Resolving W boson mass shift and CKM unitarity violation in left-right symmetric models with a universal seesaw mechanism

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We explore the possibility of resolving simultaneously the W-boson mass shift reported by the CDF Collaboration and the apparent deviation from unitarity in the first row of the CKM mixing matrix in a class of left-right symmetric models. The fermion masses are generated in these models through a universal seesaw mechanism, utilizing vectorlike partners of the usual fermions. We find a unique solution to the two anomalies where the mixing of the down quark with a vectorlike quark (VLQ) resolves the CKM unitarity puzzle, and the mixing of the top quark with a VLQ partner explains the W boson mass shift. The validity of this setup is tested against the stability of the Higgs potential up to higher energies. We find upper bounds of (4, 4) TeV on the masses of the top-partner VLQ, and a neutral scalar associated with $SU(2)_R$ gauge symmetry breaking, respectively. This class of models can solve the strong CP problem via parity symmetry without the need for an axion.

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I. INTRODUCTION

Left-right symmetric models (LRSM) are well-motivated extensions of the Standard Model (SM) that provide a natural understanding of the origin of parity violation [1–3]. In these models, the gauge symmetry is enhanced to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, with the left-handed quarks and leptons transforming as doublets under $SU(2)_L$ while the right-handed ones are doublets of $SU(2)_R$. The observed V-A structure of weak interactions is a low energy manifestation of spontaneous breaking of parity (P) symmetry, which is well defined in LRSM. Apart from providing insight into the origin of parity violation, these models, owing to the presence of right-handed neutrinos to complete the gauge multiplets, also explain small neutrino masses via the seesaw mechanism, either type-I [4–8], or type-II [9–12]. Moreover, they give a physical interpretation of the hypercharge quantum numbers of fermions as a quantity arising from the $B-L$ (baryon number minus lepton number) charges and the third components of the right-handed isospin. A class of left-right symmetric models also provides a solution to the strong CP problem via parity symmetry alone, without the need to introduce the axion [13]. It is this class of models that is the focus of the present paper.

The strong CP problem could very well be called the strong P problem since the large neutron electric dipole moment (EDM) induced by the QCD θ-parameter is odd under parity symmetry. In a left-right symmetric framework where parity can be defined, the QCD θ-parameter would vanish owing to P. Even after spontaneous (and soft) symmetry breaking, the quark flavor contribution to the observable $\bar{\theta}$ can be zero at the tree level. In the class of models studied in Ref. [13], it was shown that $\bar{\theta}$ remains zero even after one-loop radiative corrections are included. Small and finite $\bar{\theta}$ would arise only via two-loop diagrams, which are consistent with neutron EDM limits [14,15]. For early work on addressing the strong CP problem via parity symmetry, see Refs. [16,17], and for related work, see Ref. [18].

The purpose of this paper is to investigate the possibility of explaining two experimental anomalies, viz., the W-boson mass shift and the apparent violation of unitarity in the first row of the CKM matrix, in the context of these LRSM models, while maintaining the parity solution to the strong CP problem. The masses of quarks and leptons in this class of models arise through a universal seesaw mechanism [19,20]. This is achieved by introducing vectorlike fermions (VLF) [21,22], which are singlets of $SU(2)_L$ and $SU(2)_R$ gauge symmetries. The SM fermions mix with these VLFs via Yukawa interactions involving a Higgs doublet χ_L of $SU(2)_L$ and its parity partner χ_R, a Higgs doublet of $SU(2)_R$. The scalar sector of the model is thus very minimal, consisting only of these two Higgs doublets. This minimality of Higgs fields plays a crucial role in

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solving the strong CP problem since, via gauge rotations, their vacuum expectation values (VEVs) can be both made real, resulting in vanishing contributions to $\bar{\theta}$ from the flavor sector at the tree level. The light fermion masses, which are induced via a generalized seesaw mechanism, are quadratically dependent on the Yukawa couplings ($\mathcal{Y}_i$), allowing for the values of $\mathcal{Y}_i$ required to explain fermion mass hierarchy to be in the range $\mathcal{Y}_i = (10^{-3}\text{–}1)$ as opposed to $\mathcal{Y}_i = (10^{-6}\text{–}1)$ in the SM or in the standard LRSM. This class of models has received considerable attention recently in the context of flavor physics and low energy experimental signals [23], gravitational waves [24], neutrino oscillations [25], and cosmological baryogenesis [26]. High-scale realizations of such models with exact parity symmetry have been developed in Refs. [15,27]. It is noteworthy that the neutrinos can be naturally light Dirac particles, or pseudo-Dirac particles in this context, with their masses arising from two-loop diagrams [14,25].

The presence of direct Yukawa couplings between VLF and SM fermions can give rise to flavor-changing neutral current (FCNC) processes arising at the tree level and modify the SM charged current interactions. These deviations from the SM could potentially be relevant to several experimental anomalies that have come to light in recent years, prominent among them being the muon anomalous magnetic moment $(g-2)_\mu$ (for a recent review, see Ref. [28]), $R_{K^{(*)}}$ (see Ref. [29] for review and Ref. [30] for updated result), $R_{D^{(*)}}$ (see Ref. [31]), the CDF W-boson mass shift [32], and the unitarity of the first row of the CKM matrix, sometimes referred to as the Cabibbo anomaly (for reviews, see Refs. [33,34]). New physics implied by models such as LRSM could, in principle, resolve one or more of these anomalies. However, with a fixed theoretical framework that solves the strong CP problem and with no room to add extra particles so that the models remain minimal, it is unsurprising that this class of LRSM models could not resolve all the anomalies. Nevertheless, we find that the new physics contributions arising from the vectorlike quarks in these models can indeed explain the W-boson mass shift and simultaneously explain the deviation from unitarity in the first row of the CKM matrix. (For attempts to resolve the $R_{D^{(*)}}$ anomaly in this context, see Ref. [35].) We investigate the model parameters resulting in a concurrent solution to these two anomalies.

One of the fundamental predictions of the standard model is the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which parametrizes the charged current weak interaction of the three generations of quarks. Each element of the CKM matrix is determined by combining experimental results with theoretical calculations that take care of relevant radiative corrections. The magnitudes of the CKM elements are well known, with enough evidence suggesting the complex nature of the matrix. The unitarity condition is a good consistency check on the otherwise overdetermined matrix. In recent years, with progress in experimental precision and a better handle on the theoretical uncertainties, the unitarity of the CKM matrix has been questioned. Namely, there is a sizeable deviation in the unitarity of the first row as a result of the precise determination of V_{ud}. There is also a slightly less significant deviation in the unitarity of the first column. With a weighted average of $|V_{ud}| = 0.97373 \pm 0.00009$ [36] and the PDG-recommended average values of [37] $|V_{us}|$ and $|V_{cd}|$, the nonunitarity appears as follows:

$$\begin{aligned}\Delta_{\mathrm{CKM}} &\equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 \\ &= (1.12 \pm 0.28) \times 10^{-3} \quad (\sim 3.9\sigma) \\ \Delta'_{\mathrm{CKM}} &\equiv 1 - |V_{ud}|^2 - |V_{cd}|^2 - |V_{td}|^2 \\ &= (3.0 \pm 1.8) \times 10^{-3} \quad (\sim 1.7\sigma). \end{aligned} \tag{1.1}$$

These discrepancies are often referred to as the Cabibbo anomaly, a nod to the Cabibbo mixing of the two-generation model. The Cabibbo anomaly could be a clear indication of new physics (NP) beyond the standard model (BSM). As we shall see, the LRSM framework with a universal seesaw can resolve this anomaly when the down quark mixes with one of the VL-down-type quarks.

Another major challenge to the SM has appeared recently with a new high-precision measurement of the W-boson mass. The prediction of W-boson mass in SM depends solely on the mass of the Z and the weak mixing angle at the lowest order. Dependence on the gauge couplings and the masses of the top quark and the Higgs boson seep in as radiative electroweak corrections. These higher-order corrections have been computed precisely in the SM, allowing a consistency check against the measured W-boson mass. Recently, the CDF Collaboration [32] at Fermilab has reported the most precise measurement of the W boson mass so far,

$$M_W^{\mathrm{CDF}} = (80.4335 \pm 0.0094)\ \mathrm{GeV} \quad (7\,\sigma\ \text{deviation}). \tag{1.2}$$

The CDF measurement deviates from the SM prediction [38] at 7σ and is also at odds with the prior PDG world average. If confirmed, this is another piece of evidence for new physics. The LRSM framework with a universal seesaw provides a unique solution to this deviation from the mixing of the top quark with one of the VL-up-type quarks.

Several NP models have been explored in the literature in resolving the CKM unitarity puzzle (see Ref. [34] for a comprehensive list of references) as well as the CDF W-boson mass shift (see Ref. [39] and references therein). Among these are solutions to the two anomalies independently with vectorlike fermions [33,40–48]. We explore the simultaneous resolution of the two anomalies in the framework of LRSM with universal seesaw, while maintaining parity symmetric solution to the strong CP problem.

This setup is more constraining compared to the general VLQ framework owing to two reasons. First, parity symmetry restricts the form of the mass matrix, as given in Eq. (2.16) below, and second, possible mixing between the usual quarks and the VLQs is constrained by the quark masses. We do find a nontrivial solution with a vectorlike down quark of mass below 5 TeV mixing with the down quark (to resolve the unitarity puzzle) and a top-partner VLQ with a mass below 4 TeV mixing with the top quark. In addition, a second scalar field associated with $SU(2)_R$ gauge symmetry breaking should be lighter than about 4 TeV for consistency of the model.

The remainder of the paper is organized as follows. In Sec. II, we provide a description of the model, where we recall the parity solution to the strong CP problem. Section III is devoted to discussing the two anomalies. In Secs. III A and III B, we discuss the anomalies associated with W-boson mass measurement and CKM unitarity to establish the parameter space that can resolve these puzzles simultaneously. The main constraints on these parameters of the model are discussed in Sec. III C, with a unique and concurrent resolution to both the anomalies and the resulting predictions on the VLQ masses discussed in Sec. III D. Finally, we conclude in Sec. IV.

