Ultraheavy FIMP dark matter and conformal sectors

Ameen Ismail [*]

Department of Physics, LEPP, Cornell University, Ithaca, New York 14853, USA

(Received 16 August 2023; accepted 10 November 2023; published 29 November 2023)

We point out a dark matter candidate that arises in a minimal extension of solutions to the hierarchy problem based on compositeness. In such models, some or all of the Standard Model fields are composites of a conformal field theory (CFT), which confines near the electroweak scale. We posit an elementary scalar field, whose mass is expected to lie near the cutoff of the CFT and whose couplings to the Standard Model are suppressed by the cutoff. Hence, it can naturally be ultraheavy and feebly coupled. This scalar can constitute all of the dark matter for masses between 10^{10} GeV and 10^{18} GeV, with the relic abundance produced by the freeze-in mechanism via a coupling to the CFT. The principal experimental constraints come from bounds on the tensor-to-scalar ratio. We speculate about future detection prospects.

DOI: 10.1103/PhysRevD.108.095049

I. INTRODUCTION

The preponderance of evidence for dark matter (DM) provides strong experimental motivation for new physics beyond the Standard Model (SM). Despite this, the microscopic nature of DM remains unknown, with a range of about 80 orders of magnitude allowed for the mass of its dominant component [1,2]. Conversely, one of the strongest theoretical motivations for new physics is the Higgs hierarchy or naturalness problem. Within the SM, the Higgs mass is a relevant operator not protected by any symmetries, and thus, it is unstable against radiative corrections.

For several decades, there has been speculation of a connection between DM and the hierarchy problem. Most solutions to the hierarchy problem involve new particles appearing near the TeV scale, and if they are stable, they can constitute the dark matter. These "weakly interacting massive particles" (WIMPs) typically have weak-scale couplings to the SM particles. Such couplings are of the right size to generate the observed DM relic abundance via thermal freeze-out, a numerical coincidence dubbed the "WIMP miracle." The WIMP paradigm has been extensively studied in the literature, manifesting as various avatars in popular solutions to the hierarchy problem [3–7]. Examples include the lightest Kaluza–Klein particle in extra-dimensional settings, stabilized by KK-parity [4,5], and the lightest neutralino in the minimal supersymmetric standard model, stabilized by R parity [3]. (Imposing R parity is also motivated by phenomenological bounds on proton decay, which perhaps renders the WIMP miracle more miraculous.) But ever-tightening direct detection bounds have now ruled out many simple WIMP models [8,9]. This has encouraged intense exploration of other DM frameworks beyond WIMPs, with varying degrees of theoretical motivation. For example, ultralight DM has seen a flurry of activity in recent years (for reviews, see Refs. [10–12]), with the prototypical candidate being the QCD axion, whose existence is motivated by the strong CP problem [13,14].

For the purposes of this paper, we will be concerned with ultraheavy particle DM, roughly defined as DM with a mass above the WIMP unitarity bound of ∼100 TeV but below the Planck scale. This regime has received relatively little attention, perhaps due to a lack of model-building efforts and the experimental challenges of detection. For a recent review including existing models and efforts for detection, see Ref. [15].

In this work, we draw attention to an ultraheavy DM candidate that arises in the context of solutions to the hierarchy problem based on a conformal sector. This popular paradigm (reviewed in [16,17]) posits that the Higgs, as well as possibly the SM gauge bosons and fermions, are composites of a conformal field theory (CFT) that confines around the TeV scale. Scale invariance allows for a large, radiatively stable hierarchy between the ultraviolet (UV) cutoff of the CFT and the infrared (IR) scale at which it confines, protecting a small Higgs mass. Our key idea is that if one introduces a scalar field that is elementary (as opposed to being a composite of the CFT), its mass will naturally lie close to the UV cutoff, which may easily be ultraheavy. This scalar can constitute the DM, with the relic

[*]ai279@cornell.edu

abundance set by the freeze-in mechanism [18,19]. The initial abundance after inflation is negligible, and the DM is generated through a coupling to the CFT, while the CFT is still in its hot, deconfined phase.

One could also describe the same scenario in the 5D dual picture through the AdS/CFT correspondence [20,21]. Composite Higgs models are dual to warped extra dimensions, involving a slice of 5D AdS capped by UV and IR branes [22,23] (see also [24] for a recent study of 5D duals of composite models). The Higgs and possibly the other SM fields propagate on the IR brane or in the 5D bulk. The model we are suggesting corresponds to a scalar DM candidate that lives on the UV brane.

For our mechanism to work and be natural, it is essential that the Higgs is composite. Otherwise there would be a renormalizable coupling of the Higgs to the DM, which would lead to an unstable hierarchy between the electroweak scale and the ultraheavy DM mass, as well as possibly affecting the DM abundance. We will see that the fermions and gauge bosons may be elementary, depending on the details of reheating.

The DM has small scattering cross sections with SM particles, since they are highly suppressed by the UV scale or the large mass. Such weak interactions are reminiscent of models of feebly interacting massive particles (FIMPs) [18,25]. FIMPs have been previously studied in the context of warped extra dimensions, but only in the case where the DM propagates on the IR brane or in the bulk. If the SM lives on the UV brane but the DM is in the bulk, the 4D dual describes an elementary SM and composite DM, which is basically opposite in spirit to this work. This setup has been explored in the conformal freeze-in scenario [26–28], as well as with non-FIMP DM in warped/conformal dark sectors [29–34]. (See also continuum freeze-out, which features a WIMP-like DM state living in the bulk while the SM lies on the UV brane [35–39].) Another case that has been studied in the literature is the SM and DM both localized on the IR brane, with a feeble SM-DM coupling [40–43]. One can also consider IR-localized WIMPs in a composite Higgs setting, which arise naturally as pseudo-Nambu–Goldstone bosons in models with extended global symmetry [44–56]. What we are proposing here is fundamentally different. Our DM is elementary and thus naturally ultraheavy and feebly coupled to the SM. Since it is heavier than a typical FIMP, it is fitting to call it a "FIMPzilla," in analogy with WIMPzillas. FIMPzillas have been considered elsewhere in the literature; the best-known model is probably Planckian interacting DM, involving Planck-scale DM interacting just gravitationally with the SM [57,58]. FIMPzillas with a mass around 10^{10} GeV in the context of a seesaw mechanism for neutrino masses were discussed in [59].

