(0)} + S_{\tilde{\text{fi}}}^{(1)} + S_{\tilde{\text{fi}}}^{(2)} + \dots, \quad (10)$$

where $S_{\tilde{\text{fi}}}^{(j)} \sim \tilde{a}^j$. The zeroth-order $S_{\tilde{\text{fi}}}^{(0)}$ is obtained by setting $\tilde{a} = 0$ in Eq. (6); it coincides with the well-known expression for the nonlinear Breit-Wheeler process in a monochromatic circularly polarized laser wave [13–15]:

$$\begin{aligned} S_{\tilde{\text{fi}}}^{(0)} &= -ieN_{\tilde{\text{fi}}} \sum_{n=-\infty}^{\infty} \bar{u}_{p',s'} M_n v_{p,s} \\ &\quad \times (2\pi)^4 \delta^4(q^\mu + q^\mu - k^\mu - nk^\mu). \end{aligned} \quad (11)$$

The effective momenta q^μ and q'^μ result from Eq. (5) by setting $\tilde{a} = 0$ therein, i.e., $q^{(\prime)\mu} = p^{(\prime)\mu} + \frac{m^2 \xi^2}{2(\kappa p^{(\prime)})} \kappa^\mu$; the normalization factor becomes $N_{\tilde{\text{fi}}} = (\frac{m}{Vq^0} \frac{m}{Vq'^0} \frac{2\pi}{V\omega})^{1/2}$, accordingly. The four-dimensional δ function displays the energy-momentum conservation in the process, and the sum over the number n of photons from the main mode A^μ originates from a Fourier series expansion of the periodic parts in the S matrix, according to the formula $e^{iz \sin(\eta - \eta_0)} = \sum_n J_{-n}(z) e^{-in(\eta - \eta_0)}$ with the ordinary Bessel functions J_n . The matrix M_n will be given in Eq. (13) below. Here and in the following, the zeroth-order contributions are displayed to facilitate a direct comparison with terms involving the dynamical assistance by the weak mode \tilde{A}^μ .

The leading-order contribution $S_{\tilde{\text{fi}}}^{(1)}$ is obtained by collecting the \tilde{A}^μ terms from the electronic and positronic Volkov states contained in Eq. (7) as well as the terms linear in \tilde{z} and \tilde{z}_α stemming from a Taylor expansion of the phase factor $e^{i\Phi_L}$ in Eq. (6). The resulting expression can be decomposed into two terms, $S_{\tilde{\text{fi}}}^{(1)} = S_{\tilde{\text{fi}}}^{(1,+)} + S_{\tilde{\text{fi}}}^{(1,-)}$, with

$$\begin{aligned} S_{\tilde{\text{fi}}}^{(1,\pm)} &= -ieN_{\tilde{\text{fi}}} \sum_{n=-\infty}^{\infty} \bar{u}_{p',s'} (\tilde{\mathcal{M}}_n^\pm \pm \tilde{M}_n^\pm) v_{p,s} \\ &\quad \times (2\pi)^4 \delta^4(q^\mu + q^\mu - k^\mu - nk^\mu \pm \tilde{k}^\mu) \end{aligned} \quad (12)$$

corresponding to the emission ($S_{\tilde{\text{fi}}}^{(1,+)}$) or absorption ($S_{\tilde{\text{fi}}}^{(1,-)}$) of one photon \tilde{k} from the assisting mode. Our approach is illustrated diagrammatically in Fig. 1.

The matrices M_n , \tilde{M}_n^\pm , $\tilde{\mathcal{M}}_n^\pm$ in Eqs. (11) and (12) can be expressed as

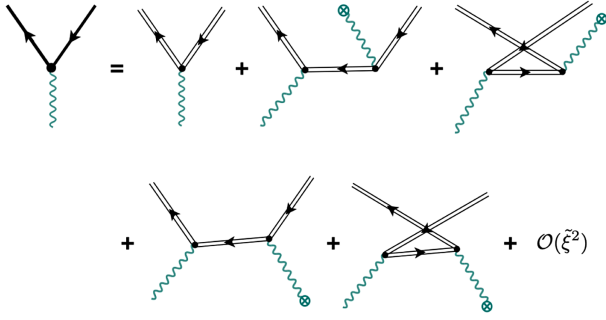


FIG. 1. Diagrammatic representation of the expansion up to linear order in the assisting mode amplitude of the S -matrix element for Breit-Wheeler pair production in the presence of a bichromatic laser field [see Eqs. (10)–(12)]. The wavy leg, common for all graphs, represents a quantized high-energy photon, whereas those starting (ending) with crossed blobs denote an absorbed (emitted) “assisting” photon. The solid arrowed lines on the left-hand side stand for the electron and positron wave functions interacting with the bichromatic background wave. Conversely, the external solid double lines on the right-hand side are the corresponding Volkov states which include the interaction with the main mode only. The internal double lines represent the electron-positron propagators in the field of the main mode. Note that when the amplitude of the main mode vanishes, the leading-order term of the S -matrix element reduces to the well-established contributions linked to the linear Breit-Wheeler process.

$$\left\{ \begin{array}{l} M_n \\ \tilde{M}_n^\pm \\ \tilde{\mathcal{M}}_n^\pm \end{array} \right\} = \left\{ \begin{array}{l} \left(\not{\epsilon}' - \frac{e^2 a^2}{2} \frac{(\epsilon' \kappa) \not{\kappa}}{(\kappa p)(\kappa p')} \right) B_{-n} \\ \left(\not{\epsilon}' - \frac{e^2 a^2}{2} \frac{(\epsilon' \kappa) \not{\kappa}}{(\kappa p)(\kappa p')} \right) \tilde{B}_{-n}^\pm \\ - \frac{e^2 a \tilde{a}}{2} \frac{(\epsilon' \kappa) \not{\kappa}}{(\kappa p)(\kappa p')} B_{-n}^\pm \end{array} \right\} - \left(\frac{\not{\epsilon}_1 \not{\kappa} \not{\epsilon}'}{2(\kappa p')} - \frac{\not{\epsilon}' \not{\kappa} \not{\epsilon}_1}{2(\kappa p)} \right) \left\{ \begin{array}{l} e a C_{-n} \\ e a \tilde{C}_{-n}^\pm \\ e \tilde{a} C_{-n}^\pm \end{array} \right\} - \left(\frac{\not{\epsilon}_2 \not{\kappa} \not{\epsilon}'}{2(\kappa p')} - \frac{\not{\epsilon}' \not{\kappa} \not{\epsilon}_2}{2(\kappa p)} \right) \left\{ \begin{array}{l} e a D_{-n} \\ e a \tilde{D}_{-n}^\pm \\ e \tilde{a} D_{-n}^\pm \end{array} \right\}, \quad (13)$$