II. MODEL DESCRIPTION

The particle spectrum of the LRSM with universal seesaw is composed of the usual SM fermions, right-handed neutrinos, and a set of vectorlike fermionic partners for each of the light fermions denoted as (U_a, D_a, E_a, N_a), where the index a is the family index. The SM fermions along with the right-handed neutrinos form left- or right-handed doublets, assigned to the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as follows:

$$\begin{aligned} \mathcal{Q}_{L,i}\left(3,2,1,+\frac{1}{3}\right) &= \begin{pmatrix} u_L \\ d_L \end{pmatrix}_i, \\ \mathcal{Q}_{R,i}\left(3,1,2,+\frac{1}{3}\right) &= \begin{pmatrix} u_R \\ d_R \end{pmatrix}_i, \\ \psi_{L,i}(1,2,1,-1) &= \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_i, \\ \psi_{R,i}(1,1,2,-1) &= \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}_i, \end{aligned} \tag{2.1}$$

with $i = 1$–3 being the family index. Here, we follow the convention $Q = T_{3L} + T_{3R} + \frac{B-L}{2}$ such that, $\frac{Y}{2} = T_{3R} + \frac{B-L}{2}$, thereby giving the hypercharge a physical meaning in terms of the $SU(2)_R$ and $U(1)_{B-L}$ quantum numbers. Vectorlike fermions (VLFs), which are singlets under both $SU(2)_{L(R)}$, are introduced to generate masses for the fermions via a generalized seesaw mechanism,

$$\begin{aligned} &U_a\left(3,1,1,+\frac{4}{3}\right), \qquad D_a\left(3,1,1,-\frac{2}{3}\right), \\ &E_a(1,1,1,-2), \qquad N_a(1,1,1,0). \end{aligned} \tag{2.2}$$

Here, the quantum numbers of vectorlike quarks $\{U_a, D_a\}$ and vectorlike leptons $\{E_a, N_a\}$ are such that they can mix with SM quarks and leptons, respectively; i.e., the VLFs have the same color charge and electric charge as SM fermions.

The Higgs sector of the model is comprised of a left-handed doublet and its parity partner, a right-handed doublet,

$$\chi_L(1,2,1,+1) = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \qquad \chi_R(1,1,2,+1) = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix}. \tag{2.3}$$

The neutral component of χ_R acquires a vacuum expectation value $\langle \chi_R^0 \rangle \equiv \kappa_R$ at a high scale, breaking the gauge symmetry down to that of the SM with the neutral component of χ_L acquiring a VEV $\langle \chi_L^0 \rangle \equiv \kappa_L \simeq 174$ GeV leading to the spontaneous breaking of the SM gauge symmetry. The Higgs potential of the model is given by

$$\begin{aligned} V = &-(\mu_L^2 \chi_L^\dagger \chi_L + \mu_R^2 \chi_R^\dagger \chi_R) + \frac{\lambda_{1_L}}{2}(\chi_L^\dagger \chi_L)^2 \\ &+ \frac{\lambda_{1_R}}{2}(\chi_R^\dagger \chi_R)^2 + \lambda_2 (\chi_L^\dagger \chi_L)(\chi_R^\dagger \chi_R). \end{aligned} \tag{2.4}$$

The physical scalar spectrum $\{h, H\}$ arises from the mixing of the neutral fields $\sigma_L = \mathrm{Re}(\chi_L^0)/\sqrt{2}$ and $\sigma_R = \mathrm{Re}(\chi_R^0)/\sqrt{2}$ with a mass matrix given by

$$\mathcal{M}^2_{\sigma_{L,R}} = \begin{bmatrix} 2\lambda_{1_L}\kappa_L^2 & 2\lambda_2 \kappa_L \kappa_R \\ 2\lambda_2 \kappa_L \kappa_R & 2\lambda_{1_R}\kappa_R^2, \end{bmatrix}, \tag{2.5}$$

resulting in the mass eigenvalues,

$$M_h^2 \simeq 2\lambda_{1_L}\left(1 - \frac{\lambda_2^2}{\lambda_{1_L}\lambda_{1_R}}\right)\kappa_L^2, \qquad M_H^2 = 2\lambda_{1_R}\kappa_R^2, \tag{2.6}$$

where in the last step, we have assumed the hierarchy $\kappa_R \gg \kappa_L$. Here, the field $h \simeq \sigma_L$ is identified as the SM-like Higgs boson of mass 125 GeV.

Under parity symmetry, which can be defined in LRSM owing to the enhanced gauge structure, the quark and lepton fields, as well as the Higgs fields transform as follows (with $F_{L,R}$ collectively denoting the vectorlike fermions):

$$\mathcal{Q}_L \leftrightarrow \mathcal{Q}_R, \qquad \psi_L \leftrightarrow \psi_R, \qquad F_L \leftrightarrow F_R, \qquad \chi_L \leftrightarrow \chi_R. \tag{2.7}$$

The gauge boson fields also transform under parity,

$$W_L^\pm \leftrightarrow W_R^\pm, \tag{2.8}$$

so that the gauge couplings of the $SU(2)_L$ and $SU(2)_R$ factors obey the relation $g_L = g_R$ above the parity breaking scale. In the parity symmetric limit, the quartic scalar couplings obey $\lambda_{1_L} = \lambda_{1_R} \equiv \lambda_1$, although μ_L may be different from μ_R in Eq. (2.4) since parity symmetry may be broken softly by these dimension-two terms, without spoiling the solution to the strong CP problem. This is what we shall assume in this work. Such a soft breaking via $d = 2$ terms is necessary to realize $\kappa_R \gg \kappa_L \neq 0$ in low-scale LRSM. (For the realization of high-scale LRSM with exact parity where $\mu_L^2 = \mu_R^2$, see Ref. [15].) The conditions for the potential to be bounded from below, with $\lambda_{1_L} = \lambda_{1_R} = \lambda_1$, are

$$\lambda_1 \geq 0, \qquad \lambda_2 \geq -\lambda_1. \tag{2.9}$$

The charged gauge bosons are unmixed at tree-level in this framework, with their masses given by

$$M^2_{W^\pm_{L(R)}} = \frac{1}{2} g^2_{L(R)} \kappa^2_{L(R)}. \tag{2.10}$$

Among the neutral gauge bosons, photon field A_μ remains massless while the two orthogonal fields Z_L and Z_R mix with a mass matrix given as [35]

$$\mathcal{M}^2_{Z_L-Z_R} = \frac{1}{2}\begin{pmatrix} (g_L^2 + g_Y^2)\kappa_L^2 & g_Y^2\sqrt{\frac{g_L^2+g_Y^2}{g_R^2-g_Y^2}}\kappa_L^2 \\ g_Y^2\sqrt{\frac{g_L^2+g_Y^2}{g_R^2-g_Y^2}}\kappa_L^2 & \frac{g_R^4\kappa_R^2+g_Y^4\kappa_L^2}{g_R^2-g_Y^2} \end{pmatrix}. \tag{2.11}$$

This gives rise to the neutral gauge bosons eigenstates $Z_{1(2)}$ with masses,

$$M^2_{Z_1} \simeq \frac{1}{2}(g_Y^2 + g_L^2)\kappa_L^2, \qquad M^2_{Z_2} \simeq \frac{g_L^4\kappa_R^2 + g_Y^4\kappa_L^2}{2(g_R^2 - g_Y^2)}, \tag{2.12}$$

where the gauge boson Z_1 or, in the limit of small mixing, Z_L, is identified as the SM Z boson of mass 91.18 GeV. Here, the hypercharge gauge coupling g_Y is related to the $B - L$ gauge coupling g_B via

$$g_Y^{-2} = g_R^{-2} + g_B^{-2}. \tag{2.13}$$

The Yukawa interactions of the charged fermions and the bare masses for the VLFs, are given in the flavor basis by the Lagrangian,

$$\begin{aligned}\mathcal{L}_{\rm Yuk} = {} & \mathcal{Y}_L^u \bar{\mathcal{Q}}_L \tilde{\chi}_L U_R + \mathcal{Y}_R^u \bar{\mathcal{Q}}_R \tilde{\chi}_R U_L + M_U \bar{U}_L U_R \\ & + \mathcal{Y}_L^d \bar{\mathcal{Q}}_L \chi_L D_R + \mathcal{Y}_R^d \bar{\mathcal{Q}}_R \chi_R D_L + M_D \bar{D}_L D_R \\ & + \mathcal{Y}_L^e \bar{\psi}_L \chi_L E_R + \mathcal{Y}_R^e \bar{\psi}_R \chi_R E_L + M_E \bar{E}_L E_R + \text{H.c.},\end{aligned} \tag{2.14}$$

with $\tilde{\chi}_{L,R} = i\tau_2\chi^*_{L,R}$. Owing to parity symmetry, the Yukawa coupling matrices and the VLF mass matrices obey the relations,

$$Y_L^{u,d,e} = Y_R^{u,d,e}, \qquad M_{U,D,E} = M^\dagger_{U,D,E}, \tag{2.15}$$

which are crucial relations to solving the strong CP problem. The Lagrangian given in Eq. (2.14) gives 6×6 mass matrices for up-type quarks (u, U), down-type quarks (d, D), and charged leptons (e, E), which can be written in the block form,

$$\mathcal{M}_F = \begin{pmatrix} 0 & \mathcal{Y}_L^f \kappa_L \\ \mathcal{Y}_R^{f\dagger}\kappa_R & M_F \end{pmatrix}. \tag{2.16}$$

The strong CP problem is solved in these models via parity symmetry as follows. The QCD θ-parameter, $\theta_{\rm QCD}$, being parity odd, vanishes in the model. The physical parameter that contributes to the neutron EDM is

$$\bar{\theta} = \theta_{\rm QCD} + \arg{\rm Det}(\mathcal{M}_U\mathcal{M}_D). \tag{2.17}$$

It is clear from the form of Eq. (2.16) that when $\mathcal{Y}_L^f = \mathcal{Y}_R^f$ via parity, $\mathrm{Det}(\mathcal{M}_U\mathcal{M}_D)$ is real, and thus, $\bar{\theta} = 0$ at tree level. Since the restriction from neutron EDM is rather severe, $\bar{\theta} \leq 10^{-10}$, it is necessary to ensure that radiative corrections do not generate a value that exceeds this limit. Reference [13] has shown that in this model the induced $\bar{\theta}$ vanishes at one-loop. Very small $\bar{\theta}$ would be induced via two-loop diagrams, which have been estimated to be consistent with neutron EDM limits [13,15].