In what follows, we illustrate our ideas with a minimal model and then explore its phenomenology. The calculation of the relic abundance is complicated by the fact that the DM freezes in before the conformal phase transition occurs, when the theory is described by a hot CFT coupled to the DM. To compute the production rate, we leverage techniques previously used to study scale-invariant sectors in the context of unparticle physics [60]. Our simple model successfully reproduces the observed DM relic abundance while being consistent with experimental bounds for masses up to 1 or 2 orders of magnitude below the Planck scale. The principal experimental constraint on the model arises from limits on the tensor-to-scalar ratio. We speculate about other approaches for detection, including the cosmological collider and direct gravitational detection.

II. MODEL

We suppose that the SM degrees of freedom are composites of a conformal sector that confines not too far above the electroweak scale. The dark matter is an elementary scalar, SM singlet χ, which is naturally expected to lie near the cutoff scale of the CFT Λ. We require a mild hierarchy between the mass of χ and the cutoff, $m \lesssim \Lambda$, so that we can reliably perform calculations in the effective theory, without the need for a UV completion. We assume that χ is odd under a discrete Z_2 symmetry so that it is stable. Lastly, since we eventually will set the relic abundance of χ through the freeze-in mechanism, we assume that the initial abundance of χ is negligible. This is possible if, for example, inflationary dynamics only reheat the conformal sector.

Ultimately, only the Higgs needs to be composite to stabilize a large hierarchy. In realistic models addressing the hierarchy problem (e.g., the minimal composite Higgs [61]), one typically takes some of the SM fields to be composite while others, particularly the light fermions, are elementary. It is quite feasible to construct a composite Higgs model in which all of the SM fermions and gauge bosons are elementary [62]. If any elementary SM states are reheated after inflation, they could contribute to and even dominate DM production. This is not the situation we are interested in, so for now, we work under the assumption that *only the conformal sector* is reheated. Then taking some of the SM particles to be elementary has no substantial effect on DM production. Later we will relax this assumption and consider the conditions for production from the CFT sector to dominate.

In the confined phase of the CFT, the holographic dual of our setup is essentially a Randall–Sundrum model [22,62]—a slice of 5D AdS space where the composite and elementary fields are, respectively, localized toward the IR and UV branes—plus a scalar χ that propagates on the UV brane. The cutoff of the CFT is identified with the location of the UV brane, as well as the inverse AdS curvature of the bulk. At high temperatures, the CFT is in its deconfined phase, and the dual theory is instead described by AdS-Schwarzschild space with a

UV brane (where χ is still localized). The usual AdS/CFT relations allow us to write the number of colors of the CFT, N, in terms of the unreduced Planck scale M_P and the cutoff [63]:

$$N = \sqrt{2\pi}\frac{M_P}{\Lambda}. \tag{1}$$

The holographic theory is only under theoretical control in the large N limit, $N \gg 1$. Note also that the 5D Planck scale M_5 is related to the 4D Planck scale and the cutoff as $M_5^3 = M_P^2\Lambda/8\pi$. We will not make any further use of the holographic dual than this, but it is still a useful picture to keep in mind.

The relic abundance of χ is generated via the freeze-in mechanism in the early Universe, when the CFT is in its hot, deconfined phase. We introduce a coupling of the DM to the CFT sector through an effective interaction of the form

$$\mathcal{L}_{\rm int} = \frac{\kappa}{2}\frac{\chi^2\mathcal{O}_{\rm CFT}}{\Lambda^{d-2}}, \tag{2}$$

where κ is a dimensionless coupling, and $\mathcal{O}_{\rm CFT}$ is a CFT operator of dimension d. In the 5D picture, this corresponds to a UV brane-localized coupling. DM can be produced through the process CFT stuff $\to \chi\chi$.

Note that the operator dimension d need not be an integer. We do require $d > 1$ from CFT unitarity. Also when the operator is relevant, $d < 4$, $\mathcal{O}_{\rm CFT}$ needs to be forbidden from appearing in the Lagrangian by itself—otherwise we would have a large explicit and relevant symmetry-breaking term, causing the CFT description to break down not far below the UV scale. The details of how this is achieved are not important to our mechanism, but one way would be to charge $\mathcal{O}_{\rm CFT}$ under a discrete symmetry.

In the next section, we will perform a detailed calculation of the relic abundance resulting from the coupling in Eq. (2). Technically we have an IR freeze-in scenario for $d \leq 2$ and UV freeze-in for $d > 2$. However, it turns out that to get the right relic abundance, the reheating temperature must be less than the DM mass. The hallmark of IR freeze-in is DM production dominantly occurring at $T \sim m$, but for our case, the temperature is never this large even at early times. Instead, the DM production rate is always suppressed by Boltzmann factors $e^{-m/T}$, so production dominantly occurs at early times, which is characteristic of UV freeze-in. Hence, the situation resembles UV freeze-in even when $d \leq 2$.

The DM mass is naturally expected to lie near the UV cutoff scale (or perhaps a loop factor below). Moreover, the UV cutoff can in principle be as large as desired so long as the energy densities involved are sub-Planckian ($m^2\Lambda^2 \lesssim M_P^4$),[1] and thus, the DM can easily be ultraheavy. In fact, the benchmark points we will study turn out to only be experimentally viable for masses larger than about 10^{10} GeV. The DM interacts very feebly with SM particles because the cross sections are suppressed by its mass (for $d > 2$, the DM-SM cross section is further suppressed by the cutoff scale). Because of the FIMP-like interactions but large mass, it is appropriate to refer to χ as a FIMPzilla.

Furthermore, the weak coupling and large mass make it infeasible to detect this DM through direct or indirect detection. Instead, the principal experimental bounds arise from constraints on the CMB. In particular, we will see that the requirement of matching the observed DM relic abundance makes sharp predictions for the reheating temperature, allowing us to constrain the model through the tensor-to-scalar ratio in the CMB power spectrum. In this regard, the phenomenology of our model is most similar to the Planckian interacting dark matter scenario [57].

III. RELIC ABUNDANCE

We now proceed to calculate the DM relic abundance in this model. For completeness, let us first write the Lagrangian for χ:

$$\mathcal{L}_\chi = \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m^2\chi^2 + \frac{\kappa}{2}\frac{\chi^2\mathcal{O}_{\rm CFT}}{\Lambda^{d-2}}. \tag{3}$$

As usual for freeze-in DM, we assume the initial abundance of χ is negligible, so we can ignore annihilation of χ into CFT stuff. The Boltzmann equation for χ is then

$$\dot{n}_\chi + 3Hn_\chi = n_{\rm CFT}^{\rm eq}\langle\sigma v({\rm CFT}\to\chi\chi)\rangle, \tag{4}$$

where n denotes a number density, $n^{\rm eq}$ an equilibrium number density, $\langle\sigma v\rangle$ a thermally averaged cross section, and H is the Hubble parameter. The right-hand side of this equation should be understood as a schematic representation of processes that produce DM pairs out of the hot CFT. It should not be taken literally—it is unclear how to even define a scattering process with CFT stuff in the initial state, due to the lack of asymptotic states in a CFT.