with the known coefficients $B_n = J_n(z) e^{-i n \eta_0}$, $C_n = \frac{1}{2}(B_{n-1} + B_{n+1})$, and $D_n = \frac{1}{2i}(B_{n-1} - B_{n+1})$ in the matrix M_n of the ordinary nonlinear Breit-Wheeler process [13–15]. The remaining coefficients

$$\begin{aligned} \tilde{B}_n^\pm &= \frac{1}{2} [\tilde{z} J_n(z) + \tilde{z}_\alpha J_{n \pm \sigma}(z)] e^{-i(n \pm \sigma) \eta_0} e^{\pm i \tilde{\eta}_\alpha} \\ \tilde{C}_n^\pm &= \frac{1}{2} (\tilde{B}_{n-1}^\pm + \tilde{B}_{n+1}^\pm), \quad \tilde{D}_n^\pm = \frac{1}{2i} (\tilde{B}_{n-1}^\pm - \tilde{B}_{n+1}^\pm) \\ \mathcal{B}_n^\pm &= B_{n \pm \sigma} e^{\pm i \tilde{\eta}_\alpha}, \quad \mathcal{C}_n^\pm = \frac{1}{2} B_{n \mp \sigma}^\pm, \quad \mathcal{D}_n^\pm = \pm \frac{\sigma}{2i} B_{n \mp \sigma}^\pm \end{aligned} \quad (14)$$

in the matrices \tilde{M}_n^\pm and $\tilde{\mathcal{M}}_n^\pm$ are associated with the first-order contribution $S_{\text{fi}}^{(1)}$. We note that the coefficients \tilde{B}^\pm , \tilde{C}^\pm , and \tilde{D}^\pm marked by a tilde vanish in the limit $\tilde{a} \rightarrow 0$. The coefficients B^\pm , C^\pm , and D^\pm do not vanish themselves in this limit, but are multiplied by a factor \tilde{a} in the matrix $\tilde{\mathcal{M}}_n^\pm$ [see Eq. (13)].

All terms in $S_{\text{fi}}^{(1, \pm)}$ scale linearly with \tilde{a} . The corresponding contributions to the pair production rate [see Eq. (22) below] will therefore contain an additional factor $\tilde{\xi}^2$ as compared to the ordinary nonlinear Breit-Wheeler process. In case of $S_{\text{fi}}^{(1, -)}$, the reduction by the factor $\tilde{\xi}^2 \ll 1$ is, however, counteracted by the modified energy-momentum balance: due to the absorption of one assisting photon \tilde{k} , the remaining barrier—that has to be overcome by additional photon absorption from the main mode—is lowered, which facilitates the pair production. For suitably chosen field parameters, the latter effect can overcompensate the reduction from the $\tilde{\xi}^2$ scaling, this way leading to enhanced pair production. This is the physical origin of the rate enhancement by dynamical assistance. Conversely, the other first-order term $S_{\text{fi}}^{(1, +)}$, involving the emission of a photon \tilde{k} into the assisting mode, corresponds to an even higher barrier that has to be overcome by photon absorption from the main laser mode. It will give a smaller contribution to the rate than the assistance-free term $S_{\text{fi}}^{(0)}$ and does not play a role for the enhancement effect that we aim for.

B. Pair production rate

From the S matrix we obtain the production rate per incident γ photon by taking the absolute square, summing over the produced particle spins, integrating over their momenta, averaging over the γ -photon polarizations, and dividing out the interaction time:

$$R = \int \frac{1}{2} \sum_{\lambda'} \sum_{s, s'} \frac{|S_{\text{fi}}|^2 V d^3 q V d^3 q'}{T (2\pi)^3 (2\pi)^3}. \quad (15)$$

The production rate R accordingly refers to a scenario where the incident beam of γ photons is unpolarized.

In general, the absolute square of S_{fi} from Eq. (10) contains—apart from diagonal terms—also cross terms that describe interferences between different contributions. In particular, the cross term of $S_{\text{fi}}^{(0)}$ with $S_{\text{fi}}^{(1)}$ leads to rate contributions linear in \tilde{a} . These terms would exist and could cause interesting effects if the ratio of $\tilde{\omega}$ and ω was an integer (e.g. $\tilde{\omega} = 2\omega$). Such two-color quantum interference effects have already been studied elsewhere [43–45]; they are not of interest in the current consideration. To be specific, we shall assume in the following that the frequency ratio $\tilde{\omega}/\omega$ is not an integer. Then the cross terms between $S_{\text{fi}}^{(0)}$ and $S_{\text{fi}}^{(1)}$ vanish identically because the associated δ functions in Eqs. (11) and (12) cannot be

satisfied simultaneously. By requiring more strictly that $2\tilde{\omega}/\omega$ is not an integer either, we can moreover exclude interferences between $S_{\text{fi}}^{(1,+)}$ and $S_{\text{fi}}^{(1,-)}$ as well as between $S_{\text{fi}}^{(0)}$ and those terms in $S_{\text{fi}}^{(2)}$ which describe the absorption (or emission) of two photons \tilde{k} [42]. And by finally imposing the stricter condition

$$6\tilde{\omega}/\omega \notin \mathbb{N}, \quad (16)$$

also interference terms of order \tilde{a}^3 between $S_{\text{fi}}^{(0)}$ and $S_{\text{fi}}^{(3)}$ as well as between $S_{\text{fi}}^{(1)}$ and $S_{\text{fi}}^{(2)}$ drop out.