Quantum gravitational corrections are expected to violate all global symmetries. Since parity symmetry falls under this category, one should worry about the quality factor of the parity solution to the strong CP problem. The leading operator that could be induced via quantum gravity that can generate $\bar{\theta}$ is the $d = 5$ operator[1],

$$\mathcal{L}^{d=5} = \frac{1}{M_{\rm Pl}}(\bar{Q}_L Q_R)\chi_R^\dagger \chi_L. \tag{2.18}$$

This operator would induce non-Hermitian entries in the quark mass matrix. Taking the coefficients of these

[1]Quantum gravity is expected to break all global symmetries through nonperturbative effects arising from black holes, worm holes, etc. These effects are naively expected to scale inversely with the Planck mass so that in the limit $M_P \to \infty$, they vanish.

operators to be of order one and demanding that the complex contribution to the up-quark mass not generate $\bar{\theta}$ larger than 10^{-10} would imply that $\kappa_R \leq 10^5$ GeV. This would lead to a relatively light W_R boson and the associated vectorlike fermions of the model, which opens up the exciting possibility of exploring them at colliders and possibly explaining some of the experimental anomalies.

The mass matrices of Eq. (2.16) can be block diagonalized by biunitary transformations. The light fermion mass matrices have the form,

$$M^f_{\rm light} \simeq -\mathcal{Y}^f_L M_F^{-1} \mathcal{Y}^{f\dagger}_R \kappa_L \kappa_R, \qquad (2.19)$$

if $|\mathcal{Y}^f_R \kappa_R| \ll |M_F|$ is assumed. For a single generation, this yields the light fermion mass as $m_f \simeq -\mathcal{Y}^f_L \mathcal{Y}^f_R \kappa_L \kappa_R / M_F$, which is the seesaw formula now applied to charged fermions. It is also possible that there could be sizable mixing between the light fermions and the vectorlike fermions. It is such mixings that enable us to explain the W-boson mass shift as well as the CKM unitarity violation. As we show in the next section, the mixing of t-quark with its vectorlike partner can generate sufficient custodial $SU(2)$ violation in order to induce the oblique parameter T and thus, explain the W-boson mass shift. The CKM unitarity violation would arise via mixing of the down quark with a vectorlike quark.

III. SIMULTANEOUS SOLUTION TO W BOSON MASS SHIFT AND CABIBBO ANOMALY

In this section, we briefly discuss the anomalies associated with the W-boson mass shift measured by the CDF Collaboration and the apparent CKM unitarity violation. We also explore the parameter space required to resolve each anomaly independently in the LRSM with a universal seesaw. The flavor structure and the mixing required in each case are summarized, along with the potential constraints arising from the model framework. Finally, we arrive at a unique flavor structure that can resolve the two anomalies simultaneously within the model. It will be shown that the down quark mixing with the VL-down-type quark and the top mixing with the VL-up-type quark provides a concurrent solution.

A. W boson mass shift

The CDF Collaboration has recently updated the measurement of W-boson mass [cf: Eq. (1.2)] with a very high precision that has not been achieved previously [32]. This measurement is considerably at odds with the PDG world average $M_W^{\rm PDG} = (80.379 \pm 0.012)$ [37] GeV, which is a combination of the LEP [49], Tevatron [50] (CDF [51] and D0 [52]), and LHCb [53] measurements, at about 3.6σ. It is also at $\sim 7\sigma$ tension with the SM prediction of $M_W^{\rm SM} = (80.357 \pm 0.004)$ GeV [38]. Assuming that the CDF measurement will be confirmed by future experiments, the LRSM model can address this mass shift from the perspective of VLQ-induced corrections to the oblique parameters, S, T, and U [54]. These radiative corrections to the gauge boson propagators, for instance, in the SM appear from the top and bottom quarks in the loop. This could easily be enhanced by the presence of a VL-top or -bottom quarks, which have significant mixings with the usual quarks. The shift in the W-boson mass arising from the oblique parameters is [54]

$$M_W^2 = M_{W^{\rm SM}}^2 + \frac{\alpha c^2}{c^2 - s^2} M_Z^2 \left[-\frac{1}{2} S + c^2 T + \frac{c^2 - s^2}{4 s^2} U \right], \qquad (3.1)$$

where, $s = \sin\theta_W$ and $c = \cos\theta_W$, θ_W being the Weinberg angle, and α is the fine structure constant. The oblique parameters are computed in terms of the two-point functions given as (see Appendix D for details)

$$\begin{aligned} \alpha S &= 4 c^2 s^2 \left[\Pi_{ZZ}{}'(0) - \frac{c^2 - s^2}{cs} \Pi_{\gamma Z}{}'(0) - \Pi_{\gamma\gamma}{}'(0) \right] \\ \alpha T &= \frac{1}{c^2 M_Z^2} [\Pi_{WW}(0) - c^2 \Pi_{ZZ}(0)] \\ \alpha U &= 4 s^2 [\Pi_{WW}{}'(0) - c^2 \Pi_{ZZ}{}'(0) - 2 cs \Pi_{\gamma Z}{}'(0) \\ &\quad - s^2 \Pi_{\gamma\gamma}{}'(0)]. \end{aligned} \qquad (3.2)$$

Violations of custodial $SU(2)$ symmetry arise in these models via the mixing of $SU(2)_L$-doublet quark fields with singlet quarks. The corrections to S and T from VLFs have been studied in great detail in Refs. [55–58]. We perform an independent analysis, verifying that a positive shift to W boson mass would require a correction from VL-top contribution to the T parameter.

For small $t - U$ and $d - D$ mixing angles, the corrections to S and U are comparable and much smaller than T. To explain the $\sim 7\sigma$ shift in W-boson mass (for a central value of $\Delta T = 0.1726$ with $\Delta S = 0 = \Delta U$), we find that the mixing, say, between top and VL-top quark has to be $\sin\theta_L \gtrsim 0.1$. We shall address this quantitatively in Sec. III D after discussing the CKM unitarity puzzle.

B. CKM unitarity puzzle

In recent years, with the precise determinations of $|V_{ud}|$ and $|V_{us}|$, the unitarity of the first row of the CKM is being tested. Currently, the most precise extraction of $|V_{ud}|$, aided by the progress in calculation of radiative corrections [59–61], comes from the superallowed $0^+ \to 0^+$ nuclear β decays [62,63]. V_{us} is determined from the semileptonic K meson decays $K \to \pi \ell \nu$ ($K_{\ell 3}$, with $\ell = e, \mu$), which involves lattice-QCD calculation of the $K \to \pi$ form factor. It can also be obtained by comparing the radiative decay rates of kaon and pion (see Ref. [64]). A significant deficit in the first row unitarity has been reported as a result of improved precision in the calculation of the radiative corrections (Δ_R) to nuclear β decay [36,65]. The SM prediction of the CKM matrix leads to the unitarity in the first row,

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \tag{3.3}$$

With the contribution from $|V_{ub}| = 3.94(36) \times 10^{-3}$ being negligible, the above expression reduces to the unitarity in a two-generation model parametrized by the Cabibbo mixing angle. Using the more precise $|V_{ud}| = 0.97373(9)$ [36,37] and the PDG average for $|V_{us}| = 0.2252(5)$, the deviation from unitarity in the first row turns out to be about 3.9σ [36] as shown in Eq. (I.1). Different approaches in determining the radiative corrections and choices of the experiments involved in estimating the CKM elements lead to inconsistency among the moduli; however, all of them show a deficit from the unitarity of about $\sim$3.3–4σ. There is also a slightly less significant deficit in the unitarity of the first column. We address both of these in our work, although the focus is primarily on the first row nonunitarity.

One possible explanation of the Cabibbo anomaly stems from the idea that the CKM mixing is a submatrix of a more general, unitary matrix arising from the mixing of SM quarks with extra undiscovered quarks. Exploiting the mixing of SM and VL-quarks (VLQ), we explore the scenarios inducing nonunitarity in the current model. The deviation can arise from mixing between the VLQ and light quarks in the up sector, down sector, or both. For illustrative purposes, let us assume that the major correction to the CKM matrix (or Cabibbo mixing matrix in case of two family mixing) arises from only one flavor of SM fermion mixing with the corresponding VLQ (referred to as VLQ-mixing, henceforth). The Cabibbo mixing can be thought of as small corrections induced by the diagonalization of the light quark matrix. The structure of the charge current interaction would heavily depend on the VLQ mixing. We can safely ignore the case where the VLQ mixing appears from the third family as the contribution to nonunitarity would be negligible owing to the smallness of V_{ub} element. VLQ mixing arising from either the first or second generations can provide large enough correction to resolve the CKM unitarity problem. For instance, the VLQ mixing appearing from down quark mixing with VL-down type quark can lead to a charge current structure given by

$$\begin{pmatrix} V_{ud}c_L & V_{us} \\ V_{cd}c_L & V_{cs} \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} V_{ud}c_L & V_{us}c_L \\ V_{cd} & V_{cs} \end{pmatrix}, \tag{3.4}$$

depending on whether the Cabibbo mixing appears from the down-sector or up-sector, respectively. Here, $c_L = \cos\theta_L$, which parametrizes the leakage of the left-handed up (down) quark into the VLQ sector, which being singlets of $SU(2)_L$ have no direct couplings to the $W^\pm$ boson. There are, in general, six different possible ways of inducing nonunitarity in the first row. A VLQ mixing of $\sin\theta_L \simeq 0.034$ arising from up or down quark mixing with the corresponding VLQ can resolve the $\sim 3.9\sigma$ deficit in the first row unitarity, while $\sin\theta_L \simeq 0.15$ would be required if the VLQ mixing arises from the charm or strange quark. Of these, only the VLQ mixing arising from the first generation will survive the constraint from hadronic decays of the Z boson (see Sec. III C 1).