Nevertheless, one can make sense of the production rate by relating it to the inverse process $\chi\chi \to$ CFT using the principle of detailed balance:

$$n_{\rm CFT}^{\rm eq}\langle\sigma v({\rm CFT}\to\chi\chi)\rangle = (n_\chi^{\rm eq})^2\langle\sigma v(\chi\chi\to{\rm CFT})\rangle \equiv \gamma. \tag{5}$$

[1] A slightly stronger but safer condition is to simply require $\Lambda < M_P$. Super-Planckian values of Λ raise difficulties in UV-completing to a theory of quantum gravity, but these issues can be avoided if the large size of Λ is generated by, for example, clockwork [64–66] or monodromy [67]. In any case, this point has no qualitative bearing on our model.

The rate γ can be computed using unparticle methods for the CFT phase space [60]; we provide the calculational details in the Appendix. One could also calculate the rate using the techniques in [68]. (Presumably, we could perform this calculation in the 5D picture, too, by considering an AdS-Schwarzschild metric with a UV brane on which χ propagates.) One finds

$$\gamma = \kappa^2 A_d \frac{T}{32\pi^4}(2m)^3\left(\frac{2m}{\Lambda}\right)^{2d-4} \times \int_1^\infty du u^{2d-4}\sqrt{u^2-1}K_1(2mu/T), \qquad (6)$$

where $A_d = 16\pi^{5/2}(2\pi)^{-2d}\Gamma(d+1/2)/(\Gamma(d-1)\Gamma(2d))$ is the usual unparticle phase space normalization, and K_1 is a modified Bessel function of the second kind.

It is convenient to write the Boltzmann equation [Eq. (4)] in terms of the abundance $Y = n_\chi/s$ (where s is the entropy density) and $x = m/T$, leading to

$$\frac{dY}{dx} = \frac{\gamma}{Hsx}. \qquad (7)$$

We then substitute in Eq. (6) for γ, use $H = 1.66\sqrt{g_*}T^2/M_P$ and $s = 2\pi^2 g_* T^3/45$, and integrate the Boltzmann equation to obtain the DM abundance today:

$$Y_\infty = \kappa^2 \frac{45A_d}{(1.66)8\pi^6 g_*^{3/2}}\frac{M_P}{m}\left(\frac{2m}{\Lambda}\right)^{2d-4} \times \int_{x_R}^\infty dx \int_1^\infty du u^{2d-4}\sqrt{u^2-1}x^3 K_1(2xu). \qquad (8)$$

Here, x_R is the value of x at the reheating temperature T_R: $x_R = m/T_R$. Also, when the CFT is in the hot phase, the effective number of relativistic degrees of freedom is controlled by the number of colors in the CFT, $g_* \sim N^2 = 2\pi M_P^2/\Lambda^2$.

The relic abundance is sensitive to the reheating temperature, which is typical of UV freeze-in. Assuming efficient reheating, the reheating temperature is related to the Hubble parameter at the end of inflation H_I as

$$H_I = \sqrt{\frac{4\pi^3 g_*}{45}}\frac{T_R^2}{M_P}. \qquad (9)$$

This is important because H_I is constrained by the CMB bound on the tensor-to-scalar ratio r.

IV. PHENOMENOLOGY

In Fig. 1, we show curves in the (m, H_I) plane that yield the observed DM relic abundance [69,70]. We fix $d = 3.5$ and $\kappa = 1$ and vary $m/\Lambda = 0.1, 0.01, 0.001$ over the three curves. We indicate the maximally allowed value of H_I from the CMB bound on the tensor-to-scalar ratio $r < 0.036$ [70,71]. We also include a bound $m > H_I$, which safely avoids DM isocurvature perturbations [57,72].

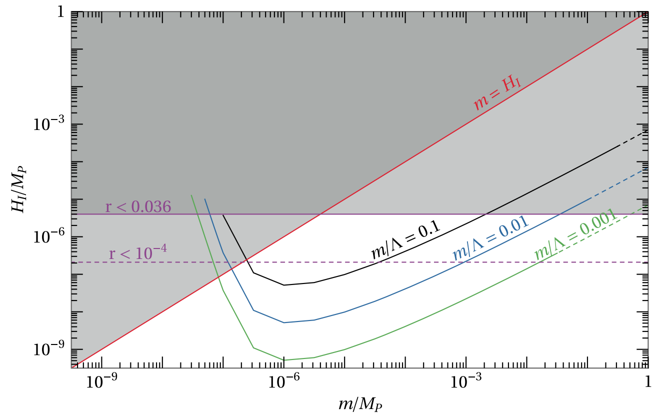

FIG. 1. The value of H_I that yields the observed DM relic abundance [69,70] as a function of the DM mass m. We fix $d = 3.5$ and $\kappa = 1$. We show results for $m/\Lambda = 0.1$ *(black line)*, $m/\Lambda = 0.01$ *(blue line)*, and $m/\Lambda = 0.001$ *(green line)*. The dashing of these lines indicates where the energy density is super-Planckian. We show bounds from the nonobservation of isocurvature perturbations, which excludes $m < H_I$ *(red line)* [57,72]; from the CMB upper limit on the tensor-to-scalar ratio, $r < 0.036$ *(purple line)* [70,71]; and a projection for a future bound $r < 10^{-4}$ *(dashed purple line)*.

Note that the DM mass cannot be taken arbitrarily close to the Planck scale. Quantum gravity effects become important as the energy density of χ, which is of order $m^2\Lambda^2$, becomes comparable to M_P^4. For this reason, we have dashed the curves in Fig. 1 for $m/M_P > \sqrt{m/\Lambda}$, where our calculations are not reliable. Also, there is a minimum mass below which, the relic abundance is too small no matter how large the reheating temperature is, which is why the curves do not extend below $m \sim 10^{-7}M_P$. We further caution that we have neglected direct gravitational production of χ, which may be important when m is very close to H_I [57].

