Corrections of Z' to the magnetic moment of the muon

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We study several previous corrections and contributions to the muon $g-2$, starting from the dark photon hypothesis to the dark Z. We explore the inputs from a dark Z boson virtual mediator in a first order loop. We consider not only the QED-like contributions in the theory but also weak interactions. We obtain a new factor that adds corrections to the form factor associated with the anomalous magnetic moment. We show that our result is favorable in new, unexplored windows in the mass-coupling parameter space.

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I. INTRODUCTION

The search for dark matter has been well established as one of the great challenges of physics today. Dark matter's eventual discovery will allow us to understand many puzzles for which the scientific community does not have an explanation based on the currently accepted Standard Model (SM) of particle physics [1–9].

Interestingly, the search for dark matter allows us to address another fundamental problem of the Standard Model, namely, the puzzle of the gyromagnetic factor of the muon [10–14], where a discrepancy between theory and experiment stands to this day, and its value is

$$a_\mu^{\text{Exp}} - a_\mu^{\text{Th}} = (251 \pm 59) \times 10^{-11}, \tag{1}$$

suggesting signals of new physics. The results of the Fermilab experiment E989 confirm those found previously at BNL and will be tested once again at the J-PARC experiment in Japan [15,16].

In the case of electrons, one of the triumphs of quantum field theory was the calculation of the anomalous magnetic moment by Schwinger in 1948 [17–19]. The "anomaly" (a) refers to the fact that, in contrast to what the Dirac equation says, the Landé factor (g) has quantum corrections $a = (g-2)/2 = \alpha/(2\pi)$; to be more specific, we note the discrepancy described above, which has a prediction by the Standard Model with a relative precision of 4×10^{-8}. In the case of the electron anomalous magnetic moment, this quantity (a) has been calculated up to fifth-loop level [20], with the best measurement being 0.25 parts per billion, providing an exquisite determination of the fine structure constant α [21]. However, for the muon, any discrepancy from what is expected by theory opens a window to new physics, which is why so many efforts have been set to measure, with better accuracy, the muon anomalous magnetic moment (a_μ). This difference has been fixed as 4.2σ, the strongest discrepancy in history for this anomaly [22,23].

Different dark matter particles remain among the candidates that could explain this discrepancy. The exact mechanism by which the visible and hidden sectors are connected is, of course, currently not known, but with some certainty, the relationship between the visible and dark sectors should be dictated by an idea similar to a kinetic mixing [24–29]. Kinetic mixing is a natural and simple procedure that, after an appropriate Lagrangian diagonalization procedure, allows us to define modified vertices that induce processes between visible and dark matter [30].

The so-called "dark photon" is a popular possible dark matter candidate for relatively small masses [O(MeV)], and it would give a contribution to the anomalous magnetic moment by coupling to the Standard Model through a kinetic mixing, as mentioned above [25]. This particle is a hypothetical vector boson, and for the most part, it has almost been ruled out by different experimental searches within the parameter space of masses which would be favorable in this scenario [31].

The next natural candidate is known in the literature as the "dark Z boson," coupled through kinetic and mass mixing with the electroweak sector of the SM [12,32,33].

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A dark Z (DZ) differentiates from the dark photon (DP) in its mass mixing with the ordinary Standard Model Z boson and in its weak interactions. The prospect of searches for this new particle has been proposed as part of the incoming International Linear Collider (ILC), suggesting that with only one month of data, it would be possible to obtain a measurement of its chiral couplings to fermions with precision of the order of percents [32].

The ideas outlined above concerning a DZ are technically implemented here as follows: Instead of the group $U(1)_Y$ of the Standard Model, we have the extension $U(1)_Y \times U'(1)$, where $U'(1)$ denotes a gauge boson, which we will call Z'_μ or dark Z throughout the text. At the Lagrangian level, for the general and commonly used kinetic mixing procedure, instead of the usual kinetic terms $F_{\mu\nu}(A)F^{\mu\nu}(A)$ and $F_{\mu\nu}(Z)F^{\mu\nu}(Z)$, the mixing $\epsilon F_{\mu\nu}(A)F^{\mu\nu}(Z)$ must be added, where ϵ is a parameter that should be set by experiments.