C. Constraints on model parameters

1. Light fermion masses

In the simplified one family mixing formulation, where the d-quark mixes with a vectorlike D-quark, or equivalently, s-quark mixes with its VLQ partner, the mass of d-quark (or s-quark) can be written as $m_i = (\kappa_L\kappa_R s_L s_R)M_i$, where s_L and s_R are the sines of left and right mixing angles among d_i and D_i quarks. The tangent functions ($t_{L,R}$) obey the relation $t_R = (\kappa_R/\kappa_L)t_L$. Therefore, with $t_L \simeq s_L$, we have

$$s_L^2 = \frac{\kappa_L}{\kappa_R}\frac{m_{d_i}}{M_{D_i}}. \tag{3.5}$$

Since $\sin\theta_L$ should be of $\mathcal{O}(0.034)$ to resolve the Cabibbo anomaly, the products $\kappa_R M_{D_i}$ are severely restricted due to the smallness of the d_i masses: $\kappa_R M_{D_i} \in [325, 1.9 \times 10^5]$ GeV^2 to fit the d- and s-quark masses. This is in direct contradiction with the experimental limits on these parameters: $\kappa_R \geq 9.14$ TeV, corresponding to the 5 TeV lower bound on M_{Z_R} [66] and $M_F > 1$ TeV, from the production and decay of pair produced VLF partners [67,68]. Note that parity symmetry plays an important role to arrive at this conclusion. Indeed, within the same model, without parity, the relation $t_R = (\kappa_R/\kappa_L)t_L$ will not hold, which would enable achieving large $d_L - D_L$ mixing. In such an attempt, however, the parity solution to the strong CP problem would be lost.

The aforementioned constraints from light fermion masses can be evaded with parity symmetry intact by considering the mixing of the down quark (or the strange quark) with two VLQs. In such a setup, large $d_L - D_L$ mixing can be realized even in the limit of vanishing d-quark mass, as will be discussed in Sec. III D. Small d-quark mass can be generated via perturbations to the mass matrix proposed there. This scenario, where the light quark masses arise as perturbative corrections, however, is not applicable in the case of the top quark, since its mass is not so small. So, we also include the fit to the mass of the top quark in our analysis.

2. Z decay width

Another major constraint on the mixing of d_L with D_L (or s_L with its VLQ partner) arises from the modification of the Z boson couplings to the fermions. The left-handed fermion interaction vertex of Z is modified by the cosine of the VLQ mixing ($\cos\theta_L$), thereby altering the total decay width of the Z boson,

$$\mathcal{L}_Z \supset \frac{g}{\cos\theta_W}\bar{f}_L\gamma_\mu(T_{3L}\cos\theta_L^2 - Q\sin\theta_W^2)f_L Z^\mu. \quad (3.6)$$

For instance, resolving the Cabibbo anomaly with VLQ mixing from the first generation requires $\sin\theta_L = 0.034$. This results in $\Delta\Gamma_Z \simeq 1(0.8)$ MeV from down (up)-quark mixing with VLF. On the other hand, $\sin\theta_L \simeq 0.15$ in case of the second family mixing, leading to $\Delta\Gamma_Z \simeq 20$ MeV, which is excluded by the experimental measurement of Z decay with an uncertainty of $\Delta\Gamma_Z \leq 2.3$ MeV at 1σ. Therefore, the Cabibbo anomaly can be resolved only by the VLQ mixing with the first family. For a general discussion of flavor nonuniversality with VLQ mixing with the usual quarks, see Ref. [33].

D. Combined solution

From the discussions so far, it is clear that the Cabibbo anomaly can only be resolved if the mixing arises from up or down quark with a corresponding VLQ. Since the mixing angle required in this case, $\sin\theta_L \simeq 0.034$, is not large enough to accommodate the explanation for W boson mass shift, which requires $\sin\theta_L \sim \mathcal{O}(0.1)$, and such large mixings are subjected to constraint from Z decay width (Sec. III C 2), the only remaining choice is to invoke top quark mixing with VLQ to explain the shift in the W-boson mass.

As mentioned previously, a simple one-family mixing between the light and VL-quark suffers from a severe constraint arising from the smallness of the light fermion masses (Sec. III C 1). To evade this, we assume that the down quark mixes with two vectorlike quarks. We can work in the limit of neglecting all SM fermion masses, except for the top quark. It turns out that top-quark mixing with a single VLQ cannot induce sufficiently large T-parameter to explain the W-boson mass shift. This is again due to parity symmetry which constrains the form of such a 2×2 mass matrix. When the top quark mixes with two VLQs, the contribution to the T-parameter can be sufficiently large, which we shall adopt.

Since there are three up-type VLQs in the model, and since the top quark mixes with two of them, there is no room for the up quark to mix significantly with any VLQs in order to explain the CKM unitarity puzzle. We are then cornered to a unique solution that simultaneously explains the W-boson mass shift and the CKM unitarity puzzle: the top quark mixes significantly with two up-type VLQs, and the down quark mixes with two down-type VLQs. The specific structures of the mass matrices in these sectors are uniquely determined and are given below,

$$\mathcal{M}_d = \begin{pmatrix} 0 & 0 & y_d\kappa_L \\ 0 & 0 & M_{1d} \\ y_d\kappa_R & M_{1d} & M_{2d} \end{pmatrix} \text{ in } \{ d \quad D_2 \quad D_3 \} \text{ basis}$$
$$\mathcal{M}_t = \begin{pmatrix} 0 & 0 & y_u\kappa_L \\ 0 & M_{3u} & M_{1u} \\ y_u\kappa_R & M_{1u} & M_{2u} \end{pmatrix} \text{ in } \{ t \quad U_2 \quad U_3 \} \text{ basis.} \quad (3.7)$$

Details of the diagonalization of $\mathcal{M}_d$ are provided in Appendix B The up-sector diagonalization has been done numerically, with the 3×3 unitary matrix represented using the standard parametrization of the CKM matrix [Eq. (B6)] (where the CP violating phase is set to zero for simplicity). Although the top-mixing also contributes to the charged current interactions, it is the down-sector mixing that resolves the Cabibbo anomaly with $\Delta_{\mathrm{CKM}} = s_{1d}^2$ with $s_{1d} \simeq 0.034$, assuming that the CKM mixing receives a contribution from the down sector. This can be seen from the extended CKM matrix with $(\bar{u}_L \quad \bar{c}_L \quad \bar{t}_L \quad \bar{U}_{2_L} \quad \bar{U}_{3_L})$ multiplied from the left and $(d_L \quad s_L \quad b_L \quad D_{2_L} \quad D_{3_L})^T$ from the right (see Appendix B for details),

$$\begin{pmatrix} c_{1d}V_{ud} & c_{1d}V_{us} & c_{1d}V_{ub} & -c_{2d}s_{1d} & -s_{1d}s_{2d} \\ V_{cd} & V_{cs} & V_{cb} & 0 & 0 \\ u_{11}V_{td} & u_{11}V_{ts} & u_{11}V_{tb} & 0 & 0 \\ u_{21}V_{td} & u_{21}V_{ts} & u_{21}V_{tb} & 0 & 0 \\ u_{31}V_{td} & u_{31}V_{ts} & u_{31}V_{tb} & 0 & 0 \end{pmatrix}. \quad (3.8)$$

From Eq. (3.8), one can also see that a combination of $u_{11} = \hat{s}_{12}^u$ when $\hat{s}_{13}^u \ll 1$ [Eq. (B6)] and s_{1d} can explain the nonunitarity of the CKM matrix in the first column, with the main correction appearing from the down-sector owing to the smallness of V_{td}. The Z interaction with left-handed light fermions gets modified owing to the mixing of d with D and t with U. New interactions involving $\bar{t}U_iZ$ and $\bar{d}D_iZ$ arise in the model proportional to these VLQ mixings. The remaining left-handed quark vertices with the Z boson and all the right-handed quark vertices with Z, however, remain unchanged. These new interactions by themselves do not cause any tree-level flavor-changing processes.

Both up- and down-sectors contribute to the corrections to the oblique parameters, but the effect from down-sector is negligible compared to that of top-mixing, since the mixing of only $s_{1d} \simeq 0.034$ is required to resolve the Cabibbo anomaly. The expressions for S, T and U arising from $t-U$ mixing has been verified against Refs. [56,57]. The exact expressions in our framework are given in Appendix D [Eqs. (D6) to (D8)]. We find that the

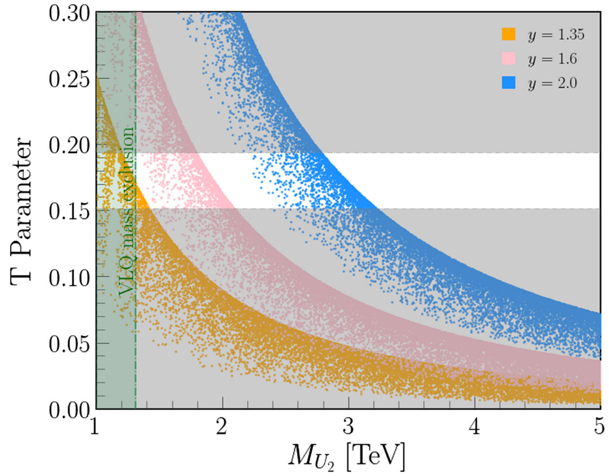

FIG. 1. Oblique parameters S, T, U as a function of the mass of the lightest vectorlike quark (M_{U_2}) for different choices of the Yukawa coupling. The unshaded region shows the required range of T parameter that can explain the CDF W-boson mass shift. The vertical dotted line shows the current experimental lower bound on VLQ mass [67,68]. This plot is consistent with the mass of the top quark. The Yukawa coupling needs to be at least $y \simeq 1.35$ to explain the W-boson mass shift while evading the current experimental bound on the mass of VLQ. The corrections to S and U parameters are seen to be much smaller than T.