Together, the benchmark points in Fig. 1 provide experimentally viable DM candidates for masses ranging from $10^{-7}M_P \sim 10^{12}$ GeV to $0.1M_P \sim 10^{18}$ GeV. As previously stated, it is difficult to search for the DM with direct or indirect detection because it is ultraheavy and couples feebly to the SM. However, the model is in principle testable through the bound on the tensor-to-scalar ratio. As the bound on r becomes tighter, one needs to fine-tune m/Λ to smaller values to evade the bound. In Fig. 1, we show a projection for a bound $r < 10^{-4}$, which would probe a substantial region of parameter space. (The specific choice of 10^{-4} was quoted as a "futuristic bound" in [57]; for comparison, the upcoming CMB-S4 experiment is projected to be sensitive to r down to about 10^{-3} to 10^{-4} [73].)

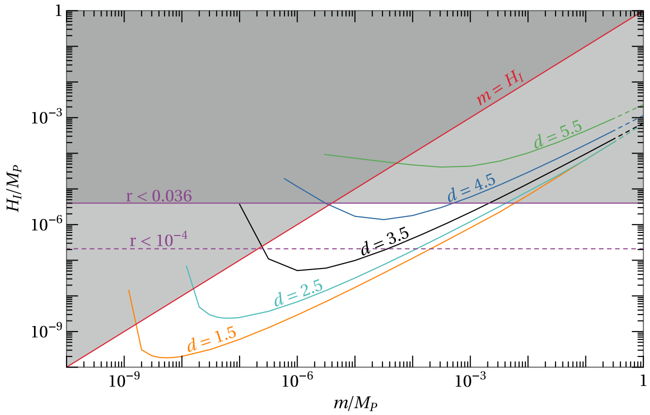

FIG. 2. Same as Fig. 1, but fixing $m/\Lambda = 0.1$ and $\kappa = 1$ while choosing different values of d: $d = 1.5$ *(orange line)*, $d = 2.5$ *(turquoise line)*, $d = 3.5$ *(black line)*, $d = 4.5$ *(blue line)*, and $d = 5.5$ *(green line)*.

Figure 2 illustrates the effect of changing the operator dimension, fixing $m/\Lambda = 0.1$ and $\kappa = 1$ while varying d from 1.5 to 5.5. The choice of half-integer values is merely to emphasize that d does not need to be an integer. The bounds are the same as in Fig. 1. As d increases, the DM production rate is suppressed by a larger power of the cutoff Λ, and so obtaining the correct relic abundance requires a larger reheating temperature and thus, a larger H_I.

For $d \gtrsim 5$, it is difficult to satisfy the existing bound on r while generating the right relic abundance. Since CFT unitarity implies $d > 1$, there is a narrow permitted window for the operator dimension $1 < d \lesssim 5$ (the exact upper bound depends weakly on the value of m/Λ). This provides a complementary view on how constraining the tensor-to-scalar ratio probes our model: As the maximum allowed r becomes smaller, the viable window for the operator dimension d shrinks.

We can also see from Fig. 2 that lowering the dimension allows for a smaller DM mass, although still firmly in the ultraheavy regime; for $d = 1.5$ (and $m/\Lambda = 0.1$), m can be as low as $\sim 10^{-9} M_P$. One might worry about direct detection bounds becoming important for $d < 2$, as the coupling between χ and the CFT becomes relevant. Recall the DM couples to the CFT like $\Lambda^{2-d}\chi^2\mathcal{O}_{\rm CFT}$, enhanced by powers of Λ when $d < 2$. To obtain the resulting interaction with a composite fermion ψ, we match the CFT operator onto the low-energy theory as $\mathcal{O}_{\rm CFT} \to \sqrt{N}/(4\pi\Lambda_{\rm IR}^{3-d})\psi^\dagger\psi$, where $\Lambda_{\rm IR}$ is the scale at which the CFT confines. This scaling is suggested by dimensional analysis, Lorentz invariance, and large-N arguments [27]. We can then calculate cross sections for direct detection in the normal way. Taking $\Lambda_{\rm IR} \sim$ TeV and using the holographic relation Eq. (1), we obtain a DM-nucleon cross section of

$$\sigma \simeq 10^{-75}\ \mathrm{cm}^2 \frac{f_N^2 m}{\Lambda}\left(\frac{\Lambda}{1\ \mathrm{TeV}}\right)^{2-d}\left(\frac{M_P}{m}\right)^3, \qquad (10)$$

where f_N is an $\mathcal{O}(1)$ nuclear form factor. Even for the smallest masses in Fig. 2 (that is, $m \sim 10^{-9} M_P$ and $d = 1.5$), the cross section is of order 10^{-45} cm^2, well below direct detection bounds.

Lastly, we speculate about further opportunities for detection. For masses near the inflationary Hubble scale, $m \sim H_I$, one could potentially see imprints of χ in cosmological collider observables [74]. A more involved analysis than what we have done in this work is required to understand the details of this. In particular, one would need to take into account gravitational production of DM when computing the relic abundance, since that is important in the very regime $m \sim H_I$ relevant for the cosmological collider.[2] Another compelling avenue for detection is to directly probe the gravitational coupling of χ to ordinary matter. This is the ultimate goal pursued by the Windchime project: gravitational detection of Planck-scale DM [75,76]. Our model provides another physics case for Windchime in the form of a well-motivated, ultraheavy particle DM candidate that is difficult to detect through other means.

V. ELEMENTARY PRODUCTION

As we emphasized earlier, we have heretofore assumed that any elementary SM particles are not reheated. We now relax this assumption, which allows for additional freeze-in DM production from annihilation of SM fermions or gauge bosons to $\chi\chi$. We want to consider under what circumstances these processes provide the leading contribution to DM production.

An elementary SM fermion ψ or gauge boson A_μ can couple to χ through a dimension-six operator:

$$\mathcal{L}_{\rm fermion} = \frac{\kappa_f}{2\Lambda^2}\bar{\psi}\not{D}\psi\chi^2, \quad \mathcal{L}_{\rm vector} = \frac{\kappa_v}{4\Lambda^2}F_{\mu\nu}F^{\mu\nu}\chi^2. \qquad (11)$$

If χ^2 can couple to a CFT operator with dimension $1 < d < 4$ [see Eq. (2)], then $\chi\chi$ production from fermions and gauge bosons is suppressed by more powers of Λ than CFT production. Hence, we expect CFT production to dominantly produce the DM relic abundance. On the other hand, if $d > 4$, then production from gauge bosons, $AA \to \chi\chi$, dominates. (We argue in the Appendix that production from fermions is subdominant.)