However, the presence of the ϵ parameter—and eventually the mass of the hidden gauge boson—adds ingredients that lead to nontrivial dynamical modifications, which can be analyzed with data from current experiments.

Future experiments previously mentioned that allow one to limit the parameters of kinetic mixing will be of central importance in the years to come for the discrepancy between theory and experiment in this case.

Nevertheless, this poses another interesting challenge; that is to say, if Z' (DZ) physics is a good idea beyond the Standard Model, then it should be able to induce significant corrections at one loop to improve the agreement between theory and experiment in the case of the muon's anomalous magnetic moment (for some recent references see, for example, Refs. [34–38]).

We review the previous work of several authors, starting from the DP scenario and continuing with the kinetic mixing setup for the dark $U(1)$ boson [$U(1)_D$]; we extend the panorama with a weak kinetic and mass mixing for the DZ [33] and look for the contributions to the muon anomalous magnetic moment that this framework would convey.

A better understanding of the anomaly a_μ could not only lead to new physics, but it could also make it possible to determine if the muon is a composite particle, as it has been theorized, in contrast to its equivalent, the electron [21]. The most recent 4.2σ discrepancy from Fermilab's experiment brings the community closer to the 5σ discovery level required to claim that the Standard Model of particle physics is not able to explain the anomaly, but it is not enough to say with certainty, although this measurement strengthens the evidence for new physics. Future experiments reaching this point doubt the existence of many beyond-the-Standard Model theories, establishing strong limits on them.

A completely different and independent approach to measuring this anomaly will be available in the future experiment currently being built at J-PARC in Japan. The main novelty of this experiment is the use of muons cooled down to a few MeV, which allows them to obtain a high-quality muon beam and enables experiments with much less systematic error than current methods [21,39]. J-PARC is the only experimental effort with current and future plans to measure the muon $g-2$, and it will start engineering and physics runs in 2027 [40].

The paper is organized as follows. In Sec. II we revisit previous constraints on the dark photon parameter space and show how our method replicates these results. In Sec. III we extend our results to a dark boson, in general, with a $U(1)_Y$ group and compare with the literature. Sec. IV shows the contributions coming from electroweak interactions among a dark boson and muons. We discuss our findings and conclude in Sec. V.

II. REVIEW OF PREVIOUS RESULTS

A minimal extension of the Standard Model is a $U(1)$ group, coupled through a kinetic mixing to the visible photon. There are many publications explaining the detail of this mechanism, and numerous examples can be found the literature. We suggest some examples in [24–29,41].

To fix the notation we follow our previously used prescription [17,42]. We identify as q_1^μ and q_2^μ the incoming and outgoing momenta; then the transferred momentum would be $p^\mu = q_1^\mu - q_2^\mu$, according to what we have set and shown in Fig. 1. Under such conditions, the structure of the vertex function has the form

$$i\mathcal{M}^\mu = -ie\bar{u}(q_2)\Gamma^\mu u(q_1), \tag{2}$$

with the Γ^μ function given by

$$\Gamma^\mu = \gamma^\mu F_1\left(\frac{p^2}{m_\mu^2}\right) + \frac{\sigma^{\mu\nu}}{2m_\mu} p_\nu F_2\left(\frac{p^2}{m_\mu^2}\right), \tag{3}$$

where F_1 and F_2 are two Lorentz invariant form factors.

If we draw our attention to the third-order interaction term in the electric charge—for example, the diagram in Fig. 1(b), with a photon in the loop—the scattering amplitude becomes

$$i\mathcal{M}_b^\mu = -e^3\epsilon^2\bar{u}(q_2)\int\frac{d^4k}{(2\pi)^4}\frac{\eta_{\alpha\beta}\gamma^\alpha(\not{p}+\not{k}+m_\mu)\gamma^\mu(\not{k}+m_\mu)\gamma^\beta}{[(k-q_1)^2+i\varepsilon][(p+k)^2-m_\mu^2+i\varepsilon][k^2-m_\mu^2]+i\varepsilon]}u(q_1), \tag{4}$$