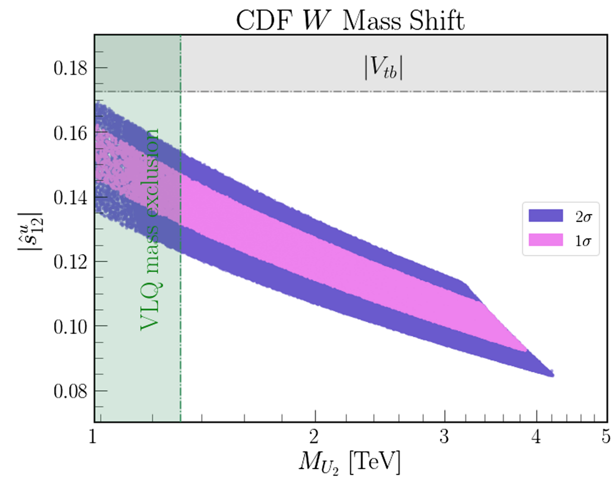

FIG. 2. 1- and 2-σ regions required to explain CDF-W mass and Cabibbo anomaly plotted as a function of the sine of the mixing angle $\hat{s}^u_{12}$ [Eq. (B6)], consistent with the stability of Higgs potential. The green shaded region is the exclusion on VLQ mass [67,68], whereas the gray shaded region corresponds to the limit on mixing angle from the determination of $|V_{tb}|$ [37] element of the CKM matrix.

corrections to S and U are negligible compared to the T parameter.

The behavior of the T parameter as a function of the lightest VLQ mass (M_{U_2}) is shown in Fig. 1, for different choices of the Yukawa coupling y, imposing the fit for top quark mass. It becomes evident that the Yukawa coupling y_u has to be at least 1.35 to explain the positive shift in W-boson mass while evading the LHC constraint on VLQ mass. The fit to the CDF W-boson mass shift using the full expressions of oblique parameters [Eqs. (D6) to (D8)] is shown in Fig. 2. The region shown in the plot is also consistent with 1 and 2σ allowed region from resolving the CKM unitarity puzzle from down-sector mixing, although this alone is not enough to explain the shift in W-boson mass and CKM nonunitarity simultaneously. In Fig. 2, we have also indicated the excluded region from $|V_{tb}|$ measurement as a shaded region on the top. In resolving the Cabibbo anomaly, it is essential to understand that the new elements of the CKM matrix are identified with those determined experimentally, i.e., $|c_{1d}V_{us}| \equiv |V^{\mathrm{PDG}}_{us}|$, etc. These redefinitions, however, do not impose any constraints on these essentially free parameters.

With only the down quark and the top quark mixing with VLQs significantly, there would not be any large FCNC mediated by the Z boson or the Higgs boson. However, once all the CKM mixing angles are induced, there could be such effects. To control these, we note that V_{us} can arise from the up-sector, while V_{cb} arises from the down-sector. These have no bearing on the light-heavy mixing. The smaller V_{ub} may arise from either sector, involving admixture of the $(u - c)$ sector in the mass matrix $\mathcal{M}_t$ of Eq. (3.7), which is not expected to lead to large FCNC effects, owing to the smallness of V_{ub}.

E. Stability of the Higgs potential

The resolution to the W-boson mass shift, along with the necessary fit to the top quark mass requires Yukawa coupling y_u to be larger than about 1.35. Although such large Yukawa couplings are not excluded by the perturbative unitarity bounds [69–72], the Higgs potential is at risk of becoming unstable as the quartic coupling λ_{1_L} [Eq. (2.4)] could turn negative when extrapolated to higher energies. Partial wave unitarity would require $y_u^2 < 8\pi/3$ [70], obtained from the condition $\mathrm{Re}(a_0) \leq 1/2$, for the elastic fermion-antifermion scattering in the color singlet channel, at the center of mass energies much above the fermion mass. While this condition was derived for a chiral fermion of the SM such as the top quark, it would equally apply to the scattering of vectorlike top quark at energies well above its bare mass. It should be noted that the heavier VLQ with order one Yukawa coupling obtains its mass primarily via the Yukawa coupling, rather than through its bare mass.

The stability of the Higgs potential is especially of concern since the heavier VLQ with order one Yukawa coupling has a mass in the range of (15–40) TeV in the model. This requires that λ_{1_L} should remain positive up to this energy scale. Here, we show that the renormalization group flow of the quartic couplings in the momentum range in which VLQs are active [∼(4–40) TeV] would keep λ_{1_L}

TABLE I. Benchmark points showing that λ_1 remain positive up to the momentum scale of the heaviest VLQ mass M_{U_3}, for Yukawa couplings y_u and y_d which are chosen at the low momentum scale equivalent to the lightest of the VLQ masses. All the masses are quoted in TeV. $M_{U_{2(3)}}$ and $M_{D_{2(3)}}$ are the light (heavy) VLQ masses in the up- and down-sectors, respectively. The heavy Higgs mass, m_H is chosen to be the lowest VLQ mass to obtain λ_1 and λ_2. The value of T parameter is given for each benchmark point.

	y_u	y_d	$\{M_{1u}, M_{2u}, M_{3u}\}$	$\{M_{1d}, M_{2d}\}$	M_{D_2}	M_{D_3}	M_{U_2}	M_{U_3}	$\lvert s_{1d}\rvert$	$\lvert s_{2d}\rvert$	$\lvert\hat{s}^u_{12}\rvert$	λ_{1_R}	λ_2	λ_{1_L}	m_H	T
BM-1	2.19	0.28	{2.9, 3.0, 1.3}	{1.5, 0.2}	1.5	3.2	3.2	22.4	0.033	0.034	0.11	0.01	0.09	0.17	1.5	0.19
BM-2	1.97	0.19	{2.8, 2.7, 1.4}	{1.4, 0.3}	1.3	2.4	3.0	20.0	0.026	0.093	0.099	0.008	0.056	0.17	1.3	0.15
BM-3	2.06	0.64	{2.6, 3.6, 1.2}	{4.6, 3.8}	4.0	9.1	2.9	21.0	0.024	0.279	0.11	0.04	0.15	0.17	2.9	0.19
BM-4	1.92	0.56	{2.3, 2.7, 1.2}	{3.8, 3.8}	3.2	8.0	2.5	19.6	0.025	0.284	0.12	0.031	0.11	0.17	2.5	0.19
BM-5	1.6	0.38	{1.7, 1.7, 1.1}	{1.7, 4.3}	14.4	38.4	3.8	24.2	0.030	0.23	0.098	0.07	0.12	0.16	3.8	0.16

positive, which would guarantee the stability of the Higgs potential up to this energy scale.

Suppose that the scalar field σ_R associated with the $SU(2)_R$ symmetry breaking has a mass of order 20 TeV. In this case, only the quartic coupling λ_{1_L} is active below 20 TeV, which by virtue of the Yukawa coupling y_u would turn negative in going from 4 TeV to this scale. On the other hand, if σ_R has a mass comparable to the lighter VLQ, of order 4 TeV, in the momentum range of VLQ masses, both λ_{1_L} and λ_{1_R} are active. λ_{1_L} would decrease in going to higher energies driven by the Yukawa coupling of σ_L, while λ_{1_R} will not feel the effect of the Yukawa coupling. This is because the Yukawa coupling of σ_R involves heavier fields. This is the scenario that is consistent with our framework. Naturally, this scheme would predict that the σ_R field should have a mass of the same order as the U_2 field, which is of order 4 TeV or lower.

To test the validity of our scenario, we have computed the renormalization group (RG) evolution of the quartic coupling and shown that the coupling $\lambda_{1_L} \geq 0$ at least up to the heaviest VLQ mass scale. We have presented the one-loop renormalization group equations for the dimensionless parameters of the model in Appendix A These are generalizations of equations given in Refs. [73,74] that can be adopted to the regime below ∼40 TeV, where parity symmetry is not exact. From the RG evolution of the Yukawa couplings, given in Eq. (A3), we find that for $y_{u,d} \simeq 1.4$, there is no significant running in $y_{u,d}$ in the momentum range of VLQ masses. Thus, it is a good approximation to take $y_{u,d}$ as constants in this regime. With this approximation, we can analytically compute the evolution of the quartic scalar couplings.