For completeness, in the Appendix, we study the latter scenario in the extreme case where all SM particles are elementary and reheated at the end of inflation. This is a standard UV freeze-in calculation, in contrast to the main focus of this paper, CFT-dominated production, where we had to use unparticle tricks. In this case, χ can

[2]This is really a trivial statement. Particles with masses near the Hubble scale have interesting effects on cosmological correlators *because* they can be produced during inflation.

still be a viable DM candidate for masses between $10^{-8}M_P$ and $10^{-2}M_P$.

We reiterate that freeze-in production from elementary SM particles is only pertinent when those particles are reheated at the end of inflation. If only the CFT is reheated, the elementary fields do not impact DM production. The central conclusion of this discussion is that even when some elementary fields are reheated, CFT production may dominate anyway if $d < 4$, in which case, the results of the previous section still apply.

VI. DISCUSSION

We have demonstrated here that solutions to the hierarchy problem based on a conformal sector easily accommodate an ultraheavy and feebly interacting DM candidate in the form of an elementary scalar. Focusing on a minimal realization of this scenario, we studied the case where freeze-in production of the DM occurs dominantly through a coupling to the CFT. This led to a viable DM candidate for ultraheavy masses upward of $10^{-9}M_P \sim 10^{10}$ GeV.

The principal experimental constraints arose from the CMB bound on the tensor-to-scalar ratio r, which bounds the inflationary Hubble scale. In the future, the parameter space will be further constrained by improved bounds on r from experiments like CMB-S4. It is possible that the cosmological collider could probe masses not too far above the inflationary Hubble scale, although we leave a detailed study of this for future work. Larger masses close to the Planck scale could provide an additional physics case for proposals for direct gravitational detection.

The importance of the conformal symmetry to our model should not be understated. Otherwise, one might simply posit an ultraheavy Dirac fermion as a DM candidate; it would only have nonrenormalizable couplings to the SM, and thus, one could obtain the right relic abundance through UV freeze-in, but this misses the point: There is a large hierarchy between the UV scale at which the ultraheavy DM lies and the IR scale at which the SM particles lie. This hierarchy is fine-tuned in the absence of any symmetries protecting it. The essential role of the conformal sector is to dynamically generate a large and stable UV/IR hierarchy.

Throughout this work, we have neglected any discussion of the conformal phase transition, and in fact, we have tacitly assumed that the phase transition occurs promptly, without supercooling. A large amount of supercooling would dilute the DM abundance, potentially posing problems for our production mechanism.[3] Whether a prompt phase transition can be achieved is dependent on the stabilization mechanism; the prototypical Goldberger–Wise mechanism often leads to a supercooled transition [77–81]. Nevertheless, alternative approaches to stabilization that avoid supercooling have been proposed [82–88]. Another option is that the CFT is always in the confined phase, as suggested in [89]. This would alter the calculation of the relic abundance, however, since we assumed freeze-in occurs when the CFT is in the deconfined phase.

We have also remained largely agnostic about the details of inflation. Freeze-in requires that the initial abundance of the DM is negligible. The obvious way to achieve this is to assume that the inflationary dynamics cause only the conformal sector to be reheated. There is a rich literature on inflation in warped throats, mainly in the context of string theory (e.g., [90–94]). In light of this, it would be interesting to study the interplay of inflationary model-building with our DM model—perhaps this could lead to additional phenomenological signatures. Furthermore, one could explore the scenario in which some of the SM fermions and/or gauge bosons are elementary, but still only the conformal sector is reheated. Would this have any consequences for the cosmological history of these models? We leave all these exciting questions for future projects.

ACKNOWLEDGMENTS

We are very grateful to Maxim Perelstein for several fruitful discussions, useful comments, and encouragement. We also thank Csaba Csáki, Sylvain Fichet, Jay Hubisz, and Seung J. Lee for helpful comments. We are supported in part by the NSF Grant No. PHY-2014071 and in part by NSERC, funding reference No. 557763.

APPENDIX A: DM PRODUCTION CALCULATION

Here, we provide some further details of the calculation of Eq. (6). We first calculate the cross section σ for the process $\chi\chi \to$ CFT. The CFT phase space factor is given in [60], and we find

$$\sigma = \frac{A_d}{4\sqrt{(p_1\cdot p_2)^2 - m^4}} \times \int \frac{d^4p}{(2\pi)^4}|\mathcal{M}|^2(2\pi)^4\delta^4(p_1+p_2-p)\theta(p^0)\theta(p^2)(p^2)^{d-2}, \tag{A1}$$

where p_1 and p_2 are the momenta of the initial χ particles, and $A_d = 16\pi^{5/2}(2\pi)^{-2d}\Gamma(d+1/2)/(\Gamma(d-1)\Gamma(2d))$. From Eq. (2), the matrix element is just $|\mathcal{M}| = \kappa/\Lambda^{d-2}$. The phase space integration is trivial, and we are left with

[3] Possibly, one could evade this issue by overproducing DM in the early Universe, such that after being diluted in a period of supercooling, one is left with the right relic abundance. It would be interesting to study how much supercooling can be accommodated in this way.

$$\sigma = \kappa^2 A_d \left(\frac{s}{\Lambda^2}\right)^{d-2} \frac{1}{2\sqrt{s(s-4m^2)}}, \qquad \text{(A2)}$$

where $s = (p_1 + p_2)^2$.

We can now use the standard formula for the thermally averaged cross section [95]:

$$(n_\chi^{\rm eq})^2 \langle \sigma v \rangle = \frac{T}{32\pi^4} \int_{4m^2}^\infty ds \sigma (s - 4m^2) \sqrt{s} K_1(\sqrt{s}/T). \qquad \text{(A3)}$$

Upon plugging in Eq. (A2) for the cross section (and changing integration variables to $u = \sqrt{s}/2m$), one readily obtains Eq. (6), as desired.

APPENDIX B: FREEZE-IN FROM ELEMENTARY PARTICLES

Here, we consider the case where production is dominated by elementary SM particles, rather than CFT production. The cross section for $AA \to \psi\psi$, where A_μ is a massless gauge boson that couples to χ as in Eq. (11), is

$$\sigma = \frac{\kappa_v^2}{8\pi} \left(\frac{s}{\Lambda^2}\right)^2 \frac{1}{2\sqrt{s(s-4m^2)}}. \qquad \text{(B1)}$$

We remark that this is identical to the CFT production cross section, Eq. (A2), with $d = 4$ and the replacements $\kappa \to \kappa_v$, $A_d \to 1/8\pi$. Thus, the production rate $(n_\chi^{\rm eq})^2 \langle \sigma v \rangle$ is simply given by Eq. (6) with those replacements.