Under this assumption, the squared S matrix becomes

$$|S_{\text{fi}}|^2 = |S_{\text{fi}}^{(0)}|^2 + |S_{\text{fi}}^{(1,+)}|^2 + |S_{\text{fi}}^{(1,-)}|^2 + \mathcal{O}(\tilde{a}^4), \quad (17)$$

with the terms $|S_{\text{fi}}^{(1,\pm)}|^2$ being of order \tilde{a}^2 . We note that the first term $|S_{\text{fi}}^{(0)}|^2$ on the right-hand side of Eq. (17) comprises the \tilde{a} -independent contribution $|S_{\text{fi}}^{(0)}|^2$ along with $\mathcal{O}(\tilde{a}^2)$ corrections to it. They stem from interferences between $S_{\text{fi}}^{(0)}$ and second-order terms in $S_{\text{fi}}^{(2)}$ associated with the simultaneous absorption and emission of an assisting photon \tilde{k} [51]. Leading to the same energy-momentum balance as in Eq. (11), these emission-absorption processes are strongly suppressed as compared with $S_{\text{fi}}^{(0)}$, since they scale with $\tilde{\xi}^2 \ll 1$ but do not lower the pair production barrier to be overcome by photon absorption from the main mode. They can therefore be safely neglected. In contrast, the dynamical assistance described by $|S_{\text{fi}}^{(1,-)}|^2$ can largely dominate over the $|S_{\text{fi}}^{(0)}|^2$ term, as will be demonstrated by numerical examples in Sec. III.

The spin summation and polarization average in Eq. (15) can be carried out in the usual way by taking traces over the involved Dirac γ matrices. The result for the ordinary nonlinear Breit-Wheeler process is

$$\begin{aligned} & \frac{1}{2} \sum_{\lambda'} \sum_{s,s'} |\bar{u}_{p',s'} M_n v_{p,s}|^2 \\ &= |B_{-n}|^2 + \xi^2 \left[(|B_{-n}|^2 - |C_{-n}|^2 - |D_{-n}|^2) \right. \\ & \quad \left. \times \left(1 - \frac{(\kappa k')^2}{2(\kappa p)(\kappa p')} \right) \right], \end{aligned} \quad (18)$$

whereas for the leading order in \tilde{a} terms we obtain

$$\begin{aligned} & \frac{1}{2} \sum_{\lambda'} \sum_{s,s'} |\bar{u}_{p',s'} (\tilde{\mathcal{M}}_n^\pm \pm \tilde{M}_n^\pm) v_{p,s}|^2 \\ &= |\tilde{B}_{-n}^\pm|^2 + \xi^2 \left[\left(|\tilde{B}_{-n}^\pm|^2 \pm \Re[\mathcal{B}_{-n}^\pm (\tilde{B}_{-n}^\pm)^*] \right) \frac{\tilde{a}}{a} \right. \\ & \quad \left. - \left| C_{-n}^\pm \frac{\tilde{a}}{a} \pm \tilde{C}_{-n}^\pm \right|^2 - \left| D_{-n}^\pm \frac{\tilde{a}}{a} \pm \tilde{D}_{-n}^\pm \right|^2 \right] \\ & \quad \times \left(1 - \frac{(\kappa k')^2}{2(\kappa p)(\kappa p')} \right). \end{aligned} \quad (19)$$

By performing afterwards the integrations over the particle momenta \mathbf{q} and \mathbf{q}' , we obtain the corresponding contributions to the pair production rate,

$$R = R^{(0)} + R^{(1,+)} + R^{(1,-)} + \mathcal{O}(\tilde{\xi}^4). \quad (20)$$

The well-established zeroth-order contribution reads [13–15]

$$\begin{aligned} R^{(0)} &= \frac{\alpha m^2}{4\omega'} \sum_{n \geq n_0}^\infty \int_1^{u_n} \frac{du}{u\sqrt{u(u-1)}} [2J_n^2 + \xi^2 \\ & \quad \times (J_{n+1}^2 + J_{n-1}^2 - 2J_n^2)(2u-1) + \mathcal{O}(\tilde{\xi}^2)], \end{aligned} \quad (21)$$

where correction terms of order \tilde{a}^2 from combined emission-absorption photon-exchange processes with the assisting mode that do not change the four-momentum balance have been neglected. Here, $\alpha = e^2$ is the fine-structure constant and $n_0 = 4m_*^2/s$ the photon number threshold, with the effective fermion mass $m_* = m\sqrt{1 + \xi^2}$ dressed by the main laser mode and the Mandelstam variable $s = 2(kk')$. Besides, the upper integration limit is $u_n = n/n_0$, and the Bessel functions $J_\nu = J_\nu(z)$ depend on the argument $z = (8m^2/s)\xi\sqrt{1 + \xi^2}\sqrt{u(u_n - u)}$.

For the leading-order contribution with respect to \tilde{a} , which involves the absorption or emission of one photon \tilde{k} from the assisting mode, we find

$$\begin{aligned} R^{(1,\pm)} &= \frac{\alpha m^2}{4\omega'} \sum_{n \geq n_0^\pm}^\infty \int_1^{u_{n^\pm}} \frac{du}{u\sqrt{u(u-1)}} \left[\frac{1}{2} (\tilde{z}^{\pm 2} J_n^2 + \tilde{z}_\alpha^2 J_{n\mp\sigma}^2 - 2\tilde{z}^\pm \tilde{z}_\alpha J_n J_{n\mp\sigma}) \right. \\ & \quad + \frac{1}{4} (\xi^2 [\tilde{z}^{\pm 2} (J_{n+1}^2 + J_{n-1}^2 - 2J_n^2) + \tilde{z}_\alpha^2 (J_n^2 + J_{n\mp 2\sigma}^2 - 2J_{n\mp\sigma}^2) - 2\tilde{z}^\pm \tilde{z}_\alpha (J_{n\pm\sigma} J_n + J_{n\mp\sigma} J_{n\mp 2\sigma} - 2J_n J_{n\mp\sigma})] \\ & \quad \left. - 4\tilde{\xi} \tilde{\xi} [\tilde{z}^\pm J_n (J_{n+\sigma} - J_{n-\sigma}) \mp \tilde{z}_\alpha (J_n^2 - J_{n\mp\sigma}^2)] + 4\tilde{\xi}^2 J_n^2 (2u-1) \right], \end{aligned} \quad (22)$$

with

$$\begin{aligned} n_0^\pm &= n_0 \pm \frac{\tilde{s}}{s}, & u_{\tilde{n}^\pm} &= \frac{ns \mp \tilde{s}}{n_0^\pm s \mp \tilde{s}}, \\ \tilde{s} &= 2(\tilde{k}k'), & \tilde{z}_\alpha &= \frac{8m^2}{\tilde{s} - \sigma s} \xi \tilde{\xi} u, \\ \tilde{z}^\pm &= \frac{8m^2}{\tilde{s}} \tilde{\xi} \sqrt{1 + \xi^2} \sqrt{u(u_{\tilde{n}^\pm} - u)}, \end{aligned} \quad (23)$$

and the Bessel functions $J_\nu = J_\nu(z^\pm)$ depending on the argument $z^\pm = (8m^2/s)\xi\sqrt{1 + \xi^2}\sqrt{u(u_{\tilde{n}^\pm} - u)}$. We note that the constant phase shift $\tilde{\eta}_\alpha$ between the laser modes has dropped out from the pair production rate.