while for Fig. 1(c), where the loop carries a massive boson, we have

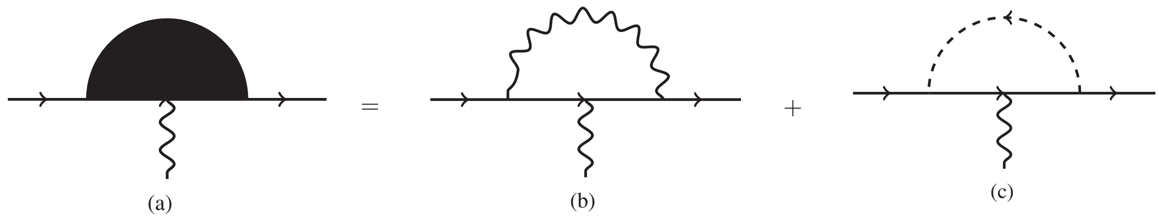

FIG. 1. Two possible vertex corrections involved in the gyromagnetic factor of dark matter (dashed lines represent Z' propagators). Diagram (a) is the total anomalous magnetic moment contribution to the muon vertex. This diagram represents the sum of all possible contributions; here we list only two. Diagram (b) is the QED type contribution from a dark vector boson, the dark photon. The last diagram (c) shows the contribution of a weak vertex from a Z' boson.

$$i\mathcal{M}_c^\mu = -e^3\bar{u}(q_2)\int\frac{d^4k}{(2\pi)^4}\frac{\eta_{\alpha\beta}\gamma^\alpha(\not{p}+\not{k}+m_\mu)\gamma^\mu(\not{k}+m_\mu)\gamma^\beta}{[(k-q_1)^2-m_Z^2+i\epsilon][(p+k)^2-m_\mu^2+i\epsilon][k^2-m_\mu^2]+i\epsilon]}u(q_1), \tag{5}$$

where m_Z represents the mass of the boson in question, independently of it being a dark photon, a Z boson, or a dark Z.

The interesting question is how to calculate the form factors, with the purpose of obtaining our desired contributions to a_μ. To do this, we first note that the physical interpretation of F_1 and F_2 is clear: F_1 leads to charge renormalization while F_2 is a direct contribution to the magnetic moment.

Taking the above into account, we will concentrate on properly isolating each diagram corresponding to Eqs. (4) and (5), so we can identify the factors proportional to $\sigma^{\mu\nu}$.

If we use Gordon's identity, after some algebra we find

$$F_2^{(c)}(p^2) = -i8m_\mu^2e^2\epsilon^2\int_0^1 dxdydz\delta(x+y+z-1) \times\int\frac{d^4k}{(2\pi)^4}\frac{z(1-z)}{(k^2-\Delta+i\epsilon)^3}+\cdots \tag{6}$$

with $\Delta = -xyp^2+(1-z)^2m_\mu^2+m_Z^2z$ and x, y, and z Feynman parameters [17].

Once $F_2(p^2)$ is given, then we evaluate it at the limit $p\to 0$, and we obtain

$$a_\mu = \frac{g_\mu-2}{2} = F_2(0). \tag{7}$$

In the limit $m_Z\to 0$ we obtain $F_2(0)=\frac{\alpha}{2\pi}$ according to the classical Schwinger result for the massless SM photon [17,18].

If $m_Z\neq 0$, the result is

$$a_\mu = \frac{\alpha}{2\pi}(\epsilon^2)f(\kappa), \tag{8}$$

with $f(\kappa)$, coming from the above integration, defined as

$$f(\kappa) = \left\{1-2\kappa^2+2(\kappa^2-2)\kappa^2\log(\kappa) - \frac{(\kappa^4-4\kappa^2+2)\kappa\tan^{-1}\left((\sqrt{4-\kappa^2}/\kappa)\right)}{\sqrt{4-\kappa^2}}\right\}. \tag{9}$$