Using the equation of motion in Eq. (11), or by direct calculation, one can show that the corresponding production rate for a fermion ψ must scale as $s m_\psi^2/\Lambda^4$, so it vanishes in the massless limit. Therefore, the contribution of an elementary fermion to the DM relic abundance is suppressed with respect to the contribution from an elementary gauge boson.

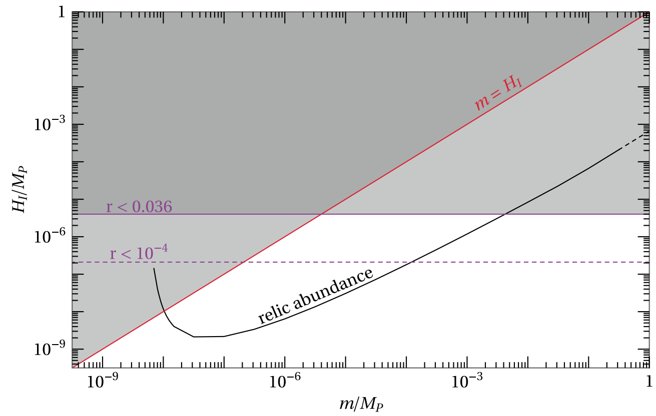

FIG. 3. Values of H_I and m that yield the observed DM relic abundance [69,70] in the case where freeze-in production from elementary gauge bosons dominates over CFT production *(black line)*. We assume all of the SM gauge bosons are elementary, are reheated at the end of inflation, and couple universally to the DM with $\kappa_v = 1$. We fix $m/\Lambda = 0.1$. The bounds are the same as in Fig. 1.

Let us study this scenario in the case where all of the SM gauge bosons are elementary and suppose that they all couple to χ with $\kappa_v = 1$. The production rate we just computed must be multiplied by 12 to account for the number of gauge boson species in the SM. In Fig. 3, we show the corresponding curve yielding the observed DM abundance, fixing $m/\Lambda = 0.1$. The bounds are the same as in Fig. 1.

From Fig. 3, we see this scenario reproduces the correct relic abundance while being consistent with all experimental bounds for masses between $10^{-8} M_P$ and $10^{-2} M_P$. The prospects for detection are similar to those discussed in the main text for the case where CFT production dominates.

[1] Wayne Hu, Rennan Barkana, and Andrei Gruzinov, Cold and fuzzy dark matter, Phys. Rev. Lett. **85**, 1158 (2000).

[2] Pablo Villanueva-Domingo, Olga Mena, and Sergio Palomares-Ruiz, A brief review on primordial black holes as dark matter, Front. Astron. Space Sci. **8**, 87 (2021).

[3] Gerard Jungman, Marc Kamionkowski, and Kim Griest, Supersymmetric dark matter, Phys. Rep. **267**, 195 (1996).

[4] Geraldine Servant and Timothy M. P. Tait, Is the lightest Kaluza-Klein particle a viable dark matter candidate?, Nucl. Phys. **B650**, 391 (2003).

[5] Hsin-Chia Cheng, Jonathan L. Feng, and Konstantin T. Matchev, Kaluza-Klein dark matter, Phys. Rev. Lett. **89**, 211301 (2002).

[6] Kaustubh Agashe, Adam Falkowski, Ian Low, and Geraldine Servant, KK parity in warped extra dimension, J. High Energy Phys. 04 (2008) 027.

[7] Andreas Birkedal, Andrew Noble, Maxim Perelstein, and Andrew Spray, Little Higgs dark matter, Phys. Rev. D **74**, 035002 (2006).

[8] Giorgio Arcadi, Maíra Dutra, Pradipta Ghosh, Manfred Lindner, Yann Mambrini, Mathias Pierre, Stefano Profumo, and Farinaldo S. Queiroz, The waning of the WIMP? A review of models, searches, and constraints, Eur. Phys. J. C **78**, 203 (2018).

[9] J. Aalbers *et al.* (LZ Collaboration), First dark matter search results from the LUX-ZEPLIN (LZ) experiment, Phys. Rev. Lett. **131**, 041002 (2023).

[10] Elisa G. M. Ferreira, Ultra-light dark matter, Astron. Astrophys. Rev. **29**, 7 (2021).

[11] D. Antypas *et al.*, New horizons: Scalar and vector ultralight dark matter, arXiv:2203.14915.

[12] Francesca Chadha-Day, John Ellis, and David J. E. Marsh, Axion dark matter: What is it and why now?, Sci. Adv. **8**, abj3618 (2022).

[13] R. D. Peccei and Helen R. Quinn, CP conservation in the presence of instantons, Phys. Rev. Lett. **38**, 1440 (1977).

[14] Steven Weinberg, A new light boson?, Phys. Rev. Lett. **40**, 223 (1978).

[15] Daniel Carney *et al.*, Snowmass2021 Cosmic Frontier White Paper: Ultraheavy particle dark matter, arXiv:2203.06508.

[16] Brando Bellazzini, Csaba Csáki, and Javi Serra, Composite Higgses, Eur. Phys. J. C **74**, 2766 (2014).

[17] Giuliano Panico and Andrea Wulzer, *The Composite Nambu-Goldstone Higgs* (Springer, Cham, 2016), Vol. 913.

[18] Lawrence J. Hall, Karsten Jedamzik, John March-Russell, and Stephen M. West, Freeze-in production of FIMP dark matter, J. High Energy Phys. 03 (2010) 080.

[19] Fatemeh Elahi, Christopher Kolda, and James Unwin, UltraViolet freeze-in, J. High Energy Phys. 03 (2015) 048.

[20] Nima Arkani-Hamed, Massimo Porrati, and Lisa Randall, Holography and phenomenology, J. High Energy Phys. 08 (2001) 017.

[21] R. Rattazzi and A. Zaffaroni, Comments on the holographic picture of the Randall-Sundrum model, J. High Energy Phys. 04 (2001) 021.

[22] Lisa Randall and Raman Sundrum, A large mass hierarchy from a small extra dimension, Phys. Rev. Lett. **83**, 3370 (1999).

[23] Roberto Contino, Yasunori Nomura, and Alex Pomarol, Higgs as a holographic pseudoGoldstone boson, Nucl. Phys. **B671**, 148 (2003).