At the momentum scale of order M_{U_3}, where parity symmetry is exact, we have $\lambda_{1_L} = \lambda_{1_R} = \lambda_1$. At lower energies, λ_{1_L} will be larger than λ_{1_R}, owing to the Yukawa contributions to their evolution. Note that σ_R couples to the heavier VLQ, while σ_L couples to the lighter VLQ in our fits. Therefore, below the parity restoration scale, λ_{1_R} does not feel the effects of its large Yukawa couplings. The difference $\delta_\lambda = \lambda_{1_L} - \lambda_{1_R}$, defined at the lower scale, is obtained by solving the relevant renormalization group equations, Eqs. (A4) and (A5), applicable to the momentum range $\sim(4\text{–}40)$ TeV.[2] The solution can be written down approximately as

$$\delta_\lambda = \frac{3}{16\pi^2}\left(y_u^4 \ln\left[\frac{M_{U_3}}{M_{U_2}}\right] + y_d^4 \ln\left[\frac{M_{U_3}}{M_{D_2}}\right]\right). \tag{3.9}$$

Only the Yukawa couplings cause a significant split in the λ_{1_L} and λ_{1_R}, since the gauge couplings g_L and g_R run identically below the heavy mass scale. We minimize the Higgs potential at a momentum scale of order 4 TeV, which roughly corresponds to the σ_R scalar mass. At this scale, λ_{1_R} is taken to be a free parameter. λ_{1_L} obtained at the low momentum scale using SM RG evolution [75] of the Higgs quartic coupling is used to compute the SM Higgs mass m_h at that scale. To ensure that the Yukawa couplings do not cross the perturbative unitarity bound of $|y| \leq \sqrt{8\pi/3}$ [70] upon RG evolution [75], the coupling was chosen to $|y_u| \lesssim 2.25$, for $|y_d| < 1$ for the entire momentum range. A few benchmark points illustrating that λ_{1_L} remains positive are given in Table I.

We find that the largest allowed mass of up-type VLQ is $M_{U_2} \lesssim 3.8(4.2)$ TeV, whereas the down-type mass can be $M_{D_2} \lesssim 14(21)$ TeV, to resolve the anomalies within $1(2)\sigma$. In getting these limits, we have allowed $|y_d| = 2.25$, its perturbative unitarity limit. However, the stability of the Higgs potential is not easily guaranteed when both $|y_u|$ and $|y_d|$ are of order 1.5 or greater. If $|y_d| < 1$ is imposed, while allowing for larger $|y_u|$, the mass of D_2 would be $M_{D_2} \lesssim 14(16)$ TeV, to resolve the Cabibbo anomaly within $1(2)\sigma$. However, the down-type VLQ can be much lighter, even as low as 1.2 TeV. These limits are the more conservative ones, consistent with the Higgs potential stability. Current experimental searches for VLQs have started to constrain models with masses up to 2 TeV. The lower bound on the mass of the singlet VL-top quark is 1.27 TeV, from the search for pair-production of VLQs decaying to Z-boson and top quark [76]. HL-LHC and a future 100 TeV proton collider are estimated to probe VLQ

[2]The Higgs potential need not always be stable up to 40 TeV, only up to the heaviest VLQ mass of the benchmark point.

masses up to 3 TeV and 15–20 TeV, respectively [77], based on the single production of the VL-top quark in association with the top quark. These searches can potentially probe the entire range of masses predicted by the model in simultaneously resolving the two anomalies. It should be noted that only one of the VLQs from both up and down sectors can be probed in the near future. The heavier VLQs in both sectors may be out of reach of near-future experiments since their masses are in the multi-TeV range, and they potentially decouple for larger values of κ_R (see discussion in Sec. III E 1).

It should be noted that there are two additional sources of corrections to the SM Higgs couplings in this model: (i) from the mixing between the left- and right-handed scalars, (ii) from the coupling of the left-handed scalar with the VLQs. The mixing between the scalars is at most of the order of 0.1, consistent with the current bounds [78,79]. Moreover, the direct couplings of Higgs to say, the light VL-up type quark of mass $\simeq$3 TeV is $\simeq$0.15, resulting in a deviation in the branching fraction of $H \to \gamma\gamma$ [80,81] less than 0.02%, which is within the current experimental uncertainty. Therefore, neither of these new contributions to Higgs couplings will lead to sizeable deviations in Higgs decays, and the SM Higgs properties are preserved.

1. Decoupling of W_R

The benchmark points in Table I were obtained for $\kappa_R = 10$ TeV, corresponding to $M_{Z_R} \simeq 5.47$ TeV (consistent with Ref. [66]) and $M_{W_R} \simeq 4.62$ TeV under parity symmetry. Currently, W_R boson masses up to 3.6 TeV [82,83] are excluded from the hadronic decays of the gauge boson produced resonantly at the LHC. There are more severe constraints on the mass of the W_R boson from direct LHC searches [84,85]; however, these are obtained under the assumption that the right-handed gauge boson decays to a right-handed heavy neutrino, which need not be the case in our model, unless the neutrinos are Dirac. There is no $W_L - W_R$ mixing at the tree level within this model, but such a mixing is induced at the quantum level, with a mixing angle of the order of $\sim 10^{-6}$ [14], so its contribution to W boson mass shift is negligible. Furthermore, our analysis cannot impose an upper bound on the right-handed scale since the larger the value of κ_R, the larger the heavy (second) VLQ. Therefore, the heavy VLQs decouple, resulting in an effective 2×2 mass mixing matrix equivalent to the upper 2–3 block of Eq. (3.7). For instance, as $\kappa_R \to \infty$, the heavy VLQs can be integrated out such that the effective mass mixing matrices would be parity asymmetric,

$$\mathcal{M}_d = \begin{pmatrix} 0 & y_d \kappa_L \\ 0 & M_{1d} \end{pmatrix}$$
$$\mathcal{M}_t = \begin{pmatrix} 0 & y_u \kappa_L \\ M_{3u} & M_{1u} \end{pmatrix}. \tag{3.10}$$

The masses of the VLQs are

$$M_{D_2}^2 = M_{1d}^2 + \kappa_L^2 y_d^2,$$
$$M_{U_2}^2 = \frac{1}{2}\Big(M_{1u}^2 + M_{3u}^2 + \kappa_L^2 y_u^2 + \sqrt{M_{1u}^4 + (M_{3u}^2 - \kappa_L^2 y_u^2)^2 + 2M_{1u}^2(M_{3u}^2 + \kappa_L^2 y_u^2)}\Big). \tag{3.11}$$

In such a scenario, the Cabibbo anomaly and the W-boson mass shift can be explained by the light VLQs in down and up sectors, respectively, which are solely dependent on $\{M_{1d}, M_{1u}\}$ and the Yukawa couplings. Because of this, resolving the two anomalies does not constrain the right-handed scale.

IV. CONCLUSION

In light of the recent CDF measurement of the W-boson mass reporting a possible 7σ shift with the SM prediction, and the improvements in determination of the CKM matrix elements resulting in an apparent violation of unitarity in the first-row as large as $\sim 3.9\sigma$, we have investigated new physics beyond the SM. In this paper, we focus on a particular class of left-right symmetric models wherein the fermion masses are generated by a universal seesaw mechanism aided by the presence of heavy vectorlike fermionic partners having Yukawa couplings to the light SM fermions. This model was proposed previously to explain the strong CP problem without the need for an axion. The solution to the strong CP problem requires a minimal Higgs sector consisting of only one doublet each, transforming under $SU(2)_L$ and $SU(2)_R$. Furthermore, the mass matrices in the fermionic sector should be Hermitian, making this a well-defined model with heavily constrained parameter space. We explore the model to provide a successful explanation of the CKM unitarity puzzle and the W-boson mass shift in a well-constrained parameter space. We have insisted that parity symmetry is broken only by dimension-two soft terms in the scalar potential, which would provide a solution to the strong CP problem without the need for an axion. Because of parity symmetry, if a light quark mixed with only one VLQ, the constraint from fermion mass would require such mixings to be too small as to explain these anomalies. We find the simplest, unique flavor structure that can explain the two anomalies where one of the SM quarks mixes with two VLQs. The CKM unitarity puzzle can be explained by the mixing of either or both of the first-generation quarks with VLQs while the W-boson mass shift requires top-quark mixing with VLQs. For a concurrent explanation, invoking the top-quark mixing eliminates the possibility of up-quark mixing with two VLQs simultaneously. This leads to a unique solution where the up-sector (top-quark mixing) and down-sector (down-quark mixing) with two VLQs each, resolve the W

mass shift and the CKM unitarity puzzle, respectively. Although we are exploring a left-right symmetric model, the resolution of the Cabibbo anomaly or explanation of the W-boson mass shift does not require the right-handed gauge bosons to be light. However, since this framework requires $\mathcal{O}(1)$ Yukawa coupling, the mass of the second Higgs boson can be constrained. Ensuring that the Higgs potential remains stable till at least up to the mass of the heaviest of the second VLQs, this model predicts an upper bound on the VL up-type quark mass of ∼4.2 TeV and the VL down-type mass of ∼16 TeV. Moreover, the second Higgs cannot be much larger than the lightest VLQ mass, predicting an upper bound of ∼4.2 TeV. The TeV scale VLQs predicted by our analysis can potentially be probed directly at the HL-LHC or a future high-energy particle collider.

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APPENDIX A: ONE-LOOP RENORMALIZATION GROUP EQUATIONS

Here, we present the full set of one-loop RGE for the dimensionless parameters of the LRSM with a universal seesaw. These generalize the equations given in Refs. [73,74]. The gauge couplings g_3, g_L, g_R, and g_B evolve with momentum according to the renormalization group equations (with $t = \ln\mu$),

$$16\pi^2 \frac{\mathrm{d}g_i}{\mathrm{d}t} = b_i g_i^3, \tag{A1}$$

where $b_i = (-3, -\frac{19}{6}, -\frac{19}{6}, \frac{41}{2})$ for $i = (3, L, R, B)$. The Yukawa coupling matrices of Eq. (2.14) evolve according to the equations,

$$16\pi^2 \frac{\mathrm{d}\mathcal{Y}_A^f}{\mathrm{d}t} = (T_A^f - G_A^f + H_A^f)\mathcal{Y}_A^f, \tag{A2}$$

with, $A = L$, R, and $f = u$, d, e. Here, T_A^f arise from fermion loops, G_A^f from gauge boson loops, and H_A^f from Higgs boson loops. The functions T_A^f and G_A^f are proportional to the unit matrix, which H_A^f is flavor dependent. The explicit forms of these functions are given by [73]

$$\begin{aligned}
T_A^u &= T_A^d = T_A^e = 3[\mathrm{Tr}(\mathcal{Y}_A^u\mathcal{Y}_A^{u\dagger}) + \mathrm{Tr}(\mathcal{Y}_A^d\mathcal{Y}_A^{d\dagger})] + \mathrm{Tr}(\mathcal{Y}_A^e\mathcal{Y}_A^{e\dagger})\\
G_A^u &= \frac{17}{8}g_B^2 + \frac{9}{4}g_{2A}^2 + 8g_3^2\\
G_A^d &= \frac{5}{8}g_B^2 + \frac{9}{4}g_{2A}^2 + 8g_3^2\\
G_A^e &= \frac{45}{8}g_B^2 + \frac{9}{4}g_{2A}^2\\
H_A^u &= -H_A^d = \frac{3}{2}(\mathcal{Y}_A^u\mathcal{Y}_A^{u\dagger} - \mathcal{Y}_A^d\mathcal{Y}_A^{d\dagger})\\
H_A^e &= \frac{3}{2}\mathcal{Y}_A^e\mathcal{Y}_A^{e\dagger}.
\end{aligned} \tag{A3}$$