This analytic expression is equivalent to that in [41] for Eq. (4), and we can reproduce those results, as can be seen in Fig. 2 in the case of a dark photon and in Fig. 3 for a dark Z. Both figures show the contribution from our analytical expression in Eq. (9). We take this point as the motivation for our work since the dark-photon-hypothesis-favored parameter space has been severely tested and not found, as shown in Fig. 2. We seek new unexplored regions that can be enlightened by searches related to a dark Z boson. Furthermore, Fig. 3 shows that the same mechanism

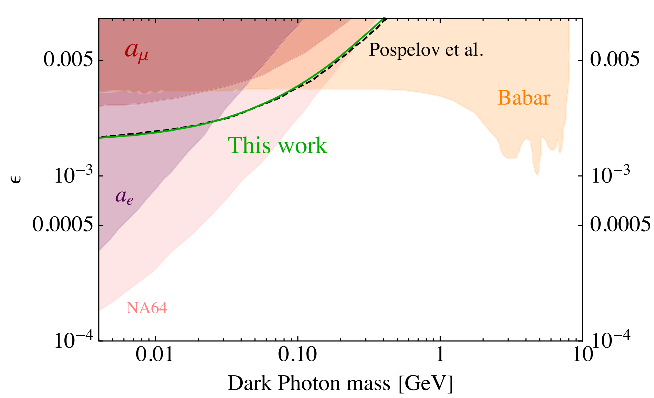

FIG. 2. Our analytic expression in Eq. (9) (green) for the numerical results previously outlined by Pospelov *et al.* (see Ref. [41]) marked with a black dashed line. This is the case of a $U(1)$ sector coupled to photons. For comparison we show known restricted zones from *BABAR* [43,44] and NA64 experiments [45], along with SM results for the exclusion in parameter space to the electron and muon anomalous magnetic moment. Constraints are adapted from [45].

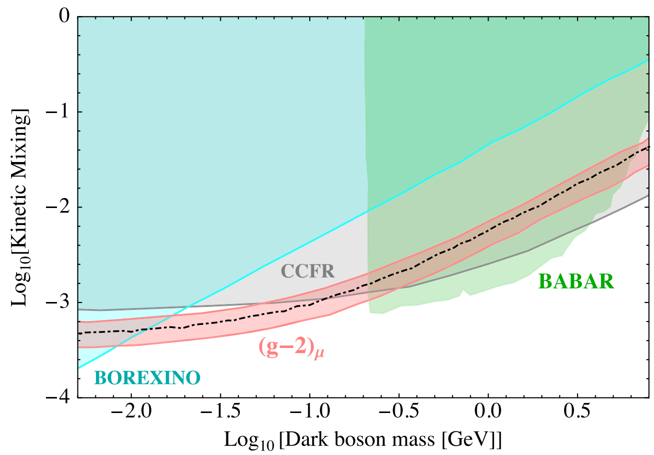

FIG. 3. Our analytic expression in Eq. (9) (black dashed line). This is the $U(1)$ coupling to SM. Constraints on the dark boson mass and kinetic mixing from [46] can be seen as shaded regions, as can their result for the muon $g-2$ using this symmetry; again (as in Fig. 2) our result explains, in a complete and legible expression, the previous result fitted to the anomaly. *BABAR*'s excluded region comes from searches of Z' or dark Z from the production of $\mu^-\mu^+Z'$ at colliders [47]. Chicago-Columbia-Fermilab-Rochester (CCFR) comes from measurements of the neutrino trident cross section [48], and Borexino comes from neutrino electron scattering [49]. In Fig. 4 we use these constraints, but we include a mass mixing and a hypercharge coupling to the SM. The figure was adapted from [46].

used to obtain constraints for the case of the DZ has a small parameter space for future searches.

It is important to highlight that this correction has been obtained assuming that only QED-like couplings contribute to Δa_μ in the case of a massive virtual boson in the loop. A similar approach has been taken by Arcadi *et al.* [50].

In the next section, we explore the contributions of electroweaklike interactions.