Equation (22) constitutes the main result of our paper. While being somewhat more involved, its general structure—containing an integral over the variable u that is related to the polar emission angle of the created particles, and a sum over the number of photons absorbed from the strong main laser mode—closely resembles the rate expression (21) for the ordinary nonlinear Breit-Wheeler process. The important difference is that our formula for $R^{(1,-)}$ (or $R^{(1,+)}$) accounts for the absorption (or emission [47]) of an additional photon from the assisting weak laser mode. Accordingly, $R^{(1,-)}$ describes nonlinear Breit-Wheeler pair production in circularly polarized laser fields proceeding via the absorption of many low-frequency photons from the main field mode and a single high-frequency photon from the assisting mode, which may have either equal or opposite helicity as the main mode. The dynamical assistance provided by the weak high-frequency mode can largely enhance the pair production rate $R^{(1,-)}$ as compared with $R^{(0)}$.

Before moving on to the next section we note that in the limit when the main laser mode vanishes ($\xi \rightarrow 0$) while the assisting laser mode has low amplitude ($\tilde{\xi} \ll 1$) and sufficiently high frequency (such that $\tilde{s} > 4m^2$), the expression for $R^{(1,-)}$ reproduces the rate for the original Breit-Wheeler process [52] of pair production by two photons (see also Fig. 1).

III. NUMERICAL RESULTS AND DISCUSSION

In this section we illustrate our findings on the dynamically assisted nonlinear Breit-Wheeler process in a bichromatic laser field by numerical examples.

The main mode intensity parameter is chosen as $\xi \approx 1$, lying in the nonperturbative multiphoton domain. Already for such moderate ξ values, the rate for nonlinear Breit-Wheeler pair production is known to closely resemble the Schwinger-like form given in the Introduction, which emerges asymptotically for $\xi \gg 1$ and $\chi \ll 1$ [53]. While closed analytical rate formulas for $\xi \approx 1$ are very difficult to obtain, a numerical treatment of this parameter regime is

well feasible and insightful. The intensity parameter of the assisting laser mode is taken as $\tilde{\xi} \sim 10^{-3}$.

We assume that the γ beam and bichromatic laser wave are counterpropagating, and perform our calculations in a frame of reference where the γ -beam and assisting laser mode frequencies are close to each other, $\omega' \approx \tilde{\omega}$ (in most cases they will exactly coincide, $\omega' = \tilde{\omega}$). This frame offers the advantage that the remaining energy gap for pair production, which has to be overcome by photon absorption from the main laser mode, becomes directly apparent. Specifically, we chose the photon frequencies as $\omega' = 0.706m$, $\tilde{\omega} \lesssim 0.706m$ and $\omega \approx 0.05m$.

By noting that the pair production rates shown below can be translated straightforwardly into another frame of reference, the chosen field parameters can be related to those of present experimental campaigns. Under a Lorentz boost with reduced velocity β and Lorentz factor γ along the laser propagation direction, the intensity parameters ξ and $\tilde{\xi}$ remain invariant, whereas the photon frequencies in the boosted frame amount to

$$\omega_b = \frac{\omega}{(1 + \beta)\gamma}, \quad \tilde{\omega}_b = \frac{\tilde{\omega}}{(1 + \beta)\gamma}, \quad \omega'_b = (1 + \beta)\gamma\omega',$$

i.e., the products $\omega\omega'$ and $\tilde{\omega}\omega'$ remain invariant.

For instance, for $\gamma = 8500$, one obtains $\omega_b \approx 1.5$ eV and $\omega'_b \approx 6$ GeV which agree with the typical parameter domains of the planned experiments on nonperturbative Breit-Wheeler pair production [23–27]. The assisting mode frequency $\tilde{\omega}_b \approx 21$ eV in this case lies in the ultraviolet domain.

The total pair production rate changes under the Lorentz boost by just an overall factor, according to

$$R_b(\omega_b, \tilde{\omega}_b, \omega'_b) = \frac{R(\omega, \tilde{\omega}, \omega')}{(1 + \beta)\gamma}. \quad (24)$$

As a consequence, the shapes of the curves shown below remain unaltered in the boosted frame; only their absolute height may change by a common factor. Moreover, rate ratios are invariant even in absolute terms.

In the following figures we present the rate $R_{\text{DA}} := R^{(0)} + R^{(1,+)} + R^{(1,-)}$ from Eqs. (20)–(22) for nonlinear Breit-Wheeler pair creation in a bichromatic laser field, including the effect of dynamical assistance. Two variants of this rate exist, depending on whether the two laser modes have equal or opposite helicities. In the figures, the corresponding rates are denoted suggestively as $R_{\text{DA}}(\sigma)$, with $\sigma = +1$ and $\sigma = -1$, respectively. Comparing them allows us to reveal the influence of the wave helicities on the exerted dynamical assistance.

The rates $R_{\text{DA}}(\sigma)$ in a bichromatic laser field will moreover be compared with the corresponding “monochromatic rates” when only one of the two laser modes is present, in order to quantify the enhancement effect.

This is, first of all, the rate $R^{(0)}$ for the ordinary nonlinear Breit-Wheeler process where the assisting mode is absent ($\tilde{\xi} = 0$); it is denoted as R_{BW} in the figures. Besides, the rate for nonlinear Breit-Wheeler pair creation by the γ beam and the assisting mode alone, when the strong field is switched off ($\xi = 0$) forms a second reference, denoted by \tilde{R}_{BW} . We note that for the chosen parameters, at least three $\tilde{\omega}$ photons need to be absorbed in the latter scenario to overcome the pair creation threshold.