The quartic scalar couplings of Eq. (2.4) evolve with momentum according to the equations,

$$\begin{aligned}
16\pi^2 \frac{\mathrm{d}\lambda_{1_L}}{\mathrm{d}t}t = {}& 12\lambda_{1_L}^2 + 4\lambda_2^2 - \lambda_{1_L}\left(9g_L^2 + \frac{9}{2}g_B^2\right)\\
&+ \frac{9}{4}\left(g_L^4 + g_L^2 g_B^2 + \frac{3}{4}g_B^4\right) + 4\lambda_{1_L}\{3\mathrm{Tr}(\mathcal{Y}_L^u\mathcal{Y}_L^{u\dagger})\\
&+ 3\mathrm{Tr}(\mathcal{Y}_L^d\mathcal{Y}_L^{d\dagger}) + \mathrm{Tr}(\mathcal{Y}_L^e\mathcal{Y}_L^{e\dagger})\}\\
&- 4\{3(\mathrm{Tr}(\mathcal{Y}_L^u\mathcal{Y}_L^{u\dagger}))^2 + 3(\mathrm{Tr}(\mathcal{Y}_L^d\mathcal{Y}_L^{d\dagger}))^2\\
&+ (\mathrm{Tr}(\mathcal{Y}_L^e\mathcal{Y}_L^{e\dagger}))^2\},
\end{aligned} \tag{A4}$$

$$\begin{aligned}
16\pi^2 \frac{\mathrm{d}\lambda_{1_R}}{\mathrm{d}t}t = {}& 12\lambda_{1_R}^2 + 4\lambda_2^2 - \lambda_{1_R}\left(9g_R^2 + \frac{9}{2}g_B^2\right)\\
&+ \frac{9}{4}\left(g_R^4 + g_R^2 g_B^2 + \frac{3}{4}g_B^4\right) + 4\lambda_{1_R}\{3\mathrm{Tr}(\mathcal{Y}_R^u\mathcal{Y}_R^{u\dagger})\\
&+ 3\mathrm{Tr}(\mathcal{Y}_R^d\mathcal{Y}_R^{d\dagger}) + \mathrm{Tr}(\mathcal{Y}_R^e\mathcal{Y}_R^{e\dagger})\}\\
&- 4\{3(\mathrm{Tr}(\mathcal{Y}_R^u\mathcal{Y}_R^{u\dagger}))^2 + 3(\mathrm{Tr}(\mathcal{Y}_R^d\mathcal{Y}_R^{d\dagger}))^2\\
&+ (\mathrm{Tr}(\mathcal{Y}_R^e\mathcal{Y}_R^{e\dagger}))^2\}.
\end{aligned} \tag{A5}$$

APPENDIX B: MASS MATRIX DIAGONALIZATION

The mass structures in Eq. (3.7) are diagonalized using a biunitary transformation of the form $U_L\mathcal{M}U_R^\dagger$. Here, $U_{L(R)}$ are parametrized by $\theta_{L(R)_{1,2}}$. Since we are interested only in the left-handed mixing angles, $U_L = U$ with $\theta_{L_{1,2}} = \theta_{1,2}$. The down-sector left-handed fields are diagonalized by $U_d\mathcal{M}_d\mathcal{M}_d^T U_d^T$, assuming all the couplings to be real,

$$\mathcal{M}_d \Rightarrow \mathrm{Diag}(0, M_{D_2}, M_{D_3}). \tag{B1}$$

$$U_d = \begin{pmatrix} 1 & 0 & 0\\ 0 & c_{2d} & -s_{2d}\\ 0 & s_{2d} & c_{2d}\end{pmatrix}\begin{pmatrix} c_{1d} & s_{1d} & 0\\ -s_{1d} & c_{1d} & 0\\ 0 & 0 & 1\end{pmatrix}. \tag{B2}$$

$c(s)$ stands for cos(sin) where, $\theta_{(1,2)d}$ are the left-mixing angles. The mixing angles are

$$\theta_{1d} = \arctan\left(\frac{-y_d\kappa_L}{M_{1d}}\right),$$
$$\theta_{2d} = \frac{1}{2}\arctan\left(\frac{2M_{2d}\sqrt{M_{1d}^2 + y_d^2\kappa_L^2}}{M_{2d}^2 + (\kappa_R^2 - \kappa_L^2)y_d^2}\right), \quad \text{(B3)}$$

and the heavy VLQ mass eigenvalues are

$$M_{D_{3,2}} = \frac{1}{2}(2M_{1d}^2 + M_{2d}^2 + (\kappa_L^2 + \kappa_R^2)y_d^2 \pm \sqrt{4M_{2d}^2(M_{1d}^2 + \kappa_L^2 y_d^2) + (M_{2d}^2 + (\kappa_R^2 - \kappa_L^2)y_d^2)^2}). \quad \text{(B4)}$$

For the up sector, we can represent a general unitary matrix under the standard CKM-like parametrization (with the CP phase set to zero for simplicity) as

$$U_u = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix} = \begin{pmatrix} \hat{c}_{12}\hat{c}_{13} & \hat{s}_{12}\hat{c}_{13} & \hat{s}_{13} \\ -\hat{s}_{12}\hat{c}_{23} - \hat{c}_{12}\hat{s}_{23}\hat{s}_{13} & \hat{c}_{12}\hat{c}_{23} - \hat{s}_{12}\hat{s}_{23}\hat{s}_{13} & \hat{s}_{23}\hat{c}_{13} \\ \hat{s}_{12}\hat{s}_{23} - \hat{c}_{12}\hat{c}_{23}\hat{s}_{13} & -\hat{c}_{12}\hat{s}_{23} - \hat{s}_{12}\hat{c}_{23}\hat{s}_{13} & \hat{c}_{23}\hat{c}_{13} \end{pmatrix}, \quad \text{(B5)}$$

such that

$$\mathcal{M}_t \Rightarrow \mathrm{Diag}(m_t, M_{U_2}, M_{U_3}). \quad \text{(B6)}$$

It should be noted that the mass matrices in Eq. (3.7) need to be modified by small corrections to fit the light fermion masses. The exact mass matrix diagonalization can proceed through a biunitary transformation with the unitary matrices $\mathcal{U}_{L(R)}$ parametrized by mixing angles $\rho_{L(R)} \ll 1$. For instance, after the diagonalization of the large couplings in the down sector, the mass matrix can take the following structure:

$$\mathcal{M}'_d = \begin{pmatrix} x & \epsilon\kappa_L \\ \epsilon'\kappa_R & X \end{pmatrix}, \quad \text{(B7)}$$

where x, ϵ, ϵ' and X are functions of the perturbative couplings and the large mixing angles $\theta_{L(R)_{1d(2d)}}$. $\epsilon' = \epsilon^\dagger$ under $\{c_{L_{1d}} \to c_{R_{1d}} c_{L_{2d}} \to c_{R_{2d}} s_{L_{2d}} \to s_{R_{2d}}, \kappa_L \to \kappa_R\}$. This matrix structure can be block diagonalized as $\mathcal{U}_L \mathcal{M}'_d \mathcal{U}_R^\dagger$, with

$$\mathcal{U}_X = \begin{pmatrix} \mathbb{1} - \frac{1}{2}\rho_X\rho_X^\dagger & \rho_X \\ -\rho_X^\dagger & \mathbb{1} - \frac{1}{2}\rho_X^\dagger\rho_X \end{pmatrix}, \qquad X = \{L, R\} \quad \text{(B8)}$$

giving rise to the SM fermion mass matrix of the form,

$$\hat{m} = x - \kappa_L\kappa_R\epsilon M^{-1}\epsilon'^\dagger, \quad \text{(B9)}$$

where $M = \mathrm{Diag}(M_1, M_2, M_3)$ are the heavy VLQ masses and

$$\rho_L = \kappa_L\epsilon M^{-1}, \qquad \rho_R = \kappa_R\epsilon'^\dagger M^{-1}. \quad \text{(B10)}$$

APPENDIX C: GAUGE BOSON INTERACTIONS

The finite contributions to gauge boson self-interactions at 1-loop level arise from their couplings to VLQs in the mass basis. The effective W_L interaction Lagrangian can be read off from the CKM matrix in Eq. (3.8) multiplied by $g/\sqrt{2}$. The effective interactions of left-handed fermions with the SM-like Z_L boson, multiplied by $g/\cos\theta_W$, in the basis $(u_L\ c_L\ t_L\ U_{2_L}\ U_{3_L})$ and $(d_L\ s_L\ b_L\ D_{2_L}\ D_{3_L})$, respectively, are $-Q_u \sin^2\theta_W \mathbb{I}_{5\times5} +$

$$T_{3L_u}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & u_{11}^2 & u_{11}u_{21} & u_{11}u_{31} \\ 0 & 0 & u_{11}u_{21} & u_{21}^2 & u_{21}u_{31} \\ 0 & 0 & u_{11}u_{31} & u_{21}u_{31} & u_{31}^2 \end{pmatrix}, \quad \text{(C1)}$$

and $-Q_d \sin^2\theta_W \mathbb{I}_{5\times5} +$

$$T_{3L_d}\begin{pmatrix} c_{1d}^2 & 0 & 0 & -c_{1d}s_{1d}s_{2d} & -c_{1d}c_{2d}s_{1d} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -c_{1d}s_{1d}s_{2d} & 0 & 0 & s_{1d}^2 s_{2d}^2 & c_{2d}s_{1d}^2 s_{2d} \\ -c_{1d}c_{2d}s_{1d} & 0 & 0 & c_{2d}s_{1d}^2 s_{2d} & c_{2d}^2 s_{1d}^2 \end{pmatrix}. \quad \text{(C2)}$$

The interactions of right-handed fields with Z_L remain the same as those in the SM.