III. CONSEQUENCES OF DARK ELECTROWEAK INTERACTIONS WITH THE Z' BOSON

In the Standard Model, the kinetic terms of the Lagrangian after electroweak symmetry breaking are

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}Z_{\mu\nu}^2, \tag{10}$$

and the neutral currents coupled to the Z and photon bosons can be written as

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \frac{e}{\sin\theta_W}Z_\mu J_\mu^Z + eA_\mu J_\mu^{\text{EM}}, \tag{11}$$

where we should keep in mind that $e = g\sin\theta_W = g'\cos\theta_W$ comes from the covariant derivative $D_\mu H = \partial_\mu H - igW_\mu^a\tau^a H - 1/2ig'B_\mu H$, with W_μ^a and B_μ the $SU(2)$ and $U(1)_Y$ gauge bosons, respectively, and θ_W the usual Weinberg angle. The explicit currents, J_μ^Z and J_μ^{EM}, are

$$J_\mu^Z = \frac{1}{\cos\theta_W}(J_\mu^3 - \sin^2\theta_W J_\mu^{\text{EM}}), \tag{12}$$

$$J_\mu^3 = \sum_i \bar{\Psi}_i^L\gamma_\mu T^3\psi_i^L, \qquad J_\mu^{\text{EM}} = \sum_i (T^3+Y)\bar{\Psi}_i^L\gamma_\mu\psi_i^L. \tag{13}$$

The Lagrangian including neutral current interactions, Eq. (11), can also be expressed as

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \frac{g}{\cos\theta_W}\bar{\Psi}\gamma^\mu\left[C_L\frac{1-\gamma^5}{2} + C_R\frac{1+\gamma^5}{2}\right]\Psi + eA_\mu J_\mu^{\text{EM}}, \tag{14}$$

where

$$C_L = T^3 - Q\sin^2\theta_W C_R = -Q\sin^2\theta_W. \tag{15}$$

If we now mix B_μ and Z'_μ which live in $U(1)_Y$ and $U(1)'$ associated with the hypercharge and dark gauge symmetry, respectively, then the dynamics of the two gauge fields interacting through a kinetic mixing is

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}(B)F^{\mu\nu}(B) - \frac{1}{4}F_{\mu\nu}(Z')F^{\mu\nu}(Z') + \frac{\epsilon}{2\cos\theta_W}F_{\mu\nu}(B)F^{\mu\nu}(Z'), \tag{16}$$

where

$$F_{\mu\nu}(B) = \partial_\mu B_\nu - \partial_\nu B_\mu, \qquad F_{\mu\nu}(Z') = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu,$$

and ϵ is a small dimensionless parameter, usually denoted as a kinetic mixing term.

The Lagrangian (16) can be diagonalized by performing the transformation $B' = B + \epsilon/\cos\theta_W Z'$, from which we obtain

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}(B)F^{\mu\nu}(B) - \frac{1}{4}\left(1 - \frac{\epsilon^2}{\cos\theta_W^2}\right)F_{\mu\nu}(Z')F^{\mu\nu}(Z'), \tag{17}$$

so the only effect of kinetic mixing is to redefine the charge.

If we include fermion fields (muons), the Lagrangian would be

$$\left(-e\epsilon J_{em}^\mu + \frac{g}{\cos\theta_W}\epsilon_Z J_\mu^Z\right)Z^{\mu'}, \tag{18}$$

where we see that the diagonalization of the Lagrangian (17) modifies the couplings, giving the next interaction term:

$$\mathcal{L} \supset \frac{-g}{2\cos\theta_W}\varepsilon_Z J_\mu^{Z'} Z^{\mu'}. \tag{19}$$

As an important observation, we point out that the inclusion of kinetic mixing induces a nontrivial redefinition of the parameters of the responsible theory—on the one hand, for the appearance of millicharges ϵe, and on the other, for the need to adjust, according to experimental data, the redefinitions of the effective coupling constants.

The above expression resembles the QED Lagrangian in calculating the muon magnetic moment for the relevant diagrams corresponding to Fig. 1. The first diagram contains the conventional electromagnetic interaction, while the second is due to the interaction with a massive boson, in our case, the Z' boson. Furthermore, the above Eq. (19) is the interaction term for a m_Z boson with photonlike coupling to the fermion. Suppose we consider the electroweak corrections to the $g-2$ regarding a Z. In that case, our result can be verified following the Particle Data Group results for electroweak contributions. Still, in this case, we have to rearrange terms depending on the unknown mass of this new particle [51,52].