A. Enhanced pair creation by dynamical assistance

Figure 2 shows the contributions to the pair creation rate stemming from the absorption of n laser photons from the main mode. Already here a pronounced rate enhancement through the dynamical assistance by the weak laser mode becomes apparent, as the contributions to $R_{\text{DA}}(\sigma)$ are much larger than those to R_{BW} . They are shifted besides to smaller photon numbers because a part of the four-momentum required for pair creation already comes from the absorption of the high-frequency photon from the weak mode. Note that the photon-number distributions in Fig. 2 are shifted to values substantially higher than the photon-number thresholds of $n_0 \approx 56.7$ and $n_0^- \approx 42.5$ [see below Eqs. (21) and (22)], respectively, which is a characteristic above-threshold feature of pair creation at $\xi \gtrsim 1$. Furthermore, a comparison of $R_{\text{DA}}(\sigma = +1)$ and $R_{\text{DA}}(\sigma = -1)$ shows an impact of the mode helicities: the number distribution for counter-rotating laser modes reaches larger maximum values and is slightly shifted to the left.

By summing the rate contributions over the number of absorbed strong-field photons, the corresponding total pair creation rates are obtained. They are shown in Fig. 3,

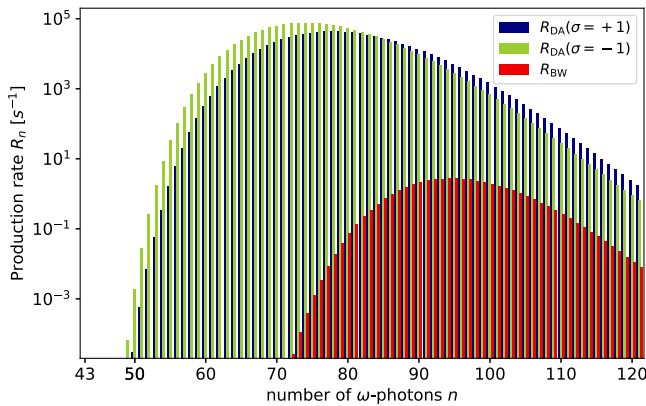


FIG. 2. Contributions to the pair creation rate in dependence on the number of photons absorbed from the main laser mode. The blue (dark gray) and green (light gray) bars refer to a bichromatic laser wave with corotating and counter-rotating modes, respectively, whereas the red (gray) bars show the unassisted case, as indicated in the legend. The parameters are $\xi = 1$, $\tilde{\xi} = 10^{-3}$, $\omega = 0.05m$, and $\tilde{\omega} = \omega' = 0.706m$.

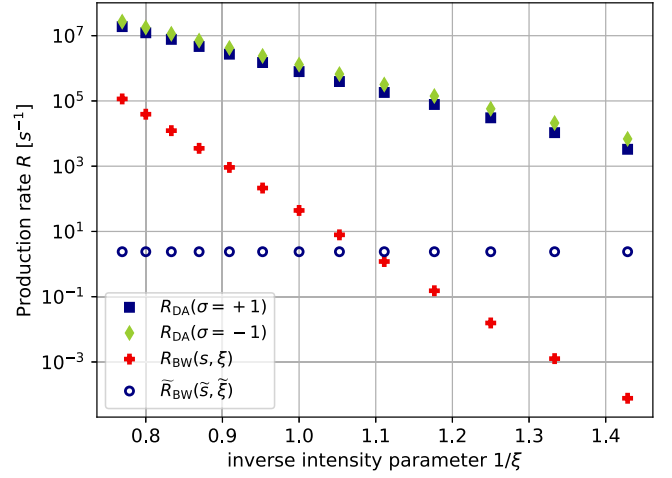


FIG. 3. Total rates for dynamically assisted Breit-Wheeler pair creation in a bichromatic laser wave, as function of the inverse intensity parameter of the main mode; blue squares (green diamonds) refer to corotating (counter-rotating) laser modes. The parameters are the same as in Fig. 2. The unassisted case ($\tilde{\xi} = 0$) is displayed by red crosses. Blue circles show the rate when instead the main mode is absent ($\xi = 0$).

as function of the inverse value of the main mode intensity parameter. In the chosen logarithmic representation, the dynamically assisted rates $R_{\text{DA}}(\sigma = \pm 1)$ and the unassisted rate R_{BW} follow to a very good approximation declining straight lines, illustrating their Schwinger-like exponential dependence on $-b/\xi$, with some process-specific parameter b . In addition, the rate \tilde{R}_{BW} for pair creation by γ beam and assisting mode is included for reference along a horizontal line.

In the chosen range of parameters, the rates for dynamically assisted nonlinear Breit-Wheeler pair creation lie far above the monochromatic rates. The relative enhancement is largest close to $1/\xi \approx 1.1$ (i.e. $\xi \approx 0.9$) and amounts to almost five orders of magnitude. The decline of the curves for assisted pair creation is much slower than for the unassisted process: by fitting our data to a Schwinger-like exponential, we obtain $b_0 \approx 35.0$ for the unassisted process [54], while $b_1 \approx 14.0$ (13.3) for the assisted process with $\sigma = +1$ ($\sigma = -1$). The reduced slope arises because the absorption of the high-frequency photon from the weak mode reduces the tunneling barrier that remains to be overcome [28,40–42]. For comparison we note that from a basic consideration within a quantum mechanical tunneling model (see Eq. (14) in [42]), one would obtain

$$b_1^{(\text{tun})} = \frac{4\sqrt{2}}{3} \frac{m}{\omega} \left(\frac{2m\Delta}{m + \tilde{\omega}} \right)^{3/2}, \quad (25)$$

with $\Delta = 1 - \tilde{\omega}\omega'/m^2$, which attains the value $b_1^{(\text{tun})} \approx 17$ for the present parameters.

Figure 3 shows moreover that the effect of dynamical assistance is stronger when the two laser modes have opposite helicity. This is remarkable because (i) the intuitive picture of dynamical assistance relies on the fact that the additional energy absorbed from the assisting field facilitates to overcome the pair creation threshold and (ii) the four-momentum of the assisting laser photon is the same for $\sigma = \pm 1$. Hence, the reduction of the tunneling-energy barrier by absorption of a high-frequency $\tilde{\omega}$ photon occurs independently of its helicity.

Mathematically, the helicity dependence of the dynamically assisted rate $R^{(1,-)}$ is attributable to terms involving $J_{n+\sigma}$ or $J_{n+2\sigma}$ in Eq. (22). They lead to larger pair production for counter-rotating laser modes ($\sigma = -1$) than for corotating laser modes ($\sigma = +1$). The appearance of such a helicity dependence can be associated with products of the form $A_\mu \tilde{A}^\mu$ which arise in Eq. (10) from the expanded Volkov states and which, setting $\tilde{\eta}_\alpha = 0$, are proportional to $\text{Re}(e^{i(k_\mu - \sigma \tilde{k}_\mu)x^\mu})$. These phase factors oscillate faster for $\sigma = -1$ and thus facilitate the pair production more effectively in that case.