[24] Johanna Erdmenger, Nick Evans, Werner Porod, and Konstantinos S. Rigatos, Gauge/gravity dynamics for composite Higgs models and the top mass, Phys. Rev. Lett. **126**, 071602 (2021).

[25] Nicolás Bernal, Matti Heikinheimo, Tommi Tenkanen, Kimmo Tuominen, and Ville Vaskonen, The dawn of FIMP dark matter: A review of models and constraints, Int. J. Mod. Phys. A **32**, 1730023 (2017).

[26] Sungwoo Hong, Gowri Kurup, and Maxim Perelstein, Conformal freeze-in of dark matter, Phys. Rev. D **101**, 095037 (2020).

[27] Sungwoo Hong, Gowri Kurup, and Maxim Perelstein, Dark matter from a conformal dark sector, J. High Energy Phys. 02 (2023) 221.

[28] Wen Han Chiu, Sungwoo Hong, and Lian-Tao Wang, Conformal freeze-in, composite dark photon, and asymmetric reheating, J. High Energy Phys. 03 (2023) 172.

[29] Kfir Blum, Mathieu Cliche, Csaba Csaki, and Seung J. Lee, WIMP dark matter through the dilaton portal, J. High Energy Phys. 03 (2015) 099.

[30] Benedict von Harling and Kristian L. McDonald, Secluded dark matter coupled to a hidden CFT, J. High Energy Phys. 08 (2012) 048.

[31] Kristian L. McDonald, Sommerfeld enhancement from multiple mediators, J. High Energy Phys. 07 (2012) 145.

[32] Kristian L. McDonald and David E. Morrissey, Low-energy signals from kinetic mixing with a warped Abelian hidden sector, J. High Energy Phys. 02 (2011) 087.

[33] Kristian L. McDonald and David E. Morrissey, Low-energy probes of a warped extra dimension, J. High Energy Phys. 05 (2010) 056.

[34] Philippe Brax, Sylvain Fichet, and Philip Tanedo, The warped dark sector, Phys. Lett. B **798**, 135012 (2019).

[35] Csaba Csáki, Sungwoo Hong, Gowri Kurup, Seung J. Lee, Maxim Perelstein, and Wei Xue, Continuum dark matter, Phys. Rev. D **105**, 035025 (2022).

[36] Csaba Csáki, Sungwoo Hong, Gowri Kurup, Seung J. Lee, Maxim Perelstein, and Wei Xue, Z-portal continuum dark matter, Phys. Rev. Lett. **128**, 081807 (2022).

[37] Csaba Csaki, Ameen Ismail, and Seung J. Lee, The continuum dark matter zoo, J. High Energy Phys. 02 (2023) 053.

[38] Sylvain Fichet, Eugenio Megias, and Mariano Quiros, Continuum effective field theories, gravity, and holography, Phys. Rev. D **107**, 096016 (2023).

[39] Sylvain Fichet, Eugenio Megias, and Mariano Quiros, Cosmological dark matter from a bulk black hole, Phys. Rev. D **107**, 115014 (2023).

[40] Nicolás Bernal, Andrea Donini, Miguel G. Folgado, and Nuria Rius, Kaluza-Klein FIMP dark matter in warped extra-dimensions, J. High Energy Phys. 09 (2020) 142.

[41] Nicolás Bernal, Andrea Donini, Miguel G. Folgado, and Nuria Rius, FIMP dark matter in clockwork/linear dilaton extra-dimensions, J. High Energy Phys. 04 (2021) 061.

[42] Arturo de Giorgi and Stefan Vogl, Dark matter interacting via a massive spin-2 mediator in warped extra-dimensions, J. High Energy Phys. 11 (2021) 036.

[43] Arturo de Giorgi and Stefan Vogl, Warm dark matter from a gravitational freeze-in in extra dimensions, J. High Energy Phys. 04 (2023) 032.

[44] Michele Frigerio, Alex Pomarol, Francesco Riva, and Alfredo Urbano, Composite scalar dark matter, J. High Energy Phys. 07 (2012) 015.

[45] Mikael Chala, $h \to \gamma\gamma$ excess and dark matter from composite Higgs models, J. High Energy Phys. 01 (2013) 122.

[46] David Marzocca and Alfredo Urbano, Composite dark matter and LHC interplay, J. High Energy Phys. 07 (2014) 107.

[47] Nayara Fonseca, Renata Zukanovich Funchal, Andre Lessa, and Laura Lopez-Honorez, Dark matter constraints on composite Higgs models, J. High Energy Phys. 06 (2015) 154.

[48] Manki Kim, Seung J. Lee, and Alberto Parolini, WIMP dark matter in composite Higgs models and the dilaton portal, arXiv:1602.05590.

[49] Yongcheng Wu, Teng Ma, Bin Zhang, and Giacomo Cacciapaglia, Composite dark matter and Higgs, J. High Energy Phys. 11 (2017) 058.

[50] Guillermo Ballesteros, Adrian Carmona, and Mikael Chala, Exceptional composite dark matter, Eur. Phys. J. C **77**, 468 (2017).

[51] Reuven Balkin, Maximilian Ruhdorfer, Ennio Salvioni, and Andreas Weiler, Charged composite scalar dark matter, J. High Energy Phys. 11 (2017) 094.

[52] Tommi Alanne, Diogo Buarque Franzosi, Mads T. Frandsen, and Martin Rosenlyst, Dark matter in (partially) composite Higgs models, J. High Energy Phys. 12 (2018) 088.

[53] Reuven Balkin, Maximilian Ruhdorfer, Ennio Salvioni, and Andreas Weiler, Dark matter shifts away from direct detection, J. Cosmol. Astropart. Phys. 11 (2018) 050.

[54] Giacomo Cacciapaglia, Haiying Cai, Aldo Deandrea, and Ashwani Kushwaha, Composite Higgs and dark matter model in SU(6)/SO(6), J. High Energy Phys. 10 (2019) 035.

[55] Maria Ramos, Composite dark matter phenomenology in the presence of lighter degrees of freedom, J. High Energy Phys. 07 (2020) 128.

[56] Haiying Cai and Giacomo Cacciapaglia, Singlet dark matter in the SU(6)/SO(6) composite Higgs model, Phys. Rev. D **103**, 055002 (2021).

[57] Mathias Garny, McCullen Sandora, and Martin S. Sloth, Planckian interacting massive particles as dark matter, Phys. Rev. Lett. **116**, 101302 (2016).