The kinetic mixing is generalized by including a mass mixing between the Standard Model Z and the Z' with the introduction of the mass matrix

$$M_0^2 = m_Z^2\begin{pmatrix} 1 & -\varepsilon_Z \\ -\varepsilon_Z & m_{Z'}^2/m_Z^2 \end{pmatrix}, \tag{20}$$

with the mixing parametrized by

$$\varepsilon_Z = \frac{m_{Z'}}{m_Z}\delta. \tag{21}$$

The degree of mass mixing between the Standard Model Z and the dark Z is given by the δ parameter above. The former conveys a decay channel for the Higgs of the form $H \to ZZ'$, which allows us to set constraints from atomic parity violation, polarized e scattering, and rare K and B decay. We initially set bounds in parameter space using values from Davoudiasl *et al.* [33], but since their work and the experimental discovery of the Higgs boson, δ has been further constrained.

IV. Z' CORRECTION

In the case of electroweak corrections, we have studied the amplitude calculations with the Feyncalc and FenyArts *Mathematica* package [53] to replicate the PDG's result on the contributions to the muon anomalous magnetic moment. We can recover the terms coming from loops involving the Higgs, Z boson, and neutrinos (or any of the weak bosons).

To procure these contributions we have followed the steps of [54–56], and as a result, we obtain the amplitude

$$\begin{aligned} i\mathcal{M}_c^\mu = -\bar{u}(p')\int \frac{d^4k}{(2\pi)^4}\frac{1}{((k+p)^2-m_\mu^2)\cdot((k+p')^2-m_\mu^2)\cdot(k^2-m_Z^2)} \\ \times\left\{\left(-ie\frac{\gamma^\nu(1-\gamma^5)(\sin\theta_W^2-1/2)}{2\sin\theta_W\cos\theta_W}-ie\frac{\gamma^\nu(1+\gamma^5)\sin\theta_W}{2\cos\theta_W}\right)((\not{k}+\not{p}')+m_\mu)\cdot\gamma^\mu\cdot((\not{k}+\not{p})+m_\mu)\right. \\ \left.\times\left(-ie\frac{\gamma^\nu(1-\gamma^5)(\sin\theta_W^2-1/2)}{2\sin\theta_W\cos\theta_W}-ie\frac{\gamma^\nu(1+\gamma^5)\sin\theta_W}{2\cos\theta_W}\right)\right\}u(p). \end{aligned} \tag{22}$$

From this result, we can obtain the form factors, as was done before in the case of a dark photon; the new interaction terms will give us new corrections to the vertex for a dark Z, but first, we will shortly review the appropriate Gordon identity in the following section.

A. Gordon identity for weak contribution

We use the Pauli form factor defined by the most general form of current conservation and CP invariance [56], which differs from what is usually seen in the QED vertex corrections.

The modified vertex has the form

$$\begin{aligned} &\bar{u}(p+q)\Lambda_\mu u(p) \\ &\quad = -ie\bar{u}(p+q)\left(\gamma_\mu F_1(q^2)+\frac{i}{2m_\mu}\sigma_{\mu\nu}q^\nu F_2(q^2)\right. \\ &\qquad \left.+(q^2\gamma_\mu - q\cdot\gamma q_\mu)\gamma_5 F_3(q^2)\right)u(p). \end{aligned} \tag{23}$$

The contribution to the form factors can be arranged in terms of the factors proportional to γ_μ, $\sigma_{\mu\nu}q^\nu$, and γ_5 [56].