We note in this context that from the different values of the fit parameter b_1 obtained for co- and counter-rotating laser waves one cannot draw the conclusion that the Schwinger-like tunneling exponent of the dynamically assisted rate depends on the wave polarization. The reason is that our fitting procedure does not take the variability of the preexponential factor with the field parameters into account. This factor may, however, contain the helicity dependence of the rate.

Physically, the more pronounced enhancement effect of an assisting mode with opposite helicity can be explained by angular-momentum conservation. The circularly polarized laser photons in our scenario carry definite angular momentum along the propagation axis of $+\hbar$ or $-\hbar$. When the modes corotate, the angular momentum of each absorbed photon points in the same direction. In contrast, when the modes counter-rotate, the absorption of the assisting photon reduces the total angular momentum of all absorbed photons. Accordingly, for fixed strong-field photon number n , the angular momentum that is transferred to the created pair amounts to $(n + \sigma)\hbar$.

In a semiclassical picture, the orbital angular momentum of the electron and positron is determined by their relative momentum. In case of the unassisted Breit-Wheeler process, it amounts to $\ell \approx 2[n_0(n - n_0)]^{1/2}\hbar$ [53]. This quantity is bounded from above according to $\ell \leq n\hbar$. Large contributions to the production rate can be expected when ℓ approximately balances the angular momentum of the absorbed photons [53].

In case of the assisted Breit-Wheeler process, we have to demand $\ell \approx (n + \sigma)\hbar$ with $\ell \approx 2[n_0^-(n - n_0^-)]^{1/2}\hbar$, accordingly. For counter-rotating waves, the condition $\ell = (n - 1)\hbar$ is very well met for $n \approx 73$ in Fig. 2 and, indeed, exactly in this region of photon numbers we find

the highest rate contributions. For corotating waves, however, the condition $\ell = (n + 1)\hbar$ cannot be satisfied because ℓ is at most $n\hbar$. In this case, the angular-momentum balance can only be fulfilled when both particle spins are oriented along the laser propagation direction, this way providing an extra contribution of one unit of \hbar to the total angular momentum of the pair. Such an additional constraint is absent for counter-rotating waves, so that the accessible spin space is larger in this case. As a result, the total pair production rate is higher when the modes counter-rotate [55,59].

Our semiclassical consideration also explains the horizontal shift between the number distributions for $\sigma = -1$ and $\sigma = +1$ in Fig. 2. Since in the latter case, $\ell \approx n\hbar$ is favorable, the region of highest rate contributions moves to larger n values than in the former case. We note that asymptotically for $\xi \gg 1$, one would expect the highest rates at $n \approx 2n_0^- \approx 85$ for corotating waves [53].

The larger effectiveness of an assisting photon of opposite helicity is also seen in Fig. 4, displaying the relative enhancement due to dynamical assistance as compared with the sum of the monochromatic rates. At $\xi \approx 0.9$, the pair creation rate for counter-rotating modes is almost twice as large as for corotating modes. The bell-shaped form of the relative enhancement curves is a consequence of the rate dependencies shown in Fig. 3. The curves reach their maximum close to the point where the monochromatic rates R_{BW} and \tilde{R}_{BW} cross. For smaller values of $1/\xi$, the relative enhancement is reduced due to the different slopes of the dynamically assisted rates $R_{\text{DA}}(\sigma)$ as compared with the unassisted rate R_{BW} .

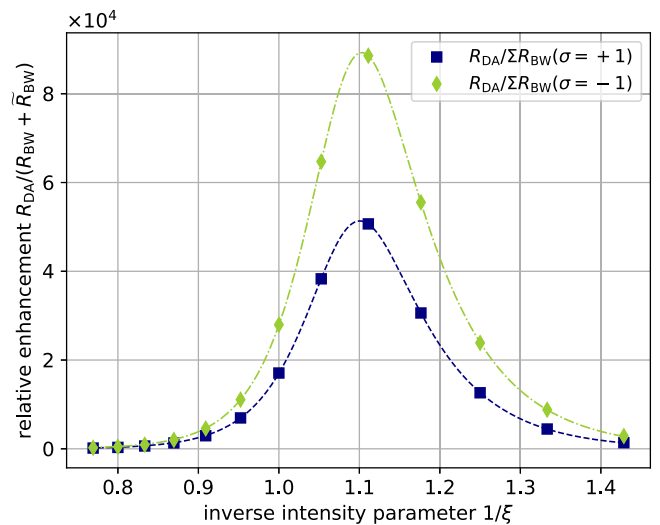


FIG. 4. Relative enhancement of the dynamically assisted Breit-Wheeler process in a bichromatic laser field over the sum of the respective monochromatic rates. Blue squares (green diamonds) refer to corotating (counter-rotating) laser modes. The parameters are the same as in Fig. 2.

For larger values of $1/\xi$, it is reduced as well, since the rates $R_{\text{DA}}(\sigma)$ are falling while \tilde{R}_{BW} is constant.

The dashed lines in Fig. 4 show fit functions of the form $[c_1 \exp(-b_1/\xi) + \exp(-b_0/\xi)]/[c_0 + \exp(-b_0/\xi)]$, with four free parameters b_0 , b_1 , c_0 and c_1 for $\sigma = \pm 1$ each. While in the first place serving to guide the eye, these fit functions have a physically motivated form, relying on the Schwinger-like exponential ξ dependencies of the assisted and unassisted nonlinear Breit-Wheeler rates and the ξ independency of \tilde{R}_{BW} . One obtains here $b_0 \approx 32.4$ and $b_1 \approx 12.7$ for $\sigma = +1$, whereas $b_1 \approx 12.1$ for $\sigma = -1$.

B. Parameter study of relative enhancement

For the field parameters considered in the previous section, we found a relative enhancement of the dynamically assisted rates $R_{\text{DA}}(\sigma)$ over the sum of the monochromatic rates R_{BW} and \tilde{R}_{BW} up to about 5×10^4 for $\sigma = +1$ and 9×10^4 for $\sigma = -1$ (see Fig. 4). In the following we will study the dependencies of the relative enhancement due to dynamical assistance on the applied parameters.