[58] Mathias Garny, Andrea Palessandro, McCullen Sandora, and Martin S. Sloth, Theory and phenomenology of Planckian interacting massive particles as dark matter, J. Cosmol. Astropart. Phys. 02 (2018) 027.

[59] Marco Chianese, Bowen Fu, and Stephen F. King, Minimal seesaw extension for neutrino mass and mixing, leptogenesis and dark matter: FIMPzillas through the right-handed neutrino portal, J. Cosmol. Astropart. Phys. 03 (2020) 030.

[60] Howard Georgi, Unparticle physics, Phys. Rev. Lett. **98**, 221601 (2007).

[61] Kaustubh Agashe, Roberto Contino, and Alex Pomarol, The minimal composite Higgs model, Nucl. Phys. **B719**, 165 (2005).

[62] Kaustubh Agashe, Antonio Delgado, Michael J. May, and Raman Sundrum, RS1, custodial isospin and precision tests, J. High Energy Phys. 08 (2003) 050.

[63] Benedict von Harling and Geraldine Servant, QCD-induced electroweak phase transition, J. High Energy Phys. 01 (2018) 159.

[64] Kiwoon Choi and Sang Hui Im, Realizing the relaxion from multiple axions and its UV completion with high scale supersymmetry, J. High Energy Phys. 01 (2016) 149.

[65] David E. Kaplan and Riccardo Rattazzi, Large field excursions and approximate discrete symmetries from a clockwork axion, Phys. Rev. D **93**, 085007 (2016).

[66] Gian F. Giudice and Matthew McCullough, A clockwork theory, J. High Energy Phys. 02 (2017) 036.

[67] Eva Silverstein and Alexander Westphal, Monodromy in the CMB: Gravity waves and string inflation, Phys. Rev. D **78**, 106003 (2008).

[68] Michele Redi and Andrea Tesi, General freeze-in and freeze-out, J. High Energy Phys. 12 (2021) 060.

[69] N. Aghanim *et al.* (Planck), Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. **641**, A6 (2020); **652**, C4(E) (2021).

[70] R. L. Workman *et al.* (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).

[71] P. A. R. Ade *et al.* (BICEP, Keck Collaborations), Improved constraints on primordial gravitational waves using Planck, WMAP, and BICEP/Keck observations through the 2018 observing season, Phys. Rev. Lett. **127**, 151301 (2021).

[72] Y. Akrami *et al.* (Planck Collaboration), Planck 2018 results. X. Constraints on inflation, Astron. Astrophys. **641**, A10 (2020).

[73] Kevork N. Abazajian *et al.* (CMB-S4 Collaboration), CMB-S4 Science Book, First Edition, arXiv:1610.02743.

[74] Nima Arkani-Hamed and Juan Maldacena, Cosmological collider physics, arXiv:1503.08043.

[75] Daniel Carney, Sohitri Ghosh, Gordan Krnjaic, and Jacob M. Taylor, Proposal for gravitational direct detection of dark matter, Phys. Rev. D **102**, 072003 (2020).

[76] Alaina Attanasio *et al.* (Windchime), Snowmass 2021 white paper: The Windchime project, arXiv:2203.07242.

[77] Walter D. Goldberger and Mark B. Wise, Modulus stabilization with bulk fields, Phys. Rev. Lett. **83**, 4922 (1999).

[78] Paolo Creminelli, Alberto Nicolis, and Riccardo Rattazzi, Holography and the electroweak phase transition, J. High Energy Phys. 03 (2002) 051.

[79] Lisa Randall and Geraldine Servant, Gravitational waves from warped spacetime, J. High Energy Phys. 05 (2007) 054.

[80] Thomas Konstandin and Geraldine Servant, Cosmological consequences of nearly conformal dynamics at the TeV scale, J. Cosmol. Astropart. Phys. 12 (2011) 009.

[81] Pietro Baratella, Alex Pomarol, and Fabrizio Rompineve, The supercooled universe, J. High Energy Phys. 03 (2019) 100.

[82] Babiker Hassanain, John March-Russell, and Martin Schvellinger, Warped deformed throats have faster (electroweak) phase transitions, J. High Energy Phys. 10 (2007) 089.

[83] Don Bunk, Jay Hubisz, and Bithika Jain, A perturbative RS I cosmological phase transition, Eur. Phys. J. C **78**, 78 (2018).

[84] Barry M. Dillon, Basem Kamal El-Menoufi, Stephan J. Huber, and Jonathan P. Manuel, Rapid holographic phase transition with brane-localized curvature, Phys. Rev. D **98**, 086005 (2018).

[85] Eugenio Megías, Germano Nardini, and Mariano Quirós, Cosmological phase transitions in warped space: Gravitational waves and collider signatures, J. High Energy Phys. 09 (2018) 095.

[86] Kaustubh Agashe, Peizhi Du, Majid Ekhterachian, Soubhik Kumar, and Raman Sundrum, Cosmological phase transition of spontaneous confinement, J. High Energy Phys. 05 (2020) 086.

[87] Csaba Csáki, Michael Geller, Zamir Heller-Algazi, and Ameen Ismail, Relevant dilaton stabilization, J. High Energy Phys. 06 (2023) 202.

[88] Cem Eröncel, Jay Hubisz, Seung. J. Lee, Gabriele Rigo, and Bharath Sambasivam, New horizons in the holographic conformal phase transition, arXiv:2305.03773.

[89] Prateek Agrawal and Michael Nee, Avoided deconfinement in Randall-Sundrum models, J. High Energy Phys. 10 (2021) 105.

[90] Shamit Kachru, Renata Kallosh, Andrei D. Linde, Juan Martin Maldacena, Liam P. McAllister, and Sandip P. Trivedi, Towards inflation in string theory, J. Cosmol. Astropart. Phys. 10 (2003) 013.
[91] Bruce A. Bassett, Shinji Tsujikawa, and David Wands, Inflation dynamics and reheating, Rev. Mod. Phys. **78**, 537 (2006).
[92] Michael R. Douglas and Shamit Kachru, Flux compactification, Rev. Mod. Phys. **79**, 733 (2007).
[93] Liam McAllister, Eva Silverstein, and Alexander Westphal, Gravity waves and linear inflation from axion monodromy, Phys. Rev. D **82**, 046003 (2010).
[94] Daniel Baumann and Liam McAllister, *Inflation and String Theory*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, England, 2015).
[95] Paolo Gondolo and Graciela Gelmini, Cosmic abundances of stable particles: Improved analysis, Nucl. Phys. **B360**, 145 (1991).