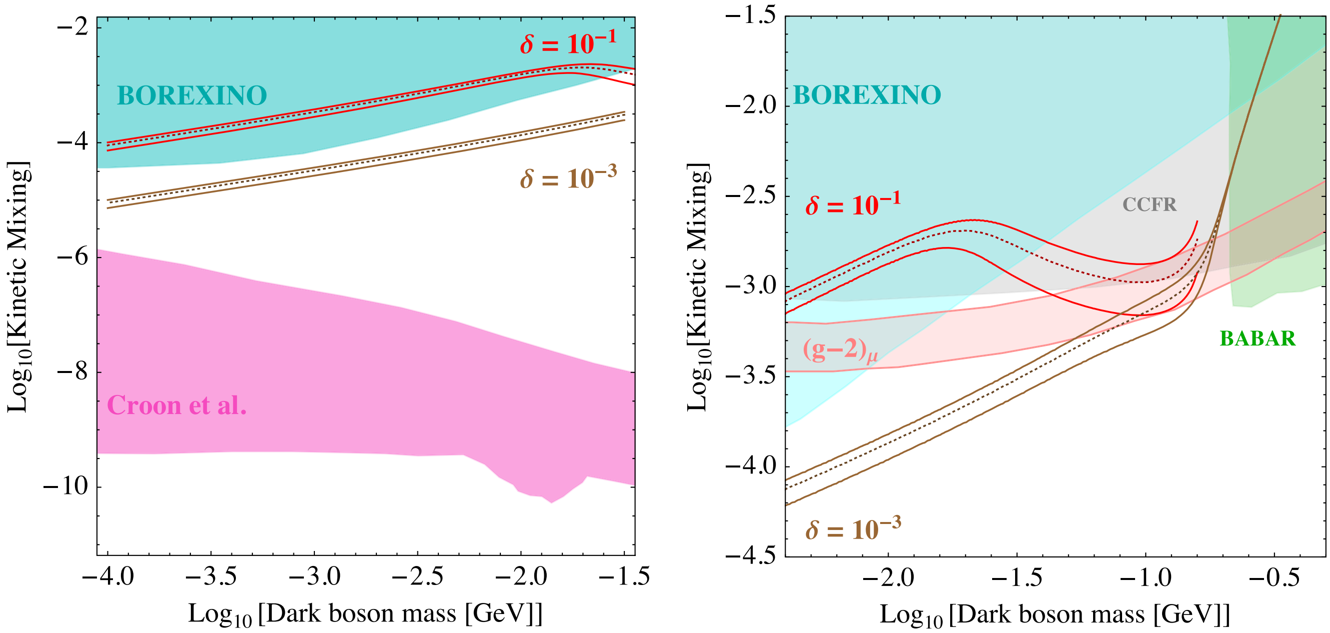

FIG. 4. Constraints for the DZ in the kinetic mixing versus boson mass parameter space in two regimes of the mass mixing parameter δ from Eq. (26). We set δ as 10^{-3} and 10^{-1}, shown as the brown and red contours, respectively. The dotted lines represent the exact comparison between our result and the anomaly, Eq. (1). The solid lines represent the 2σ allowed region. Left: in fuchsia, constraints outlined by Croon *et al.* [58] on supernova muons coupled to Z' and Borexino, from Fig. (3). Right: in pink, exclusion zone from previous work in the dark photon-QED-like approach to a DZ [46], as shown in Fig. (3) and coming from simply setting the masses for the boson in Eq. (9), plus the constraints from *BABAR*, Borexino, and CCFR as described in the Sec. V.

From Eq. (22) we can see that several terms would contribute to F_3, but, after some algebra, some terms will take the shape of our desired form factor F_2, which will contribute to the correction of the magnetic momentum.

We start by selecting the Feynman diagrams associated with (1), and we work with the amplitude shown in (22). To the resulting expression, we apply the Gordon identity as described in Eq. (23), and we identify the corresponding contribution to the form factor F_2.

We expect to obtain a contribution proportional to the one in the Standard Model correction of a_μ since the propagators have a similar shape and the main differences are the masses and constants. This is an important contrast with respect to our previous work [42]; in the case of a dark-QED, we go from a massless photon to a massive one, and the propagators have a different form.

With the help of `FeynCalc`, we obtain the corresponding form factor F_2 in terms of Passarino-Veltman integrals, here shown as the different C_i coefficients:

$$i\mathcal{M}_c^\mu = C_0 + 2C_1 + (8\sin^4\theta_W - 4\sin^2\theta_W + 1)(C_1 + C_{11} + C_{12}) \propto F_2, \tag{24}$$

where C_0, C_1, C_{11}, and C_{12} are the integrals depending on combinations of the masses involved [57], that is, the muon and Z in the present case.