Figure 5 shows the relative enhancement for various values of the weak mode intensity parameter in the interval $5 \times 10^{-4} \leq \tilde{\xi} \leq 5 \times 10^{-3}$. The dashed green lines with diamond symbols refer to $\tilde{\xi} = 10^{-3}$, as considered in Sec. III A. One sees that the maxima of the relative enhancement curves shift to larger values of $1/\xi$ and increase in magnitude, when $\tilde{\xi}$ decreases. These trends can be understood by taking reference to Fig. 3 and noting that the dominant contributions $R^{(1,-)}$ to the dynamically assisted rates $R_{\text{DA}}(\sigma)$ scale with $\tilde{\xi}^2$, while the monochromatic rate \tilde{R}_{BW} scales more strongly with $\tilde{\xi}^6$; the unassisted rate R_{BW} remains unaltered when $\tilde{\xi}$ is varied. Accordingly, when the value of $\tilde{\xi}$ is reduced, the rate \tilde{R}_{BW} decreases much more strongly than the rates $R_{\text{DA}}(\sigma)$, while R_{BW} stays the same (see Fig. 3). The position of the maximum relative enhancement thus shifts to the right towards larger values of $1/\xi$ and grows in magnitude. We note, moreover, that the $\tilde{\xi}$ dependence for corotating and counter-rotating laser modes in Figs. 5(a) and 5(b), respectively, has very similar appearance.

In Fig. 6, the relative enhancement is displayed when the main mode frequency is varied in the range $0.04m \leq \omega \leq 0.06m$. The maxima of the curves decrease and move towards larger values of $1/\xi$ when ω grows. That the relative enhancement reaches higher values when ω is small can be attributed to the growing ratio of $\tilde{\omega}/\omega$ so that a single high-frequency photon corresponds to an increasing number of low-frequency photons. Interestingly, the product $\omega\tilde{\xi}$ attains an approximately constant value at all curve maxima, corresponding to an electric field strength of the main mode of $E \approx 0.045E_c$. Thus, close to this field strength, the dynamical assistance is most efficient in the present scenario.

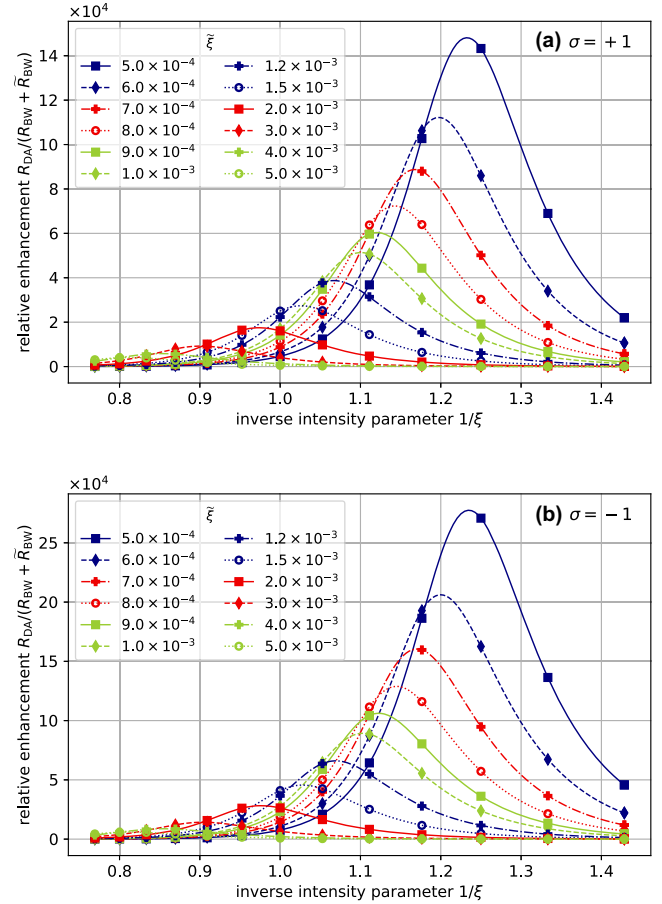


FIG. 5. Relative rate enhancement due to dynamical assistance for different values of the weak mode intensity parameter. The γ -photon and laser frequencies are the same as in Fig. 2. Panels (a) and (b) refer to co- and counter-rotating laser modes, respectively.

The influence of the main mode frequency is more pronounced for corotating laser modes [see Fig. 6(a)]. The maximum relative enhancement decreases from left to right by about 40%, reaching 6.4×10^4 for $\omega = 0.04m$ and 4.0×10^4 for $\omega = 0.06m$. In contrast, the relative enhancement for counterpropagating laser modes in panel (b) decrease only by about 20%, from nearly 10^5 for $\omega = 0.04m$ to 7.8×10^4 for $\omega = 0.06m$.

Very interesting structures arise when the relative enhancement is considered under variation of the assisting mode frequency $\tilde{\omega}$. The results are shown in Fig. 7 within the interval $0.706m \geq \tilde{\omega} \geq 0.354m$. Note that the figure legend contains selected $\tilde{\omega}$ values in order to not overload it. The top curve on the left corresponds to the frequency $\tilde{\omega} = 0.706m$ that has been considered so far. When this value is lowered, the relative enhancement curves go down and slightly shift to the right until $\tilde{\omega} \approx 0.5m$ is reached. Their decrease is due to the fact that the reduction of the tunneling barrier is less and less pronounced when the assisting mode frequency becomes smaller.