APPENDIX D: OBLIQUE PARAMETERS

The expressions for oblique parameters in Eq. (3.2) are obtained from [54]

$$\alpha S \equiv 4e^2[\Pi'_{33}(0) - \Pi'_{3Q}(0)]$$
$$\alpha T \equiv \frac{e^2}{s^2c^2M_Z^2}[\Pi_{11}(0) - \Pi_{33}(0)]$$
$$\alpha U \equiv 4e^2[\Pi'_{11}(0) - \Pi'_{33}(0)] \quad \text{(D1)}$$

and converting these to the gauge boson basis using [86]

$$\Pi_{\gamma\gamma} = e^2\Pi_{QQ}, \qquad \Pi_{WW} = \frac{e^2}{s^2}\Pi_{11},$$
$$\Pi_{\gamma Z} = \frac{e^2}{sc}(\Pi_{3Q} - s^2\Pi_{QQ}),$$
$$\Pi_{ZZ} = \frac{e^2}{c^2s^2}(\Pi_{33} - 2s^2\Pi_{3Q} + s^4\Pi_{QQ}). \tag{D2}$$

Here, $s = \sin\theta_W$ and $c = \cos\theta_W$, θ_W being the Weinberg angle, and α is the electromagnetic coupling strength. In deriving Eq. (3.2) we have used $\Pi_{\gamma\gamma}(0) = 0 = \Pi_{\gamma Z}(0)$. Other variations of these expressions can be found in [55,86–90] among others. The difference between the expressions which are written in terms of slopes and derivatives are negligible under the approximation that Π_{ij} are linear functions of q^2 near $q^2 = 0$, i.e.,

$$\Pi_{ij}(q^2) = \Pi_{ij}(0) + q^2\Pi'_{ij}(0). \tag{D3}$$

See Ref. [86] for a detailed discussion on the oblique parameters. Some expressions may be in terms of

$$\Pi_{3Y} = 2(\Pi_{3Q} - \Pi_{33}). \tag{D4}$$

Wherever the definitions of U differ in terms of M_W and M_Z, care must be taken to use the relevant expression for W mass shift. The complete expressions for the oblique parameters in our framework are given below(using the short-hand $\sin\theta_{if} = s_{if}$ and $\cos\theta_{if} = c_{if}$). Note that the contributions from down-sector mixing can be obtained under the transformation,

$$\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix} \to \begin{pmatrix} c_{1d} & s_{1d} & 0 \\ -c_{2d}s_{1d} & c_{1d}c_{2d} & -s_{2d} \\ -s_{1d}s_{2d} & c_{1d}s_{2d} & c_{2d} \end{pmatrix} \tag{D5}$$

with $\{M_{U_2} \to M_{D_2}, M_{U_3} \to M_{D_3}, m_t \to m_d$, and $m_b \to m_u\}$.

$$T = \frac{N_c}{16\pi s^2 M_W^2}\Bigg[(u_{11}^4 - 1)m_t^2 + (u_{21}^4 M_{U_2}^2 + u_{31}^4 M_{U_3}^2)$$
$$+ 2\left\{u_{11}^4 - 1 + m_t^2\left(\frac{1 - u_{11}^2}{m_t^2 - m_b^2} - \frac{u_{11}^2 u_{21}^2}{M_{U_2}^2 - m_t^2} - \frac{u_{11}^2 u_{31}^2}{M_{U_3}^2 - m_t^2}\right)\right\} m_t^2 \ln\left[\frac{m_t^2}{M_{U_3}^2}\right]$$
$$+ 2u_{21}^2\left\{u_{21}^2 + M_{U_2}^2\left(-\frac{1}{M_{U_2}^2 - m_b^2} + \frac{u_{11}^2}{M_{U_2}^2 - m_t^2} - \frac{u_{31}^2}{M_{U_3}^2 - M_{U_2}^2}\right)\right\} M_{U_2}^2 \ln\left[\frac{M_{U_2}^2}{M_{U_3}^2}\right]$$
$$+ 2\left\{\frac{u_{21}^2 + u_{31}^2}{m_t^2 - m_b^2} - \frac{u_{21}^2}{M_{U_2}^2 - m_b^2} - \frac{u_{31}^2}{M_{U_3}^2 - m_b^2}\right\} m_b^4 \ln\left[\frac{m_b^2}{M_{U_3}^2}\right]\Bigg], \tag{D6}$$

$$U = \frac{N_c}{18\pi}\Bigg[\frac{7}{2}(1 - u_{11}^4) - \frac{3}{2}(u_{21}^4 + u_{31}^4) - 2(u_{21}^2 + u_{31}^2 - u_{21}^2 u_{31}^2)$$
$$+ 12\left\{m_b^2\left((u_{11}^2 - 1)\frac{m_t^2}{(m_t^2 - m_b^2)^2} + u_{21}^2\frac{M_{U_2}^2}{(M_{U_2}^2 - m_b^2)^2} + u_{31}^2\frac{M_{U_3}^2}{(M_{U_3}^2 - m_b^2)^2}\right)\right.$$
$$\left. + m_t^2 u_{11}^2\left(u_{21}^2\frac{M_{U_2}^2}{(M_{U_2}^2 - m_t^2)^2} - u_{31}^2\frac{M_{U_3}^2}{(M_{U_3}^2 - m_t^2)^2}\right) - u_{21}^2 u_{31}^2\frac{M_{U_2}^2 M_{U_3}^2}{(M_{U_3}^2 - M_{U_2}^2)^2}\right\}$$
$$+ 6\left\{(u_{11}^2 - 1)\frac{3m_t^2 - m_b^2}{(m_t^2 - m_b^2)^3} + u_{21}^2\frac{3M_{U_2}^2 - m_b^2}{(M_{U_2}^2 - m_b^2)^3} + u_{31}^2\frac{3M_{U_3}^2 - m_b^2}{(M_{U_3}^2 - m_b^2)^3}\right\} m_b^4 \ln\left[\frac{m_b^2}{M_{U_3}^2}\right]$$
$$- 6\{(u_{11}^2 - 1)\frac{m_t^2 - 3m_b^2}{(m_t^2 - m_b^2)^3} + u_{11}^2 u_{21}^2\frac{3M_{U_2}^2 - m_t^2}{(M_{U_2}^2 - m_t^2)^3} + u_{11}^2 u_{31}^2\frac{3M_{U_3}^2 - m_t^2}{(M_{U_3}^2 - m_t^2)^3}\Big\} m_t^4 \ln\left[\frac{m_t^2}{M_{U_3}^2}\right]$$
$$+ 6u_{21}^2\left\{\frac{M_{U_2}^2 - 3m_b^2}{(M_{U_2}^2 - m_b^2)^3} - u_{11}^2\frac{M_{U_2}^2 - 3m_t^2}{(M_{U_2}^2 - m_t^2)^3} - u_{31}^2\frac{3M_{U_3}^2 - M_{U_2}^2}{(M_{U_3}^2 - M_{U_2}^2)^3}\right\} M_{U_2}^4 \ln\left[\frac{M_{U_2}^2}{M_{U_3}^2}\right]$$
$$+ 3(1 - u_{11}^4)\ln\left[\frac{m_t^2}{M_{U_3}^2}\right] - 3u_{21}^2 u_{21}^2 \ln\left[\frac{M_{U_2}^2}{M_{U_3}^2}\right]\Bigg], \tag{D7}$$

$$S = \frac{N_c}{18\pi}\Bigg[5(-u_{11}^2 + u_{11}^4 - u_{21}^2 u_{31}^2) - \Bigg\{1 + 2u_{11}^2 - 3u_{11}^4 - 6u_{11}^2 m_t^4\Bigg(u_{21}^2 \frac{3M_{U_2}^2 - m_t^2}{(M_{U_2}^2 - m_t^2)^3} + u_{31}^2 \frac{3M_{U_3}^2 - m_t^2}{(M_{U_3}^2 - m_t^2)^3}\Bigg)\Bigg\} \ln\Bigg[\frac{m_t^2}{M_{U_3}^2}\Bigg]$$
$$+ u_{21}^2\Bigg\{3u_{21}^2 - 4 + 6M_{U_2}^4\Bigg(u_{31}^2 \frac{3M_{U_3}^2 - M_{U_2}^2}{(M_{U_3}^2 - M_{U_2}^2)^3} + u_{11}^2 \frac{M_{U_2}^2 - 3m_t^2}{(M_{U_2}^2 - m_t^2)^3}\Bigg)\Bigg\} \ln\Bigg[\frac{M_{U_2}^2}{M_{U_3}^2}\Bigg]$$
$$+ 12\Bigg\{u_{11}^2 u_{21}^2 \frac{m_t^2 M_{U_2}^2}{(M_{U_2}^2 - m_t^2)^2} + u_{11}^2 u_{31}^2 \frac{m_t^2 M_{U_3}^2}{(M_{U_3}^2 - m_t^2)^2} + u_{21}^2 u_{31}^2 \frac{M_{U_2}^2 M_{U_3}^2}{(M_{U_3}^2 - M_{U_2}^2)^2}\Bigg\}\Bigg]. \tag{D8}$$

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