We rapidly check the Standard Model result using `Package X` [57] on these integrals. By approximating and expanding in a series on the ratio of the masses m_μ/m_Z, we recover the result below, for contributions of weak interactions to the F_2, as can be seen in [12]:

$$a_\mu \propto \frac{\pi^2}{\sqrt{2}} G_W m_\mu^2 \frac{1}{3}(5 - (1 - 4\sin^2\theta_W)^2). \tag{25}$$

To take all the contributions adding up to the Δa_μ from dark matter in Eq. (24), we define the ratio $\tau = M_{Z'}/m_\mu$, and each component takes the form

$$a_\mu(Z) = \frac{\alpha}{2\pi}(\varepsilon^2)\{\bar{f}(\tau) + [8\sin^4\theta'_W - 4\sin^2\theta'_W + 1]f(\tau)\}, \tag{26}$$

with $f(\kappa)$ given by Eq. (9), $\bar{f}(\tau)$:

$$\bar{f}(\tau) = -\frac{\tan^{-1}\left(\frac{(\sqrt{4-\tau^2})}{\tau^2}\right)}{\sqrt{4-\tau^2}} + 1 - (\tau^2 - 1)\log(\tau) \tag{27}$$

and, according to [33],

$$\sin^2\theta'_W = \sin^2\theta_W - \frac{\varepsilon}{\varepsilon_Z}\sin\theta_W\cos\theta_W, \tag{28}$$

such that, by taking the definition of $\varepsilon_Z = \frac{m_{Z'}}{m_Z}\delta$ for simplicity, we obtain

$$\sin^2\theta'_W = \sin^2\theta_W - \frac{m_Z\varepsilon}{m'_Z\delta}\sin\theta_W\cos\theta_W. \qquad (29)$$

We compare Eq. (26) with the expected value from the SM for a_μ and show this in the parameter space of kinetic mixing versus dark boson mass. Our final results are shown in Fig. 4.

V. RESULTS AND DISCUSSION

To set the constraints shown in Fig. 4 for these two parameter space windows, we vary the kinetic mixing ε and the dark boson mass (the DZ) using our result in (26). Consequently, since we have a mass mixing, we set the value of δ for the mass matrix. We have fixed δ to span the two extremes of parameters described in [33], which are 10^{-3} and 10^{-1}, shown in Fig. 4 as the brown and red contours, respectively. These come from atomic low-energy parity-violating experiments, polarized electron scattering, and bounds on the mixing obtained from rare K and B decays. The dashed lines in Fig. 4 represent the correspondence between our result in Eq. (26) to the most recent 4σ experimental discrepancy in Eq. (1). This means that we adjust our result to the *whole* anomaly. The solid curved lines, forming a band, represent the 2σ deviation from that result in each δ extreme described above, following results previously outlined in [33]. Our analytical expression and results allow us to set discovery lines for a longer range of parameters.

We focus on two regions of interest, with previously restricted regions set up by Croon *et al.* [58] in pink and Borexino, represented in both windows of Fig. 4, in cyan shades. The right panel of Fig. 4 shows CCFR and *BABAR* constraints from the measurement of the neutrino trident cross section [48] and from neutrino electron scattering [49], respectively, in addition to the favored zone by [46] regarding their results on a_μ.

We have set new windows for the search of a hypothetical dark matter candidate, the dark Z. This prospective dark matter particle still interacts with the SM through kinetic mixing but through a hypercharge, as well as mass mixing.

The anomalously behaving muon is a good case to investigate and probe the limits of our knowledge. Its anomaly is much more interesting than the electron's anomaly, being short lived and unstable and much more sensitive to hypothetical physics beyond the SM, compared to its counterpart, the τ, for which we do not have the technology even to measure a possible anomaly. The possibility of new physics coming from the Δa_μ has kept the scientific community working on an explanation for years, and the fact that its equivalent, the electron, has a so well-understood and precise calculation and measurement makes us think that this possibility is a reality. A chance to understand any physics beyond the Standard Model predictions opens a window to connect with other phenomena and mysteries, like the dark matter particle composition itself.

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