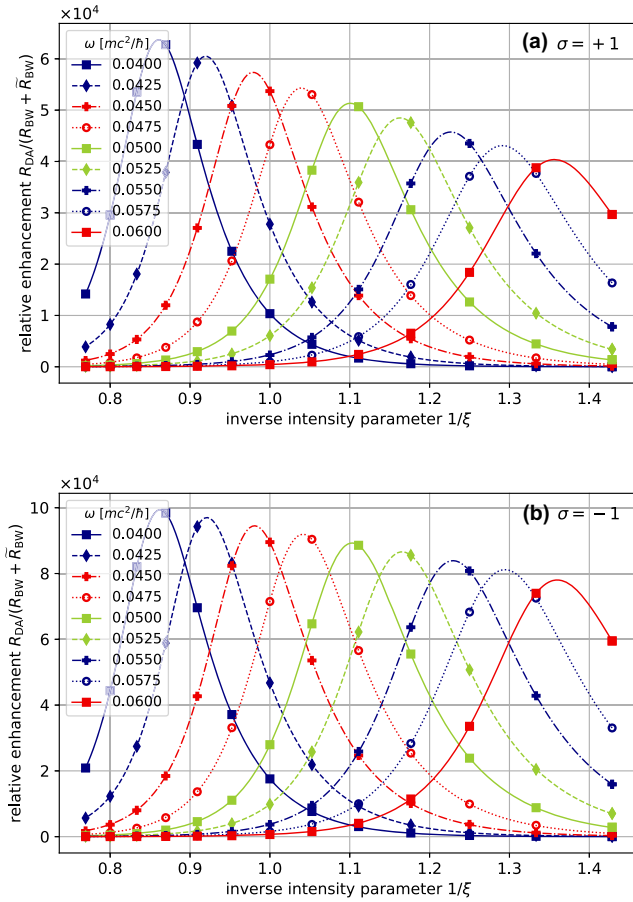


FIG. 6. Relative rate enhancement due to dynamical assistance for different values of the main mode frequency. The γ -photon frequency and assisting mode parameters are the same as in Fig. 2. Panels (a) and (b) refer to co- and counter-rotating laser modes, respectively.

However, when $\tilde{\omega}$ is decreased further, the relative enhancement curves start to grow again and shift considerable to the right, until the value $\tilde{\omega} = 0.472m$ is reached. At this frequency, the monochromatic process of nonlinear Breit-Wheeler pair creation by the γ beam and the assisting mode alone changes its character in the sense that now at least four (rather than three) $\tilde{\omega}$ photons are required to overcome the creation threshold. The corresponding rate \tilde{R}_{BW} therefore drops down considerably, being suppressed by an additional factor of $\tilde{\xi}^2 \ll 1$. As a consequence, the crossing point with the unassisted rate R_{BW} shifts to the right (see Fig. 3), close to which the maximum relative enhancement occurs, in agreement with the shift of the curve maxima arising in Fig. 7. When the assisting mode frequency is even further decreased, the same transition takes place again. The curves decrease until $\tilde{\omega} \approx 0.38m$ is reached, afterwards start to increase again up to the value $\tilde{\omega} = 0.354m$, from which point onwards at least five $\tilde{\omega}$ photons must be absorbed to produce pairs via the monochromatic channel associated with the rate \tilde{R}_{BW} .

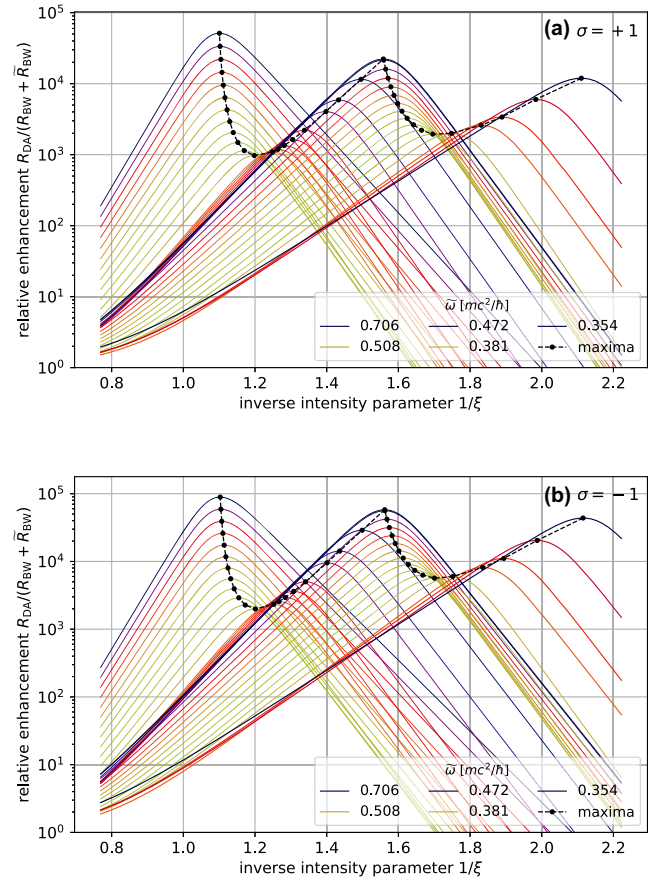


FIG. 7. Relative rate enhancement due to dynamical assistance for a sequence of decreasing values of the weak mode frequency between $\tilde{\omega} = 0.706m$ and $\tilde{\omega} = 0.354m$ (from left to right). The γ -photon frequency, main mode frequency and assisting mode intensity parameter are the same as in Fig. 2. Black circles mark the curve maxima. Panels (a) and (b) refer to co- and counter-rotating laser modes, respectively.

In order to highlight the gradual transitioning due to multiphoton-channel closings, we have marked the maxima of the relative enhancement curves in Fig. 7 by black circles. The distinguished frequencies of the assisting mode $\tilde{\omega} \in \{0.706m, 0.472m, 0.354m\}$, where the main maxima in Fig. 7 arise, are of the form $\tilde{\omega} \approx \sqrt{2}m/\tilde{n}_0$ with $\tilde{n}_0 \in \{2, 3, 4\}$, corresponding to $\tilde{s} = 4\tilde{\omega}\omega' \approx 4m^2/\tilde{n}_0$.

IV. CONCLUSION

Dynamically assisted nonlinear Breit-Wheeler pair creation has been studied in a bichromatic laser field, consisting of a strong main mode of low frequency and a weak assisting mode of high frequency, both with circular polarization. By taking the weak mode in leading order into account, an integral expression for the corresponding pair creation rate with absorption of one high-frequency laser photon has been obtained, whose structure resembles the well-known formula for the ordinary (i.e. unassisted) nonlinear Breit-Wheeler process [13–15].

We have shown that the assistance from the weak laser mode can enhance the pair creation rate by several orders of magnitude. In particular, the enhancement is more pronounced when the two laser modes have opposite helicities. This interesting effect can be attributed to a lowering of the angular-momentum barrier of the process. The relative enhancement due to dynamical assistance, as compared with the rates for nonlinear Breit-Wheeler pair creation by each laser mode separately, was found to be the larger, the smaller the main mode frequency and the assisting mode intensity parameter are. Quite complex structures have arisen when instead the assisting mode frequency is varied, which are caused by multiphoton-channel closings.

Our analysis complements a previous study where dynamically assisted nonlinear Breit-Wheeler pair creation of scalar particles was considered for mutually orthogonal, linearly polarized laser waves [42]. It is of potential relevance for future experiments on strong-field pair creation at high-intensity laser facilities [23–27].

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